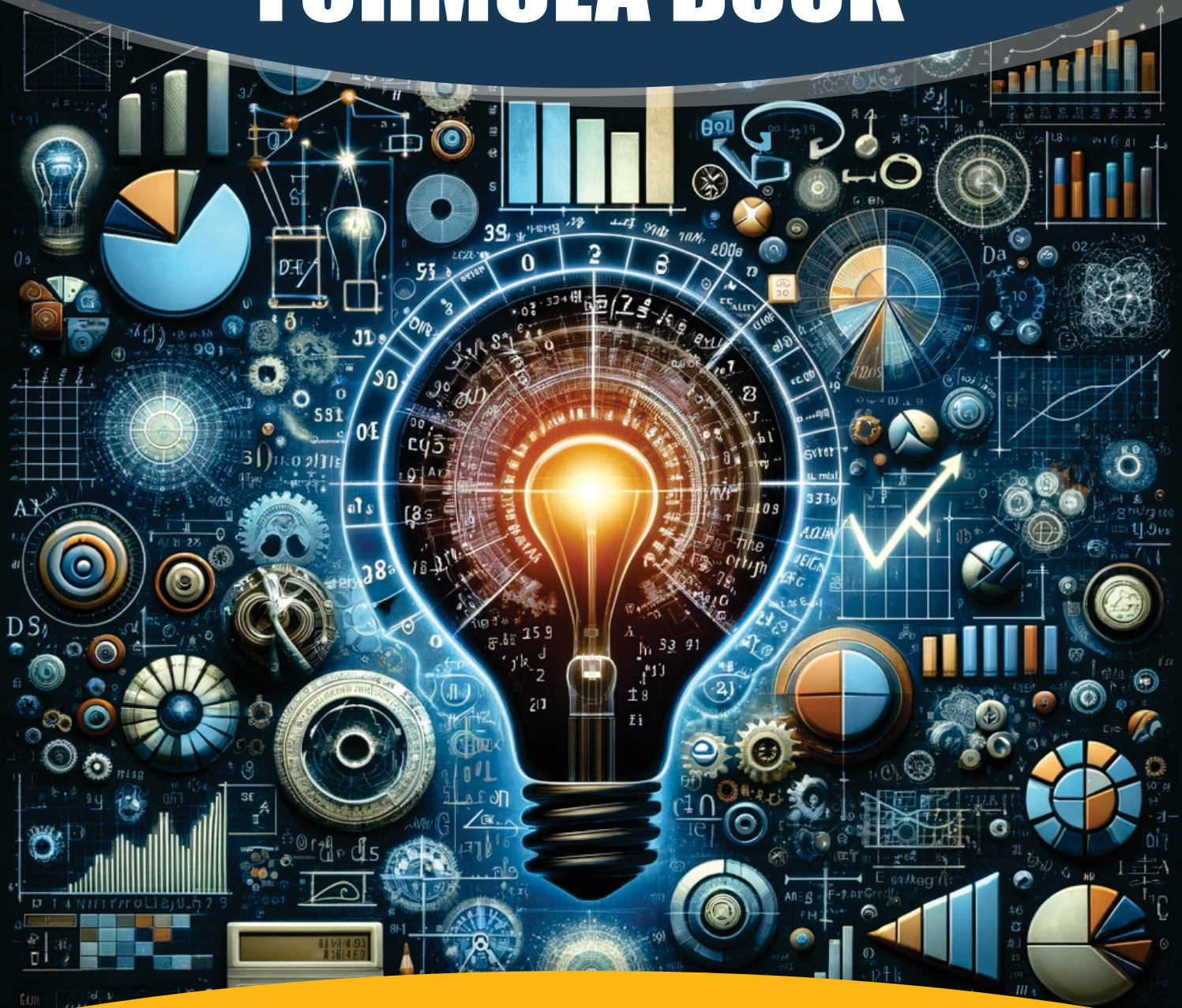




**MBA
WALLAH**

QUANTITATIVE APTITUDE FORMULA BOOK



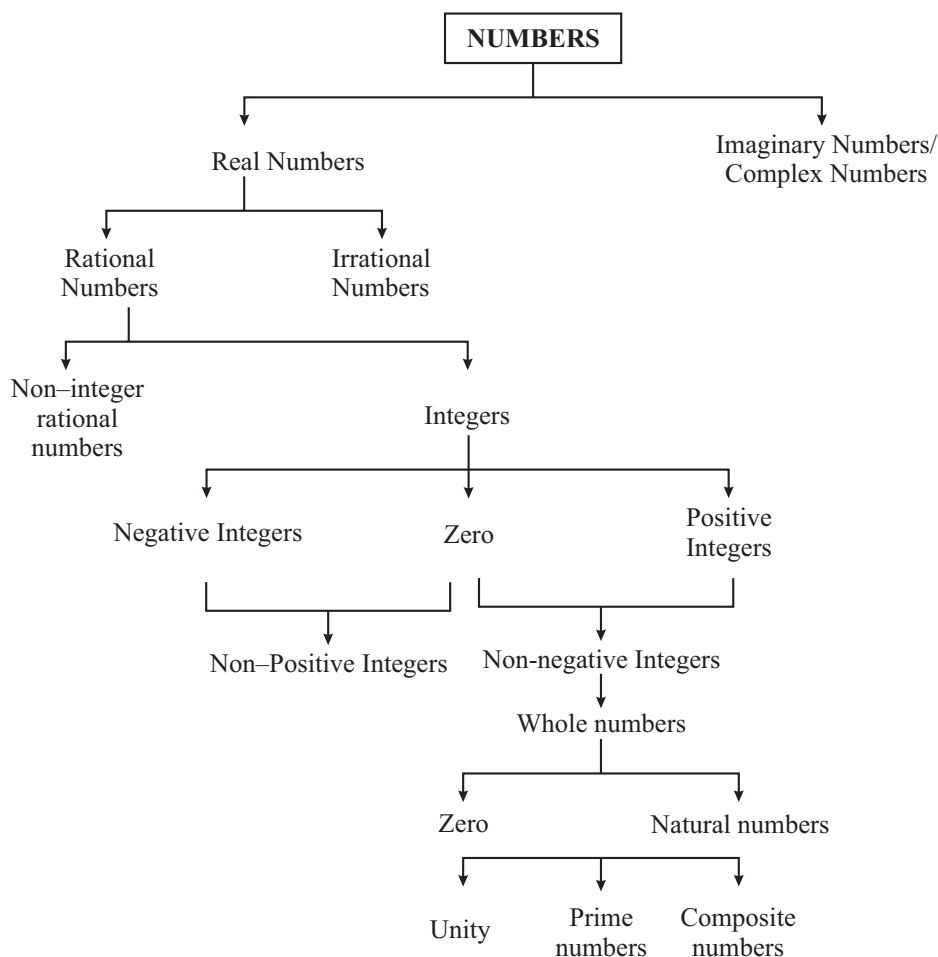
**Quick Revision of Quantitative Aptitude for
CAT and Other MBA Entrance Exams**

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NUMBER SYSTEM

CLASSIFICATION OF NUMBERS



- 1. Real Numbers:** These can be denoted on a number line. Eg. $\rightarrow +2, -8, +19, \frac{13}{11}, \sqrt{6}, \sqrt{32}$. These are denoted by the symbol “ R ”.
- 2. Imaginary Numbers:** These cannot be denoted on a number line and are expressed in the form of $b + ic$. Here ‘ b ’ and ‘ c ’ are real numbers and “ i ” is an imaginary number also known as “iota”. Eg. $\rightarrow \sqrt{11}, \sqrt{18}, \sqrt{-42} \cdot \sqrt{-1} = i, -1 = i^2$
- 3. Rational Numbers:** Those numbers which can be expressed in the form of $\frac{p}{q}$ where q (the denominator) is not equal to zero and p, q are integers. The set of rational numbers is denoted by the letter “ Q ”.
- 4. Irrational Numbers:** These cannot be expressed in the form of $\frac{p}{q}$.
- 5. Integers:** Integers are those numbers with no decimal or fractional part and can further be divided into zero, negative and positive integers. The set of Integers is represented by the letter “ Z ”. **Even Integers** – Integers divisible by 2 and can be expressed in “ $2k$ ” form. **Odd Integers** – Integers which are not divisible by 2 can be expressed in “ $2k \pm 1$ ” form.

Some Useful Results with Respect to Integers

- $\text{Odd} \times \text{Odd} = \text{Odd}$
- $\text{Odd} \times \text{Even} = \text{Even}$

- $\text{Odd} \pm \text{Odd} = \text{Even}$
 - $\text{Even} \pm \text{Even} = \text{Even}$
 - $\text{Odd} \pm \text{Even} = \text{Odd}$
 - When any two consecutive integers are taken, one of them is odd and the other is even. Hence the product of any two consecutive integers is always even i.e. divisible by 2
 - Out of any 3 consecutive integers, one of them is divisible by 3 and at least one of the three is definitely even. Hence, the product of any 3 consecutive integers is always divisible by 6
 - Out of any 'n' consecutive integers, exactly one number will be divisible by 'n' and the product of 'n' consecutive integers will always be divisible by n!
 - Square of a number can neither end with an odd number of zeroes nor with 2, 3, 7 or 8.
6. **Natural numbers:** These are all the positive integers starting from 1. The set of Natural numbers is represented by the letter " N ".
 7. **Whole Numbers:** These are all the integers beginning with 0 and include natural numbers. The set of whole numbers is represented by the letter " W ".
 8. **Factor:** If a given positive integer gets completely divided by another positive integer i.e. without leaving any remainder, then such divisor is said to be a "factor" of the dividend.
 9. **Prime Numbers:** These numbers are those positive integers which have only two factors i.e. the number itself and 1. 2 is the smallest prime number and the only even prime number. Each prime number greater than 3 can be written in the form of $6k \pm 1$ but every number of such form may not necessarily be prime.
 10. **Relatively prime/Co-prime Numbers:** These are those two numbers whose no factor other than 1 is common i.e. their HCF = 1.
 11. **Twin prime Numbers:** Two prime numbers with a gap of 2 such as (3,5), (5,7), (11,13), (17,19), (41,43)
 12. **Perfect Numbers:** If the sum of all the factors excluding the number itself is equal to that number, then it is called a perfect number. 6 is the smallest perfect number. Factors of 6 other than 6 itself are 1, 2 and 3, when we add them up i.e. $1 + 2 + 3 = 6$. Other such examples are 28, 496, 8198 etc.
 13. **Composite Numbers:** These numbers are those positive integers which have more than two factors.
 14. **Factorial:** A factorial of a natural number ' N ' is defined as a product of all the natural numbers upto " N " such that $N! = N \times (N-1) \times (N-2) \times \dots \times 3 \times 2 \times 1$. Please note that the value of $0! = 1$.
 15. **Absolute Value:** The absolute value or modulus of a number ' a ' is the distance of ' a ' from 0 along the number line. Absolute value is expressed as $|a|$ such that $|a|$ is always positive.

DIVISIBILITY RULES

1. **Divisibility by 2:** A number is divisible by 2 if the last digit of the number is 0, 2, 4, 6 or 8.
2. **Divisibility by 4:** A number is divisible by 4 if the last two digits of the number are divisible by 4.
3. **Divisibility by 8:** A number is divisible by 8 if the last three digits of the number are divisible by 8.
4. **Divisibility by 16:** A number is divisible by 16 if the last four digits of the number are divisible by 16.
5. **Divisibility by 3:** A number is divisible by 3 if the sum of the digits of the number is divisible by 3.
6. **Divisibility by 9:** A number is divisible by 9 if the sum of the digits of the number is divisible by 9.
7. **Divisibility by 5:** A number is divisible by 5 if the last digit of the number is either 0 or 5.
8. **Divisibility by 25:** A number is divisible by 25 if the last two digits of the number are divisible by 25.
9. **Divisibility by 125:** A number is divisible by 125 if the last three digits of the number are divisible by 125.
10. **Divisibility by 6:** Express 6 as a product of co-prime factors (2 and 3). Therefore, if a number is divisible by both 2 and 3, it must also be divisible by 6.
11. **Divisibility by 7:** Take the unit digit of the number, double it and subtract this figure from the remaining part of the number. If the result thus obtained is divisible by 7, then the original number is divisible by 7.
12. **Divisibility by 11:** If the difference between the sum of the digits at odd places and the sum of the digits at even places is 0 or a multiple of 11, then the number is said to be divisible by 11.
13. A number of the form $ABAB$ is always divisible by 101 and a number of the form $ABCABC$ is always divisible by 1001.

RECURRING DECIMALS

1. **Recurring Decimal:** A decimal in which a digit or a set of digits is repeated continuously. A bar is placed over the entire set of digits that recur.
2. **Pure Recurring Decimal:** A recurring decimal such as $0.\overline{8}$ is termed a pure recurring decimal because all the digits after the decimal point are recurring.
3. **Mixed Recurring Decimal:** A recurring decimal such as $0.1\overline{6}$ is termed a mixed recurring decimal because some of the digits after the decimal are not recurring.

SOME USEFUL CONVERSIONS WITH RESPECT TO RECURRING DECIMALS

1. **Conversion of a pure recurring decimal into a vulgar fraction:** A pure recurring decimal is equivalent to a vulgar fraction which has the number formed by the recurring digits for its numerator, and for its denominator the number which has for its digits as many nines as there are digits in the period.

Eg. $0.\overline{85}$ can directly be written as equivalent to $\frac{85}{99}$ or $0.1\overline{26}$ can directly be written as $\frac{126}{999}$ which can further be simplified as $\frac{14}{111}$.

2. **Conversion of a mixed recurring decimal into a vulgar fraction:** For the numerator part, write the entire given number formed by the recurring and the non-recurring parts and subtract from it the part of the decimal that is not recurring. Now for the denominator part, write as many nines as the number of digits recurring and then place next to it as many zeroes as there are digits without recurring in the given decimal.

Eg. $0.2\overline{85} = \frac{285 - 2}{990} = \frac{283}{990}$

FACTORS

Any natural number N can be expressed in the form of $N = a^p \cdot b^q \cdot c^r \dots$, such that a, b and c are prime factors of N and p, q, r are positive integers.

Eg. $1620 = 2^2 \cdot 3^4 \cdot 5^1$

Now, let us see some important formulae for $N = a^p \cdot b^q \cdot c^r$

1. **Number of factors of N is equal to** $= (p + 1)(q + 1)(r + 1)$
2. **Number of ways of expressing N as a product of two factors (Even number of total factors)** $= \frac{1}{2}(p + 1)(q + 1)(r + 1)$
3. **Number of ways of expressing N as a product of two factors (Odd number of total factors)** = If a number has an odd number of factors, it must be a perfect square. Therefore, we have two different cases here:
 - (i) as a product of two “different” factors (i.e. excluding $\sqrt{N} \times \sqrt{N}$) $= \frac{1}{2}\{(p + 1)(q + 1)(r + 1) - 1\}$
 - (ii) as a product of two factors (i.e. including $\sqrt{N} \times \sqrt{N}$) $= \frac{1}{2}\{(p + 1)(q + 1)(r + 1) + 1\}$
4. **Sum of all the factors of N** $= \frac{a^{p+1} - 1}{a - 1} \cdot \frac{b^{q+1} - 1}{b - 1} \cdot \frac{c^{r+1} - 1}{c - 1}$
5. **Product of all the factors of N** $= N^{\frac{\text{Number of factors}}{2}} = N^{\frac{1}{2}(p+1)(q+1)(r+1)}$
6. **Number of ways of writing N as a product of two Co-primes** $= 2^{n-1}$, where ‘ n ’ is the number of distinct prime factors of N .
7. **Number of Co-primes to N less than N** $= N \left(1 - \frac{1}{a}\right) \left(1 - \frac{1}{b}\right) \left(1 - \frac{1}{c}\right)$
8. **Sum of Co-primes to N less than N** $= \frac{N}{2} \times \text{Number of Co-primes to } N \text{ less than } N$

RULES PERTAINING TO NATURAL NUMBERS

1. Sum of first ' n ' natural numbers = $\frac{n(n+1)}{2}$
2. Sum of first ' n ' odd natural numbers = n^2
3. Sum of first ' n ' even natural numbers = $n(n+1)$
4. Sum of the squares of first ' n ' natural numbers = $\frac{n(n+1)(2n+1)}{6}$
5. Sum of the cubes of first ' n ' natural numbers = $\left[\frac{n(n+1)}{2}\right]^2$

HCF AND LCM

1. The **least common multiple (LCM)** is defined as the smallest multiple that two or more numbers have in common.
2. The **highest common factor (HCF)** is the largest factor of two or more given numbers
3. Product of two numbers = LCM $\times$ HCF
4. **LCM of fractions** = $\frac{\text{LCM of numerators}}{\text{HCF of denominators}}$
5. **HCF of fractions** = $\frac{\text{HCF of numerators}}{\text{LCM of denominators}}$
6. A number which when divided by x , y or z leaves remainder ' r ' in each case will be of the form **LCM (x , y and z) $k + r$** , when $k = 0$, we get smallest such number
7. A number which when divided by a , b and c gives remainders x , y and z respectively such that the common difference (d) = $a - x = b - y = c - z$, then the number will be of the form **LCM (a , b and c) $k - d$** , when $k = 1$, we get smallest such number
8. The largest number with which the numbers a , b and c are divided giving remainders of x , y and z respectively will be the **HCF ($a - x$, $b - y$, $c - z$)**
9. The largest number with which the numbers a , b and c are divided giving the same remainder in each case = **HCF ($a - b$, $a - c$) = HCF ($a - b$, $b - c$) = HCF ($b - c$, $a - c$)**

LARGEST POWER OF A NUMBER IN N!

1. The largest power of X in $Y!$ can be found when X is prime by dividing Y by X . The quotient obtained is then divided by X . The resulting quotient is once again divided by X . This procedure is repeated until the quotient obtained is lesser than X . Finally, all the quotients are added up and the result equals the required power.

Eg. Finding the highest power of 3 that divides 279!

3	279
3	93
3	31
3	10
3	3
	1

Therefore, the largest power of 3 that divides 279! Is

$$93 + 31 + 10 + 3 + 1 = 138$$

2. Largest power of 2 in $2^n! = 2^n - 1$
3. If the number whose largest power is to be found out is not prime, then we need to write the number as the product of prime numbers, and find the largest power of each of the factors separately. Then the smallest among the largest powers of all these factors of the given number will give the largest power required.

FINDING THE UNIT/LAST DIGIT

1. The unit digit of an expression is the same as getting the remainder when the expression is divided by 10. The last digits of the powers of any number follow a cyclic pattern – i.e. they repeat after a certain number of steps. We just need to find out after how many steps the last digit of the powers of a number repeat, then we can find the last digit of any power of any number.

Eg. Last digit of 3^{1001}

Last digit of $3^1 = 3$

Last digit of $3^2 = 9$

Last digit of $3^3 = 7$

Last digit of $3^4 = 1$

Last digit of $3^5 = 3$

Last digit of $3^6 = 9$

We see that the powers start getting repeated after 3^4 , this means that the cyclicity of powers of 3 is 4. Now we divide 1001 by 4 and get the remainder as 1, this means that the last digit of 3^{1001} will be the same as the last digit of 3^1 which is 3.

2. Sometimes, we could be asked to find the last digit of an expression containing various mathematical operations – we will adopt the same process as discussed above to get the unit digit for each of the terms and apply the mathematical operations accordingly on them, the unit digit will be the last digit of the resultant number obtained after performing all such operations.

REMAINDER THEOREM

1. Remainder theorem states that when $f(x)$, a polynomial function in x , is divided by $x - a$, the remainder is $f(a)$. A polynomial function in x is a function where x will appear only in the form of x^n and not in any other form, where n is a positive integer.

Eg. Find the remainder when the function $x^2 + 6x - 16$ is divided by $x - 2$.

Applying the above theorem here, the remainder will be the same as $f(2)$ which can be obtained by putting the value of $x = 2$ in the given expression. Putting $x = 2$, we see that the same results in 0, this means that the remainder when $x^2 + 6x - 16$ is divided by $x - 2$ is 0 or $x - 2$ is a factor of $x^2 + 6x - 16$

IMPORTANT THEOREMS RELATED TO REMAINDERS

1. **Binomial theorem:** For any natural number n ,

$$(a + b)^n = {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + \dots + {}^nC_{n-1} a^1 b^{n-1} + {}^nC_n a^0 b^n$$

where ${}^nC_r$ is the number of ways of choosing ' r ' objects out of ' n ' distinct objects and is given by ${}^nC_r = \frac{n!}{r!(n-r)!}$

2. **Fermat's little theorem:** If ' p ' is prime and $\text{HCF}(a, p) = 1$, then $a^{p-1} - 1$ is a multiple of p .
3. **Wilson's theorem:** If ' p ' is prime, $(p-1)! + 1$ is a multiple of p .
4. **Euler's theorem:** When $y^{E(z)}$ is divided by z , the remainder is 1. Here y and z must be co-prime to each other whereas $E(z)$ represents the euler number of z . Euler number is nothing but the number of co-primes to z less than z which is calculated by
$$N\left(1 - \frac{1}{a}\right)\left(1 - \frac{1}{b}\right)\left(1 - \frac{1}{c}\right)$$
5. **Chinese Remainder Theorem:** If a number $N = x * y$, where x and y are prime to each other, and K is a number such that the remainders obtained when K is divided by x and y are z_1 and z_2 respectively, then the remainder obtained when K is divided by N is the smallest value in the form of $xa + z_1$ or $yb + z_2$ such that $xa + z_1 = yb + z_2$, where ' a ' and ' b ' are non-negative integers.

FINDING THE LAST TWO DIGITS OF a^b

The last two digits of a^b are the same as the remainder when it is divided by 100. We have the following 4 cases for different values of ' a '

1. If a ends with 0, the square and all the higher powers end in at least two zeroes;
2. If a ends with 5, all the powers end with 25 when the digit at the ten's place is even or end alternatively with 25 and 75 when the digit at the ten's place is odd;

3. If the base ends in 1, 3, 7 or 9, there is a cycle of at the most 20 distinct remainders. The cycle length could also be some factor of 20 i.e. 1, 2, 4, 5 or 10;
4. If the base ends in 2, 4, 6 or 8, there is a cycle of at the most 20 distinct remainders. The twentieth power ends in 76. The cycle length could also be some factor of 20.

REMAINDER OF A NUMBER WHEN DIVIDED BY $10^n - 1$

To find the remainder when a number say 'K' is divided by $D_n = 99 \dots 9$ (n nines) or $10^n - 1$, we start at the right end of K , group the digits ' n ' at a time and add up all the groups, the result is finally divided by the divisor again to get the required remainder.

IMPORTANT RULES FOR $x^n + y^n$ & $x^n - y^n$ (x, y and n being positive integers)

1. Rules applicable to $x^n + y^n$
 - (i) When n is odd, it is divisible by $x + y$;
 - (ii) When n is odd, it is divisible by $x - y$, if $x - y$ is a factor of $2 \cdot y^n$;
 - (iii) When n is even, it is divisible by $x + y$, if $x + y$ is a factor of $2 \cdot y^n$
2. Rules applicable to $x^n - y^n$
 - (i) It is divisible by $x - y$ (irrespective of whether n is odd or even);
 - (ii) When n is even, it is also divisible by $x + y$;
 - (iii) When n is odd, it is divisible by $x + y$, if $x + y$ is a factor of $2 \cdot y^n$

SOME IMPORTANT RESULTS

1. If $x, y \geq 0$, then $\frac{x+y}{2} \geq \sqrt{xy}$
2. $\frac{x}{y} + \frac{y}{x} \geq 2$, where ' x ' and ' y ' are positive numbers
3. $\frac{w}{x} + \frac{x}{y} + \frac{y}{z} + \frac{z}{w} \geq 4$, where ' w ', ' x ', ' y ' and ' z ' are positive numbers
4. $w^4 + x^4 + y^4 + z^4 \geq 4wxyz$, where ' w ', ' x ', ' y ' and ' z ' are positive numbers
5. If $x > y$ and $z > 0$, then $x + z > y + z$,

$$x - z > y - z, xz > yz \text{ and } \frac{x}{z} > \frac{y}{z}$$

6. If $x < y$ and $z > 0$, then $\frac{x+z}{y+z} > \frac{x}{y}$

7. If $x > y$ and $z > 0$, then $\frac{x+z}{y+z} < \frac{x}{y}$

8. $|x-y| = |y-x|$

$$|x \cdot y| = |x| \cdot |y|$$

$$|x + y| \leq |x| + |y|$$

$$|x + y| \geq |x| - |y|$$

BASE SYSTEM

The most commonly used number system is the 'Decimal Number System' or the numbers with base 10 involving ten symbols namely 0, 1, 2, ..., 7, 8 and 9.

Eg. $(8213)_{10} = 8 \times 10^3 + 2 \times 10^2 + 1 \times 10^1 + 3 \times 10^0$

$(11100101)_2 = 1 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = (229)_{10}$

Base: also known as radix or scale of notation is an integer greater than 1, which decides the place value of a symbol or a digit in a number. It is the number of distinct symbols that are used in a particular number system. **Eg.** In the binary number system, we have two symbols 0 and 1, in septenary we have 7 symbols from 0 to 6, in octal we have 8 from 0 to 7.

Conversion from Decimal to any base 'N' number system

Eg. Convert $(373)_{10}$ into binary system and $(426)_{10}$ into octal system

2	373	8	426
2	186 → 1	8	53 → 2
2	93 → 0		6 → 5
2	46 → 1		
2	23 → 0		
2	11 → 1		
2	5 → 1		
2	2 → 1		
	1 → 0		

Therefore $(373)_{10} = (101110101)_2$ and $(426)_{10} = (652)_8$

PERCENTAGES

“Percent” consists of two words “Per” and “cent” which means for every hundred. The concept of percentages facilitates easier comparison of fractions by equalising the denominators of all fractions to hundred.

1. To convert a fraction or decimal into a percentage, we multiply the same by 100. Following are some important fractions and their percentage equivalents.

Fraction	Percentage Equivalent	Fraction	Percentage Equivalent
1/1	100%	1/11	9.09%
1/2	50%	1/12	8.33%
1/3	33.33%	1/13	7.69%
1/4	25%	1/14	7.14%
1/5	20%	1/15	6.66%
1/6	16.66%	1/16	6.25%
1/7	14.28%	1/17	5.88%
1/8	12.5%	1/18	5.55%
1/9	11.11%	1/19	5.26%
1/10	10%	1/20	5%

2. To convert a percentage value into fractional or decimal form, divide the same by 100

3. **Multiplying factor:** To increase a number by a given percentage, multiply the number by the factor $\frac{100 + \text{rate}}{100}$

Similarly, To decrease a number by a given percentage, multiply the number by the factor $\frac{100 - \text{rate}}{100}$

4. To find % increase in a number, Percentage increase = $\frac{\text{Actual increase}}{\text{Initial quantity}} \times 100$, Actual Increase = Final quantity – Initial quantity
5. To find % decrease in a number, Percentage decrease = $\frac{\text{Actual decrease}}{\text{Initial quantity}} \times 100$, Actual decrease = Initial quantity – Final quantity
6. If the percentage increase is $x\%$, then the new value is $\left(\frac{x}{100} + 1\right)$ times the original value
7. If the new value is ‘ a ’ times the old value, then the percentage increase/decrease is $(a - 1) \times 100$
8. If the price of an article increases by $a\%$, then the percentage reduction in consumption such that the total expenditure remains the same is given by $\left(\frac{a}{100 + a} \times 100\right)\%$
9. If the price of an article decreases by $a\%$, then the percentage increase in consumption such that the total expenditure remains the same is given by $\left(\frac{a}{100 - a} \times 100\right)\%$
10. If X ’s income is $a\%$ more than that of Y , then Y ’s income is less than that of X by $\left(\frac{a}{100 + a} \times 100\right)\%$

11. If X 's income is $a\%$ less than that of Y , then Y 's income is more than that of X by $\left(\frac{a}{100-a} \times 100\right)\%$
12. If a given quantity is first increased by $a\%$ and then increased by $b\%$, then the net increase can be calculated using the concept of successive percentages given by the formula $\left(a + b + \frac{ab}{100}\right)\%$
13. If a given quantity is first increased by $a\%$ and then decreased by $b\%$, then the net increase can be calculated using the concept of successive percentages given by the formula $\left(a - b - \frac{ab}{100}\right)\%$
14. Net percentage change of three consecutive increments of $a\%$, $b\%$ and $c\%$ is given by the formula $\left(a + b + c + \frac{ab + bc + ca}{100} + \frac{abc}{10000}\right)\%$
15. On increasing/decreasing the cost of a certain commodity by $x\%$, a person can buy ' a ' kg commodity less/more in ' y ' rupees, then the Increased/decreased cost of the commodity $= \frac{xy}{100 \times a}$ and initial cost $= \frac{xy}{(100 \pm x) \times a}$
(Negative sign when decreasing and positive sign when increasing)
16. **Percentage Points:** Percentage points is simply the difference of two percentage figures. Please note that it is different from Percentage increase/decrease.

PROFIT, LOSS AND DISCOUNT

1. **Cost Price(CP)** = The amount spent to acquire/purchase a product;

Selling Price(SP) = Revenue generated from selling the product;

If $SP > CP$, then there is a profit given by $SP - CP$

If $SP < CP$, then there is a loss given by $CP - SP$

If $SP = CP$, then there is no profit and no loss.

$$2. \text{Profit}\% = \frac{\text{Profit}}{\text{CP}} \times 100 = \frac{SP - CP}{CP} \times 100 = \left[\frac{SP}{CP} - 1 \right] \times 100$$

$$3. \text{Loss}\% = \frac{\text{Loss}}{\text{CP}} \times 100 = \frac{CP - SP}{CP} \times 100 = \left[1 - \frac{SP}{CP} \right] \times 100$$

4. **Marked Price:** The printed price of a product or the percentage by which the cost price is marked up by the shopkeeper.

$$\text{Mark-up}\% = \frac{\text{Mark-up}}{\text{CP}} \times 100 = \frac{MP - CP}{CP} \times 100 = \left[\frac{MP}{CP} - 1 \right] \times 100$$

$$5. \text{Discount} = \text{Marked Price} - \text{Selling Price}, \text{Discount}\% = \frac{\text{Discount}}{\text{Marked Price}} \times 100$$

$$6. \text{ If an object is sold for } x\% \text{ profit then, } SP = CP \left(\frac{100 + x}{100} \right) \text{ and if it sold for } x\% \text{ loss then, } SP = CP \left(\frac{100 - x}{100} \right)$$

7. On selling 'a' articles the profit or loss is equal to selling of 'y' articles, then

$$\text{Profit}\% = \frac{100y}{x - y} \text{ or loss}\% = \frac{100y}{x + y}$$

8. If the selling price of two articles is the same such that one is sold at a profit of $x\%$ and the other at a loss of $x\%$ then there is an overall loss given by $\frac{x^2}{100}\%$

9. If the cost price of two articles is the same such that one is sold at a profit of $x\%$ and the other at loss of $x\%$ then overall there is no profit no loss

10. If 'a + b' articles are sold at a cost price of 'a' articles i.e. Buy 'a' and get 'b' articles free, then the percentage discount $= \frac{b}{a + b} \times 100$

11. **Dishonest shopkeeper:** A dishonest shopkeeper sells his goods at cost price but uses false weight, then his Profit % $= \frac{\text{True weight} - \text{false weight}}{\text{False weight}} \times 100$ or $\frac{\text{Error}}{\text{True value} - \text{Error}} \times 100$

PARTNERSHIP

1. Two or more persons can get together to do business by pooling their resources. The money put in by each of the partners is called his “**Investment**” or “**Capital**” and the persons who have invested their money are known as **Partners**.
2. **Active/ Working Partner:** Invests money as well as takes part in business activity for which he is paid salary from the profit.
3. **Sleeping/ Dormant Partner:** Only invests money and does not take part in business activities.
4. The profit left after paying the working partners’ remuneration/commission is shared amongst all the partners. Sometimes, the partners also take interest on their investments and only the remaining profit is shared by the partners.
5. If the partners invest **DIFFERENT** amounts each for the same period of time, then the profit at the end of the year is shared in the ratio of their investments.
6. If the partners invest the same amounts but for **DIFFERENT** periods of time, then the profit at the end of the year is shared in the ratio of the time periods for which their respective investments have been in business
7. If the partners invest **DIFFERENT** amounts for **DIFFERENT** time periods, then the profit at the end of the year is shared in the ratio of products of (investment × time period) calculated for each partner

STOCKS AND SHARES

1. A limited company raises capital by floating shares also referred to as stock. The total capital requirement of the company is divided into small units referred to as shares. People who are holding the shares are known as shareholders.
2. **Face Value/Par Value:** A stock’s face value is the initial cost of the stock, as listed on the certificate.
3. **Market Value:** The price of a share keeps fluctuating, therefore the rate at which a share is bought or sold in the market is the market value of the share.
4. If the market value is more than the face value of the share, then we say that such a share is quoting at a “premium”
5. If the market value is less than the face value of the share, then we say that such a share is quoting at a “discount”
6. **Dividend:** Part of the profits distributed by the company to its shareholders is known as dividend, Dividend%
$$= \frac{\text{Dividend amount}}{\text{PAR Value}} \times 100$$

Dividend is always calculated on the Face value or the PAR value irrespective of the purchase price of the share.

SIMPLE AND COMPOUND INTEREST

1. If X borrows money from Y , then X is termed as the borrower and Y is the lender. The sum borrowed is known as the Principal denoted by P .
2. Money paid by the borrower for using the lender's money is known as Interest, denoted by I .
3. Time for which money is borrowed is called a period, denoted by n (n is expressed in number of periods, which is normally one year)
4. The interest paid per 100 rupees in a year is called rate percent per annum denoted by r .
5. Amount is the sum of Principal and Interest. Amount = Principal + Interest

6. **Simple Interest** = $\frac{P \cdot r \cdot n}{100}$, where P is the Principal sum borrowed, r is the rate of interest and n is the number of years for which the sum had been borrowed.

7. Compound Interest differs from Simple Interest due to what is termed as **Interest on Interest**. This means that in compound interest, interest is calculated on the interest part as well apart from the principal amount. Therefore, for a given sum, rate of interest and time period, Compound Interest is always greater than or equal to Simple Interest. The equality occurs only for the first year's interest amount.

8. Let ' P ' be the Principal kept at $r\%$ per annum compound rate of interest for a period of n years, then the Amount (A) is calculated by the formula

$$A = P \left(1 + \frac{r}{100} \right)^n$$

9. If the compounding is done ' k ' times a year, then the formula for calculating the amount remains the same, we just divide the rate of interest by ' k ' and multiply with ' k ' the number of years.
10. If the number of times compounding is done in a year is increased to infinity which means that the compounding is done **every moment**, then ' r ' being the rate of interest per annum and n as the number of years, then

$$\text{Amount} = P \cdot e^{nr/100}$$

11. Difference between Compound Interest and Simple Interest for 2 years on the same sum and rate of interest = Interest calculated for one year on one year's Simple Interest

$$= P \left(\frac{R}{100} \right)^2$$

12. Difference between Compound Interest and Simple Interest for 3 years on the same sum and rate of interest = $P \left(\frac{R}{100} \right)^2 \left(\frac{300 + R}{100} \right)$

13. Formula for calculation of Each equal Instalment under Simple Interest is given by, **Each Instalment** = $\frac{P \times 200}{n[200 + (n-1)r]}$, where P is the sum borrowed, n is the number of years and r is the rate of interest

14. Formula for calculation of Each equal Instalment under Compound Interest is given by, **Each Instalment** = $\frac{P \cdot r}{100[1 - \{100/100 + r\}^n]}$ where P is the sum borrowed, which is repaid in n equal instalments at $r\%$ per period of instalment

15. A certain sum becomes ' x ' times itself in t years on compound interest then the time it will take to become x^y times itself will be $(t * y)$ years.

16. A certain sum at compound interest becomes ' a ' times in k_1 years and ' b ' times in k_2 years, then $a^{1/k_1} = b^{1/k_2}$

RATIO, PROPORTION & VARIATION

1. The comparative relation between two amounts/quantities of the same type is called **Ratio**. Let an amount be 'a' and another be 'b', then the ratio between them is $a : b$ or $\frac{a}{b}$, here 'a' is known as the 'antecedent' and 'b' is called 'consequent'

2. If an amount 'Z' is to be divided between A and B in the ratio $a : b$ respectively, then

$$\text{Share of A} = \frac{a}{a+b} \times Z$$

$$\text{Share of B} = \frac{b}{a+b} \times Z$$

3. If $a : b :: c : d$, then a and d are called **extremes** whereas b and c are called **means**. When two ratios are equal, then the four quantities involved are said to be in **proportion**.

Therefore, product of extremes = product of means or $ad = bc$

4. If $a < b$, then $\frac{a+x}{b+x} > \frac{a}{b}$; where 'x' is a positive quantity.

5. If $a > b$, then $\frac{a+x}{b+x} < \frac{a}{b}$; where 'x' is a positive quantity.

6. If $a = b$, then $\frac{a+x}{b+x} = \frac{a}{b}$

7. If $\frac{a}{b} = \frac{c}{d}$, then

$$\frac{b}{a} = \frac{d}{c} \text{ (Invertendo)}$$

$$\frac{a}{c} = \frac{b}{d} \text{ (Alternendo)}$$

$$\frac{a+b}{b} = \frac{c+d}{d} \text{ (Componendo)}$$

$$\frac{a-b}{b} = \frac{c-d}{d} \text{ (Dividendo)}$$

$$\frac{a+b}{a-b} = \frac{c+d}{c-d} \text{ (Componendo - Dividendo)}$$

8. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} \dots\dots = k$, then

$$\frac{a+c+e\dots\dots}{b+d+f\dots\dots} = k$$

9. If $a : b :: b : c$, then a, b and c are said to be in continued proportion such that $b^2 = ac$. b is said to be the mean proportional of a and c whereas c is said to be the third proportional of a and b.

10. **Direct Variation:** If $x = ky$ where k is a constant, then we say that x is directly proportional to y. This implies that when one of x and y increases/decreases in a certain ratio then the other also increases/decreases in the same ratio. This is denoted as $x \propto y$.

11. Inverse Variation: If $x = \frac{k}{y}$ where k is a constant, then we say that x is inversely proportional to y . This implies that when one of x and y increases/decreases in a certain ratio then the other decreases/increases in the same ratio. This is denoted as $x \propto \frac{1}{y}$

12. Joint Variation: If there are three quantities x , y and z such that x varies with y when z is constant and varies with z when y is constant, then x is said to vary jointly with y and z when both y and z are varying *i.e.* $x \propto y$ when z is constant and $x \propto z$ when y is constant, therefore $x \propto yz$ or $x = kyz$ where k is the constant of proportionality.

13. Let x be a number which is added to a , b , c and d to make them proportional, then, $x = \frac{bc - ad}{(a + d) - (b + c)}$

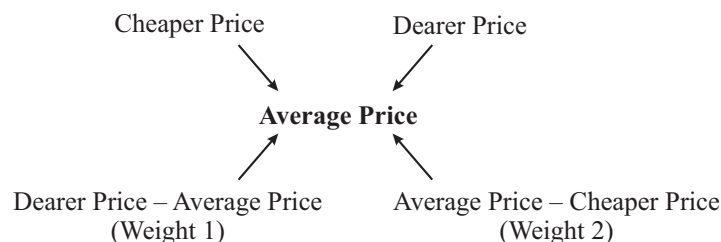
14. Let x be a number which is subtracted from a , b , c and d to make them proportional, then, $x = \frac{ad - bc}{(a + d) - (b + c)}$

MIXTURES & ALLIGATIONS

- Mixture:** Mixing two or more qualities of a particular commodity. Upon mixing, the value of the resultant mixture lies in between the values of the original constituents i.e. higher than the lowest quality but lower than the highest quality. Here, the average or the resultant value obtained is the **weighted average** of the constituents.
- If two commodities valued at P_1 and P_2 are mixed in the ratio Q_1 and Q_2 respectively, then the average price of the resultant commodity is given by $\frac{P_1Q_1 + P_2Q_2}{Q_1 + Q_2}$

The above concept can be used even when the constituent items are greater than two.

- Concentration:** Percentage of a particular quantity in the full mixture. **Eg.** If 20 litres of milk is mixed with 30 litres of water, then the concentration of milk in the mixture will be $\frac{20}{20 + 30} \times 100 = 40\%$
- Alligation:** This is one of the faster methods of calculating the weighted averages. The ratio of the weights of the two items mixed will be inversely proportional to the difference of each of these two items from the average attribute of the resultant mixture.



- If there is ' I ' volume of pure liquid initially and in each operation, R volume is taken out and replaced by R volume of water, then at the end of ' n ' such operations, the concentration (F) of the liquid in the solution is given by

$$\left(\frac{I - R}{I}\right)^n = F$$

This gives the concentration (F) of the liquid as a proportion of the total volume of the solution.

If the concentration has to be expressed in terms of percentage, then it will be equal to $100F$.

If the volume of the liquid is to be found out at the end of ' n ' operations, it is given by $F \times I$ (Concentration multiplied by the total volume)

- If from ' n ' litres of liquid P , x litres is withdrawn and same quantity of liquid R is added. Again from the mixture, y litre mixture is withdrawn and same quantity of liquid R is added. Again from the mixture, z litre mixture is withdrawn and same quantity of liquid R is added, then in final mixture,

$$\text{Liquid } P = n \left(\frac{n - x}{n}\right) \left(\frac{n - y}{n}\right) \left(\frac{n - z}{n}\right)$$

If the same process is repeated ' k ' times, then in the final mixture,

$$\text{Liquid } P = n \left(\frac{n - x}{n}\right)^k$$

Liquid $R = n - (\text{Liquid } P \text{ in the final mixture})$

- If ' a ' is initial amount of liquid. ' b ' is the amount of liquid which is drawn, and this process is repeated n -times such that the resultant mixture is in the ratio $x : y$, then,

$$\frac{x}{x + y} = \left(\frac{a - b}{a}\right)^n$$

AVERAGES

1. Average of two or more numbers/values is also known as the mean of these values, and is given by the formula

$$\text{Average} = \frac{\text{Sum of all the observations}}{\text{Total number of observations}}$$

It is an effective way of representing an entire group by a single value.

2. The average of a group of items will always lie between the smallest value in the group and largest value in the group – i.e. the average will be greater than the smallest value and lesser than the largest value in the group.
3. If the given observations (x) are occurring with certain frequency (F), then

$$\text{Average} = \frac{F_1x_1 + F_2x_2 + F_3x_3 + \dots + F_nx_n}{x_1 + x_2 + x_3 + \dots + x_n}$$

4. **Assumed Mean Method:** If there are ' n ' items and they are denoted by $F_1, F_2, F_3, \dots, F_n$ then the average of these ' n ' items is given by the formula,

$$\text{Average} = A + \frac{1}{n} \sum_{i=1}^n (F_i - A)$$

Here ' A ' is an arbitrary number or the assumed mean, further we take deviations of the given items (F_i) from the assumed mean and then calculate the average deviation. Finally, we add it to the assumed mean (A) to arrive at the correct average of the given items.

5. If the average of ' a ' numbers is ' x ' and the average of ' b ' numbers is ' y ', then the average of all the numbers combined,

$$\text{Average} = \frac{ax + by}{a + b}$$

6. The average of first ' n ' consecutive natural numbers = $\frac{n+1}{2}$

7. The average of squares of first ' n ' consecutive natural numbers = $\frac{(n+1)(2n+1)}{6}$

8. The average of cubes of first ' n ' consecutive natural numbers = $\frac{n(n+1)^2}{4}$

9. The average of first ' n ' even consecutive natural numbers = $n + 1$

10. The average of first ' n ' odd consecutive natural numbers = n

11. The average of first ' n ' multiples of a given number ' k ' = $\frac{k(1+n)}{2}$

12. The average of squares of first ' n ' even numbers = $\frac{2n(n+1)(2n+1)}{3}$

13. The average of squares of first ' n ' odd numbers = $\frac{n(n+1)(2n-1)}{3}$

14. The average of cubes of first ' n ' even numbers = $2n(n+1)^2$

15. The average of cubes of first ' n ' odd numbers = $n(2n^2 - 1)$

16. The average of consecutive numbers $a, b, c, \dots, n$ = $\frac{\text{First Number} + \text{Last Number}}{2}$

17. If a person goes from Point X to Point Y at a speed of ' a ' km/hr and then comes back from Point Y to Point X at a speed of ' b ' km/hr, then his average speed during the entire journey = $\frac{2ab}{a+b}$

18. If same distance is travelled with three different speeds of ' a ' km/hr, ' b ' km/hr and ' c ' km/hr, then the average speed during the entire journey = $\frac{3abc}{ab + bc + ca}$

19. IMPORTANT POINTS IN RESPECT OF AVERAGES

- (i) If the value of each item is increased by the same value ' x ', then the new average of all the items will also increase by ' x ';
- (ii) If the value of each item is decreased by the same value ' x ', then the new average of all the items will also decrease by ' x ';
- (iii) If the value of each item is multiplied by the same value ' x ', then the new average of all the items will also get multiplied by ' x ';
- (iv) If the value of each item is divided by the same value ' x ' ($x \neq 0$), then the new average of all the items will also get divided by ' x '.

TIME AND WORK

1. Work may be constructing a wall or a road, filling or emptying a tank, writing a certain number of pages etc. It is generally considered as one unit. Unless mentioned otherwise, if a person does some work in a certain number of days then we assume that he must have done it uniformly i.e. he must have completed the same amount of work everyday.
2. If A can do a piece of work in ' x ' days and B can do it in ' y ' days then A and B can together complete the same work in $\frac{xy}{x+y}$ days.
3. If A can do a piece of work in ' x ' days, B can do it in ' y ' days and C can do it in ' z ' days, then A , B and C can together complete the same work in $\frac{xyz}{xy + yz + zx}$ days.
4. If A is ' x ' times as efficient as B , then A would take $\frac{1}{x}$ (time taken by B) to complete the work since time and efficiency are inversely proportional to each other.
5. If A and B together can do a work in ' x ' days, B and C can do the same work in ' y ' days, C and A can do it in ' z ' days, then the total time required by A , B and C to complete the work = $\frac{2xyz}{xy + yz + zx}$ days
6. If A can complete a work in ' x ' days, whereas A and B can together complete the same work in ' y ' days, then B can complete the same work in $\frac{xy}{x-y}$ days
7. If A men or B boys or C women can complete a certain work in ' x ' days, then A_1 men, B_1 boys and C_1 women can together complete the same work in $\frac{x}{\frac{A_1}{A} + \frac{B_1}{B} + \frac{C_1}{C}}$ days.
8. If A can complete a work in ' x ' days and B is ' $M\%$ ' more efficient than A , then B can complete the same work in $\frac{x}{100+M} *$ 100 days
9. If A working alone takes ' x ' days more than A and B together and B working alone takes ' y ' days more than A and B together, then the number of days in which A and B can complete the work together = $\sqrt{xy}$ days.
10. If M_1 men can do W_1 work in D_1 days working H_1 hours per day and M_2 men can do W_2 work in D_2 days working H_2 hours per day, then

$$\frac{M_1 D_1 H_1}{W_1} = \frac{M_2 D_2 H_2}{W_2}$$
11. **Wages:** Money received upon completion of a work. Generally, the distribution of the wages is done in the ratio of total work completed by each of the members.
12. If A can complete a certain work in ' x ' days and B can do the same work in ' y ' days. They complete a work together and receive ' W ' as the total wages, then

$$\text{Share of } A = \frac{y}{x+y} * W$$

$$\text{Share of } B = \frac{x}{x+y} * W$$
13. If a pipe can fill a tank in ' x ' hours but it takes ' t ' more hours to fill the tank due to a leakage in the tank, then the time required by the leakage to empty a completely filled tank = $\frac{x(x+t)}{t}$ hours.
14. If a Pipe A can fill a tank in ' x ' hours, the same tank can be filled by Pipe B in ' y ' hours. Both the pipes are opened together, then the time after which Pipe B should be closed so that the tank is full in ' t ' hours is given by $\left[y \left(1 - \frac{t}{x} \right) \right]$ hours.

TIME, SPEED AND DISTANCE

- Distance = Speed $\times$ Time
- If two objects travel with the same speed, then distance covered is directly proportional to time taken.
- If two objects travel for the same period of time, then distance covered is directly proportional to speed.
- If two objects travel for the same distance, then time is inversely proportional to speed
- To convert speed in km/hr to m/sec, multiply it by $\frac{5}{18}$.
- When a train passes a stationary object, the distance covered in passing is equal to the length of the train.
- When a train crosses a platform or a bridge, the distance covered by it in crossing is the sum of the lengths of the train and that of the platform/bridge as the case may be;
- If two trains pass each other (travelling in same or in opposite directions), the total distance covered in crossing or overtaking, as the case may be, is equal to the sum of the lengths of two trains;
- When a train passes a person sitting in another moving train, the distance covered by the train is equal to its own length.
- From stations X and Y , two trains start travelling towards each other at speeds of ' x ' and ' y ' respectively. When they met each other, it was found that one of the trains had covered distance ' d ' more than the other train. Then, the distance between the stations X and Y is given by $\frac{d}{x-y} (x+y)$
- Excluding stoppages, the average speed of a train is ' x ' kmph and with stoppages its average speed is ' y ' kmph ($x > y$ and $x, y \neq 0$).
Then, the stoppage time per hour is given by $= \frac{\text{Difference in average speeds}}{\text{Speed without stoppage}} = \frac{x-y}{x}$
- Average Speed = $\frac{\text{Total Distance travelled}}{\text{Total time taken}}$
- If an object travels from Point X to Point Y with a speed of ' a ' and back to Point Y with a speed of ' b ', then the average speed of the object is given by $\frac{2ab}{a+b}$ irrespective of the distance between the two points
- Similarly, if the object covers a part of the journey at speed ' a ' and the remaining part of the journey at speed ' b ' and the distances covered in the two parts of the journey is $x : y$, then the average speed for the entire journey is given by $\frac{(x+y)ab}{bx+ay}$
- In general, if a person travelling between two points reaches ' a ' hours late travelling at a speed of ' x ' kmph and reaches ' b ' hours early travelling at ' y ' kmph, then the distance between the two points is given by $\frac{yx(a+b)}{y-x}$
- If X and Y start walking towards each other. After meeting each other, X covers his remaining distance in t_1 time and Y covers his remaining distance in t_2 time, then the ratio of speeds of X and Y is given by $\sqrt{\frac{t_2}{t_1}}$
If they meet each other after ' t ' hours, then $t = \sqrt{\frac{t_1}{t_2}}$
- Relative Speed:** The speed of one moving object in relation to another moving object is known as the relative speed of these two bodies i.e. the speed of one moving object as observed, from the other moving object.
- If the two objects are moving in the same direction, then the relative speed is given by the difference in speeds of the two objects
- If the two objects are moving in opposite directions, then the relative speed is given by the sum of the speeds of both the objects
- When two persons are running around a circular track in the **same direction**, then
 - The relative speed is equal to the difference of the speeds of the persons
 - From one meeting point to the next meeting point, the faster person covers one complete round more than the slower person

21. When two persons are running around a circular track but in opposite directions, then
- The relative speed is equal to the sum of the speeds of the two persons;
 - From one meeting to the next meeting point, both of them together cover a distance equal to the length of the track.
22. If two persons X and Y with respective speeds of ' x ' and ' y ' ($x > y$) are running around a circular track of length L starting at the same point and at the same time. Then, the time taken to meet for the **first time ever** is given by $\frac{L}{x-y}$ in case the two persons are running in the **same direction** and $\frac{L}{x+y}$ in case both the persons are running in **opposite** directions.
23. If two persons X and Y with respective speeds of ' x ' and ' y ' ($x > y$) are running around a circular track of length L starting at the same point and at the same time. Then, the time taken to meet for the first time at the starting point is given by LCM of $\left\{\frac{L}{x}, \frac{L}{y}\right\}$ in case of both the **same direction and opposite directions**.
24. If three persons X , Y and Z with respective speeds of ' x ', ' y ' and ' z ' ($x > y > z$) are running around a circular track of length L starting at the same point at the same time in **same directions**, then, time taken to meet for the **first time ever** is given by LCM of $\left\{\frac{L}{x-y}, \frac{L}{y-z}\right\}$ and the time taken to meet for the **first time at the starting point** is given by LCM of $\left\{\frac{L}{x}, \frac{L}{y}, \frac{L}{z}\right\}$.
- The above logic can accordingly be extended when the directions of the persons are altered.
25. If in a race of length L , the time taken by X and Y be t_x and t_y respectively such that $t_y > t_x$, then the distance (d) by which X beats Y is given by $\frac{L}{t_y} (t_y - t_x)$
26. If in a race of length L , X can give Y , a start of ' y ' and Z , a start of ' z ' such that $z > y$, then the start that Y can give Z is given by $L \left(\frac{Z-y}{L-y} \right)$
27. If X gives Y a start of distance ' d ' and still beats him by time ' t ' in a race of length L , then Y 's speed is given by
- $$\frac{\text{Distance covered by } Y}{\text{Total time taken by } Y} = \frac{L-d}{\frac{L}{\text{Speed of } X} + t}$$
28. **Minute hand** covers 360° in 1 hour, i.e. in 60 minutes it covers 360° or every minute it covers 6° . **Hour hand** covers 360° in 12 hours i.e. in 720 minutes it covers 360° or every minute it covers 0.5°
29. In a period of 12 hours, the hands make an angle of
- 0° (coincide) with each other exactly 11 times and hence the time gap between two successive coincidences is $\frac{12}{11}$ hours or $65 \frac{5}{11}$ minutes;
 - 180° (lie in the same straight line) exactly 11 times;
 - 90° or any other angle with each other exactly 22 times.
30. Let the speed of a boat in still water = ' x ' kmph and the speed of the stream = ' y ' kmph. The speed of the boat in the same direction of the stream or downstream (D) is given by $x+y$ and the speed of the boat in the opposite direction of the stream or upstream (U) is given by $x-y$.
31. The speed of the boat in still water (x) = $\frac{D+U}{2}$
32. The speed of the stream (y) = $\frac{D-U}{2}$

33. If the speed of a boat in still water is 'x' kmph and the speed of the stream is 'y' kmph then the average speed in going to a certain point and coming back to the starting point is given by $\frac{(x+y)(x-y)}{x}$
34. If a boat takes same time to travel 'd₁' km downstream and 'd₂' km upstream, then, $\frac{\text{Speed of the boat}}{\text{Speed of the stream}} = \frac{d_1 + d_2}{d_1 - d_2}$
35. A boat travels a certain distance upstream in t₁ hours and the same distance downstream in t₂ hours, then $\frac{\text{Speed of the boat}}{\text{Speed of the stream}} = \frac{t_1 + t_2}{t_1 - t_2}$

LINEAR EQUATIONS

- In general, if the number of equations is less than the number of variables, then we say that the variables are indeterminate or we have an indeterminate system of equations.
- But, there may be some exceptions to the above rule as listed below:
 - If the ratio of the coefficients of two variables in one equation is the same as the ratio of the coefficients of the same two variables in the second equation – then the value of the third (remaining) variable can uniquely be determined whereas the other two variables will have infinite solutions;
 - If some additional conditions are either implicitly built into the problem or explicitly stated by specifying some constraints on the values of the variables;
 - At times, it may be possible to find out the value of some specific combinations of variables even though we cannot individually determine the values of these variables;
 - If the equations are **dependent** – at times, even if the number of equations are equal to the number of variables, we may not be able to uniquely determine the values of the variables i.e. one of the given equations can be written as a linear combination of the other two;
 - If the equations are **inconsistent** – Let $3x + 4y = 9$, then the value of $24x + 32y$ has to be 72 and it cannot take any other value. If some other value is given, there will be inconsistency in the data because that would be tantamount to saying $3x + 4y = 9$ and $3x + 4y \neq 9$ at the same time;
- Let there be two equations such that $ax + by = z$ and $mx + ny = p$ where a, b, z, m, n, p are constants. Then,
 - If $\frac{a}{m} = \frac{b}{n} = \frac{z}{p}$, then the equations are represented by the same line and thus will have **infinite number of solutions**;
 - If $\frac{a}{m} = \frac{b}{n} \neq \frac{z}{p}$, then the equations are represented by parallel lines and thus will have **no solutions**;
 - If $\frac{a}{m} \neq \frac{b}{n}$, then the equations are represented by intersecting lines and thus will have a **unique solution**.

QUADRATIC EQUATIONS

- An algebraic expression of the form $a_0 + a_1x + a_2x^2 + a_3x^3 \dots + a_nx^n$, where $n \in \mathbb{N}$, is called a polynomial, generally denoted by $f(x)$;
- The degree of polynomial $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 \dots + a_nx^n$ is 'n' such that $a^n \neq 0$. A polynomial of second degree is known as a quadratic polynomial. A quadratic polynomial when equated to zero is called a quadratic equation
- Standard form** of a quadratic equation is $ax^2 + bx + c = 0$, where $a \neq 0$, and a, b, c are real;
- The values of variable x which satisfy the quadratic equation are called the **roots** of the quadratic equation.
- The roots of the quadratic equation can be found out either by factorisation method or by using the standard formula.
 - Factorisation Method:** Let a quadratic equation $ax^2 + bx + c = 0$ be written in the form of $(x - \alpha)(x - \beta) = 0$, then the roots/solutions of the quadratic equation will be α and β ;

- (ii) **Standard Formula:** Let a quadratic equation be $ax^2 + bx + c = 0$, then its roots (α and β) can be found out using the below formula;

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-b + \sqrt{D}}{2a};$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-b - \sqrt{D}}{2a};$$

Where 'D' is known as the discriminant of the equation.

6. For a quadratic equation $ax^2 + bx + c = 0$, we have,

$$\text{Sum of the roots} = \frac{-b}{a} \text{ and}$$

$$\text{Product of the roots} = \frac{c}{a}$$

7. If 's' is the sum of the roots and 'p' is the product of the roots of a quadratic equation, then the equation can be written as:
 $x^2 - (s)x + p = 0$

8. Nature of the roots:

For a standard quadratic equation $ax^2 + bx + c = 0$

- (i) If $b^2 - 4ac > 0$, then the roots of the quadratic equation will be real and unequal
- (ii) If $b^2 - 4ac = 0$, then the roots of the quadratic equation are real and equal
- (iii) If $b^2 - 4ac < 0$, then the roots of the quadratic equation are complex conjugates.

Nature of the roots when a, b and c are all rational

- (i) If $b^2 - 4ac > 0$ and a perfect square, then the roots are rational and unequal;
- (ii) If $b^2 - 4ac > 0$ but not a perfect square, then the roots are irrational and unequal;
- (iii) If $b^2 - 4ac = 0$, then the roots are rational and equal;
- (iv) If $b^2 - 4ac < 0$, then the roots are complex and unequal.

Whenever the roots of a quadratic equation are irrational such that a, b, c are rational, they will be of the form $x + \sqrt{y}$ and $x - \sqrt{y}$ i.e. they will always occur in conjugate pairs.

9. Let there be a quadratic equation $ax^2 + bx + c = 0$ with α and β as its roots

- (i) A new quadratic equation whose roots are 'm' more than the roots of the equation $ax^2 + bx + c = 0$ can be formed by substituting $(x - m)$ in place of 'x' in the said equation;
- (ii) A new quadratic equation whose roots are 'm' less than the roots of the equation $ax^2 + bx + c = 0$ can be formed by substituting $(x + m)$ in place of 'x' in the said equation;
- (iii) A new quadratic equation whose roots are 'm' times the roots of the equation $ax^2 + bx + c = 0$ can be formed by substituting $\frac{x}{m}$ in place of 'x' in the said equation;
- (iv) A new quadratic equation whose roots are ' $\frac{1}{m}$ ' times the roots of the equation $ax^2 + bx + c = 0$ can be formed by substituting 'mx' in place of 'x' in the said equation;
- (v) A new quadratic equation whose roots are reciprocals of roots of the equation $ax^2 + bx + c = 0$ can be formed by substituting $\frac{1}{x}$ in place of 'x' in the said equation; In such a newly formed equation, the coefficient of x^2 and the constant term get interchanged i.e. the new equation will look like $cx^2 + bx + a = 0$

10. Minimum and Maximum value of a quadratic expression $ax^2 + bx + c$

- (i) Whenever $a > 0$, we have the minimum value of the expression at $x = \frac{-b}{2a}$ and the minimum value can be obtained by using the formula $\frac{4ac - b^2}{4a}$;
- (ii) Whenever $a < 0$, we have the maximum value of the expression at $x = \frac{-b}{2a}$ and the maximum value can be obtained by using the formula $\frac{4ac - b^2}{4a}$.

11. For an equation of degree ' n ',

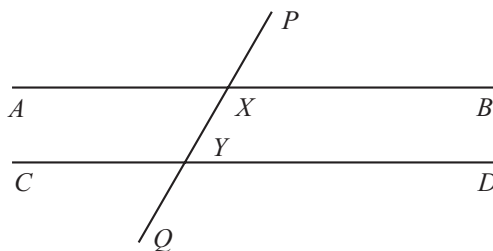
- (i) Sum of all the roots = $-\frac{\text{coefficient of } x^{n-1}}{\text{coefficient of } x^n}$
- (ii) Sum of roots taken two at a time = $\frac{\text{coefficient of } x^{n-2}}{\text{coefficient of } x^n}$
- (iii) In general, sum of roots taken ' k ' (where $k \leq n$) at a time = $(-1)^k \frac{\text{coefficient of } x^{n-k}}{\text{coefficient of } x^n}$

12. Descartes' Rule: The number of positive roots of an equation say $f(x) = 0$ is at the most equal to the number of sign changes. If there are ' n ' sign changes in $f(x)$, then the number of positive roots can be ' n ', ' $n-2$ ', ' $n-4$ '...

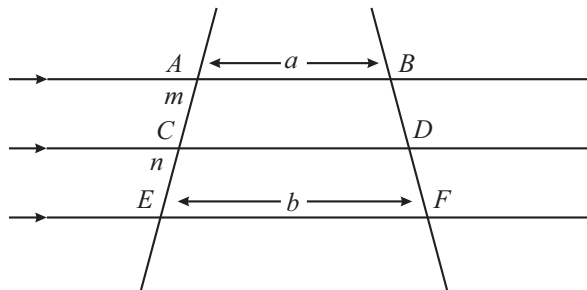
13. The number of negative roots of $f(x) = 0$ is equal to the number of positive roots of $f(-x) = 0$

LINES AND ANGLES

- Point:** A circle of zero radius;
- Collinear Points:** If three or more points lie on the same straight line, such points are said to be collinear;
- Line:** It is made up of an infinite number of points and has only length *i.e.* it does not have any thickness or width. It is endless as it can be extended in both directions. **Line Segment** – It has both the end points. **Ray** – It has one only end point.
- Intersecting lines:** Two or more lines which share exactly one common point are known as Intersecting lines. The common point is called the point of intersection.
- Parallel lines:** Two straight lines are parallel if they lie in the same plane and do not intersect even when produced indefinitely. The perpendicular distance between two parallel lines remains constant at all the places.
- Perpendicular lines:** If two lines intersect at right angles, then such lines are said to be perpendicular to each other.
- Transversal:** A line which intersects two or more given lines at distinct points is called a transversal of the given lines.
- Angle:** A figure formed by the intersection of two rays having a common endpoint. The rays are known as the arms of the angle whereas the common endpoint is known as the vertex.
 - Acute angle:** Angles measuring less than 90°
 - Obtuse angle:** Angles measuring more than 90° but less than 180°
 - Right angle:** Angles measuring 90°
 - Straight angle:** Angles measuring 180°
 - Reflex angle:** Angles measuring more than 180° but less than 360°
- Complementary Angles:** If the sum of two angles is equal to 90° , then they are complementary to each other.
- Supplementary Angles:** If the sum of two angles is 180° , then they are supplementary to each other.
- Linear Pair:** Pair of adjacent angles that are supplementary.



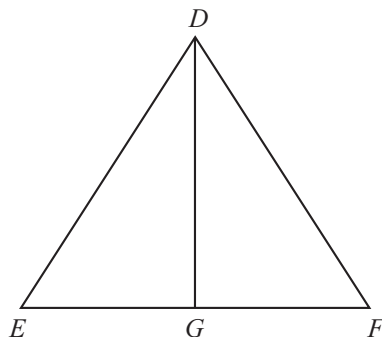
- In the above figure, there are two straight lines AB and CD , which are parallel to each other. Transversal PQ intersects these two lines at points X and Y respectively, then,
 - $\angle AXY = \angle XYD$ and $\angle BXY = \angle XYC$ (Alternate angles are equal);
 - $\angle PXA = \angle XYC$; $\angle PXB = \angle XYD$; $\angle AXY = \angle CYQ$; $\angle BXY = \angle DYQ$ (Corresponding angles are equal);
 - $\angle AXY + \angle XYC = 180^\circ$ and $\angle BXY + \angle XYD = 180^\circ$ (Sum of interior angles on the same side of the transversal is equal to 180°)
 - $\angle PXA + \angle CYQ = 180^\circ$ and $\angle PXB + \angle DYQ = 180^\circ$ (Sum of exterior angles on the same side of the transversal is equal to 180°)
 - $\angle PXA = \angle BXY$; $\angle PXB = \angle AXY$; $\angle XYC = \angle DYQ$; $\angle XYD = \angle CYQ$ (Vertically opposite angles are equal)
- If three or more parallel lines make intercepts on a transversal in a certain proportion, then they make intercepts in the same proportion on any other transversal as well.



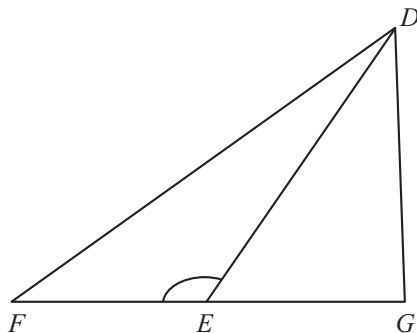
In the figure above, $\frac{AC}{CE} = \frac{BD}{DF} = \frac{m}{n}$ and $CD = \frac{an+bm}{m+n}$

TRIANGLES

- Triangle:** A three sided polygon with sum of all three angles equal to 180°
- If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles
 - 3. In a triangle, the angle between the internal bisector of a base angle and the external bisector of the other base angle is half the remaining vertex angle.
 - 4. The angle between the perpendicular drawn by a vertex to opposite side and angle bisector of the vertex angle is half of the difference between the other two remaining vertex angles.
- Sum of any two sides of a triangle is always greater than the third side or the difference of any two sides of a triangle is always lesser than the third side.
- Angles opposite to the equal sides of a triangle are equal. The angle opposite to the greater side is always greater than the angle opposite to the smaller side.
- Types of triangles:**
 - Equilateral triangle:** All three sides are equal with each angle equal to 60°
 - Isosceles triangle:** Two sides are equal to each other.
 - Scalene triangle:** All the sides are unequal.
 - Acute angled triangle:** All three angles are less than 90°
 - Obtuse angled triangle:** Any one angle is greater than 90°
 - Right angled triangle:** One angle is equal to 90°
- Pythagoras theorem:** In a right angled triangle, the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides. Some of the most frequently used Pythagorean triplets in the examinations are (3, 4, 5), (5, 12, 13) (7, 24, 25), (8, 15, 17), (9, 40, 41), (11, 60, 61), (1, 1, $\sqrt{2}$ – Represents a 45, 45, 90 triangle), (1, $\sqrt{3}$, 2 – Represents a 30, 60, 90 triangle)

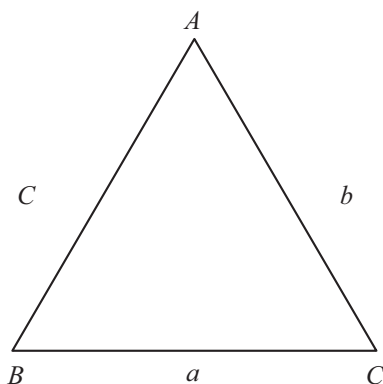


- In an **acute angled triangle**, the square of the side opposite the acute angle is less than the sum of the squares of the other two sides by a quantity equal to twice the product of one of these two sides and the projection of the second side on the first side. Kindly refer to the figure above. $DF^2 = DE^2 + EF^2 - 2 EF.EG$



10. In an **obtuse angled triangle**, the square of the side opposite the obtuse angle is greater than the sum of the squares of the other two sides by a quantity equal to twice the product of one of the sides containing the obtuse angle and the projection of the second side on the first side. Kindly refer to the figure above.

$$DF^2 = DE^2 + EF^2 + 2 EF \cdot EG$$

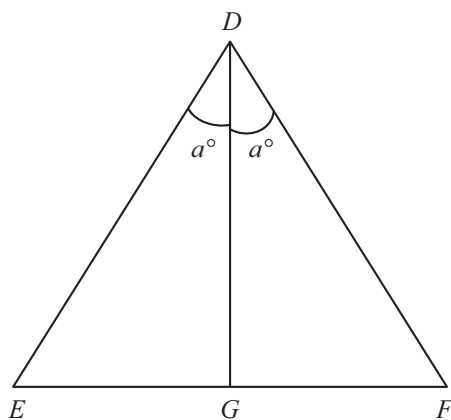


11. **Sine Rule:** Ratio of side and sine of opposite angle of a triangle is equal to double the circumradius.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R \text{ (Refer to the above figure)}$$

12. **Cosine Rule:** If two sides and angle between the sides is given, then we can find the length of the opposite side by Cosine Rule.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}, \cos B = \frac{a^2 + c^2 - b^2}{2ac}, \cos C = \frac{a^2 + b^2 - c^2}{2ab} \text{ (Refer to the above diagram)}$$

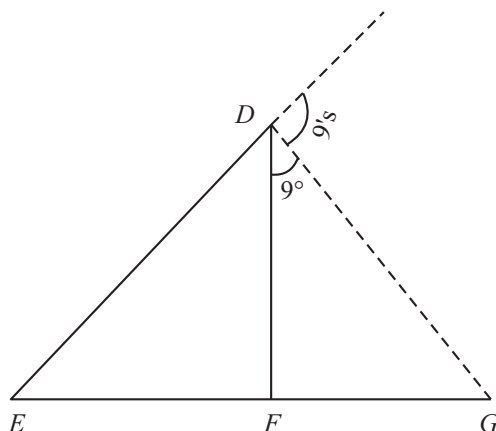


13. **Internal Angle bisector theorem:** In a triangle, the internal bisector of an angle intersects the opposite side in the ratio of the other two sides. Kindly refer to the above figure, in triangle DEF , DG is the angular bisector of angle D , then

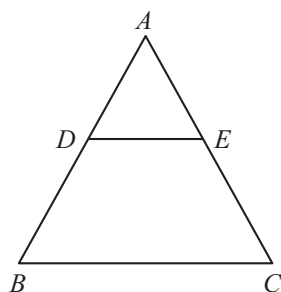
$$\frac{EG}{GF} = \frac{DE}{DF}$$

Length of the angle bisector (DG) is given by the following formula

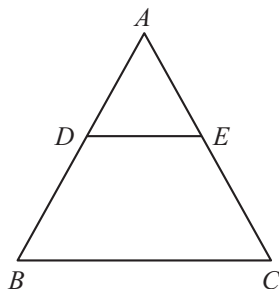
$$DG^2 = DE \times DF - EF \times GF$$



- 14. External Angle bisector theorem:** The external bisector of an angle of a triangle divides the opposite side externally in the ratio of the sides containing the angle. Kindly refer to the above figure. $\frac{EG}{GF} = \frac{DE}{DF}$



- 15. Apollonius Theorem:** Kindly refer to the above figure. In triangle DEF , DG is the median from D to EF bisecting EF into two equal parts ($EG = GF$), then,
 $2(DG^2 + EG^2) = DE^2 + DF^2$



- 16. Basic Proportionality Theorem:** A line drawn parallel to one side of a triangle divides the other two sides in the same ratio or If a line divides any two sides of a triangle in the same ratio then the line must be parallel to the third side. Kindly refer to the figure above. Here,

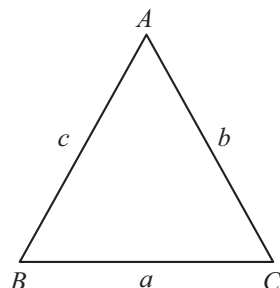
- (i) $\frac{AD}{DB} = \frac{AE}{EC}$
- (ii) $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$
- (iii) $\triangle ADE \sim \triangle ABC$

$$(iv) \frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle ABC)} = \left(\frac{AD}{AB}\right)^2 = \left(\frac{AE}{AC}\right)^2 = \left(\frac{DE}{BC}\right)^2$$

(v) **Mid – Point Theorem** is just an extension of Basic Proportionality Theorem (BPT).

17. Area of Triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$. Any of the sides of a triangle can be taken as base and the perpendicular from the opposite vertex to that side is the height.

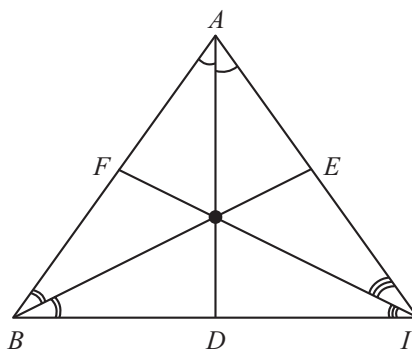
(i) **Area of Triangle** = $\sqrt{s(s-a)(s-b)(s-c)}$, where s = semi-perimeter of the triangle and a, b, c are the lengths of the sides of the triangle.



(ii) **Area of triangle** = Half the product of two sides and sine of included angle. Kindly refer to the figure above. Area of triangle ABC is given by

$$\frac{1}{2} bc \sin A = \frac{1}{2} ac \sin B = \frac{1}{2} ab \sin C.$$

18. Geometric Centres of a triangle.



(i) **In centre:** Refer to the above figure.

(a) The point of intersection of the internal bisectors of the angles of a triangle is known as the in centre. Here 'I' is the in centre.

$$\angle BIC = 90 + \frac{\angle A}{2}, \angle AIC = 90 + \frac{\angle B}{2}, \angle AIB = 90 + \frac{\angle C}{2}$$

(b) In centre divides each angle bisector in the ratio of length of sum of two adjacent sides and the opposite side.

$$AI : ID = \frac{AB + AC}{BC}; BI : IE = \frac{AB + BC}{AC}; CI : IF = \frac{AC + BC}{AB}$$

(c) The radius of the in circle of such a triangle is known as the in radius and is denoted by r .

In radius of a triangle = $\frac{\text{Area of the triangle}}{\text{Semi - Perimeter}}$ or Area of triangle = rs , where ' r ' is the in radius and ' s ' is the semi-perimeter

(d) In radius of a right angled triangle = $\frac{\text{Perpendicular} + \text{Base} - \text{Hypotenuse}}{2}$

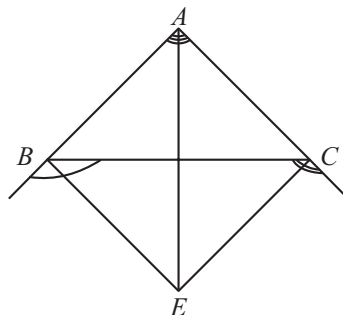
(e) In radius of an equilateral triangle = $\frac{\text{Side}(a)}{2\sqrt{3}}$

(f) In radius is less than half of any of the three altitudes of the triangle.

(g) If altitudes h_1 , h_2 , and h_3 are given, then,

$$\frac{1}{r} = \frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3}$$

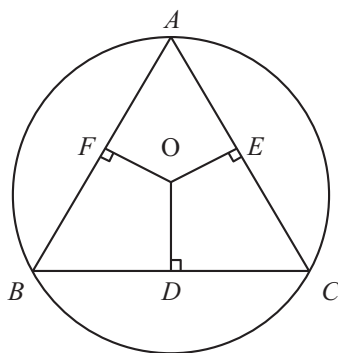
(h) Distance between Incentre and orthocentre in a right triangle is given by $\sqrt{2} r$ where 'r' is the inradius.



(ii) **Excentre – Refer to the above figure.**

(a) The point of intersection of the internal bisector of one of the angles and the external bisectors of the other two opposite angles is known as the Excentre.

(b) $\angle BEC = 90 - \frac{\angle A}{2}$



(iii) **Circumcentre – Refer to the above figure.**

(a) The point of intersection of the perpendicular bisectors of the sides of a triangle is known as the circumcentre. Here 'O' is the circumcentre such that

$OD \perp BC$ and $BD = DC$, $OE \perp AC$ and $AE = EC$, $OF \perp AB$ and $AF = FB$

(b) $\angle BOC = 2A$, $\angle COA = 2\angle B$, $\angle AOB = 2\angle C$

(c) In an acute triangle, the circumcentre lies inside the triangle.

In an obtuse triangle, the circumcentre lies outside the triangle and is at the front of the angle which is obtuse.

In a right angled triangle, the circumcentre lies at the mid-point of the hypotenuse.

(d) The radius of the circumcircle of a triangle is called the circumradius and is denoted by R .

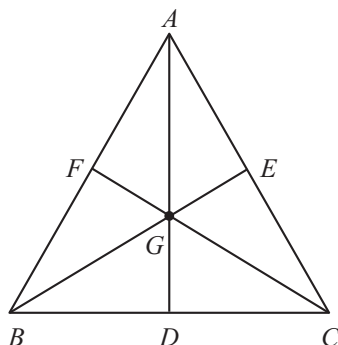
$$\text{Circumradius of a triangle} = \frac{\text{Product of all the sides}}{4 \times \text{Area of the triangle}}$$

(e) Circumradius of a right-angled triangle = $\frac{\text{Hypotenuse}}{2}$

(f) Circumradius of an equilateral triangle = $\frac{\text{Side}(a)}{\sqrt{3}}$

(g) $P + B = 2(r + R)$, where 'P' and 'B' are respectively the perpendicular and base of a right angled triangle whereas 'r' and 'R' represent the inradius and circumradius of the right angled triangle.

(h) The distance between the incentre and the circumcentre of any triangle can be calculated using the formula $\sqrt{R^2 - 2Rr}$



(iv) **Centroid - Refer to the above figure.**

(a) The point of intersection of the medians of a triangle is known as the Centroid. It is denoted by 'G'.

(b) $\frac{AG}{GD} = \frac{BG}{GE} = \frac{CG}{GF} = \frac{2}{1}$

(c) When the three medians of a triangle are drawn, the resulting six triangles are equal in area and the area of each such triangle in turn is equal to one-sixth of the area of the original triangle.

(d) In a right angled triangle, the length of the median drawn to the hypotenuse is equal to half the hypotenuse.

(d) The sum of any two sides of the triangle is greater than twice the median drawn to the third side.

$$AB + AC > 2 AD; AC + BC > 2 CF; AB + BC > 2 BE$$

Adding up all the above three inequalities, we have,

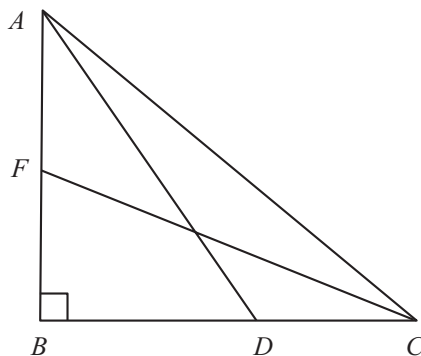
$$2(AB + AC + BC) > 2(AD + CF + BE)$$

$$(AB + AC + BC) > (AD + CF + BE)$$

Thus, we can conclude that, **Perimeter of a triangle is always greater than the sum of lengths of all the medians drawn to it.**

(e) $\frac{AB^2 + BC^2 + AC^2}{AD^2 + BE^2 + CF^2} = \frac{4}{3}$

(f) $2(AD^2 + BD^2) = AB^2 + AC^2$ (This will follow for all the medians in that order).

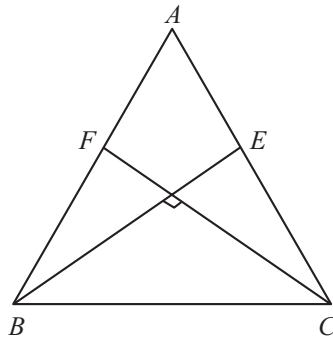


(g) Let ABC be a Right angled triangle such that AD and CF are the medians Refer to the above figure, then,

$$4(AD^2 + CF^2) = 5 AC^2;$$

$$AD^2 + CF^2 = 5 DF^2;$$

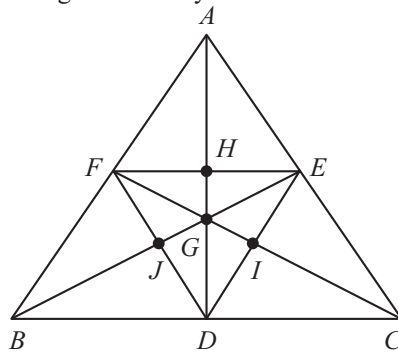
$$AD^2 + CF^2 = AC^2 + DF^2$$



(h) Let ABC be a triangle with BE and CF as its medians meeting each other at right angles. Refer to the above figure, then, $AB^2 + AC^2 = 5 BC^2$

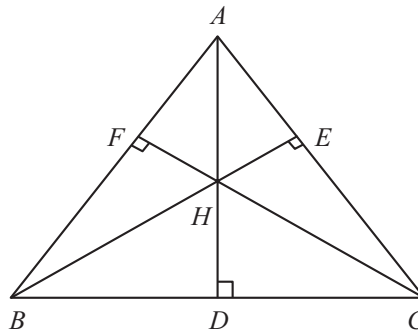
(i) Area of the triangle formed by joining the mid-points of any two sides and the centroid is $\frac{1}{12}$ th the area of the triangle.

(j) Area of triangle = $\frac{4}{3}$ (Area of the triangle formed by the medians as sides)



(k) The line segment joining the mid-points of any two sides divides the line joining the vertex in between line to the centroid in the ratio 3 : 1. Refer to the above figure.

Here, $AH : HG = BJ : JG = CI : IG = 3 : 1$



(v) **Orthocentre – Refer to the above figure.**

(a) The point of intersection of the altitudes of a triangle is called the orthocentre. It is generally denoted by ' H '.

(b) In an acute-angled triangle, the orthocentre lies inside the triangle

In an obtuse-angled triangle, it lies outside the triangle and is at the back of the angle which is obtuse

In a right angled triangle, it is at the vertex which is right angled.

(c) $\angle BHC = 180 - \angle A$, $\angle CHA = 180 - \angle B$, $\angle AHB = 180 - \angle C$

(d) $\angle CHE = \angle BHF = \angle A$

$\angle CHD = \angle AHF = \angle B$

$\angle BHD = \angle AHE = \angle C$

$$(e) AH \times HD = BH \times HE = CH \times HF$$

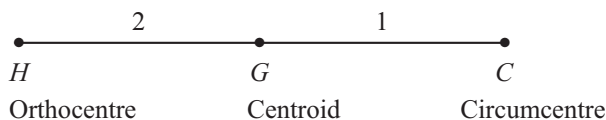
$$(f) AD \times HD = BD \times DC$$

$$BE \times HE = CE \times EA$$

$$CF \times HF = AF \times FB$$

(g) Sum of the lengths of the sides is greater than the sum of the altitudes i.e.

$$AB + BC + CA > AD + BE + CF$$



(h) In any triangle, the orthocentre, centroid and circumcentre are collinear and the centroid divides such line in the ratio of 2 : 1 (Refer to the above figure).

1. In an isosceles triangle, the centroid, the orthocentre, the circumcentre and the incentre, all lie on the median to the base. Let the equal sides of an isosceles triangle be 'a' and the unequal side be 'b', then,

$$\text{Area of the triangle} = \frac{b}{4} \sqrt{4a^2 - b^2}$$

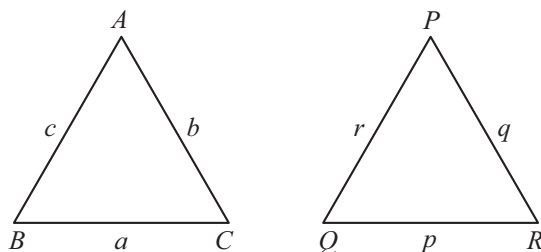
$$\text{Length of the Median} = \frac{\sqrt{4a^2 - b^2}}{2} \quad (\text{Median is drawn to the base 'b'})$$

2. In an equilateral triangle, the centroid, the orthocentre, the circumcentre and the incentre, all coincide. Let each side of an equilateral triangle be 'a', then,

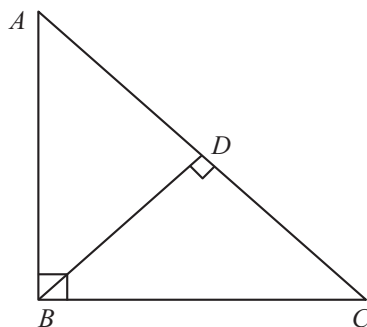
$$\text{Area of the triangle} = \frac{\sqrt{3}}{4} a^2$$

$$\text{Length of the median} = \frac{\sqrt{3}a}{2}$$

3. Similarity of triangles



- (i) Two triangles are said to be similar if the three angles of one triangle are equal to the three angles of the other triangle. Similar triangles are alike in shape only i.e. though the corresponding angles of two similar triangles are equal but the corresponding sides are only proportional and not equal.
- (ii) In the above figure, let $\triangle ABC \sim \triangle PQR$, then,
 - (a) $\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$
 - (b) Ratio of corresponding sides = Ratio of Perimeters = Ratio of Corresponding medians = Ratio of inradii = Ratio of circumradii = Ratio of heights = Ratio of the lengths of angular bisectors
 - (c) Ratio of areas = (Ratio of Corresponding sides)²
- (iii) Following are the conditions in which similarity of triangles can occur:
 - (a) If all three angles of one triangle are equal to all three angles of the other triangle;
 - (b) If all three sides of one triangle are proportional to all three sides of the other triangle;
 - (c) If two sides of one triangle are proportional to the two sides of the other triangle and the included angles are equal.



(iv). In the above figure, ABC is a right triangle right angled at B , whereas BD is the perpendicular drawn from B to the hypotenuse AC , then we have,

(a) $\triangle ABC \sim \triangle ADB \sim \triangle BDC$

(b) $BD^2 = AD \times DC$

$$BC^2 = AC \times CD$$

$$AB^2 = AC \times AD$$

$$AB \times BC = BD \times AC$$

$$\frac{1}{BD^2} = \frac{1}{AB^2} + \frac{1}{BC^2}$$

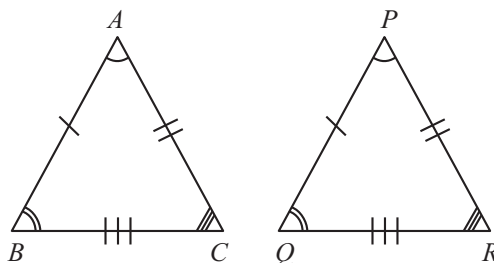
4. Congruency of Triangles

(i) Two triangles are said to be congruent if

(a) The corresponding sides are equal;

(b) The corresponding angles are equal

(c) The areas of both the triangles are equal.



In the above figure, $\triangle ABC \cong \triangle PQR$

(ii) Following are the conditions for congruency of triangles;

(a) SSS (side–side–side) – Corresponding sides are equal;

(b) SAS (side–angle–side) – Two sides and the included angle are equal;

(c) ASA (angle–side–angle) – Two angles and corresponding sides are equal;

(d) RHS (Right angle–hypotenuse–side) – Hypotenuse and one side of both the triangles are equal.

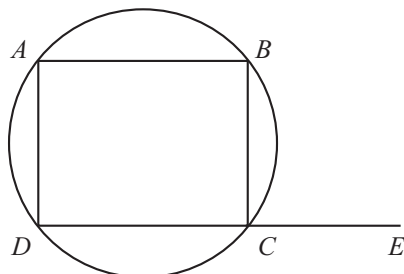
5. Mass Point Geometry



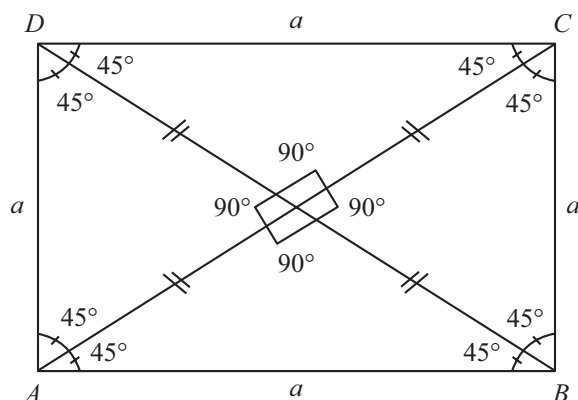
$$\frac{M_1}{M_2} = \frac{L_2}{L_1}$$

QUADRILATERALS

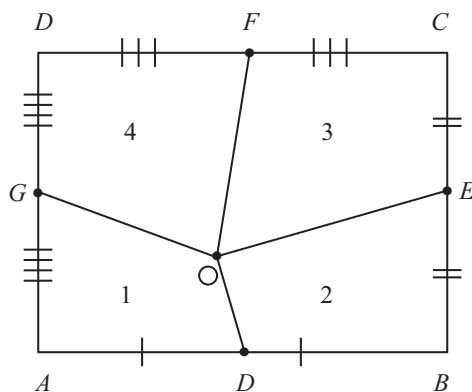
1. Quadrilateral is a four-sided closed figure. Various figures such as Rectangle, Square, Parallelogram, Trapezium etc. can be obtained by imposing certain conditions on the sides/angles of a quadrilateral;
2. Sum of the four angles of a quadrilateral equals 360°
3. **Cyclic Quadrilateral**



- (a) If the four vertices of a quadrilateral lie on the circumference of a circle i.e. if the quadrilateral can be inscribed in a circle then it is called a cyclic quadrilateral (refer to the above figure).
 - (b) Sum of the opposite angles equals 180° i.e. $\angle A + \angle C = \angle B + \angle D = 180^\circ$
 - (c) Exterior angle is equal to the interior opposite angle, $\angle BCE = \angle BAD$
 - (d) **Ptolemy's theorem:** $AC \times BD = AD \times BC + AB \times CD$
4. **Square**



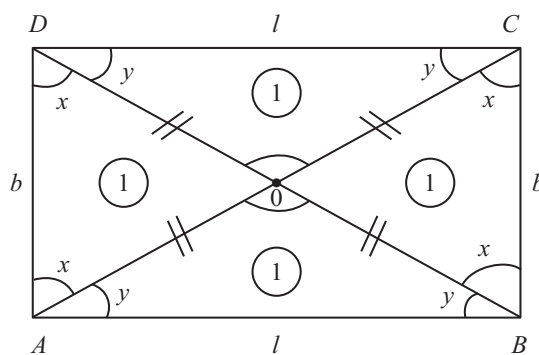
- (a) All four sides are equal and all angles are equal to 90°
- (b) Perimeter = $4 \times \text{side}$
- (c) Area = $(\text{side})^2 = \frac{\text{Diagonal}^2}{2}$
- (d) Diagonals bisect the vertex angles
- (e) Diagonal = $\sqrt{2}$ side
- (f) Diagonals are equal and bisect each other at 90°
- (g) All the four triangles are congruent to each other.
- (g) Inradius of a square with side ' a ' = $\frac{a}{2}$
- (h) Circumradius of a square with side ' a ' = $\frac{a}{\sqrt{2}}$
- (i) The largest rectangle that can be inscribed in a given circle will be a square.
- (j) All the properties of a rectangle as well as those of a rhombus hold good for a square.
- (k) Straight lines joining the mid points of adjacent sides of any square forms a square and the area of the newly formed square is half the area of the original square.



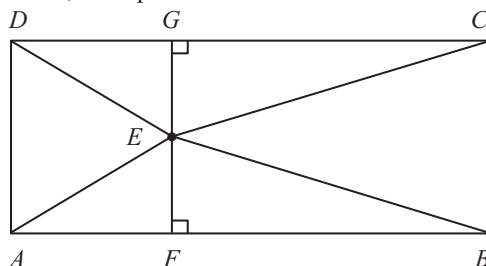
(l) Refer to the above figure where $ABCD$ can be a square or a rectangle such that D, E, F and G are the mid points, then,

$$\text{Area}(1 + 3) = \text{Area}(2 + 4) = \frac{\text{Area}(ABCD)}{2}$$

5. Rectangle

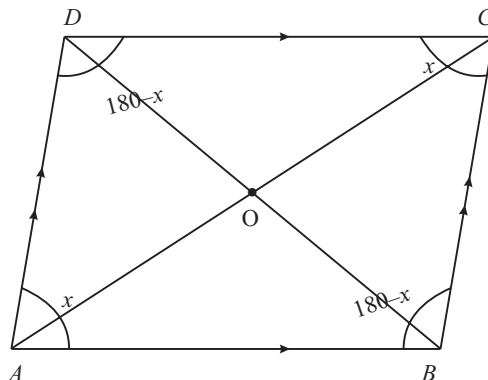


- Opposite sides are equal and parallel.
- All the angles equal 90°
- Diagonals do not bisect the vertex angles
- Diagonals are equal and bisect each other but not at right angles
- The triangles which are opposite to each other are congruent
- Bisectors of the four angles enclose a square
- $\text{Area } \triangle AOB = \text{Area } \triangle BOC = \text{Area } \triangle COD = \text{Area } \triangle DOA$
- $\text{Area} = \text{length} \times \text{breadth}$
- $\text{Perimeter} = 2(\text{length} + \text{breadth})$
- $\text{Diagonal} = \sqrt{\text{length}^2 + \text{breadth}^2}$
- If a rectangle and a triangle are on the same base and between the same parallels, then the area of the triangle will be equal to half the area of the rectangle.
- Of all the rectangles of given perimeter, the square will have the maximum area

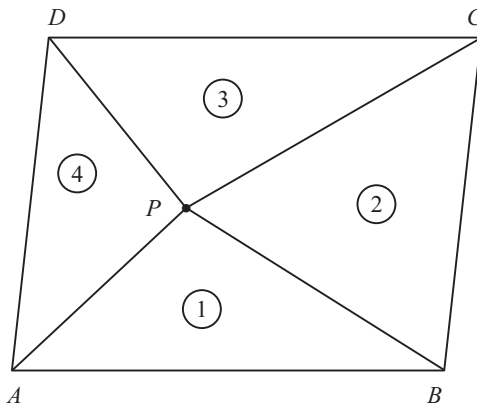


- (i) Refer to the above figure – let ‘E’ be any point inside the rectangle $ABCD$, then, $EA^2 + EC^2 = EB^2 + ED^2$
- (j) All the properties of a parallelogram apply to a rectangle.

6. Parallelogram



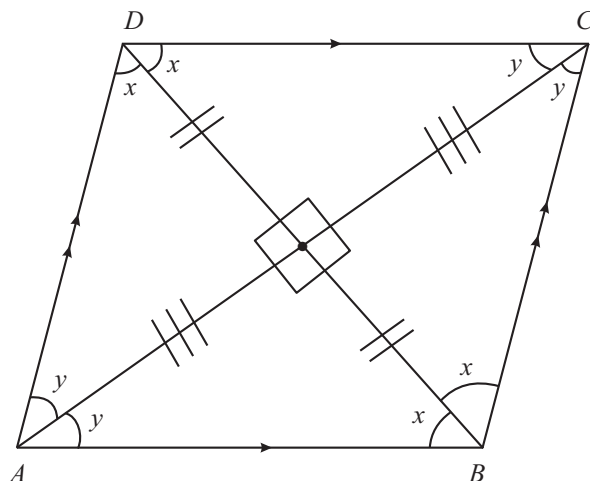
- (a) Opposite sides are equal and parallel.
- (b) Opposite angles are equal.
- (b) Diagonals may or may not be equal.
- (d) Diagonals bisect each other
- (d) Sum of any two adjacent angles = 180°
- (e) Bisectors of the four angles enclose a rectangle.
- (f) Each diagonal divides the parallelogram into two triangles of equal area
- (g) A parallelogram inscribed in a circle is always a rectangle
- (h) A parallelogram circumscribed about a circle is always a rhombus.
- (i) If two adjacent angles of a parallelogram are equal, then all four angles will be equal and each in turn will be equal to 90° . Thus, the figure will be a rectangle.
- (j) If any two adjacent sides of a parallelogram are equal, then all the four sides will be equal to each other and thus the figure will be a rhombus.
- (k) Area of parallelogram = base $\times$ height
- (l) Area of parallelogram = $AB \times AD \times \sin 'x'$
- (m) Area of parallelogram = $2 \sqrt{s(s-a)(s-b)(s-d)}$, where $s = \frac{AB+BC+AC}{2}$
- (n) $AC^2 + BD^2 = 2 (AB^2 + BC^2)$
- (n) The figure formed by joining the midpoints of the sides of any quadrilateral is a parallelogram.
- (o) If there is a parallelogram and a triangle with the same base and between the same parallel lines, then the area of the triangle will be half that of the parallelogram



(p) Refer to the above figure – let ‘P’ be any point inside the parallelogram ABCD, then,

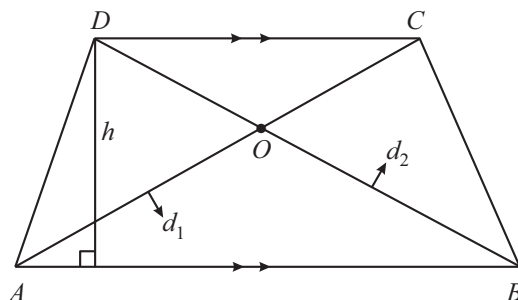
$$\text{Area } (1 + 3) = \text{Area } (2 + 4) = \frac{\text{Area of } \square ABCD}{2}$$

7. RHOMBUS



- (a) Rhombus is a type of parallelogram with all equal sides and diagonals bisecting each other at 90°
- (b) Opposite angles are equal
- (c) Diagonals bisect the vertex angles
- (d) Each of the diagonals divides the rhombus into two equal areas
- (e) Diagonals are unequal
- (f) All the four triangles made by the two diagonals are congruent to each other
- (g) Perimeter of a rhombus with side ‘s’ = $4s$
- (h) Area of a rhombus with its diagonals measuring d_1 and $d_2 = \frac{1}{2} \times d_1 \times d_2$
- (i) $(d_1)^2 + (d_2)^2 = 4s^2$

8. Trapezium



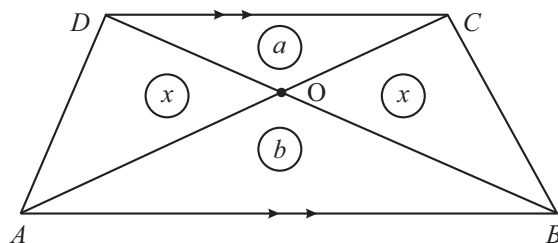
- (a) A quadrilateral with one pair of parallel opposite sides. The parallel sides are known as the bases and the non-parallel sides are known as the oblique sides. (Refer to the above figure)
- (b) If the mid-points of the two oblique sides are joined, it is equal in length to the average of the two parallel sides
- (c) If a line is drawn in between the two parallel sides of the trapezium such that it is parallel to the parallel sides and also divides the distance between the two parallel sides in the ratio $x : y$ such that the portion closer to the shorter of the two parallel sides is x , then the length of such lines is given by

$$\frac{x}{x+y} \times \text{longer parallel side} + \frac{y}{x+y} \times \text{shorter parallel side}$$

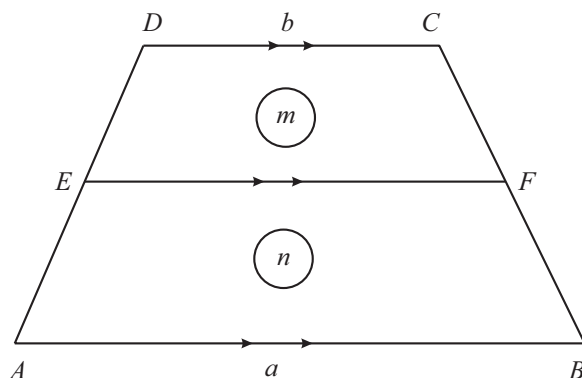
$$(d) (d_1)^2 + (d_2)^2 = AD^2 + BC^2 + 2AB.CD$$

$$(e) \text{ Area of a trapezium} = \frac{1}{2} (\text{sum of parallel sides}) \times \text{distance between them} = \frac{1}{2} (AB + CD) (h)$$

$$(f) \triangle AOB \sim \triangle COD$$



$$(g) \text{ Refer to the above figure, } x^2 = ab$$



(h) Refer to the above figure,

$$\frac{\text{Area of DCFE}}{\text{Area of EFBA}} = \frac{m}{n}, \text{ then,}$$

$$EF = \sqrt{\frac{ma^2 + nb^2}{m+n}}$$

(i) If the oblique sides are equal to each other then it is known as an isosceles trapezium.

(j) Each isosceles trapezium is a cyclic quadrilateral or if a trapezium is inscribed in a circle, it must be an isosceles trapezium

(k) Diagonals are equal in an isosceles trapezium

POLYGONS

- Polygon:** A closed figure having 3 or more sides.
- Convex Polygon:** Each interior angle is less than 180° . All the **regular polygons** are convex polygons i.e. each side is equal and each of the interior and exterior angles are also equal to each other
- Concave Polygon:** Any one angle is greater than 180°
- An ' n ' sided polygon has ' n ' vertices, ' n ' interior angles and ' n ' exterior angles
- Sum of all interior angles of a polygon with ' n ' sides $= (n - 2) \times 180^\circ$
- Each interior angle of a regular polygon with ' n ' sides $= \frac{(n-2) \times 180^\circ}{n}$
- Sum of all exterior angles of a polygon with ' n ' sides $= 360^\circ$
- Each exterior angle of a regular polygon with ' n ' sides $= \frac{360}{n}$
- Interior angle + Exterior angle $= 180^\circ$

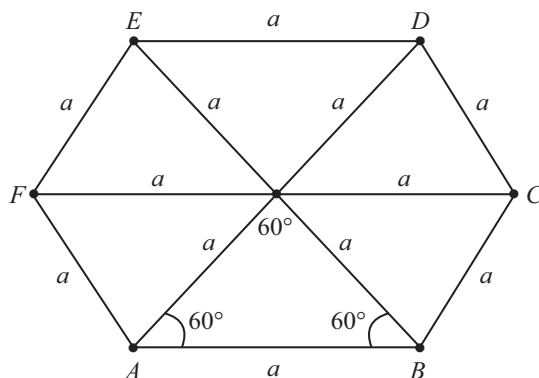
10. Number of diagonals in a polygon with 'n' sides = $\frac{n(n-3)}{2}$

11. Perimeter of a regular polygon with 'n' sides with each side equal to 'x' = nx

12. Area of a regular polygon with 'n' sides where length of each side is 'x'

$$n \frac{x^2}{4} \cot \frac{180}{n}$$

13. **Regular Hexagon:** A regular polygon with 6 equal sides.



(a) Perimeter = $6a$

(b) Area (when length of a side (a) is known) = $\frac{3\sqrt{3}}{2} a^2$

(c) Area (when length of a smaller diagonal (d) is known) = $\frac{\sqrt{3}}{2} d^2$

(d) Area (when length of a longer diagonal (D) is known) = $\frac{3\sqrt{3}}{8} D^2$

(e) Length of the smaller diagonal (d) = $\sqrt{3}a$

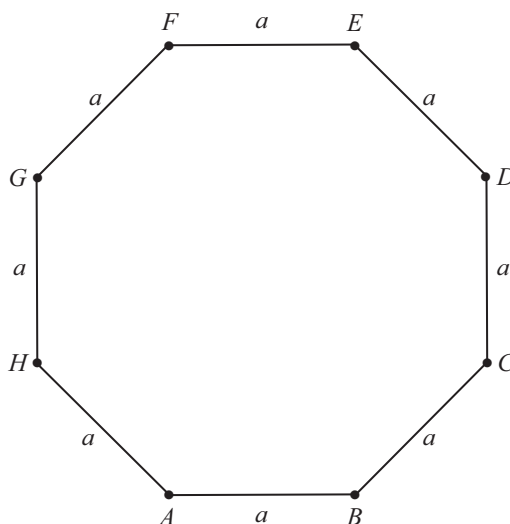
(f) Length of the longer diagonal (D) = $2a$

(g) Inradius of regular hexagon = $\frac{\sqrt{3}}{2} a$

(h) Circumradius of regular hexagon = a

(i) There are 6 equilateral triangles of equal area.

14. **Regular Octagon:** A regular polygon with 8 equal sides



(a) Perimeter = $8a$

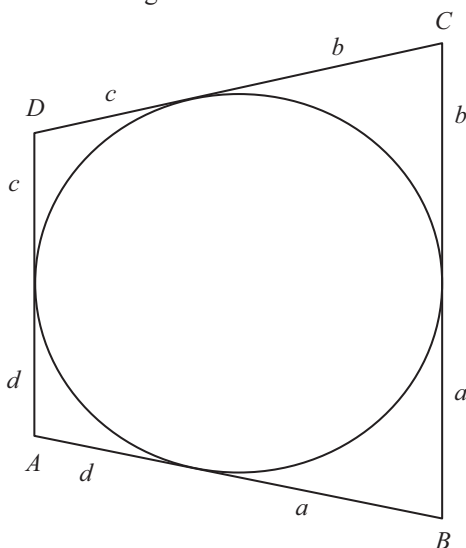
(b) Area = $2(\sqrt{2} + 1)a^2$

(c) Inradius of regular octagon = $\frac{a}{2\sqrt{2} - 2}$

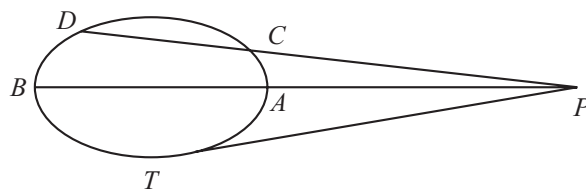
(d) Circumradius of regular octagon = $\frac{\sqrt{2+\sqrt{2}}}{2}$

CIRCLES

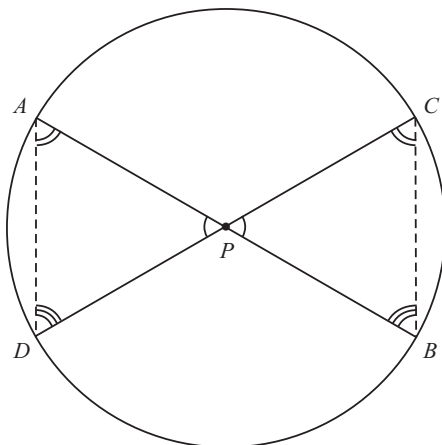
1. A circle is a closed curve such that any point on the curve is equidistant from a fixed point. The fixed point is known as the centre of the circle and the distance from the centre to any point on the circle is known as the radius of the circle.
2. One and only one circle passes through any three given non-collinear points.
3. Two circles are said to be concentric if they have the same centre.
4. **Diameter:** A line segment passing through the centre of the circle and joining any two points on the circumference of the circle.
5. **Chord:** A line segment joining any two points on the circumference of the circle. Diameter is the longest chord in the circle.
6. **Secant:** A line intersecting a circle at two distinct points and extending outside the circle also.
7. **Tangent:** A line that touches the circle at only one point is known as a tangent to the circle.
8. **Sector:** Area formed by an arc and the two radii at the two end points of the arc is called a sector.
9. **Circumference** – $2\pi r$
10. **Area** – πr^2
11. Two tangents can be drawn to a circle from any given point outside the circle and both these tangents will always be of equal length.
12. A tangent is perpendicular to the radius drawn at the point of tangency. Conversely, if a perpendicular is drawn to the tangent at the point of tangency, then the perpendicular passes through the centre of the circle.
13. A perpendicular drawn from the centre of a circle to a chord bisects the chord. Conversely, the perpendicular bisector of a chord passes through the centre of the circle.
14. Equal chords are equidistant from the centre. Conversely, two chords which are equidistant from the centre will be of equal length.
15. Equal chords subtend equal angles at the centre.
16. Angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
17. Angle subtended by a diameter on any point of a circle is 90° or angle made in a semicircle is a right-angle.
18. Angles in the same segment are equal.
19. **Alternate Segment Theorem:** Angle formed between a tangent and a chord through the point of contact of the tangent is equal to the angle formed by the chord in the alternate segment.



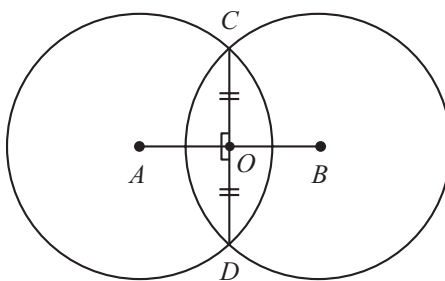
20. If a circle is inscribed in a quadrilateral or a quadrilateral is circumscribing a circle, then, $AB + CD = AD + BC$ (Refer to the above figure)



21. **Tangent-Secant Theorem:** If PAB and PCD are two secants and PT is a tangent to a circle Please refer to the above figure, then, $PT^2 = PA \times PB = PC \times PD$



22. Two chords AB and CD intersect each other at point P internally Please refer to the above figure, then,
 $PA \times PB = PC \times PD$



23. If there are two intersecting circles, then the line joining their centres perpendicularly bisects the line joining their points of intersection, thus,
 $AB \perp CD$ and $CO = OD$
24. When two circles touch each other internally, then the distance between their centres is equal to the difference in the radii of the two circles.
25. When two circles touch each other externally, then the distance between their centres is equal to the sum of the radii of the two circles
26. **Direct Common Tangent:** It is either parallel to the line of centres or cuts the line joining the centres not between the two circles but on one side of the circles
27. **Transverse Common Tangent:** It cuts the line joining the centres in between the two circles.
28. For two circles, there can be 0 to 4 common tangents drawn depending upon the position of one circle with respect to the other.
- If one circle lies completely inside the other without touching it at any point, then there will be no common tangent to these circles;
 - If two circles touch each other internally, then there will be one common tangent to these circles which will at the point of contact of these two circles;

- (c) If two circles intersect each other, there will be two common tangents to these circles, both of which will be direct common tangents.
- (d) If two circles touch each other externally, there will be three common tangents, two direct and one transverse.
- (e) If two circles are non-intersecting and non-enclosing, there will be four common tangents, two direct and two transverse.

29. If r_1 and r_2 are the radii of two non-intersecting and non-enclosing circles, then,

(a) Length of direct common tangent = $\sqrt{(\text{Distance between centres})^2 - (r_1 - r_2)^2}$

(b) Length of transverse common tangent = $\sqrt{(\text{Distance between centres})^2 - (r_1 + r_2)^2}$

MENSURATION (2D)

1. **Triangle:** Let three angles of a triangle be A , B and C and the sides opposite these angles be a , b and c respectively, then,

$$\text{Perimeter} = a + b + c$$

$$\text{Semi - Perimeter} = \frac{a + b + c}{2}$$

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where 's' is the semi-perimeter. This is also known as Heron's formula.}$$

$$\text{Area} = \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ac \sin B$$

$$\text{Area} = rs, \text{ where 'r' is the inradius and 's' is the semi-perimeter.}$$

$$\text{Area} = \frac{abc}{4R}, \text{ where 'R' is the circumradius of the triangle.}$$

2. **Equilateral triangle:** Let all the equal sides of the triangle ABC be equal to a , then,

$$\text{Perimeter} = 3a$$

$$\text{Area} = \frac{\sqrt{3}}{4} a^2$$

$$\text{Height (h)} = \frac{\sqrt{3}a}{2}$$

$$\text{Inradius} = \frac{a}{2\sqrt{3}}$$

$$\text{Circumradius} = \frac{a}{\sqrt{3}}$$

Let the lengths of perpendiculars drawn from any point inside an equilateral triangle be h_1 , h_2 and h_3 , then,

$$h_1 + h_2 + h_3 = h = \frac{\sqrt{3}a}{2}$$

3. **Isosceles triangle:** Let the equal sides of the triangle be equal to a and the remaining side be equal to b , then,

$$\text{Perimeter} = 2a + b$$

$$\text{Area} = \frac{b}{4} \sqrt{4a^2 - b^2}$$

$$\text{Height} = \frac{\sqrt{4a^2 - b^2}}{2}$$

$$\text{Circumradius} = \frac{a^2}{\sqrt{4a^2 - b^2}}$$

4. **Right angled triangle:** Let the Perpendicular, Base and Hypotenuse be represented by ' P ', ' B ' and ' H ' respectively, then,

$$\text{Perimeter} = P + B + H$$

$$\text{Area} = \frac{1}{2} \times \text{Perpendicular} \times \text{Base}$$

$$\text{Inradius} = \frac{P + B - H}{2}$$

$$\text{Circumradius} = \frac{H}{2}$$

5. Area of the quadrilateral = $\frac{1}{2} \times \text{one diagonal} \times \text{sum of the offsets drawn to that diagonal}$. The perpendiculars drawn to a diagonal from the opposite vertices are known as offsets.

6. Area of the quadrilateral made by joining the mid points of all the sides of a quadrilateral is half of the original quadrilateral.

7. **Square:** Let each side of the square be equal to a , then,

$$\text{Perimeter} = 4a$$

$$\text{Area} = a^2$$

$$\text{Diagonal} = \sqrt{2}a$$

$$\text{Inradius} = \frac{a}{2}$$

$$\text{Circumradius} = \frac{a}{\sqrt{2}}$$

8. **Rectangle:** Let length and breadth of a rectangle be equal to l and b respectively, then,

$$\text{Perimeter} = 2(l + b)$$

$$\text{Area} = lb$$

$$\text{Diagonal} = \sqrt{l^2 + b^2}$$

$$\text{Radius of the maximum possible circle} = \frac{b}{2}$$

9. **Parallelogram:** Let each of the opposite sides which are parallel and equal be equal to a and b respectively, then,

$$\text{Perimeter} = 2(a + b)$$

$$\text{Area} = \text{Base} \times \text{Height}$$

Let the length of one of the diagonals be ' d ', then,

$$\text{Area} = 2 \sqrt{s(s-a)(s-b)(s-d)}, \text{ where } s = \frac{a+b+d}{2}$$

10. **Rhombus:** If each side of a parallelogram is equal, then it is a rhombus. Let each of the equal sides be ' a ' and the lengths of the two diagonals be d_1 and d_2 respectively, then,

$$\text{Perimeter} = 4a$$

$$\text{Area} = \frac{1}{2} \times d_1 \times d_2$$

$$(d_1)^2 + (d_2)^2 = 4a^2$$

11. **Trapezium:** Let the lengths of one pair of parallel sides be equal to a and b respectively and the lengths of oblique sides be equal to c and d respectively, then,

$$\text{Perimeter} = a + b + c + d$$

$$\text{Area} = \frac{1}{2} (\text{Sum of parallel sides}) \times \text{distance between them} = \frac{1}{2} (a + b) (h)$$

Let the lengths of the diagonals be equal to d_1 and d_2 respectively, then,

$$(d_1)^2 + (d_2)^2 = c^2 + d^2 + 2ab$$

12. **Regular polygon**

$$\text{Perimeter} = \text{number of sides} \times \text{length of a side}$$

$$\text{Area} = \frac{1}{2} \times \text{Perimeter} \times \text{Perpendicular distance from the centre of the polygon to any side}$$

13. Circles: Let ' r ' be the radius of a circle then

$$\text{Circumference} = 2\pi r$$

$$\text{Area} = \pi r^2$$

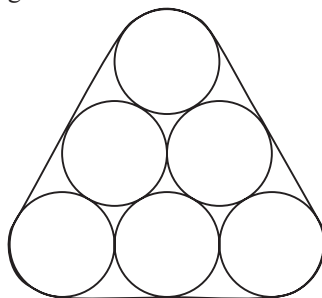
$$\text{Circumference of semi-circle} = \pi r + 2r$$

$$\text{Area of semi-circle} = \frac{\pi r^2}{2}$$

$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2 \text{ (where } \theta \text{ is the central angle subtended by the arc)}$$

$$\text{Length of an arc (l)} = \frac{\theta}{360} \times 2\pi r$$

$$\text{Area of sector} = \frac{1}{2} \times l \times r, \text{ where 'l' is the length of the arc and 'r' is the radius of the circle.}$$



14. Length of string $= 2\pi r + 2r \times \text{number of circles}$ (Refer to the above figure)

15. Ellipse: Let ' a ' be the semi-major axis and ' b ' be the semi-minor axis.

$$\text{Perimeter} = \pi(a + b)$$

$$\text{Area} = \pi ab$$

MENSURATION (3D)

1. Euler's rule (for any 3D flat surface figure) – Number of faces + Number of vertices = Number of edges + 2

2. Lateral Surface Area (for figures having flat surface) – surrounding area except top and bottom. In case of curved surfaces like cone, cylinder etc. it is known as **Curved Surface Area**.

3. Total Surface Area = Lateral Surface Area + Area of top and bottom

4. Cube: A solid figure in which length, breadth and height are equal or a right prism whose base is a square and height is equal to the side of the base. It has 6 faces, 8 vertices and 12 edges. Let length of each side of a cube be a , then,

$$\text{Lateral surface area} = 4a^2$$

$$\text{Total Surface area} = 6a^2$$

$$\text{Volume} = a^3$$

Length of the longest diagonal of a cube i.e. the line joining one vertex on the top face to the diagonally opposite vertex on the bottom face is $\sqrt{3}a$

5. Cuboid: A right prism whose base is a rectangle is known as a cuboid. Let the length and breadth of the rectangular base be l and b respectively and the height of the cuboid be h , then,

$$\text{Lateral Surface Area} = 2(l + b) \times h$$

$$\text{Total Surface Area} = 2(lb + bh + hl)$$

$$\text{Volume} = lbh$$

$$\text{Length of the longest diagonal of a cuboid} = \sqrt{l^2 + b^2 + h^2}$$

6. Prism: A right prism is a solid whose top and bottom faces are parallel to each other and are identical polygons. The bottom face is also known as the base of the prism. The faces joining the top and bottom faces are rectangles and are called lateral faces. There are as many lateral faces as there are sides in the base. The distance between the base and the top is known as the height of the prism.

In a right prism, if a perpendicular is drawn from the centre of the top face, it passes through the centre of the base.

Lateral Surface Area = Perimeter of the base $\times$ Height of the prism

Total Surface Area = Lateral Surface Area + $2 \times$ Area of the base

Volume = Area of the base $\times$ height of the prism

7. **Pyramid:** A solid whose base is a polygon and whose faces are triangles is known as a pyramid. The triangular faces meet at a common point called vertex. The perpendicular drawn from the vertex to the base is called the height of the pyramid.

A pyramid whose base is a regular polygon and the foot of the perpendicular from the vertex to the base coincides with the centre of the base, is called a right pyramid.

The length of the perpendicular from the vertex to any side of the base along the slant lateral surface is known as the slant height of the prism.

Lateral Surface Area = $\frac{1}{2} \times$ Perimeter of the base $\times$ slant height

Total Surface Area = Lateral Surface Area + Area of the base

Volume = $\frac{1}{3} \times$ Area of the base $\times$ height

8. **Cylinder:** It is equivalent to a right prism whose base is a circle, its lateral face is a curved surface. Let ' r ' be the radius of the base and ' h ' be the height of the cylinder.

Curved Surface Area = $2\pi rh$

Total Surface Area = $2\pi r (r + h)$

Volume = $\pi r^2 h$

Hollow Cylinder: Let there be a hollow cylinder with ' R ' as its outer radius and ' r ' as its inner radius

Curved Surface Area = $2\pi h (R + r)$

Total Surface Area = Curved Surface Area + $2\pi(R^2 - r^2)$

Volume of hollow cylinder = $\pi r^2 h$

Volume of material contained in a hollow cylinder = $\pi (R^2 - r^2) \times h$

9. **Cone:** A cone is equivalent to a right pyramid whose base is a circle. The lateral surface of a cone is a curved surface. Let ' r ' be the radius of the base, ' h ' be the height of the cone and ' l ' be the slant height, then,

$$l^2 = r^2 + h^2$$

Curved Surface Area = πrl

Total Surface Area = $\pi r (r + l)$

Volume = $\frac{1}{3} \pi r^2 h$

10. **Cone Frustum:** If a cone is cut into two parts by a plane parallel to the base, the portion that contains the base is known as the frustum of a cone. Let ' r ' be the top radius, ' R ' be the radius of the base, ' h ' be the height and ' l ' be the slant height of the frustum of a cone, then,

$$l^2 = (R - r)^2 + h^2$$

Curved Surface Area = $\pi l (R + r)$

Total Surface Area = Curved Surface Area + $\pi (R^2 + r^2)$

Volume = $\frac{1}{3} \pi h (R^2 + r^2 + Rr)$

Let ' H ' be the height of the complete cone from which the frustum is cut, then, $\frac{r}{R} = \frac{H - h}{H}$

11. **Pyramid Frustum:** A pyramid left after cutting off a portion at the top by a plane parallel to the base is known as the frustum of a pyramid. Let A_1 be the area of the top and A_2 be the area of the base and ' h ' be the height of the frustum, then,

Lateral Surface Area = $\frac{1}{2} \times$ (Sum of perimeters of the top and base) $\times$ slant height

Total Surface Area = Lateral Surface Area + Sum of the areas of top and bottom

$$\text{Volume} = \frac{1}{3} \times h \times (A_1 + A_2 + \sqrt{A_1 A_2})$$

- 12. Sphere:** A round solid figure, with every point on its surface equidistant from its centre. The distance is the radius of the sphere. Let the radius of a sphere be 'r', then,

$$\text{Surface Area} = 4\pi r^2$$

$$\text{Volume} = \frac{4}{3} \pi r^3$$

Hemisphere -

$$\text{Curved Surface Area} = 2\pi r^2$$

$$\text{Total Surface Area} = 3\pi r^2$$

$$\text{Volume} = \frac{2}{3} \pi r^3$$

- 13. Torus:** It is a doughnut shaped figure produced by the revolution of a circle about an axis lying in its plane but not intersecting the circle. Let 'r' be the radius of the circle that rotates and 'R' be the distance between the centre of the circle and the axis of revolution, then,

$$\text{Surface Area} = 4\pi^2 r R$$

$$\text{Volume} = 2\pi^2 r^2 R$$

- 14. Regular tetrahedron:** It is composed of four equilateral triangular faces with all its edges of equal length. The slant edge is the same as the side of the base. Let the length of each such edge be equal to 'a', then,

$$\text{Curved Surface Area} = \frac{3\sqrt{3}}{4} a^2$$

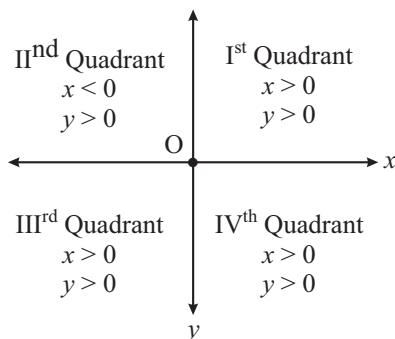
$$\text{Total Surface Area} = \sqrt{3} a^2$$

$$\text{Height} = \sqrt{\frac{2}{3}} a$$

$$\text{Volume} = \frac{1}{3} \times \text{Area of the base} \times \text{height} = \frac{a^3}{6\sqrt{2}}$$

COORDINATE GEOMETRY

- 1. Rectangular coordinate system:** Let us draw two lines which are mutually perpendicular to each other, then the horizontal line is termed as the x-axis and the vertical line is termed as the y-axis. The point of intersection of the axes is known as the origin (O). To the right of O, we have positive real numbers and to the left of O, we have negative real numbers. Similarly, we have positive real numbers above O and negative real numbers below O (Refer to the below figure).



- 2. Ordered Pair:** Let the ordered pair (x, y) denote the coordinates of a point P. Thus, the coordinates help us in locating a given point.

3. Distance Formula: The distance between the two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

4. Let the coordinates of vertices of a triangle be (x_1, y_1) , (x_2, y_2) and (x_3, y_3) respectively, then the area of the triangle is given by the formula,

$$\frac{1}{2} \{(x_1 - x_2)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_2)\}$$

5. Let the consecutive coordinates of vertices of a quadrilateral be (x_1, y_1) , (x_2, y_2) , (x_3, y_3) and (x_4, y_4) , then its area is given by the formula,

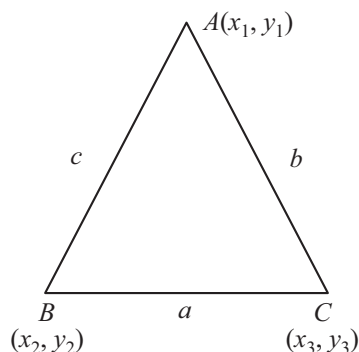
$$\frac{1}{2} \{(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)\}$$

6. Section Formulae:

(a) **Internal division:** Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be the two given points, then the coordinates of a point R , which divides the line segment joining PQ internally in the ratio $m : n$ is given by the formula $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

The midpoint formula is just an extension of the above concept.

(b) **External division:** Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be the two given points, then the coordinates of a point R , which divides the line segment joining PQ externally in the ratio $m : n$ is given by the formula $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$



7. (Refer to the above figure) Let the coordinates of vertices of a triangle be $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, then the coordinates of,

(i) Centroid of the triangle will be given by:

$$\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}$$

(ii) Incentre of the triangle will be given by:

$$\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}$$

(iii) Circumcentre of the triangle will be given by:

$$\frac{x_1 \sin 2A + x_2 \sin 2B + x_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C}, \frac{y_1 \sin 2A + y_2 \sin 2B + y_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C}$$

(iv) Orthocentre of the triangle will be given by:

$$\frac{x_1 \tan A + x_2 \tan B + x_3 \tan C}{\tan A + \tan B + \tan C}, \frac{y_1 \tan A + y_2 \tan B + y_3 \tan C}{\tan A + \tan B + \tan C}$$

8. Slope of a line: The slope of a line is a measure of its steepness or “rise over run”. It is the change in y -coordinate with respect to the change in x -coordinate. Let A and B be two distinct points with coordinates (x_1, y_1) and (x_2, y_2) respectively, then the slope of the line joining these two points is given by

$$\text{Slope } (m) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2} = \tan \theta; \text{ if } x_1 \neq x_2$$

If $x_1 = x_2$, then the slope of the line would be undefined, thus it will be a vertical line.

9. $ax + by + c = 0$ is a first degree equation in x and y

(i) If $a = 0$ and $b \neq 0$, then the line will be a horizontal line.

(ii) If $a \neq 0$ and $b = 0$, then the line will be a vertical line.

(iii) If $c = 0$, then the line passes through the origin.

(iv) Slope of such a line $(m) = -\left(\frac{\text{coefficient of } x}{\text{coefficient of } y}\right) = \left(\frac{-a}{b}\right)$

(v) x -intercept $= \frac{-c}{a}$

(vi) y -intercept $= \frac{-c}{b}$

10. **Equation of a vertical line:** The equation of a vertical line passing through a point $(k, 0)$ is given by the equation $x = k$, where ' k ' is a real number.

11. **Intercept form:** $\frac{x}{a} + \frac{y}{b} = 1$

The portions cut off by a line on the coordinate axes are known as intercepts. In the above equation, ' a ' is the x -intercept and ' b ' is the y -intercept.

12. **Slope – Intercept form:** $y = mx + c$

The coefficient of ' x ' is the slope and the constant term gives the y -intercept of the line.

13. **Point – Slope form:** The equation of a non-vertical line of slope m and passing through the point (x_1, y_1) is $y - y_1 = m(x - x_1)$

14. **Two point form:** Let the given two points be (x_1, y_1) and (x_2, y_2) , then,

(i) If $x_1 \neq x_2$, then the equation is given by $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

(ii) If $x_1 = x_2$, then the equation is given by $x = x_1$ as it will be a vertical line.

15. The coordinates of point of intersection of two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is given by $\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right)$

16. Given two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

(i) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the lines are overlapping each other and thus have infinite solutions.

(ii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the lines are parallel to each other and thus there is no solution. For such lines, the slopes will be equal and thus $m_1 = m_2$

(iii) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then these are intersecting lines and thus will have a unique solution

17. Given two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then the angle θ between them is given by

$$\cos \theta = \frac{a_1a_2 + b_1b_2}{\sqrt{(a_1^2 + b_1^2)(a_2^2 + b_2^2)}}$$

18. If m_1 and m_2 are the slopes of two lines, the angle θ between them is given by

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1m_2} \right|$$

19. For parallel lines, we have $m_1 = m_2$ or $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

For perpendicular lines, we have $m_1 \times m_2 = -1$ or $a_1 a_2 + b_1 b_2 = 0$

20. Perpendicular distance of a point (x_1, y_1) from the line $ax + by + c = 0$ is given by the formula $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

21. The distance between two parallel straight lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is given by $\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$

22. The equation of a circle centered at (a, b) with radius r units is given by $(x - a)^2 + (y - b)^2 = r^2$

23. The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle with centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$

24. **Translation of axes:** The origin is shifted to a new point, keeping the direction of the axes intact. The new axes are parallel to the original axes.

Let the origin be shifted to (a, b) , then if the original coordinates of a point M are (x, y) and (X, Y) denote the coordinates of M with reference to the new axes, then,

$$X = x - a \text{ and } Y = y - b$$

25. **Rotation of axes:** The origin is kept intact and the axes are rotated about the origin through an angle.

If θ is the angle of rotation and (x, y) are the coordinates of a point M with reference to the original axes and (X, Y) with reference to the new axes, then,

$$x = X \cos \theta - Y \sin \theta, y = X \sin \theta + Y \cos \theta$$

$$X = x \cos \theta + y \sin \theta, Y = -x \sin \theta + y \cos \theta$$

26. **Mirror Image/Reflection of a point with respect to a straight line:** Perpendicular distance from a point to the mirror is equal to the perpendicular distance of the image from the mirror and both are in the same straight line.

(i) Reflection over x -axis then x -coordinate remains the same but y -coordinate changes its sign

Therefore, (x, y) would become $(x, -y)$

(ii) Reflection over y -axis then y -coordinate remains the same but x -coordinate changes its sign

Therefore, (x, y) would become $(-x, y)$

(iii) Reflection over $y = x$, the x -coordinate and y -coordinate interchange their places

Therefore, (x, y) would become (y, x)

(iv) Reflection over $y = -x$, both the coordinates change their places as well as their sign

Therefore, (x, y) would become $(-y, -x)$

(v) Reflection about the origin, signs of both the coordinates will be changed.

Therefore (x, y) would become $(-x, -y)$

ALGEBRA

1. $(a + b)^2 = a^2 + b^2 + 2ab$

2. $(a - b)^2 = a^2 + b^2 - 2ab$

3. $a^2 - b^2 = (a + b)(a - b)$

4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

5. $(a + b)(b + c)(c + a) + abc = (a + b + c)(ab + bc + ca)$

6. $ab(a - b) + bc(b - c) + ca(c - a) = -(a - b)(b - c)(c - a)$

7. $a^2(b - c) + b^2(c - a) + c^2(a - b) = -(a - b)(b - c)(c - a)$

8. $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2) = (a - b)(b - c)(c - a)$

9. $a^3(b - c) + b^3(c - a) + c^3(a - b) = -(a - b)(b - c)(c - a)(a + b + c)$

10. If $a + \frac{1}{b} = b + \frac{1}{c} = c + \frac{1}{a}$ (where $a \neq b \neq c$) then $abc = \pm 1$
11. If $x + \frac{1}{y} = a, y + \frac{1}{z} = b, z + \frac{1}{x} = c$, then $xyz + \frac{1}{xyz} = abc - (a + b + c)$
12. If $x - \frac{1}{y} = a, y - \frac{1}{z} = b, z - \frac{1}{x} = c$, then $xyz - \frac{1}{xyz} = abc + (a + b + c)$
13. If $x^2 + y^2 = z + 1, y^2 + z^2 = x + 1$ and $z^2 + x^2 = y + 1$, then $xyz = 1$ or $-\frac{1}{8}$
14. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
15. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
16. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
17. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
18. If $a^2 + a + 1 = 0$, then $a^3 = 1$
19. If $ab(a + b) = 1$, then $\frac{1}{a^3b^3} - a^3 - b^3 = 3$
20. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

(i) If $a = b = c$,

$$\text{then } a^2 + b^2 + c^2 = ab + bc + ca$$

Thus, RHS becomes 0 and hence,

$$a^3 + b^3 + c^3 = 3abc$$

(ii) If $a + b + c = 0$,

Then also RHS becomes 0 and hence,

$$a^3 + b^3 + c^3 = 3abc$$

21. If $x + \frac{1}{x} = y$, then $x^2 + \frac{1}{x^2} = y^2 - 2$
22. If $x - \frac{1}{x} = y$, then $x^2 + \frac{1}{x^2} = y^2 + 2$
23. If $x^2 + \frac{1}{x^2} = y$, then $x + \frac{1}{x} = \pm \sqrt{y + 2}$
24. If $x^2 + \frac{1}{x^2} = y$, then $x - \frac{1}{x} = \pm \sqrt{y - 2}$
25. If $x + \frac{1}{x} = y$, then $x - \frac{1}{x} = \pm \sqrt{y^2 - 4}$
26. If $x - \frac{1}{x} = y$, then $x + \frac{1}{x} = \pm \sqrt{y^2 + 4}$
27. If $x - \frac{1}{x} = y$, then $x^3 - \frac{1}{x^3} = y^3 + 3y$
28. If $x + \frac{1}{x} = y$, then $x^3 + \frac{1}{x^3} = y^3 - 3y$
29. If $x + \frac{1}{x} = \pm \sqrt{3}$, then $x^3 + \frac{1}{x^3} = 0$ and $x^6 = -1$
30. If $x + \frac{1}{x} = 1$, then $x^3 = -1$

31. If $x + \frac{1}{x} = -1$, then $x^3 = 1$

32. $a^5 - b^5 = (a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$

33. $a^5 + b^5 = (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$

34. $(a^2 - ab + b^2)(a^2 + ab + b^2) = a^4 + a^2b^2 + b^4$

35. $a^2 + b^2 + ab = (a + b + \sqrt{ab})(a + b - \sqrt{ab})$

36. $x^4 + x^2 + 1 = (x^2 + x + 1)(x^2 - x + 1)$

TRIGONOMETRY

1. **Sexagesimal system:** A right angle is taken as the unit of measurement

$$\frac{1}{90} \text{th of a right angle} = 1^\circ \text{ (1 degree)}$$

$$1^\circ = 60 \text{ minutes denoted as } 60'$$

$$1' = 60 \text{ seconds denoted as } 60''$$

2. **Circular System:** A radian is taken as the unit of this measure.

$$\pi \text{ radians} = 180^\circ, 1 \text{ revolution} = 2\pi \text{ radians}$$

3. $\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

4. $\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}}$

5. $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\text{Perpendicular}}{\text{Base}}$

6. $\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$

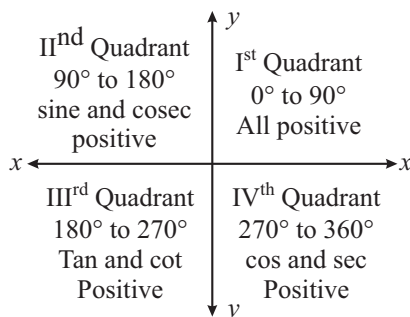
7. $\sec \theta = \frac{1}{\cos \theta} = \frac{\text{Hypotenuse}}{\text{Base}}$

8. $\cot \theta = \frac{1}{\tan \theta} = \frac{\text{Base}}{\text{Perpendicular}}$

9. **Pythagoras Theorem:** $P^2 + B^2 = H^2$

Some of the most frequently used Pythagorean triplets in the examinations are (3, 4, 5), (5, 12, 13), (7, 24, 25), (8, 15, 17), (9, 40, 41), (11, 60, 61), (1, 1, $\sqrt{2}$ – Represents a 45, 45, 90 triangle), (1, $\sqrt{3}$, 2 – Represents a 30, 60, 90 triangle)

10. **Signs of Trigonometric Ratios**



11. Trigonometric Ratios of some special angles

θ	0°	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\operatorname{cosec} \theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot \theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

12. Change of Trigonometric Ratios

- (i) $\sin(90 - \theta) = \cos \theta$
- (ii) $\cos(90 - \theta) = \sin \theta$
- (iii) $\tan(90 - \theta) = \cot \theta$
- (iv) $\cot(90 - \theta) = \tan \theta$
- (v) $\sec(90 - \theta) = \operatorname{cosec} \theta$
- (vi) $\operatorname{cosec}(90 - \theta) = \sec \theta$
- (vii) $\sin(-\theta) = -\sin \theta$
- (viii) $\cos(-\theta) = \cos \theta$
- (ix) $\tan(-\theta) = -\tan \theta$
- (x) $\operatorname{cosec}(-\theta) = -\operatorname{cosec} \theta$
- (xi) $\sec(-\theta) = \sec \theta$
- (xii) $\cot(-\theta) = -\cot \theta$

In general, the trigonometric ratios change at 90° and 270° and there is no change at 180° and 360°

13. Basic Trigonometric Identities:

- (i) $\sin^2 \theta + \cos^2 \theta = 1$
- (ii) $\sec^2 \theta - \tan^2 \theta = 1$
- (iii) $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$

14. Double Angle Formulae:

$$(i) \sin 2\theta = 2 \sin \theta \cos \theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

$$(ii) \cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \frac{\cot^2 \theta - 1}{\cot^2 \theta + 1}$$

$$(iii) \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$(iv) \cot 2\theta = \frac{\cot^2 \theta - 1}{2 \cot \theta}$$

$$(v) \operatorname{Cosec} 2\theta = \frac{\sec \theta \operatorname{Cosec} \theta}{2}$$

$$(v) \cos \theta = \sqrt{\frac{1 + \cos 2\theta}{2}}$$

$$(vi) \sin \theta = \sqrt{\frac{1 - \cos 2\theta}{2}}$$

15. Triple Angle Formulae:

$$(i) \sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

$$(ii) \cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$(iii) \tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$$

$$(iv) \cot 3\theta = \frac{3 \cot \theta - \cot^3 \theta}{1 - 3 \cot^2 \theta}$$

16. Addition, Subtraction and Other Miscellaneous Formulae

$$(i) \sin (A + B) = \sin A \cos B + \cos A \sin B$$

$$(ii) \sin (A - B) = \sin A \cos B - \cos A \sin B$$

$$(iii) \cos (A + B) = \cos A \cos B - \sin A \sin B$$

$$(iv) \cos (A - B) = \cos A \cos B + \sin A \sin B$$

$$(v) \tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$(vi) \tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$(vii) \cot (A + B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$$

$$(viii) \cot (A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

$$(ix) \sin C + \sin D = 2 \sin \left(\frac{C + D}{2} \right) \cos \left(\frac{C - D}{2} \right)$$

$$(x) \sin C - \sin D = 2 \cos \left(\frac{C + D}{2} \right) \sin \left(\frac{C - D}{2} \right)$$

$$(xi) \cos C + \cos D = 2 \cos \left(\frac{C + D}{2} \right) \cos \left(\frac{C - D}{2} \right)$$

$$(xii) \cos C - \cos D = 2 \sin \left(\frac{C + D}{2} \right) \sin \left(\frac{D - C}{2} \right)$$

$$(xiii) \tan (A + B + C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - (\tan A \tan B + \tan B \tan C + \tan C \tan A)}$$

$$(xiv) \sin (60 - \theta) \sin \theta \sin (60 + \theta) = \frac{1}{4} \sin 3\theta$$

- (xv) $\cos(60 - \theta) \cos \theta \cos(60 + \theta) = \frac{1}{4} \cos 3\theta$
- (xvi) $\tan(60 - \theta) \tan \theta \tan(60 + \theta) = \tan 3\theta$
- (xvii) If $A + B + C = 180^\circ$, then,
 $\tan A + \tan B + \tan C = \tan A \tan B \tan C$
 $\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$
- (xviii) If $A + B + C = 90^\circ$, then,
 $\tan A \tan B + \tan B \tan C + \tan C \tan A = 1$
 $\cot A + \cot B + \cot C = \cot A \cot B \cot C$
- (xix) $\tan 4\theta = \frac{4 \tan \theta (1 - \tan^2 \theta)}{1 - 6 \tan^2 \theta + \tan^4 \theta}$
- (xx) If $\sec \theta \pm \tan \theta = x$, then $\sec \theta = \frac{x^2 + 1}{2x}$
- (xxi) If $\sin \theta + \cos \theta = x$, then $\sin \theta - \cos \theta = \sqrt{2 - x^2}$
- (xxii) If $\sin \theta + \operatorname{cosec} \theta = x$, then $\sin^n \theta + \operatorname{cosec}^n \theta = x$
- (xxiii) If $\tan \theta + \cot \theta = x$, then $\tan^n \theta + \cot^n \theta = x$

17. Trigonometric ratios of some special angles:

- (i) $\sin 15^\circ = \cos 75^\circ = \frac{\sqrt{3} - 1}{2\sqrt{2}}$
- (ii) $\sin 75^\circ = \cos 15^\circ = \frac{\sqrt{3} + 1}{2\sqrt{2}}$
- (iii) $\sin 18^\circ = \cos 72^\circ = \frac{\sqrt{5} - 1}{4}$
- (iv) $\sin 72^\circ = \cos 18^\circ = \frac{\sqrt{10 + 2\sqrt{5}}}{4}$
- (v) $\sin 36^\circ = \cos 54^\circ = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$
- (vi) $\sin 54^\circ = \cos 36^\circ = \frac{\sqrt{5} + 1}{4}$
- (vii) $\sin 22.5^\circ = \frac{\sqrt{2 - \sqrt{2}}}{2}$
- (viii) $\cos 22.5^\circ = \frac{\sqrt{2 + \sqrt{2}}}{2}$

18. Maximum and Minimum values of Trigonometric functions:

- (i) The values of $\sin \theta$ and $\cos \theta$ range from -1 to 1
- (ii) The values of $\tan \theta$ and $\cot \theta$ range from $-\infty$ to ∞
- (iii) The values of $\sec \theta$ and $\operatorname{cosec} \theta$ range from $-\infty$ to ∞ but excluding values from -1 to 1
- (iv) $a \sin \theta + b \cos \theta$
 Maximum value $= + \sqrt{a^2 + b^2}$
 Minimum value $= - \sqrt{a^2 + b^2}$

- (v) $a \sin^2 \theta + b \cos^2 \theta$
 Maximum value = Max. $[a, b]$
 Minimum value = Min. $[a, b]$
- (vi) $a \sin^2 \theta + b \operatorname{cosec}^2 \theta$
 Minimum value = $2\sqrt{ab}$ (when $a > b$)
- (vii) $a \cos^2 \theta + b \sec^2 \theta$
 Maximum value = ∞
 Minimum value = $a + b$ (when $a < b$)
- (viii) $a \sec^2 \theta + b \operatorname{cosec}^2 \theta$
 Minimum value = $(\sqrt{a} + \sqrt{b})^2$
- (ix) $\sin^n \theta \cos^n \theta$ (where n is odd)
 Maximum value = $\frac{1}{2^n}$
 Minimum value = $-\frac{1}{2^n}$
- (x) $\sin^n \theta \cos^n \theta$ (where n is even)
 Maximum value = $\frac{1}{2^n}$
 Minimum value = 0
- (xi) $\sin^{2m} \theta + \cos^{2n} \theta$ (where m and n are natural numbers)
 Maximum value = 1
- (xii) $a \tan^2 \theta + b \cot^2 \theta$
 Maximum value = ∞
 Minimum value = $2\sqrt{ab}$

SEQUENCE AND SERIES

1. Sequence: A sequence is an arrangement of numbers in definite order according to a given set of rules.

2. Arithmetic Progression: An arithmetic progression looks like the following

$$a, a + d, a + 2d, a + 3d, \dots, a + (n - 1)d$$

A sequence of numbers in which any term is greater than or smaller than its previous terms by a fixed quantity. Such a fixed quantity is known as the common difference denoted by ' d '. Let ' a ' be the first term of an arithmetic progression containing total ' n ' terms and ' d ' be the common difference, then,

$$T_n \text{ (} n^{\text{th}} \text{ term)} = a + (n - 1)d = \text{Sum of first 'n' terms} - \text{Sum of first (n - 1) terms}$$

$$S_n \text{ (Sum to 'n' terms)} = \frac{n}{2} \{2a + (n - 1)d\} = n \times \frac{a + l}{2} \text{ (Here 'l' denotes the last term of the AP)}$$

The average of all the terms of an Arithmetic Progression is known as the Arithmetic mean, hence it is given by the sum of

all the terms divided by the number of terms or simply $\frac{a + l}{2}$ Arithmetic mean of an AP can also be obtained by taking the

average of any two terms of the progression which are equidistant from the two ends i.e. the Arithmetic mean can be calculated by taking the average of taking n^{th} term from the beginning and n^{th} term from the end.

If an AP has an odd number of terms, then the middle term will be the Arithmetic mean.

3. Geometric Progression: A geometric progression looks like the following

$$a, ar, ar^2, ar^3, \dots, ar^{n-1}$$

A sequence of numbers in which the ratio of any term to its previous term is the same. Such a ratio is known as the common ratio denoted by ' r '. Let the first term of the progression be ' a ' and the common ratio be ' r ', then,

$$T_n (n^{\text{th}} \text{ term}) = ar^{n-1}$$

$$S_n (\text{Sum to 'n' terms}) = \frac{a(1-r^n)}{1-r} \text{ (when } r < 1) \text{ and } \frac{a(r^n-1)}{r-1} \text{ (when } r > 1) \text{ or}$$

$$\frac{a-lr}{1-r} \text{ (when } r < 1) \text{ and } \frac{lr-a}{r-1} \text{ (when } r > 1) \text{ (Here 'l' is the last term of the GP)}$$

$$\text{Sum of an infinite GP (defined only for } |r| < 1) = \frac{a}{1-r}$$

The product of n^{th} term from the beginning and n^{th} term from the end in a GP is equal to the product of the first and the last terms.

If n terms $a_1, a_2, a_3, \dots, a_n$ are in GP, then the Geometric Mean of these ' n ' terms is given by $\sqrt[n]{a_1 \cdot a_2 \cdot a_3 \cdot a_4 \cdot \dots \cdot a_n}$

If three terms are in GP, then the middle term will be the geometric mean. Eg. If a, b and c are in GP then ' b ' will be their geometric mean such that $b^2 = ac$.

For any two unequal positive numbers ' x ' and ' y ', their Arithmetic Mean is always greater than their Geometric Mean

$$\frac{x+y}{2} > \sqrt{xy}$$

4. Harmonic Progression: A sequence $a_1, a_2, a_3, \dots, a_n$ of non-zero numbers is said to be in a Harmonic Progression (HP) if

the sequence $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \dots, \frac{1}{a_n}$ is in Arithmetic Progression (AP).

$$T_n (n^{\text{th}} \text{ term}) = \frac{a_1 a_2}{a_2 + (n-1)(a_1 - a_2)} = \frac{1}{\frac{1}{a_1} + (n-1)\left(\frac{1}{a_2} - \frac{1}{a_1}\right)} \text{ (where } a \text{ and } d \text{ are the first term and common difference respectively of}$$

the corresponding AP)

For two numbers x and y , their Harmonic Mean is given by $\frac{2xy}{x+y}$

$$\text{Harmonic Mean} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}}$$

Here, the total number of observations is divided by the sum of reciprocals of all observations.

If a, b and c are in harmonic progression, then b is said to be the harmonic mean of a and c .

If $a_1, a_2, a_3, \dots, a_n$ are in harmonic progression, then $a_2, a_3, \dots, a_{n-1}$ are the $n-2$ harmonic means between a_1 and a_n .

For any two positive numbers a and b , we have,

Arithmetic Mean (AM) $\geq$ Geometric Mean (GM) $\geq$ Harmonic Mean (HM)

$$(GM)^2 = AM \times HM$$

SURDS AND INDICES

1. Whenever an expression is written in the form say x^4 , it means that x is multiplied 4 times, here ' x ' is called the base and ' 4 ' is known as the exponent.

2. Laws of Indices

$$(i) a \times a \times a \dots n \text{ times} = a^n$$

$$(ii) a^m \times a^n = a^{m+n}$$

$$(iii) \frac{a^m}{a^n} = a^{m-n}$$

$$(iv) (a^m)^n = a^{mn} = (a^n)^m$$

$$(v) (ab)^n = a^n \times b^n$$

$$(vi) \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \text{ (} b \neq 0 \text{)}$$

$$(vii) a^{-m} = \frac{1}{a^m}$$

$$(viii) \sqrt[n]{a} = a^{\frac{1}{n}}$$

$$(ix) a^0 = 1$$

$$(x) (a^m)^n \text{ need not necessarily be equal to } a^{m^n}$$

$$(2^3)^4 = 2^{12}$$

$$2^{3^4} = 2^{81}$$

(xi) When the bases of two equal numbers are equal, then their powers also will be equal (if the bases are neither 0, -1 and 1)

(xii) When the powers of two equal numbers are equal and not equal to zero, then,

(a) If the power is an odd number, then the bases are equal.

(b) If the powers are even numbers, then the bases are numerically equal but can have different signs.

$$(xiii) \left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}$$

$$(xiv) \sqrt{x\sqrt{x\sqrt{x\sqrt{x\cdots\infty}}}} = x$$

$$(xv) \sqrt{x\sqrt{x\sqrt{x\sqrt{x\cdots n \text{ times}}}}} = x^{1-\frac{1}{2^n}}$$

$$(xvi) \sqrt[n]{x\sqrt[n]{x\sqrt[n]{x\sqrt[n]{x\cdots\infty}}}} = n\sqrt[n]{x}$$

$$(xvii) \sqrt{x+\sqrt{x+\sqrt{x+\sqrt{x+\cdots\infty}}}} = \frac{\sqrt{4x+1}+1}{2} = a$$

$$\sqrt{x-\sqrt{x-\sqrt{x-\sqrt{x-\cdots\infty}}}} = \frac{\sqrt{4x+1}-1}{2} = b$$

$$a-b=1 \text{ and } ab=x$$

$$(xviii) \sqrt{x+\sqrt{x-\sqrt{x+\sqrt{x-\cdots\infty}}}} = \frac{\sqrt{4x-3}+1}{2} = a$$

$$\sqrt{x-\sqrt{x+\sqrt{x-\sqrt{x+\cdots\infty}}}} = \frac{\sqrt{4x-3}-1}{2} = b$$

$$a-b=1 \text{ and } ab=2(x-1)$$

$$(xix) \sqrt{x+\sqrt[y]{x+\sqrt[y]{x+\cdots\infty}}}} = \frac{\sqrt{4x+y^2}+y}{2} = a$$

$$\sqrt{x-\sqrt[y]{x-\sqrt[y]{x-\cdots\infty}}}} = \frac{\sqrt{4x+y^2}-y}{2} = b$$

$$a-b=y, ab=x \text{ and } a+b=\sqrt{4x+y^2}$$

$$(xx) \sqrt{x+\sqrt[y]{x-\sqrt[y]{x+\cdots\infty}}}} = \frac{\sqrt{4x-3y^2}+y}{2}$$

$$(xxi) \sqrt{x-\sqrt[y]{x+\sqrt[y]{x-\cdots\infty}}}} = \frac{\sqrt{4x-3y^2}-y}{2}$$

3. If $a+b$ is a constant then the product of a and b can be maximised when the difference of a and b is kept minimum possible. Similarly, when the product needs to be minimised, the difference needs to be kept maximum possible.

4. If $a \times b$ is a constant then the sum of a and b can be minimised when the difference of a and b is kept minimum possible. The sum of squares of a and b can also be minimised at such values of a and b respectively.
5. If there is a surd of the form $a + \sqrt{b}$, then a surd of the form $\pm (a - \sqrt{b})$ is called the conjugate of the surd $a + \sqrt{b}$. The product of a surd and its conjugate always results in a rational number.
6. To convert the denominator of a surd into a rational number, multiply the numerator and the denominator simultaneously with the conjugate of the surd in the denominator, this process is known as rationalisation.
7. If there exists a square root of a surd of the type $a + \sqrt{b}$, then it has to be of the form $\sqrt{x} + \sqrt{y}$. Whenever there is an equation with rational and irrational terms, the rational part on the left hand side is equal to the rational part on the right hand side and the irrational part on the left hand side is equal to the irrational part on the right hand side of the equation.

LOGARITHMS

1. The logarithm of any number to a given base is the power to which the base must be raised in order to equal the given number.

$$\text{If } x^y = Z, \text{ then } y = \log_x Z$$

Here, Z is a positive number and x is a positive number but not 1.

2. Important Logarithmic Formulae

$$(i) \log_x x = 1 \text{ (log of any number to the same base is 1)}$$

$$(ii) \log_x 1 = 0 \text{ (log of 1 to any base other than 1 is 0)}$$

$$(iii) \log(mn) = \log m + \log n$$

$$(iv) \log\left(\frac{m}{n}\right) = \log m - \log n$$

$$(v) \log m^n = n \log m$$

$$(vi) \log_n m = \frac{1}{\log_m n}$$

$$(vii) \log_n m = \frac{\log_x m}{\log_x n}$$

$$(viii) \log_x y m^n = \frac{n}{y} \log_x m$$

$$(ix) m^{\log m} N = N$$

$$(x) m^{\log n} = n^{\log m}$$

3. Given $\log_x m = \log_y n$, then,

$$(i) \text{ If } m = n, \text{ then } x = y \text{ (If } m \neq 1 \text{ and } n \neq 1)$$

$$(ii) \text{ If } x = y, \text{ then } m = n.$$

PERMUTATIONS AND COMBINATIONS

1. **Permutations:** Permutation refers to arrangement. Each of the arrangements which can be made by taking some or all of a given number of items at a time is called a permutation. Here the “order of items” is important *i.e.* ab and ba are different and hence counted separately.

The number of permutations of n things taking r at a time is given by n_{P_r}

2. **Combinations:** Combination refers to selection. Each of the selections which can be made by taking some or all of a given number of items at a time is called a combination. Here the “order of items” does not matter *i.e.* ab and ba are the same and hence included only once.

The number of combinations of n things taking r at a time is given by n_{C_r}

3. Number of arrangements of ‘ n ’ distinct items taking ‘ r ’ at a time (without repetition) $= n_{P_r} = \frac{n!}{(n-r)!}$

4. Number of arrangements of ‘ n ’ items out of which ‘ a ’ are alike, ‘ b ’ are alike, ‘ c ’ are alike and the rest all are distinct $= \frac{n!}{a!b!c!}$

5. Number of arrangements of ' n ' distinct items taking ' r ' at a time (with repetition) $= n^r$
6. Number of combinations of ' n ' distinct items taking ' r ' at a time $= n_{cr} = \frac{n!}{(n-r)!} \times r! = n_{c-r}$
7. Number of ways of selecting one or more items from ' n ' given items $= 2^n - 1$
8. Number of ways of dividing $(m+n)$ items into two groups of m and n items respectively $= \frac{(m+n)!}{m!n!}$
9. Number of ways of dividing $2n$ items into two equal groups of n each where the two groups have distinct identity $= \frac{(2n)!}{(n!)^2}$
10. Number of ways of dividing $2n$ items into two equal groups of n each where the two groups do not have distinct identity $= \frac{(2n)!}{2!(n!)^2}$
11. Number of circular arrangements of ' n ' distinct items when clockwise and anticlockwise arrangements are different $= (n-1)!$
12. Number of circular arrangements of ' n ' distinct items when clockwise and anticlockwise arrangements are the same $= \frac{(n-1)!}{2}$
13. Number of positive integral solutions of $a_1 + a_2 + a_3 + a_4 + \dots + a^r = n$ where $n \geq r = n-1 C_{r-1}$
14. Number of non-negative integral solutions of $a_1 + a_2 + a_3 + a_4 + \dots + a_r = n$ where $n \geq r = n+r-1 C_{r-1}$
15. Sum of all possible n -digit numbers using n distinct digits $= (n-1)! \times (\text{sum of the } n \text{ digits}) \times (1111\dots 11) \text{ } n \text{ times}$
16. The number of derangements of n objects $= n! \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \frac{1}{n!} \right]$
17. The number of ways in which a selection can be made by taking some or all out of $a + b + c + \dots$ things where ' a ' are alike, ' b ' are alike, ' c ' are alike and so on $= \{[(a+1)(b+1)(c+1)\dots] - 1\}$
18. $n+1 C_r = n C_r + n C_{r-1}$
19. $n_{pr} = r \cdot n-1 C_{r-1} + n-1 p_r$
20. **Multiplication Principle:** If an operation can be performed in ' a ' ways following which another operation can be performed in ' b ' ways, then the number of ways in which the two operations can be performed is given by $a \times b$. This can be extended to any number of operations. We multiply when the two operations are performed in succession *i.e.* they are connected by 'AND'.
21. **Addition Principle:** If an operation can be performed in a ways and another operation, which is independent of the first, can be performed in b ways. Then, either of the two operations can be performed in $a + b$ ways. This can be extended to any number of mutually exclusive operations. We add when only one of the two operations is performed at a time *i.e.* the tasks are connected by 'OR'.

PROBABILITY

1. Probability of an event $= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$ denoted as $P(E)$
2. The probability of each event lies between 0 and 1
3. **Complement of an event:** Event of non-occurrence of E denoted as $P(\bar{E})$ and is equal to $1 - P(E)$
4. **Mutually Exclusive:** A set of events are termed as "mutually exclusive" such that if any of them happens, none of the others can happen.
If X, Y are mutually exclusive, then,
 $P(X \cap Y) = 0$
5. **Collectively Exhaustive:** A set of events are termed "collectively exhaustive" such that at least one of them is bound to happen.
If a set of events are both mutually exclusive and collectively exhaustive, then the sum of their probabilities will equal 1.

6. If X and Y are two events, then,

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$

This is known as **addition law of probability**.

7. $P\left(\frac{X}{Y}\right) = \frac{P(X \cap Y)}{P(Y)}$

$P\left(\frac{X}{Y}\right)$ denotes the probability of X , given Y .

This is known as **conditional probability**.

8. **Bayes' Theorem:** Let $X_1, X_2, X_3, \dots, X_n$ be mutually exclusive and collectively exhaustive events with respective probabilities of $c_1, c_2, c_3, \dots, c_n$

Let Y be an event such that $P(Y) \neq 0$ and $P\left(\frac{Y}{X_i}\right)$ for $i = 1$ to n be $k_1, k_2, k_3, \dots, k_n$

Then, the conditional probability of X_i given $Y = \frac{c_i k_i}{c_1 k_1 + c_2 k_2 + c_3 k_3 + \dots + c_n k_n}$

SETS AND FUNCTIONS

1. **Set:** A well defined collection of objects which are known as elements of the set.

A set can be represented in below two forms:

(i) **Set builder form:** The set is described by a characterising property.

(ii) **Roster form:** The set is described by actually listing out the elements.

2. **Empty Set:** A set is said to be an empty set if it has no elements. It is also called a null set or a void set.
3. **Finite Set:** A set is said to be finite, if it is either an empty set or contains a finite number of elements, otherwise it is said to be infinite.
4. **Cardinality or Order of a Set:** It is the number of distinct elements in a set.
5. **Disjoint Sets:** Two sets X and Y are said to be disjoint if they do not have any element in common.
6. **Difference of Sets:** It is the set of all the elements which are there in one set but not in the other.
7. **Symmetric difference of two sets:** Let X and Y be two sets. The symmetric difference of sets X and Y is the set $(X - Y) \cup (Y - X)$
8. **Cartesian product of two sets:** Let X and Y be two sets. Then the cartesian product of X and Y is the set of all ordered pairs (x, y) where $x \in X$ and $y \in Y$, denoted as $X \times Y$.
9. **Complement of a Set:** The complement of Set X is the set of all those elements that do not belong to Set X i.e. the complement of Set X is the difference of the universal set and Set X .
10. **Singleton Set:** A set with a cardinality of 1 i.e. containing only a single element.
11. **Equal Sets:** Two sets X and Y are said to be equal if they have the same elements, i.e. every element of Set X is an element of Set Y and every element of Set Y is an element of Set X .
12. **Subset and Superset:** If every element of Set X belongs to Set Y , then Set X is said to be a subset of Set Y , denoted as $X \subseteq Y$.
If Set X is a subset of Set Y and there is at least one element in Set Y that is not there in Set X , then Set X is said to be a proper subset of Set Y , denoted as $X \subset Y$.
If X is a subset of Y , it means that Y contains X , thus Y is the superset of X , denoted as $Y \supseteq X$.
Every set is a subset of itself.
The empty set is a subset of every set.
If X is a finite set having n elements, then the total number of subsets of $X = 2^n$
13. **Power Set:** If X is any set, then the set of all the subsets of X is known as the power set of X .
The set itself and the empty set are included in the power set of X .
If X is a finite set having n elements, then the cardinality of Power Set of $X = 2^n$
14. **Universal Set:** The set that contains all the elements in a given context.

15. Operations on Sets:

- (i) **Union of Sets:** Let X and Y be two sets, then the union of X and Y is the set of all those elements which belong to either X or to Y or both X and Y , denoted as $X \cup Y$.

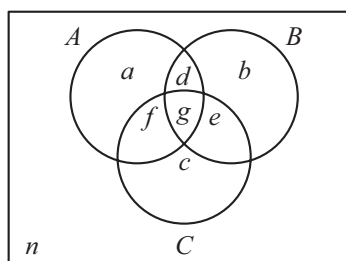
$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

$$n(X \cup Y \cup Z) = n(X) + n(Y) + n(Z) - n(X \cap Y) - n(Y \cap Z) - n(X \cap Z) + n(X \cap Y \cap Z)$$

- (ii) **Intersection of Sets:** Let X and Y be two sets, then the intersection of X and Y is the set of all those elements which belong to both X and Y denoted as $X \cap Y$.

$$n(X \cap Y) = n(X) + n(Y) - n(X \cup Y)$$

16. **Venn Diagram:** A venn diagram is also known as a set diagram and is used to visually represent the relationship between various sets.



a represents only A

b represents only B

c represents only C

d represents A and B but not C

e represents B and C but not A

f represents A and C but not B

g represents A , B and C

h represents none of A , B or C

17. **Modulus Function:** The function defined by $f(x) = |x|$ i.e. $f(x)$ takes only the magnitude of x . Here the domain of f is R and the range of f is the set of all non-negative real numbers.
18. **Exponential function:** A function $f(x) = e^x$ is called an exponential function. Here the domain of f is R and the range of f is the set of all positive real numbers.
19. **Logarithmic function:** A function $f(x) = \log x$ is called a logarithmic function. Here the domain of f is set of all positive real numbers and the range of f is R .
20. **Even and Odd functions:** A function $f(x)$ is said to be an even function if $f(-x) = f(x)$ for all x in its domain.
A function $f(x)$ is said to be odd if $f(-x) = -f(x)$ for every x in its domain.
21. **Greatest Integer Function:** $f(x) = [x]$ is the greatest integer less than or equal to x . Here the domain of f is R and the range of f is the set of all integers.
22. **Signum Function:** The function f defined by

$$sg(x) = \frac{|x|}{x} \text{ if } x \neq 0 \text{ and}$$

$$0 \text{ if } x = 0$$

Here the domain of f is R and the range of f is $\{-1, 0, 1\}$

STATISTICS

1. Following are the measures of central tendencies:

- (i) Arithmetic Mean or Mean
(ii) Geometric Mean

(iii) Harmonic Mean

(iv) Median

(v) Mode

2. **Arithmetic Mean (AM)** = $\frac{\text{Sum of all the observations}}{\text{Total number of observations}}$

3. **Geometric Mean (GM)**: Let $a_1, a_2, a_3, \dots, a_n$ be ' n ' individual observations all being positive, then their Geometric mean = $(a_1 \times a_2 \times a_3 \times \dots \times a_n)^{1/n}$

4. **Harmonic Mean (HM)**: Let $a_1, a_2, a_3, \dots, a_n$ be ' n ' individual observations such that none of them is equal to 0, then,

$$\text{Harmonic Mean} = \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \dots + \frac{1}{a_n}}$$

5. For any two positive numbers a and b ,

(i) $AM \geq GM \geq HM$

(ii) $(GM)^2 = AM \times HM$

6. **Median**: The "middle most" item in a series of values arranged in ascending or descending order.

(i) When the number of observations is odd

$$\text{Median} = \left(\frac{n+1}{2} \right) \text{th term}$$

(ii) When the number of observations is even

$$\text{Median} = \frac{\left(\frac{n}{2} \right) \text{th term} + \left[\left(\frac{n}{2} \right) + 1 \right] \text{th term}}{2}$$

(iii) Median formula for grouped data

$$I + \left[\frac{\frac{n}{2} - c}{f} \right] \times h$$

where,

l = lower limit of the class

n = total number of observations

c = cumulative frequency of the preceding class

f = frequency of median class

h = class size (upper limit – lower limit)

7. **Mode**: It is the most frequently occurring value in a given set of observations i.e. the value occurring the highest number of times.

(i) Mode of grouped data formula

$$I + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h$$

where,

l = lower limit of the modal class

f_1 = frequency of the modal class

f_0 = frequency of the class preceding the modal class

f_2 = frequency of the class succeeding the modal class

h = size of the class interval

8. Empirical formula: **Mode = 3 Median – 2 Mean**

9. **Range:** For a given set of observations,

$$\text{Range} = \text{Maximum value} - \text{Minimum value}$$

10. **Quartile Deviation:** Quartiles are values that divide a given distribution into four equal parts, when the values are arranged in ascending or descending order of magnitude.

Q_1 is the first quartile, Q_2 is the second quartile or the median and Q_3 is the third quartile.

$$\text{Quartile Deviation} = \frac{Q_3 - Q_1}{2}$$

$$Q_1 = \text{size of } \left(\frac{n+1}{4} \right) \text{th item}$$

$$Q_3 = \text{size of } 3 \left(\frac{n+1}{4} \right) \text{th item}$$

11. **Mean Deviation:** It is the average of deviations of each item in the data set from the mean.

$$\text{Mean Deviation} = \frac{\sum_{i=1}^n |a_i - A|}{n}$$

Where 'A' is the mean and 'n' is the number of items.

12. **Standard Deviation:** It is a measure of how spread out the observations are and is the root mean squared deviation taken about the mean.

$$\text{Standard deviation} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} = \sqrt{\frac{\sum (x_i)^2}{n} - (\bar{x})^2}$$

$$\text{Standard deviation} = \sqrt{\text{variance}} .$$



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