Q1 The equation of the line joining the points (–3, 4, 11) and (1, –2, 7) is

(A) $\frac{x+3}{2} = \frac{y-4}{3} = \frac{z-11}{4}$

(B) $\frac{x+3}{-2} = \frac{y-4}{3} = \frac{z-11}{2}$

(C) $\frac{x+3}{-2} = \frac{y+4}{3} = \frac{z+11}{4}$

(D) $\frac{x+3}{2} = \frac{y+4}{-3} = \frac{z+11}{2}$

Q2 The angle between the lines whose direction cosines are $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2}\right)$ and $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{-\sqrt{3}}{2}\right)$

(A) π (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

Q3 If a plane meets the coordinate axes at A, B and C in such a way that the centroid of ABC is at the point (1, 2, 3), then the equation of the plane is

(A) $\frac{x}{1} + \frac{y}{2} + \frac{z}{3} = 1$

(B) $\frac{x}{3} + \frac{y}{6} + \frac{z}{9} = 1$

(C) $\frac{x}{1} + \frac{y}{2} + \frac{z}{3} = \frac{1}{3}$

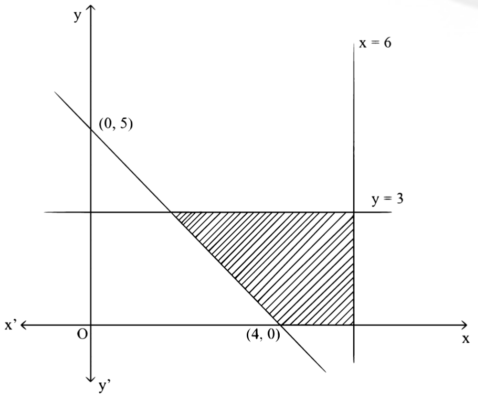
(D) $\frac{x}{1} - \frac{y}{2} + \frac{z}{3} = -1$

Q4 The area of the quadrilateral ABCD when A(0, 4, 1), B(2, 3, -1), C(4, 5, 0) and D(2, 6, 2) is equal to

(A) 9 sq units (B) 18 sq units

(C) 27 sq units (D) 81 sq units

Q5 The shaded region is the solution set of the inequalities



(A) $5x + 4y \geq 20, x \leq 6, y \geq 3, x \geq 0, y \geq 0$

(B) $5x + 4y \leq 20, x \leq 6, y \leq 3, x \geq 0, y \geq 0$

(C) $5x + 4y \geq 20, x \leq 6, y \leq 3, x \geq 0, y \geq 0$

(D) $5x + 4y \geq 20, x \geq 6, y \leq 3, x \geq 0, y \geq 0$

Q6 Given that, A and B are two events such that $P(B) = \frac{3}{5}, P\left(\frac{A}{B}\right) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$ then P(A) is equal to

(A) $\frac{3}{10}$ (B) $\frac{1}{2}$

(C) $\frac{1}{5}$ (D) $\frac{3}{5}$

Q7 If A, B and C are three independent events such that P(A) = P(B) = P(C) = P, then P (at least two of A, B and C occur) is equal to

(A) $P^3 - 3P$

(B) $3P - 2P^2$

(C) $3P^2 - 2P^3$

(D) $3P^2$

Q8 Two dice are thrown. If it is known that the sum of numbers on the dice was less than 6 the probability of getting a sum as 3 is

(A) $\frac{1}{18}$ (B) $\frac{5}{18}$

(C) $\frac{1}{5}$ (D) $\frac{2}{5}$

Q9 A car manufacturing factory has two plants X and Y. Plant X manufactures 70% of cars and plant Y manufactures 30% of cars. 80% of cars at plant X and 90 of cars at plant Y are rated as standard quality. A car is chosen at random and is found to be standard quality. The probability that it has come from plant X is:

(A) $\frac{56}{73}$ (B) $\frac{56}{84}$

(C) $\frac{56}{83}$ (D) $\frac{56}{79}$

Q10 In a certain two 65% families own cell phones, 15000 families own scooter and 15% families own both. Taking into consideration that the families own at least one of the two, the total number of families in the town is

(A) 20000 (B) 30000

(C) 40000 (D) 50000

Q11 A and B are non-singleton sets and n(A × B) = 35 If B⊂A, then ${}^{n(A)}C_{n(B)}$ is equal to

(A) 28 (B) 35

(C) 42 (D) 21

Q12 Domainof $f(x) = \frac{x}{1-|x|}$ is

(A) $R - [-1, 1]$

(B) $(-\infty, 1)$

(C) $(-\infty, 1) \cup (0, 1)$

(D) $R - \{-1, 1\}$

Q13 The value of cos 1200° + tan 1485° is

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$

(C) $-\frac{3}{2}$ (D) $-\frac{1}{2}$

Q14 The value of tan 1° tan 2 tan 3°...tan 89° is

(A) 0 (B) 1

(C) $\frac{1}{2}$ (D) –1

Q15 If $\left(\frac{1+i}{1-i}\right)^x = 1,$ then

(A) $x = 4n + 1, n \in N$

(B) $x = 2n + 1, n \in N$

(C) $x = 2n, n \in N$

(D) $x = 4n, n \in N$

Q16 The cost and revenue functions of a product are given by c(x) = 20x + 4000 and R(x) = 60x + 2000 respectively, where x is the number of items produced and sold. The value of x to earn profit is

(A) > 50 (B) > 60
(C) > 80 (D) > 40

Q17 A student has to answer 10 questions, choosing at least 4 from each of the parts A and B. If there are 6 questions in part A and 7 in part B, then the number of ways can the student choose 10 questions is

(A) 256 (B) 352
(C) 266 (D) 426

Q18 If the middle term of the AP is 300, then the sum of its first 51 terms is

(A) 15300 (B) 14800
(C) 16500 (D) 14300

Q19 The equation of straight line which passes through the point $\left(a\cos^3\theta,\ a\sin^3\theta\right)$ and perpendicular to $x\ \sec\ \theta + y\ \operatorname{cosec}\ \theta = a$ is

(A) $\frac{x}{a} + \frac{y}{a} = a\cos\theta$
(B) $x\cos\theta - y\sin\theta = a\cos 2\theta$
(C) $x\cos\theta + y\sin\theta = a\cos 2\theta$
(D) $x\cos\theta - y\sin\theta = a\cos\theta$

Q20 The mid points of the sides of triangle are (1, 5, - 1)(0, 4, - 2) and (2, 3, 4) then centroid of the triangle

(A) (1, 4, 3)
(B) (1, 4, $\frac{1}{3}$)
(C) (–1, 4, 3)
(D) ($\frac{1}{3}$, 2, 4)

Q21 Consider the following statements

Statement 1 : $\lim\limits_{x\to 1} \frac{ax^2+bx+c}{x^2+bx+a}$ is 1 $\left(\text{where } a+b+c \neq 0\right)$.

Statement 2 : $\lim\limits_{x\to -2} \frac{\frac{1}{x}+\frac{1}{2}}{x+2}$ is $\frac{1}{4}$.

(A) Only statement 2 is true.
(B) Only statement 1 is true.
(C) Both statements 1 and 2 are true.
(D) Both statements 1 and 2 are false.

Q22 If a and b are fixed non $-$ zero constants, then the derivative of $\frac{a}{x^4} - \frac{b}{x^2} + \cos x$ is $ma + nb - p$, where

(A) $m = 4x^3, n = \frac{-2}{x^3}$ and $p = \sin x$
(B) $m = \frac{-4}{x^5}, n = \frac{2}{x^3}$ and $p = \sin x$
(C) $m = \frac{-4}{x^5}, n = \frac{-2}{x^3}$ and $p = \sin x$
(D) $m = 4x^3, n = \frac{2}{x^3}$ and $p = -\sin x$

Q23 The standard deviation of the numbers 31, 32, 33... 46, 47 is

(A) $\sqrt{\frac{17}{12}}$ (B) $\sqrt{\frac{47^2-1}{12}}$
(C) $2\sqrt{6}$ (D) $4\sqrt{3}$

Q24 If P(A) = 0.59, P(B) = 0.30 and P(A B) = 0.21 then P (A' B') is equal to

(A) 0.11 (B) 0.38
(C) 0.32 (D) 0.35

Q25

$f : R \to R$ defined by $f\left(x\right)$ is equal to

$\begin{cases} 2x, x > 3 \\ x^2, 1 < x \le 3 \\ 3x, x \le 1 \end{cases}$, then $f\left(-2\right) + f\left(3\right) + f\left(4\right)$ is

(A) 14 (B) 9
(C) 5 (D) 11

Q26 Let A $=\{x : x \in R, x$ is not a positive integer) Define $f : A \to R$ as $f\left(x\right) = \frac{2x}{x-1}$, then f is

(A) injective but not surjective.
(B) surjective but not injective.
(C) bijective.
(D) neither injective nor surjective.

Q27 The function $f\left(x\right) = \sqrt{3}\sin 2x - \cos 2x + 4$ is one $-$ one in the interval

(A) $\left[\frac{-\pi}{6}, \frac{\pi}{3}\right]$
(B) $\left[\frac{\pi}{6}, \frac{-\pi}{3}\right]$
(C) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
(D) $\left[\frac{-\pi}{6}, \frac{-\pi}{3}\right]$

Q28 Domain of the function $f\left(x\right) = \frac{1}{\sqrt{[x^2]-[x]-6}}$, where $\left[x\right]$ is greatest integer $\le x$ is

(A) $(-\infty, -2) \cup [4, \infty)$
(B) $(-\infty, -2 \cup [3, \infty]$
(C) $[-\infty, -2] \cup [4, \infty]$
(D) $[-\infty, -2] \cup [3, \infty)$

Q29 $\cos\left[\cot^{-1}\left(-\sqrt{3}\right) + \frac{\pi}{6}\right]$ is equal to

(A) 0 (B) 1
(C) $\frac{1}{\sqrt{2}}$ (D) –1

Q30

$\tan^{-1}\left[\frac{1}{\sqrt{3}}\sin\frac{5\pi}{2}\right]\sin^{-1}\left[\cos\left(\sin^{-}\frac{\sqrt{3}}{2}\right)\right]$ is equal to

(A) $\left(\frac{\pi}{6}\right)^2$ (B) $\frac{\pi}{6}$
(C) $\frac{\pi}{3}$ (D) π

Q31 If $A=\begin{bmatrix}1 & -2 & 1\\ 2 & 1 & 3\end{bmatrix}$ $B=\begin{bmatrix}2 & 1\\ 3 & 2\\ 1 & 1\end{bmatrix}$, then

(AB)′ is equal to

(A) $\begin{bmatrix}-3 & -2\\ 10 & 7\end{bmatrix}$

(B) $\begin{bmatrix}-3 & 10\\ -2 & 7\end{bmatrix}$

(C) $\begin{bmatrix}-3 & 7\\ 10 & 2\end{bmatrix}$

(D) $\begin{bmatrix}-3 & 7\\ 10 & -2\end{bmatrix}$

Q32 Let M be 2 x 2 symmetric matrix with integer entries, then M is invertible if

(A) the first column of M is the transpose of second row of M.

(B) the second row of M is the transpose of first column of M.

(C) M is diagonal matrix with non-zero entries in the principal diagonal.

(D) The product of entries in the principal diagonal of M is the product of entries in the other diagonal.

Q33 If A and B are matrices of order 3 and |A| = 5, |B| = 3, then |3AB| is

(A) 425 (B) 405

(C) 565 (D) 585

Q34 If A and B are invertible matrices then which of the following is not correct?

(A) $\operatorname{adj}A=|A|A^{-1}$

(B) $\det\left(A^{-1}\right)=[\det(A)]^{-1}$

(C) $(AB)^{-1}=B^{-1}A^{-1}$

(D) $(A+B)^{-1}=B^{-1}+A^{-1}$

Q35 If $f\left(x\right)=\begin{vmatrix}\cos x & 1 & 0\\ 0 & 2\cos x & 3\\ 0 & 1 & 2\cos x\end{vmatrix}$, then

$\lim\limits_{x\to\pi} f\left(x\right)$ is equal to

(A) –1 (B) 1

(C) 0 (D) 3

Q36 If $x^3-2x^2-9x+18=0$ and A then the $=\begin{vmatrix}1 & 2 & 3\\ 4 & x & 6\\ 7 & 8 & 9\end{vmatrix}$

maximum value of A is

(A) 96 (B) 36

(C) 24 (D) 120

Q37 At $x=1$, the function $f\left(x\right)$

$=\begin{cases}x^3-1, & 1<x<\infty\\ x-1, & -\infty<x\le 1\end{cases}$ is

(A) continuous and differentiable.

(B) continuous and non-differentiable.

(C) discontinuous and differentiable.

(D) discontinuous and non-differentiable.

Q38 If $y=\left(\cos x^2\right)^2$, then $\frac{dy}{dx}$ is equal to

(A) $-4x\sin 2x^2$

(B) $-x\sin x^2$

(C) $-2x\sin 2x^2$

(D) $-x\cos 2x^2$

Q39 For constant a, $\frac{d}{dx}(x^x+x^a+a^x+a^a)$ is

(A) $x^x\left(1+\log x\right)+ax^{a-1}$

(B) $x^x\left(1+\log x\right)+ax^{a-1}+a^x\log a$

(C) $x^x(1+\log x)+a^a(1+\log x)$

(D) $x^x\left(1+\log x\right)+a^a\left(1+\log a\right)+ax^{a-1}$

Q40 Consider the following statements

Statement 1 : If $y=\log_{10}x+\log_e x$,

then $\frac{dy}{dx}=\frac{\log_{10}e}{x}+\frac{1}{x}$

Statement 2 : If $\frac{d}{dx}\left(\log_{10}x\right)$

$=\frac{\log x}{\log 10}$ and $\frac{d}{dx}\left(\log_e x\right)=\frac{\log x}{\log e}$

(A) Statement 1 is true, Statement 2 is false.

(B) Statement 1 is false, statement 2 is true.

(C) Both statements 1 and 2 are true.

(D) Both statements 1 and 2 are false.

Q41 If the parametric equation of curve is given by $x=\cos\theta+\log\tan\frac{\theta}{2}$ and $y=\sin\theta$ then the points for which $\frac{dy}{dx}=0$ are given by

(A) $\theta=\frac{n\pi}{2}, n\in Z$

(B) $\theta=\left(2n+1\right)\frac{\pi}{2}, n\in Z$

(C) $\theta=(2n+1)\pi, n\in Z$

(D) $\theta=n\pi, n\in z$

Q42 If $y=(x-1)^2(x-2)^3(x-3)^5$, then $\frac{dy}{dx}$ at $x=4$ is equal to

(A) 108 (B) 54

(C) 36 (D) 516

Q43 A particle starts form rest and its angular displacement (in radians) is given by $\theta=\frac{t^2}{20}+\frac{t}{5}$. the angular velocity at the end of t = 4 is k, then the value of 5k is

(A) 0.6 (B) 5

(C) 5k (D) 3

Q44 If the parabola $y=\alpha x^2-6x+\beta$ passes through the point (0, 2) and has its tangent at $x=\frac{3}{2}$ parallel to X-axis, then

(A) $\alpha=2, \beta=-2$

(B) $\alpha=2, \beta=2$

(C) $\alpha=-2, \beta=2$

(D) $\alpha=-2, \beta=-2$

Q45 The function $f(x) = x^2 - 2x$ is strictly decreasing in the interval

(A) $(-\infty, 1)$
(B) $(1, \infty)$
(C) R
(D) $(-\infty, \infty)$

Q46 The maximum slope of the curve $y = -x^3 + 3x^2 + 2x - 27$ is

(A) 1 (B) 23
(C) 5 (D) –23

Q47 $\int \frac{x^3 \sin(\tan^{-1}(x^4))}{1+x^8} dx$ is equal to

(A) $\frac{-\cos(\tan^{-1}(x^4))}{4} + C$
(B) $\frac{\cos(\tan^{-1}(x^4))}{4} + C$
(C) $\frac{-\cos(\tan^{-1}(x^3))}{3} + C$
(D) $\frac{\sin(\tan^{-1}(x^4))}{4} + C$

Q48 The value of $\int \frac{x^2 dx}{\sqrt{x^6+a^6}}$ is equal to

(A) $\log\left|x^3 + \sqrt{x^6 + a^6}\right| + C$
(B) $\log\left|x^3 - \sqrt{x^6 + a^6}\right| + C$
(C) $\frac{1}{3}\log\left|x^3 + \sqrt{x^6 + a^6}\right| + C$
(D) $\frac{1}{3}\log\left|x^3 - \sqrt{x^6 + a^6}\right| + C$

Q49 The value of $\int \frac{xe^x dx}{(1+x)^2}$ is equal to

(A) $e^x(1 + x) + C$ (B) $e^x\left(1 + x^2\right) + C$
(C) $e^x(1 + x)^2 + C$ (D) $\frac{e^x}{1+x} + C$

Q50 The value of $\int e^x \left[\frac{1+\sin x}{1+\cos x}\right] dx$ is equal to

(A) $e^x \tan\frac{x}{2} + C$ (B) $e^x \tan x + C$
(C) $e^x(1 + \cos x) + C$ (D) $e^x(1 + \sin x) + C$

Q51 If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x dx$, where n is positive integer, then $I_{10} + I_8$ is equal to

(A) 9 (B) $\frac{1}{7}$
(C) $\frac{1}{8}$ (D) $\frac{1}{9}$

Q52 The value of $\int_0^{4042} \frac{\sqrt{x}\, dx}{\sqrt{x}+\sqrt{4042-x}}$ is equal to

(A) 4042 (B) 2021
(C) 8084 (D) 1010

Q53 The area of the region bounded by $y = -\sqrt{16 - x^2}$ and X-axis is

(A) 8π sq units
(B) 20π sq units
(C) 16π sq units
(D) 256π sq units

Q54 If the area of the ellipse is $\frac{x^2}{25} + \frac{y^2}{\lambda^2} = 1$ is 20π sq units, then λ is

(A) ± 4 (B) ± 3
(C) ± 2 (D) ± 1

Q55 Solution of differential equating xdy – ydx = 0 represents

(A) A rectangular hyperbola.
(B) Parabola whose vertex is at origin.
(C) Straight line passing through origin.
(D) A circle whose centre is origin.

Q56 The number of solutions of $\frac{dy}{dx} = \frac{y+1}{x-1}$, when $y(1) = 2$ is

(A) three (B) one
(C) infinite (D) two

Q57 A vector a makes equal acute angles on the coordinate axis. Then the projection of vector $b = 5\hat{i} + 7\hat{j} + \hat{k}$ on a is

(A) $\frac{11}{15}$ (B) $\frac{11}{\sqrt{3}}$
(C) $\frac{4}{5}$ (D) $\frac{3}{5\sqrt{3}}$

Q58 The diagonals of a parallelogram are the vectors $3\hat{i} + 6\hat{j} - 2\hat{k}$. and $-\hat{i} - 2\hat{j} - 8\hat{k}$. Then the length of the shorter side of parallelogram is

(A) $\sqrt{29}$ (B) $\sqrt{14}$
(C) $3\sqrt{5}$ (D) $4\sqrt{3}$

Q59 If a.b = 0 and a + b makes an angle 60° with a, then

(A) |a| = 2|b| (B) 2|a| = |b|
(C) $|a| = \sqrt{3}|b|$ (D) $\sqrt{3}|a| = |b|$

Q60 If the area of the parallelogram with a and b as two adjacent sides is 15 sq units, then the area of the parallelogram having 3a + 2b and a + 3b as two adjacent sides in sq units is

(A) 45 (B) 75
(C) 105 (D) 120

Vernacular

Answer Key

Q1	B	Q31	B
Q2	C	Q32	C
Q3	B	Q33	B
Q4	A	Q34	D
Q5	C	Q35	A
Q6	B	Q36	A
Q7	C	Q37	B
Q8	C	Q38	C
Q9	C	Q39	B
Q10	B	Q40	A
Q11	D	Q41	D
Q12	D	Q42	D
Q13	A	Q43	D
Q14	B	Q44	B
Q15	D	Q45	A
Q16	A	Q46	C
Q17	C	Q47	A
Q18	A	Q48	C
Q19	B	Q49	D
Q20	B	Q50	A
Q21	B	Q51	D
Q22	B	Q52	B
Q23	C	Q53	A
Q24	C	Q54	A
Q25	D	Q55	C
Q26	A	Q56	B
Q27	A	Q57	B
Q28	A	Q58	A
Q29	D	Q59	D
Q30	A	Q60	C

Hints & Solutions

Note: scan the QR code to watch video solution

Q1 Text Solution:

Given, points are (–3, 4, 11) and (1, –2, 7).

Let A(–3, 4, 11) and B(1, –2, 7)

Direction ratios of AB = 1 - (–3), –2 – 4, 7 – 11

= 4 , –6, –4 = –23, 2

Now, we have one point (–3, 4, 11) = (x_1, y_1, z_1)

and direction ratios (a, b, c) = –2, 3, 2

∵ Equation of line :

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

$$\Rightarrow \frac{x+3}{-2}=\frac{y-4}{3}=\frac{z-11}{2}$$

Q2 Text Solution:

Given, direction cosines of line 1 : (l_1, m_1, n_1)

$$=\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2}\right)$$

direction cosines of line 2 $\Rightarrow (l_2, m_2, n_2)$

$$=\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, -\frac{\sqrt{3}}{2}\right)$$

We know that,

angle between two lines is given by $\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$

$$\cos\theta = \left|\frac{\sqrt{3}}{4}\times\frac{\sqrt{3}}{4}+\frac{1}{4}\times\frac{1}{4}-\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}\right|$$

$$=\left|\frac{3}{16}+\frac{1}{16}-\frac{3}{4}\right|=\left|\frac{3+1-12}{16}\right|$$

$$=\left|-\frac{8}{16}\right|=\frac{1}{2}$$

$$\therefore \theta = \frac{\pi}{3}$$

Q3 Text Solution:

Given, plane meets the coordinate axes at A, B and C.

Let the plane meets the coordinate axes at the points A(a, 0, 0), B(0, 0, 0), C(0, 0, c)

The equation of plane is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

Centroid formula says,

$$x=\frac{x_1+x_2+x_3}{3}, y=\frac{y_1+y_2+y_3}{3}$$

$$z=\frac{z_1+z_2+z_3}{3}$$

Centroid of $\Delta ABC = \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

Given centroid $= (1, 2, 3)$

$$\Rightarrow \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$$

Therefore, a = 3 b = 6 and c = 9

Put the value of a, b and c in equation of plane (i), we get

$$\frac{x}{3}+\frac{y}{6}+\frac{z}{9}=1$$

Q4 Text Solution:

In quadrilateral ABCD, given that A(0, 4, 1), B(2, 3, -1), C(4, 5, 0) and D(2, 6, 2).

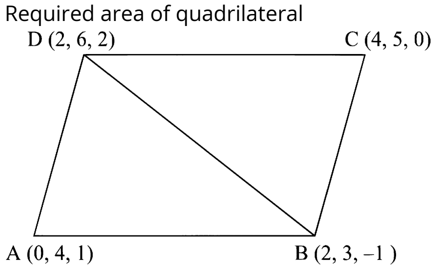
$$AB = 2\hat{i}-\hat{j}-2\hat{k}$$

$$AD = 2\hat{i}+2\hat{j}+\hat{k}$$

$$CB = -2\hat{i}-2\hat{j}-\hat{k}$$

$$CD = -2\hat{i}+\hat{j}+2\hat{k}$$

Required area of quadrilateral



$$=\frac{1}{2}|AB\times AD|+\frac{1}{2}|CB\times CD|$$

$$=\frac{1}{2}|3\hat{i}-6\hat{j}+6\hat{k}|+\frac{1}{2}|-3\hat{i}+6\hat{j}-6\hat{k}|$$

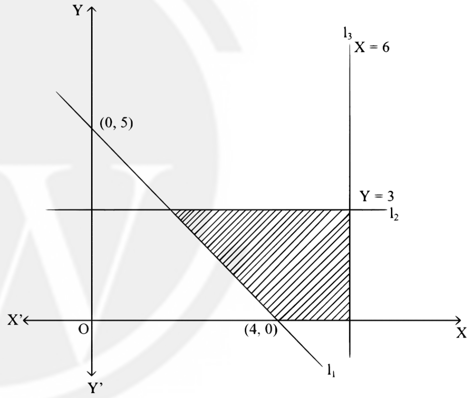
$$=\frac{1}{2}\sqrt{3^2+(-6)^2+6^2}$$

$$+\frac{1}{2}\sqrt{(-3)^2+6^2+(-6)^2}n$$

$$=\frac{1}{2}\times 9+\frac{1}{2}\times 9 = 9 \text{ sq units}$$

Q5 Text Solution:

Lets draw the given shaded region,



Line $l_1 \Rightarrow \frac{x}{4}+\frac{y}{5}=1$ (interceptform)

$$\Rightarrow \frac{5x+4y}{4\times 5}=1$$

$$\Rightarrow 5x+4y=20$$

As, origin is not in the feasible region.

$$\therefore 5x+4y\geq 20$$

Line $l_2 \Rightarrow y\leq 3$ (from the graph)

Line $l_3 \Rightarrow x\leq 6$ (from the graph)

and coordinate axes $x\geq 0, y\geq 0$

Hence, inequalities are $5x+4y\geq 20, y\leq 3, x\leq 6, x\geq 0, y\geq 0$.

Q6 Text Solution:

$\text{Given, } P(B) = \frac{3}{5}, P\left(\frac{A}{B}\right)$

$= \frac{1}{2} \text{ and } P(A \cup B) = \frac{4}{5}$

$\because P\left(\frac{A}{B}\right) = \frac{1}{2}$

$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{1}{2}$

$\Rightarrow P(A \cap B) = \frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$

$\text{Now, } P(A \cup B) = \frac{4}{5}$

$\Rightarrow P(A) + P(B) - P(A \cap B) = \frac{4}{5}$

$\Rightarrow P(A) + \frac{3}{5} - \frac{3}{10} = \frac{4}{5}$

$\Rightarrow P(A) = \frac{4}{5} + \frac{3}{10} - \frac{3}{5}$

$= \frac{8+3-6}{10} = \frac{5}{10}$

$\therefore P(A) = \frac{1}{2}$

Q7 Text Solution:

$\text{Given, } P(A) = P(B) = P(C) = P$

$P\left(\text{At least two of } A, B, C \text{ occur}\right)$

$= \left(P \cap B \cap C'\right) + P\left(A \cap B' \cap C\right)$

$+ P\left(A' \cap B \cap C\right) + P\left(A \cap B \cap C\right)$

$= P(A)P(B)P(C') + P(A)P(B')P(C)$

$+ P(A')P(B)P(C) + P(A)P(B)P(C)$

$= P(A)P(B)[1 - P(C)]$

$+ P(A)[1 - (B)]P(C) + [1 - P(A)]P(B)P(C)$

$+ P(A)P(B)P(C$

$= P \times P \times (1 - P) + (1 - P) \times P \times P + P$

$\times (1 - P) \times P + P \times P \times P$

$= P^2\left[(1 - P) + (1 - P) + (1 - P) + P\right]$

$= P^2\left[3 - 2P\right]$

$= 3P^2 - 2P^3$

$P\left(\text{At least two of } A, B,\right.$

$\left. C \text{ occur}\right) \text{ is } 3P^2 - 2P^3.$

Q8 Text Solution:

$\text{Let } E_A$

$= \text{The event that the sum of numbers on the dice was less than 6}$

$E_A = \{(1,4), (4,1), (2,3), (3,2), (2,2),$

$(1,3), (3,1), (1,2), (2,1), (1,1)\}$

$\Rightarrow n(E_A) = 10$

E_B

$= \text{The event that the sum of numbers on the dice is 3}$

$E_B = \{(1,2), (2,1)\}$

$\Rightarrow n(E_2) = 2$

$\therefore \text{Required probability } = \frac{2}{10} = \frac{1}{5}$

Q9 Text Solution:

$\text{Let } E$

$= \text{the event that the car is of standard quality}$

$\text{Let } A_1$

$= \text{the event that the car is manufactured in plant X}$

$\text{and } A_2$

$= \text{the event that the car is manufactured in plant Y}$

$\text{Now, } P(A_1) = \frac{70}{100} = \frac{7}{10}, P(A_2) = \frac{30}{100} = \frac{3}{10}$

$P\left(\frac{E}{A_1}\right)$

$= \text{Probability that a standard quality car is manufactured in plant X}$

$= \frac{80}{100} = \frac{8}{10}$

$P\left(\frac{E}{A_2}\right)$

$= \text{Probability that a standard quality car is manufactured in plant Y}$

$= \frac{90}{100} = \frac{9}{10}$

$P\left(\frac{A_1}{E}\right)$

$= \text{Probability that a standard quality car has come from plant X}$

$= \frac{P(A_1) \times P\left(\frac{E}{A_1}\right)}{P(A_1) \cdot P\left(\frac{E}{A_1}\right) + P(A_2) \cdot P\left(\frac{E}{A_2}\right)}$

$= \frac{\frac{7}{10} \times \frac{8}{10}}{\frac{7}{10} \times \frac{8}{10} + \frac{3}{10} \times \frac{9}{10}}$

$= \frac{7 \times 8}{7 \times 8 + 3 \times 9}$

$= \frac{56}{56+27} = \frac{56}{83}$

$\text{Hence, the required probability is } \frac{56}{83}.$

Q10 Text Solution:

Let the total number of families be x.

Let A = number of families that own cell phones

n(A) = $\frac{65}{100} \times x$

Let B = number of families that own scooter

n(B) = 15000

and (A ∩ B) = number of families that own cell phones and scooter both

$(A \cap B) = \frac{15}{100} \times x$

$\text{Here, } n(A \cup B) = x$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$x = \frac{65x}{100} + 15000 - \frac{15x}{100}$

$\Rightarrow 100x - 65x = 1500000 - 15x$

$\Rightarrow 100x - 50x = 1500000$

$\Rightarrow 50x = 1500000$

$\Rightarrow x = 30000$

Total number of families in the town is 30000.

Q11 Text Solution:

Given,

$n(A \times B) = 35, B \subset A$

$n(A \times B) = 35$

$n(A \times B) = 7 \times 5$

$n(A \times B) = n(A) \times n(B)$

$\therefore n(A) = 7 \text{ and } n(B) = 5$

${}^{n(A)}C_{n(B)} = {}^7C_5 = {}^7C_2 = 21$

Q12 Text Solution:

Given function,

$f\left(x\right) = \frac{x}{1-|x|}$

$\because$ Denominator can't be zero.

$\therefore 1 - |x| \neq 0$

$\Rightarrow |x| \neq 1$

$\Rightarrow x \neq \pm 1$

Hence, domainis $R - \{-1, 1\}$.

Q13 Text Solution:

Given, $\cos 1200^\circ + \tan 1485^\circ$

$= \cos 1200^\circ + \tan 1485^\circ$

$= \cos(3 \times 360^\circ + 120^\circ)$
$+ \tan(4 \times 360^\circ + 45^\circ)$

$= \cos(120^\circ) + \tan(45^\circ)$

$= \cos(180^\circ - 60^\circ) + 1$

$= -\cos 60^\circ + 1$

$= -\frac{1}{2} + 1 = \frac{1}{2}$

Q14 Text Solution:

Given, $\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots \tan 89^\circ$

$= \tan 1^\circ \tan 2^\circ \tan 3^\circ$
$\ldots \tan 87^\circ \tan 88^\circ \tan 89^\circ$

$= \tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots \tan(90 - 3^\circ)$
$\tan(90 - 2)^\circ \tan(90 - 1)^\circ$

$= \tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots \cot 3^\circ \cot 2^\circ \cot 1^\circ$

$= \tan 1^\circ \cot 1^\circ \tan 2^\circ \cot 2^\circ \tan 3^\circ \cot 3^\circ$
$\ldots \tan 45^\circ$

$= (1)(I)(I) \ldots (I)$

$= 1$

Q15 Text Solution:

Given, $\left(\frac{1+i}{1-i}\right)^x = 1$

$\Rightarrow \left[\frac{(1+i)}{1-i)} \times \frac{(1+i)}{(1+i)}\right]^x = 1$

$\Rightarrow \left[\frac{1+i^2+2i}{1^2-i^2}\right]^x = 1$

$\Rightarrow \left[\frac{1-1+2i}{1+1}\right]^x = 1$

$\Rightarrow i^x = 1$

$\Rightarrow i^x = 1 = i^{4n}$ where $n \in N$

$\therefore x = 4n, n \in N.$

Q16 Text Solution:

Given, C(x) = 20x + 4000

R(x) = 60x + 2000

x = number of items produced and sold

To earn profit, R(x) - C(x) > 0

60x + 2000 - 20x - 4000 > 0

40x - 2000 > 0

40x > 2000

x > 50

Q17 Text Solution:

Given,

Total number of questions in part A = 6

Total number of questions in part B = 7

Pattern to choose 10 questions from 13 questions is

4 from part A and 6 from part B or

5 from part A and 5 from part B or

6 from part A and 4 from part B.

$\therefore$ Total required number of ways

$= \left({}^6C_4 \times {}^7C_6\right) + \left({}^6C_5 \times {}^7C_5\right)$
$+ \left({}^6C_6 \times {}^7C_4\right)$

$= (15 \times 7) + (6 \times 21) + (1 \times 35)$

$= 105 + 126 + 35 = 266$

Q18 Text Solution:

Given, number of terms, $n = 51$

$\because$ n is odd.

$\therefore$ Middle term will be $\left(\frac{n+1}{2}\right)^{th}$ term.

$= \left(\frac{51+1}{2}\right)$ th term $= 26$ th term

$\therefore T_{26} = 300$

$a + 25d = 300 \quad [\because T_n = a + (n-1)d]$

$\Rightarrow \quad T_1 + 25d = 300 \quad [\because T_1 = a]$

$\Rightarrow \quad T_1 = 300 - 25d$

and $T_{51} = a + 50d = T_1 + 50d = 300$
$- 25d + 50d$

$= 300 + 25d$

$\therefore S_{51} = \frac{51}{2}\left[300 - 25d + 300 + 25d\right]$

$= \frac{51}{2}\left[600\right] = 15300$

Q19 Text Solution:

Given,

Point $(x_1, y_1) = \left(a\cos^3\theta, a\sin^3\theta\right)$

Other line $\Rightarrow x\sec\theta + y\operatorname{cosec}\theta = a$

$y = -\frac{x\sec\theta}{\operatorname{cosec}\theta} + \frac{a}{\operatorname{cosec}\theta}$

$\Rightarrow y = \left(-\frac{\sin\theta}{\cos\theta}\right)x + a\sin\theta$

$\Rightarrow y = m_1 x + c$

$\therefore m_1 = -\frac{\sin\theta}{\cos\theta}$

Equation of required line

$\Rightarrow (y - y_1) = m(x - x_1)$

where m

$= \frac{\cos\theta}{\sin\theta}\left[\because m_1 \cdot m = -1, m = -\frac{1}{m_1} = -\frac{1}{\left(-\frac{\sin\theta}{\cos\theta}\right)} = \frac{\cos\theta}{\sin\theta}\right]$

$\left(y - a\sin^3\theta\right) = \frac{\cos\theta}{\sin\theta}\left(x - a\cos^3\theta\right)$

$\Rightarrow \quad y\sin\theta - a\sin^4\theta = x\cos\theta - a\cos^4\theta$

$\Rightarrow \quad x\cos\theta - y\sin\theta = a\left(\cos^4\theta - \sin^4\theta\right)$

$= a\left(\cos^2\theta + \sin^2\theta\right)\left(\cos^2\theta - \sin^2\theta\right)$

$= a \cdot (1) \cdot (\cos 2\theta) = a\cos 2\theta$

$\Rightarrow \quad x\cos\theta - y\sin\theta = a\cos 2\theta$

Q20 Text Solution:

Given,

mid points of the sides of triangle are $(1, 5, -1)$, $(0, 4, -2)$ and $(2, 3, 4)$.

Let,

$(x_1, y_1, z_1) = (1, 5, -1)$,

$(x_2, y_2, z_2) = (0, 4, -2)$,

$(x_3, y_3, z_3) = (2, 3, 4)$.

Then centroid of triangle, $G(x, y, z)$

$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

$G(x, y, z) = \left(\frac{1+0+2}{3}, \frac{5+4+3}{3}, \frac{-1-2+4}{3}\right)$

$= \left(1, 4, \frac{1}{3}\right)$

Q21 Text Solution:

Given the limit: $\lim_{x\to1} \frac{ax^2+bx+c}{x^2+bx+a}$

Evaluating this limit at x = 1 gives: $\frac{a\times1^2+b\times1+c}{1^2+b\times1+a} = \frac{a+b+c}{a+b+c} = 1$

As long as a + b +c≠0, dividing the numerator and the denominator by the same non-zero quantity results in 1. Therefore, Statement 1 is true.

Explanation for Statement 2:

Consider the limit: $\lim_{x\to-2} \frac{\frac{1}{x}+\frac{1}{2}}{x+2}$

This can be rewritten as: $\lim_{x\to-2} \frac{\frac{2+x}{2x}}{x+2}$

Simplifying further, we have: $\lim_{x\to-2} \frac{1}{2x}$

Substituting x = –2, we find: $\frac{1}{2\times(-2)} = -\frac{1}{4}$

Thus, the correct limit is $-\frac{1}{4}$,

not $\frac{1}{4}$ as stated. Therefore,

Statement 2 is false.

Q22 Text Solution:

Given, functionis $\frac{a}{x^4} - \frac{b}{x^2} + \cos x$.

Let $f(x) = \frac{a}{x^4} - \frac{b}{x^2} + \cos x$

Differentiating w.r.t. x, $\frac{d}{dx}f(x)$

$= \frac{d}{dx}\left(\frac{a}{x^4} - \frac{b}{x^2} + \cos x\right)$

$= -\frac{4a}{x^5} + \frac{2b}{x^3} - \sin x$

$= ma + nb - P$

$m = -\frac{4}{x^5}, n = \frac{2}{x^3}$ and $p = \sin x$.

Q23 Text Solution:

Given, numbers: 31, 32, 33, 46, 47.

The standard deviation of 31, 32, 33, ..., 47 will be the same as those of 1, 2, 3, ..., 17.

(decreasing each item by 30)

$\because SD = \sqrt{\frac{n^2-1}{12}}$

Here, $n = 17$

$\therefore SD = \sqrt{\frac{17^2-1}{12}} = \sqrt{\frac{288}{12}}$

$= \sqrt{24} = 2\sqrt{6}$

Q24 Text Solution:

$Given,\ P(A) = 0.59, P(B) = 0.30$

$and\ P(A \cap B) = 0.21$

$P(A' \cap B')\& = 1$

$- P(A \cup B)\quad [\therefore DeMorgan's law]$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - [0.59 + 0.30 - 0.21]$

$= 1 - [0.89 - 0.21]$

$= 1 - 0.68 = 0.32$

Q25 Text Solution:

Given, $f\left(x\right) = \begin{cases} 2x, & x > 3 \\ x^2, & 1 < x \le 3 \\ 3x, & x \le 1 \end{cases}$

$\therefore (-2) + f(3) + f(4)$

$= 3(-2) + (3)^2 + 2(4) = -6 + 9 + 8 = 11$

Q26 Text Solution:

Given function, $f(x) = \frac{2x}{x-1}$

On differentiating w.r.t. x, we get

$f'(x) = \frac{(x-1)(2-2x(1-0)}{(x-1)^2}$

$= \frac{2x-2-2x}{(x-1)^2} = \frac{-2}{(x-1)^2} < 0$

Function is strictly decreasing.

Function is injective

$\frac{2x}{x-1} = y \Rightarrow 2x = yx - y$

$\Rightarrow y = x(y-2)$

Let $x = \frac{y}{y-2} \notin \pi$ for $y = 2$

f is not surjective.

Q27 Text Solution:

Given function,

$f(x) = \sqrt{3}\sin 2x - \cos 2x + 4$

$f(x) = 2\left(\sin 2x \cdot \frac{\sqrt{3}}{2} - \cos 2x \cdot \frac{1}{2}\right) + 4$

$= 2\left(\sin 2x \cdot \cos\frac{\pi}{6} - \cos 2x \cdot \sin\frac{\pi}{6}\right) + 4$

$= 2\left[\sin\left(2x - \frac{\pi}{6}\right)\right] + 4$

f is one – one.

$-\frac{\pi}{2} \le 2x - \frac{\pi}{6} \le \frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{2} + \frac{\pi}{6} \le 2x \le \frac{\pi}{2} + \frac{\pi}{6}$

$\Rightarrow \frac{-3\pi+\pi}{6} \le 2x \le \frac{4\pi}{6} \Rightarrow -\frac{\pi}{6} \le x \le \frac{\pi}{3}$

$\therefore x \in \left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$

Q28 Text Solution:

Given function, $f\left(x\right) = \frac{1}{\sqrt{[x]^2-[x]-6}}$

where $[x]$ is greatest integer $\le x$.

$[x]^2 - [x] - 6 > 0$

$\Rightarrow [x]^2 - 3[x] + 2x]-6 > 0$

$\Rightarrow ([x]-3)([x]+2 > 0$

$\Rightarrow ([x]-3) > 0, [x]+2 < 0$

$\Rightarrow [x] > 3, [x] < -2$

$\Rightarrow x \in [4, \infty), x \in (-\infty, -2)$

$\therefore x \in (-\infty, -2) \cup [4, \infty).$

Q29 Text Solution:

Given,

$\cos\left[\cot^{-1}\left(-\sqrt{3}\right) + \frac{\pi}{6}\right]$

$= \cos\left[\pi - \cot^{-1}\left(\sqrt{3}\right) + \frac{\pi}{6}\right]$

$[\because \cot^{-1} - x = \pi - \cot^{-1} x]$

$= \cos\left[\pi - \frac{\pi}{6} + \frac{\pi}{6}\right]$

$= \cos[\pi] = -1$

Q30 Text Solution:

Given,

$= \tan^{-1}\left[\frac{1}{\sqrt{3}}\sin\frac{5\pi}{2}\right]\sin^{-1}\left[\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right]$

$= \tan^{-1}\left[\frac{1}{\sqrt{3}}\sin\left(2\pi + \frac{\pi}{2}\right)\right]\sin^{-1}\left[\cos\left(\frac{\pi}{3}\right)\right]$

$= \tan^{-1}\left[\frac{1}{\sqrt{3}}\sin\frac{\pi}{2}\right]\cdot\sin^{-1}\left[\cos\frac{\pi}{3}\right]$

$= \tan^{-1}\left[\frac{1}{\sqrt{3}} \times 1\right]\cdot\sin^{-1}\left[\frac{1}{2}\right]$

$= \frac{\pi}{6} \times \frac{\pi}{6} = \left(\frac{\pi}{6}\right)^2$

Q31 Text Solution:

Given, $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix} B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2-6+1 & 1-4+1 \\ 4+3+3 & 2+2+3 \end{bmatrix}$

$= \begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$

$AB' = \begin{bmatrix} -3 & 10 \\ -2 & 7 \end{bmatrix}$

Q32 Text Solution:

Let a symmetric matrix $M = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$

For matrix to be invertible, determinant must not be equal to zero.

$|M| = ab - c^2 \neq 0$

$\Rightarrow ab \neq c^2$

Therefore, M is a diagonal matrix with non-zero entries in the main diagonal and the product of entries in the main diagonal of M is not the square of an integer.

Q33 Text Solution:

Given order of matrix A and B are 3,

$|A| = 5, |B| = 3$

$\Rightarrow$ The order of AB matrix is also 3.

$|AB| = |A| \cdot |B| = 15$

Using the property $|KA| = K^n|A|$,

where n is the order of square matrix

$\Rightarrow |3\,AB| = 3^3|AB|$

$= 27 \times 15 = 405$

Q34 Text Solution:

If A and B are invertible matrices, then

(a) $(AB)^{-1} = B^{-1}A^{-1}$

(b) $\text{adj}A = |A| \cdot A^{-1}$

(c) $|A^{-1}| = |A|^{-1}$

Hence, $(A+B)^{-1} \neq B^{-1} + A^{-1}$

Q35 Text Solution:

If $f(x) = \begin{vmatrix} \cos x & 1 & 0 \\ 0 & 2\cos x & 3 \\ 0 & 1 & 2\cos x \end{vmatrix}$

Expand along C_1,

$= \cos x\left(4\cos^2 x - 3\right)$

$= 4\cos^3 x - 3\cos x$

$= \cos 3x$

$\therefore \lim_{x\to\pi} \cos 3x = \cos 3\pi = -1$

Q36 Text Solution:

Given, $x^3 - 2x^2 - 9x + 18 = 0$ and A

$= \begin{vmatrix} 1 & 2 & 3 \\ 4 & x & 6 \\ 7 & 8 & 9 \end{vmatrix}$

$x^3 - 2x^2 - 9x + 18 = 0$

$\Rightarrow x^2(x-2) - 9(x-2) = 0$

$\Rightarrow (x^2-9)(x-2) = 0$

$\Rightarrow (x-3)(x+3)(x-2) = 0$

$\therefore x = 2, 3, -3$

$A = \begin{vmatrix} 1 & 2 & 3 \\ 4 & x & 6 \\ 7 & 8 & 9 \end{vmatrix}$

$= 1(9x-48) - 2(36-42+3(32-7x)$

$= 9x - 48 + 12 + 96 - 21x$

$= -12x + 60$

$A(\text{when } x = 2) = -12 \times 2 + 60 = 36$

$A(\text{when } x = 3) = -12 \times 3 + 60 = 24$

$A(\text{when } x = -3) = -12 \times (-3) + 60 = 96$

More value of A at $x = -3$ is 96.

Q37 Text Solution:

$$f(x)=\begin{cases} x^3-1, & 1<x<\infty \\ x-1, & -\infty<x\le 1\end{cases}$$

We have to check the continuity at $x = 1$.

$$\text{RHL} \Rightarrow \lim_{x\to1^+} f(x) = \lim_{x\to1^+}(x^3-1) = 1-1 = 0$$

$$\text{LHL} \Rightarrow \lim_{x\to1^-} f(x) = \lim_{x\to1^-}(x-1) = 1-1 = 0$$

$f(1) = 1-1 = 0$

Thus the function is continuous at $x = 1$.

$$f'(x)=\begin{cases} 3x^2, & 1<x<\infty \\ 1, & -\infty<x\le 1\end{cases}$$

Now, check the differentiability at $x = 1$. LHD at $x = 1 \Rightarrow 1$

RHD at $x = 1 \Rightarrow 3(1)^2 = 3$

As, LHD $\neq$ RHD,

function is not differentiable.

Q38 Text Solution:

$y = (\cos x^2)^2$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = 2\cos x^2 \frac{d}{dx}(\cos x^2)$

$= 2\cos x^2 \times (-\sin x^2)\frac{d}{dx}(x^2)$

$= -4x\cos x^2 \sin x^2$

$= -2x\sin 2x^2$

Q39 Text Solution:

$\frac{d}{dx}(x^x + x^a + a^x + a^a)$

$= \frac{d}{dx}(x^x) + \frac{d}{dx}(x^a) + \frac{d}{dx}(a^x) + \frac{d}{dx}(a^a)$

$= \frac{d}{dx}(y) + ax^{a-1} + a^x \log a + 0$

$[\text{let } x^x = y]$

$y = x^x$

Taking log on both sides,

$\log y = x\log x$

On differentiating w.r.t. x, we get

$\frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{x} + \log x$

$\Rightarrow \frac{dy}{dx} \& = x^x(1+\log x)$

Substitute the value of $\frac{dy}{dx}$ in Eq. (i),

$\frac{d}{ax}(x^x + x^a + a^x + a^a)$

$= x^x(1+\log x) + ax^{a-1} + a^x \log a$

Q40 Text Solution:

Statement 1,

$y = \log_{10} x + \log_e x$

$\Rightarrow y = \frac{\log_e x}{\log_e 10} + \log_e x$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = \frac{1}{x\log_e 10} + \frac{1}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{\log_{10} e}{x} + \frac{1}{x}$

Statement 2,

$\frac{d}{dx}\log_{10} x = \frac{d}{dx}\frac{\log_e x}{\log_e 10}$

$= \frac{1}{x\log_e 10}$

$\Rightarrow \frac{d}{dx}(\log_e x) = \frac{d}{dx}\frac{\log_e x}{\log_e e} = \frac{1}{x}$

Hence,

statement 1 is true but statement 2 is false.

Q41 Text Solution:

$x = \cos\theta + \log\tan\frac{\theta}{2}$

On differentiating w.r.t. θ, we get

$\frac{dx}{d\theta} = -\sin\theta + \frac{1}{\tan\frac{\theta}{2}} \times \sec^2\frac{\theta}{2} \times \frac{1}{2}$

$= -\sin\theta + \frac{1}{\sin\theta}$

$= \frac{1-\sin^2\theta}{\sin\theta}$

$= \frac{\cos^2\theta}{\sin\theta}$ ..... (i)

Now, $y = \sin\theta$

On differentiating w.r.t. θ

$\frac{dy}{d\theta} = \cos\theta$ ....(ii)

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

$= \frac{\cos\theta}{\frac{\cos^2\theta}{\sin\theta}} = \tan\theta$

[from Eqs. (i) and (ii)]

Q42 Text Solution:

$y = (x-1)^2(x-2)^3(x-3)^5$

Taking log on ths ides,

$\Rightarrow \log y = \log[(x-1)^2(x-2)^3(x-3)^5]$

$\log y = 2\log(x-1) + 3\log(x-2) + 5\log(x-3)$

On both sides differentiating w.r.t. x, we get

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x-1} + \frac{3}{x-2} + \frac{5}{x-3}$

$\Rightarrow \frac{dy}{dx} = (x-1)^2(x-2)^3(x-3)^5\left[\frac{2}{(x-1)} + \frac{3}{(x-2)} + \frac{5}{(x-3)}\right]$

$\therefore \left(\frac{dy}{dx}\right)_{x=4} = 3^2 \times 2^3 \times 1^5\left[\frac{2}{3} + \frac{3}{2} + 5\right]$

$= 9 \times 8 \times \left(\frac{4+9+30}{6}\right) = 516$

Q43 Text Solution:

$\theta = \frac{t^2}{20} + \frac{t}{5}$

On differentiating θ w.r.t. t,

$\frac{d\theta}{dt} = \frac{t}{10} + \frac{1}{5}$

$\left(\frac{d\theta}{dt}\right)_{t=4} = \frac{4}{10} + \frac{1}{5}$

$\Rightarrow k = \frac{3}{5}$

$\therefore 5k = 3$

Q44 Text Solution:

Given the parabola $y = \alpha x^2 - 6x + \beta$ . ...(i)

Since the curve passes through the point $(0, 2)$, we substitute $x = 0$ and $y = 2$ into the equation.

$(0, 2) \Rightarrow 2 = 0 - 0 + \beta$

$\Rightarrow \beta = 2$

Now differentiate equation (i) with respect to x to find the slope of the tangent .

$\frac{dy}{dx} = 20x - 6$

Substitute $x = \frac{3}{2}$ into the derivative.

$\therefore \left(\frac{dy}{dx}\right)_{x=\frac{3}{2}} = 2 \times \alpha \times \frac{3}{2} - 6$

$= 3\alpha - 6$

The tangent is parallel to the $X-$axis, so its slope is 0.

$\frac{dy}{dx} = 0$

$\Rightarrow 3\alpha - 6 = 0$

$\Rightarrow \alpha = 2$

Q45 Text Solution:

$f(x) = x^2 - 2x$

$\therefore f'(x) = 2x - 2$

$f(x)$ is strictly decreasing, when $f'(x) < 0$

$f'(x) < 0$

$\Rightarrow 2(x-1) < 0$

$\Rightarrow x < 1$

Hence,

$f(x)$ is strictly decreasing in the interval $(-\infty, 1)$.

Q46 Text Solution:

$y = -x^3 + 3x^2 + 2x - 27$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = -3x^2 + 6x + 2$

Slope of the curve $= -3x^2 + 6x + 2$

Let $m = -3x^2 + 6x + 2$

$\therefore \frac{dm}{dx} = -6x + 6$

Put $\frac{dm}{dx} = 0$ to find the critical points,

$\Rightarrow -6x + 6 = 0$

$\Rightarrow x = 1$

$\left(\frac{d^2m}{dx^2}\right)_{x=1} = -6 < 0$

Slope is maximum at $x = 1$

The maximum slope,

$m_{max} = -3(1)^2 + 6(1) + 2$

$= -3 + 6 + 2 = 5$

Q47 Text Solution:

Let $I = \int \frac{x^3 \sin[\tan^{-1}(x^4)]}{1+x^8}$

Let $\tan^{-1}(x^4) = t$.

On differentiating w.r.t. t, we get

$\frac{1}{1+x^8} \times 4x^3 dx = dt$

$\Rightarrow \frac{x^3}{1+x^8} dx = \frac{1}{4} dt$

$I = \frac{1}{4} \int \sin\ t dt$

$= -\frac{1}{4} \cos t + C$

$= -\frac{1}{4} \cos(\tan^{-1}(x^4)) + C$

Q48 Text Solution:

Let $I = \int \frac{x^2}{\sqrt{x^6 + a^6}} dx$

Let $x^3 = t$

$\Rightarrow 3x^2 dx = dt$

$I = \frac{1}{3} \int \frac{1}{\sqrt{t^2 + (a^3)^2}} dt$

$= \frac{1}{3} \log\left|t + \sqrt{t^2 + a^6}\right| + C$

$= \frac{1}{3} \log\left|x^3 + \sqrt{x^6 + a^6}\right| + C$

Q49 Text Solution:

Let $I = \int \frac{xe^x\, dx}{(1+x)^2}$

$= \int \frac{e^x(x+1-1)}{(1+x)^2} dx$

$= \int e^x \left[\frac{1}{1+x} + \left(\frac{-1}{(1+x)^2}\right)\right] dx$

Let $\frac{1}{1+x} = f(x)$

$\therefore f'(x) = -\frac{1}{(1+x)^2}$

Using the formula, $\int e^x [f(x) + f'(x)] dx$

$= e^x f(x) + C$

$\Rightarrow I = e^x \left(\frac{1}{1+x}\right) + C = \frac{e^x}{1+x} + C$

Q50 Text Solution:

Let $I = \int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$

$= \int e^x \left(\frac{1+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right) dx$

$= \int e^x \left[\frac{\sec^2\frac{x}{2}}{2} + \tan\frac{x}{2}\right] dx$

Let $f(x) = \tan\frac{x}{2}$

$\therefore f'(x) = \frac{\sec^2\frac{x}{2}}{2}$

Using the formula, $\int e^x [f(x) + f'(x)] dx$

$= e^x f(x) + C$

$\Rightarrow I = e^x \times \tan\frac{x}{2} + C$

Q51 Text Solution:

$I_n = \int_0^{\frac{\pi}{4}} \tan^n x dx \; \ldots (i)$

$I_{n+2} = \int_0^{\frac{\pi}{4}} \tan^{n+2} x dx \; \ldots (ii)$

Adding Eqs. (i) and (ii),

$I_n + I_{n+2} \& = \int_0^{\frac{\pi}{4}} \tan^n x dx + \int_0^{\frac{\pi}{4}} \tan^{n+2} x dx$

$= \int_0^{\frac{\pi}{4}} \tan^n x \cdot \sec^2 x dx$

$= \left[\frac{\tan^{n+1} x}{n+1}\right]_0^{\frac{\pi}{4}} = \frac{1}{n+1}$

Thus, $I_n + I_{n+2} = \frac{1}{(n+1)}$

Substitute 8 for x.

$I_8 + I_{10} = \frac{1}{9}$

Q52 Text Solution:

Let $I = \int_0^{4042} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4042-x}} dx \; \ldots (i)$

Using the property, $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

$I = \int_0^{4042} \frac{\sqrt{4042-x}}{\sqrt{4042-x}+\sqrt{x}} \ldots (ii)$

Adding Eqs. (i) and (ii),

$2I = \int_0^{4042} \frac{\sqrt{x}+\sqrt{4042-x}}{\sqrt{4042-x}+\sqrt{x}} dx$

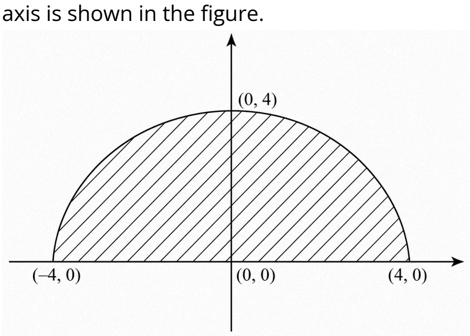
$\Rightarrow 2I = \int_0^{4042} I\, dx$

$\Rightarrow 2I = 4042$

$\Rightarrow I = 2021$

Q53 Text Solution:

The area bounded by $y = -\sqrt{16 - x^2}$ and X-axis is shown in the figure.



Area $= \int_{-4}^{4} y\, dx$

$= \int_{-4}^{4} \sqrt{4^2 - x^2}\, dx$

$= \left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_{-4}^{4}$

$= [0 + 8\sin^{-1}(1)] - [(8\sin^{-1}(-1))]$

$= 8\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right]$

$= 8\pi$

Q54 Text Solution:

Area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\pi|ab|$

$a^2 = 25, b^2 = \lambda^2$

Area of the ellipse $\frac{x^2}{25} + \frac{b^2}{\lambda^2} = 1$ is $\pi \times 5 \times |\lambda|$.

$\Rightarrow 20\pi \& = 5\pi|\lambda|$

$\Rightarrow |\lambda| = 4$

$\Rightarrow \lambda = \pm 4$

Q55 Text Solution:

Given, $x dy = y dx$

$\Rightarrow \frac{1}{y} dy = \frac{1}{x} dx$

Integrating both sides, $\int \frac{1}{y} dy = \int \frac{1}{x} dx$

$\Rightarrow \log y = \log x + \log c$

$\Rightarrow y$

$= xc$ which is the equation of line passing through origin

.

Q56 Text Solution:

$\frac{dy}{dx} = \frac{y+1}{x-1}$

$\Rightarrow \frac{1}{x-1} dx = \frac{1}{y+1} dy$

Integrating both sides,

$\int \frac{1}{x-1} dx = \int \frac{1}{y+1} dy$

$\Rightarrow \log|x - 1| = \log|y + 1| + \log C$

$\Rightarrow (x - 1) = c(y + 1)$

As, $y(1) = 2$

$0 = c(3)$

$c = 0$

Hence, $x - 1 = 0$ or $x = 1$

The given differential equation has only one solution.

Q57 Text Solution:

Let the equal angles of a with the coordinate axis be α.

$l = \cos\alpha, m = \cos\alpha, n = \cos\alpha$

$\because l^2 + m^2 + n^2 = 1$

$\Rightarrow \quad 3\cos^2\alpha = 1$

$\Rightarrow \quad \cos\alpha = \frac{1}{\sqrt{3}}$

Direction ratios are 1, 1, 1.

$\therefore a = \hat{i} + \hat{j} + \hat{k}$

Projection of b on $a = \frac{b \cdot a}{|a|}$

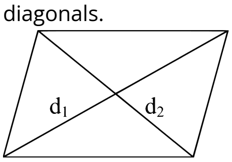
$= \frac{(5\hat{i}+7\hat{j}-\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k})}{\sqrt{1^2+1^2+1^2}}$

$= \frac{5+7-1}{\sqrt{3}}$

$= \frac{11}{\sqrt{3}}$

Q58 Text Solution:

Let a and b be the length of the sides of the parallelogram and d_1, d_2 be the length of diagonals.



Given, $d_1 = 3\hat{i} + 6\hat{j} - 2\hat{k}$

$d_2 = -\hat{i} - 2\hat{j} - 8\hat{k}$

$\therefore a = \frac{d_1+d_2}{2} = \frac{2\hat{i}+4\hat{j}-10\hat{k}}{2} = \hat{i} + 2\hat{j} - 5\hat{k}$

$\Rightarrow |a| = \sqrt{1+4+25} = \sqrt{30}$

and $b = \frac{d_1-d_2}{2} = \frac{4\hat{i}+8\hat{j}+6\hat{k}}{2}$

$= 2\hat{i} + 4\hat{j} + 3\hat{k}$

$\Rightarrow \quad |b| = \sqrt{4+16+9} = \sqrt{29}$

Q59 Text Solution:

Given, $a \cdot b$

$= 0$ and $(a + b)$ makes 60° angle with a.

$\cos 60^\circ = \frac{(a+b)\cdot a}{|a+b||a|}$

$\Rightarrow \frac{1}{2} = \frac{|a|^2+a\cdot b}{|a+b||a|}$

$\Rightarrow |a + b|\& = 2|a|$

$\Rightarrow |a + b|^2 = 4|a|^2$

$\Rightarrow (a + b)(a + b) = 4|a|^2$

$\Rightarrow |a|^2 + |b|^2 + 2a \cdot b = 4|a|^2$

$\Rightarrow b^2 = 3|a|^2$

$\Rightarrow |b| = \sqrt{3}|a|$

Q60 Text Solution:

Area of parallelogram having a and b as its adjacent sides is 15 sq units.

$\Rightarrow |a \times b| = 15$

Area of parallelogram having (3a + 2b) and a + 3b) as two adjacent side

$= |(3a + 2b) \times (a + 3b)|$

$= |3a \times a + 2b \times a + 9a \times b + 6b \times b|$

$= 2b \times a + 9a \times b|$

$[\therefore a \times a = b \times b = 0]$

$[\therefore b \times a = -a \times b]$

$= 7|a \times b|$

$= 7 \times 15$

$= 105$