Q1 If A = {x : x is an integer and x^2 - 9 = 0} B = {x : x is a natural number and $2 \leq x < 5$}, C = {x : x is a prime number $\leq$ 4} Then (B - C) U A is :

(A) {-3, 3, 4} (B) {2, 3, 4}
(C) {3, 4, 5} (D) {2, 3, 5}

Q2 A and B are two sets having 3 and 6 elements respectively. Consider the following statements.

Statement (I): Minimum number of elements in AUB is 3

Statement (II): Maximum number of elements in AB is 3 Which of the following is correct?

(A) Statement (I) is true, statement (II) is false
(B) Statement (I) is false, statement (II) is true
(C) Both Statements (I) and (II) are true
(D) Both Statements (I) and (II) are false

Q3 Domain of the function f, given by $f(x) = \frac{1}{\sqrt{(x-2)(x-5)}}$ is

(A) $(-\infty,\ 2] \cup [5,\ \infty)$
(B) $(-\infty,\ 2) \cup (5,\ \infty)$
(C) $(-\infty,\ 3) \cup [5,\ \infty)$
(D) $(-\infty,\ 3] \cup (5,\ \infty)$

Q4 If $f(x) = \sin[\pi^2]x - \sin[-\pi^2]x$, where [x] = greatest integer $\leq$ x, then which of the following is not true?

(A) f(0) = 0
(B) f(p/2) = 1
(C) $f\left(\frac{\pi}{4}\right) = 1 + \frac{1}{\sqrt{2}}$
(D) $f(\pi) = -1$

Q5 Which of the following is not correct?

(A) $\cos 5\pi = \cos 4\pi$
(B) $\sin 2\pi = \sin(-2\pi)$
(C) $\sin 4\pi = \sin 6\pi$
(D) $\tan 45^o = \tan(-315^o)$

Q6 If cos x + $\cos^2$x = 1, then the value of $\sin^2$x + $\sin^4$x is

(A) -1 (B) 1
(C) 0 (D) 2

Q7 The mean deviation about the mean for the date $4, 7, 8, 9, 10, 12, 13, 17$ is

(A) 10 (B) 3
(C) 8.5 (D) 4.03

Q8 A random experiment has five outcomes w_1, w_2, w_3, w_4 and w_5. The probabilities of the occurrence of the outcomes w_1, w_2, w_3, w_4 and w_5 are respectively $\frac{1}{6}, a, b$ and $\frac{1}{12}$ such that $12a + 12b - 1 = 0$. Then the probabilities of occurrence of the outcome w_3 is

(A) 2/3 (B) 1/3
(C) 1/6 (D) 1/12

Q9 A die has two face each with number '1', three faces each with number '2' and one face with number '3'. If the die is rolled once, then P(1 or 3) is

(A) 2/3 (B) 1/2
(C) 1/3 (D) 1/6

Q10 Let A = {a, b, c}, then the number of equivalence relations on A containing (b, c) is

(A) 1 (B) 3
(C) 2 (D) 4

Q11 Let the functions "f" and "g" be $f : \left[0,\ \frac{\pi}{2}\right] \rightarrow R$ given by f(x) = sin x and $g : \left[0,\ \frac{\pi}{2}\right] \rightarrow R$ where R is the set of real numbers

Consider the following statements :

Statement (I) : f and g are one-one

Statement (II) : f + g is one-one

Which of the following is correct?

(A) Statement (I) is true, statement (II) is false
(B) Statement (I) is false, statement (II) is true
(C) Both statements (I) and (II) are true
(D) Both statements (I) and (II) are false

Q12 $\sec^2(\tan^{-1} 2) + \operatorname{cosec}^2(\cot^{-1} 3) =$

(A) 1 (B) 5
(C) 15 (D) 10

Q13 $2\cos^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ is valid for all values of 'x' satisfying

(A) $0 \le x \le \frac{1}{\sqrt{2}}$

(B) $-1 \le x \le 1$

(C) $0 \le x \le 1$

(D) $\frac{1}{\sqrt{2}} \le x \le 1$

Q14 Consider the following statements:

Statement (I): In a LPP, the objective function is always linear.

Statement (II): Ina LPP, the linear inequalities on variables are called constraints. Which of the following is correct?

(A) Statement (I) is true, statement (II) is true

(B) Statement (I) is true, Statement (II) is false

(C) Both Statements (I) and (II) are false

(D) Statement (I) is false, Statement (II) is true

Q15 The maximum value of z = 3x + 4y, subject to the constraints $x + y \le 40,\ x + 2y \le 60 \text{ and } x,\ y \ge 0$ is

(A) 130 (B) 120

(C) 140 (D) 40

Q16 Consider the following statements.

Statement (I): If E and F are two independent events, then E' and F' are also independent.

Statement (II): Two mutually exclusive events with non-zero probabilities of occurrence cannot be independent.

Which of the following is correct?

(A) Statement (I) is true and Statement (II) is false

(B) Statement (I) is false and statement (II) is true

(C) Both the statements are true

(D) Both the statements are false

Q17 If A and B are two non-mutually exclusive events such that P(A|B| = P(B|A, then $\mathrm{P}(\mathrm{A} \mid \mathrm{B}) = \mathrm{P}(\mathrm{B} \mid \mathrm{A})$

(A) $A \subset B \text{ but } A \neq B$

(B) A = B

(C) $A \cap B = \phi$

(D) P(A) = P(B)

Q18 If A and B are two events such that $A \subset B \text{ and } P(B) \neq 0,$ then which of the following is correct?

(A) $P(A|B) = \frac{P(B)}{P(A)}$

(B) P(A | B) < P(A)

(C) P(A | B) $\ge$ P(A)

(D) P(A) = P(B)

Q19 Meera visits only one of the two temples A and B in her locality. Probability that she visits temple A is 2/5. If she visits temple A, 1/3 is the probability that she meets her friend, whereas it is 2/7 if she visits temple B. Meera met her friend at one of the two temples. The probability that she met her at temple B is

(A) 7/16 (B) 5/16

(C) 3/16 (D) 9/16

Q20 If Z_1 and Z_2 are two non-zero complex numbers, then which of the following is not true?

(A) $\overline{Z_1 + Z_2} = \overline{Z}_1 + \overline{Z}_2$

(B) $|Z_1 Z_2| = |Z_1| \cdot |Z_2|$

(C) $\overline{Z_1 Z_2} = \overline{Z}_1 \cdot \overline{Z}_2$

(D) $|Z_1 + Z_2| \ge |Z_1| + |Z_2|$

Q21 Consider the following statements :

Statement(I) : The set of all solutions of the linear inequalities 3x + 8 < 17 and 2x + 8 $\ge$ 12 are x < 3 x $\ge$ 2 respectively.

Statement(II) : The common set of solutions of linear inequalities 3x + 8 < 17 and 2x + 8 $\ge$ 12 is (2, 3) Which of the following is true?

(A) Statement (I) is true but statement (II) is false

(B) Statement (I) is false but statement (II) is true

(C) Both the statements are true

(D) Both the statements are false

Q22 The number of four digit even number that can be formed using the digits 0, 1, 2 and 3 without repetition is

(A) 6 (B) 10

(C) 4 (D) 12

Q23 The number of diagonals that can be drawn in an octagon is

(A) 15 (B) 20
(C) 28 (D) 30

Q24 If the number of terms in the binomial expansion of $(2x + 3)^{3n}$ is 22, then the value of n is

(A) 8 (B) 6
(C) 7 (D) 9

Q25 If 4th, 10th and 16th terms of a G.P. are x, y and z respectively, then

(A) $z = \sqrt{xy}$ (B) $y = \sqrt{xz}$
(C) $x = \sqrt{yz}$ (D) $y = \frac{x+z}{2}$

Q26 If A is a square matrix such that $A^2 = A$, then $(I - A)^3$ is

(A) I - A (B) A - I
(C) I + A (D) - I - A

Q27 If A and B are two matrices such that AB is an identity matrix and the order of matrix B is 3 x 4, then the order of matrix A is

(A) 3 x 4 (B) 3 x 3
(C) 4 x 3 (D) 4 x 4

Q28 Which of the following statements is not correct?

(A) A row matrix has only one row
(B) A diagonal matrix has all diagonal elements equal to zero
(C) A symmetric matrix A is a square matrix satisfying A' = A
(D) A skew symmetric matrix has all diagonal elements equal to zero

Q29 If a matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ satisfies $A^6 = kA'$, then the value of k is

(A) 32 (B) 1
(C) 1/32 (D) 6

Q30 If $A = \begin{bmatrix} k & 2 \\ 2 & k \end{bmatrix}$ and $|A^3| = 125$, then the value of k is

(A) ±2 (B) ±3
(C) -5 (D) -4

Q31 If A is a square matrix satisfying the equation $A^2 - 5A + 7I = 0$, where I is the identity matrix and 0 is null matrix of same order, then $A^{-1} =$

(A) 1/7(5I - A) (B) 1/7(A - 5I)
(C) 7(5I - A) (D) 1/5(7I - A)

Q32 if A is a square matrix of order 3 x 3, det A = 3, then the value of det $(3A^{-1})$ is

(A) 1/3 (B) 3
(C) 27 (D) 9

Q33 If $B = \begin{bmatrix} 1 & 3 \\ 1 & \alpha \end{bmatrix}$ be the adjoint of a matrix A and |A| = 2, then the value of a is

(A) 4 (B) 5
(C) 2 (D) 3

Q34 The system of equations 4x + 6y = 5 and 8x + 12 y = 10 has

(A) No solution
(B) Infinitely many solutions
(C) A unique solution
(D) Only two solutions

Q35 If $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \hat{j} + \hat{k}$ āre such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then the value of l is

(A) 1 (B) ±1
(C) -1 (D) 0

Q36 If $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$, then the value of $|\vec{a} \times \vec{b}|$ is

(A) -5 (B) 10
(C) 14 (D) 16

Q37 Consider the following statements :

Statement (I) : If either $\vec{a} = 0$ or $\left|\vec{b}\right| = 0,$ then $\vec{a} \cdot \vec{b} = 0$

Statement (II) : If $\vec{a} \times \vec{b} = \vec{0}$, then a is perpendicular to b. Which of the following is correct?

(A) Statement (I) is true but Statement (II) is false

(B) Statement (I) is false but Statement (II) is true

(C) Both Statement (I) and Statement (II) is true

(D) Both Statement (I) and Statement (II) is false

Q38 If a line makes angles 90^o, 60^o and θ with x, y and z axes respectively, where q is acute, then the value of q is

(A) p/6 (B) p/4

(C) p/3 (D) p/2

Q39 The equation of the line through the point (0, 1, 2) and perpendicular to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$ is

Vasu

(A) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{-3}$

(B) $\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

(C) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{3}$

(D) $\frac{x}{3} = \frac{y-1}{-4} = \frac{z-2}{3}$

Q40 A line passes through (-1, -3) and perpendicular to x + 6y = 5. Its x intercept is

(A) 1/2 (B) -1/2

(C) -2 (D) 2

Q41 The length of the latus rectum of $x^2 + 3y^2 = 12$ is

(A) 2/3 units (B) 1/3 units

(C) $\frac{4}{\sqrt{3}}$ units (D) 24 units

Q42 $\lim\limits_{x\to 1} \frac{x^4-\sqrt{x}}{\sqrt{x}-1}$ is

(A) 0 (B) 7

(C) Does not exist (D) 1/2

Q43 If $y = \frac{\cos x}{1+\sin x}$, then

(i) $\frac{dy}{dx} = \frac{-1}{1+\sin x}$

(ii) $\frac{dy}{dx} = \frac{1}{1+\sin x}$

(iii) $\frac{dy}{dx} = -\frac{1}{2}\sec^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$

(iv) $\frac{dy}{dx} = \frac{1}{2}\sec^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$

(A) Only b is correct

(B) Only a is correct

(C) Both a and c are correct

(D) Both b and d are correct

Q44 Match the following:

In the following, [x] denotes the greatest integer less than or equal to x.

	Column - I		Column - II
(a)	x|x|	(i)	continuous in (-1, 1)
(b)	$\sqrt{\|x\|}$	(ii)	differentiable in (-1, 1)
(c)	x + [x]	(iii)	Strictly increasing in (-1, 1)
(d)	|x - 1| + |x + 1|	(iv)	not differentiable at, at least one point in (-1, 1)

(A) a - i, b - ii, c- iv, d - iii

(B) a - iv, b - iii, c - i, d - ii

(C) a - ii, b - iv, c - iii, d - i

(D) a - iii, b - ii, c - iv, d - i

Q45 The function $f(x)=\begin{cases} e^x + ax, & x < 0 \\ b(x-1)^2 & x \geq 0 \end{cases}$ is differentiable at x = 0. Then

(A) a = 1, b = 1 (B) a = 3, b = 1

(C) a = -3, b = 1 (D) a = 3, b = -1

Q46 A function $f(x)=\begin{cases} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

(A) continuous at x = 0
(B) not continuous at x = 0
(C) differentiable at x = 0
(D) differentiable at x = 0, but not continuous at x = 0

Q47 If $y = a\sin^3 t,\ x = a\cos^3 t,$ then $\frac{dy}{dx}$ at t is $= \frac{3\pi}{4}$

(A) -1
(B) $\frac{1}{\sqrt{3}}$
(C) $-\sqrt{3}$
(D) 1

Q48 The derivative of sin x with respect to log x is

(A) cos x
(B) x cos x
(C) $\frac{\cos x}{\log x}$
(D) $\frac{\cos x}{x}$

Q49 The minimum value of 1 - sin x is

(A) 0
(B) -1
(C) 1
(D) 2

Q50 The function f(x) = tan x - x

(A) always increases
(B) always decreases
(C) never increases
(D) neither increases nor decreases

Q51 The value of $\int \frac{dx}{(x+1)(x+2)}$ is

(A) $\log\left|\frac{x-1}{x+2}\right|+c$
(B) $\log\left|\frac{x-1}{x-2}\right|+c$
(C) $\log\left|\frac{x+2}{x+1}\right|+c$
(D) $\log\left|\frac{x+1}{x+2}\right|+c$

Q52 The value of $\int_{-1}^{1} \sin^5 x\ \cos^4 x\ dx$ is

(A) -p/2
(B) p
(C) p/2
(D) 0

Q53 The value of $\int_{0}^{2x} \sqrt{1+\sin\left(\frac{x}{2}\right)}dx$ is

(A) 8
(B) 4
(C) 2
(D) -2

Q54 $\int \frac{dx}{x^2\left(x^4+1\right)^{3/4}}$ equals

(A) $\left(\frac{x^4+1}{x^4}\right)^{1/4}+c$
(B) $\left(x^4+1\right)^{1/4}+c$
(C) $-\left(x^4+1\right)^{1/4}+c$
(D) $-\left(\frac{x^4+1}{x^4}\right)^{1/4}+c$

Q55 $\int_0^1 \log\left(\frac{1}{x}-1\right)dx$ is

(A) 1
(B) 0
(C) $\log_e 2$
(D) $\log_e(1/2)$

Q56 The area bounded by the curve $y = \sin\left(\frac{x}{3}\right)$, x axis, the lines x = 0 and x = 3p is

(A) 9 sq. units
(B) 1/3 sq. units
(C) 6 sq. units
(D) 3 sq. units

Q57 The area of the region bounded by the curve y = x^2 and the line y = 16 is

(A) 32/3 sq. units
(B) 256/3 sq. units
(C) 64/3 sq. units
(D) 128/3 sq. units

Q58 General solution of the differential equation $\frac{dy}{dx} + y\tan x = \sec x$ is

(A) y sec x = tan x + c
(B) y tan x = sec x + c
(C) cosec x = y tan x + c
(D) x sec x = tan y + c

Q59 If 'a' and 'b' are the order and degree respectively of the differentiable equation. $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 + x^4 = 0$, then a - b =

(A) 1
(B) 2
(C) -1
(D) 0

Q60 The distance of the point P(-3, 4, 5) from yz plane is

(A) 4 units
(B) 5 units
(C) -3 units
(D) 3 units

Answer Key

Q1	A
Q2	B
Q3	B
Q4	D
Q5	A
Q6	B
Q7	B
Q8	A
Q9	B
Q10	C
Q11	A
Q12	C
Q13	D
Q14	A
Q15	C
Q16	C
Q17	D
Q18	C
Q19	D
Q20	D
Q21	A
Q22	B
Q23	B
Q24	C
Q25	B
Q26	A
Q27	C
Q28	B
Q29	A
Q30	B
Q31	A
Q32	D
Q33	B
Q34	B
Q35	C
Q36	D
Q37	A
Q38	A
Q39	B
Q40	B
Q41	C
Q42	B
Q43	C
Q44	C
Q45	C
Q46	B
Q47	D
Q48	B
Q49	A
Q50	A
Q51	D
Q52	D
Q53	A
Q54	D
Q55	B
Q56	C
Q57	B
Q58	A
Q59	D
Q60	D

Hints & Solutions

Note: scan the QR code to watch video solution

Q1 Text Solution:

$A = \{x : x \text{ is an integer and } x^2 - 9 = 0\}$

$x^2 = 9 \Rightarrow x = \pm 3 = \{-3, 3\}$

B

$= \{x : x \text{ is a natural number and } 2 \leq x < 5\}$

$= \{2, 3, 4\}$

$C = \{x : x \text{ is a prime number} \leq 4\} = \{2, 3\}$

$(B - C) \cup A = \{4\} \cup \{-3, 3\} = \{-3, 3, 4\}$

Q2 Text Solution:

$|A| = 3$

$|B| = 6$

$|A \cup B| = |A| + |B| - |A \cap B|$

$\min |A \cup B| = |A| + |B| - \max |A \cap B|$

$= 3 + 6 - 3 = 6$

$|A \cap B| = |A| + |B| - |A \cup B|$

$\max |A \cap B| = |A| + |B| - \min |A \cup B|$

$= 3 + 6 - 6 = 3$

Q3 Text Solution:

$f\left(x\right) = \frac{1}{\sqrt{(x-2)(x-5)}}$

$\Rightarrow (x-2)(x-5) > 0$

$\Rightarrow x \in (-\infty, 2) \cup (5, \infty)$

Q4 Text Solution:

$f\left(x\right) = \sin\left[\pi^2\right]x - \sin\left[-\pi^2\right]x$

$= \sin 9x - \sin(-10)x$

$= \sin 9x + \sin 10x$

$f(\pi) = \sin 9\pi + \sin 10\pi = 0$

Q5 Text Solution:

Let's check each identity:

$\cos 5\pi = \cos(\pi + 4\pi) = -1, \ \cos 4\pi = 1$

These are not equal, so A is false.

$\sin 2\pi = 0, \ \sin(-2\pi) = -\sin 2\pi = 0$

$\sin 4\pi = 0, \ \sin 6\pi = 0$

$\tan 45^{\circ} = 1, \ \tan\left(-315^{\circ}\right) = \tan\left(45^{\circ}\right) = 1$

Only Option A fails, so the incorrect statement is Option A.

Q6 Text Solution:

$\cos x + \cos^2 x = 1$

$\Rightarrow 1 - \cos^2 x = \cos x \quad \Rightarrow \sin^2 x = \cos x$

$\Rightarrow \sin^2 x + \sin^4 x$

$\Rightarrow \cos x + (\cos x)^2 = 1$

Q7 Text Solution:

Mean derivation about mean

$\mu = \frac{4+7+8+9+10+12+13+17}{8}$

$\frac{1}{N} \sum\left(xi - \mu\right) \quad = \frac{80}{8} = 10$

$= \frac{1}{8}\left(6+3+2+1+2+3+7\right) \quad = \frac{24}{8}$

$= 3$

Q8 Text Solution:

$p(w_1) = \frac{1}{6}$

$p(w_2) = a \qquad \Rightarrow \frac{1}{6} + a + b + x + \frac{1}{12} = 1$

$p(w_3) = b \qquad \Rightarrow 12(a+b+x) = 9$

$p(w_4) = c \qquad \Rightarrow a + b + x = \frac{3}{4}$

$p(w_5) = \frac{1}{12} \qquad \Rightarrow \frac{1}{12} + x = \frac{3}{4} \Rightarrow x = \frac{2}{3}$

Q9 Text Solution:

To determine the probability of rolling a 1 or a 3 on this die, let's first outline the probability of each individual outcome:

Probability of rolling a 1, $P(1)$: Since there are 2 faces showing the number 1, the probability is $\frac{2}{6}$.

Probability of rolling a 3, $P(3)$: There is 1 face showing the number 3, so the probability is $\frac{1}{6}$.

To find the probability of rolling a 1 or a 3, we use the rule for the union of two mutually exclusive events:

P(1 U 3) = P(1) + P()

Substituting the known probabilities:

$P(1 \cup 3) = \frac{2}{6} + \frac{1}{6}$

$P(1 \cup 3) = \frac{3}{6}$

Thus, the probability of rolling a 1 or a 3 is 1/2.

Q10 Text Solution:

To determine the number of equivalence relations on the set A = {a, b, c} that include the pair (b, c), we must ensure the relations satisfy the reflexivity, symmetry, and transitivity properties of equivalence relations.

Reflexivity requires that every element is related to itself: (a, a), (b, b) and (c, c) must be included.

Symmetry necessitates that if (b, c) is included, then (c, b) must also be included.

Transitivity requires that for the inclusion of (b, c)

$(b, c)$the set must also consider the relationships between (a, b), (b, a), (a, c) and (c, a)

There are two potential equivalence relations:

Minimal relation involving (b, c):

{(a, a), (b, b), (c, c), (b, c), (c, b)}

Full equivalence connecting all elements:

{(a, a), (b, b), (c, c), (a, b), (b, a), (a, c), (c, a), (b, c), (c, b)}

Thus, there are 2 equivalence relations on A containing (b, c).

Q11 Text Solution:

$f: \text{ one} - \text{one } \left[0, \frac{\pi}{2}\right] \to R.\ f(x) = \sin x$

$g: \text{ one} - \text{one } \left[0, \frac{\pi}{2}\right] \to R.\ f(x) = \cos x$

Statement I is true

$$\left.\begin{array}{c} (f+g):\left[0, \frac{\pi}{2}\right] \to R \\ f+g(x) = \sin x + \cos x \\ (f+g)(0) = 0 \\ (f+g)(\pi/2) = 0 \end{array}\right\} \Rightarrow f$$

$+ g$ is not one $-$ one

Q12 Text Solution:

$$= 1 + \tan^2\left(\tan^{-1} 2\right) + 1 + \cot^2\left(\cot^{-1} 3\right)$$

$$= 1 + 4 + 1 + 9 = 15$$

Q13 Text Solution:

To determine the values of 'x' that satisfy the equation $2\cos^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$, let's analyze the components:

Let x = cos q. This implies q = $\cos^{-1}$x.

The equation can be expressed as $\sin^{-1}\left(2\cos\theta\ \sin\theta\right)$, which simplifies to $\sin^{-1}(\sin 2\theta)$.

Hence, $2\theta = \sin^{-1}(\sin 2\theta)$

For

$\sin^{-1}(\sin 2\theta)$ to equal $2\ \cos^{-1} x,\ i.e.,\ 2\theta,$
 2θ

must lie within the range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Given :

$\theta = \cos^{-1} x,$

 the valid range for θ is $\left[0, \pi/4\right]$

Therefore, $\cos^{-1} x \in \left[0, \pi/4\right]$

This implies :

x lies within $\left[\frac{1}{\sqrt{2}}, 1\right]$ because $\cos(\pi/4) = \frac{1}{\sqrt{2}}$

Thus, the equation is satisfied for values of x in the interval

$$\left[\frac{1}{\sqrt{2}}, 1\right]$$

Q14 Text Solution:

Both statements are true.

Reasoning:

By definition, a Linear Programming Problem (LPP) has a linear objective function, for example

$\max\ z = c_1x_1 + c_2x_2 + \ldots.. + c_nx_n.$

The restrictions on the variables take the form of linear inequalities (or equalities), such as

$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n \le b_i,$

and these are called constraints.

Q15 Text Solution:

$z = 3x + 4y$

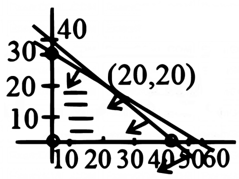
$x + y \le 40$

$x + 2y \le 60$

$y = 20$

$x = 20$

Z	(40,0)	(0,30)	(20,20)
3x+4y	120	120	140



Q16 Text Solution:

To enhance understanding of the statements, let's examine each one in detail:

Statement (I): If events E and F are independent, the complements of these events, E' and F', are also independent. This statement is true.

When two events, E and F, are independent, the occurrence of one does not affect the likelihood of the other occurring, mathematically expressed as $P(E \cap F) = P(E) \times P(F)$.
Similarly, the complements of these events, E' and F', retain this independence property.
Statement (II): Two mutually exclusive events cannot be independent if they have non-zero probabilities. This statement holds true.
Mutually exclusive events are defined by $A \cap B = \phi$ which implies $P(A \cap B) = 0$. For these events to be independent, it would require $P(A \cap B) = P(A) \times P(B)$. However, $P(A) \neq 0$ and $\mathrm{P}(B) \neq 0$, then $P(A \cap B) \neq 0$, contradicting the independence condition. Thus, two non-zero mutually exclusive events cannot be independent.
Conclusively, **Statement I is true and Statement II is also true.**

Q17 Text Solution:

A and B are non mutually exclusive

$$P(A|B) = P(B|A)$$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(B) = P(A)[\because P(A \cap B) \neq 0]$$

Q18 Text Solution:

$$A \subset B \Rightarrow A \cap B = A$$

$$P\left(A \mid B\right) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$$

$$\Rightarrow P(A \mid B)P(B) = P(A) \Rightarrow P(A) \geq P(A \mid B)$$

$$[\because P(B) \neq 0]$$

Q19 Text Solution:

$\mathrm{P}(\mathrm{A}) = \frac{2}{5}$ F : The events of meera meets her friend.

$$\mathrm{P}(\mathrm{F/A}) = \frac{1}{3}$$

$$\mathrm{P}(\mathrm{F/B}) = \frac{2}{7}$$

$$\mathrm{P}(\mathrm{B}) = 1 - \mathrm{P}(\mathrm{A}) = 1 - \frac{2}{5} = \frac{3}{5}$$

The probability she meet her at temple B

$$P\left(B/F\right) = \frac{P(F \cap B)}{P(F)}$$

$$= \frac{P(B) \times P(B/F)}{P(A)P(F/A) + P(B)P(B/F)}$$

$$= \frac{3/5 \times 2/7}{(2/5 \times 1/3) + (3/5 \times 5/7)} = \frac{9}{16}$$

Q20 Text Solution:

$$|Z_1 + Z_2| \leq |Z_1| + |Z_2|$$

Q21 Text Solution:

$$3x + 8 < 17 \Rightarrow 3x < 9 \Rightarrow x < 3$$

$$2x + 8 \geq 12 \Rightarrow 2x \geq 4 \Rightarrow x \geq 2$$

Statement I is correct.
The common set of solution $\Rightarrow \{2\}$
$\rightarrow$ Statement II is false

Q22 Text Solution:

Here's a quick breakdown by cases (last digit must be 0 or 2):
Last digit = 0
First digit can be 1, 2 or 3 3 choices
The two middle places are filled by the remaining 2 digits in 2! = 2 ways$2! = 2$
$\Rightarrow 3 \times 2 = 6$ numbers
Last digit = 2
First digit can be 1 or 3 (can't be 0 or 2) 2 choices
The two middle places are filled by the remaining 2 digits in 2! = 2 ways
$\Rightarrow$ $2 \times 2 = 4$ numbers
$\text{Total} = 6 + 4 = 10$

Q23 Text Solution:

To find the number of diagonals in an octagon (a polygon with 8 sides), use the formula for the number of diagonals in a polygon, which is: $\frac{n(n-3)}{2}$
where n is the number of sides of the polygon.
For an octagon, n = 8. Plug this value into the formula:

$$\frac{8(8-3)}{2} = \frac{8 \times 5}{2} = \frac{40}{2} = 20$$

Therefore, an octagon has 20 diagonals.

Q24 Text Solution:

A binomial expansion $(ax+b)^k$ has $k+1$ distinct terms. Here k = 3n, so :

Number of terms = 3n + 1

Set 3n + 1 = 22

3n + 1 = 22

3n = 21

n = 7

Q25 Text Solution:

In a G.P. the nth term is ar^{n-1}. Thus,

4th term : x = ar^3

10th term : y = ar^9

16th term z = ar^{15}

Now

$xz = (ar^3)(ar^{15}) = a^2r^{18} = (ar^9)^2 = y^2$

So $y^2 = xz \quad \Rightarrow y = \sqrt{xz}$

Q26 Text Solution:

$A^2 = A. \quad (I-A)^3$

$= (I-A)(I-2A+A^2)$

$= (I-A)(I-2A+A)$

$= (I-A)(I+A)$

$= I - A^2 = I - A$

Q27 Text Solution:

Let's find the size of A step by step:

B has size 3 x 4.

For AB to be defined, if A is m x n, then n must equal 3.

So A is m x 3, and AB has size $(m\times 3)(3\times 4)$=$(m\times 4)$

Therefore

A is 4 x 3.

Q28 Text Solution:

A diagonal matrix need not be contains only zero as it diagonal element.

Q29 Text Solution:

$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$A^3 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 2^2 & 2^2 \\ 2^2 & 2^2 \end{bmatrix}$

$A^4 = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix} = \begin{bmatrix} 2^3 & 2^3 \\ 2^3 & 2^3 \end{bmatrix}$

$A^6 = \begin{bmatrix} 2^5 & 2^5 \\ 2^5 & 2^5 \end{bmatrix} = 2^5\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow 2^5 = 32$

$\therefore k = 32$

Q30 Text Solution:

$A = \begin{bmatrix} k & 2 \\ 2 & k \end{bmatrix}$

$|A| = k^2 - 4$

$|A|^3 = |A|\cdot|A|\cdot|A| = 125, \text{ given}$

$\Rightarrow (k^2-4)^3 = 125$

$\Rightarrow k^2 - 4 = 5 \Rightarrow k^2 = 9$

$\Rightarrow k = \pm 3$

Q31 Text Solution:

$A^2 - 5A + 7I$

$= 0 \ (\because \text{Multiply by} A^{-1} \text{both sides} |A| \neq 0)$

$A - 5I + 7A^{-1} = 0$

$A^{-1} = \frac{(5I-A)}{7}$

Q32 Text Solution:

First note for a 3×3 matrix A with det A = 3:

$\det(A^{-1}) = \frac{1}{\det A} = \frac{1}{3}$

When you multiply a 3×3 matrix by a scalar 3, its determinant is multiplied by 3^3.

So $\det(3A^{-1}) = 3^3 \det(A^{-1}) = 27 \times \frac{1}{3} = 9$

Hence the correct choice is Option D: 9.

Q33 Text Solution:

Let B = adj (A) for a 2 x 2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

Recall that for 2 x 2.

$\text{adj}(A) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

Here we're given $B = \begin{pmatrix} 1 & 3 \\ 1 & \alpha \end{pmatrix}$.

By matching entries:

$d = 1$

$-b = 3 \quad \Rightarrow b = -3$

$-c = 1 \quad \Rightarrow c = -1$

$a = \alpha$

So $A = \begin{pmatrix} \alpha & -3 \\ -1 & 1 \end{pmatrix}, \quad |A| = \alpha \cdot 1$

$- (-3)(-1) = \alpha - 3.$

Since |A| = 2, we get

$\alpha - 3 = 2 \quad \Rightarrow \alpha = 5.$

Q34 Text Solution:

Notice that the second equation is just twice the first:

First equation:

4x + 6y = 5

Multiply both sides by 2:

8x + 12y = 10

Since both equations represent the same line, there are infinitely many solutions.

Q35 Text Solution:

Given

$\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + 4\hat{k}$, $\vec{c} = \hat{i} + \hat{j} + \hat{k}$

We need the expression $\vec{a} + \lambda \vec{b}$ to be perpendicular to $\vec{c}$.

$\left(\vec{a} + \lambda \vec{b}\right) \cdot \vec{c} = 0$

First, calculate $\vec{a} + \lambda \vec{b}$

$\vec{a} + \lambda \vec{b} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + 4\hat{k})$

Expanding this, we get:

$= \hat{i} + 2\hat{j} + \hat{k} + \lambda\hat{i} - \lambda\hat{j} + 4\lambda\hat{k}$

$= (1 + \lambda)\hat{i} + (2 - \lambda)\hat{j} + (1 + 4\lambda)\hat{k}$

Next, calculate the dot product with $\vec{c}$:

$\left((1 + \lambda)\hat{i} + (2 - \lambda)\hat{j} + (1 + 4\lambda)\hat{k}\right) \cdot (\hat{i} + \hat{j} + \hat{k})$

Performing the dot product:

$= (1 + \lambda) \cdot 1 + (2 - \lambda) \cdot 1 + (1 + 4\lambda) \cdot 1$

$= (1 + \lambda) + (2 - \lambda) + (1 + 4\lambda)$

$= 1 + \lambda + 2 - \lambda + 1 + 4\lambda$

$= 4 + 4\lambda$

Set the expression to zero for perpendicularity:

$4 + 4\lambda = 0$

Solving for l:

4l = -4

l = -1

Thus, the correct value of l is -1.

Q36 Text Solution:

To find the value of $|\vec{a} \times \vec{b}|$ remember that the relationship with the dot product and magnitudes is given by:

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta$

where q is the angle between the vectors $\vec{a}$ and $\vec{b}$.

We use the formula for the dot product to calculate cos q.

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Given $|\vec{a}| = 10$

$|\vec{b}| = 2$

$\vec{a} \cdot \vec{b} = 12$

Substitute these values into the dot product formula:

$12 = 10 \times 2 \times \cos\theta$

Thus,

$12 = 20\cos\theta \Rightarrow \cos\theta = \frac{12}{20} = \frac{3}{5}$

Since $\sin^2\theta + \cos^2\theta = 1$, we find $\sin\theta$:

$\sin^2\theta = 1 - \left(\frac{3}{5}\right)^2 = 1 - \frac{9}{25} = \frac{16}{25}$

$\sin\theta = \frac{4}{5}$

Now, substitute back to find the magnitude of the cross product:

$|\vec{a} \times \vec{b}| = 10 \times 2 \times \frac{4}{5} = 20 \times \frac{4}{5} = 16$

Therefore, the value of $|\vec{a} \times \vec{b}|$ is 16.

Q37 Text Solution:

Let's check each statement in turn:

Statement (I): "If either $\vec{a} = 0$ or $|\vec{b}| = 0$, then $\vec{a} \cdot \vec{b} = 0$

- If one of the vectors is the zero vector, say $\vec{a} = \vec{0}$, then for any $\vec{b}$

$\vec{0} \cdot \vec{b} = 0$

Hence (I) is true.

Statement (II) : If $\vec{a} \times \vec{b} = \vec{0}$, then $\vec{a}$-is perpendicular to $\vec{b}$.

Recal $\left\|\vec{a}\times\vec{b}\right|=\left|\vec{a}\right|\left|\vec{b}\right|\sin\theta$, where θ is the angle between them.

$\vec{a}\times\vec{b}=\vec{0}$-means

$\sin\theta=0,$ so $\theta=0$ or π :the vectors are parallel (or nor is zero), not perpendicular.

Hence (II) is false

Conclusion:

Statement (I) is true, Statement (II) is false.

Q38 Text Solution:

A line's direction cosines along the x, y, z axes are

$\cos\alpha,\ \ \cos\beta,\ \ \cos\gamma$

where $\alpha=90^{\circ},\ \beta=60^{\circ}$ and $\gamma=\theta.$

They satisfy $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1.$

Compute the known cosines:

$\cos\alpha=\cos 90^{\circ}=0$

$\cos\beta=\cos 60^{\circ}=\frac{1}{2}$

Plug in and solve for cos g:

$0^2+\left(\frac{1}{2}\right)^2+\cos^2\theta=1 \quad\Rightarrow \frac{1}{4}+\cos^2\theta$

$=1 \quad\Rightarrow \cos^2\theta=\frac{3}{4}$

Since θ is acute, $\cos\theta>0,$ so

$\cos\theta=\frac{\sqrt{3}}{2}$

Therefore $\theta=\text{arc}\ \cos\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{6}$

Q39 Text Solution:

The given line has direction vector

v = (2, 3, -2)

We seek a directi on $d=(d_1,\ d_2,\ d_3)$ with

$v\cdot d=2d_1+3d_2-2d_3=0$

One convenicent choice is

$d_1=-3,\ d_2=4,\ d_3=3$

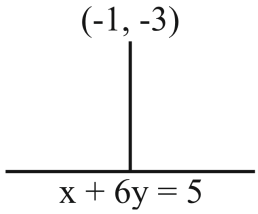
Since

2(-3) + 3(4) - 2(3) = -6 + 12 - 6 = 0

Hence the required line through (0, 1, 2) is $\frac{x-0}{-3}=\frac{y-1}{4}=\frac{z-2}{3},$

Which is option B.

Q40 Text Solution:



y + 3 = 6(x + 1)

x intercept y = 0

x = -1/2

Q41 Text Solution:

$\frac{x^2}{12}+\frac{y^2}{4}=1$

$\text{L.R}=\frac{2b^2}{a}=\frac{2\times4}{2\sqrt{3}}=\frac{4}{\sqrt{3}}$

Q42 Text Solution:

$\lim\limits_{x\to1}\frac{x^4-\sqrt{x}}{\sqrt{x}-1}=\lim\limits_{x\to1}\frac{4x^3-\frac{1}{2\sqrt{x}}}{\frac{1}{2\sqrt{x}}}=\lim\limits_{x\to1}\frac{\frac{8x^3\sqrt{x}-1}{2\sqrt{4}}}{\frac{1}{2\sqrt{x}}}$

$=7$

Q43 Text Solution:

$y=\frac{\cos x}{1+\sin x}$

$y'=\frac{-\sin x(1+\sin x)-\cos x(\cos x)}{(1+\sin x)^2}=\frac{-(1+\sin x)}{(1+\sin x)^2}$

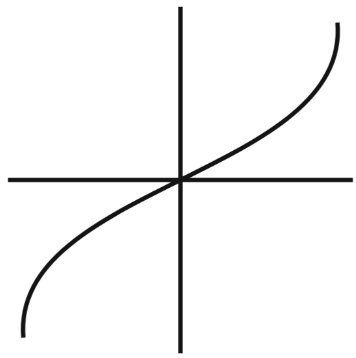
$y'=\frac{-1}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}=\frac{-1}{1+\sin x}$

$=\frac{-1}{2\cdot\left(\frac{1}{\sqrt{2}}\cdot\cos\frac{x}{2}+\frac{1}{\sqrt{2}}\sin\frac{x}{2}\right)^2}=\frac{-1}{2\cdot\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}$

$=\frac{-1}{2}\cdot\sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)$

Q44 Text Solution:

(a) $x|x|f(x)=\begin{cases} x^2, & x\geq0 \\ -x^2 & x<0\end{cases}$ differentiable in (-1, 1)



(b) $\sqrt{|x|}=\begin{cases}\sqrt{x}, & x\geq0 \\ \sqrt{-x}, & x<0\end{cases}$ Not differentiable at x = 0

(c) x + [x] strictly increasing in (-1, 1)

(d)

$|x-1|+|x+1|$

$=\{ \quad -x+1-x-1 \qquad x<-1 \quad -x+1+x+1$

Continuous

$$(-1,\ 1)=\begin{cases} -2x, & x<-1 \\ 2 & -1<x<1 \\ 2x & x>1 \end{cases}$$

Q45 Text Solution:

$$f(x)=\begin{cases} e^x+ax, & x<0 \\ b(x-1)^2 & x\geq 0 \end{cases}$$

Continuity LHL = 1

RHL = b ⇒ b = 1

Differentiability LHD = 1 + a

RHD = -2b

1 + a = -2b

a = -3

Q46 Text Solution:

$$f(x)=\begin{cases} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} & \text{if } x\neq 0 \\ 0, & \text{if } x=0 \end{cases}$$

LHL = -1, RHL = 1, not continuous.

Q47 Text Solution:

$y=a\sin^3 t,\ x=a\cos^3 t$

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{3a\sin^2 t\cos t}{3a\cos^2 t(-\sin t)}=-\tan t$$

$at\ t=\frac{3\pi}{4}$

$-\tan\frac{3\pi}{4}=1$

Q48 Text Solution:

To find the derivative of sin x with respect to log x, we need to use the concept of implicit differentiation and the chain rule.

Let f(x) = sin x

We want to find $\frac{d(\sin x)}{d(\log x)}$

Using the chain rule, we have:

$$\frac{d(\sin x)}{d(\log x)}=\frac{d(\sin x)}{dx}\cdot\frac{dx}{d(\log x)}$$

First, we calculate each part separately:

$$\frac{d(\sin x)}{dx}=\cos x$$

To find $\frac{dx}{d(\log x)}$, consider that $\frac{d(\log x)}{dx}=\frac{1}{x}$,

hence : $\frac{dx}{d(\log x)}$

$$\frac{dx}{d(\log x)}=x$$

Putting it all together, we get:

$$\frac{d(\sin x)}{d(\log x)}=\cos x\cdot x=x\cos x$$

Therefore, the derivative of sin x with respect to log x is x cos x.

Q49 Text Solution:

To find the minimum value of the expression f(x) = 1 - sin x,

we start by considering the range of the sine function.

Since $-1\leq\sin x\leq 1$, this means the minimum value of sin x is -1.

Now, compute f(x) when sin x = -1:

f(x) = 1 - (-1) = 1 + 1 = 2

Check if sin x = 1:

f(x) = 1 - 1 = 0

Thus, the minimum value of f(x) = 1 - sin x occurs when sin x = 1, making the minimum value :

f_{min} = 0

Q50 Text Solution:

The function given is f(x) = tan x - x.

To determine whether this function is increasing, decreasing, or neither, we need to analyze its derivative:

$$f'(x)=\sec^2 x-1=\tan^2 x$$

Since $\tan^2 x$ is always greater than or equal to zero $(\tan^2 x\geq 0)$ the derivative f'(x) is non-negative across its domain. This implies that the function f(x) is always increasing or at the very least, non-decreasing.

Thus, the function f(x) = tan x - x consistently increases.

Q51 Text Solution:

$$\int\frac{dx}{(x+1)(x+2)}=\int\frac{(x+2)-(x+1)}{(x+1)(x+2)}dx$$

$$=\int\frac{1}{x+1}dx-\int\frac{1}{x+2}dx$$

$$=\log\left|\frac{x+1}{x+2}\right|+c$$

Q52 Text Solution:

$$\int_{-1}^{1}\sin^5 x\cos^4 x\,dx=0$$

Since it is odd function.

Q53 Text Solution:

$\int_0^{2\pi}\sqrt{1+\sin\frac{x}{2}}$

$\int_0^{2\pi}\left|\cos\frac{x}{4}+\sin\frac{x}{4}\right| = \left[4\sin\frac{x}{4}-4\cos\frac{x}{4}\right]_0^{2\pi}$

$= (4(1) - 0(0-4) = 8$

Q54 Text Solution:

$\int\frac{dx}{x^2\left(x^4+1\right)^{3/4}} = \int\frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}$

$1+\frac{1}{x^4} = T$

$-4x^{-5}dx = dt$

$= -\frac{1}{4}\int t^{-3/4}dt = -\frac{1}{4}\frac{t^{1/4}}{\frac{1}{4}}+c =$

$-\left(1+\frac{1}{x^4}\right)^{1/4}$

Q55 Text Solution:

$\int_0^1\log\left(\frac{1}{x}-1\right)dx = \int_0^1\log\left(\frac{1-x}{x}\right)dx$

$I = \int_0^1\log\left(\frac{1}{1+0-x}-1\right) =$

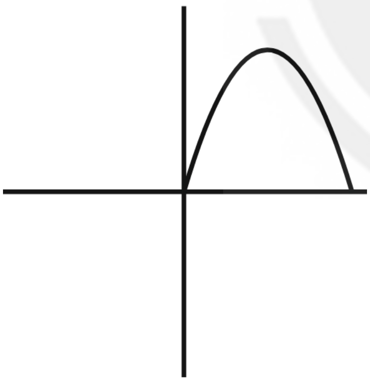
$\int_0^1\log\left(\frac{1}{1-x}-1\right)dx$

$I = \int_0^1\log\left(\frac{1-1+x}{1-x}\right)dx = \int_0^1\log\left(\frac{x}{1-x}\right)dx$

$2I = \int_0^1\left(\log\left(\frac{1-x}{x}\right)+\log\left(\frac{x}{1-x}\right)\right)dx$

$2I = 0 \quad \Rightarrow I = 0$

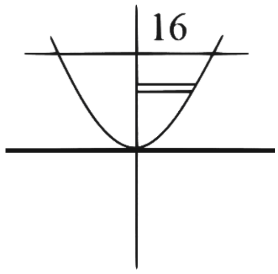
Q56 Text Solution:



$\int_0^{3\pi}\sin\left(\frac{x}{3}\right)dx$

$\left(-3\cdot\cos\frac{x}{3}\right)_0^{3\pi} = (-3(-1))-(-3(1)) = 6$

Q57 Text Solution:



$y = x^2,\ y = 16$

$2\int_0^{16}\sqrt{y}dy = 2\frac{y^{3/2}}{3/2}$

$= \frac{4}{3}(4)^3 = \frac{256}{3}$

Q58 Text Solution:

$\frac{dy}{dx}+\left(\tan x\right)y = \sec x$

$I\cdot F = e^{(-)\int-\frac{\sin x}{\cos x}dx} = e^{-\log_e\cos x} = \sec x$

$\therefore y\cdot\sec x = \int\sec^2 x dx$

$y\sec x = \tan x + c$

Q59 Text Solution:

Here the highest derivative is $\frac{d^2y}{dx^2}$ so the order a = 2.

The equation is a polynomial in derivatives and the highest derivative appears as $\left(\frac{d^2y}{dx^2}\right)^2$,

So the degree b = 2.

Hence a - b = 2 - 2 = 0.

Q60 Text Solution:

The yz-plane is given by x = 0. So the distance from $P\left(x_0,\ y_0,\ z_0\right)$ to that plane is

$d = |x_0| = |-3| = 3.$