CHAPTER

3 Motion in a Straight Line

INTRODUCTION

A body is at rest when it does not change its position with time and is in motion if it changes its position with time in the frame of reference of the observer.

All motion is relative. There is no meaning of rest or motion without reference to the observer.

A passenger in a moving train is at rest with respect to another passenger in the same train while both are in motion with respect to observer on the ground. Therefore nothing is at absolute rest or in absolute motion.

To describe the motion of a particle, we introduce four important quantities namely position, displacement, velocity and acceleration. In general motion of a particle in three dimension these quantities are vectors which have direction as well as magnitude. But for a particle moving in a straight line, there are only two directions, distinguished by designating one as positive and one negative.

DISTANCE AND DISPLACEMENT

Distance

The length of the actual path between initial and final positions of a particle in a given interval of time is called distance covered by the particle. It is the actual length of the path covered by the body.

Characteristics of Distance

- It is a scalar quantity
- It depends on the path
- It never reduces with time
- Distance covered by a particle is always positive or zero and can never be negative
- Dimension: $[M^0 L^1 T^0]$
- Unit: In C. G. S. centimeter (cm), In S.I. system meter (m).

Displacement

The shortest distance from the initial position to the final position of the particle is called displacement. The displacement of a particle is measured as the change in the position of the particle in a particular direction over a given time interval. It depends only on final and initial positions.

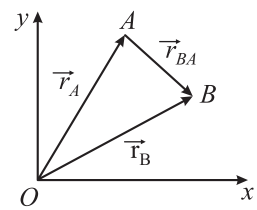
Displacement of a particle is a position vector of its final position w.r.t, initial position.

Position vector of A w.r.t. $O = \overrightarrow{OA}$

$$\Rightarrow \vec{r}_A = x_1\,\hat{i} + y_1\,\hat{j} + z_1\,\hat{k}$$

Position Vector of B w.r.t. $O = \overrightarrow{OB}$

$$\Rightarrow \vec{r}_B = x_2\,\hat{i} + y_2\,\hat{j} + z_2\,\hat{k}$$



Displacement $= \overrightarrow{AB} = \vec{r}_{BA}$

$$= (x_2 - x_1)\,\hat{i} + (y_2 - y_1)\,\hat{j} + (z_2 - z_1)\,\hat{k}$$

For a particle moving in a straight line, if we take x-axis along direction of motion, then displacement has only one component $S = x_2 - x_1$

Characteristics of Displacement

- It is a vector quantity.
- The displacement of a particle between any two points is equal to the shortest distance between them.
- The displacement of an object in a given time interval may be +ve, –ve or zero.
- The actual distance travelled by a particle in the given interval of time is always equal to or greater than the magnitude of the displacement and in no case, is it less than the magnitude of the displacement, i.e. Distance ≥ |Displacement|
- Dimension: $[M^0 L^1 T^0]$
- Unit: In C. G. S. centimeter (cm), In S.I. system meter (m).

Note: Distance is always positive but displacement may be +ve, – ve or zero.

AVERAGE SPEED AND INSTANTANEOUS SPEED

Average speed is the ratio of total distance covered by a particle in a given time interval divided by the time interval.

$$v_{av.} = \frac{\Delta s}{\Delta t} \qquad \text{(where } \Delta t = t_2 - t_1\text{)}$$

Instantaneous speed at any instant t is

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

- The slope of the distance-time graph provides the value of instantaneous speed.
- The average speed is defined for a time interval while the instantaneous speed is defined at an instant. The word speed normally implies instantaneous speed.
- Average speed and instantaneous speed both are scalar quantities.
- For any moving object, the average speed can never be zero or negative, i.e., $v_{av} > 0$, as total distance covered is always +ve only.
- If a particle travels distances s_1, s_2, s_3, ..., etc., at different speeds v_1, v_2, v_3, ... etc., respectively.

  $$v_{av} = \frac{\Delta s}{\Delta t} = \frac{\Sigma s_i}{\Sigma (s_i / v_i)} \quad \text{If } s_1 = s_2 = ... = s_n = s,$$

  Then $$\frac{1}{v_{av.}} = \frac{1}{n}\left[\frac{1}{v_1} + \frac{1}{v_2} + ...\right] = \frac{1}{n}\Sigma\frac{1}{v_i}$$

  Special case: If a particle moves a distance at speed v_1 and comes back to initial position with speed v_2, then

  $$v_{av} = \frac{2v_1v_2}{v_1 + v_2}$$

- If a particle travels at speeds v_1, v_2, ..., etc., for time intervals t_1, t_2, ...

  $$v_{av} = \frac{\Delta s}{\Delta t} = \frac{v_1t_1 + v_2t_2 + ... + v_nt_n}{t_1 + t_2 + ...t_n} = \frac{\Sigma v_it_i}{\Sigma t_i}$$

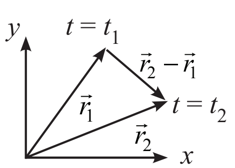
- If a particle moves for two equal intervals of time at different speeds, then $v_{av} = \frac{v_1 + v_2}{2}$.

AVERAGE VELOCITY

The average velocity is the ratio of displacement from time t_1 to t_2 and the time interval $t_2 - t_1$.

$$\vec{v}_{av.} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta \vec{r}}{\Delta t}$$

In one dimension, $v_{av} = \frac{x_2 - x_1}{t_2 - t_1}$



INSTANTANEOUS VELOCITY

The instantaneous velocity at any instant t is

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

The magnitude of instantaneous velocity is always equal to the instantaneous speed.

Instantaneous speed

$$= |\vec{v}| = \lim_{\Delta t \to 0} \frac{|\Delta \vec{r}|}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Instantaneous velocity is called simply velocity.

In one dimension, we can simply write velocity as

$$v = v_x = \frac{dx}{dt}$$

If an object moves along a straight line without changing its direction, then the magnitude of the average velocity is equal to the average speed, otherwise magnitude of average velocity < average speed.

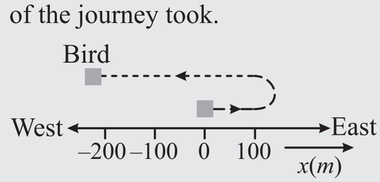
- Velocity can be +*ve* or –*ve* as it is a vector but speed can never be negative as it is the magnitude of velocity.
- If a body is moving with a constant velocity, then the average velocity and instantaneous velocity are equal.
- The velocity of a body is uniform, if both magnitude and direction do not change.
- If a body moves with non-uniform velocity, then magnitude of velocity may change or direction of velocity may change or both.
- A body can have non-zero speed and zero average velocity when a body completes one revolution around a circle, the average velocity is zero since the displacement is zero. But the average speed is not zero since the distance travelled ≠ 0.
- If a body is moving with constant speed then its velocity may or may not be constant. In case of uniform circular motion though speed remains constant but velocity changes instant to instant because of change in direction.

Train Your Brain

Example 1: A bird flies towards east at 10 m/s for 100 m. It then turns around and flies at 20 m/s for 15s. Find

(*a*) Its average speed (*b*) Its average velocity

Sol. (*a*) Let us take the x axis to point eastwards A sketch of the path is shown in the figure. To find the required quantities, we need the total time interval. The first of the journey took.



Δt_1 = (100 m)/(10 m/s) = 10s, and we are given t_2 = 15s for the second part. Hence the total time interval is $\Delta t = \Delta t_1 + \Delta t_2 = 25$ s

The bird flies 100 m east and then (20 m/s) × (15s) = 300 m west.

(a) Average speed $= \dfrac{\text{Distance}}{\Delta t}$

$= \dfrac{100\,\text{m} + 300\,\text{m}}{25\text{s}} = 16\,\text{m/s}$

(b) The net displacement is

$\Delta x = -200$ m

So, $v_{av} = \dfrac{\Delta x}{\Delta t} = \dfrac{-200\text{m}}{25\text{s}} = -8$ m/s

The negative sign means that v_{av} is directed toward the west.

Example 2: A particle moves with speed v_1 along a particular direction. After some time it turns back and reaches the starting point again travelling with speed v_2. Find (for the whole journey).

(a) Average velocity (b) Average speed

Sol. (a) Since the particle reaches the starting point again, its displacement is zero.

$\therefore$ Average velocity $= \dfrac{\text{Net displacement}}{\text{total time}} = 0$

(b) Let it travelled distance x while moving away as well as while moving towards the starter point.

Time taken to go away is $t_1 = \dfrac{x}{v_1}$

Time taken while return journey $t_2 = \dfrac{x}{v_2}$

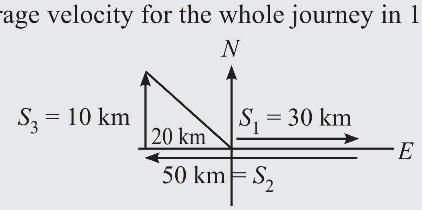
$\therefore$ Average speed $= \dfrac{x+x}{t_1+t_2} = \dfrac{2x}{\dfrac{x}{v_1}+\dfrac{x}{v_2}}$

$$v_{av} = \frac{2v_1v_2}{v_1+v_2}$$

i.e., harmonic mean of individual speeds.

Example 3: A person goes 30 km East, then he walks 50km west and then he goes 10 km N. Find Average speed and average velocity for the whole journey in 15 hrs.

Sol.



Average speed $= \dfrac{\text{Total Distance}}{\text{Total Time}} = \dfrac{30+50+10}{15}$

$= 6$ km/hr

Average velocity $= \dfrac{\text{Total Displacement}}{\text{Total Time}}$

$= \dfrac{\sqrt{(20^2+10^2)}}{15} = \dfrac{10\sqrt{5}}{15}$

Average velocity $= \dfrac{2\sqrt{5}}{3}$ km/hr

Concept Application

1. A car moves 30 km with 20 km/hr and then 30 km with 30 km/hr. Find average speed in Average velocity for the whole journey in the same straight line.

ACCELERATION

The rate of change of velocity of an object with time is called acceleration of the object.

Let v and v' be the velocity of the object at time t and t' respectively, then acceleration of the body is given by

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

For one dimensional motion,

$$a = a_x = \frac{dv_x}{dt} = \frac{d\vec{v}}{dt}$$

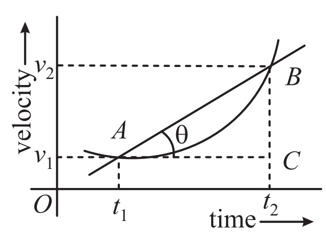
- Acceleration is a vector quantity.
- It is positive if the velocity is increasing and is negative if the velocity is decreasing.
- The negative acceleration is also called retardation or deceleration.
- **Unit:** In S.I. System m/s^2
  In C.G.S. System cm/s^2
- **Dimension :** $[M^0L^1T^{-2}]$

Average Acceleration

When an object is moving with a variable acceleration in a straight line, then the average acceleration of the object for the given motion is defined as the ratio of the total change in velocity of the object during motion to the total time taken i.e.,

Average Acceleration $= \dfrac{\text{total change in velocity}}{\text{total time taken}}$

Suppose the velocity of a particle is v_1 at time t_1 and v_2 at time t_2.



Then, Change in velocity $= v_2 - v_1 = \Delta v$

Elapsed time in changing the velocity $= t_2 - t_1 = \Delta t$

Thus, $a_{av} = \dfrac{v_2 - v_1}{t_2 - t_1} = \dfrac{\Delta v}{\Delta t}$

$\Rightarrow$ $a_{av} = \dfrac{BC}{AC} = \tan\theta =$ the slope of chord of $v - t$ graph is average acceleration.

Note: If any body is accelerated with acceleration a_1 till time t_1 and acceleration a_2 up to time t_2 then average acceleration will $a_{av} = \dfrac{a_1t_1 + a_2t_2}{t_1 + t_2}$.

Instantaneous Acceleration

The acceleration of the object at a given instant of time or at a given point of motion, is called its instantaneous acceleration.

Suppose the velocity of a particle at time $t_1 = t$ is $v_1 = v$ and becomes $v = v + \Delta v$ at time $t_2 = t + \Delta t$,

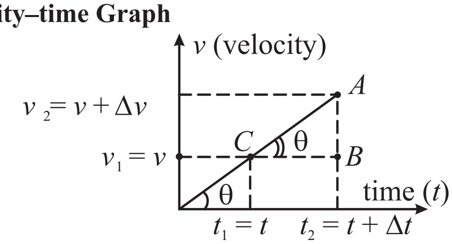
Then, $a_{av} = \dfrac{\Delta v}{\Delta t}$

If Δt approaches to zero, then the rate of change of velocity will be instantaneous acceleration.

Instantaneous acceleration $a_{\text{inst}} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$

Instantaneous acceleration at a point is equal to slope of tangent at that point on velocity time graph in the graph shown.

Velocity–time Graph



$$\text{Slope} = \frac{AB}{BC} = \frac{\Delta v}{\Delta t} = \text{acceleration}$$

As $v = \dfrac{dx}{dt}$ therefore $a = \dfrac{d}{dt}\left(\dfrac{dx}{dt}\right) = \dfrac{d^2x}{dt^2}$

Thus, instantaneous acceleration of an object is equal to the second derivative of the position w.r.t. time of the object at the given instant.

Note:

(i) It is not essential that when velocity is zero acceleration must be zero.

e.g. In vertical motion under gravity at the top point $v = 0$ but $a \neq 0$

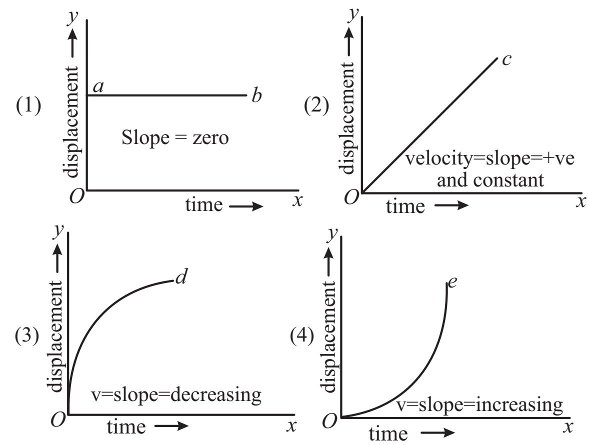
(ii) If a and v are both positive or both negative, speed of a body increases. If a and v have opposite signs then speed decreases, this is called retardation.

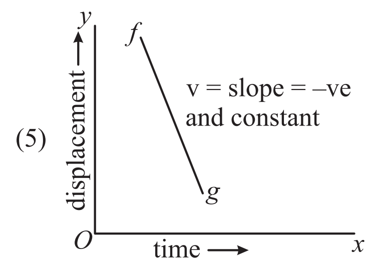
(iii) For curvilinear motion velocity may vary even if speed is constant.

DISPLACEMENT-TIME GRAPHS AND THEIR CHARACTERISTICS

If the graph is:

- A straight line parallel to time-axis, shown by line *ab*, it means that the body is at rest, i.e., $v = 0$.
- A straight line inclined to x-axis (such as *Oc* and *fg*) shows that body is moving with a constant velocity.
- A straight line inclined to x-axis by an angle > 90° (line *f g*) represent negative velocity.
- The curve is of the type *Od* (graph-3) whose slope decreases with time, the velocity goes on decreasing, i.e., motion is retarded.
- The curve is of the type *Oe* (graph-4) whose slope increases with time, the velocity goes on increasing, i.e., motion is accelerated.



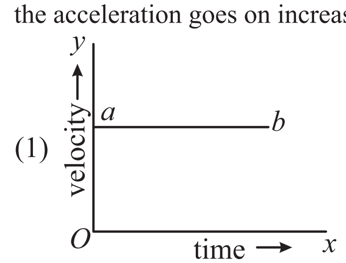


No line can ever be perpendicular to the time axis because it implies infinite velocity.

VELOCITY-TIME GRAPHS AND THEIR CHARACTERISTICS

If the graph is:

- A straight line parallel to time axis shown by line *ab*, it means that the body is moving with a constant velocity or acceleration (a) is zero.
- A straight line inclined to the x-axis with +*ve* slope (line *Oc*) it means that the body is moving with constant acceleration.
- A straight line inclined to x-axis with negative slope it means that the body is under retardation.
- A curve like *Od* (graph 3) whose slope decreases with time, the acceleration goes on decreasing.
- A curve like *Oe* (graph 4) whose slope increases with time, the acceleration goes on increasing.



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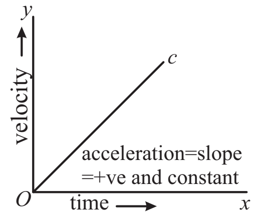

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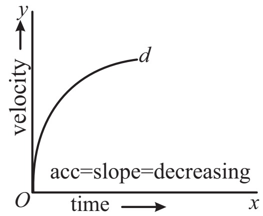

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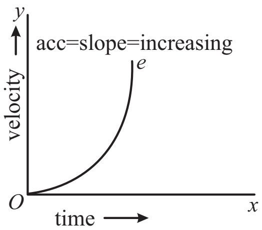

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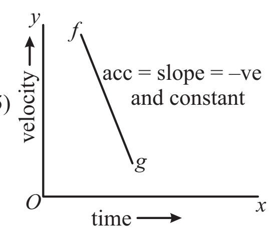


Note:

1. No velocity-time graph can ever be perpendicular to the time-axis because it implies infinite acceleration.
2. The area of velocity-time graph with time axis represents the displacement of that body.

MOTION WITH CONSTANT ACCELERATION

In many types of motion, the acceleration is either constant or approximately so. For example, near the surface of earth all objects fall vertically with constant acceleration if air resistance is neglected. Even when acceleration is not constant, we can learn something about the motion of the body by using constant acceleration results to be developed later in this section.

Let the velocity of body at $t = 0$ is u and it moves with constant acceleration a and acquires velocity v at time t.

$$\frac{dv}{dt} = a \quad \text{or} \quad dv = adt$$

$$\Rightarrow \int_u^v dv = \int_0^t adt = a\int_0^t dt$$

$$\Rightarrow v\Big|_u^v = at\Big|_0^t$$

$$\Rightarrow v - u = at \quad \text{or} \quad v = u + at \qquad \text{...(i)}$$

To find the displacement, we again integrate

Let body be at x_0 at $t = 0$ and reaches x at time t

$$v = u + at$$

$$\frac{dx}{dt} = u + at$$

or $dx = (u + at)dt$

$$\int_{x_0}^x dx = \int_0^t (u+at)dt = u\int_0^t dt + a\int_0^t tdt$$

$$x\Big|_{x_0}^x = ut\Big|_0^t + a\frac{t^2}{2}\Big|_0^t$$

$$x - x_0 = ut + \frac{1}{2}at^2$$

$$S = x - x_0 = ut + \frac{1}{2}at^2 \qquad \text{...(ii)}$$

We can also find, relation between velocity and displacement. By using chain rule

$$a = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = \frac{vdv}{dx}$$

$$\Rightarrow vdv = adx$$

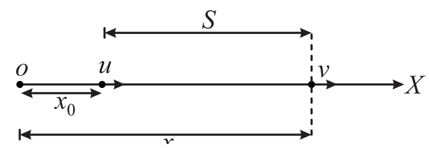
$$\int_u^v vdv = a\int_{x_0}^x dx$$

$$\frac{v^2}{2}\Big|_u^v = a\,x\Big|_{x_0}^x$$

$$\frac{v^2}{2} - \frac{u^2}{2} = a(x - x_0)$$

$$\Rightarrow v^2 - u^2 = 2aS \qquad \text{...(iii)}$$

These relation are very helpful in solving the problems of motion in one dimension. All these relations are given in table below for easy reference.



Equation	Contains		
	s	***v***	***t***
$v = u + at$	No	Yes	Yes
$s = ut + \frac{1}{2}at^2$	Yes	No	Yes
$v^2 - u^2 = 2as$	Yes	Yes	No

In simple problems in uniformly accelerated motion, two parameters are given third is to be found. Depending on convenience one can choose any one of the three relation. The following two relation are also helpful in solving problems.

Displacement of the Body in the n^{th} Second:

$$S_n = S(t = n) - s(t = n - 1)$$

$$= \left(un + \frac{1}{2}an^2\right) - \left(u(n-1+\frac{1}{2}a(n-1)^2\right)$$

$$= u + \frac{a}{2}(2n-1)$$

Average velocity:

$$V_{avg} = \frac{S}{t} = \frac{ut + \frac{1}{2}at^2}{t} = u + \frac{at}{2}$$

$$= u + \frac{v-u}{2} = \frac{u+v}{2}$$

$$\text{or} \quad S = \left(\frac{u+v}{2}\right)t$$

This relation is only valid for uniform acceleration.

Note:

1. These equations can be applied only when acceleration is constant.

- If a body moves with uniform acceleration and velocity changes from u to v in a time interval, then average velocity = $\frac{v+u}{2}$.
- If a body moving with uniform acceleration has velocities u and v at two points in its path, then the velocity at the midpoint of its path = $\sqrt{\frac{u^2+v^2}{2}}$.
- In position time graph, slope is equal to velocity.
- In velocity time graph area under the curve is displacement and slope is equal to acceleration.
- In acceleration time graph area under the curve is equal to change in velocity.
- **For a body starting from rest and moving with uniform acceleration,**
  - (*a*) The ratio of distances covered in first one sec, two sec, three sec, ... is :

    $1^2 : 2^2 : 3^2 : ...$, i.e., 1 : 4 : 9 : ...

    Ratio of distances covered in

    1st, 2nd, 3rd sec, ... is 1 : 3 : 5 : ...
  - (*b*) The ratio of velocities after

    1 sec, 2 sec, 3 sec, ... is 1 : 2 : 3 : ...

Train Your Brain

Example 4: The displacement of a particle, moving in a straight line, is given by $S = 2t^2 + 2t + 4$ where s is in metres and t in seconds. The acceleration of the particle is

(*a*) 2 m/s^2

(*b*) 4 m/s^2

(*c*) 6 m/s^2

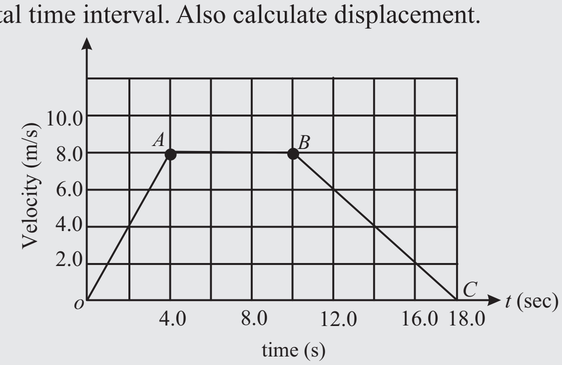
(*d*) 8 m/s^2

Sol. (*b*) Given $S = 2t^2 + 2t + 4$

$\therefore$ Velocity (v) = $\frac{dS}{dt} = 4t + 2$

Acceleration (a) = $\frac{dv}{dt} = 4(1) + 0 = 4\text{m/s}^2$

Example 5: What is the acceleration for each graph segment in figure? Describe the motion of the object over the total time interval. Also calculate displacement.



Sol. Segment OA; $a = \frac{8-0}{4-0}$ = 2 m/s^2

Segment AB; graph horizontal i.e., slope zero i.e., $a = 0$

Segment BC; $a = \frac{0-8}{18-10}$ = – 1 m/s^2

The graph is trapezium. Its area between t = 0 to t = 18s is displacement.

Area of v-t graph = displacement = $\frac{1}{2}(18+6)\times 8 = 96$ m

Particle accelerates uniformly for first 4 sec., then moves with uniform velocity for next 6 sec. and then retards uniformly to come to rest in next 8 sec.

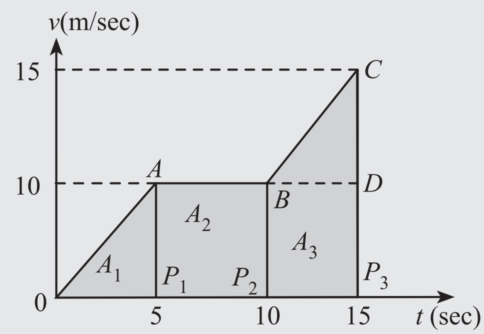
Example 6: The motion of a body is described in (v–t) graph as given under.

Find the followings:

(*a*) Max and Min acceleration

(*b*) Displacement from t = 10 to t = 15

(*c*) Av-velocity for the whole journey.



Sol. (*a*) We know slope ($v - t$) graph gives acceleration

$$\text{Slope}_{OA} = \frac{AP_1}{OP_1} = \frac{10}{5} = 2\,\text{m/sec}^2 \text{ (Max-acceleration)}$$

$$\text{Slope}_{A\to B} = 0 \text{ m/sec}^2 \qquad (\text{min}-\text{acceleration})$$

$$\text{Slope}_{B\to C} = \frac{CD}{BD} = \frac{5}{5}$$

$= 1\text{m/sec}^2$

(*b*) $\underset{\text{from } t=10 \text{ to } t=15 \text{ sec}}{\text{Displacement}} = \textit{Area}\,(v-t)\textit{graph}$

$$= \frac{1}{2}(10+15)\times 5 = 62.5 \text{ m}$$

(*c*) $\text{Average Velocity} = \frac{\text{Total Displacement}}{\text{Total Time}} = \frac{\text{Area }(v-t)\text{graph}}{t_{total}}$

$$= \frac{A_1 + A_2 + A_3}{t_{total}} = \frac{25+50+62.5}{15}$$

$$= \frac{137.5}{15} = 9.16 \text{ m/sec}$$

Example 7: How long does it take for a particle to travel 100 m if it begins from rest and accelerates at 10 m/s^2? What is its velocity when it has travelled 100 m? What is the average velocity during this time.

Sol. $u = 0$, $a = 10$ m/s^2, $S = 100$ m

Applying $S = ut + \frac{1}{2}at^2$

we get $100 = \frac{1}{2} \times 10 \times t^2$

$\Rightarrow t = \sqrt{20} = 2\sqrt{5}$ s

$v = u + at = 0 + 10 \times 2\sqrt{5} = 20\sqrt{5}$ m

$v_{avg} = \frac{u+v}{2} = \frac{0+20\sqrt{5}}{2} = 10\sqrt{5}$ m/s

Example 8: A car travelling with 72 km/hr is 30 m from a barrier when the driver slams the breaks. The car hits barrier 2.0 seconds later.

(*a*) What is the car's constant deceleration before impact?

(*b*) How fast is car travelling at impact?

Sol. (*a*) $u = 72$ km/hr $= 72 \times \frac{5}{18}$ m/s $= 20$ m/s

$S = 30$ m

$t = 2$s

$a = ?$

$S = ut + \frac{1}{2}at^2$

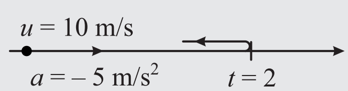
$30 = 20 \times 2 + \frac{1}{2} \times a \times 2^2$

$\Rightarrow a = -\frac{10}{2} = -5$ m/s^2

(*b*) $v = u + at = 20 + (-5) \times 2 = 10$ m/s

Example 9: A particle moving with initial velocity of 10 m/s towards East has an acceleration of 5 m/s^2 towards west. Find the displacement and distance travelled by the particle in first 4 seconds?

Sol.



$v = u + at = 10 - 5t$

The direction of velocity changes after two seconds.

$S = 10 \times 4 + \frac{1}{2}(-5) \times 4^2 = 0 =$ displacement

Distance travelled is not equal to displacement because during course of journey, velocity changes direction.

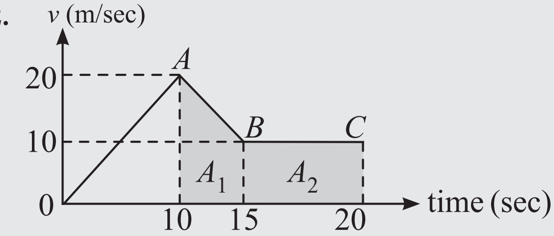
$D = S(t = 2) + |S(t = 4) - S(t = 2)|$

$= \left(10 \times 2 - \frac{1}{2} \times 5 \times 2^2\right) + \left|0 - (10 \times 2) - \frac{1}{2} \times 5 \times 2^2\right|$

$= 10 + 10 = 20$ m

Concept Application

2.



(*a*) Find the ratio of Acceleration to Retardation in the graph shown.

(*b*) Total distance covered between 10 to 20 sec.

VERTICAL MOTION UNDER GRAVITY (FREE FALL)

Motion that occurs solely under the influence of gravity is called free fall. Thus a body projected upward or downward or released from rest are all under free fall.

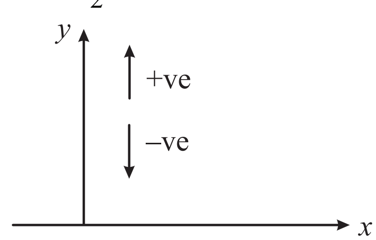
In the absence of air resistance all falling bodies have the same acceleration due to gravity, regardless of their sizes or shapes.

The value of the acceleration due to gravity depends on both latitude and altitude. It is approximately 9.8 m/s^2 near the surface of the earth. For simplicity a value of 10 m/s^2 used. To do calculations regarding motion under gravity, we follow a proper sign convention. If we take upward direction as positive then $a = -g$

Thus the equation of kinematics may be modified as

$v = u - gt$ ...(i)

$\Delta y = y - y_0 = ut - \frac{1}{2}gt^2$ ...(ii)



$v^2 = u^2 - 2g(y - y_0)$ ...(iii)

y_0 = position of particle at time $t = 0$

y = position of particle at time t.

u = velocity of particle at time $t = 0$

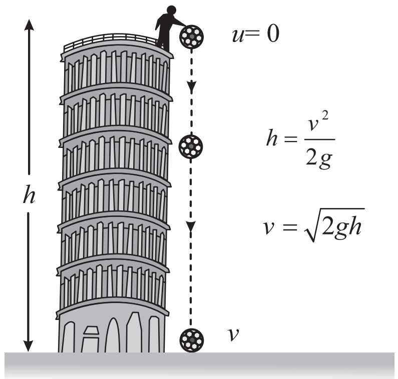
v = velocity of particle at time t.

(i) **A body dropped from some height (initial velocity zero)**

- Equation of motion: Taking initial position as origin and downward direction as negative. Here we have,

$u = 0$ [As body starts from rest]

$a = -g$ as acceleration is in the downward direction



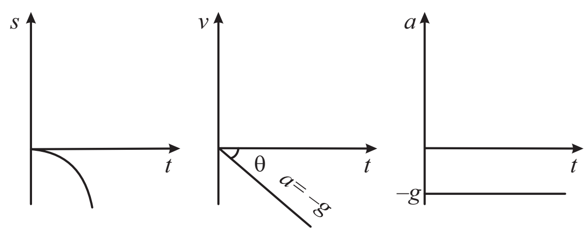
$v = -gt$ ...(i)

$\Delta y = -h = -\frac{1}{2}gt^2$ ...(ii)

$v^2 = 2(-g)(-h) = 2gh$ ...(iii)

$h_n = \frac{g}{2}(2n-1)$ = Height covered in n^{th} second. ...(iv)

- Graph of displacement velocity and acceleration with respect to time:



- As $h = (1/2)gt^2$, i.e., $h \propto t^2$, distance covered in time t, $2t$, $3t$, etc., will be in the ratio of $1^2 : 2^2 : 3^2$, i.e., square of integers.
- The distance covered in the nth sec, $h_n = \frac{1}{2}g(2n-1)$

  So distance covered in I, II, III sec, etc., will be in the ratio of 1 : 3 : 5. This is called 'Galileo's Law of odd numbers'.

(ii) **A body projected vertically downward with some initial velocity:** The initial velocity is downward and will be negative

Equation of motion: $v = -u - gt$

$\Delta y = -h = -ut - \frac{1}{2}gt^2$

$v^2 = u^2 + 2gh$

$h_n = u + \frac{g}{2}(2n-1)$

(iii) **A body is projected vertically upward:**

Equation of motion: Taking initial position as origin and vertically up as positive

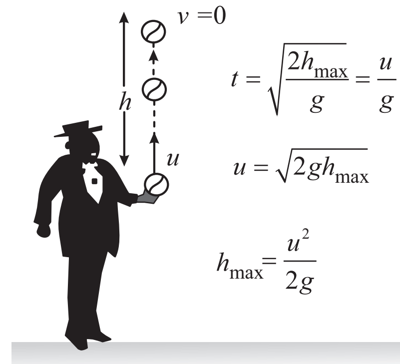
$a = -g$ [As acceleration is downwards]

So, if the body is projected with velocity u and after time t it reaches up to height h then

$v = u - gt$; $h = ut - \frac{1}{2}gt^2$; $v^2 = u^2 - 2gh$;

$h_n = u - \frac{g}{2}(2n-1)$

For maximum height $v = 0$

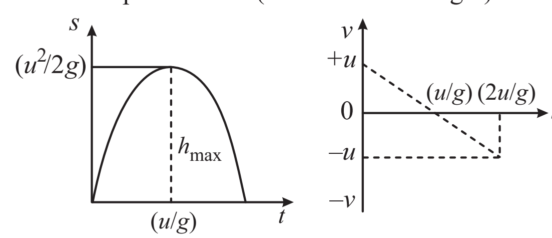


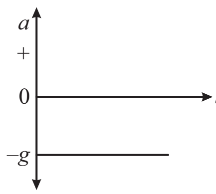
So from above equation

$u = gt$,

$h_{max} = \frac{1}{2}gt^2$ and $u^2 = 2gh_{max}$

- Graph of displacement, velocity ansssd acceleration with respect to time (for maximum height):





Important Points

(i) In case of motion under gravity for a given body, mass, acceleration, and mechanical energy remains constant while speed, velocity, momentum, kinetic energy and potential energy changes.

(ii) The motion is independent of the mass of the body, as in any equation of motion, mass is not involved. That is why a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity i.e., $t = \sqrt{(2h/g)}$ and $v = \sqrt{2gh}$

(iii) In case of motion under gravity time taken to go up is equal to the time taken to fall down through the same distance. Time of ascent (t_1) = time of descent (t_2) = u/g

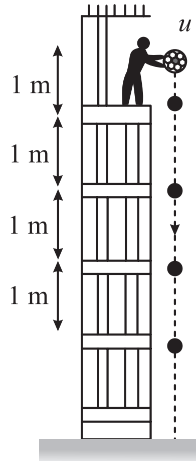
Total time of flight $T = t_1 + t_2 = \frac{2u}{g}$

(iv) In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.

Acceleration at any point on the path is same whether the body is moving in upwards or downward direction.

(v) A particle is dropped vertically from rest from a height. The time taken by it to fall through successive distance of 1 m each will then be in the ratio of the difference in the square roots of the integers i.e.

$\sqrt{1}, (\sqrt{2}-\sqrt{1}), (\sqrt{3}-\sqrt{2}), (\sqrt{4}-\sqrt{3}), \ldots\ldots$



$t_1 = \sqrt{2/g}$

$t_2 = \sqrt{4/g} - \sqrt{2/g}$

$t_3 = (\sqrt{3}-\sqrt{2})(\sqrt{2/g})$

$t_4 = (\sqrt{4}-\sqrt{3})(\sqrt{2/g})$

Notes:

(i) During ascent, $a = -g$, velocity becomes less positive i.e., speed decreases velocity and acceleration are in opposite direction.

(ii) During descent, $a = -g$, but now it is in the direction of velocity so it is not retardation. It makes velocity becomes more negative i.e. increases v in negative direction. Velocity and acceleration are in the same direction.

Train Your Brain

Example 10: A man is standing on the top of a building, throws a ball with speed 5 m/s in upward direction from 30 m height above the ground level. How much time does it takes to reach the ground.

Sol. $u = 5$ m/s

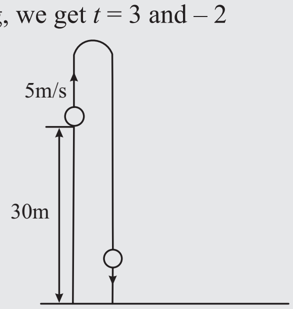
When it reaches the ground, $\Delta y = -30$ m

$\therefore$ From above equation (ii)

$-30 = 5t - \frac{1}{2}(10)t^2$

$\Rightarrow t^2 - t - 6 = 0$

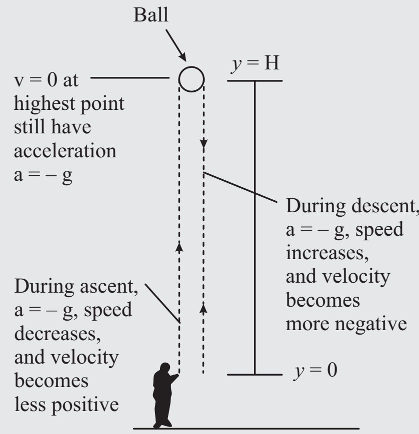
On solving, we get $t = 3$ and -2



Rejecting $t = -2$ sec, we get $t = 3$ sec

Example 11: A kid throws a ball up, with some initial speed. Comment on magnitudes and signs of acceleration and velocity of the ball.

Sol.



Example 12: If a body is thrown up with the velocity of 15 m/s then maximum height attained by the body is ($g = 10$ m/s^2)

(*a*) 11.25 m (*b*) 16.2 m

(*c*) 24.5 m (*d*) 7.62 m

Sol. (*a*) $H_{max} = \frac{u^2}{2g} = \frac{(15)^2}{2\times 10} = 11.25\,\text{m}$

Example 13: A body falls from rest in the gravitational field of the earth. The distance travelled in the fifth second of its motion is ($g = 10$ m/s^2)

(*a*) 25 m (*b*) 45 m (*c*) 90 m (*d*) 125 m

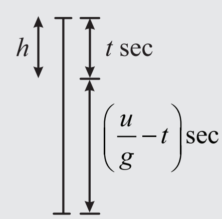
Sol. (*b*) $h_n = \frac{g}{2}(2n-1) \Rightarrow h_{5th} = \frac{10}{2}(2\times 5 - 1) = 45$ m

Example 14: If a ball is thrown vertically upwards with speed u, the distance covered during the last t seconds of its ascent is

(*a*) $\frac{1}{2}gt^2$ (*b*) $ut - \frac{1}{2}gt^2$

(*c*) $(u - gt)t$ (*d*) ut

Sol. (*a*) If ball is thrown with velocity u, then time of flight $= \frac{u}{g}$



Velocity after $\left(\frac{u}{g} - t\right)$ sec $\quad v = u - g\left(\frac{u}{g} - t\right) = gt.$

So, distance in last 't' sec $\quad 0^2 = (gt)^2 - 2(g)h.$

$h = \frac{1}{2}gt^2.$

Example 15: A man drops a ball downside from the roof of a tower of height 400 m. At the same time another ball is thrown upside with a velocity 50 m/s from the foot of tower. What is the height from the foot of the tower where the two balls would meet?

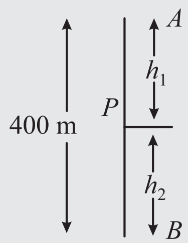
(*a*) 100 meters (*b*) 320 meters

(*c*) 80 meters (*d*) 240 meters

Sol. (*c*) Let both balls meet at point *P* after time *t*.

The distance travelled by ball *A*

$$h_1 = \frac{1}{2}gt^2 \quad ...(i)$$



The distance travelled by ball *B*

$$h_2 = ut - \frac{1}{2}gt^2 \quad ...(ii)$$

By adding (i) and (ii) $h_1 + h_2 = ut = 400$

(Given $h = h_1 + h_2 = 400$)

$\therefore$ $t = 400/50 = 8$ s and $h_1 = 320$ m, $h_2 = 80$ m

Example 16: Water drops fall at regular intervals from a tap which is 5 m above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant

(*a*) 2.50 m (*b*) 3.75 m

(*c*) 4.00 m (*d*) 1.25 m

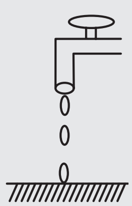
Sol. (*b*) Let the interval be *t* then from question

For first drop $\frac{1}{2}g(2t)^2 = 5$ …(i)

For second drop $x = \frac{1}{2}gt^2$ …(ii)

By solving (i) and (ii) $x = \frac{5}{4}$ and

Hence required height $h = 5 - \frac{5}{4} = 3.75m.$



Example 17: A balloon is at a height of 81 m and is ascending upwards with a velocity of 12 m/sec. A body of 2 kg weight is dropped from it. If $g = 10$ m/s^2 the body will reach the surface of the earth in

(*a*) 1.5 s (*b*) 4.025 s (*c*) 5.4 s (*d*) 6.75 s

Sol. (*c*) As the balloon is going up we will take initial velocity of falling body = + 12 m/s,

$\Delta y = -81$ m; $a = -g = -10$ m/s^2

By applying $h = ut + \frac{1}{2}gt^2$; $-81 = 12t - \frac{1}{2}(10)t^2$

$\Rightarrow 5t^2 - 12t - 81 = 0$

$$\Rightarrow t = \frac{12 \pm \sqrt{144+1620}}{10} = \frac{12 \pm \sqrt{1764}}{10}$$

$= 5.4$ s

Example 18: A particle is dropped under gravity from rest from a height *h* ($g = 9.8$ m/s^2) and it travels a distance $\frac{9h}{25}$ in the last second, the height *h* is

(*a*) 100 m

(*b*) 122.5 m

(*c*) 145 m

(*d*) 167.5 m

Sol. (*b*) Distance travelled in *n* sec = $\frac{1}{2}gn^2 = h$ …(i)

Distance travelled in n^{th} sec.

$$= \frac{g}{2}(2n-1) = \frac{9h}{25} \quad …(ii)$$

Solving (i) and (ii)

We get. $h = 122.5$ m

Example 19: A stone thrown upward with a speed *u* from the top of the tower reaches the ground with a velocity 3*u*. The height of the tower is

(*a*) $3u^2/g$ (*b*) $4u^2/g$

(*c*) $6u^2/g$ (*d*) $9u^2/g$

Sol. (*b*) For vertical downward motion we will consider initial velocity = $-u$.

By applying $v^2 = u^2 + 2gh$,

$(3u)^2 = (-u)^2 + 2gh$,

$$\Rightarrow h = \frac{4u^2}{g}$$

Example 20: A body is released from a great height and falls freely towards the earth. Another body is released from the same height exactly one second later. The separation between the two bodies, two seconds after the release of the second body is

(*a*) 4.9 m (*b*) 9.8 m

(*c*) 19.6 m (*d*) 24.5 m

Sol. (*d*) The separation between two bodies, two second after the release of second body is given by

$$s = \frac{1}{2}g(t_1^2 - t_2^2)$$

$$= \frac{1}{2} \times 9.8 \times (3^2 - 2^2) = 24.5\ m$$

Concept Application

3. A body is projected from Top of a tower vertically upward with 5 m/sec, the body lands the ground in 4 sec. Total height of tower would be
   (*a*) 45 m (*b*) 60 m
   (*c*) 30 m (*d*) 50 m
4. A body is thrown vertically upwards and if returns back to hand in 3 sec; its velocity of throw and max-height attained are,
   (*a*) 15 m/sec, 11.25 m (*b*) 10 m/sec, 12 m
   (*c*) 30 m/sec, 22.5 m (*d*) 20 m/sec, 30 m
5. Water drops are released from the bottom of an overhead tank at regular internal. When 1^{st} drop touches the ground, fifth drop is just about to release. The height of overhead tank is 8m from the ground. Find the separation between 2^{nd} and 3^{rd} drop just when 1^{st} drop touches the ground.
   (*a*) 1.5 m (*b*) 2 m
   (*c*) 2.5 m (*d*) 4.5 m

MOTION WITH VARIABLE ACCELERATION

In previous section, we studied rectilinear motion when acceleration is constant. In general acceleration can vary and depend on time, position and velocity of the particle.

Let us consider some simple cases

(i) **Acceleration only depends on time *t*.**

$$\frac{dv}{dt} = a(t)$$

$$\int_u^v dv = \int_0^t a(t)dt$$

$$\Rightarrow v - u = \int_0^t a(t)dt$$

$$\text{or } v = u + \int_0^t a(t)dt$$

(ii) **Acceleration only depends on position *x*.**

$$\frac{dv}{dt} = a(x)$$

We can use chain rule to eliminate time.

$$\frac{dv}{dx}\frac{dx}{dt} = a(x)$$

$$\Rightarrow v\frac{dv}{dx} = a(x)$$

$$\int_u^v vdv = \int_{x_0}^x a(x)dx$$

$$\text{or } \frac{v^2}{2} - \frac{u^2}{2} = \int_{x_0}^x a(x)dx$$

(iii) **Acceleration only depends on velocity.**

$$\frac{dv}{dt} = a(v)$$

$$\Rightarrow \int_u^v \frac{dv}{a(v)} = \int_0^t dt$$

$$\text{or } \int_u^v \frac{dv}{a(v)} = t$$

This gives us velocity as a function of time.

In case we want velocity as a function of position, we can use chain rule.

$$\frac{dv}{dx}\frac{dx}{dt} = a(v) \qquad \frac{v\,dv}{a(v)} = dx$$

$$\int_u^v \frac{v\,dv}{a(v)} = \int_{x_0}^x dx = x - x_0$$

Train Your Brain

Example 21: The acceleration of a particle moving in one dimension is given by $a = 6 - 2t$. If the particle is initially at $x = 0$ and its velocity is 2 m/s, find it position and velocity at time t?

Sol. $\frac{dv}{dt} = 6 - 2t$

$$\int_2^v dv = \int_0^t (6-2t)dt$$

$$\Rightarrow v - 2 = (6t - t^2)\Big|_0^t = 6t - t^2$$

$$\Rightarrow v(t) = 2 + 6t - t^2$$

To find position, we integrate velocity.

$$v = \frac{dx}{dt} = 2 + 6t - t^2$$

$$dx = (2 + 6t - t^2)dt$$

$$\int_0^x dx = \int_0^t (2 + 6t - t^2)dt = 2t + 3t^2 - \frac{t^3}{3}$$

$$\text{or } x(t) = 2t + 3t^2 - \frac{t^3}{3}$$

Example 22: The retardation of a car when its engine is shut off depends on its velocity as $a = -\alpha v$ where α is constant. Find the total distance travelled by the car if its initial velocity is 20 m/s and $\alpha = 0.5$/s

Sol. $\frac{dv}{dt} = -\alpha v \quad \frac{dv}{dx}\left(\frac{dx}{dt}\right) = -\alpha v$

$\frac{vdv}{dx} = -\alpha v$

or $dv = -\alpha dx$

$\int_{20}^{0} dv = -\alpha \int_{0}^{d} dx$

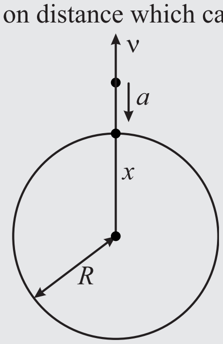
$v\big|_{20}^{0} = -\alpha\, x\big|_{0}^{d}$

$-20 = -\alpha d$

$d = \frac{20}{\alpha} = \frac{20}{0.5} = 40 \text{ m}$

Example 23: With what velocity in vertical upward direction should a body be projected from the surface of earth so that it reaches a height equal to radius of earth? The acceleration of body is given by $a = -\frac{GM}{x^2}$ where x is the distance from centre of earth and M is the mass of earth.

Sol. Acceleration due to gravity is nearly constant near the surface of earth. But if the height become too large its dependence on distance which can not be ignored.



$a = \frac{dv}{dt} = -\frac{GM}{x^2}$

or $\frac{dv}{dx}.\frac{dx}{dt} = -\frac{GM}{x^2}$

$vdv = = -\frac{GM}{x^2}dx$

At the highest point, velocity is zero. Also note $x_i = R$ and $x_f = 2R$.

$\int_{u}^{0} vdv = -GM\int_{R}^{2R}\frac{dx}{x^2}$

$\frac{v^2}{2}\Big|_{u}^{0} = -GM\int_{R}^{2R} x^{-2}dx = \frac{GM}{x}\Big|_{R}^{2R}$

$\Rightarrow -\frac{u^2}{2} = GM\left[\frac{1}{2R} - \frac{1}{R}\right]$

$\Rightarrow \quad u^2 = \frac{GM}{R} \quad \Rightarrow \quad u = \sqrt{\frac{GM}{R}} = \sqrt{\frac{GM}{R^2}R}$

$= \sqrt{gR} = 8 \text{ km/s}$ [$\because R = 6400$ km, g = 10 m/s^2]

Concept Application

6. Starting from rest at $t = 0$, a particle moves in a straight line with an acceleration a given by $a = t^3$ m/s^2 where t is in seconds. Then the velocity of particle after 4 seconds is

(*a*) 32 m/s (*b*) 64 m/s (*c*) 128 m/s (*d*) 16 m/s

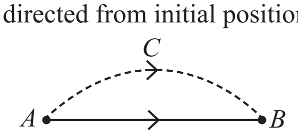
7. A particle moves in a straight line with acceleration $a = -\frac{1}{3v^2}$ where v is its velocity at time t. If initial velocity is 5 m/s then time t at which its velocity becomes zero is

(*a*) 5 sec (*b*) 25 sec (*c*) 125 sec (*d*) 50 sec

Short Notes

Distance versus Displacement

Total length of path (ACB) covered by the particle is called distance. Displacement vector or displacement is the minimum distance (AB) and directed from initial position to final position.



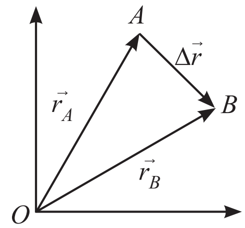
Displacement is Change of Position Vector

From ΔOAB $\Delta\vec{r} = \vec{r}_B - \vec{r}_A$

$\vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

and $\vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$



Average velocity $= \dfrac{\text{Displacement}}{\text{Time interval}} \Rightarrow \vec{v}_{av} = \dfrac{\Delta\vec{r}}{\Delta t}$

Average speed $= \dfrac{\text{Distance travelled}}{\text{Time interval}}$

For uniform motion

Average speed = | average velocity | = | instantaneous velocity|

Velocity $\vec{v} = \dfrac{d\vec{r}}{dt} = \dfrac{d}{dt}(x\hat{i} + y\hat{j} + z\hat{k})$

$$= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

AverageAcceleration $= \dfrac{\text{Total change in velocity}}{\text{Total time taken}} = \vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$

Acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v_x\hat{i} + v_y\hat{j} + v_z\hat{k})$$

$$= \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

Important Points About 1D Motion

- Distance ≥ | displacement | and Average speed ≥ | average velocity |
- If distance > | displacement | this implies
  (*a*) atleast at one point in path, velocity is zero.

Displacement —Differentiation→ **Velocity** —Differentiation→ **Acceleration**

Displacement ←Integration— **Velocity** ←Integration— **Acceleration**

Motion with Constant Acceleration: Equations of Motion

- **In vector form**

  $\vec{v} = \vec{u} + \vec{a}t$ and

  $$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1,\ \vec{s} = \left(\frac{\vec{u}+\vec{v}}{2}\right)t = \vec{u}t + \frac{1}{2}\vec{a}t^2 = \vec{v}t - \frac{1}{2}\vec{a}t^2$$

  $$v^2 = u^2 + 2\vec{a}.\vec{s} \text{ and } \vec{s}_{n^{th}} = \vec{u} + \frac{\vec{a}}{2}(2n-1)$$

  ($S_{nth} \rightarrow$ displacement in n^{th} second)

- **In scalar form (for one dimensional motion):**

  $$v = u + at \qquad s = \left(\frac{u+v}{2}\right)t = ut + \frac{1}{2}at^2 = vt - \frac{1}{2}at^2$$

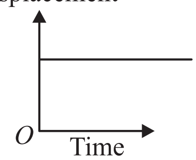
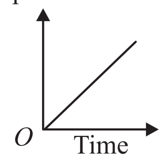
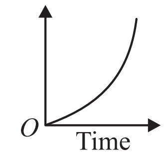
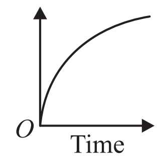
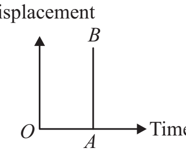
  $$v^2 = u^2 + 2as \qquad s_n = u + \frac{a}{2}(2n-1)$$

Uniform Motion

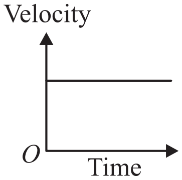
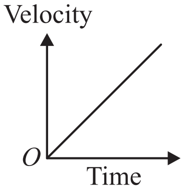
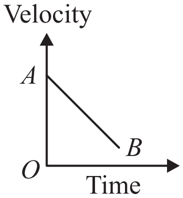
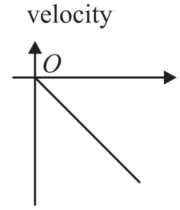
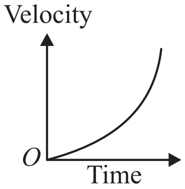
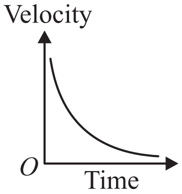
If an object is moving along the straight line covers equal distance in equal interval of time, it is said to be in uniform motion along a straight line.

Different Graphs of Motion

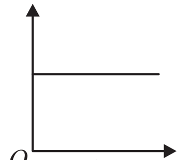
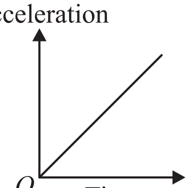
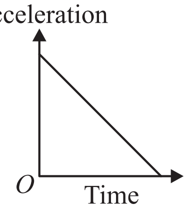
Displacement-Time Graph

(*a*) For a stationary body Displacement  O Time	(*b*) Body moving with a constant velocity Displacement  O Time	(*c*) Body moving with a constant acceleration Displacement  O Time
(*d*) Body moving with a constant retardation Displacement  O Time	(*e*) Body moving with infinite velocity. But such motion of body is never possible Displacement  B O A Time	

Velocity-Time Graph

(*a*) Moving with a constant velocity Velocity  O Time	(*b*) Moving with a constant acceleration having zero initial velocity Velocity  O Time	(*c*) Body moving with a constant retardation and its initial retardation and its initial velocity is not zero Velocity  A B O Time
(*d*) Moving with negative acceleration with zero initial velocity  O	(*e*) Moving with increasing acceleration Velocity  O Time	(*f*) Moving with decreasing retardation Velocity  O Time

Note: Slope of velocity-time graph gives acceleration.

Acceleration-Time Graph		
(*a*) When object is moving with constant acceleration Acceleration  O Time	(*b*) When object is moving with constant increasing acceleration Acceleration  O Time	(*c*) When object is moving with constant decreasing acceleration Acceleration  O Time

Motion under Gravity (No Air Resistance)

If an object is falling freely under gravity and downward direction is taken as positive, then equations of motion becomes

(i) $v = u + gt$ (ii) $h = ut + \frac{1}{2}gt^2$ (iii) $v^2 = u^2 + 2gh$

Note: If upward direction is taken as positive then g is replaced by $-g$ in above three equations.

If a body is thrown vertically up with a velocity u in the uniform gravitational field then

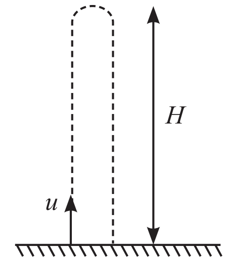
(i) Maximum height attained $H = \frac{u^2}{2g}$

(ii) Time of ascent = time of descent = $\frac{u}{g}$

(iii) Total time of flight = $\frac{2u}{g}$

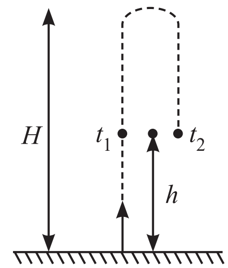
(iv) Final velocity at the point of projection = u (downwards)

(v) **Gallileo's law of odd numbers:** For a freely falling body ratio of successive distance covered in equal time interval 't'



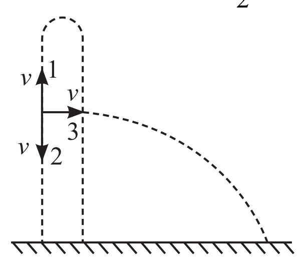
$S_1 : S_2 : S_3 : \ldots, S_n = 1 : 3 : 5 : \ldots, 2n - 1$

(vi) At any point on its path the body will have same speed for upward journey and downward journey.



(vii) If a body thrown upwards crosses a point in time t_1 and t_2 respectively then height of point $h = ½\, gt_1t_2$. Maximum height $H = \frac{1}{2}g(t_1 + t_2)^2$.

(viii) A body is thrown upward, downward and horizontally with same speed takes time t_1, t_2 and t_3 respectively to reach the ground then $t_3 = \sqrt{t_1t_2}$ and height from where the particle was thrown is $H = \frac{1}{2}gt_1t_2$.



Solved Examples

1. A body is thrown down from the top of a tower of height h with velocity 10 m/s. Simultaneously, another body is projected upward from bottom. They meet at a height $2h/3$ from the ground level. If $h = 60$ m, find the initial velocity of the lower body.

Sol. Let us choose the origin at the ground level with +ve X-axis pointing in the upward direction.

Let us refer lower and upper body 1 and 2 respectively

Then,

$a = -g$

$x_1 = 0, x_1 = 2h/3$

$x_2 = h, x_2 = 2h/3, u_2 = -10$ m/s

From eqn of motion , we have

$$x_1 = 0 + u_1 t - \frac{1}{2} \times 9.8 \times t^2 \qquad ...(i)$$

$$x_2 = h - 10t - \frac{1}{2} \times 9.8 \times t^2 \qquad ...(ii)$$

But , $x_1 = x_2 = 2h/3$

Hence, equating eqn (i) and (ii) we have

$u_1 t = h - 10\,t \qquad \Rightarrow t = \dfrac{h}{u_1 + 10}$

Putting this value in eqn (ii) , we get

$$\frac{h}{3} = \frac{10h}{u_1 + 10} - 4.9\left(\frac{h}{u_1 + 10}\right)^2$$

$$\Rightarrow 20 = \frac{200}{u_1 + 10} - 4.9\frac{60^2}{(u_1 + 10)^2}$$

$$\Rightarrow (u_1 + 10)^2 - 10(u_1 + 10) - 882 = 0$$

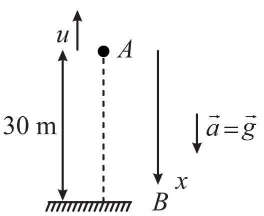
Solving this quadratic eqn, we find

$$u_1 + 10 = \frac{10 \pm \sqrt{100 + 3528}}{2} \qquad \Rightarrow u_1 = 30.11 \text{ m/s}$$

The other value is not possible because body is thrown upwards and is positive in the chosen coordinate system.

2. A stone is dropped from a rising balloon at a height of 30 m above the ground and it reaches the ground in 10 seconds. What was the velocity of the balloon at the moment the stone was dropped.

Sol.



Let the balloon be at point A when the stone is dropped. Let its velocity be u. The velocity of the stone will be equal to the velocity of balloon when it is dropped.

Let us choose origin at point A and take +ve X-direction to be vertically downward direction. In this case

$a = g$ and $S = 30$

Using equation, $S = ut + \frac{1}{2}at^2$

we get

$$30 = ut + \frac{1}{2} \times 9.8 \times (10)^2$$

$\Rightarrow u = -46$ m/s

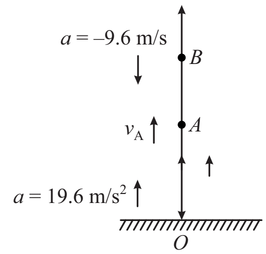
minus sign indicates that the velocity is in the upward direction as we have choosen vertically downward direction to be positive.

3. A rocket is fired vertically and ascends with constant vertical acceleration of 19.6 m/s^2 for 30 seconds. Its fuel is then all used up and it continues as a free particle.

(a) What is the maximum altitude reached ?

(b) What is total time after which it strikes the ground again?

Sol.



Let us choose the vertical upward to be the positive direction. Let A be the point at which fuel is exhausted and let point B represent the maximum altitude.

From $O \to A$

$u = 0, \quad a = 19.6 \text{ m/s}^2, \; t = 30\text{s}$

$$OA = ut + \frac{1}{2}at^2 = 0 + 9.8 \times 900 = 8820 \text{ m}$$

$$v_A = u + at = 0 + 19.6 \times 30 = 588 \text{ m/s}$$

Form $A \to B$

$u = v_A = 588$ m/s, $a = -9.8$ m/s^2

$v = v_B = 0$

using $v^2 - u^2 = 2as$ we get

$$0 - (588)^2 = 2 \times -9.8 \times AB$$

$$\Rightarrow AB = 17640 \text{ m}$$

Hence,

maximum altitude $= OA + AB = 26.46$ km

To find time, let us consider the path $A \to B \to O$

$a = -9.8 \text{ m/s}^2, \; s = -OA = -8820, \; u = 588$ m/s

$$s = ut + \frac{1}{2}at^2$$

$$\Rightarrow -8820 = 588 \times t - 4.9t^2$$

$$\Rightarrow t^2 - 120t - 1800 = 0$$

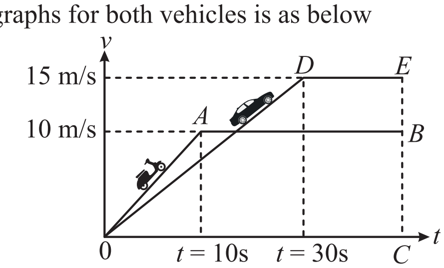
$$t = \frac{120 + \sqrt{14400 + 7200}}{2}$$

$= 133.5$ sec

So, the total time $= 30 + 133.5 = 163.5$ sec

4. A motorcycle and a car start their rectilinear motion from rest from the same place at the same time and travel in the same direction. The motorcycle accelerates at 1.0 m/s^2 up to a speed of 36 km/hr and the car at 0.5 m/s^2 up to a speed of 54 km/hr. Their velocities remain constant after that. Draw v-t graph of both. Calculate the time and distance at which the car would overtake the motorcycle.

Sol. v-t graphs for both vehicles is as below



For motorcycle

$10\ m/s = 0 + (1 m/s^2)\ t$

$t = 10s$

For car

$15\ m/s = 0 + (0.5)\ m/s^2)\ t$

$t = 30\ s$

Suppose after t time car over takes motorcycle.

Area under v-t graph till that time for both will be same

Area of $OABC$ = Area of $ODEC$

$$\frac{1}{2}(t + t - 10)\ 10 = \frac{1}{2}(t + t - 30)\ 15$$

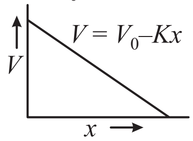
$4t - 20 = 6t - 90$

$2t = 70$

$t = 35$ sec.

Area under the graph = 300 m = distance at which car overtakes motorcycle.

5. A particle is moving along x-axis with velocity V which varies according to the law $V = V_0 - Kx$ here V_0 and K are constant. Plot acceleration *vs* time plot for the time interval when particle moves from $x = 0$ to $x = \frac{V_0}{K}$.



Sol. $V = V_0 - Kx$

$$\frac{dx}{dt} = (V_0 - Kx)$$

$$\int_0^x \frac{dx}{(V_0 - Kx)} = \int_0^t dt$$

$$x = \frac{V_0}{K}(1 - e^{-Kt})$$

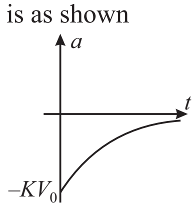
$$\therefore \frac{dx}{dt} = + V_0\, e^{-Kt}$$

$$a = \frac{d^2x}{dt^2} = KV_0\, e^{-Kt}$$

At $t = 0$, $a = -KV_0$

At $t = \infty$, $a = 0$

Therefore, graph is as shown



6. Two trains A and B are approaching each other on a straight track, the former with a uniform velocity of 15 m/s and the latter with 25 m/s. When they are 225 m apart brakes are simultaneously applied to both of them. The deceleration given by the brakes to the train A increases as $0.3t$ where t is time in sec. while the train B is given a uniform deceleration.

(*a*) What must be the minimum deceleration of B so that the trains do not collide.

(*b*) What is the times taken by the trains to come to stop.

Sol. $a_A = -0.3\ t$

$$\frac{dv}{dt} = -0.3\,t \Rightarrow \int_u^v dv = -0.3\int_0^t t\,dt \Rightarrow v = 15 - \frac{0.3t^2}{2}$$

Train A stops when $v = 0$

$\Rightarrow t = 10$ sec

Now, displacement of A in 10 sec is

$$S(t) = 15t - \frac{0.3}{2}\times\frac{t^3}{3}$$

$$S = 15 \times 10 - \frac{0.3}{2}\times\frac{10^3}{3} = 100\ m$$

Train B must stop in 225 – 100 = 125 m

$v^2 - u^2 = 2aS$

$$0 - 25^2 = 2a \times 125 \Rightarrow a = -\frac{625}{2\times 225} = -2.5\ m/s^2$$

Therefore, deceleration of train B must be greater than 2.5 m/s^2

7. A steel ball bearing is released from the roof of a building. An observer standing in front of a window 120 cm high observes that the ball takes 0.125 sec to fall from top to the bottom of the window. The ball continues to fall and makes a completely elastic collision with side walk and reappears at the bottom of the window 2 s after passing it on the way down. How tall is the building?

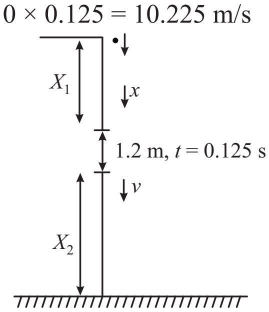
Sol. $1.2 = ut + \frac{1}{2}gt^2$

$$1.2 = u \times 0.125 + \frac{1}{2} \times 10 \times (0.125)^2$$

$$1.2 = u \times 0.125 + 5 \times (0.125)^2$$

$$u = \frac{1.2 - 0.078125}{0.125} = 8.975\ m/s$$

$v = 8.975 + 10 \times 0.125 = 10.225$ m/s



$$X_2 = 10.225 \times 1 + \frac{1}{2} \times 10 \times 1^2 = 15.225\ m$$

$$u^2 = 0 + 2 \times 10 \times X_1 \quad \Rightarrow \quad X_1 = \frac{u^2}{20} = \frac{(8.975)^2}{20} = 4.027\ m$$

$$H_{total} = X_1 + X_2 + 1.2 = 4.027 + 15.225 + 1.2 = 20.452 = 20.5\ m$$

8. A car starts from rest at $t = 0$ and for the first 4 seconds of its rectilinear motion the acceleration 'a' (ms^{-2}) at time 't' (sec.) after starting is given by $a = 6 - 2t$.

(*a*) Find the maximum velocity of the car

(*b*) Find the velocity of the car after 4 seconds, and the distance travelled up to this time.

Sol. (*a*) $a = 6 - 2t = \frac{dv}{dt}$

For maximum velocity $\frac{dv}{dt} = 0$

$6 - 2t = 0$

$t = 3$sec

$$\frac{dv}{dt} = 6 - 2t$$

$$\int_0^v dv = \int_0^t (6-2t)\, dt$$

$v = 6t - t^2 = 18 - 9 = 9$m/s

(*b*) after 4 sec $\quad v = 6t - t^2$

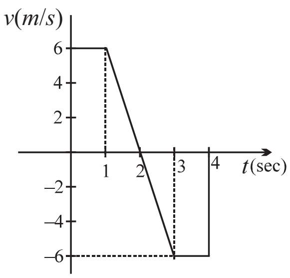
$= 6 \times 4 - 16 = 24 - 16 = 8$ m/s

$$\int_0^v dx = \int_0^t (6t - t^2)\, dt$$

$x = 3t^2 - \frac{t^3}{3} = \frac{80}{3}$ m = distance travelled

[(putting $t = 4$ sec)]

9. A particle moves along a straight line along *x*-axis. At time $t = 0$, its position is at $x = 0$. The velocity *v* m/s of the object changes as a function of time *t* seconds as shown in the figure.



(*a*) What is *x* at $t = 1$ sec?

(*b*) What is the acceleration at $t = 2$ sec?

(*c*) What is *x* at $t = 4$ sec?

(*d*) What is the average speed between $t = 0$ and $t = 3$ sec?

Sol. (*a*) *x* is displacement at $t = 1$ sec.

Area under the $v - t$ curve gives displacement

From $t = 0$ to $t = 1$ sec.

$x = 6 \times 1 = 6$ m

(*b*) Slope of the *v*–*t* curve gives acceleration from the given *v*–*t* curve

Slope at $t = 2$ sec. gives acceleration at $t = 2$ sec.

$\tan\theta = a = -\frac{6}{1} = -6$ m/s^2

(*c*) *x* (at $t = 4$ sec):

Area under the curve from $t = 0$ to $t = 4$ sec

$= 6 \times 1 + \frac{1}{2} \times 6 \times 1 - \frac{1}{2} \times 6 \times 1 - 6 \times 1 = 0$

$\Rightarrow x\,(t = 4) = 0$ m

(*d*) Average speed from $t = 0$ to $t = 3$ sec.

Displacement from $t = 0$ to $t = 2$ sec. = Area under the curve = $6 + \frac{1}{2} \times 6 \times 1 = 9$ m

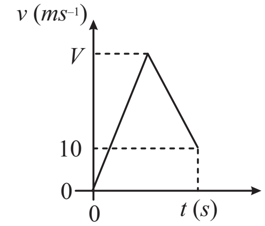
Displacement form $t = 2$ to $t = 3$ sec. = $-\frac{1}{2} \times 6 \times 1 = -3$ m

Distance from $t = 0$ to $t = 3$ sec = $|9| + |-3| = 12$ m

Average speed = $\frac{\text{Distance}}{\text{Speed}} = \frac{12}{3} = 4$ m/s

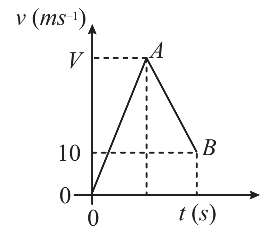
10. The figure shows the (v, t) graph for the train accelerating from rest up to a maximum speed of V ms^{-1} and then decelerating to a speed of 10 ms^{-1}. The acceleration and deceleration have the same magnitude which is equal to 0.5 m/s^2.

Show that the distance travelled is $(2V^2 - 100)$ metre.



Sol. $v^2 - u^2 = 2as \qquad s = \frac{v^2 - u^2}{2a}$

For the motion between *A* and *B*



Distance $(s_1) = \frac{V^2 - 0}{2 \times 0.5} = V^2$

For the motion between *O* and *A*

Distance $(s_2) = \frac{10^2 - V^2}{2 \times (-0.5)} = V^2 - 100$

Total distance = $s_1 + s_2 = 2V^2 - 100$

11. When a situation demands our immediate action, it takes some time before we really respond. Reaction time is the time a person takes to observe, think and act. For example, if a person is driving and suddenly a boy appears on the road, then the time elasped before he slams the breaks of the car is the reaction time. Reaction time depends on complexity of the situation and on individual. You can measure your reaction time by a simple experiment. Take a ruler and ask your friend to drop it vertically through the gap between your thumb and forefinger. After you catch it, find the distance d travelled by the ruler. In a particular case, *d* was found to be 20 cm. [$g = 10$ m/s^2]

(*a*) Estimate reaction time.

(*b*) Now if you are driving a car at a speed of 54 km/h and the brakes cause a deceleration of 6.0 m/s^2, find the distance travelled by the car after you see the need to put the brakes on.

Sol. (*a*) $0.2 = \frac{1}{2} \times 10t^2 \Rightarrow t^2 = \frac{0.4}{10} \Rightarrow t = 0.2$

Reaction time = 0.2 sec

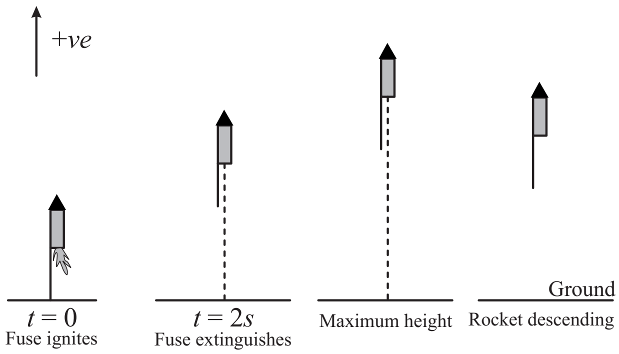
(*b*) 54 km/hr = $\frac{54 \times 5}{18} = 15$ m/s

Total distance = 15 × reaction time + $(v^2/2a)$

Total distance = $0.2 \times 15 + \frac{15^2}{2 \times 6} = 3 + \frac{225}{12}$

$= \frac{261}{12}$ m

12. A Diwali rocket is launched vertically with its fuse ignited at time $t = 0$, as shown. The engine provides constant acceleration for 2 sec. till rocket attains $V = 40\ \text{ms}^{-1}$. Afterwards rocket continues to move freely under gravity.



(*a*) Draw labelled acceleration-time graph from launching till it reaches ground.

(*b*) Draw labelled velocity-time graph from launching till it reaches ground.

[Take vertically upward direction as positive]

Sol. $v = u + at$

$\Rightarrow 40 = 0 + a \times 2$

$\Rightarrow a = 20\ \text{m/s}^2$

$v^2 = u^2 + 2ah$

$\Rightarrow (40)^2 = 0 + 2 \times 20h$

$\Rightarrow h = \dfrac{40 \times 40}{40} = 40\ \text{m}$

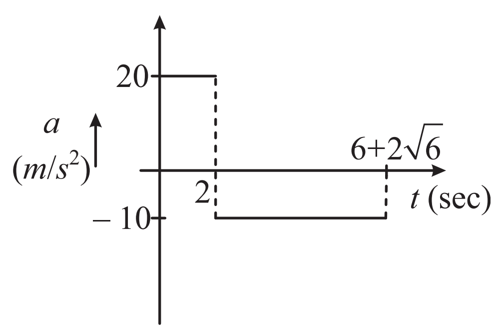
Time taken in reaching from $h = 40$ m to ground

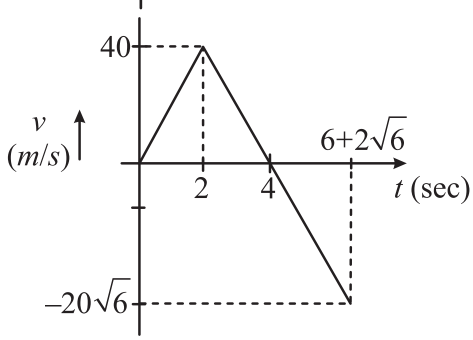
$h = ut - \dfrac{1}{2}gt^2$

$\Rightarrow -40 = 40t - \dfrac{1}{2} \times 10 \times t^2$

On solving

$t = 6 + 2\sqrt{6}$ sec





13. The force acting on a body moving in a straight line is given by $F = (3t^2 - 4t + 1)$ Newton where t is in sec. If mass of the body is 1kg and initially it was at rest at origin. Find

(*a*) Displacement between time $t = 0$ and $t = 2$ sec.

(*b*) Distance travelled between time $t = 0$ and $t = 2$ sec.

Sol. $a = F/m = 3t^2 - 4t + 1\ \text{m/s}^2$

$$\therefore \int_0^v dv = \int_0^t (3t^2 - 4t + 1)\,dt$$

$\Rightarrow v = (t^3 - 2t^2 + t)$ m/s

(*a*) $\displaystyle\int_0^s ds = \int_0^2 v\,dt$

$$\therefore \text{Displacement } (s) = \left[\frac{t^4}{4} - \frac{2t^3}{3} + \frac{t^2}{2}\right]_0^2 = 2/3\ \text{m}$$

(*b*) Now, $v = 0 = t^3 - 2t^2 + t$

$\therefore t(t-1)^2 = 0$

i.e. $v = 0$ at $t = 0$ and $t = 1$ sec

$$\therefore \text{Distance} = \left|\int_0^1 v\,dt\right| + \left|\int_1^2 v\,dt\right| = 2/3\ \text{m}$$

[here distance = displacement, since velocity is positive $\forall\ t > 0$

14. A block of mass m is fired horizontally along a level surface that is lubricated with oil. The oil provides a viscous resistance that varies as the 3/2 power of the speed. If the initial speed of the block is v_0 at $x = 0$, find the maximum distance reached by the block. Assume no resistance to motion other than that provided by the oil.

Sol. $F = -v^{3/2}$

$a = -\dfrac{1}{m}v^{3/2}$

$v\dfrac{dv}{dx} = -\dfrac{1}{m}v^{3/2}$

$$\int_{v_0}^{0} v^{-\frac{1}{2}}dv = -\frac{1}{m}\int_0^d dx$$

$2mv_0^{1/2} = d$ or $d = 2mv_0^{½}$

15. Acceleration of particle moving rectilinearly is $a = 4 - 2x$ (where x is position in metre and a in m/s^2). It is at instantaneous rest at $x = 0$. At what position x (in metre) will the particle again come to instantaneous rest?

Sol. $\dfrac{vdv}{dx} = 4 - 2x$

$$\int_0^v vdv = \int_0^x (4 - 2x)dx$$

$\Rightarrow \dfrac{v^2}{2} = 4x - x^2$

when $v = 0$, $4x - x^2 = 0$

$x = 0, 4$

$\therefore$ At $x = 4$ m, the particle will again come to rest.

Exercise-1 (Topicwise)

POSITION, DISTANCE AND DISPLACEMENT

1. A Body moves 6 m north. 8 m east and 10 m vertically upwards, what is its resultant displacement from initial position.

(*a*) $10\sqrt{2}$ m (*b*) 10 m

(*c*) $\frac{10}{\sqrt{2}}$ m (*d*) 20 m

2. A man goes 10 m towards North, then 20 m towards East then displacement is

(*a*) 22.5 m (*b*) 25 m

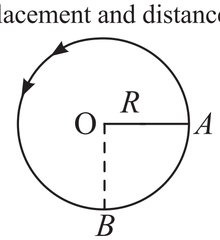
(*c*) 25.5 m (*d*) 30 m

3. An aeroplane flies from P(–4m, –5m, +8m) to Q (7m, –2m, –3m) in *xyz* coordinate system. The position vector of aeroplane.

(*a*) $11\hat{i}+3\hat{j}+11\hat{k}$ (*b*) $11\hat{i}-3\hat{j}+11\hat{k}$

(*c*) $11\hat{i}+3\hat{j}-11\hat{k}$ (*d*) $11\hat{i}-3\hat{j}-11\hat{k}$

4. A body moves in circular path of radius R from A to B as shown. Its displacement and distance covered are



(*a*) $R, \frac{3\pi R}{2}$ (*b*) $\sqrt{2}R, \frac{\pi R}{2}$

(*c*) $\sqrt{2}R, \frac{3\pi R}{2}$ (*d*) None of these

5. A particle covers half of the circle of radius r. Then the displacement and distance of the particle are respectively

(*a*) $2\pi r$, 0 (*b*) $2r$, πr

(*c*) $\frac{\pi r}{2}$, $2r$ (*d*) πr, r

SPEED AND VELOCITY

6. A person travels along a straight road for half the distance with velocity v_1 and the remaining half distance with velocity v_2 The average velocity is given by

(*a*) $v_1 v_2$ (*b*) $\frac{v_2^2}{v_1^2}$

(*c*) $\frac{v_1+v_2}{2}$ (*d*) $\frac{2v_1v_2}{v_1+v_2}$

7. A car travels the first half of a distance between two places at a speed of 30 km/hr and the second half of the distance at 50 km/hr. The average speed of the car for the whole journey is

(*a*) 42.5 km/hr (*b*) 40.0 km/hr

(*c*) 37.5 km/hr (*d*) 35.0 km/hr

8. A person travels along a straight road for the first half time with a velocity v_1 and the next half time with a velocity v_2. The mean velocity V of the man is

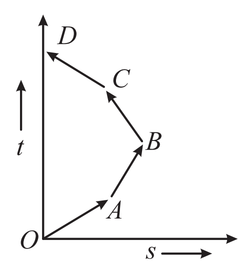
(*a*) $\frac{2}{V}=\frac{1}{v_1}+\frac{1}{v_2}$ (*b*) $V=\frac{v_1+v_2}{2}$

(*c*) $V=\sqrt{v_1v_2}$ (*d*) $V=\sqrt{\frac{v_1}{v_2}}$

9. If a car covers 2/5th of the total distance with v_1 speed and 3/5th distance with v_2 then average speed is

(*a*) $\frac{1}{2}\sqrt{v_1v_2}$ (*b*) $\frac{v_1+v_2}{2}$ (*c*) $\frac{2v_1v_2}{v_1+v_2}$ (*d*) $\frac{5v_1v_2}{3v_1+2v_2}$

10. Which of the following options is correct for the object having a straight line motion represented by the following graph



(*a*) The object moves with constantly increasing velocity from O to A and then it moves with constant velocity

(*b*) Velocity of the object increases uniformly

(*c*) Average velocity is zero

(*d*) The graph shown is impossible

CONSTANT ACCELERATION

11. A particle experiences a constant acceleration for 20 sec after starting from rest. If it travels a distance S_1 in the first 10 sec and a distance S_2 in the next 10 sec, then

(*a*) $S_1 = S_2$ (*b*) $S_1 = S_2/3$

(*c*) $S_1 = S_2/2$ (*d*) $S_1 = S_2/4$

12. A body is moving from rest under constant acceleration and let S_1 be the displacement in the first $(p-1)$ sec and S_2 be the displacement in the first p sec. The displacement in $(p^2-p+1)^{th}$ sec. will be

(*a*) $S_1 + S_2$ (*b*) $S_1 S_2$

(*c*) $S_1 - S_2$ (*d*) S_1 / S_2

13. A body starts from the origin and moves along the X-axis such that the velocity at any instant is given by, $(4t^3 - 2t)$, where t is in sec and velocity is in m/s. What is the acceleration of the particle, when it is 2 m from the origin?
(*a*) 28 m/s^2 (*b*) 22 m/s^2
(*c*) 12 m/s^2 (*d*) 10 m/s^2

14. A motor car moving with a uniform speed of 20 m/sec comes to stop on the application of brakes after travelling a distance of 10 m Its acceleration is
(*a*) 20 m/sec^2 (*b*) –20 m/sec^2
(*c*) –40 m/sec^2 (*d*) +2 m/sec^2

15. Which of the following four statements is false
(*a*) A body can have zero velocity and still be accelerated.
(*b*) A body can have a constant velocity and still have a varying speed.
(*c*) A body can have a constant speed and still have a varying velocity.
(*d*) The direction of the velocity of a body can change when its acceleration is constant.

16. A car moving with a velocity of 10 m/s can be stopped by the application of a constant force F in a distance of 20 m. If the velocity of the car is 30 m/s, it can be stopped by this force in
(*a*) $\frac{20}{3}$ m (*b*) 20 m
(*c*) 60 m (*d*) 180 m

17. A car starts from rest and moves with uniform acceleration a on a straight road from time $t = 0$ to $t = T$. After that, constant deceleration brings it to rest. In this process the average speed of the car is
(*a*) $\frac{aT}{4}$ (*b*) $\frac{3aT}{2}$
(*c*) $\frac{aT}{2}$ (*d*) aT

18. A car, starting from rest, accelerates at constant rate f through a distance S, then continues at constant speed for time t and then decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance traversed is 15 S, then value of S
(*a*) $S = \frac{1}{2}ft^2$ (*b*) $S = \frac{1}{4}ft^2$
(*c*) $S = \frac{1}{72}ft^2$ (*d*) $S = \frac{1}{6}ft^2$

19. A body is starts from rest with acceleration 2 m/s^2 till it attains the maximum velocity then retards to rest with 3 m/s^2. If total time taken is 10 second then maximum speed attained is
(*a*) 12 m/s (*b*) 8 m/s
(*c*) 6 m/s (*d*) 4 m/s

MOTION UNDER GRAVITY

20. A stone falls from a balloon that is descending at a uniform rate of 12 m/s. The displacement of the stone from the point of release after 10 sec is
(*a*) 490 m (*b*) 510 m
(*c*) 610 m (*d*) 725 m

21. Two bodies of different masses m_a and m_b are dropped from two different heights a and b. The ratio of the time taken by the two to cover these distances are
(*a*) $a : b$ (*b*) $b : a$
(*c*) $\sqrt{a} : \sqrt{b}$ (*d*) $a^2 : b^2$

22. A ball P is dropped vertically and another ball Q is thrown horizontally with the same velocities from the same height and at the same time. If air resistance is neglected, then
(*a*) Ball P reaches the ground first
(*b*) Ball Q reaches the ground first
(*c*) Both reach the ground at the same time
(*d*) The respective masses of the two balls will decide the time

23. A body is released from the top of a tower of height h. It takes t sec to reach the ground. Where will be the ball after time $t/2$ sec
(*a*) At $h/2$from the ground
(*b*) At $h/4$ from the ground
(*c*) Depends upon mass and volume of the body
(*d*) At $3h/4$ from the ground

24. A body is slipping from an inclined plane of height h and length l. If the angle of inclination is θ, the time taken by the body to come from the top to the bottom of this inclined plane is
(*a*) $\sqrt{\frac{2h}{g}}$ (*b*) $\sqrt{\frac{2h}{g}}$ (*c*) $\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$ (*d*) $\sin\theta\sqrt{\frac{2h}{g}}$

25. A man in a balloon rising vertically with an acceleration of 4.9 m/sec^2 releases a ball 2 sec after the balloon is let go from the ground. The greatest height above the ground reached by the ball is ($g = 9.8$ m/sec^2)
(*a*) 14.7 m (*b*) 19.6 m
(*c*) 9.8 m (*d*) 24.5 m

26. A rocket is fired upward from the earth's surface such that it creates an acceleration of 19.6 m/sec^2. If after 5 sec its engine is switched off, the maximum height of the rocket from earth's surface would be
(*a*) 245 (*b*) 490 m
(*c*) 980 m (*d*) 735 m

27. A ball of mass m_1 and another ball of mass m_2 are dropped from equal height. If time taken by the balls are t_1 and t_2 respectively, then
(*a*) $t_1 = \frac{t_2}{2}$ (*b*) $t_1 = t_2$
(*c*) $t_1 = 4t_2$ (*d*) $t_1 = \frac{t_2}{4}$

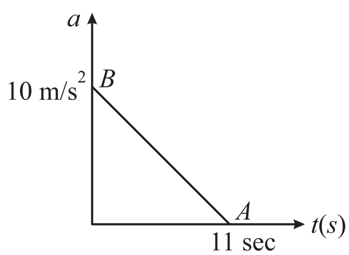
28. Three different objects of masses m_1, m_2 and m_3 are allowed to fall from rest and from the same point 'O' along three different frictionless paths. The speeds of the three objects, on reaching the ground, will be in the ratio of

(*a*) $m_1 : m_2 : m_3$ (*b*) $m_1 : 2m_2 : 3m_3$

(*c*) 1 : 1 : 1 (*d*) $\frac{1}{m_1} : \frac{1}{m_2} : \frac{1}{m_3}$

VARIABLE ACCELERATION

29. A particle starts from rest. Its acceleration (a) versus time (t) is as shown in the figure. The maximum speed of the particle will be



(*a*) 110 m/s

(*b*) 55 m/s

(*c*) 550 m/s

(*d*) 660 m/s

30. If the velocity of a particle is given by $v = (180 - 16x)^{1/2}$ m/s, then its acceleration will be

(*a*) Zero (*b*) 8 m/s^2

(*c*) –8 m/s^2 (*d*) 4 m/s^2

31. The displacement x of a particle varies with time t as $x = ae^{-\alpha t} + be^{\beta t}$, where a, b, α and β are positive constants. The velocity of the particle will

(*a*) Go on decreasing with time

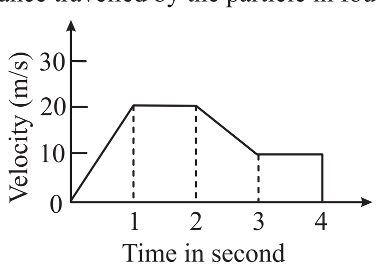
(*b*) Will be independent of α and β

(*c*) Drop to zero when $\alpha = \beta$

(*d*) Go on increasing with time

GRAPHS

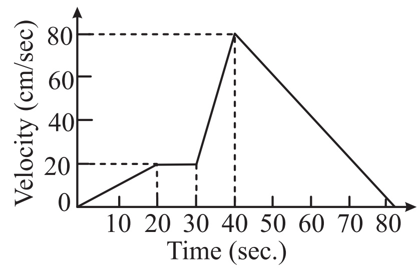
32. The variation of velocity of a particle with time moving along a straight line is illustrated in the following figure. The distance travelled by the particle in four seconds is



(*a*) 60 m (*b*) 55 m

(*c*) 25 m (*d*) 30 m

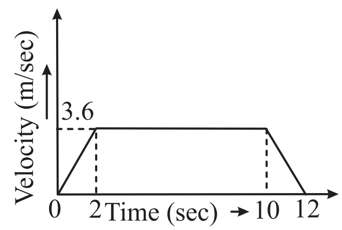
33. The $v - t$ graph of a moving object is given in figure. The maximum acceleration is



(*a*) 1 cm/sec^2 (*b*) 2 cm/sec^2

(*c*) 3 cm/sec^2 (*d*) 6 cm/sec^2

34. A lift is going up. The variation in the speed of the lift is as given in the graph. What is the height to which the lift takes the passengers?



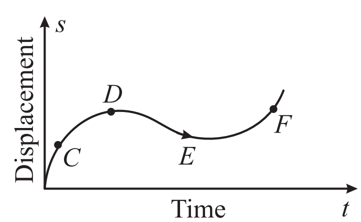
(*a*) 3.6 m

(*b*) 28.8 m

(*c*) 36.0 m

(*d*) Cannot be calculated from the above graph

35. The displacement-time graph of moving particle is shown below

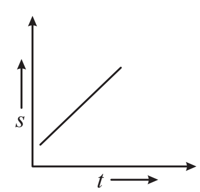


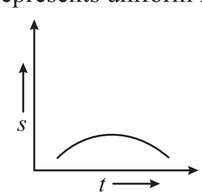
The instantaneous velocity of the particle is negative at the point

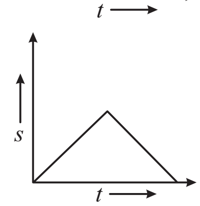
(*a*) D (*b*) F

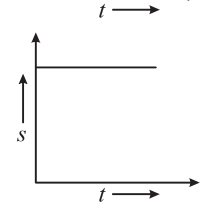
(*c*) C (*d*) E

36. Which of the following graph represents uniform motion

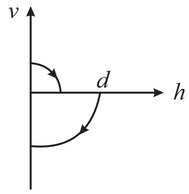
(*a*)


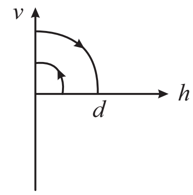
(*b*)


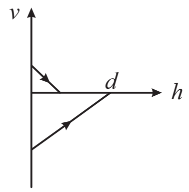
(*c*)


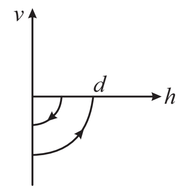
(*d*)


37. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as

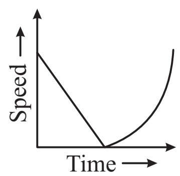
(*a*) 

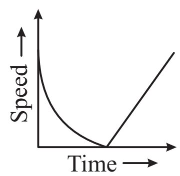
(*b*) 

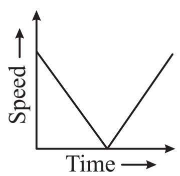
(*c*) 

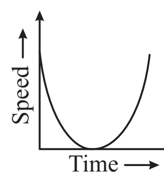
(*d*) 

38. A ball is thrown vertically upwards. Which of the following plots represents the speed-time graph of the ball during its height if the air resistance (constent) is not ignored

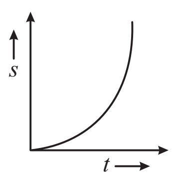
(*a*) 

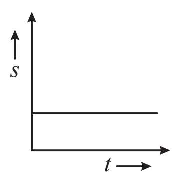
(*b*) 

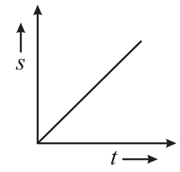
(*c*) 

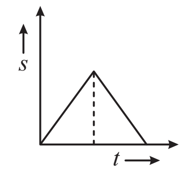
(*d*) 

39. Which graph represents the uniform acceleration

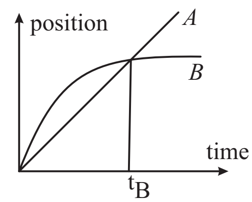
(*a*) 

(*b*) 

(*c*) 

(*d*) 

40. The graph shows position as a function of time for two trains running on parallel tracks. Which one of the following statements is true?



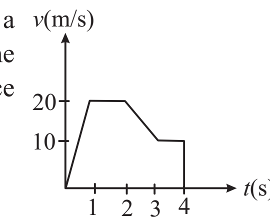
(*a*) At time t_B, both trains have the same velocity

(*b*) Both trains have the same velocity at some time after t_B

(*c*) Both trains have the same velocity at some time before t_B

(*d*) Somewhere on the graph, both trains have the same acceleration

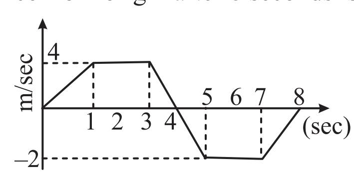
41. The variation of velocity of a particle moving along straight line is shown in the figure. The distance travelled by the particle in 4 s is



(*a*) 25m (*b*) 30m

(*c*) 55 m (*d*) 60m

42. The *v*-*t* graph of a linear motion is shown in adjoining figure. The distance from origin after 8 seconds is



(*a*) 18 meters (*b*) 16 meters

(*c*) 8 meters (*d*) 6 meters

Exercise-2 (Learning Plus)

1. A body starts from rest with constant acceleration, the ratio of distances travelled by the body during 4^{th} and 5^{th} seconds is

(*a*) 7/5 (*b*) 7/9 (*c*) 7/3 (*d*) 3/7

2. A particle, after starting from rest, experiences, constant acceleration for 20 seconds. If it covers a distance of S_1, in first 10 seconds and distance S_2 in next 10 sec, then

(*a*) $S_2 = S_1/2$ (*b*) $S_2 = S_1$

(*c*) $S_2 = 2S_1$ (*d*) $S_2 = 3S_1$

3. A body sliding on a smooth inclined plane requires 4sec to reach the bottom after starting from rest at the top. How much time does it take to cover one fourth the distance starting from the top

(*a*) 1 sec (*b*) 2 sec

(*c*) 0.4 sec (*d*) 1.6 sec

4. A body is dropped from a height *h* under acceleration due to gravity *g*. If t_1 and t_2 are time intervals for its fall for first half and the second half distance, the relation between them is

(*a*) $t_1 = t_2$ (*b*) $t_1 = 2t_2$

(*c*) $t_1 = 2.414\ t_2$ (*d*) $t_1 = 4t_2$

5. The motion of a body is given by the equation

$\frac{dv(t)}{dt} = 6.0 - 3v(t)$, where *v* (*t*) is speed in m/s and *t* in sec.

If body was at rest at $t = 0$ find the wrong option.

(*a*) The terminal speed is 2.0 m/s

(*b*) The speed varies with the time as $v(t) = 2(1 - e^{-3t})$ m/s

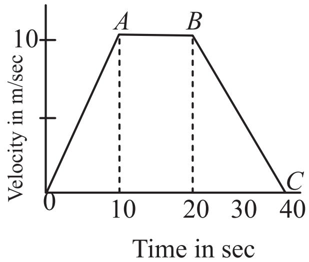
(*c*) The speed is 0.1 m/s when the acceleration is half the initial value

(*d*) The magnitude of the initial acceleration is 6.0 m/s^2

6. The displacement time graphs of two particles *A* and *B* are straight lines making angles of respectively 30º and 60º with the time axis. If the velocity of *A* is v_A and that of *B* is v_B then the value of $\frac{v_A}{v_B}$ is

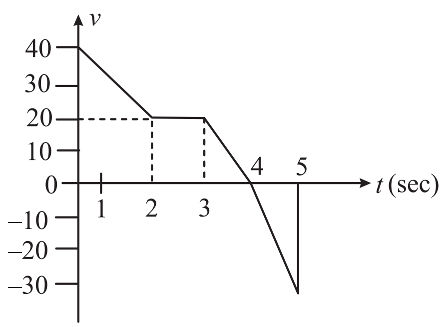
(*a*) 1/2 (*b*) $1/\sqrt{3}$ (*c*) $\sqrt{3}$ (*d*) 1/3

7. The adjoining curve represents the velocity-time graph of a particle, its acceleration values along *OA*, *AB* and *BC* in metre/sec^2 are respectively



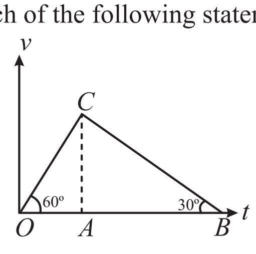
(*a*) 1, 0, –0.5 (*b*) 1, 0, 0.5 (*c*) 1, 1, 0.5 (*d*) 1, 0.5, 0

8. In the following velocity-time graph of a body, the distance and displacement travelled by the body in 5 second in meters will be



(*a*) 75,115 (*b*) 105, 75 (*c*) 45, 75 (*d*) 95, 55

9. The velocity-time graph of body is shown in figure. The ratio of the ______ during the intervals *OA* and *AB* is ______. Which of the following statement is wrong?



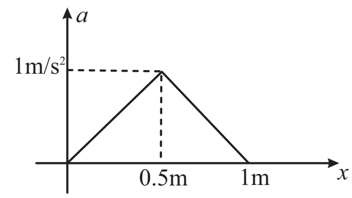
(*a*) Magnitude of average velocities, 1

(*b*) $\frac{OA}{OB}, \frac{1}{4}$

(*c*) Magnitude of average accelerations, inverse of ratio of distances covered

(*d*) Distance covered, 1 : 3

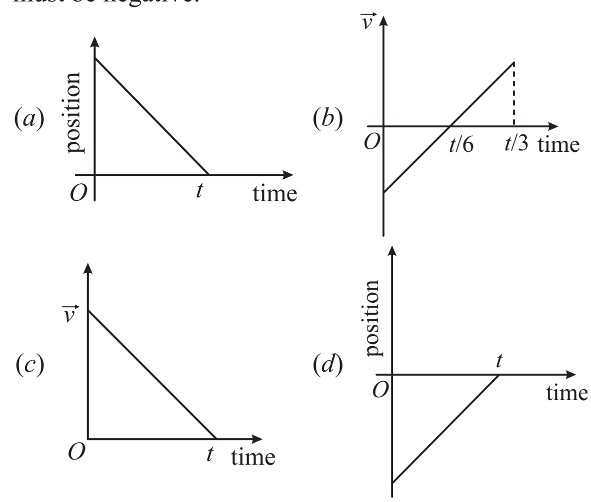
10. A body initially at rest, starts moving along *x*-axis in such a way so that its acceleration *vs* displacement plot is as shown in figure. The maximum velocity of particle is



(*a*) 1 m/s (*b*) 6 m/s

(*c*) 2 m/s (*d*) None of these

11. For which of the following graphs the average velocity of a particle moving along a straight line for time interval (0, *t*) must be negative.



12. Each of four particles move along *x*-axis. Their coordinates (in meters) as functions of time (in seconds) are given by

Particle 1 : $x(t) = 3.5 - 2.7t^3$

Particle 2 : $x(t) = 3.5 + 2.7t^3$

Particle 3 : $x(t) = 3.5 + 2.7t^2$

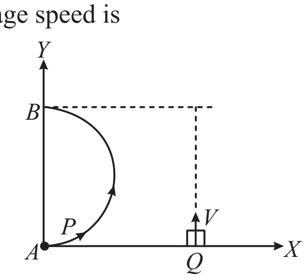
Particle 4 : $x(t) = 3.5 - 3.4t - 2.7t^2$

Which of these particles have constant acceleration?

(*a*) All four (*b*) Only 1 and 2

(*c*) Only 2 and 3 (*d*) Only 3 and 4

13. A particle *P* starts from origin as shown and moves along a circular path. Another particle *Q* crosses *x*-axis at the instant particle *P* leaves origin. *Q* moves with constant speed *v* parallel to *y*-axis and is all the time having *y*-coordinate same as that of *P*. When *P* reaches diametrically opposite at point *B*, its average speed is



(*a*) πV (*b*) $\frac{\pi V}{2}$ (*c*) $\frac{V}{2}$ (*d*) None of these

14. A particle is projected up from ground with initial speed v_0. Starting from time $t = 0$ to $t = t_1$,

(*a*) Distance travelled and magnitude of displacement are not equal if $t_1 < \frac{v_0}{g}$

(*b*) Distance travelled and magnitude of displacement are equal if $\frac{v_0}{g} < t_1 < \frac{2v_0}{g}$

(*c*) Distance travelled and magnitude of displacement may not be equal if $0 < t_1 < \frac{2v_0}{g}$

(*d*) The magnitude of displacement is greater than the distance travelled if $\frac{v_0}{g} < t_1 < \frac{2v_0}{g}$

15. Two bodies P and Q have to move equal distances starting from rest. P is accelerated with 2a for first half distance then its acceleration becomes a for last half, where as Q has acceleration a for first half and acceleration $2a$ for last half, then for whole journey.

(*a*) Average speed of P is more than that of

(*b*) Average speed of both will be same

(*c*) Maximum speed during the journey is more for P

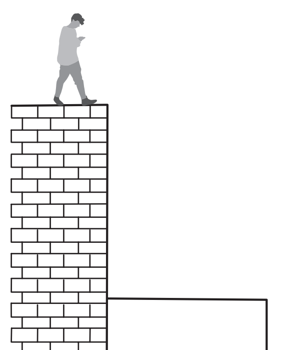
(*d*) Maximum speed during the journey is more for

16. A particle moves along the x-axis from x_i to x_f . Of the following values of the initial and final coordinates, which results in a negative displacement?

(*a*) $x_i = 4$ m, $x_f = 6$ m (*b*) $x_i = -4$ m, $x_f = -8$ m

(*c*) $x_i = -4$ m, $x_f = 2$ m (*d*) $x_i = -4$ m, $x_f = -2$ m

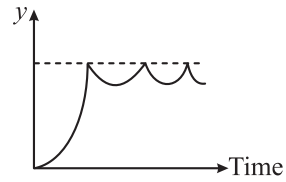
17. Suppose that a man jumps off a building 202 m high onto cushions having a total thickness of 2 m. If the cushions are crushed to a thickness of 0.5 m, what is the man's acceleration as he slows down?



(*a*) 10 m/s^2 (*b*) $\frac{4000}{3}$ m/s^2

(*c*) 50 m/s^2 (*d*) 20 m/s^2

18. The graph below describes the motion of a ball rebounding from a horizontal surface being released from a point above the surface. The quantity represented on the y-axis is the balls

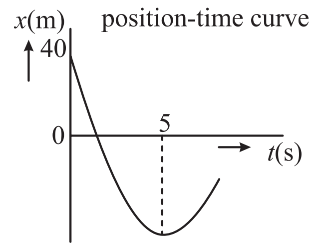


(*a*) Displacement (*b*) Velocity

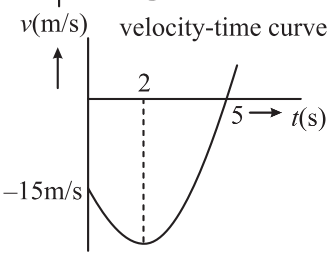
(*c*) Acceleration (*d*) Momentum

19. The position of a particle which moves along a straight line is defined by the relation $x = t^3 - 6t^2 - 15t + 40$, where x is expressed in meters and t in seconds. Which of the graph does not represent the motion of the particle?

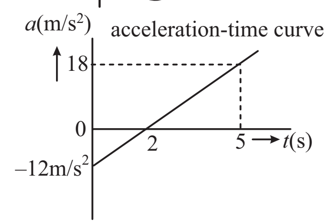
(*a*)



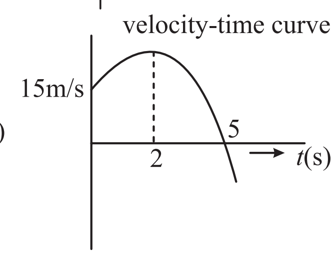
(*b*)



(*c*)



(*d*)



20. A Trolley is moving away from a stop with an acceleration $a = 0.2$ m/s^2. After reaching the velocity $u = 36$ km/h, it moves with a constant velocity for the time of 2 min. Then, it uniformly slows down, and stops after further travelling a distance of 100 m. Find the average speed all the way between stops.

(*a*) $\frac{76}{17}$ m/s (*b*) $\frac{208}{21}$ m/s (*c*) $\frac{85}{12}$ m/s (*d*) $\frac{155}{19}$ m/s

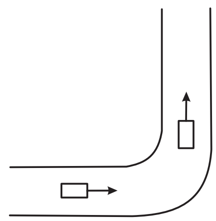
21. Two cars start to move simultaneously with the same speed from point A to point B. The first moves in a straight line connecting A and B, uniformly, and the second - on the bypass road, made as a half-circle connecting the same points. The speed of the second uniformly increases so that at the end of the path its speed is doubled. Which car will arrive earlier at point B?

(*a*) 1st car
(*b*) 2nd car
(*c*) Both will reach simultaneously
(*d*) Depends on values of R and v

22. A particle is dropped from rest. The particle first covers a distance x_1 in time t_1 and then a distance x_2 in further time t_2. If ratio of time $\frac{t_1}{t_2} = \frac{1}{(\sqrt{2}-1)}$, find the correct option:

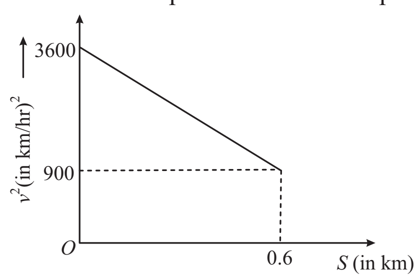
(*a*) $x_1 = x_2$ (*b*) $x_1 > x_2$
(*c*) $x_1 < x_2$ (*d*) Unpredictable

23. A car is moving with 20 ms^{-1} from west to east and takes left turn in 5 sec without changing its speed. Find average acceleration of the car during this period of 5 sec.



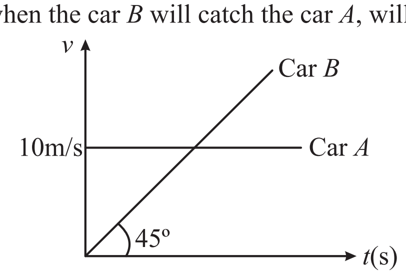
(*a*) $4\sqrt{2}$ ms^{-2} $N-E$
(*b*) $4\sqrt{2}$ ms^{-2} $N-W$
(*c*) 4 ms^{-2} $N-E$
(*d*) Zero

24. A graph between the square of the velocity of a particle and the distance 'S' moved by the particle is shown in the figure. The acceleration of the particle in kilometer per hour square is



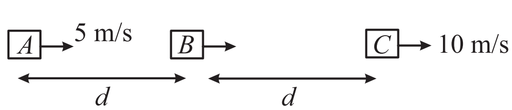
(*a*) 2250 (*b*) 225 (*c*) –2250 (*d*) –225

25. Initially car A is 10.5 m ahead of car B. Both start moving at time $t = 0$ in the same direction along a straight line. The velocity time graph of two cars is shown in figure. The time when the car B will catch the car A, will be



(*a*) $t = 21$ sec (*b*) $t = 2\sqrt{5}$ s
(*c*) $t = 20$ sec (*d*) None of these

26. Three persons A, B, C are moving along a straight line as shown with constant and different speeds. When B catches C, the separation between A and C becomes $4d$, then the speed of B is

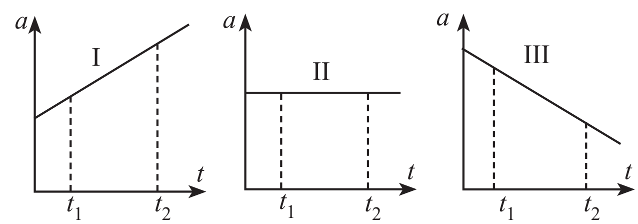


(*a*) 15 m/s (*b*) 10 m/s
(*c*) 12.5 m/s (*d*) Not possible

27. Two balls are projected simultaneously with the same speed from the top of a tower, one vertically upwards and the other vertically downwards. If the first ball strikes the ground with speed 20 m/s then speed of second ball when it strikes the ground is.

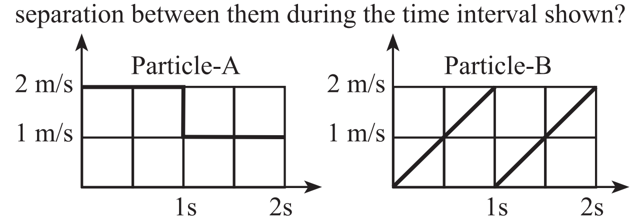
(*a*) 10 m/s (*b*) 20 m/s
(*c*) 40 m/s (*d*) Data insufficient

28. Each of the three graphs represents acceleration versus time for an object that already has a positive velocity at time t_1. Which graphs show an object whose speed is increasing for the entire time interval between t_1 and t_2?



(*a*) graph I, only (*b*) graphs I and II, only
(*c*) graphs I and III, only (*d*) graphs I, II, and

29. Two particles A and B starts from the same point and move in the positive x-direction. Their velocity-time relationships are shown in the following figures. What is the maximum separation between them during the time interval shown?



(*a*) 1.00 m (*b*) 1.25 m (*c*) 1.50 m (*d*) 2.00 m

30. Two balls are projected simultaneously with the same speed from the top of a tower, one vertically upwards and the other vertically downwards. They reach the ground in 9*s* and 4*s*, respectively. The height of the tower is (g = 10 m/s^2)

(*a*) 90 m (*b*) 180 m (*c*) 270 m (*d*) 360 m

31. From the top of a tower, a stone is thrown up. It reaches the ground in time t_1. A second stone thrown down with the same speed reaches the ground in time t_2. A third stone released from rest reaches the ground in time t_3. Then:

(*a*) $t_3 = \frac{t_1 + t_2}{2}$ (*b*) $t_3 = \sqrt{t_1 t_2}$
(*c*) $\frac{1}{t_3} = \frac{1}{t_1} - \frac{1}{t_2}$ (*d*) $t_3^2 = t_1^2 - t_2^2$

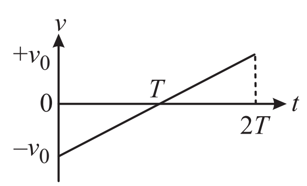
32. A particle is thrown upwards from ground. It experiences a constant resistance force which can produce retardation 2 m/s^2. The ratio of time of ascent to the time of descent is [g = 10 m/s^2]

(*a*) 1 : 1 (*b*) $\sqrt{\frac{2}{3}}$ (*c*) $\frac{2}{3}$ (*d*) $\sqrt{\frac{3}{2}}$

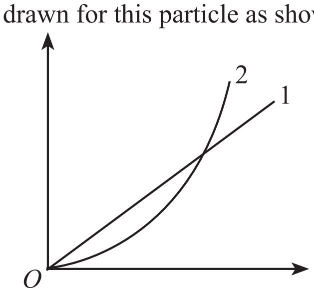
Exercise-3 (JEE Advanced Level)

MULTIPLE CORRECT TYPE QUESTIONS

1. Mark the correct statements for a particle going on a straight line
   (*a*) If the velocity is zero at any instant, the acceleration should also be zero at that instant.
   (*b*) If the velocity is zero for a time interval, the acceleration is zero at any instant within the time interval.
   (*c*) If the velocity and acceleration have opposite sign, the object is slowing down.
   (*d*) If the position and velocity have opposite sign, the particle is moving towards the origin.
2. Let $\vec{v}$ and $\vec{a}$ denote the velocity and acceleration respectively of a body in one-dimensional motion then which among the following is/are true
   (*a*) $|\vec{v}|$ must decrease when $\vec{a} < 0$.
   (*b*) Speed must increase when $\vec{a} > 0$.
   (*c*) Speed will increase when both $\vec{v}$ and $\vec{a}$ are < 0.
   (*d*) Speed will decrease when $\vec{v} < 0$ and $\vec{a} > 0$.
3. A particle has initial velocity 10 m/s. It moves due to constant retarding force along the line of velocity which produces a retardation of 5 m/s^2. Then
   (*a*) The maximum displacement in the direction of initial velocity is 10 m.
   (*b*) The distance travelled in first 3 seconds is 7.5 m.
   (*c*) The distance travelled in first 3 seconds is 12.5 m.
   (*d*) The distance travelled in first 3 seconds is 17.5 m.
4. The displacement x of a particle depend on time t as $x = \alpha t^2 - \beta t^3$
   (*a*) Particle will return to its starting point after time α/β.
   (*b*) The particle will come to rest after time $\frac{2\alpha}{3\beta}$.
   (*c*) The initial velocity of the particle was zero but its initial acceleration was not zero.
   (*d*) No net force act on the particle at time $\frac{\alpha}{3\beta}$.
5. The figure shows the velocity (v) of a particle plotted against time (t)



   (*a*) The particle changes its direction of motion at some point.
   (*b*) The acceleration of the article remains constant.
   (*c*) The displacement of the particle is zero.
   (*d*) The initial and final speeds of the particle are the same.
6. A particle moves with constant speed v along a regular hexagon *ABCDEF* in the same order. Then the magnitude of the average velocity for its motion from A to
   (*a*) F is $v/5$ (*b*) D is $v/3$
   (*c*) C is $v\sqrt{3/2}$ (*d*) B is v
7. Path of a particle moving in x-y plane is $y = 3x + 4$. At some instant suppose x-component of velocity is 1 m/s and it is increasing at a rate of 1 m/s^2. Then at this instant
   (*a*) Speed of particle is $\sqrt{10}$ m/s.
   (*b*) Acceleration of particle is $\sqrt{10}$ m/s^2.
   (*c*) Velocity time graph is a straight line.
   (*d*) Acceleration-time graph is a straight line.
8. A particle having a velocity $v = v_0$ at $t = 0$ is decelerated at the rate $|a| = \alpha\sqrt{v}$, where α is a positive constant.
   (*a*) The particle comes to rest at $t = \frac{2\sqrt{v_0}}{\alpha}$
   (*b*) The particle will come to rest at infinity.
   (*c*) The distance travelled by the particle is $\frac{2v_0^{3/2}}{\alpha}$.
   (*d*) The distance travelled by the particle is $\frac{2}{3}\frac{v_0^{3/2}}{\alpha}$.
9. A particle is resting over a smooth horizontal floor. At $t = 0$, a horizontal force starts acting on it. Magnitude of the force increases with time as $F = kt$, where k is a constant. The two curves are drawn for this particle as shown.



   (*a*) Curve-1 shows acceleration versus time.
   (*b*) Curve-2 shows velocity versus time.
   (*c*) Curve-2 shows velocity versus acceleration.
   (*d*) Curve-1 shows velocity versus acceleration.

COMPREHENSION BASED QUESTIONS

Comprehension (Q. 10 to 12): A boy is standing on a open truck. Truck is moving with an acceleration 2 m/s^2 on horizontal road. When speed of truck is 10 m/s and reaches to a electric pole, boy projected a ball with a velocity 10 m/s in vertical upward direction relative to himself (take $g = 10$ m/s^2). Neglect the height of boy and truck.

10. The distance of ball from pole where ball land is

(a) 20 m (b) 10 m (c) 30 m (d) 40 m

11. Maximum height of ball from ground is

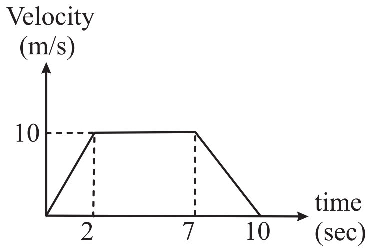
(a) 5 m (b) 7.5 m (c) 2.5 m (d) 10 m

12. Speed of truck at the instant when boy see that ball is moving backward horizontally is

(a) 14 m/s (b) 10 m/s

(c) 12 m/s (d) Data is insufficient

Comprehension (Q. 13 to 15): The velocity-time graph of a car moving on a straight track is given below. The car weighs 1000 kg. (Use $F = ma$)



13. The distance travelled by the car during the whole motion is

(a) 50 m (b) 75 m (c) 100 m (d) 150 m

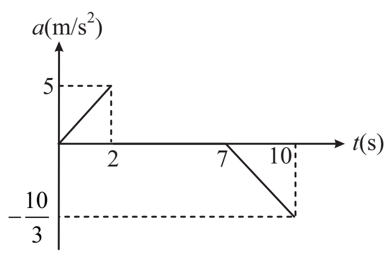
14. The braking force required to bring the car to a stop with in one second from the maximum speed is

(a) $\frac{10000}{3} N$ (b) 5000 N

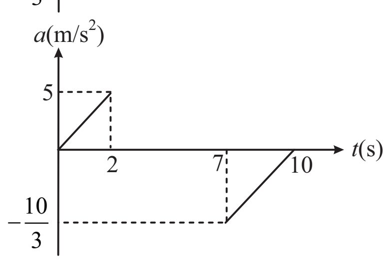
(c) 10000 N (d) $\frac{5000}{3} N$

15. Correct acceleration-time graph representing the motion of car is

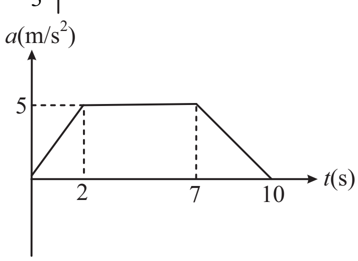
(a)



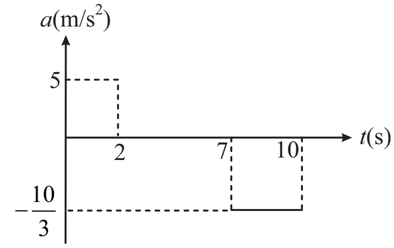
(b)



(c)

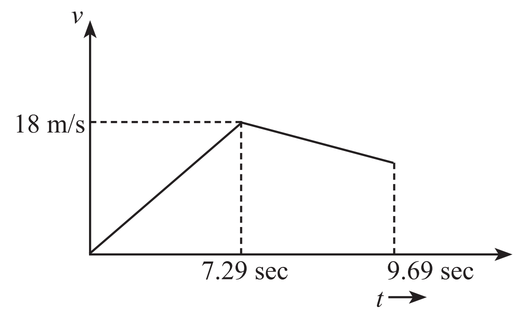


(d)



Comprehension (Q. 16 to 17): In the 2008 Olympic 100 m final, Usain Bolt broke new ground, winning in 9.69 s (unofficially 9.683 s). This was an improvement upon his own world record, and he was well ahead of second-place finisher Richard Thompson, who finished in 9.89 s. Not only was the record set without a favourable wind (+0.0 m/s), but he also visibly slowed down to celebrate before he finished and his shoelace was untied. Bolt's coach reported that, based upon the speed of Bolt's opening, he could have finished with a time of 9.52 s. After scientific analysis of Bolt's run by the Institute of Theoretical Astrophysics at the University of Oslo, Hans Eriksen and his colleagues also predicted a 9.60 s time. Considering factors such as Bolt's position, acceleration and velocity in comparison with second-place-finisher Thompson, the team estimated that Bolt could have finished in 9.55 ± 0.04 s had he not slowed to celebrate before the finishing line.

Let us also analyse the motion of Bolt. Assume that the velocity time graph of Usain Bolt is as shown below.



16. What was the initial acceleration of Bolt.

(a) 4.5 m/s^2 (b) 3.1 m/s^2 (c) 2.5 m/s^2 (d) 1.2 m/s^2

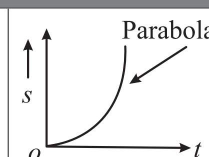
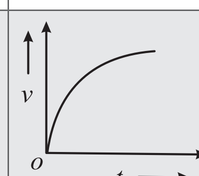
17. What was the final velocity of Bolt.

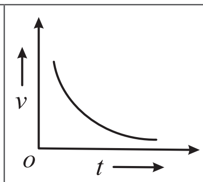
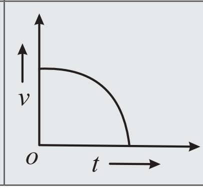
(a) 10.1 m/s (b) 10.6 m/s

(c) 13.4 m/s (d) 14.6 m/s

MATCH THE COLUMN TYPE QUESTIONS

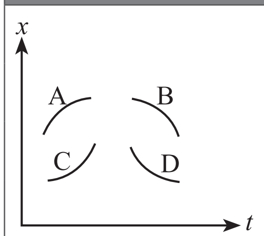
18. Match Column-I with Column-II and select the correct answer using the codes given below the lists.

	Column-I		Column-II
A.	Acceleration decreasing with time	p.	
B.	Velocity increasing with time	q.	

C.	Magnitude of acceleration increasing with time	r.	
D.	Body going farther away from the starting point with time	s.	

(*a*) A-(r); B-(p,q); C-(s); D-(p,q,r,s)
(*b*) A-(r,s); B-(r,q); C-(p,q,r,s); D-(s)
(*c*) A-(r,q); B-(q); C-(s,r); D-(p,q,r,s)
(*d*) A-(r,s); B-(p,r); C-(p,q,r,s); D-(p,q)

19. Column-I shows the position-time graph of particles moving along a straight line.

Column-I		Column-II
	p.	Acceleration $a > 0$
	q.	Acceleration $a < 0$
	r.	Speeding up
	s.	Slowing down

(*a*) A-(q,p); B-(r,p); C-(s,q); D-(r,s)
(*b*) A-(q,s); B-(q,r); C-(p,r); D-(p,s)
(*c*) A-(r,q); B-(s,r); C-(q,r); D-(p,r)
(*d*) A-(p,s); B-(r,s); C-(p,r); D-(p,q)

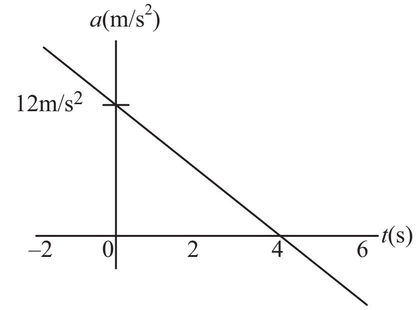
20. The position of a particle along *x*-axis is given by $x = (2t^3 - 21t^2 + 60t)m$. Then match the Column-I with Column-II.

	Column-I		Column-II
A.	Velocity of particle is zero	p.	2 sec
B.	Acceleration of particle is zero	q.	3 sec
C.	Acceleration of particle is negative	r.	3.5 sec
D.	Velocity of particle is towards the origin	s.	4 sec
		t.	5 sec

(*a*) A-(p,r,t); B-(r); C-(p,q); D-(q,r,s)
(*b*) A-(p,t,r); B-(r,t); C-(q); D-(s)
(*c*) A-(p,r); B-(s); C-(p,r,q); D-(r,s)
(*d*) A-(p,t); B-(r); C-(p,q); D-(q,r,s)

NUMERICAL TYPE QUESTIONS

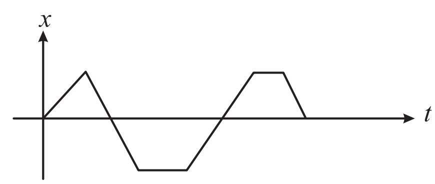
21. Figure gives the acceleration a versus time *t* for a particle moving along an *x*-axis. At $t = -2.0$ s, the particle's velocity is 7.0 m/s. What is its velocity (m /s) at $t = 6.0$ s?



22. A particle moves in *xy*-plane according to the equation $x = 3t$, $y = 25 - 4t$. What is the minimum distance of the particle (in m) from the origin? Both *x* and *y* are in *m*.

23. A rocket rises vertically up from the surface of earth so that it's distance from the earth's surface is $l = ct^2$ where *c* is a constant. After 10 sec. the rocket has travelled 2 km. Determine it's speed (in m/s) at that moment.

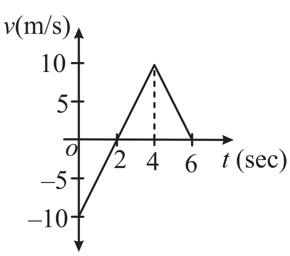
24. Consider a particle moving on a straight line with varying velocity. Its position time graph is as shown.



Find the number of times its velocity changes during motion.

25. Acceleration of particle moving rectilinearly is $a = 4 - 2x$ (where *x* is position in metre and *a* in m/s^2). It is at instantaneous rest at $x = 0$. At what position *x* (in metre) will the particle again come to instantaneous rest?

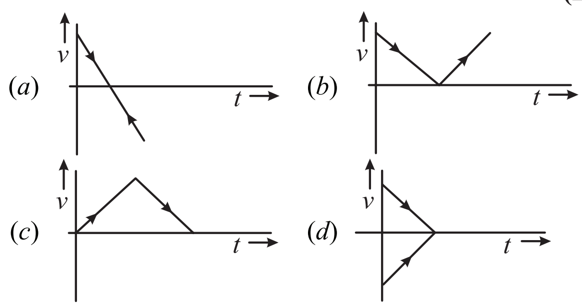
26. The figure shows the graph of velocity-time for a particle moving in a straight line. If the average speed for 6 sec is '*b*' and the average acceleration from 0 sec to 4 sec is '*c*' find magnitude of *bc* (in m^2/s^3).



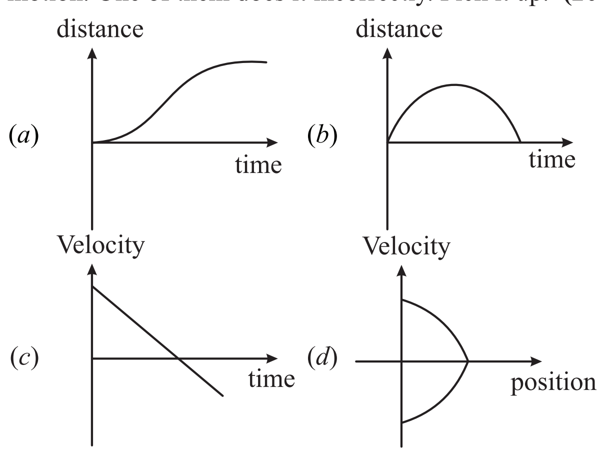
Exercise-4 (Past Year Questions)

JEE MAIN

1. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time? **(2017)**



2. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up. **(2018)**



3. A particle moves from the point $(2.0\hat{i}+4.0\hat{j})\text{m}$, at $t = 0$ with an initial velocity $(5.0\hat{i}+4.0\hat{j})\text{ms}^{-1}$. It is acted upon by a constant force which produces a constant acceleration $(4.0\hat{i}+4.0\hat{j})\text{ms}^{-2}$. What is the distance of the particle from the origin at time 2s? **(2019)**

(*a*) 15 m (*b*) $20\sqrt{2}$ m

(*c*) 5 m (*d*) $10\sqrt{2}$ m

4. The position co-ordinates of a particle moving in a 3-D coordinates system is given by

$x = a\cos\omega t$

$y = a\sin\omega t$

and $z = a\omega t$

The speed of the particle is: **(2019)**

(*a*) $\sqrt{2}\,a\omega$

(*b*) $a\omega$

(*c*) $\sqrt{3}\,a\omega$

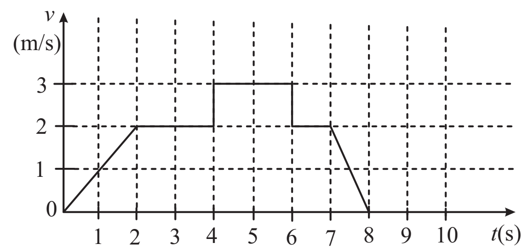
(*d*) $2a\omega$

5. In a car race on straight road, car A takes a time 't' less than car B at the finish and passes finishing point with a speed 'V' more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'v' is equal to **(2019)**

(*a*) $\frac{2a_1a_1}{a_1+a_2}t$ (*b*) $\sqrt{2a_1a_2}\,t$

(*c*) $\sqrt{a_1a_2}\,t$ (*d*) $\frac{a_1+a_2}{2}t$

6. A particle starts from the origin at time $t = 0$ and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time $t = 5s$? **(2019)**



(*a*) 10 m (*b*) 6 m

(*c*) 3 m (*d*) 9 m

7. The position of a particle as a function of time t, is given by $x(t) = at + bt^2 - ct^3$ where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be: **(2019)**

(*a*) $a+\frac{b^2}{4c}$ (*b*) $a+\frac{b^2}{c}$

(*c*) $a+\frac{b^2}{2c}$ (*d*) $a+\frac{b^2}{3c}$

8. The position vector of a particle changes with time according to the relation $\vec{r}(t) = 15t^2\hat{i}+(4-20t^2)\hat{j}$. What is the magnitude of the acceleration at $t = 1$? **(2019)**

(*a*) 40 (*b*) 100

(*c*) 25 (*d*) 50

9. A bullet of mass 20 g has an initial speed of 1 ms^{-1}, just before it starts penetrating a mud wall of thickness 20 cm. If the wall offers a mean resistance of 2.5×10^{-2} N, the speed of the bullet after emerging from the other side of the wall is close to **(2019)**

(*a*) 0.4 ms^{-1}

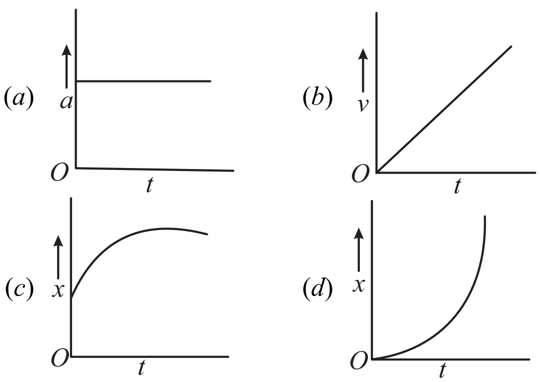
(*b*) 0.1 ms^{-1}

(*c*) 0.3 ms^{-1}

(*d*) 0.7 ms^{-1}

10. A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify the figure that is not correctly representing the motion qualitatively. **(2019)**

(a = acceleration, v = velocity, x = displacement, t = time)



(a) (a), (b), (c) (b) (a)

(c) (c) (d) (b), (c)

11. A particle is moving with speed $v = b\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time $t = \tau$ (assume that the particle is at origin at $t = 0$) **(2019)**

(a) $\frac{b^2\tau}{4}$ (b) $\frac{b^2\tau}{2}$

(c) $b^2\tau$ (d) $\frac{b^2\tau}{\sqrt{2}}$

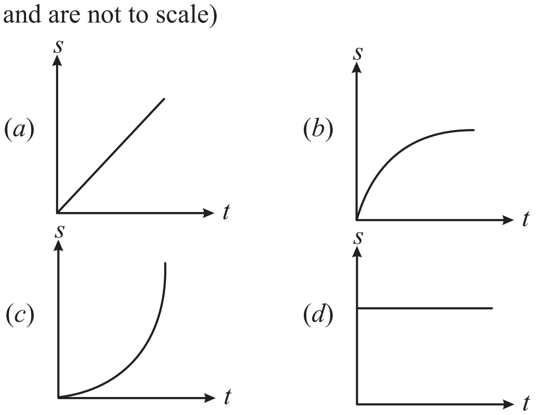
12. The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + 2bt + c$. If the acceleration of the particle depends on x as x^{-n}, where n is an integer, the value of n is **(2019)**

13. A particle starts from the origin at $t = 0$ with an initial velocity of 3.0 $\hat{i}$ m/s and moves in the x-y plane with a constant acceleration $(6.0\hat{i} + 4.0\hat{j})$ m/s^2. The x-coordinate of the particle at the instant when its y-coordinate is 32 m is D meters. The value of D is **(2020)**

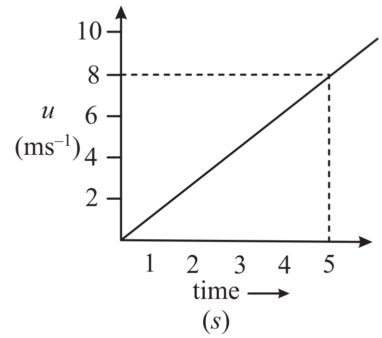
(a) 50 (b) 40

(c) 32 (d) 60

14. A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement (s) - time (t) graph that describes the motion of the particle is (graphs are drawn schematically and are not to scale) **(2020)**



15. The speed versus time graph for a particle is shown in the figure. The distance travelled (in m) by the particle during the time interval $t = 0$ to $t = 5$ s will be ___________. **(2020)**

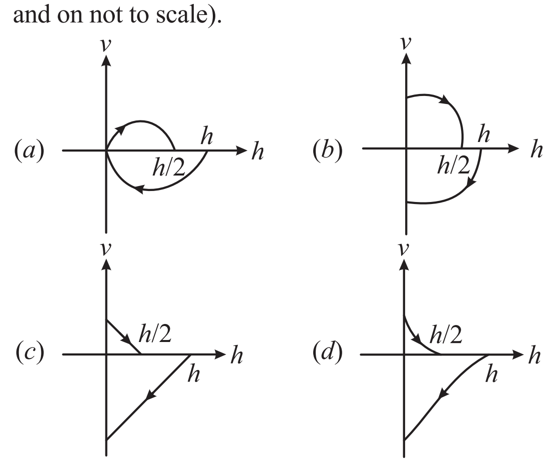


16. Starting from the origin at time $t = 0$, with initial velocity $5\hat{j}$ m/s, a particle moves in the x-y plane with a constant acceleration of $(10\hat{i} + 4\hat{j})$ m/s^2. At time t, its coordinates are (20 m, y_0 m). The values of t and y_0 are, respectively **(2020)**

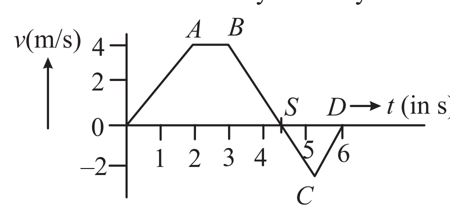
(a) 4 s and 52 m (b) 2 s and 24 m

(c) 5 s and 25 m (d) 2 s and 18 m

17. A tennis ball is released from a height h and after freely falling on a wooden floor it rebounds and reaches height $h/2$. The velocity versus height of the ball during its motion may be represented graphically by (graph are drawn schematically and on not to scale). **(2020)**



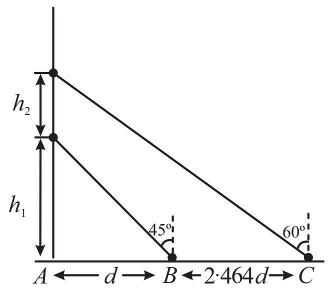
18. Th velocity (v) and time (t) graph of a body in a straight line motion is shown in the figure. The point S is at 4.333 second. The total distance covered by the body in 6 s is **(2020)**



(a) $\frac{37}{3}$ m (b) 11 m

(c) 12 m (d) $\frac{49}{4}$ m

19. A balloon is moving up in air vertically above a point A on the ground. When it is at a height h_1, a girl standing at a distance d (point B) from A (see figure) sees it at an angle 45° with respect to the vertical. When the balloon climbs up a further height h_2, it is seen at an angle 60° with respect to the vertical if the girl moves further by a distance 2.464 d (point C). Then the height h_2 is (given tan 30° = 0.5774) **(2020)**



(*a*) d (*b*) 0.732 d

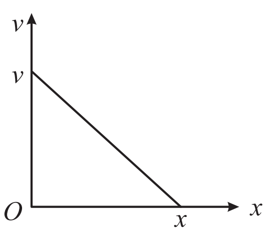
(*c*) 1.464 d (*d*) 0.464 d

20. A helicopter rises from rest on the ground vertically upwards with a constant acceleration g. A food packet is dropped from the helicopter when it is at a height h. The time taken by the packet to reach the ground is close to [g is the acceleration due to gravity] **(2020)**

(*a*) $t = 3.4\sqrt{\left(\frac{h}{g}\right)}$ (*b*) $t = 1.8\sqrt{\frac{h}{g}}$

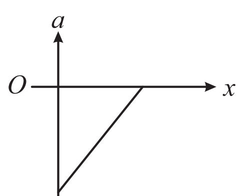
(*c*) $t = \sqrt{\frac{2h}{3g}}$ (*d*) $t = \frac{2}{3}\sqrt{\left(\frac{h}{g}\right)}$

21. The velocity - displacement graph of a particle is shown in the figure. **(2021)**

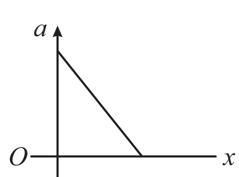


The acceleration - displacement graph of the same particle is represented by:

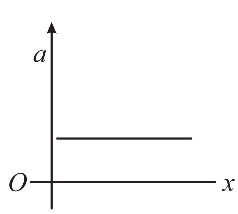
(*a*)



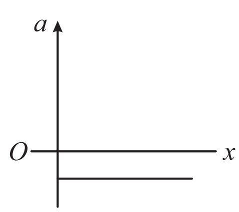
(*b*)



(*c*)

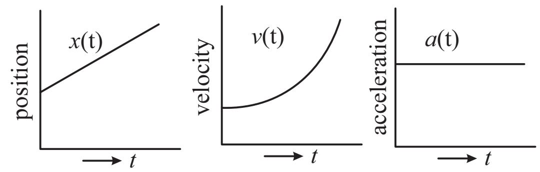


(*d*)

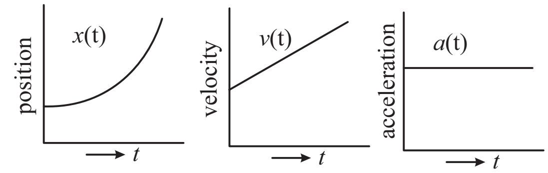


22. The position, velocity and acceleration of a particle moving with a constant acceleration can be represented by **(2021)**

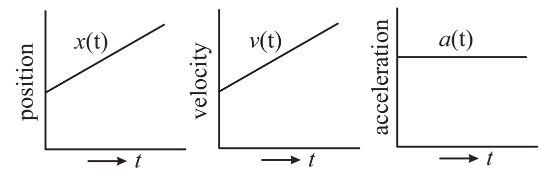
(*a*)



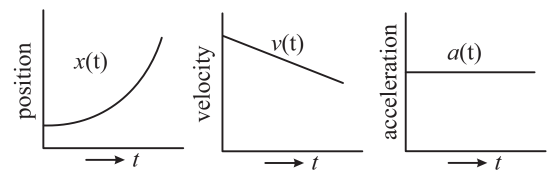
(*b*)



(*c*)



(*d*)



23. A rubber ball is released from a height of 5 m above the floor. It bounces back repeatedly, always rising to $\frac{81}{100}$ of the height through which it falls. Find the average speed of the ball. (Take $g = 10\ ms^{-2}$) **(2021)**

(*a*) 2.50 ms^{-1} (*b*) 3.0 ms^{-1}

(*c*) 2.0 ms^{-1} (*d*) 3.50 ms^{-1}

24. The velocity of a particle is $v = v_0 + gt + Ft^2$. Its position is $x = 0$ at $t = 0$; then its displacement after time ($t = 1$) is: **(2021)**

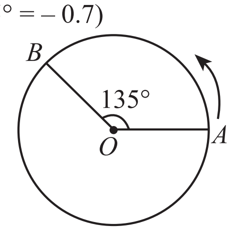
(*a*) $v_0 + g + f$

(*b*) $v_0 + \frac{g}{2} + \frac{F}{3}$

(*c*) $v_0 + 2g + 3F$

(*d*) $v_0 + \frac{g}{2} + F$

25. A person moved from A to B on a circular path a shown in figure. If the distance travelled by him is 60 m, then the magnitude of displacement would be:
(Given cos135° = – 0.7) **(2022)**



(*a*) 42 m (*b*) 47 m

(*c*) 19 m (*d*) 40 m

26. A car is moving with speed of 150 km/h and after applying the brake it will move 27 m before it stops. If the same car is moving with a speed of one third the reported speed then it will stop after travelling ______ m distance. **(2022)**

27. A particle is moving in a straight line such that its velocity is increasing at 5 ms^{-1} per meter. The acceleration of the particle is_____ms^{-2} at a point where its velocity is 20 ms^{-1}. **(2022)**

28. A ball is thrown vertically upwards with a velocity of 19.6 ms^{-1} from the top of a tower. The ball strikes the ground after 6 s. The height from the ground up to which the ball can rise will be $\left(\frac{k}{5}\right)$ m. The value of k is ______

(use $g = 9.8$ m/s^2) **(2022)**

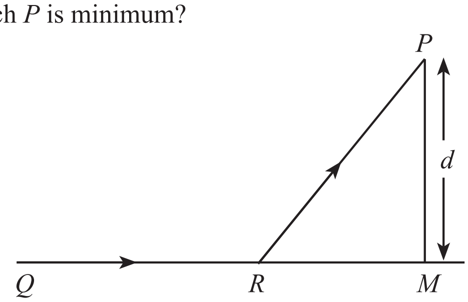
29. A ball is thrown up vertically with a certain velocity so that, it reaches a maximum height h. Find the ratio of the times in which it is at height $\frac{h}{3}$ while going up and coming down respectively. **(2022)**

(*a*) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ (*b*) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(*c*) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (*d*) $\frac{1}{3}$

JEE ADVANCED

30. A man in a car at location Q on a straight highway is moving with speed u. He decides to reach a point P in a field at a distance d from highway (point M) as shown in the figure. Speed of the car in the field is half to that on the highway. What should be the distance RM so that the time taken to reach P is minimum? **(2018)**



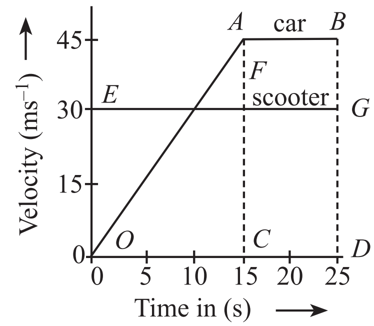
(*a*) $\frac{d}{\sqrt{3}}$

(*b*) $\frac{d}{2}$

(*c*) $\frac{d}{\sqrt{2}}$

(*d*) d

31. The velocity-time graphs of a car and a scooter are shown in the figure, (i) the difference between the distance travelled by the car and the scooter in 15s and (ii) the time at which the car will catch up with the scooter are, respectively **(2018)**



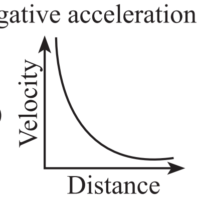
(*a*) 337.5 m and 25 s

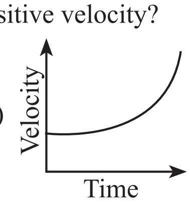
(*b*) 225.5 m and 10 s

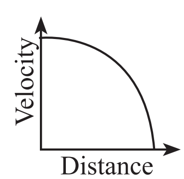
(*c*) 112.5 m and 22.5 s

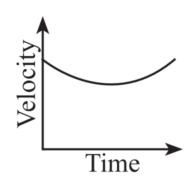
(*d*) 11.2.5 m and 25 s

32. Which graph corresponds to an object moving with a constant negative acceleration and a positive velocity? **(2018)**

(*a*)


(*b*)


(*c*)


(*d*)


ANSWER KEY

CONCEPT APPLICATION

1. (24 km/hr) **2.** (*a*) 1, (*b*) 50 **3.** (*b*) **4.** (*a*) **5.** (*c*) **6.** (*b*) **7.** (*c*)

EXERCISE-1 (TOPICWISE)

1. (*a*)	**2.** (*a*)	**3.** (*c*)	**4.** (*c*)	**5.** (*b*)	**6.** (*d*)	**7.** (*c*)	**8.** (*b*)	**9.** (*d*)	**10.** (*c*)
11. (*b*)	**12.** (*a*)	**13.** (*b*)	**14.** (*b*)	**15.** (*b*)	**16.** (*d*)	**17.** (*c*)	**18.** (*c*)	**19.** (*a*)	**20.** (*c*)
21. (*c*)	**22.** (*c*)	**23.** (*d*)	**24.** (*c*)	**25.** (*a*)	**26.** (*d*)	**27.** (*b*)	**28.** (*c*)	**29.** (*b*)	**30.** (*c*)
31. (*d*)	**32.** (*b*)	**33.** (*d*)	**34.** (*c*)	**35.** (*d*)	**36.** (*a*)	**37.** (*a*)	**38.** (*d*)	**39.** (*a*)	**40.** (*c*)
41. (*c*)	**42.** (*a*)								

EXERCISE-2 (LEARNING PLUS)

1. (*b*)	**2.** (*d*)	**3.** (*b*)	**4.** (*c*)	**5.** (*c*)	**6.** (*d*)	**7.** (*a*)	**8.** (*b*)	**9.** (*b*)	**10.** (*a*)
11. (*a*)	**12.** (*d*)	**13.** (*b*)	**14.** (*c*)	**15.** (*a*)	**16.** (*b*)	**17.** (*b*)	**18.** (*a*)	**19.** (*d*)	**20.** (*d*)
21. (*a*)	**22.** (*a*)	**23.** (*b*)	**24.** (*c*)	**25.** (*a*)	**26.** (*c*)	**27.** (*b*)	**28.** (*d*)	**29.** (*b*)	**30.** (*b*)
31. (*b*)	**32.** (*b*)								

EXERCISE-3 (JEE ADVANCED LEVEL)

1. (*b,c,d*)	**2.** (*c,d*)	**3.** (*a,c*)	**4.** (*a,b,c,d*)	**5.** (*a,b,c,d*)	**6.** (*a,c,d*)	**7.** (*a,b*)	**8.** (*a,d*)	**9.** (*a,b,c*)	**10.** (*a*)
11. (*a*)	**12.** (*c*)	**13.** (*b*)	**14.** (*c*)	**15.** (*d*)	**16.** (*c*)	**17.** (*b*)	**18.** (*a*)	**19.** (*b*)	**20.** (*d*)
21. [0055]	**22.** [0015]	**23.** [0400]	**24.** [0006]	**25.** [0004]	**26.** [0025]				

EXERCISE-4 (PAST YEAR QUESTIONS)

JEE Main

1. (*a*)	**2.** (*a*)	**3.** (*b*)	**4.** (*a*)	**5.** (*c*)	**6.** (*d*)	**7.** (*d*)	**8.** (*d*)	**9.** (*d*)	**10.** (*c*)
11. (*b*)	**12.** [3]	**13.** (*d*)	**14.** (*c*)	**15.** [20]	**16.** (*d*)	**17.** (*b*)	**18.** (*a*)	**19.** (*a*)	**20.** (*a*)
21. (*a*)	**22.** (*b*)	**23.** (*a*)	**24.** (*b*)	**25.** (*b*)	**26.** [3]	**27.** [100]	**28.** [392]	**29.** (*b*)	

JEE Advanced

30. (*a*) **31.** (*c*) **32.** (*c*)