Subject : Physics

Class : XI

Kannada

Rotational Dynamics

CENTRE OF MASS OF A SYSTEM OF 'N' DISCRETE PARTICLES

$$\vec{r}_{cm}=\frac{m_1\vec{r}_1+m_2\vec{r}_2+\cdots+m_n\vec{r}_r}{m_1+m_2+\cdots+m_n};\vec{r}_{cm}=\frac{\sum_{i=1}^{n}m_i\vec{r}_i}{\sum_{i=1}^{n}m_i},\vec{r}_{cm}=\frac{1}{M}\sum_{i=1}^{n}m_i\vec{r}_i$$

CENTRE OF MASS OF A CONTINUOUS MASS DISTRIBUTION

$$x_{cm}\aleph\frac{\int xdm}{\int dm},y_{cm}\frac{\int ydm}{\int dm},z_{cm}\frac{\int zdm}{\int dm}$$

$\int dm = M$ (mass of the body)

MOTION OF CENTRE OF MASS AND CONSERVATION OF MOMENTUM

Velocity of Centre of Mass of System

$$\vec{v}_{cm}=\frac{m_1\frac{\overrightarrow{dv_1}}{dt}+m_2\frac{\overrightarrow{dr_2}}{dt}+m_3\frac{\overrightarrow{dr_3}}{dt}\cdots+m_n\frac{\overrightarrow{dr_n}}{dt}}{M}$$

$$=\frac{m_1\vec{v}_1+m_2\vec{v}_2+m_3\vec{v}_3\ldots+m_n\vec{v}_n}{M}$$

$$\vec{P}_{sys}=M\vec{v}_{cm}$$

Acceleration of Centre of Mass of System

$$\vec{a}_{cm}=\frac{m_1\frac{\overrightarrow{dV_1}}{dt}+m_2\frac{\overrightarrow{dV_2}}{dt}+m_3\frac{\overrightarrow{dV_3}}{dt}\cdots+m_n\frac{\overrightarrow{dV_n}}{dt}}{M}$$

$$=\frac{m_1\vec{a}_1+m_2\vec{a}_2+m_3\vec{a}_3\ldots+m_n\vec{a}_n}{M}$$

$$=\frac{\text{Net force on system}}{M}=\frac{\text{Net external force + Net internal force}}{M}$$

$$=\frac{\text{Net External Force}}{M}\qquad(\because\Sigma\text{ Internal force = 0})$$

$$\vec{F}_{\text{ext}}=M\vec{a}_{cm}$$

Impulse of a force F on a body is defined as:

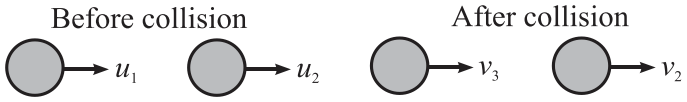
$$\vec{J}=\int_{t_i}^{t_f}\vec{F}dt=\int_{t_i}^{t_f}d\vec{P}=\Delta\vec{P}$$ (Area under the Force vs time curve gives the impulse)

$$\vec{J}=\Delta\vec{P}$$ (Impulse – momentum theorem)

Principle of conservation of linear momentum

- If $\left(\sum\vec{F}_{\text{ext}}\right)_{\text{system}}=0\Rightarrow\left(\vec{P}_{\text{i}}\right)_{\text{system}}=\left(\vec{P}_{\text{f}}\right)_{\text{system}}$
- $(KE)_{\text{system}}=\frac{1}{2}\left(m_1v_1^2+m_2v_2^2+\ldots m_nv_n^2\right)\neq\frac{1}{2}MV_{\text{com}}^2$

COEFFICIENT OF RESTITUTION (E)



$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$e = \frac{\text{Im } pulse\ of\ \ reformation}{\text{Im } pulse\ of\ \ deformation} = \frac{\int F_r dt}{\int F_d dt}$$

$$= \frac{\text{Velocity of separation along line of impact}}{\text{Velocity of approach along line of impact}}$$

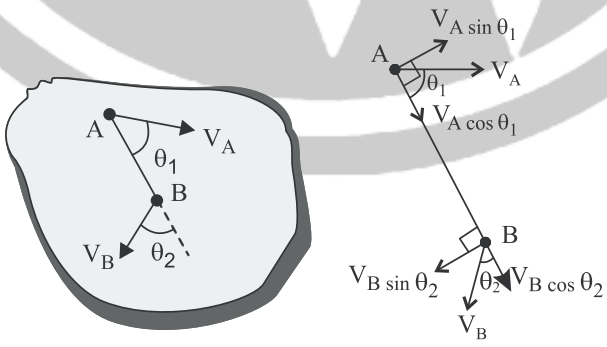
$$V_1 = \frac{P_i + m_2 e(u_2 - u_1)}{m_1 + m_2}, V_2 = \frac{P_i + m_1 e(u_1 - u_2)}{m_1 + m_2}$$

- $e = 1$
  Impulse of Reformation = Impulse of Deformation
  Velocity of separation = Velocity of approach
  Kinetic Energy is conserved in elastic collision.

- $e = 0$
  Impulse of Reformation = 0
  Velocity of separation = 0
  Kinetic Energy is not conserved
  Perfectly Inelastic collision.

- $0 < e < 1$
  Impulse of Reformation < Impulse of Deformation
  Velocity of seperation < Velocity of approach
  Kinetic Energy is not conserved
  Inelastic collision.

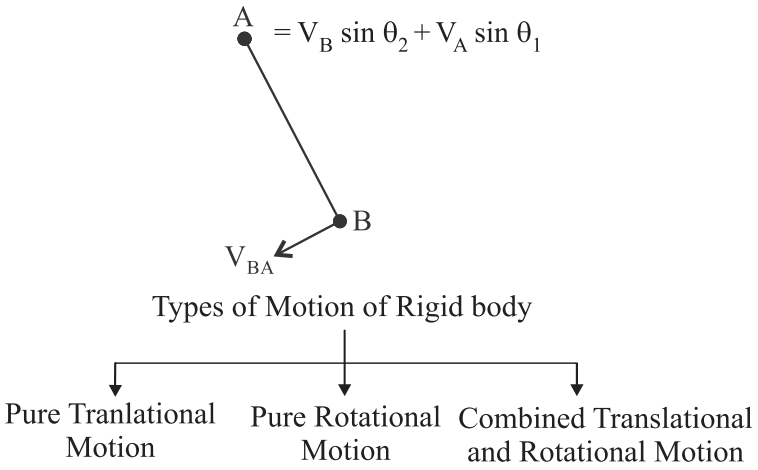
RIGID BODY



If the above body is rigid

$V_A \cos\theta_1 = V_B \cos\theta_2$

V_{BA} = relative velocity of point B with respect to point A.



MOMENT OF INERTIA (I)

Definition: Moment of Inertia is defined as the capability of system to oppose the change produced in the rotational inertia of a body. Moment of Inertia is a scalar (positive quantity).

$$I = m_1 r_1^2 + m_2 r_2^2 + \ldots \qquad = I_1 + I_2 + I_3 + \ldots$$

SI unit of Moment of Inertia is Kgm^2.

Moment of Inertia of

A single particle

$I = mr^2$

Where m = mass of the particle

r = perpendicular distance of the particle from the axis about which moment of Inertia is to be calculated

For a continuous object

$$I = \int dI = \int r^2 dm$$

Where, dI = Moment of inertia of a small element

dm = mass of a small element

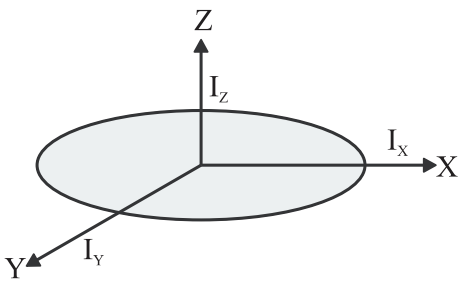
r = perpendicular distance of the particle from the axis

TWO IMPORTANT THEOREMS ON MOMENT OF INERTIA

Perpendicular Axis Theorem

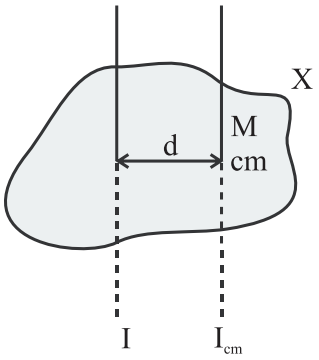
Only applicable to plane lamina (that means for 2-D objects only)

$I_Z = I_X + I_Y$ (when object is in x– y plane).

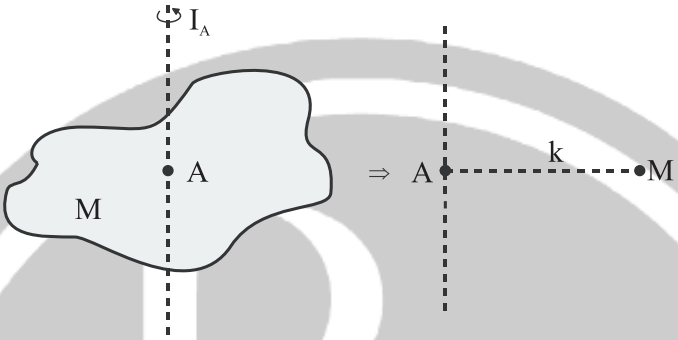


Parallel Axis Theorem

(Applicable to any type of object) :



RADIUS OF GYRATION (k)

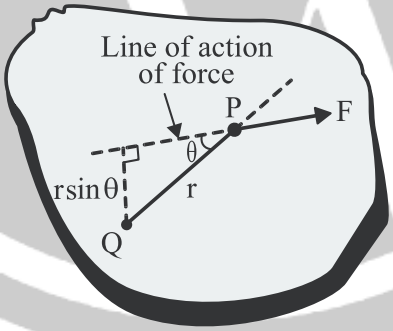


$$I_A = Mk^2$$

$$k = \sqrt{\frac{I_A}{M}}$$

TORQUE

$$\vec{\tau} = \vec{r} \times \vec{F}$$



$$\vec{P} = M\vec{v}_{CM} \quad \vec{F}_{\text{extermal}} = M\vec{a}_{CM}$$

Net external force acting on the body has two components tangential and centripetal.

$$F_C = ma_C = m\frac{v^2}{r_{CM}} = m\omega^2 r_{CM} \quad F_t = ma_t = m\alpha r_{CM}$$

ROTATIONAL EQUILIBRIUM

For translational equilibrium

$$\sum F_x = 0$$

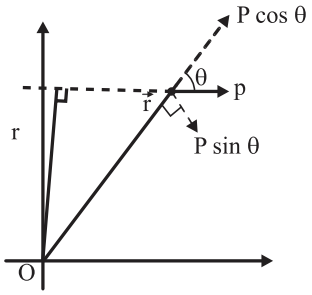
and $\sum F_y = 0$

The condition of rotational equilibrium is

$$\sum \vec{\tau} = 0$$

ANGULAR MOMENTUM ($\vec{L}$)

Angular Momentum of a Particle About a Point



$\vec{L} = \vec{r} \times \vec{P}$

$L = rP\sin\theta$

$|L| = r_{\perp} \times P$

$|L| = P_{\perp} \times r$

Angular Momentum of a Rigid Body Rotating about Fixed Axis

$L_H = I_H\omega$

L_H = angular momentum of object about axis H.

I_H = Moment of Inertia of rigid object about axis H.

ω = angular velocity of the object.

Conservation of Angular Momentum

Angular momentum of a particle or a system remains constant if $\tau_{ext} = 0$ about that point or axis of rotation.

$L_i = L_f \Rightarrow I_i\omega_i = I_f\omega_f$

Relation Between Torque and Angular Momentum

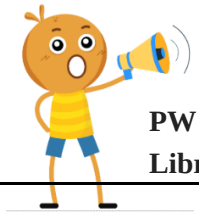
$\tau = \frac{dL}{dt}$

Torque is change in angular momentum

IMPULSE OF TORQUE

$\int \vec{\tau} dt = \Delta\vec{J}$

$\overrightarrow{\Delta J} =$ Change in angular momentum



PW Web/App - https://smart.link/7wwosivoicgd4
Library- https://smart.link/sdfez8ejd80if