Subject : Mathematics
Class : XI

Kannada

Conic Section

Standard Equations of the Circle

(a) **Central Form :** If (h, k) is the centre and r is the radius of the circle then its equation is $(x-h)^2 + (y-k)^2 = r^2$.

(b) **General equation circle :** $x^2 + y^2 + 2gx + 2fy + c = 0$, where g, f, c are constants and centre is $(-g, -f)$

i.e., $\left(-\frac{\text{coefficient of } x}{2}, -\frac{\text{coefficient of } y}{2}\right)$

and radius $r = \sqrt{g^2 + f^2 - c}$

Intercepts cut by the circle on axes:

The intercepts cut by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ on :

(i) x-axis $= 2\sqrt{g^2 - c}$

(ii) y-axis $= 2\sqrt{f^2 - c}$

Diameter form of circle:

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

The parametric forms of the circle:

(i) The parametric equation of the circle $x^2 + y^2 = r^2$ are $x = r\cos\theta$, $y = r\sin\theta$; $\theta \in [0, 2\pi]$.

(ii) The parametric equation of the circle $(x-h)^2 + (y-k)^2 = r^2$ is $x = h + r\cos\theta$, $y = k + r\sin\theta$ where θ is parameter.

General Equation of A Conic : Focal Directrix Property :

The general equation of a conic with focus (p, q) & directrix $lx + my + n = 0$ is:

$(l^2 + m^2)[(x-p)^2 + (y-q)^2] = e^2(lx + my + n)^2$

$\equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

Case (i) When the focus lines on the directrix

In this case $D \equiv abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ & the general equation of a conic represents a pair of straight lines and if:

$e > 1$, $h^2 > ab$ the lines will be real & distinct intersecting at S.

$e = 1$, $h^2 = ab$ the lines will coincident.

$e < 1$, $h^2 < ab$ the lines will be imaginary.

When the focus does not lie on the directix

The conic represents:

A parabola	An ellipse	A hyperbola	A rectangular hyperbola
$e = 1$; $D \neq 0$ $h^2 = ab$	$0 < e < 1$; $D \neq 0$ $h^2 < ab$	$D \neq 0$; $e > 1$ $h^2 > ab$	$e > 1$; $D \neq 0$ $h^2 > ab$; $a + b = 0$

Standard equation of a parabola is y2 = 4ax. For this parabola:

(i) Vertex is (0, 0)

(ii) Focus is (a, 0)

(iii) Axis is y = 0

(iv) Directrix is x + a = 0

Latus Rectum

A focal chord perpendicular to the axis of a parabola is called the LATUS RECTUM. For $y^2 = 4ax$.

(i) Length of the latus rectum = $4a$.

(ii) Length of the semi latus rectum = $2a$.

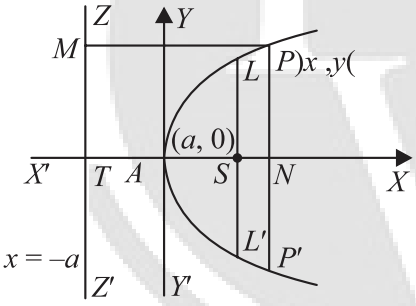
(iii) Ends of the latus rectum are $L(a, 2a)$ & $L'(a, -2a)$.

Parametric Representation

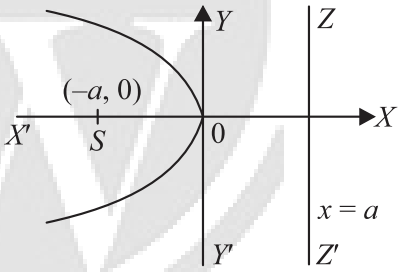
The simplest & the best form of representing the co-ordinates of a point on the parabola $y^2 = 4ax$ is $(at^2, 2at)$. The equation $x = at^2$ & $y = 2at$ together represents the parabola $y^2 = 4ax$, t being the parameter.

Types of Parabola:

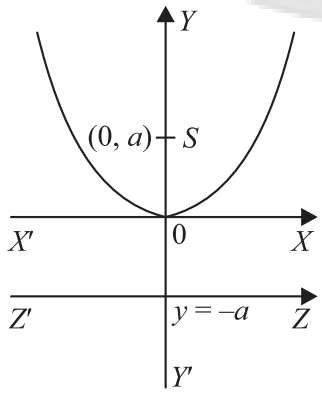
Four standard forms of the parabola are $y^2 = 4ax$; $y^2 = -4ax$; $x^2 = 4ay$; $x^2 = -4ay$.



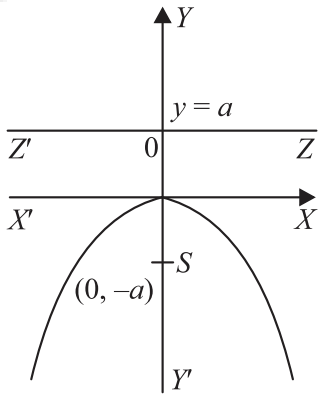
$y^2 = 4ax$



$y^2 = -4ax$



$x^2 = 4ay$



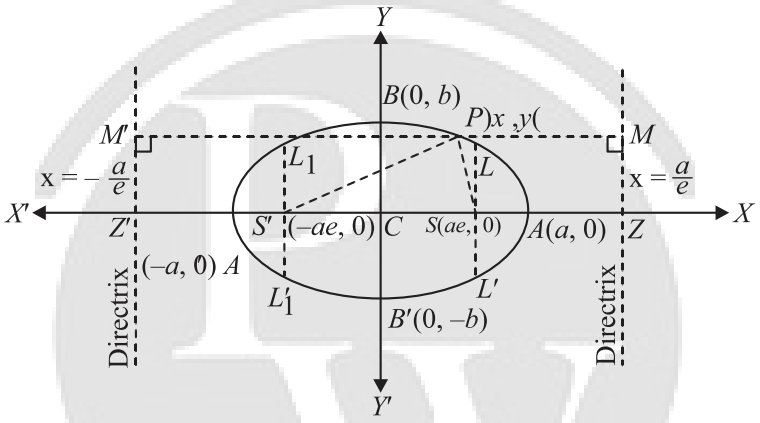
$x^2 = -4ay$

Parabola	Vertex	Focus	Axis	Directrix	Length of Latus rectum	Ends of Latus rectum	Para-metric equation	Focal length
$y^2 = 4ax$	$(0, 0)$	$(a, 0)$	$y = 0$	$x = -a$	$4a$	$(a, \pm 2a)$	$(at^2, 2at)$	$x + a$
$y^2 = -4ax$	$(0, 0)$	$(-a, 0)$	$y = 0$	$x = a$	$4a$	$(-a, \pm 2a)$	$(-at^2, 2at)$	$x - a$
$x^2 = +4ay$	$(0, 0)$	$(0, a)$	$x = 0$	$y = -a$	$4a$	$(\pm 2a, a)$	$(2at, at^2)$	$y + a$
$x^2 = -4ay$	$(0, 0)$	$(0, -a)$	$x = 0$	$y = a$	$4a$	$(\pm 2a, -a)$	$(2at, -at^2)$	$y - a$
$(y-k)^2 = 4a(x-h)$	(h, k)	$(h + a, k)$	$y = k$	$x + a - h = 0$	$4a$	$(h + a, k \pm 2a)$	$(h + at^2, k + 2at)$	$x - h + a$
$(x-p)^2 = 4b(y-q)$	(p, q)	$(p, b + q)$	$x = p$	$y + b - q = 0$	$4b$	$(p \pm 2a, q + a)$	$(p + 2at, q + at^2)$	$y - q + b$

The co-ordinate axis is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Where $a > b$ and $b^2 = a^2(1 - e^2)$

$\Rightarrow a^2 - b^2 = a^2e^2$.

Where e = eccentricity $(0 < e < 1)$.



FOCI : S = (ae, 0) and $S' \equiv (-ae,\ 0)$.

(j) **Latus Rectum:** The focal chord perpendicular to the major axis is called the latus rectum.

(i) Length of latus rectum

$$(LL') = \frac{2b^2}{a} = \frac{(\text{minor axis})^2}{\text{major axis}} = 2a(1 - e^2)$$

(ii) Equation of latus rectum : $x = \pm ae$.

(iii) Ends of the latus rectum are $L\left(ae, \frac{b^2}{a}\right), L'\left(ae, -\frac{b^2}{a}\right)$,

$L_1\left(-ae\frac{b^2}{a}\right)$ and $L_1'\left(-ae, -\frac{b^2}{a}\right)$

(k) **Focal Radii:** $SP = a - ex$ and $S''P = a + ex$

$\Rightarrow SP + S'P = 2a = \text{Major axis.}$

(l) **Eccentricity:** $e = \sqrt{1\frac{b^2}{a^2}}$

Position of a Point W.r.t. an Ellipse

The point $P(x_1, y_1)$ lines outside, inside or on the ellipse according as; $\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1 >< \text{or} = 0$.

Paramatric Representation:

The equations $x = a\cos\theta$ and $y = b\sin\theta$ together represent the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where θ is parameter (eccentric angle).

Note that if $P(\theta) \equiv (a\cos\theta, b\sin\theta)$ is on the ellipse then; $Q(\theta) \equiv (a\cos\theta, a\sin\theta)$ is on the auxiliary circle.

Standard equation of the hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$,

Where $b^2 = a^2(e^2 - 1)$

Or $a^2e^2 = a^2 + b^2$ i.e., $e^2 = 1+\frac{b^2}{a^2} = 1+\left(\frac{\text{Conjugate Axis}}{\text{Transverse Axis}}\right)^2$

(a) Foci : $S \equiv (ae, 0)$ and $S' \equiv (-ae, 0)$.

(b) Equations of Directrices: $x=\frac{a}{e}$ and $x=-\frac{a}{e}$.

(c) Vertices: $A \equiv (a, 0)$ and $A' \equiv (-a, 0)$

(d) Latus Rectum:

(i) Equation: $x = \pm ae$

(ii) Length $=\frac{2b^2}{a}=\frac{(\text{Conjugate Axis})}{(\text{Transverse Axis})}=2a(e^2-1)$

$= 2e$ (distance from focus to directrix)

(iii) Ends: $\left(ae, \frac{b^2}{a}\right), \left(ae, \frac{-b^2}{a}\right); \left(-ae, \frac{b^2}{a}\right), \left(-ae, \frac{-b^2}{a}\right)$

(e) Focal Property :

The difference of the focal distances of any point on the hyperbola is constant and equal to transverse axis i.e. $|\,|PS| - |PS'|\,| = 2a$. The distance $SS' =$ focal length.

(f) Focal Distance:

Distance of any point $P(x, y)$ on hyperbola from foci $PS = ex - a$ & $PS' = ex + a$.

Conjugate Hyperbola : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ & $-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are conjugate hyperbola of each.

Auxillary Circle : $x^2 + y^2 = a^2$.

Parametric Representation : $x = a\sec\theta$ & $y = b\sec\theta$

Position of *A* point '*P*' w.r.t. *A* Hyperbola :

$S_1 \equiv \frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}-1 > = \text{or } < 0$ according as the point (x_1, y_1) lines inside, on or outside the curve.

Tangents :

(i) Slope Form : $y = m\times \pm\sqrt{a^2m^2 - b^2}$

(ii) Point Form : at the point (x_1, y_1) is $\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$.

(iii) Parametric Form : $\frac{x\sec\theta}{a}-\frac{y\tan\theta}{b}=1$

Normal to The Hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$:

(a) **Point form :** Equation of the normal to the given hyperbola at the point $P(x_1, y_1)$ on it is

$$\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2=a^2e^2.$$

(b) **Slope form :** The equation of normal of slope m to the given hyperbola is $y=mx\mp\frac{m(a^2+b^2)}{\sqrt{(a^2-m^2b^2)}}$ foot of normal are $\left(\pm\frac{a^2}{\sqrt{(a^2-m^2b^2)}},\mp\frac{mb^2}{\sqrt{(a^2-m^2b^2)}}\right)$

(c) **Parametric form :** The equation of the normal at the point $P(a\sec\theta, b\tan\theta)$ to the given hyperbola is

$$\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2=a^2e^2.$$

