ULTIMATE KCET

CRASH COURSE 2026

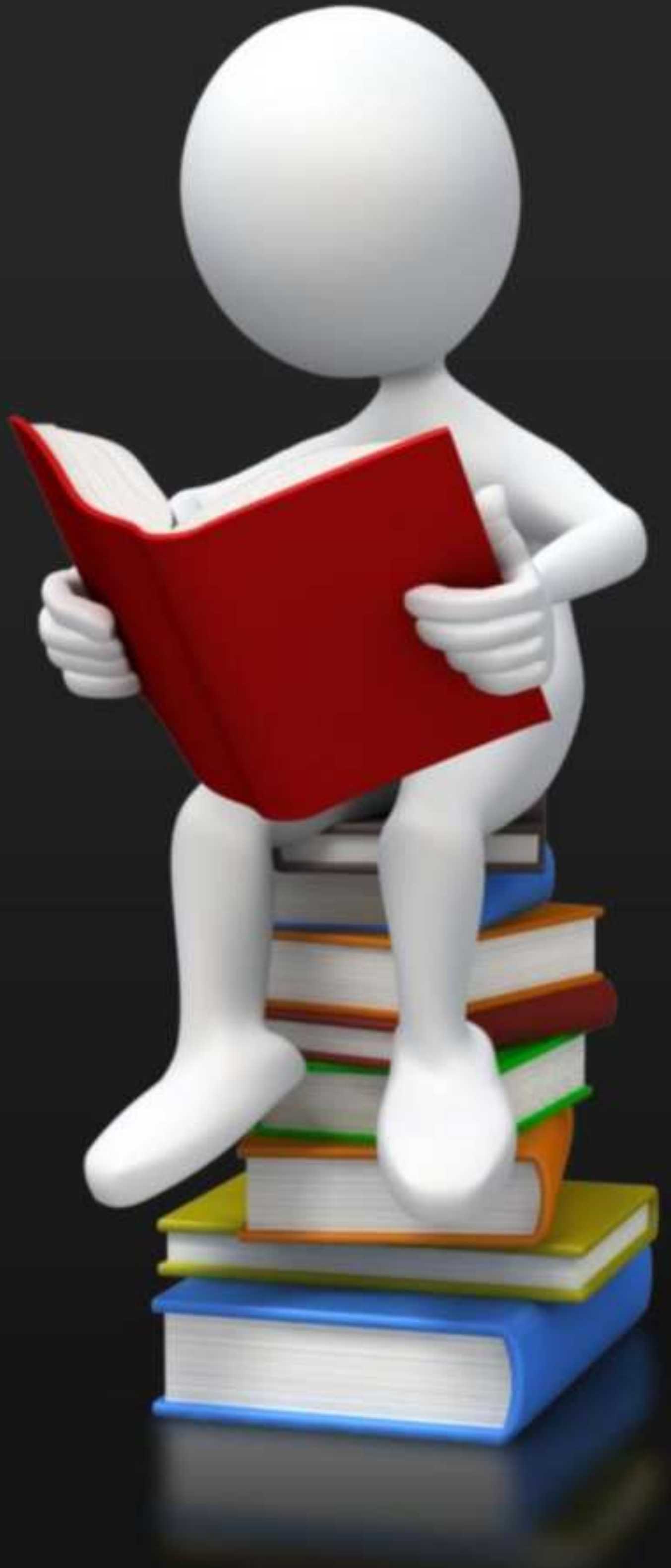
Mathematics | Lecture – 01

Applications of Integrals

By – Guru sir

Topics *to be covered*

1. AOI – MCQ Discussion
2.
3.
4.

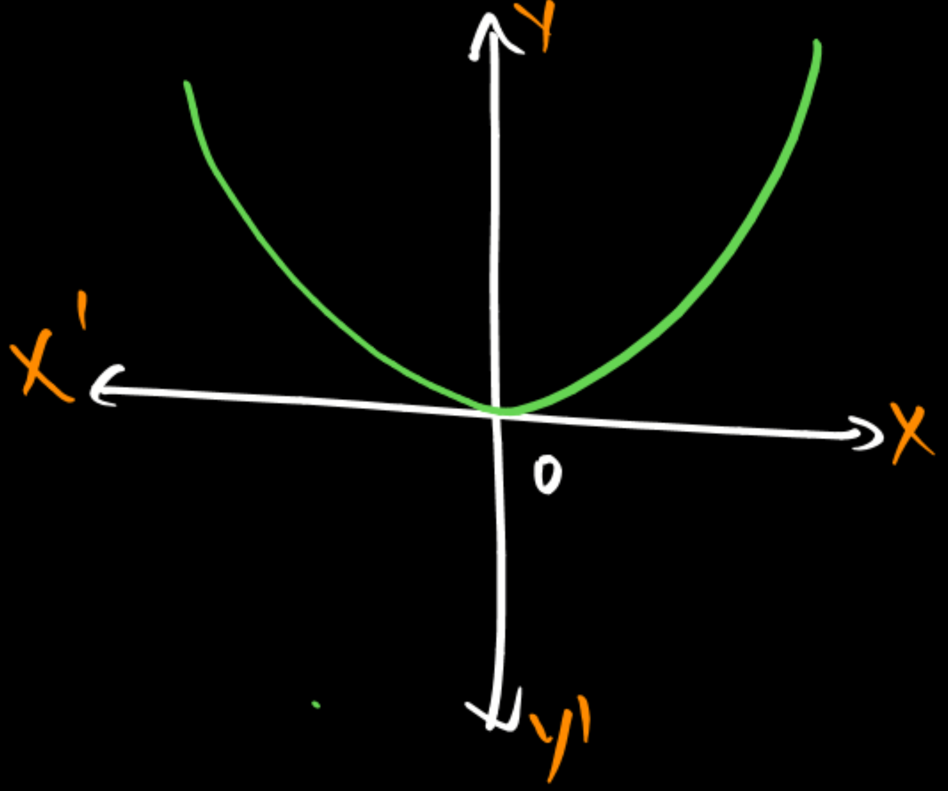


$x^2 = 4ay$

(upper Parabola)

Ex:

$x^2 = y$	$x^2 = 3y$
$x^2 = 2y$	$x^2 = 4y$

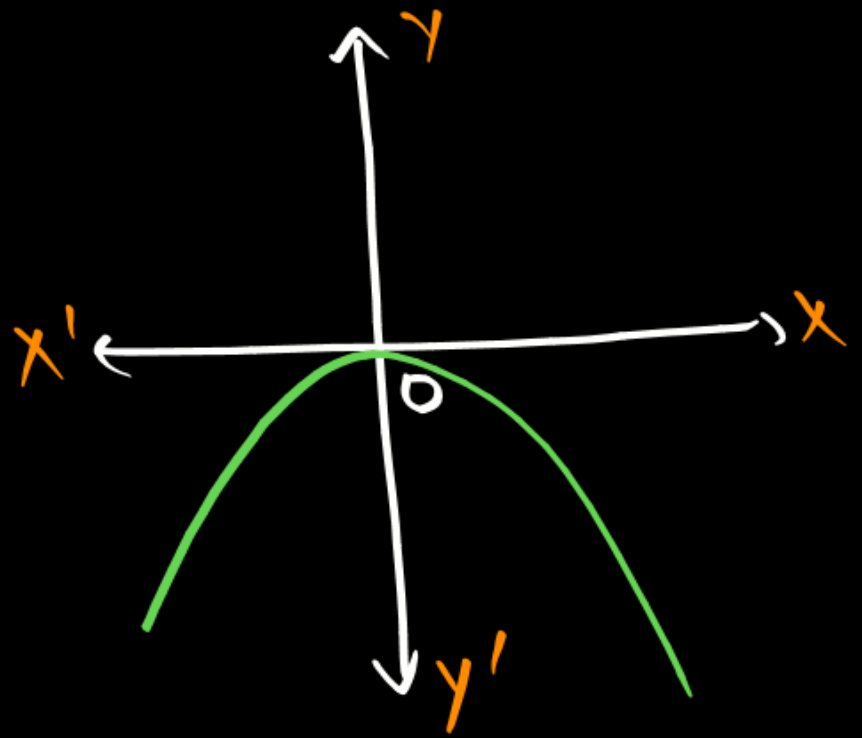


$x^2 = -4ay$

(lower Parabola)

$x^2 = -y$

$x^2 = -2y$

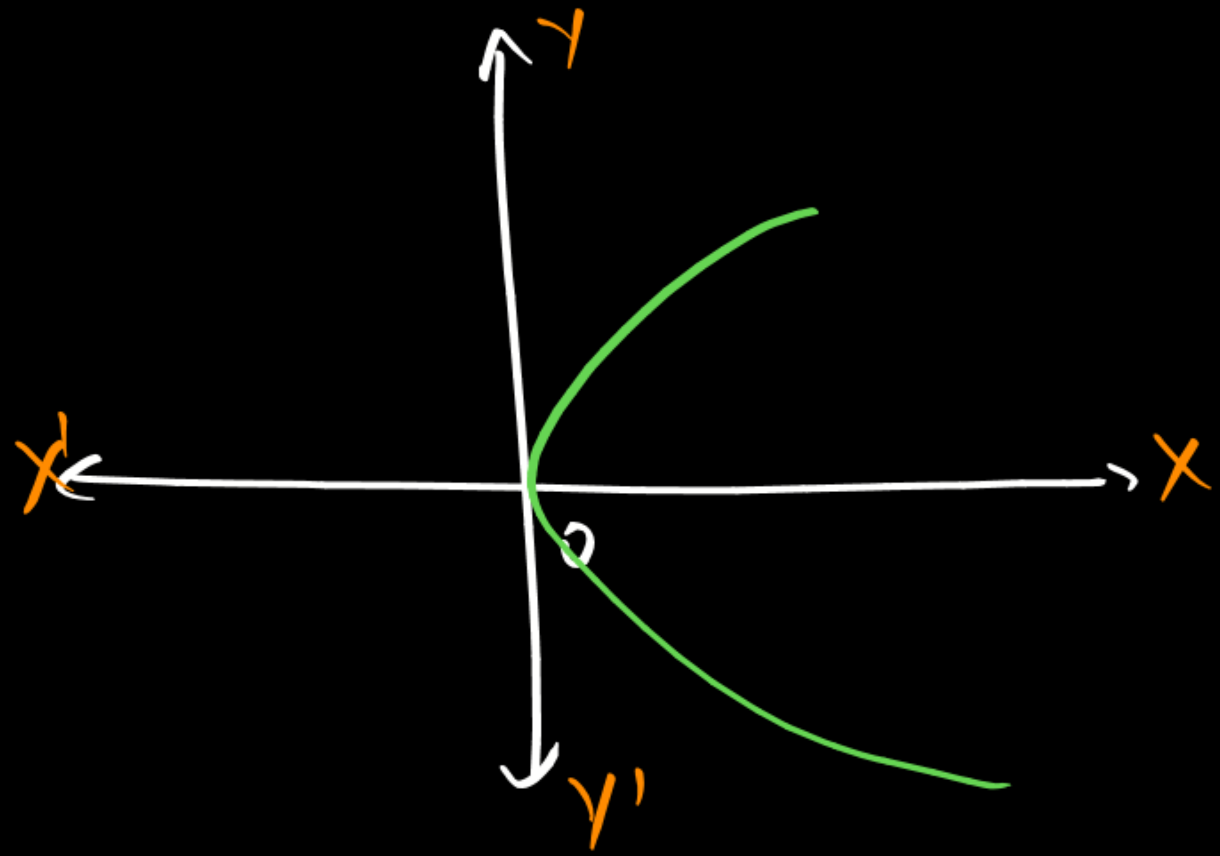


③ $y^2 = 4ax$

(Right Parabola)

Ex:

$y^2 = x$	$y^2 = 3x$
$y^2 = 2x$	$y^2 = 4x$



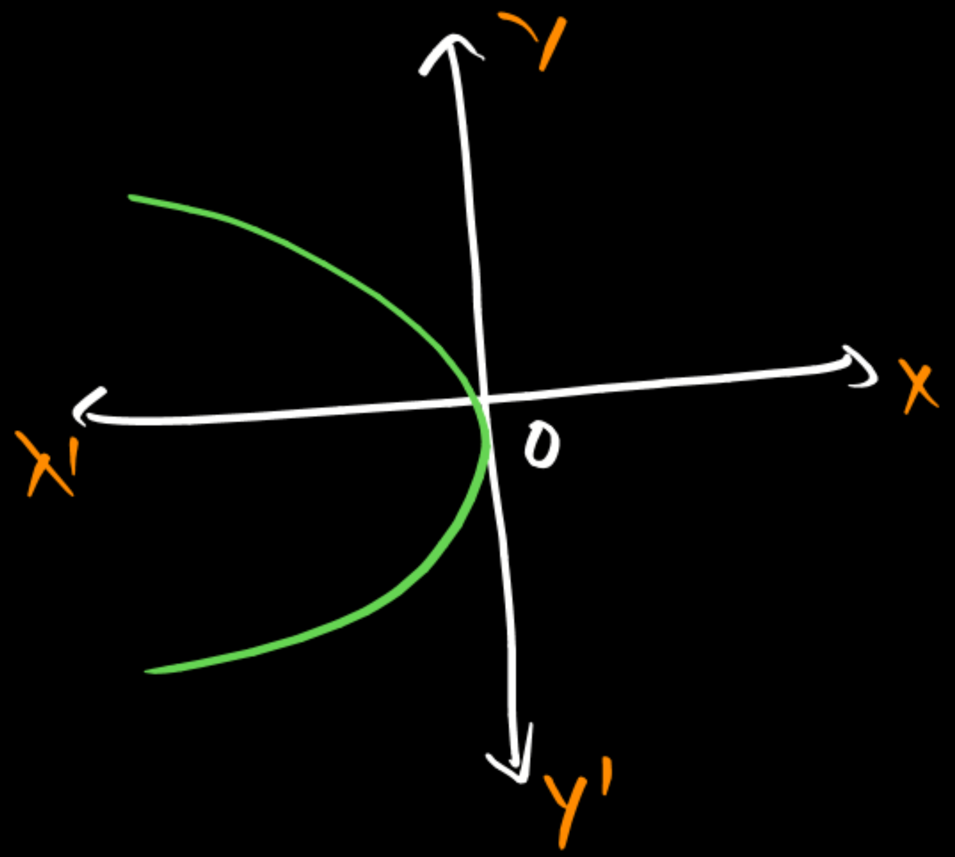
④ $y^2 = -4ax$

(Left Parabola)

Ex:

$y^2 = -x$

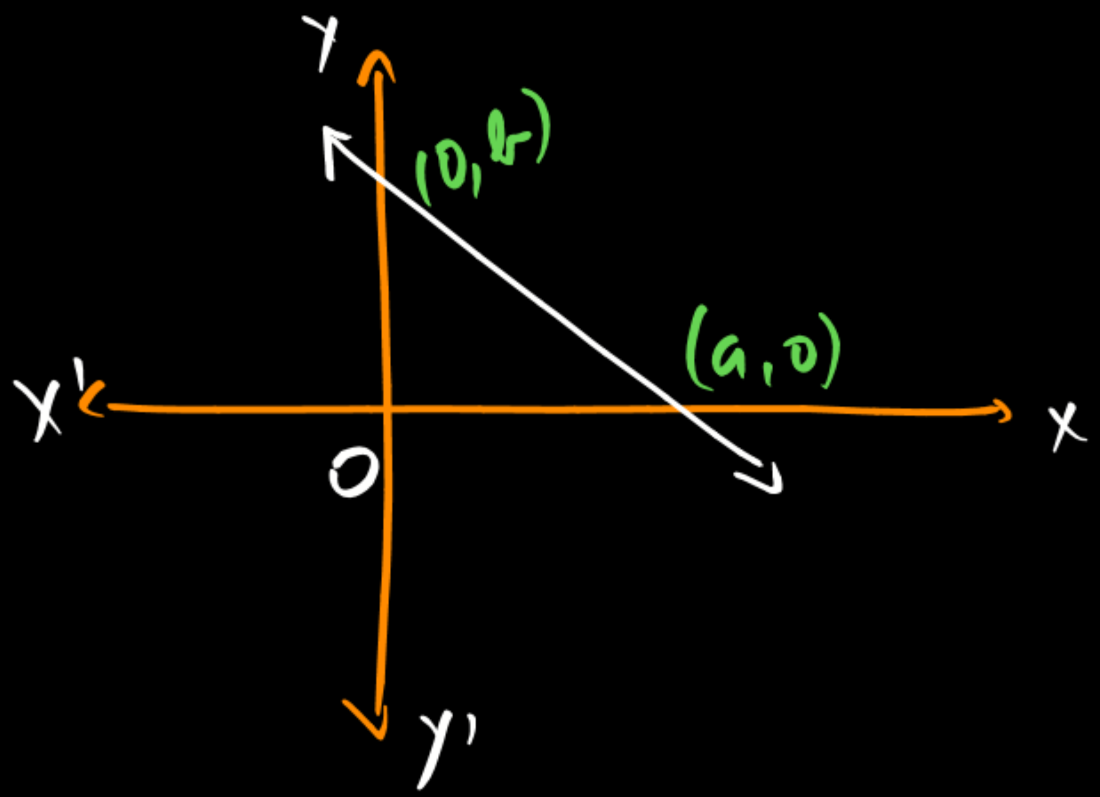
$y^2 = -2x$



(*) Straight line :-

① Intercept form :-

$$\frac{x}{a}+\frac{y}{b}=1$$



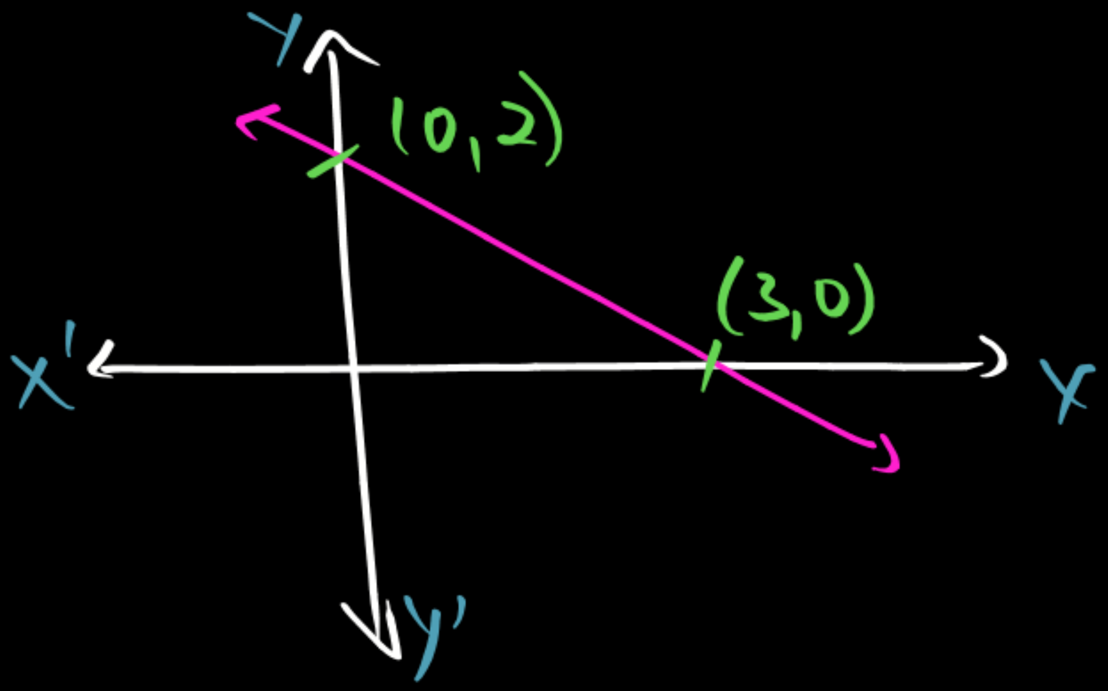
Ex:- $2x+3y-6=0$ ↳ Power of x & y is one

$\Downarrow$

$2x+3y=6$

$\div$ by 6

$$\frac{x}{3}+\frac{y}{2}=1$$

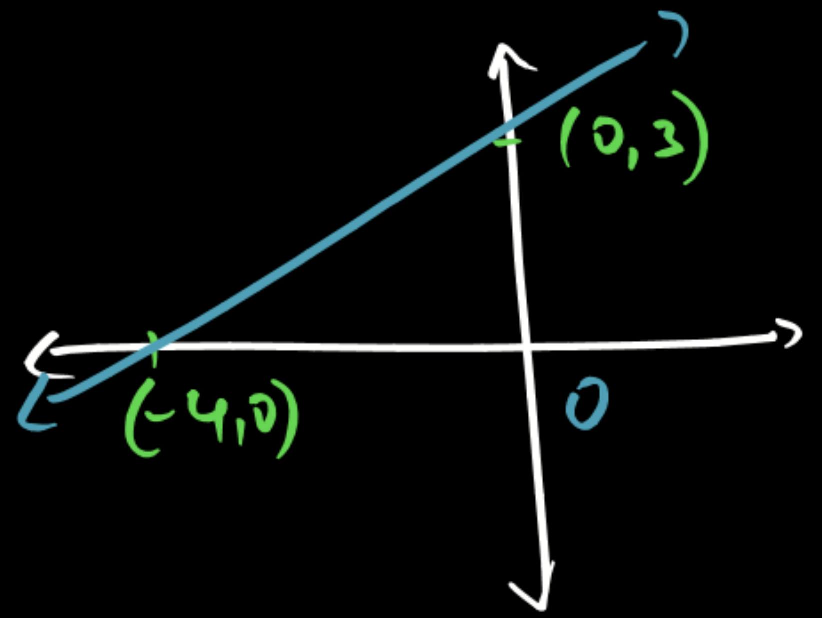


② $3x-4y+12=0$

$3x-4y=-12$

$\div$ by -12

$$\frac{x}{-4}+\frac{y}{3}=1$$



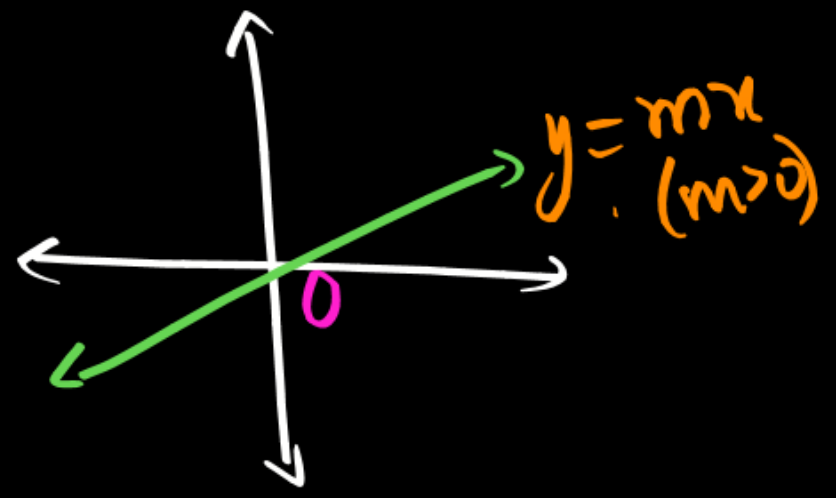
⊛ Straight line Passing through origin

⊛ $y=mx$

① $m>0$

Ex: $y=x$

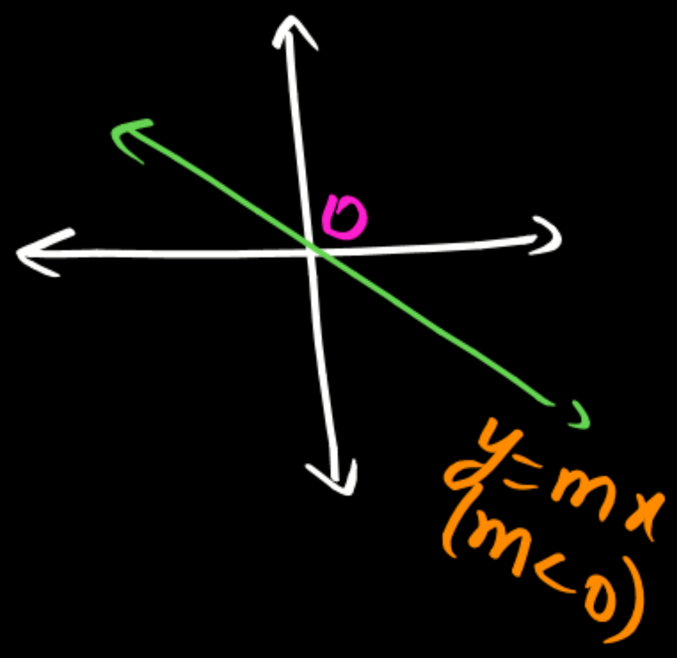
$y=2x$



② $m<0$

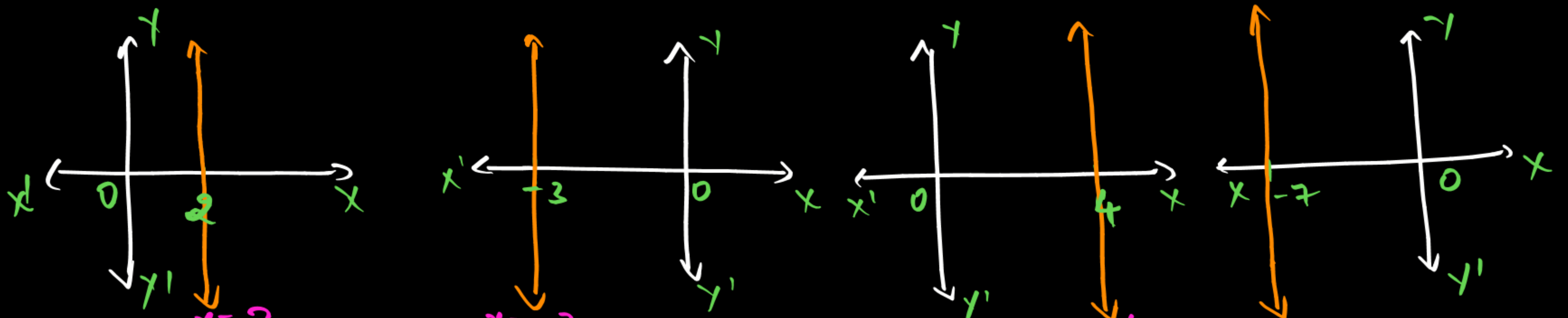
Ex: $y=-x$

$y=-2x$



(*) $x = 2$ $\quad x = -3$ $\quad x = 4$ $\quad x = -7$

Straight lines Parallel to y-axis



$x = 2 \Rightarrow x - 2 = 0$

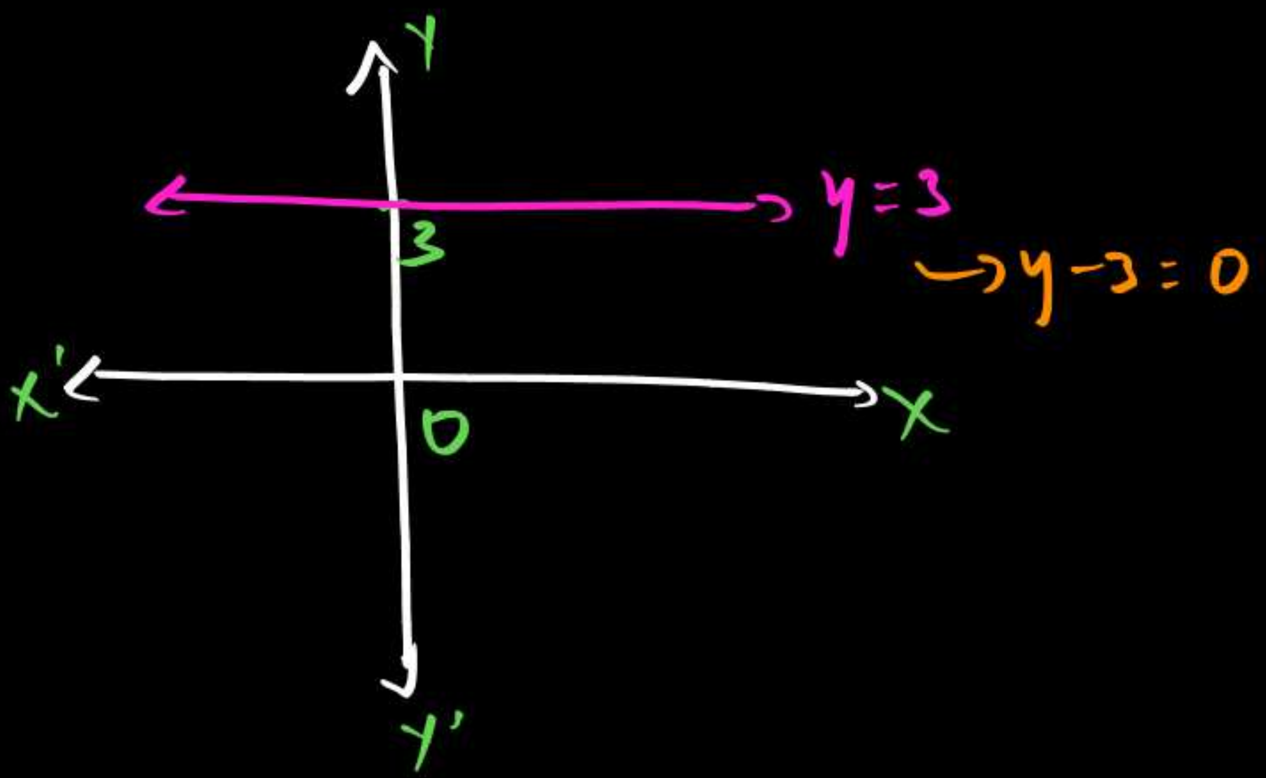
$x = -3 \Rightarrow x + 3 = 0$

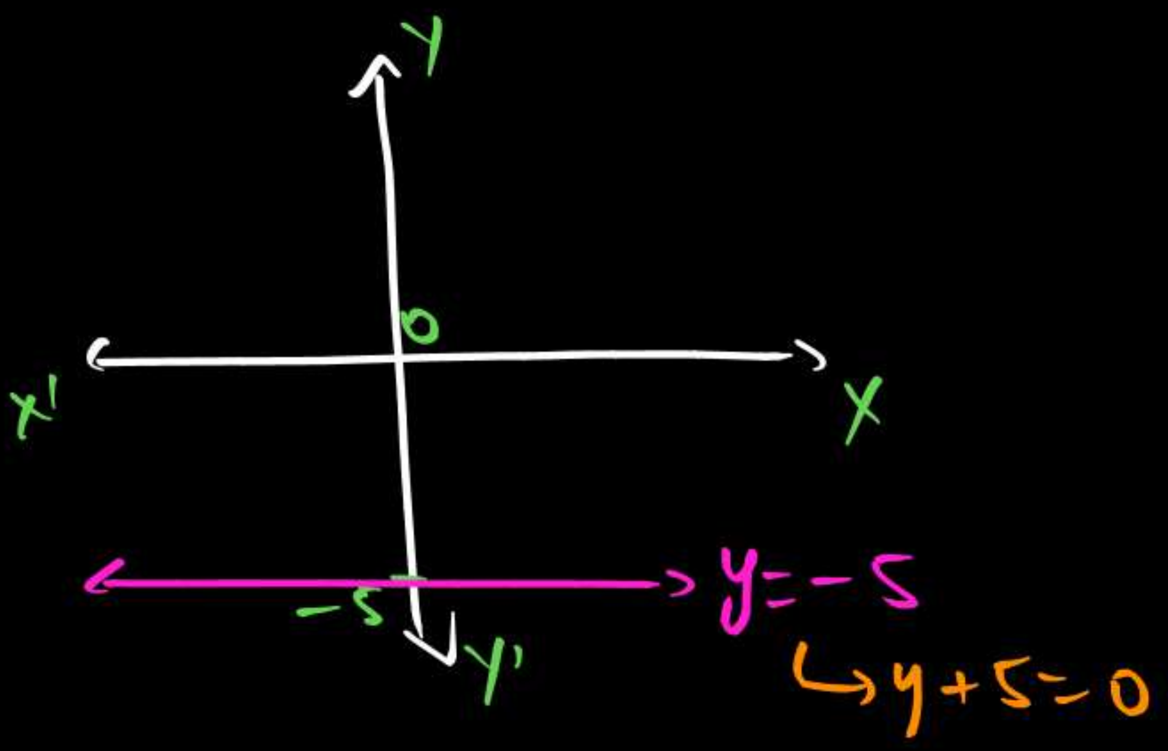
$x = 4 \Rightarrow x - 4 = 0$

$x = -7 \Rightarrow x + 7 = 0$

$y = 3$ $\qquad\qquad$ $y = -5$

Straight lines parallel to x-axis





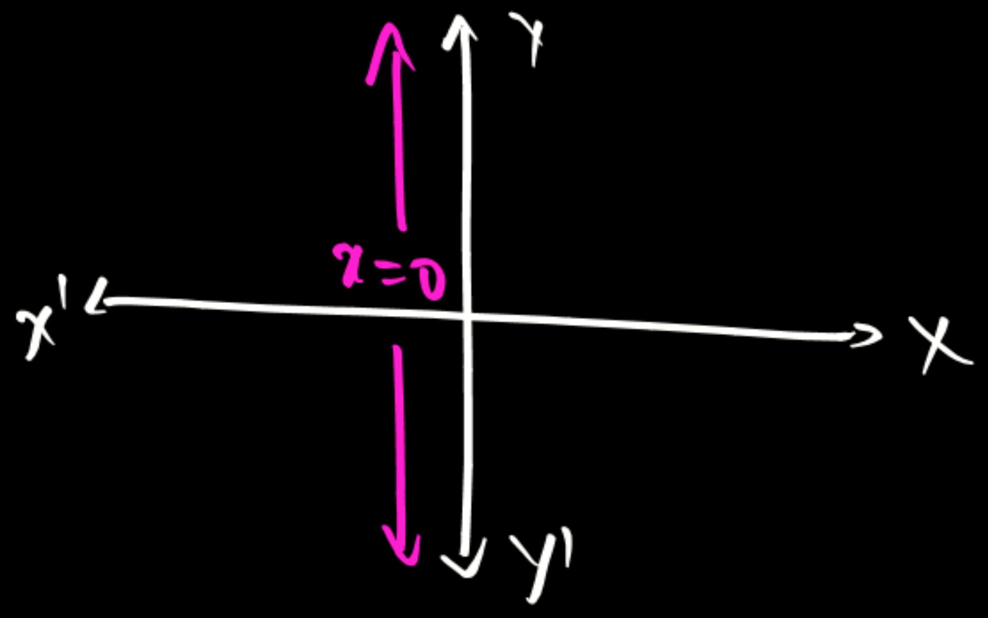
* $x = 0$

$\Downarrow$

represents y-axis

$\Downarrow$

eq^n of y-axis



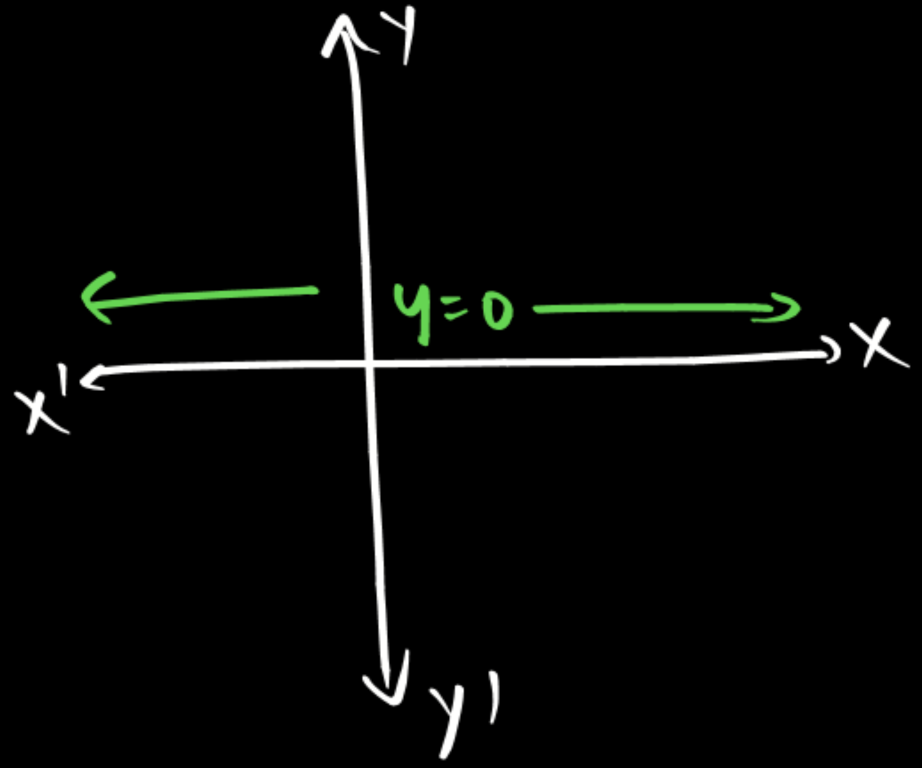
* $y = 0$

$\Downarrow$

represents x-axis

$\Downarrow$

eq^n of x-axis



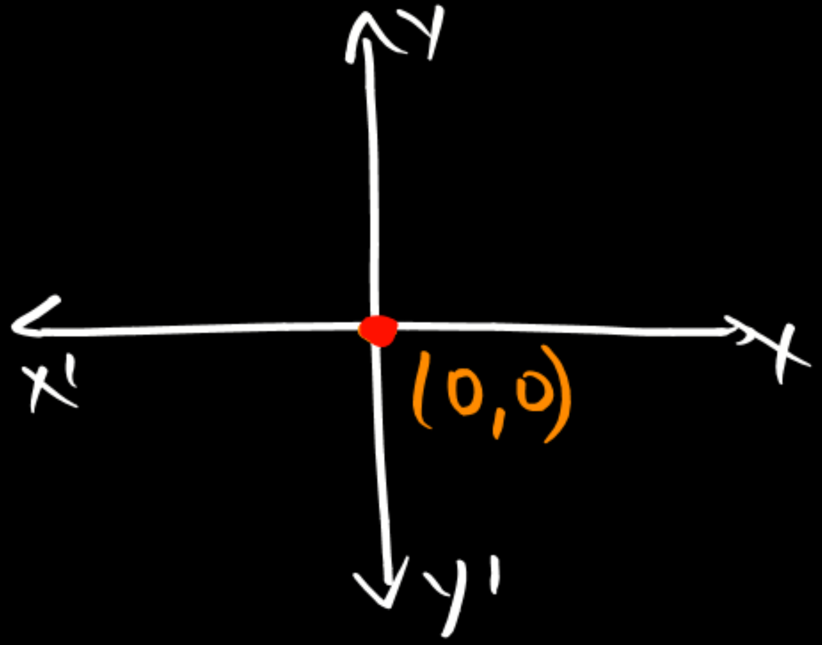
* $(0,0)$

$\Downarrow$

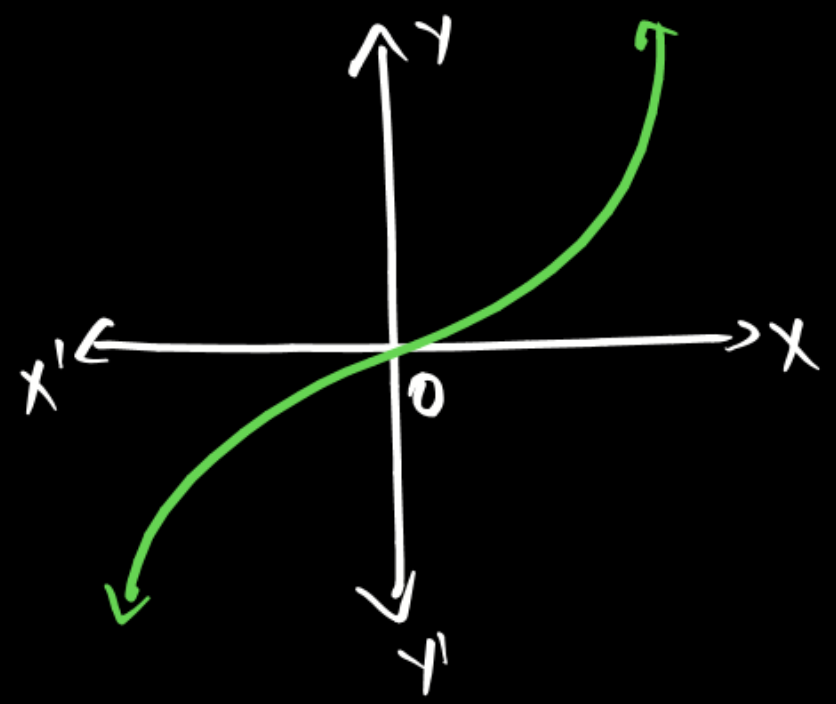
represents origin

$\Downarrow$

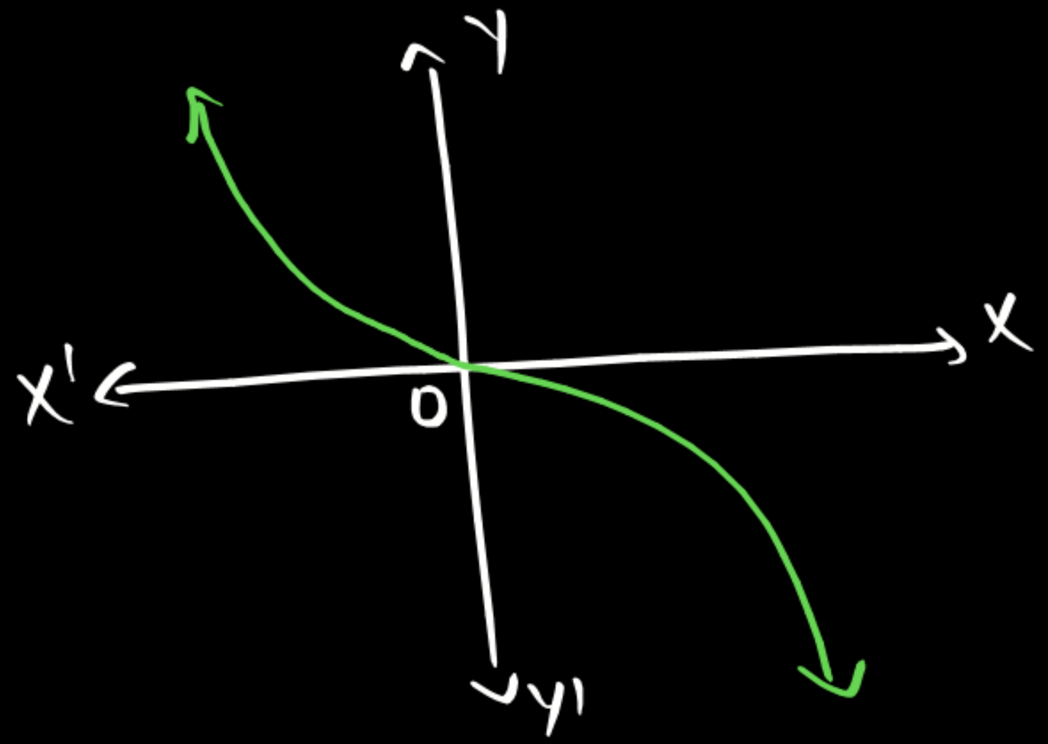
Point (coordinate)



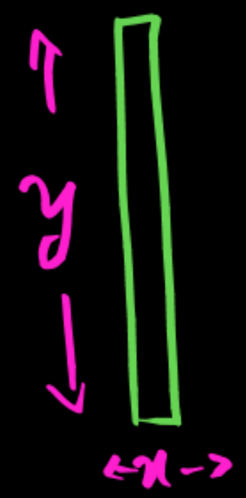
* $y = x^3$



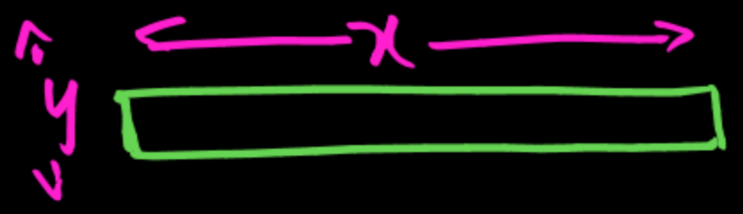
* $y = -x^3$



Area of rectangle = length × breadth



Area = yx



Area = xy

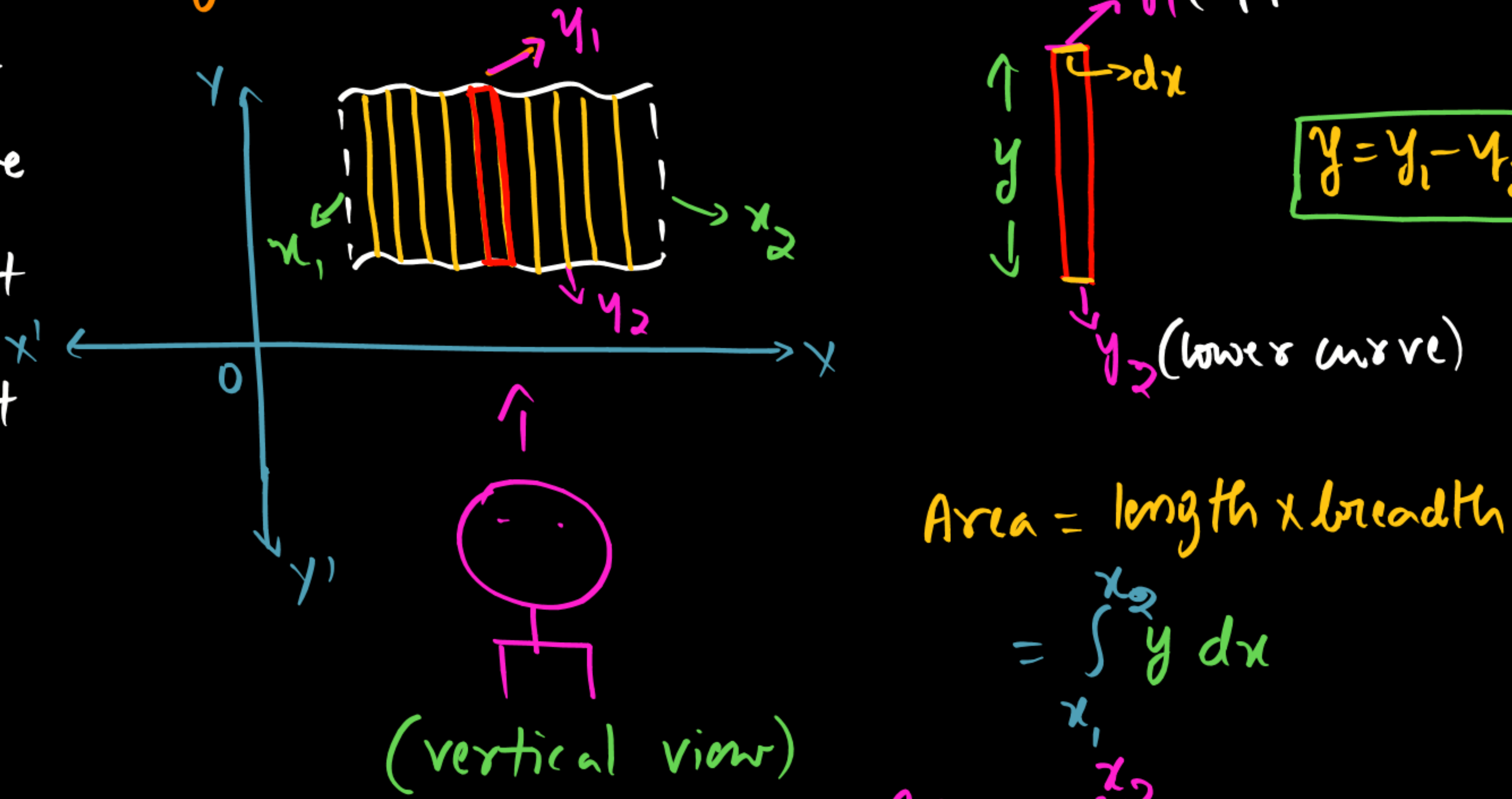
How to consider the upper limit, lower limit & f(x) in the region.

$y_1 \rightarrow$ upper curve

$y_2 \rightarrow$ lower curve

$x_1 \rightarrow$ lower limit

$x_2 \rightarrow$ upper limit

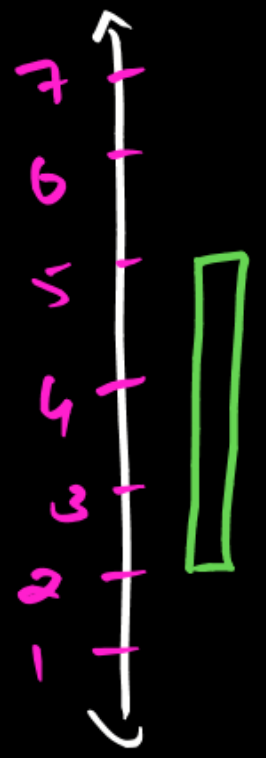


$$\boxed{y = y_1 - y_2}$$

Area = length × breadth

$$= \int_{x_1}^{x_2} y \, dx$$

$$\text{Area} = \int_{x_1}^{x_2} (y_1 - y_2)\, dx$$



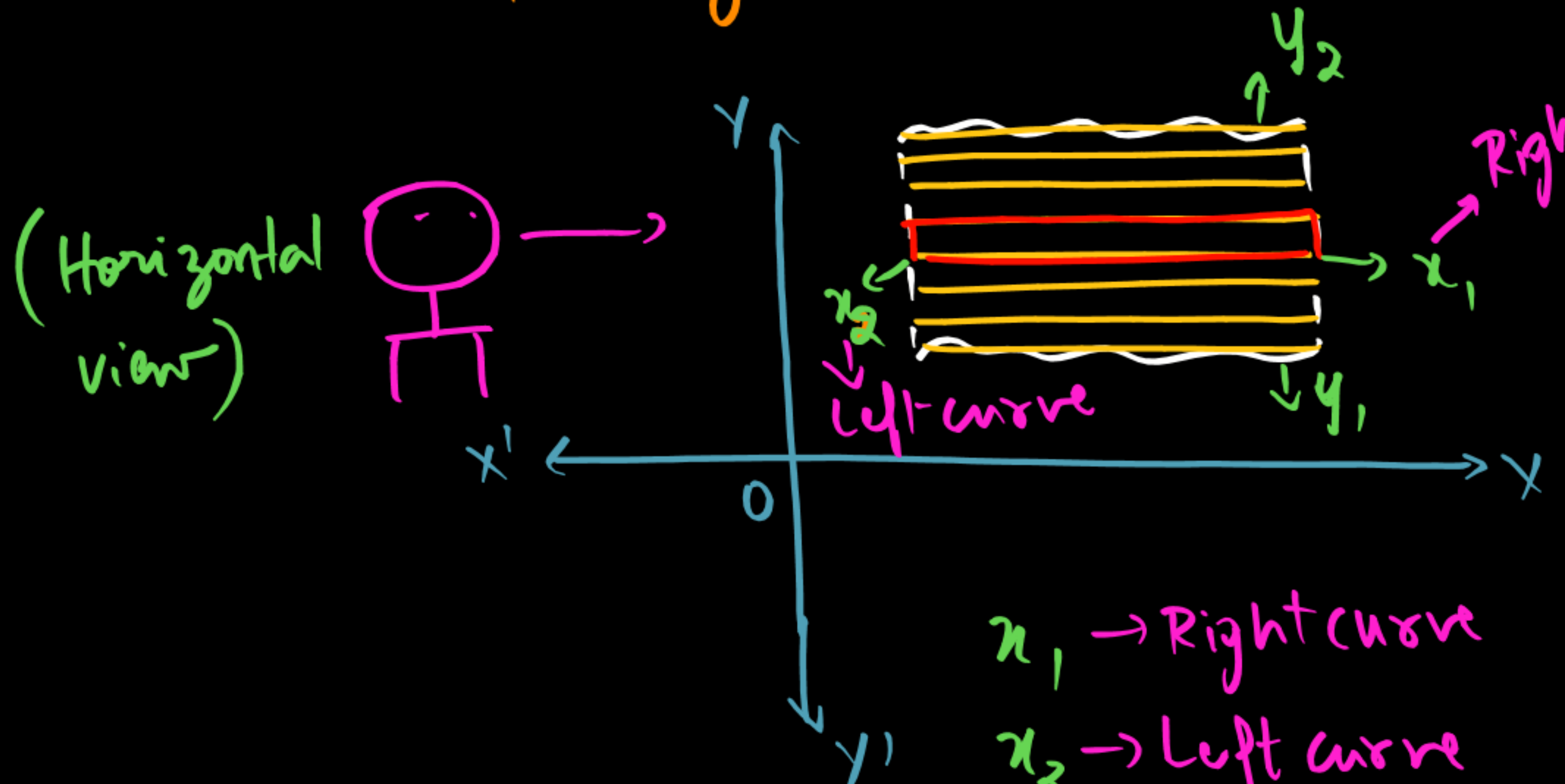
upper Part

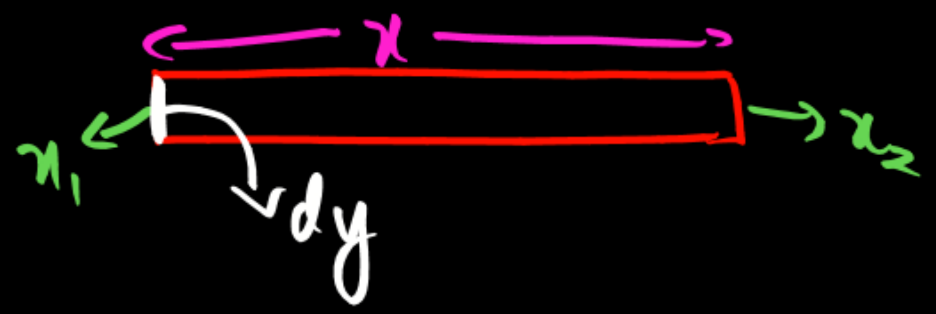
Lower Part

length $= 5 - 2$

$= 3$

How to consider the upper limit, lower limit & f(x) in the region.





$x_1 \rightarrow$ Right curve

$x_2 \rightarrow$ Left curve

$y_1 \rightarrow$ Lower Limit

$y_2 \rightarrow$ upper limit

$$A = \int x\,dy$$

$$= \int_{y_1}^{y_2} (x_1 - x_2)\,dy$$

QUESTION

Closed area

St. line passing through origin

upper Parabola

#Q. The area of the region $\boxed{\text{bounded}}$ by the line $y = 3x$ and the curve $y = x^2$ in sq. units is

(A) 10

(B) $\frac{9}{2}$ ✓

(C) 9

(D) 5

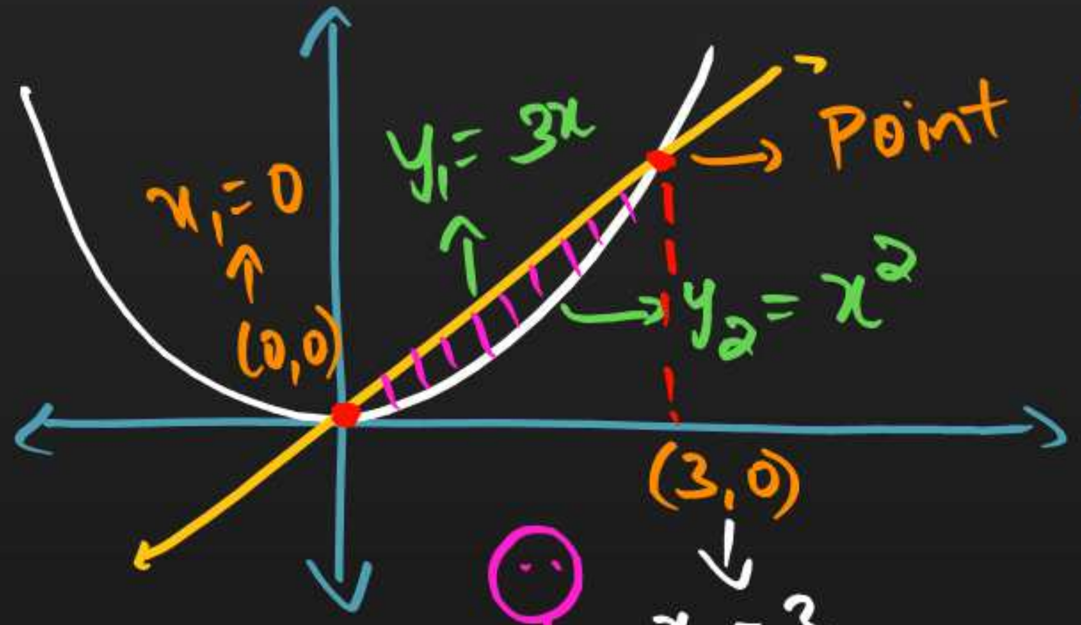
$$A = \int_{x_1}^{x_2} (y_1 - y_2)\,dx$$

$$= \int_0^3 (3x - x^2)\,dx$$

$$= \frac{3x^2}{2} - \frac{x^3}{3}\Big]_0^3$$

$$= \frac{3}{2}(9) - \frac{27}{3} = \frac{27}{2} - 9$$

$$= \frac{9}{2}$$



$x_2 = 3$

of $y = 3x$ & $y = x^2$

$3x = x^2$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0 \mid x = 3$

U.C $y_1 = 3x$

L.C $y_2 = x^2$

L.L $x_1 = 0$

U.L $x_2 = 3$

QUESTION

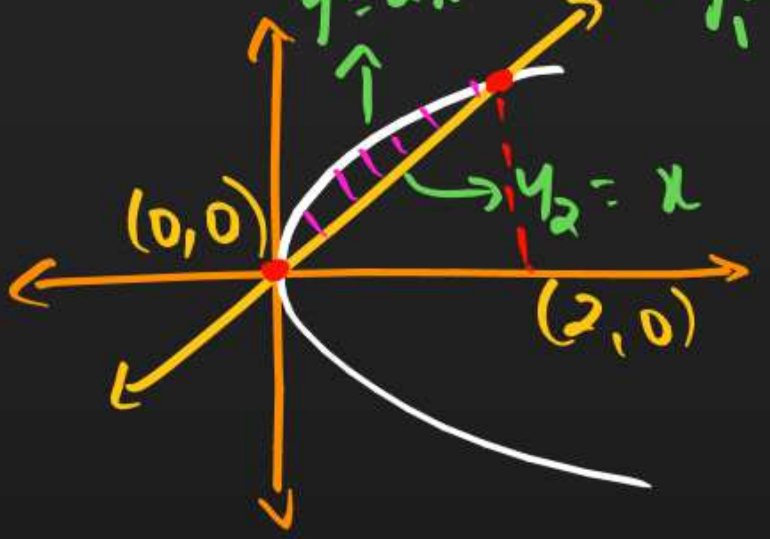
#Q. Area lying between the curves $y^2 = 2x$ and $y = x$ is **[2016]**

(A) $\frac{2}{3}$ sq. units

(B) $\frac{1}{3}$ sq. units

(C) $\frac{1}{4}$ sq. units

(D) $\frac{3}{4}$ sq. units



$y_1 = \sqrt{2}\sqrt{x} = \sqrt{2}x^{1/2}$

$y_2 = x$

$x_1 = 0$

$x_2 = 2$

$y = x$ & $y^2 = 2x$

$y = \sqrt{2}\sqrt{x}$

$x = \sqrt{2}\sqrt{x}$

$x^2 = 2x$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0 \mid x = 2$

$$A = \int_0^2 \sqrt{2}x^{1/2} - x \, dx$$

$$= \sqrt{2}\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\Big]_0^2$$

$$= \frac{2}{3}\sqrt{2}(x)\sqrt{x}\Big]_0^2 - \left(\frac{4}{2}\right)$$

$$= \frac{2}{3}\sqrt{2}(2)\sqrt{2} - 2$$

$$= \frac{8}{3} - 2$$

$$= \frac{2}{3}.$$

QUESTION

#Q. The area of the region bounded by the curve $y^2 = 8x$ and the line $y = 2x$ is

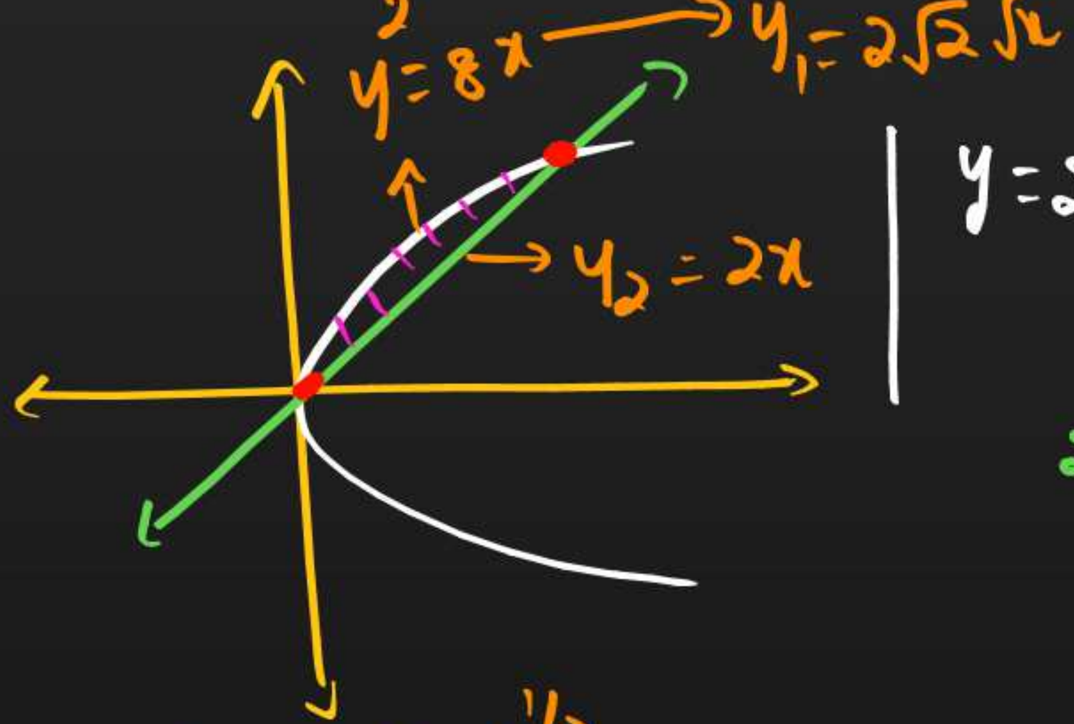
[2020]

(A) $\frac{4}{3}$ sq. units

(B) $\frac{3}{4}$ sq. units

(C) $\frac{8}{3}$ sq. units

(D) $\frac{16}{3}$ sq. units



$y_1 = 2\sqrt{2}\, x^{1/2}$

$y_2 = 2x$

$x_1 = 0$

$x_2 = 2$

$y = 2\sqrt{2}\sqrt{x}$ & $y = 2x$

$2x = 2\sqrt{2}\sqrt{x}$

$x = \sqrt{2}\sqrt{x}$

$x^2 = 2x$

$x = 0$ & $x = 2$

$$A = \int_0^2 2\sqrt{2}x^{1/2} - 2x \, dx$$

$$= \left. \frac{2\sqrt{2}\,x^{3/2}}{3/2} - x^2 \right]_0^2$$

$$= \left. \frac{4}{3}\sqrt{2}(x)\sqrt{x} \right]_0^2 - 4$$

$$= \frac{4}{3}\sqrt{2}(2)\sqrt{2} - 4$$

$$= \frac{16}{3} - 4$$

$$= \frac{4}{3}.$$

QUESTION

#Q. The area of the region bounded by lines $y = mx$, $x = 1$, $x = 2$, and x axis is 6 sq. units, then 'm' is [2014]

(A) 3

(B) 1

(C) 2

(D) 4

$y_1 = mx$

$y_2 = 0$

$x_1 = 1$

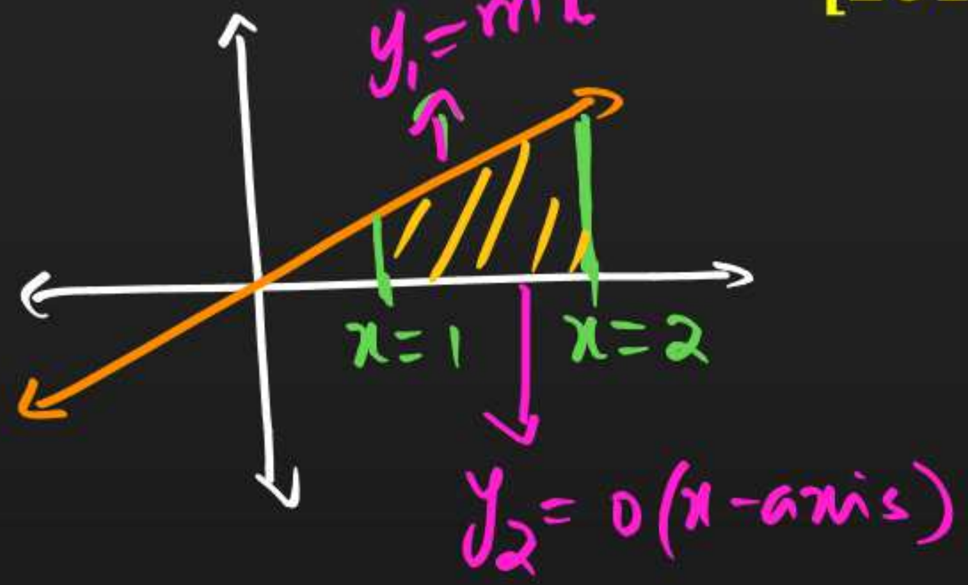
$x_2 = 2$

$A = \int_1^2 mx - 0\,dx = 6$

$\left.\frac{mx^2}{2}\right]_1^2 = 6$

$m(3) = 12$

$m = 4$



QUESTION

$x = x^3$

$x^3 - x = 0$

$x(x^2-1) = 0$

$x = 0$ | $x^2 = 1$, $x = \pm 1$

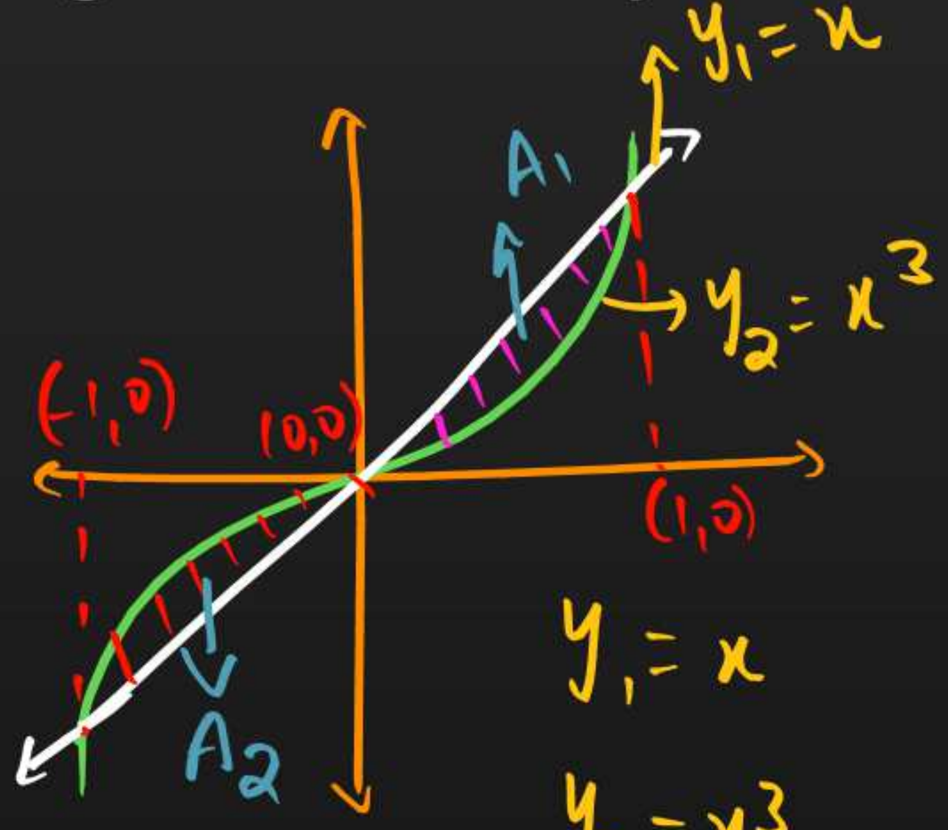
#Q. The area of the region bounded by the line $y = x$ and the curve $y = x^3$ is

(A) 0.2 sq. units

(B) 0.3 sq. units

(C) 0.4 sq. units

(D) ✓ 0.5 sq. units



$y_1 = x$

$y_2 = x^3$

$x_1 = 0$

$x_2 = 1$

$A = A_1 + A_2$

Here $A_2 = A_1$

$A = 2A_1$

$A = 2\int_0^1 x - x^3\,dx$

$= 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$

$= 2\left[\frac{1}{2} - \frac{1}{4}\right] = 2\left[\frac{1}{4}\right] = \frac{1}{2} = 0.5$

QUESTION

$8 = x^3$

$x^3 = 8$

$x = 2$

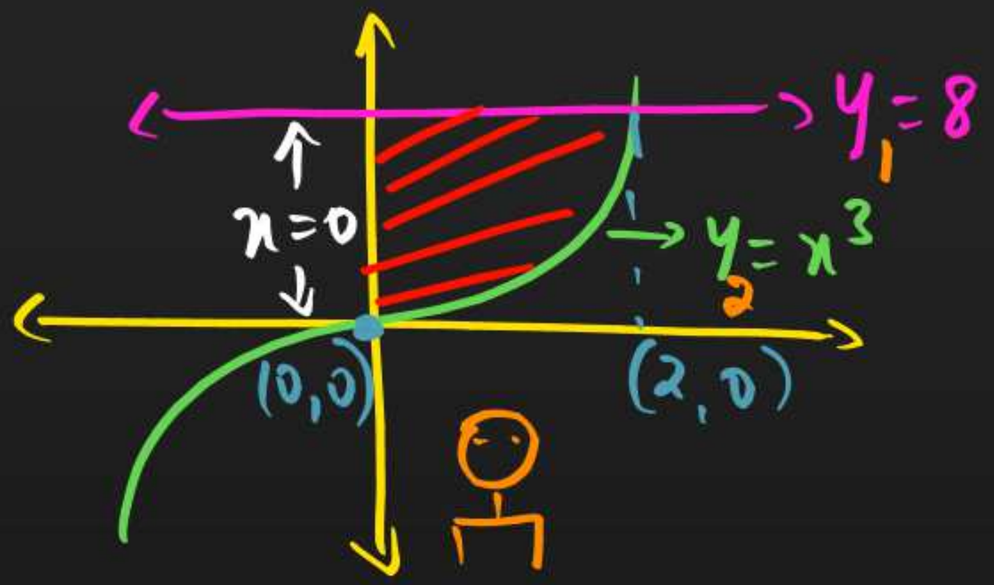
#Q. Area bounded by $y = x^3$, y = 8 and $x = 0$ is ________. [2015]

(A) 14 sq. units

(B) 6 sq. units

(C) 2 sq. units

(D) 12 sq. units



$y_1 = 8$

$y_2 = x^3$

$x_1 = 0$

$x_2 = 2$

$$A = \int_0^2 8 - x^3 \, dx$$

$$= 8x - \frac{x^4}{4}\Big]_0^2$$

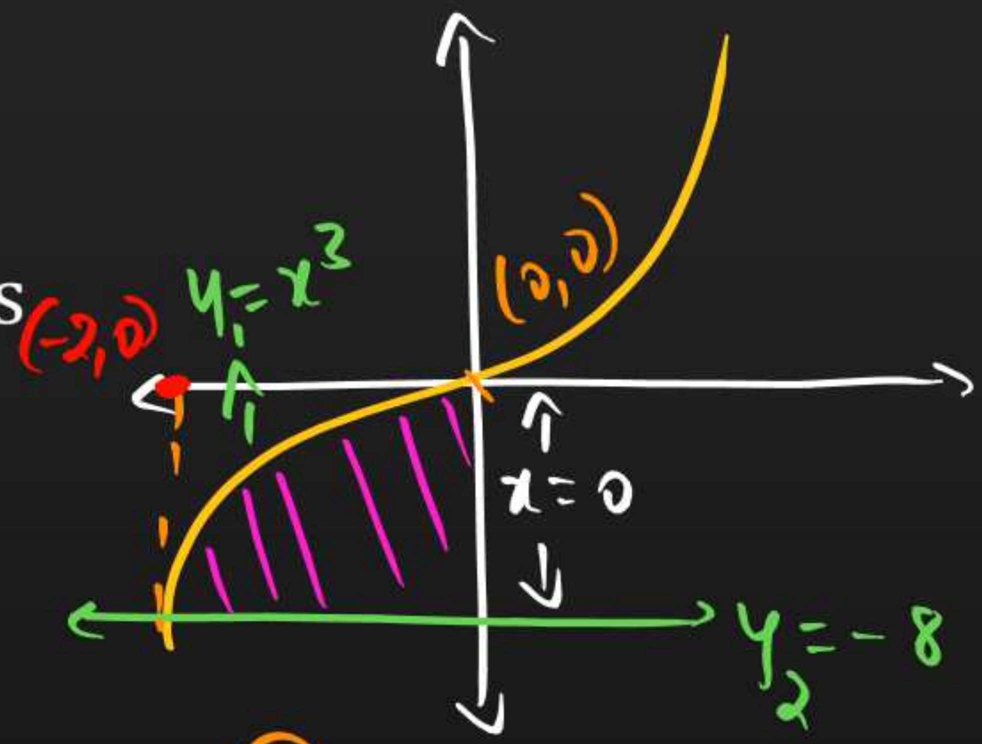
$$= 16 - \frac{2^4}{4} = 16 - 4 = \underline{12}$$

QUESTION

$y = x^3$ & $y = -8$

$x = -2$

#Q. Area bounded by $y = x^3$, $y = -8$ and $x = 0$ is ________. **[2015]**



(A) 14 sq. units

(B) 6 sq. units

(C) 2 sq. units

(D) 12 sq. units

$y_1 = x^3 \mid x_1 = -2$

$y_2 = -8 \mid x_2 = 0$

$$A = \int_{-2}^{0} x^3 - (-8)\,dx = \left.\frac{x^4}{4} + 8x\right]_{-2}^{0}$$

$$= \frac{1}{4}[0 - 16] + 8[0 + 2]$$

$$= -\frac{16}{4} + 16$$

$$= -4 + 16$$

$$= \underline{12}$$

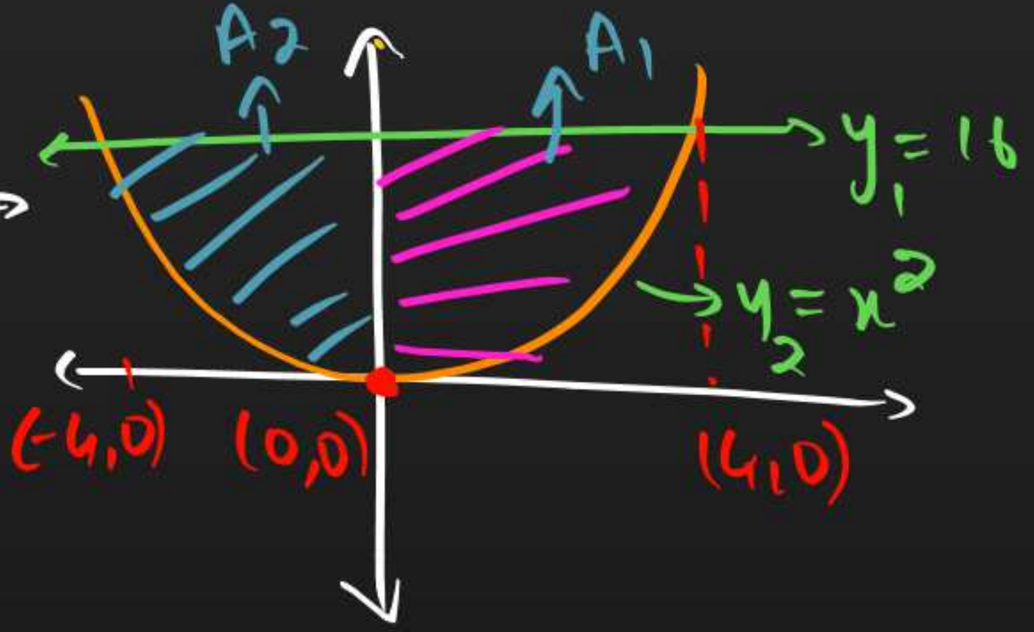
QUESTION

$x^2 = 16$

$x = \pm 4$

#Q. The area of the region bounded by the curve $y = x^2$ and the line $y = 16$ is

[2017]



(A) $\frac{256}{3}$ sq. units

(B) $\frac{128}{3}$ sq. units

(C) $\frac{32}{3}$ sq. units

(D) $\frac{64}{3}$ sq. units

$y_1 = 16$

$y_2 = x^2$

$x_1 = 0$

$x_2 = 4$

$A = A_1 + A_2$

But $A_2 = A_1$

$A = 2A_1$

$A = 2\int_0^4 16 - x^2 \, dx$

$= 2\left[16x - \frac{x^3}{3}\right]_0^4$

$= 2\left[64 - \frac{64}{3}\right] = 2\left[\frac{128}{3}\right] = \frac{256}{3}$

QUESTION

$4 = x^2$

$x = \pm 2$

$\Downarrow$

$(-2, 0) \to -ve\ x\text{-axis}$

$y = 4 \mid y = x$

$\Downarrow$

$x = 4$

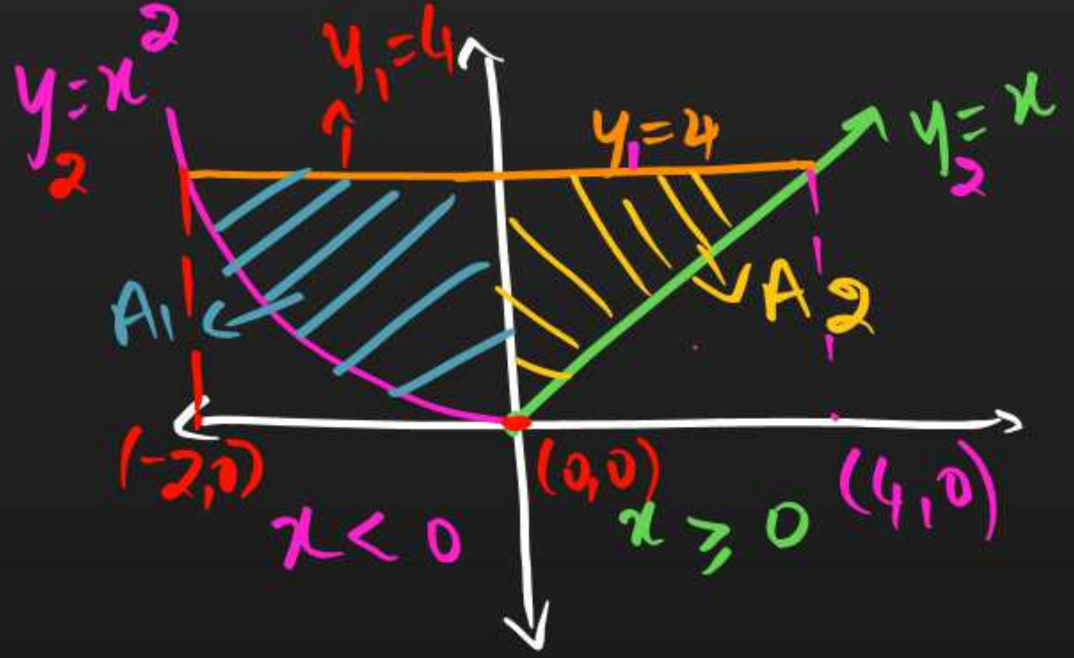
#Q. The area bounded by the curve $y = \begin{cases} x^2, & x < 0 \\ x, & x \geq 0 \end{cases}$ & the line $y = 4$ [2010]

(A) $\frac{8}{3}$

(B) $\frac{32}{3}$

(C) $\frac{16}{3}$

(D) $\frac{40}{3}$



A_1	A_2
$y_1 = 4$	$y_1 = 4$
$y_2 = x^2$	$y_2 = x$
$x_1 = -2$	$x_1 = 0$
$x_2 = 0$	$x_2 = 4$

$$A = A_1 + A_2$$

$$= \int_{-2}^{0} 4 - x^2\, dx + \int_{0}^{4} 4 - x\, dx$$

$$= \left[4x - \frac{x^3}{3}\right]_{-2}^{0} + \left[4x - \frac{x^2}{2}\right]_{0}^{4}$$

$$= 4(2) - \frac{1}{3}(8) + 16 - 8$$

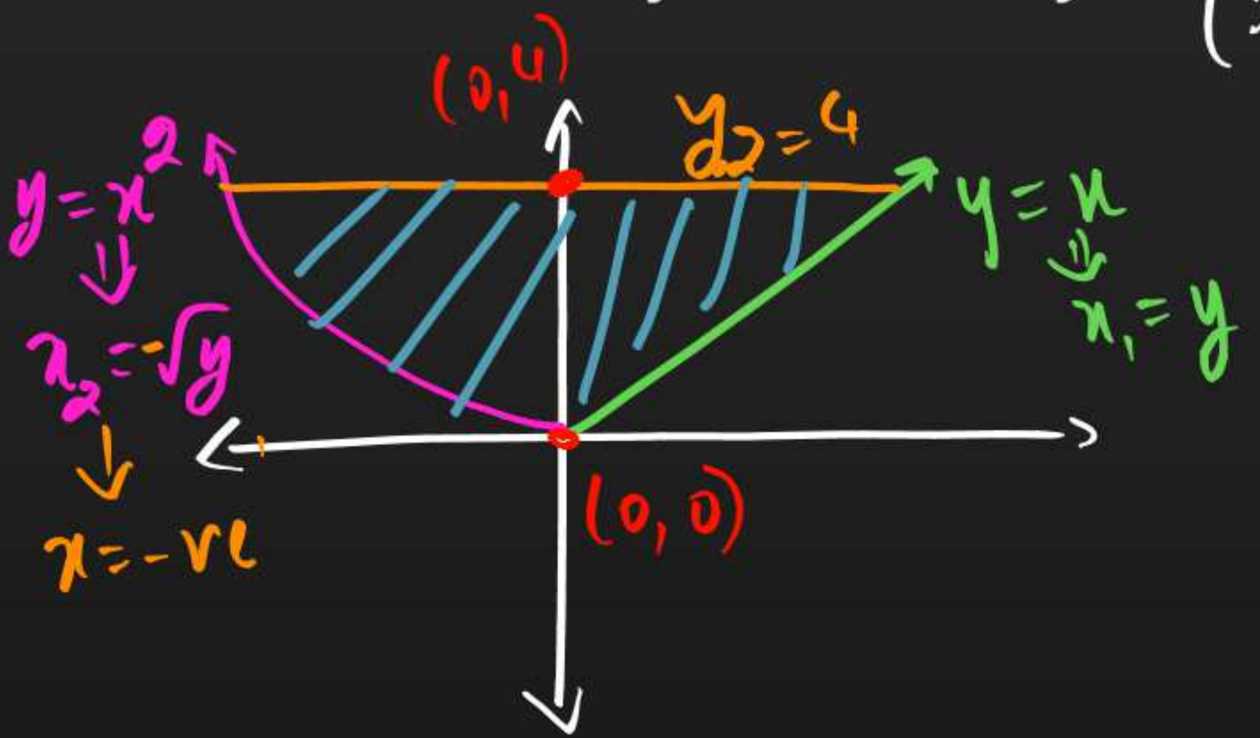
$$= 16 - \frac{8}{3} = \frac{40}{3}$$

QUESTION

$A = A_1 + A_2$

$= \int_0^4 (y-0) + (0-(-\sqrt{y}))\,dy = \int_0^4 y + \sqrt{y}\,dy$

#Q. The area bounded by the curve $y = \begin{cases} x^2, & x < 0 \\ x, & x \geq 0 \end{cases}$ & the line $y=4$ [2010]



(A) $\frac{8}{3}$

(B) $\frac{32}{3}$

(C) $\frac{16}{3}$

(D) $\frac{40}{3}$

$x_1 = y$

$x_2 = \sqrt{y}$

$y_1 = 0$

$y_2 = 4$

$A = \int_0^4 y - (-\sqrt{y})\,dy$

$= \left.\frac{y^2}{2} + \frac{y^{3/2}}{3/2}\right]_0^4$

$= \frac{16}{2} + \frac{2}{3}(y\sqrt{y})_0^4 = 8 + \frac{2}{3}(4(2))$

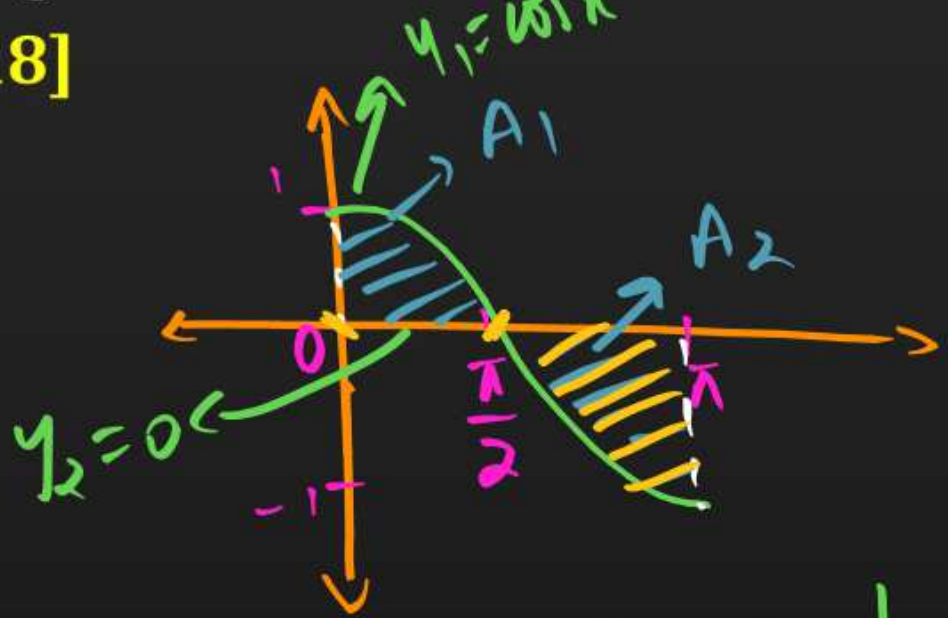
$= 8 + \frac{2}{3}(8)$

$= \frac{24+16}{3} = \frac{40}{3}$

QUESTION

#Q. Area of the region bounded by the curve $y = \cos x$, $x = 0$ and $x = \pi$ is

[2017, 2018]



(A) 2 sq. units

(B) 3 sq. units

(C) 4 sq. units

(D) 1 sq. units

$y_1 = \cos x$

$y_2 = 0$

$x_1 = 0$

$x_2 = \frac{\pi}{2}$

$A = A_1 + A_2$

But $A_2 = A_1$

$A = 2A_1$

$A = 2\int_0^{\pi/2} y\,dx$

$= 2\int_0^{\pi/2} \cos x\,dx$

$= 2\,(\sin x)_0^{\pi/2}$

$= 2(\sin x)_0^{\pi/2}$

$= 2(1-0)$

$= \underline{2}$

QUESTION

#Q. The area of the region bounded by Y-axis, $y = \cos x$ and $y = \sin x$; $0 \le x \le \frac{\pi}{2}$ is

$x = 0$

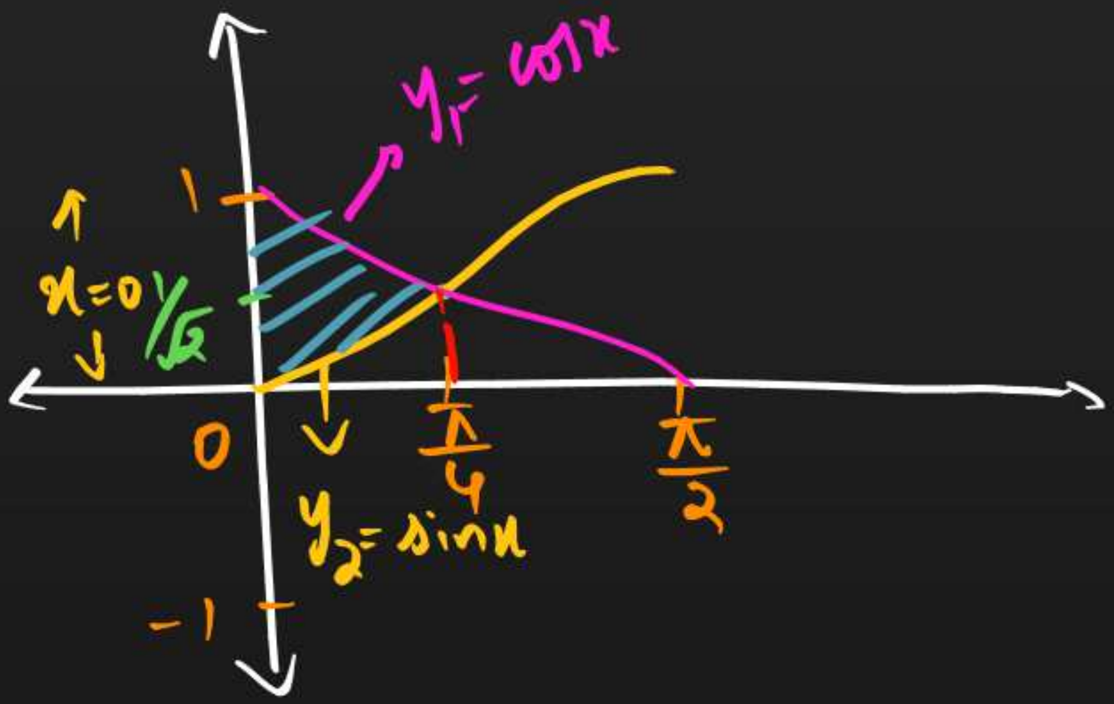
[2019]

(A) $(\sqrt{2} - 1)$ sq. units

(B) $(\sqrt{2} + 1)$ sq. units

(C) $\sqrt{2}$ sq. units

(D) $(2 - \sqrt{2})$ sq. units



$y_1 = \cos x \mid x_1 = 0$

$y_2 = \sin x \mid x_2 = \frac{\pi}{4}$

$$A = \int_0^{\pi/4} \cos x - \sin x \, dx$$

$$A = \sin x + \cos x \Big]_0^{\pi/4}$$

$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (0 + 1)$$

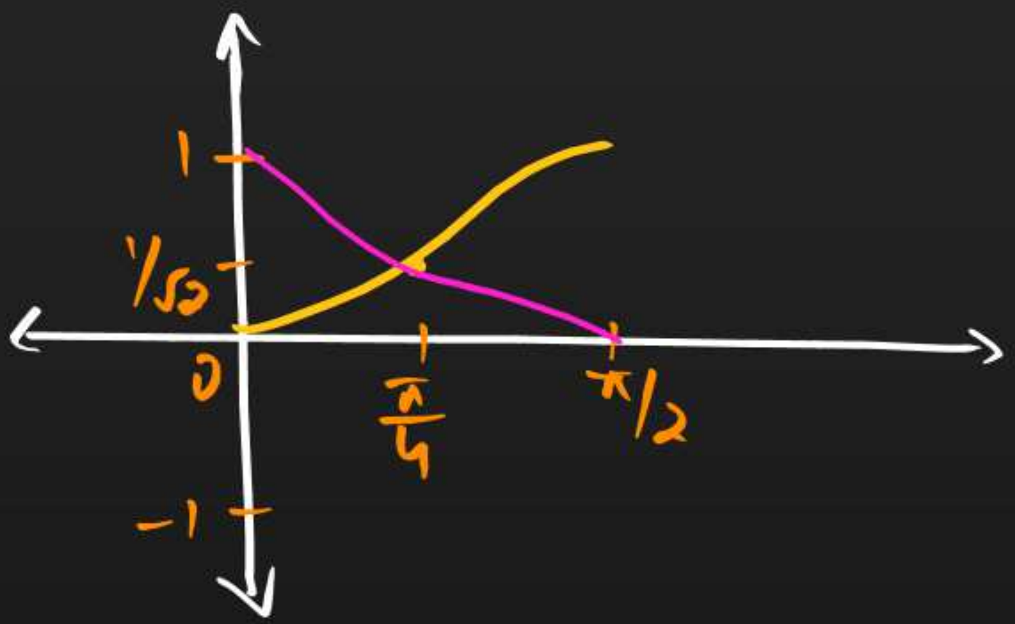
$$= \frac{2}{\sqrt{2}} - 1 = \sqrt{2} - 1$$

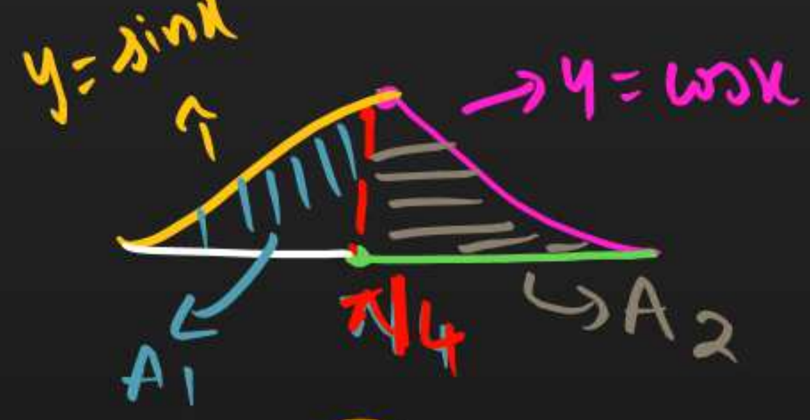
QUESTION

$\rightarrow y = 0$

#Q. The area of the region bounded by x-axis, $y = \cos x$ and $y = \sin x$; $0 \leq x \leq \frac{\pi}{2}$ is

[2019]





(A) $(\sqrt{2} - 1)$ sq. units

(B) $(\sqrt{2} + 1)$ sq. units

(C) $\sqrt{2}$ sq. units

(D) $(2 - \sqrt{2})$ sq. units

(roof is changing here)

A_1	A_2
$y_1 = \sin x$	$y_1 = \cos x$
$y_2 = 0$	$y_2 = 0$
$x_1 = 0$	$x_1 = \pi/4$
$x_2 = \pi/4$	$x_2 = \pi/2$

$$A = A_1 + A_2$$

$$= \int_0^{\pi/4} \sin x \, dx + \int_{\pi/4}^{\pi/2} \cos x \, dx$$

$$= -\left(\cos x\right)_0^{\pi/4} + \left(\sin x\right)_{\pi/4}^{\pi/2}$$

$$= -\left[\frac{1}{\sqrt{2}} - 1\right] + \left[1 - \frac{1}{\sqrt{2}}\right]$$

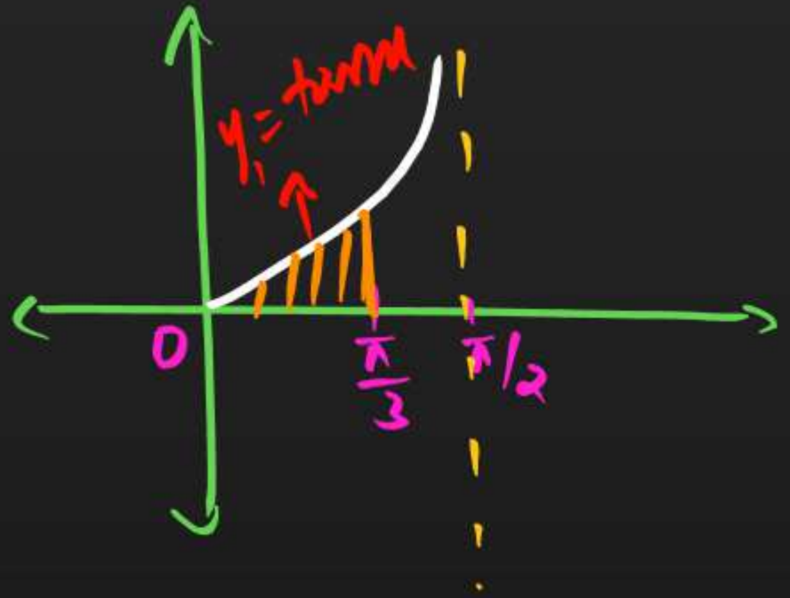
$$= -\frac{1}{2} + 1 + 1 - \frac{1}{\sqrt{2}}$$

$$= 2 - \frac{2}{\sqrt{2}}$$

$$= \underline{2 - \sqrt{2}}$$

QUESTION

#Q. Area of the region bounded by the curve $y = \tan x$, the x-axis and the line $x = \frac{\pi}{3}$ is [2022]



(A) $\log 1/2$

(B) 0

(C) $\log 2$

(D) $+\log 3$

$$A = \int_0^{\pi/3} \tan x \, dx$$

$$= \log \sec x \Big]_0^{\pi/3}$$

$$= \log \sec \frac{\pi}{3} - \log \sec 0$$

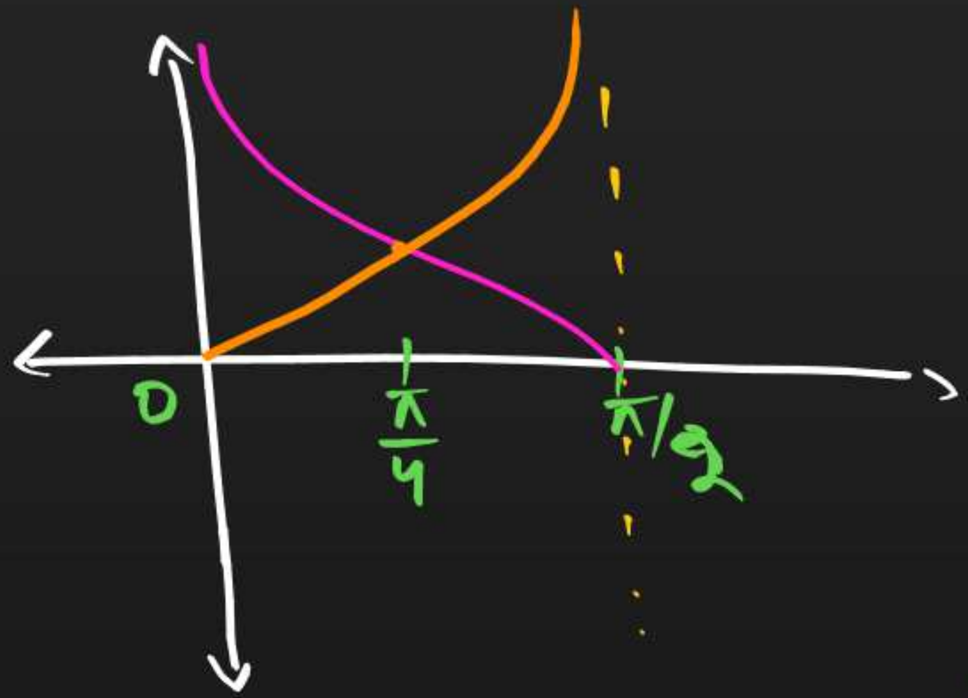
$$= \log 2 - \log 1$$

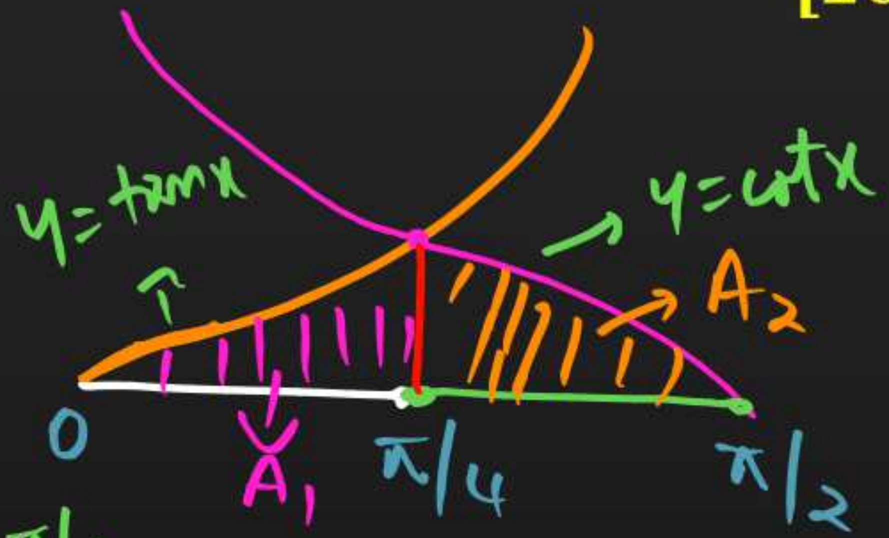
$$= \underline{\log 2}$$

QUESTION

#Q. In the interval $\left(0, \frac{\pi}{2}\right)$, area lying between the curves $y = \tan x$ and $y = \cot x$ and the X-axis is **[2023]**

$y = 0$





(A) $4 \log 2$ sq. units

(B) $3 \log 2$ sq. units

(C) $\log 2$ sq. units

(D) $2 \log 2$ sq. units

$$A = \int_0^{\pi/4} \tan x \, dx + \int_{\pi/4}^{\pi/2} \cot x \, dx$$

$$= \log \sec x \Big]_0^{\pi/4} + \log \sin x \Big]_{\pi/4}^{\pi/2}$$

$$= \log \sec x \Big]_0^{\pi/4} + \log [\sin x]_{\pi/4}^{\pi/2}$$

$$= \left(\log \sec \frac{\pi}{4} - \log \sec 0\right) + \left(\log \sin \frac{\pi}{2} - \log \sin \frac{\pi}{4}\right)$$

$$= \log \sqrt{2} - \log 1 + \log 1 - \log \frac{1}{\sqrt{2}}$$

$$= \log \sqrt{2} - 0 + 0 + \log \sqrt{2}$$

$$= 2 \log \sqrt{2}$$

$$= \log (\sqrt{2})^2$$

$$= \log 2$$

QUESTION

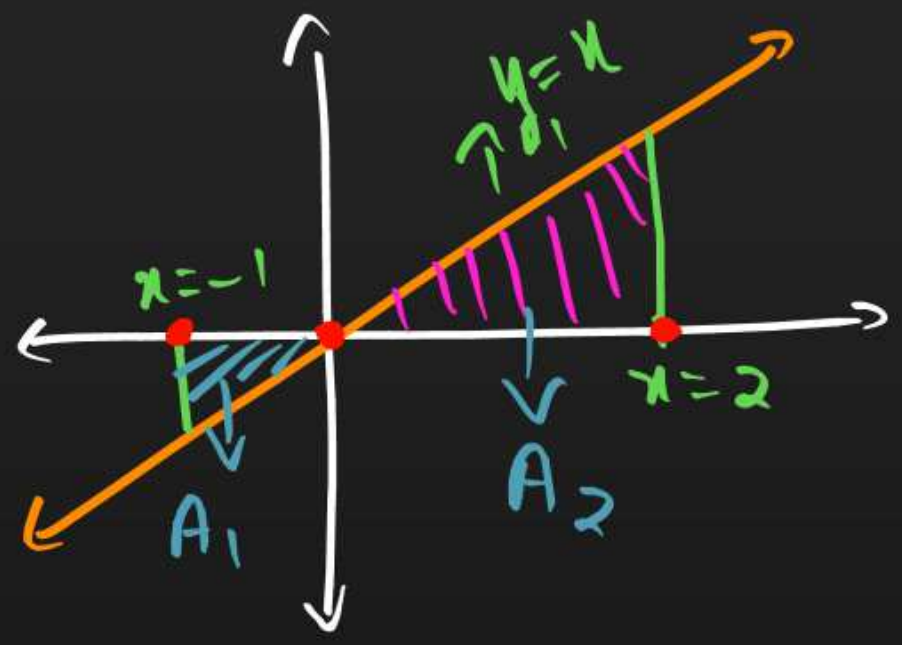
#Q. The area bounded by the line $y = x$, x-axis and ordinates $x = -1$ and $x = 2$ is **[2018]**

A 3/2 sq. units

B 5/2 sq. units

C 2 sq. units

D 3 sq. units



A_1	A_2
$y_1 = 0$	$y_1 = x$
$y_2 = x$	$y_2 = 0$
$x_1 = -1$	$x_1 = 0$
$x_2 = 0$	$x_2 = 2$

$$A = -\frac{1}{2}[0-1] + 2$$

$$= \frac{+1}{2} + 2$$

$$= \frac{5}{2}$$

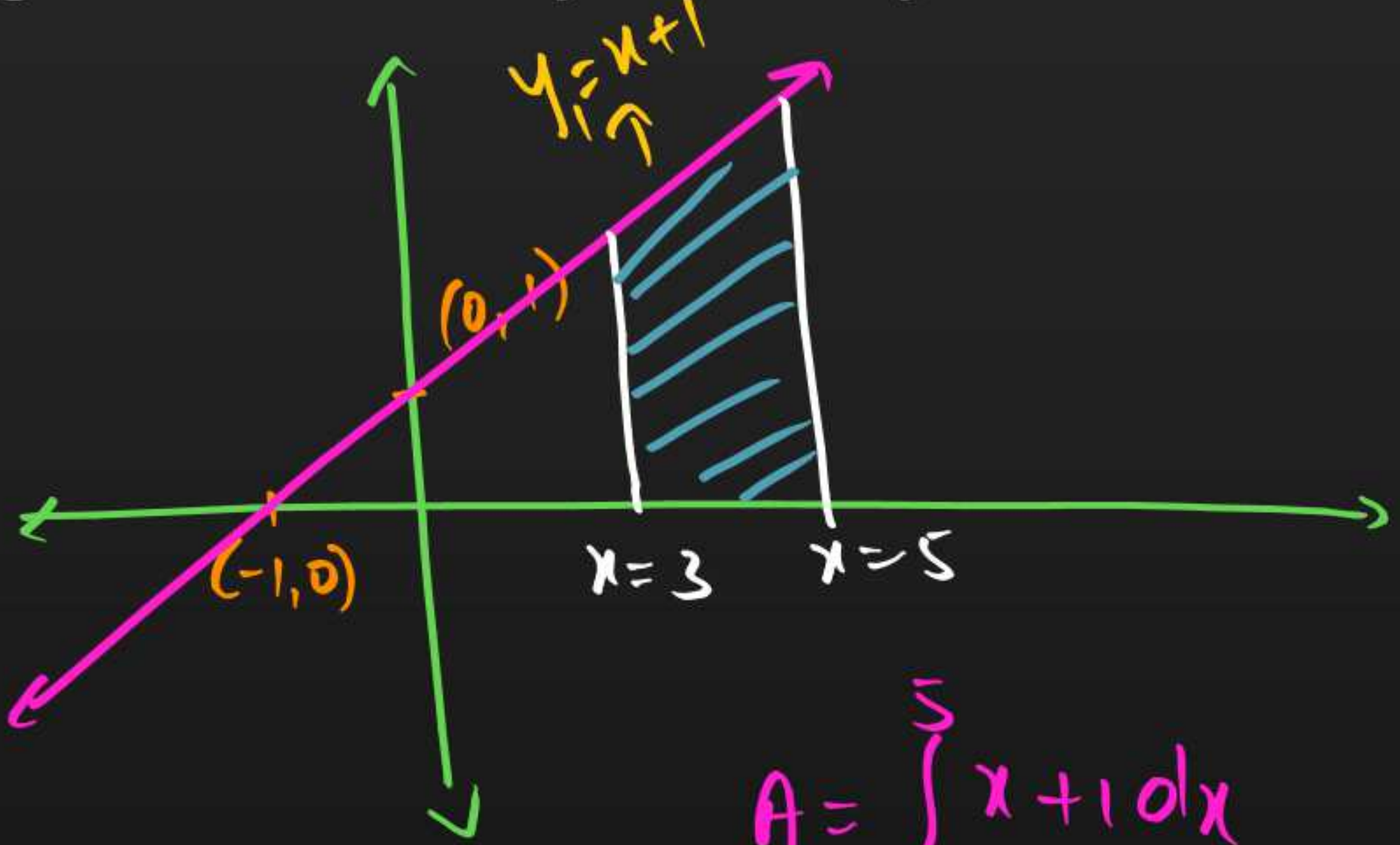
$$A = \int_{-1}^{0} (0-x)\,dx + \int_{0}^{2} x\,dx$$

$$= -\left[\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{2}$$

QUESTION

$x - y = -1 \longrightarrow \frac{x}{-1} + \frac{y}{1} = 1$

#Q. The area of the region bounded by the line $y = x + 1$, and the lines $x = 3$ and $x = 5$ is [2023]



(A) $\frac{11}{2}$ sq. units

(B) 10 sq. units

(C) 7 sq. units

(D) $\frac{7}{2}$ sq. units

$y_1 = x + 1$

$y_2 = 0$

$x_1 = 3$

$x_2 = 5$

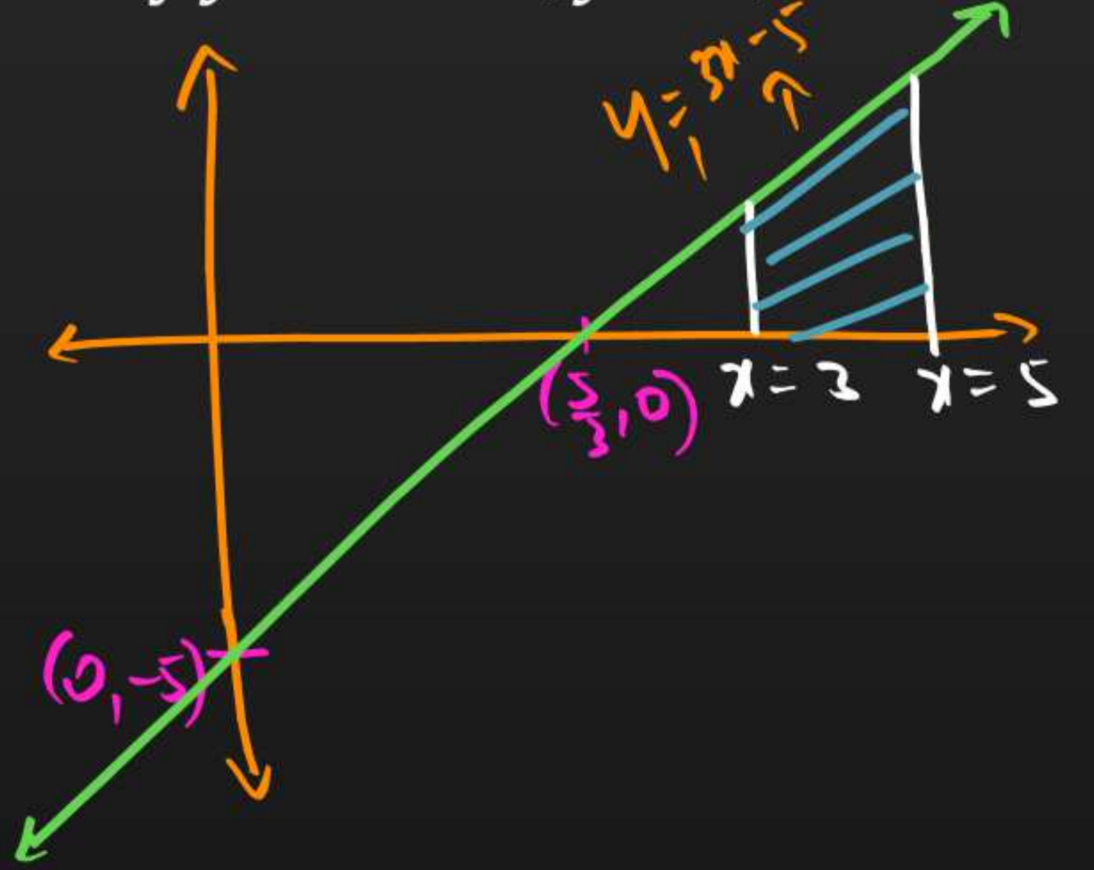
$A = \int_3^5 x + 1 \, dx$

$= \frac{x^2}{2} + x \Big]_3^5 = \frac{1}{2}(16) + 2 = \underline{10}$

QUESTION

$3x - y = 5 \;\Big|\; \frac{x}{5/3} + \frac{y}{-5} = 1$

#Q. The area enclosed by $y = 3x - 5$, $y = 0$, $x = 3$ and $x = 5$ is [2023]



$$A = \int_3^5 3x - 5\, dx$$

$$= \frac{3x^2}{2} - 5x \Big]_3^5$$

$$= \frac{3}{2}(16) - 5(2)$$

$$= 24 - 10$$

$$= 14.$$

(A) 12 sq. units

(B) 13 sq. units

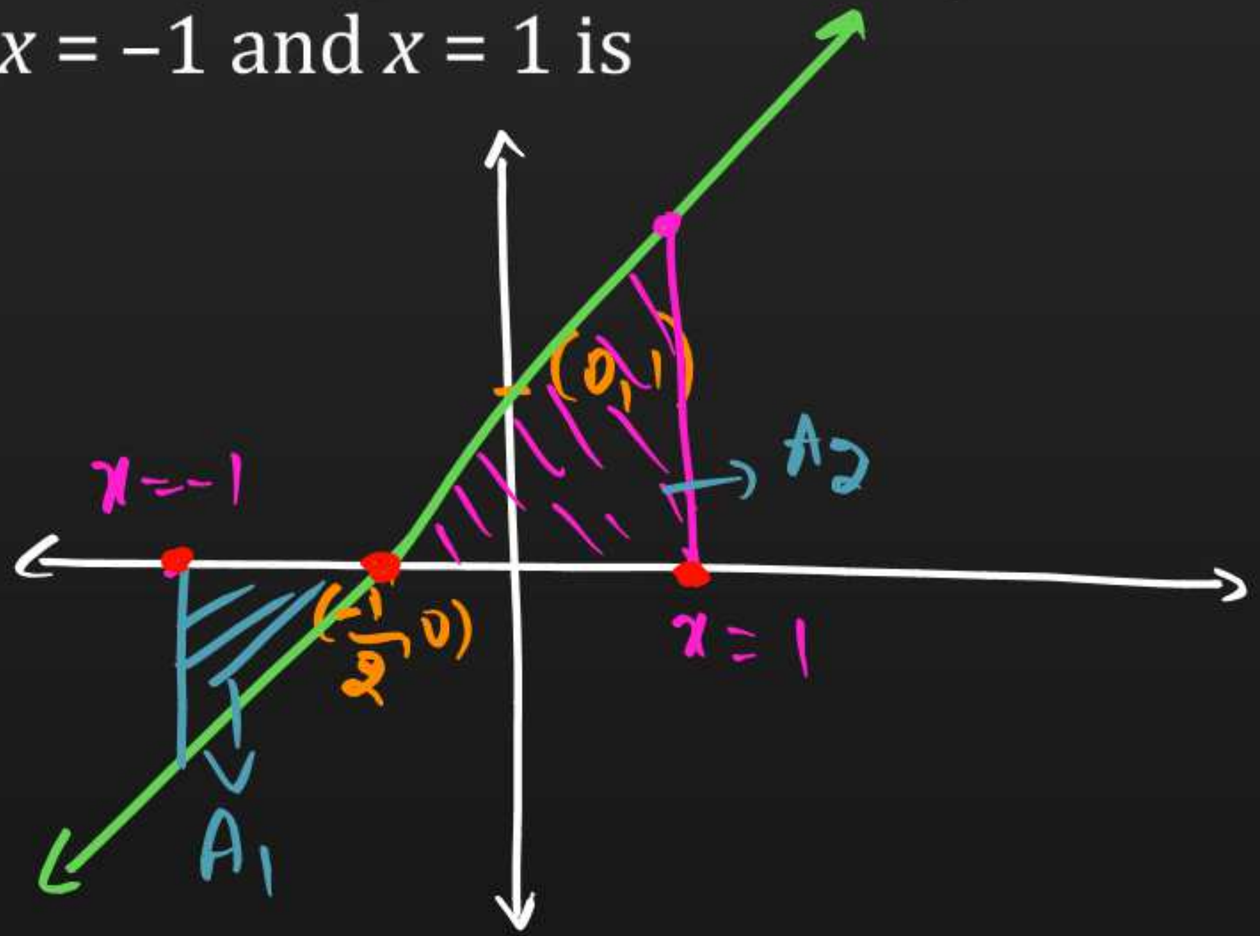
(C) $13\frac{1}{2}$ sq. units

(D) 14 sq. units ✓

QUESTION

$2x-y=-1 \hookrightarrow \frac{x}{-1/2}+\frac{y}{1}=1$

#Q. The area of the region bounded by the line $y = 2x + 1$, x-axis and the ordinates $x = -1$ and $x = 1$ is [2020]



A_1	A_2
$y_1 = 0$	$y_1 = 2x+1$
$y_2 = 2x+1$	$y_2 = 0$
$x_1 = -1$	$x_1 = -\frac{1}{2}$
$x_2 = -\frac{1}{2}$	$x_2 = 1$

(A) 2

(B) 5/2

(C) 5

(D) 9/4

$$A = \int_{-1}^{-1/2} 0-(2x+1)\,dx + \int_{-1/2}^{1} 2x+1\,dx$$

$$= -\left[x^2+x\right]_{-1}^{-1/2} + \left(x^2+x\right)_{-1/2}^{1}$$

$$= -\left[\left(\frac{1}{4}-1\right)+\left(-\frac{1}{2}+1\right)\right] + \left[\left(1-\frac{1}{4}\right)+\left(1+\frac{1}{2}\right)\right]$$

$$= -\left[-\frac{3}{4}+\frac{1}{2}\right] + \frac{3}{4} + \frac{3}{2}$$

$$= +\frac{1}{4}+\frac{3}{4}+\frac{3}{2}$$

$$= \frac{1+3+6}{4} = \frac{10}{4} = \frac{5}{2}$$

QUESTION

Area of circle $= \pi r^2$

Area of semicircle $= \frac{\pi r^2}{2}$

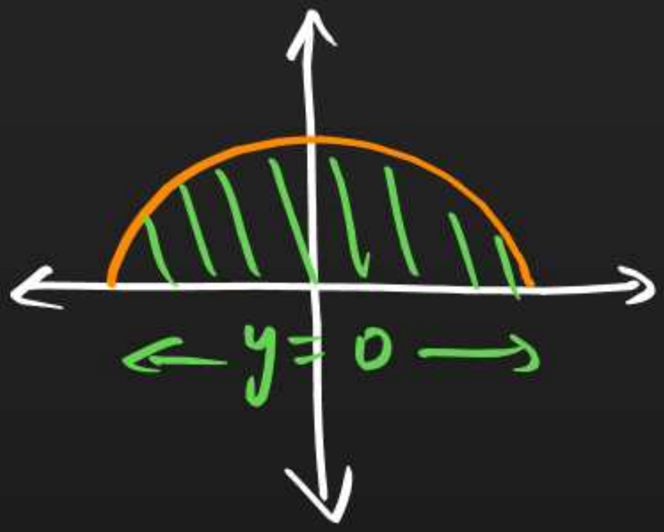
#Q. The area of the region bounded by $y = \sqrt{16 - x^2}$ and x-axis is $\to y = 0$ [2021]

$\Downarrow$

$y^2 = 16 - x^2$

$x^2 + y^2 = 4^2 \to r = 4$

$\to$ eq^n of circle



A) 8π sq. units

B) 20π sq. units

C) 16π sq. units

D) 256π sq. units

$\therefore$ Area $= \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi(4^2)$

$= \frac{16\pi}{2} = 8\pi$

① Find the area of the region $y = \sqrt{16 - x^2}$

$\Downarrow$

$y^2 = 16 - x^2$

$x^2 + y^2 = 16$

$\downarrow$

$r^2 = 16$

$A = \pi r^2$

$A = 16\pi$

Area of ellipse :- πab

$\therefore$ if given $\frac{x^2}{4} + \frac{y^2}{9} = 1$

$a^2 = 4$ | $b^2 = 9$
$a = 2$ | $b = 3$

Area $= \pi(2)(3)$

$= \underline{6\pi}$

QUESTION

#Q. If the area of the ellipse is $\frac{x^2}{25}+\frac{y^2}{\lambda^2}=1$ is 20π square units, then λ is [2021]

$a=5 \mid b=\lambda$

(A) ± 4

(B) ± 3

(C) ± 2

(D) ± 1

$A = 20\pi$

$\pi(5)(\lambda) = 20\pi$

$5\lambda = 20$

$\lambda = 4$

Thank

You