# RMO Test Paper-2025

**DURATION : 180 Minutes** | **DATE: 16/11/2025** | **M.MARKS : 102**

## IMPORTANT INSTRUCTIONS

1. The test is of **180 Minute** duration and the Test Booklet contains **6** questions from **Maths.** All questions are compulsory.
2. Question No. **1 to 6** carry **17** marks for each correct response.
3. **Calculators (in any form) and protractors are not allowed.**
4. **Rulers and compasses are allowed.**
5. **Answer all the questions.**
6. **All questions carry equal marks. The maximum marks are 102.**
7. **Answer to each should start on a new page. Clearly indicate the question number.**

## Test Syllabus

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|---|---|
| **Maths :** | Full Syllabus |

# RMO Test Paper-2025

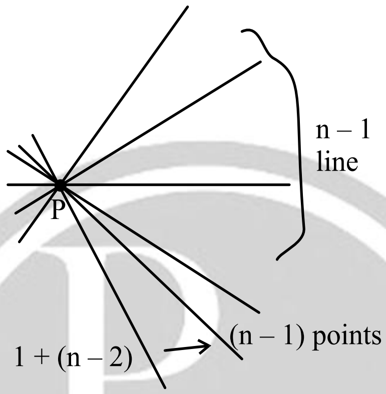
**1.** **(a)** Let $n \geq 3$ be an integer. Find a configuration of n lines in the plane which has exactly

(i) $n - 1$ distinct points of intersection;

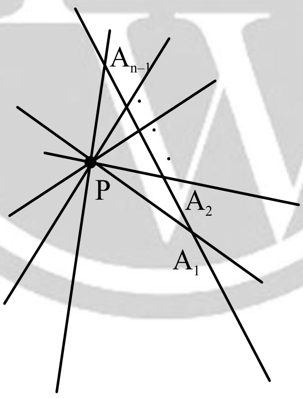
(ii) n distinct points of intersection;

**(b)** Give configurations of n lines that have exactly $n + 1$ distinct points of intersection for (i) $n = 8$ and (ii) $n = 9$.

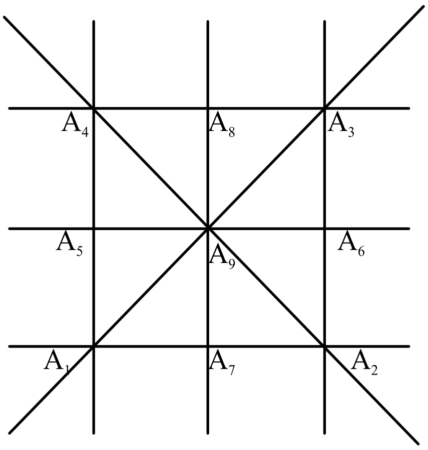
**Sol.** (i) $n - 1$



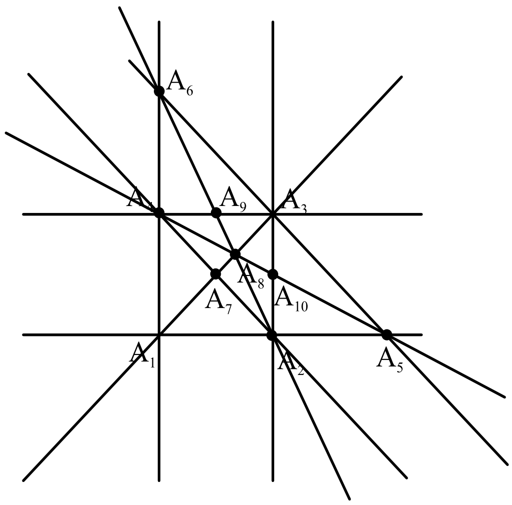

(ii) $1 + (n - 2) \rightarrow n - 1$ points




(b) (i) $n = 8$, point of intersection = 9




(ii) $n = 9$, point of intersection = 10




**2.** Let a, b, c be distinct nonzero real numbers satisfying $a+\frac{2}{b}=b+\frac{2}{c}=c+\frac{2}{a}$.

Determine the value of $\left|a^2b+b^2c+c^2a\right|$.

**Sol.** $a-b=\frac{2}{c}-\frac{2}{b}$

$$a-b=2\frac{(b-c)}{bc} \quad \&$$

$$b-c=2\frac{(c-a)}{ac}$$

$$c-a=\frac{2(a-b)}{ab}$$

$$(a-b)(b-c)(c-a)=8\frac{(a-b)(b-c)(c-a)}{(abc)^2}$$

$$(abc)^2=8$$

$$abc=\pm2\sqrt{3}$$

$$a-b=\frac{2(b-c)}{bc}$$

let $abc=p,\ bc=\frac{p}{a}$

$$a-b=\left(\frac{2(b-c)a}{p}\right)\times c$$

$$b-c=\left(\frac{2(c-a)b}{p}\right)\times a$$

$$c-a=\left(\frac{2(a-b)c}{p}\right)\times b$$

$$0=\frac{b-c}{b}+\frac{c-a}{c}+\frac{a-b}{a}$$

$$3=\frac{c}{b}+\frac{a}{c}+\frac{b}{a}$$

$$\frac{b}{a}+\frac{a}{c}+\frac{c}{b}=3$$

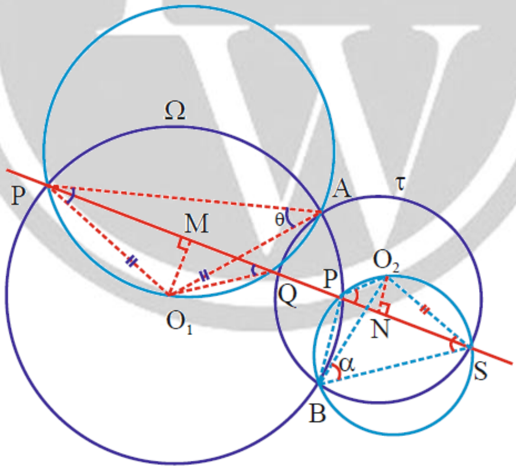
$$\frac{b^2a+a^2b+c^2a}{abc}=3$$

$$b^2a+a^2b+c^2a=\pm 6\sqrt{2}$$

$$|b^2a+a^2b+c^2a|=6\sqrt{2}$$

**3.** Let $\Omega$ and $\Gamma$ be circles centred at $O_1$, $O_2$ respectively. Suppose that they intersect in distinct points A, B. Suppose $O_1$ is outside $\Gamma$ and $O_2$ is outside $\Omega$. Let $\ell$ be a line not passing through A and B that intersects $\Omega$ at P, R and $\Gamma$ at Q, S so that P, Q, R, S lie on the line in this order. Furthermore, the points $O_1$, B lie on one side of $\ell$ and the points $O_2$, A lie on the other side of $\ell$. Given that the points A, P, Q, $O_1$ are concyclic and B, R, S, $O_2$ are concyclic as well, prove that AQ = BR.

**Sol.**




$\angle ARP = \angle QAR$

AQ = QR

Now similarly

BR = QR $\Rightarrow \angle BQR = \angle QBR$

comment : AQ = BR

**4.** Prove that there do not exist positive rational numbers x and y such that

$$x+y+\frac{1}{x}+\frac{1}{y}=2025.$$

**Sol.** $x=\frac{p}{q}, y=\frac{r}{s}$

$$x+\frac{1}{x}=\frac{p}{q}+\frac{q}{p},\ y+\frac{1}{y}=\frac{r^2+s^2}{rs}$$

$$=\frac{p^2+q^2}{pq}$$

$\gcd(p, q) \equiv 1,\qquad \gcd(r, s) \equiv 1$

$\gcd(p, q^2) \equiv 1$

$\gcd(q, p) \equiv 1$

$\gcd(p, p^2 + q^2) \equiv 1$

$\gcd(q, p^2 + q) \equiv 1$

$$\frac{p^2+q^2}{pq}+\frac{r^2+s^2}{rs}=2025$$

reduced

$\gcd(pq, p^2 + q^2) \equiv 1$

similar

$\gcd(rs, r^2 + s^2) \equiv 1$

$p^2 + q^2 = A,\ \gcd(A, pq) \equiv 1$

$r^2 + s^2 = B,\ \gcd(B, rs) \equiv 1$

$$\frac{A}{pq}+\frac{B}{rs}=2025$$

$A\cdot rs + B\cdot pq = 2025\ pqrs$

$A\cdot rs + B\cdot pq = pq(2025rs)$

$pq|rs$

similarly

$rs|pq$

$rs = pq$

$A+ B = 2025\ pq$

$p^2 + q^2 + r^2 + s^2 = 2025pq$

$(p\cdot s)$ unit digit 2

(0, 0, 0, 0), (1, 1), (1, 0) not possible

p, q = gcd(3) not possible

**5.** Let ABC be an acute angled triangle with AB < AC, orthocentre H and circumcircle $\Omega$. Let M be the midpoint of minor arc BC of $\Omega$. Suppose that MH is equal to the radius of $\Omega$. Prove that $\angle BAC = 60^\circ$.

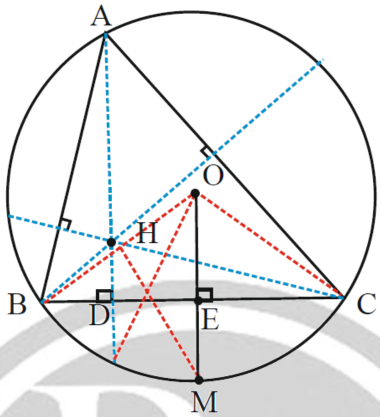
**Sol.** Radius of $\Omega$ = R

$\Rightarrow$ MH = R

MO || AH

$\Rightarrow$ AOMH is either

a rhombus or isosceles trapezoid




Case-I: Let with the sake of contradiction that AOMH is an isosceles trapezoid

$\Rightarrow \angle AHM < 90^\circ$, because $\angle AOM = \angle AOB + \angle BOM$

$= 2\angle C + \angle A > 90^\circ$

but $\angle A$ HM cannot be less than 90° as H lies inside $\Delta ABC$ as $\Delta ABC$ is acute angled $\Delta$.

$\Rightarrow \angle AMM \neq 90^\circ$

$\Rightarrow$ AOMH is not an isosceles trapezoid

$\Rightarrow$ AOMH is a Rhombus

$\Rightarrow$ AH = R

AH = 2RGA

R = 2RGA

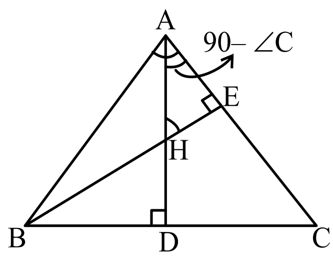
1 = 2GA

$\frac{1}{2} = \cos A$

$60^\circ = \angle A$

$\Rightarrow \angle BAC = 60^\circ$ Hence proved.

**Alternate:**



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

c = 2RsinC

In ΔAEB

$$\cos A = \frac{AE}{AB}$$

C cosA = AE

In ΔAEH

$$\sin C = \frac{AE}{AH}$$

$$AH = \frac{\cos A}{\sin C}$$

$$= \frac{2R \sin C . \cos A}{\sin C} = 2R \cos A$$ Hence Proved

**6.** Let p(x) be a non-constant polynomial with integer coefficients, and let $n \geq 2$ be an integer such that no term of the sequence p(0), p(p(0)), p(p(p(0))),… is divisible by n. Show that there exist integers a, b such that $0 \leq a < b \leq n-1$ and n divides p(b) – p(a)

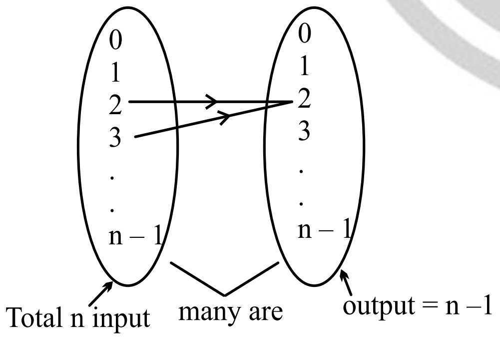
**Sol.** Let $p(x) = a_0x^n + a_1x^n + a_2x^{n-2} + \ldots + a_n$

$\because$ p(0), p(p(0)), p(p(p(0))) …… $\not\equiv$ 0 (mnod n)

$p(x) \not\equiv 0 \ \forall \ x \in \{0, 1, 2, \ldots, n-1\}$

$p(2) \equiv p(3) \pmod n$

$n \mid p(2) \cdot p(3)$




Possible range ≡ 0, 1, 2, ….. n – 1 (mod n)