PYP - 2022

Mathematics

Q1 If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is co-factor of a_{ij}, then value of Δ is equal to

(A) $a_{11}\ A_{31} + a_{12}\ A_{32} + a_{13}\ A_{33}$

(B) $a_{11}\ A_{11} + a_{12}\ A_{21} + a_{13}\ A_{31}$

(C) $a_{21}\ A_{11} + a_{22}\ A_{12} + a_{23}\ A_{13}$

(D) $a_{11}\ A_{11} + a_{21}\ A_{21} + a_{31}\ A_{31}$

Q2 If the matrix $\begin{bmatrix} 0 & -1 & 3x \\ 1 & y & -5 \\ -6 & 5 & 0 \end{bmatrix}$ is skew symmetric, then $6x + y$ is equal to

(A) 6 (B) 12

(C) 18 (D) 4

Q3 If $\begin{vmatrix} 3 & -4 \\ 2 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 5 \\ 1 & x \end{vmatrix}$ then $|x|$ is equal to

(A) $\sqrt{\frac{5}{2}}$ (B) 4

(C) $2\sqrt{2}$ (D) 6

Q4 Consider the following statements:

1. A square matrix A is said to be non-singular if $|A| = 0$.
2. A square matrix A is invertible if and only if $|A| \neq 0$.
3. If elements of a row are multiplied with cofactors of any other row, then their sum is zero.
4. A is square matrix of order 3 then $|Adj(A)| = |A|^4$

Which of the statements given above is/are correct ?

(A) 1 and 3 only (B) 2 and 3 only

(C) 3 and 4 only (D) All of these

Q5 The interval in which $y = x^2e^{2x}$ is increasing is

(A) $(-\infty, -1)$

(B) $(-1, -\infty)$

(C) $(-\infty, -1) \cup (0, \infty)$

(D) $(-\infty, 0) \cup (1, \infty)$

Q6 If $x = t^3, y = t^4$ then $\frac{d^2y}{dx^2}$ at $t = 2$ is

(A) $\frac{8}{3}$ (B) $\frac{1}{9}$

(C) $\frac{2}{9}$ (D) $\frac{9}{16}$

Q7 Match List-I with List-II

List-I	List-I
A. $\int \frac{dx}{\sqrt{9x^2-16}}$	(i). $\frac{1}{3}\sin^{-1}\frac{3x}{4} + C$
B. $\int \frac{dx}{\sqrt{16-9x^2}}$	(ii). $\log(e^x + 1) + C$
C. $\int \frac{e^{2\ln x}+1}{e^{2\ln x}-1}dx$	(iii). $\frac{1}{3}\log_e\left\|3x + \sqrt{9x^2-16}\right\| + C$
D. $\int \frac{1}{1+e^{-x}}dx$	(iv). $x + \log_e\left\|\frac{x-1}{x+1}\right\| + C$

Choose the correct answer from the options given below:

(A) $A-(i), B-(iii), C-(ii), D-(iv)$

(B) $A-(i), B-(iii), C-(iv), D-(ii)$

(C) $A-(iii), B-(i), C-(iv), D-(ii)$

(D) $A-(iii), B-(i), C-(ii), D-(iv)$

Q8 The area (in sq. units) bounded by the parabola $y^2 = 8x$ and the line $x = 2$ is

(A) $\frac{16}{3}$ (B) $\frac{32}{3}$

(C) $\frac{32\sqrt{2}}{3}$ (D) $\frac{16\sqrt{3}}{3}$

Q9 $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2\pi-x}}dx$ is equal to

(A) $\pi/3$ (B) π

(C) $\pi/2$ (D) 1

Q10 The solution curve $y = y(x)$ of the differential equation $ydx - xdy = 0$, passing through $(2, 4)$, will pass through the point

(A) (-1,2) (B) $\left(\frac{-1}{2}, -1\right)$

(C) (0,-4) (D) $\left(1, \frac{1}{2}\right)$

Q11 The solution curve of the differential equation $\log_e\left(\frac{dy}{dx}\right) = 3x + 4y$, passing through the origin is

(A) $4e^{3x} - 3e^{-4y} - 1 = 0$ (B) $4e^{3x} + 3e^{-4y} - 7 = 0$

(C) $3e^{3x} + 4e^{-4y} - 1 = 0$ (D) $3e^{3x} - 4e^{-4y} + 7 = 0$

Q12 If the probability distribution of a random variable X is

X	1	2	3	4	5
P(X=x)	k	2k	3k	3k	k

Value of $P(X > 2)$ is

(A) $\frac{5}{6}$ (B) $\frac{9}{10}$

(C) $\frac{4}{5}$ (D) $\frac{7}{10}$

Q13 The curve represented by the differential equation $\frac{dy}{dx} = \frac{2(x+1)y}{x^2+2x+2}, y > 0$, passes through the point (0, 4). If it also passes through $P(-1, k)$ and $Q(\lambda, 10)$ then $(PQ)^2$ is equal to

(A) 148 (B) 66

(C) 68 (D) 72

Q14 If $\int \frac{dx}{\sqrt{x}-\sqrt{x-1}} = \lambda\left(x^{\frac{3}{2}} + (x-1)^{\frac{3}{2}}\right) + C$, then the value of λ is

(A) 2 (B) 3 / 2

(C) 1 (D) 2 / 3

Q15 The corner points of the feasible region of an LPP are $(0,0), (30,0), (20,30) and (0,50)$. If the objective function maximize $(Z) = ax + by$, where $a \neq b$ and $a, b > 0$, has optimal solution at both points $(30,0)$ and $(20,30)$ then which of the following is true?

(A) $a = 2b$ (B) $a = 3b$

(C) $2a = b$ (D) $3a = b$

Q16 Which of the following is not an equivalence relation on Z ?

(A) $a\mathbf{R}b \Leftrightarrow a + b$ is an even integer

(B) $a\mathbf{R}b \Leftrightarrow a - b$ is an even integer

(C) $a\mathbf{R}b \Leftrightarrow a \leq b$

(D) $a\mathbf{R}b \Leftrightarrow a = b$

Q17 The function $f : R - \{-1\} \to R - \{1\}$ defined by $f(x) = \frac{x}{x+1}$ is

(A) Both one-one and onto

(B) Only one-one

(C) Only onto

(D) Neither one-one nor onto

Q18 If $X + Y = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 0 & 3 \\ 7 & 9 \end{bmatrix}$ then Y is equal to

(A) $\begin{bmatrix} -1 & 1 \\ -2 & -2 \end{bmatrix}$ (B) $\begin{bmatrix} -1 & 1 \\ 2 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & -1 \\ -2 & -2 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$

Q19 If A and B are any two matrices such that $(A-B)^2 = A^2 - 2AB + B^2$ then

(A) $B^2 A^3 + A^3 B^2 = 0$

(B) $AB + BA = 0$

(C) $B^3 A^4 - A^4 B^3 = 0$

(D) $AB(B - A) = 0$

Q20 If A and B are two matrix such that $AB = O$ (null matrix) then

A. A may be a null matrix, but B may not be a null matrix

B. B may be a null matrix, but A may not be a null matrix.

C. Both A and B may be null matrices.

D. Both A and B may be non-null matrices.

Choose the correct answer from the option given below:

(A) A, B only (B) A, B, C only

(C) C, D only (D) All of these

Q21

$\int \frac{x^2+3x-1}{(x+1)^2}dx$ is equal to

(A) $\frac{1}{x+1}+\frac{1}{2}\log_e|x+1|+C$

(B) $x+\frac{3}{x+1}+\log_e|x+1|+C$

(C) $x-\frac{1}{x+1}+\log_e|x+1|+C$

(D) $x+\frac{3}{x+1}+\frac{1}{2}\log_e|x+1|+C$

Q22 The area (in sq. units) of the region bounded by the curve $y=x^2$ and line $y=4$ is

(A) 16/3 (B) 32/3

(C) 32 (D) 24

Q23 If $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}dx = ax+b\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)+c\tan^{-1}\left(\frac{x}{2}\right)+d$

Where d is a constant of integration then $\frac{a+c}{b}$ is equal to

(A) $-\sqrt{3}$ (B) $\sqrt{3}$

(C) $2\sqrt{3}$ (D) -2

Q24 The function $f(x)$

$= \begin{cases} \frac{\sqrt{1+px}-\sqrt{1-px}}{x}, & -1\le x<0, \\ \frac{2x+1}{x-2}, & 0\le x\le 1, \end{cases}$ is

continuous in the interval $[-1,1]$, then p is equal to

(A) 1 (B) -1/2

(C) 1/2 (D) -1

Q25 If $y=\log_e\left(\sin\left(\frac{x^2}{3}-1\right)\right)$ then $\frac{d^2y}{dx}$ is equal to:

(A) $\frac{2}{3}\left(-\cot\left(\frac{x^2}{3}-1\right)+\text{cosec}^2\left(\frac{x^2}{3}-1\right)\right)$

(B) $\frac{2}{3}\cot\left(\frac{x^2}{3}-1\right)-\frac{4x^2}{9}\text{cosec}^2\left(\frac{x^2}{3}-1\right)$

(C) $-\frac{2}{3}\cot\left(\frac{x^2}{3}-1\right)+\frac{4x^2}{9}\text{cosec}^2\left(\frac{x^2}{3}-1\right)$

(D) $\frac{2}{3}\left(\cot\left(\frac{x^2}{3}-1\right)-x\text{cosec}^2\left(\frac{x^2}{3}-1\right)\right)$

Q26 If $z=\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$ then $6z$ equal to:

(A) 1/3 (B) 16

(C) 17 (D) 29

Q27 If $x=\sqrt{t}, y=\sin t$ and $\frac{d^2y}{dx^2}=a\cos x^2+bx^2\sin x^2$ then a^2+b^2 is equal to

(A) 4 (B) 16

(C) 20 (D) 48

Q28 If $x\sqrt{1+y}+y\sqrt{1+x}=0, x\neq y$ then $2\frac{dy}{dx}+y$ at $x=-3$ equal to

(A) -1 (B) 1

(C) 2 (D) -2

Q29 The radius of a sphere is increasing at the rate of 0.5 cm/ minute. The rate of change of the surface area of (in cm^2/ minute) the sphere when the radius is $20cm$ is

(A) 20π (B) 40π

(C) 160π (D) 80π

Q30 The equation of the tangent to the curve $x(\theta)=2\sqrt{2}(\cos\theta+\theta\sin\theta), y(\theta)$, at $\theta=\pi/4$ is $=2\sqrt{2}(\sin\theta-\theta\cos\theta)$

equal to

(A) $x-y=\pi/2$ (B) $x-y=\pi$

(C) $x-y=2$ (D) $x-y=4$

Q31 Match List-I with List-II

List-I	List-II
A. Distance of the point (1,-1,1) from the plane $\vec{r}\cdot\left(\hat{i}-\hat{j}+\hat{k}\right)=2$	(i). $x+y+z=2$
B. $\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=2$	(ii). $-x+y+z=5$
C. Distance of the point (1,-1,1) from the plane $\vec{r}\cdot\left(\hat{i}+\hat{j}-\hat{k}\right)=5$	(iii). $\frac{1}{\sqrt{3}}$
D. $\vec{r}\cdot\left(-\hat{i}+\hat{j}+\hat{k}\right)=5$	(iv). $2\sqrt{3}$

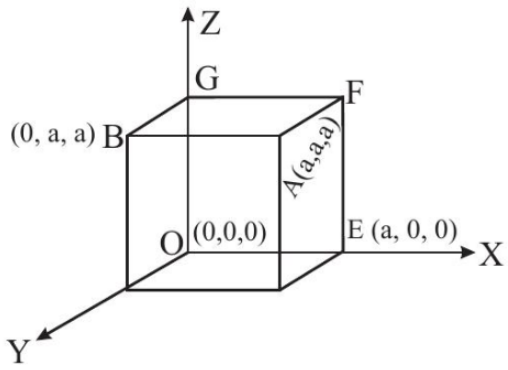
Choose the correct answer from the options given below:

(A) $A-(iv), B-(ii), C-(i), D-(iii)$

(B) $A-(iv), B-(ii), C-(iii), D-(i)$

(C) $A-(iv), B-(i), C-(iii), D-(ii)$
(D) $A-(iii), B-(i), C-(iv), D-(ii)$

Q32



Given figure is cuboid then the acute angle between the diagonal $\overrightarrow{OA}$ and $\overrightarrow{BE}$ is

(A) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (B) $\cos^{-1}\left(\frac{1}{2}\right)$
(C) $\cos^{-1}\left(\frac{2}{3}\right)$ (D) $\cos^{-1}\left(\frac{1}{3}\right)$

Q33 A solider fire a bullet in the air and at time 't' the bullet travels along the path $\mathrm{x}=2\mathrm{t}, \mathrm{y}=\mathrm{t}-2, \mathrm{z}=5-\mathrm{t}$. Then path of the bullet is a

(A) Parabola
(B) Straight line with direction ratios $2, 1, -1$
(C) Straight line with direction ratios $2, -1, 1$
(D) Ellipse centred at (0,2,)

Q34 Which of the following is NOT a corner point of the feasible set $S=\{(x, y): 3x+4y \leq 24, 8x+6y \leq 48, x \leq 5, \mathrm{x}, \mathrm{y} \geq 0\}$ of some LPP?

(A) $\left(\frac{24}{7}, \frac{24}{7}\right)$ (B) $\left(5, \frac{4}{3}\right)$
(C) $\left(\frac{24}{7}, \frac{33}{7}\right)$ (D) (0,6)

Q35 If the sum of two unit vectors is a unit vector, then the magnitude of their difference is

(A) $\sqrt{2}$ (B) 3
(C) $\sqrt{3}$ (D) 2

Q36 $\hat{\mathrm{i}} \cdot\left(\hat{\mathrm{j}} \times \widehat{\mathrm{k}}\right)+\hat{\mathrm{j}} \cdot\left(\widehat{\mathrm{k}} \times \hat{\mathrm{i}}\right)+\widehat{\mathrm{k}} \cdot\left(\hat{\mathrm{i}} \times \hat{\mathrm{j}}\right)$ is equal to

(A) 1 (B) 0
(C) 3 (D) -3

Q37 If $\left|\overrightarrow{a}-\overrightarrow{b}\right|=10,\left|\overrightarrow{a}\right|=8,\left|\overrightarrow{b}\right|=6$, then the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$ is:

(A) 0 (B) $\cos^{-1}\left(\frac{29}{48}\right)$
(C) $\cos^{-1}\left(\frac{5}{24}\right)$ (D) $\pi/2$

Q38 The linear programing problem
Max. $(\mathrm{z})=5\mathrm{x}+7\mathrm{y}$
Subject to
$\mathrm{x}+\mathrm{y} \geq 10$
$3x+5y \leq 15,$
$\mathrm{x} \geq 0, \mathrm{y} \geq 0$

(A) Has a unique optimal solution
(B) Is infeasible
(C) Has multiple optimal solutions
(D) Is feasible and unbounded

Q39 A bag contains 5 red and 7 black balls. Three balls are chosen at random one by one without replacement. What is the probability that the third ball is a Red ball given that first ball is red and second ball is black?

(A) $\frac{2}{5}$
(B) $\frac{3}{5}$
(C) $\frac{2}{7}$
(D) $\frac{5}{7}$

Q40 If two independent events A and B are such that $\mathrm{P}(\mathrm{A})=\frac{1}{2}$ and $\mathrm{P}\left(\left(\mathrm{A}' \cap \mathrm{B}'\right)'\right)=\frac{3}{5}$, then $\mathrm{P}(\mathrm{B})$ is equal to

(A) $\frac{3}{10}$
(B) $\frac{1}{5}$
(C) $\frac{2}{5}$
(D) $\frac{3}{5}$

Directions (41-45) Read the following passage and answer the given questions.

Read the following passage and answer the questions that follow

By using equations of the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z - 3 = 0$, answer the following questions.

Q41 The acute angle between the line and the plane is
(A) $\cos^{-1}\left(\frac{8}{21}\right)$
(B) $\sin^{-1}\left(\frac{8}{21}\right)$
(C) $\sin^{-1}\left(\frac{6}{7}\right)$
(D) $\cos^{-1}\left(\frac{6}{7}\right)$

Q42 Co-ordinate of point of intersection of line and plane
(A) $\left(\frac{33}{10}, \frac{69}{20}, \frac{39}{10}\right)$
(B) $\left(\frac{-33}{10}, \frac{-69}{20}, \frac{-39}{10}\right)$
(C) $\left(0, \frac{3}{2}, 6\right)$
(D) $\left(-2, -\frac{3}{2}, 0\right)$

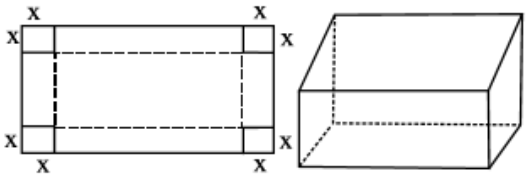
Q43 Distance of plane $10x + 2y - 11z - 3 = 0$, from the origin is:
(A) 1
(B) $\frac{1}{10}$
(C) $\frac{1}{15}$
(D) $\frac{1}{5}$

Q44 The foot of perpendicular from the origin to the given plane is
(A) $\left(\frac{2}{75}, \frac{2}{15}, -\frac{11}{75}\right)$
(B) $\left(\frac{2}{15}, \frac{2}{75}, -\frac{11}{75}\right)$
(C) $\left(\frac{2}{5}, \frac{-2}{25}, \frac{11}{25}\right)$
(D) $\left(\frac{2}{5}, \frac{2}{25}, -\frac{11}{25}\right)$

Q45 The plane $10x + 2y - 11z - 3 = 0$ intersect the Z-axis at the point
(A) $\left(0, 0, \frac{3}{10}\right)$
(B) $\left(0, 0, \frac{-3}{10}\right)$
(C) $\left(0, 0, \frac{3}{11}\right)$
(D) $\left(0, 0, -\frac{3}{11}\right)$

Directions (46-50) Read the following passage and answer the given questions.

Read the following passage and answer the questions that follow



An online retail company ships its product in cartons. Each carton is made by a rectangular sheet of fibre board with dimension of $8\ m$ by $3\ m$. While making the carton, equal squares of side x metres are cut off from each corner of the rectangular sheet of fibre board. After that the resulting flaps are folded to form the carbon (see that figure to identify the side of the carton).

Q46 The volume (V) of the carton is given by $V = f(x)$, where $\mathrm{f(x)}$ is:
(A) $4\mathrm{x}^3 - 24\mathrm{x}^2 + 22\mathrm{x}$
(B) $4\mathrm{x}^3 + 24\mathrm{x}^2 - 22\mathrm{x}$
(C) $-4x^3 + 22x^2 + 24x$
(D) $4x^3 - 22x^2 + 24x$

Q47 The derivative $f'(x)$ of the volume function $f(x)$ is equal to
(A) $24 + 44x - 12x^2$
(B) $24 - 44x + 12x^2$
(C) $-22 + 48x + 12x^2$
(D) $22 - 48x + 12x^2$

Q48 The value of x (in meters) for which the volume V of carton is maximum is
(A) 3
(B) $\frac{2}{3}$
(C) 1
(D) $\frac{1}{2}$

Q49 The length (in meters) of carton box having maximum volume is

(A) 6
(B) $\frac{5}{3}$
(C) $\frac{20}{3}$
(D) $\frac{22}{3}$

Q50 The maximum volume (in m^3) of carton box is
(A) $\frac{200}{9}$
(B) $\frac{154}{9}$
(C) $\frac{154}{27}$
(D) $\frac{200}{27}$

Answer Key

Q1	(D)
Q2	(B)
Q3	(C)
Q4	(B)
Q5	(C)
Q6	(B)
Q7	(C)
Q8	(A)
Q9	(C)
Q10	(B)
Q11	(B)
Q12	(D)
Q13	(C)
Q14	(D)
Q15	(B)
Q16	(C)
Q17	(A)
Q18	(C)
Q19	(C)
Q20	(D)
Q21	(B)
Q22	(B)
Q23	(A)
Q24	(B)
Q25	(B)
Q26	(C)
Q27	(C)
Q28	(D)
Q29	(D)
Q30	(B)
Q31	(D)
Q32	(D)
Q33	(B)
Q34	(C)
Q35	(C)
Q36	(C)
Q37	(D)
Q38	(B)
Q39	(A)
Q40	(B)
Q41	(B)
Q42	(B)
Q43	(D)
Q44	(B)
Q45	(D)
Q46	(D)
Q47	(B)
Q48	(B)
Q49	(C)
Q50	(D)

Hints & Solutions

Q1 Text Solution:

Expanding along C_1, we get

$\Delta = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$

$+ a_{31}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$

or, $\Delta = a_{11}\ A_{11} + a_{21}\ A_{21} + a_{31}\ A_{31}$

Q2 Text Solution:

Let $A = \begin{bmatrix} 0 & -1 & 3x \\ 1 & y & -5 \\ -6 & 5 & 0 \end{bmatrix}$

Since, A is skew symmetric. So, $A = -A'$

i.e., $\begin{bmatrix} 0 & -1 & 3x \\ 1 & y & -5 \\ -6 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 6 \\ 1 & -y & -5 \\ -3x & 5 & 0 \end{bmatrix}$

On comparing, we get

$3x = 6$ and $y = -y \Rightarrow x = 2$ and $y = 0$

Thus, $6x + y = 6 \times 2 + 0 = 12$

Therefore, otpion(B) is the correct option.

Q3 Text Solution:

Given, $\begin{vmatrix} 3 & -4 \\ 2 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 5 \\ 1 & x \end{vmatrix}$

$\Rightarrow 3 - (-8) = 2x^2 - 5$

$\Rightarrow 2x^2 = 16$

$\Rightarrow x^2 = 8$

$\Rightarrow x = 2\sqrt{2}$

Q4 Text Solution:

(1) A square matrix A is said to be non-singular if $|A| \neq 0$. Thus, (1) is incorrect.

(2) A square matrix A is invertible if and only if A is non-singular matrix. Thus, (2) is correct.

(3) If elements of a row are multiplied with cofactors of any other row, then their sum is zero. Thus, (3) is correct.

(4) A is a square matrix is of order 3 , then $|Adj(A)| = |A|^2$

$\left[\because |\text{Adj}(A)| = |A|^{n-1}\right.$, where n is the order of the matrix A]

Thus, statement 4 is incorrect.

Therefore, option(B) is the correct option.

Q5 Text Solution:

Given, $y = x^2 e^{2x}$

$\therefore \ \frac{dy}{dx} = x^2\left(2e^{2x}\right) + e^{2x}\left(2x\right)$

$\Rightarrow \frac{dy}{dx} = 2x^2 e^{2x} + 2xe^{2x}$

Now, y is increasing, if $\frac{dy}{dx} > 0$

i.e., $2x^2 e^{2x} + 2xe^{2x} > 0$

$\Rightarrow 2\,xe^{2x}(x+1) > 0$

$\Rightarrow x\,(x+1) > 0 \qquad (\because e^{2x} > 0 \forall x)$

$\Rightarrow x \in (-\infty, -1) \cup (0, \infty)$

$\therefore x \in (-\infty, -1) \cup (0, \infty)$

Therefore, option(C) is the correct option.

Q6 Text Solution:

Given, $x = t^3, y = t^4$

$\therefore \ \frac{dx}{dt} = 3t^2, \frac{dy}{dt} = 4t^3$

Thus, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4}{3}t$

Differentiating again, we get

$\frac{d^2y}{dx^2} = \frac{4}{3} \times 1 \times \frac{dt}{dx}$

$= \frac{4}{3} \times \frac{1}{3t^2}$

Now, $\left.\frac{d^2y}{dx^2}\right|_{t=2} = \frac{4}{9} \times \frac{1}{(2)^2} = \frac{1}{9}$

Therefore, option(B) is the correct option.

Q7 Text Solution:

(c) (A) Let $I = \int \frac{dx}{\sqrt{9x^2 - 16}} = \frac{1}{3}\int \frac{dx}{\sqrt{\left(x\right)^2 - \left(\frac{4}{3}\right)^2}}$

$= \frac{1}{3}\left[\log\left|x + \sqrt{x^2 - \left(\frac{4}{3}\right)^2}\right|\right] + C'$

$\left[\because \int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + C\right]$

$= \frac{1}{3}\log\left|3x + \sqrt{9x^2 - 16}\right| + C$

(B) Let $\int \frac{dx}{\sqrt{16 - 9x^2}} = \frac{1}{3}\int \frac{dx}{\sqrt{\left(\frac{4}{3}\right)^2 - \left(x\right)^2}}$

$= \frac{1}{3}\sin^{-1}\frac{x}{\frac{4}{3}} + C = \frac{1}{3}\sin^{-1}\left(\frac{3x}{4}\right) + C$

(C) Let $I = \int \frac{x^2+1}{x^2-1}dx$

$= \int \left(1 + \frac{2}{x^2-1}\right) dx$

$= \int 1dx + 2\int \frac{1}{x^2-1}dx$

$= x + \log\left|\frac{x-1}{x+1}\right| + C$

(D) Let $I = \int \frac{e^x}{1+e^x}dx$

Substitute $e^x + 1 = u \Rightarrow e^x dx = du$

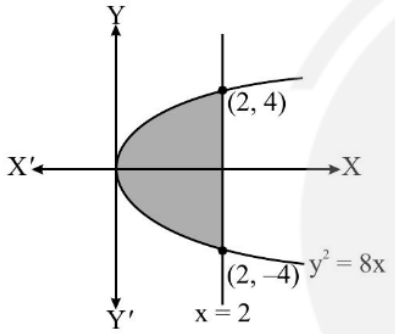
$\therefore \quad I = \int \frac{du}{u} = \log|u| + C = \log|e^x + 1| + C$

Hence, the correct option is (C).

Q8 Text Solution:

We have, $x = 2$...(i)

and $y^2 = 8x$....(ii)



Solving equations (i) & (ii), we get intersection points (2,-4) and (2,4)

Required area = area of the shaded region

$= 2 \times \int_0^2 ydx$

$= 2 \times \int_0^2 \sqrt{8x}dx = 2\sqrt{8} \times \int_0^2 \left(x\right)^{1/2} dx$

$= 2\sqrt{8}\left[\frac{x^{3/2}}{3/2}\right]_0^2 = 2\sqrt{8} \times \frac{2}{3}\left[(2)^{3/2} - 0\right]$

$= \frac{32}{3}$ sq. units

Q9 Text Solution:

Let $I = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2\pi-x}}dx$.....(i)

or, $I = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{\sqrt{2\pi-x}}{\sqrt{2\pi-x}+\sqrt{x}}dx$.....(ii)

$\left[\text{Using } \int_a^b f\left(x\right)dx = \int_a^b f\left(a+b-x\right)dx\right]$

On adding eqs. (i) and (ii), we get

$2I = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 1 \cdot dx = \left[x\right]_{\pi/2}^{3\pi/2} = \frac{3\pi}{2} - \frac{\pi}{2}$

$\therefore I = \frac{\pi}{2}$

Therefore, option(C) is the correct option.

Q10 Text Solution:

We have, $ydx - xdy = 0 \Rightarrow \frac{dx}{x} = \frac{dy}{y}$

$\Rightarrow \int \frac{dx}{x} = \int \frac{dy}{y} \Rightarrow \log x = \log y + \log c$

$\Rightarrow x = yc$, where c is a constant

Since, it passes through (2,4)

So, $2 = 4.c \Rightarrow c = 1/2$

[from eq. (i)]

$\therefore x = \frac{y}{2}$ or $y = 2x$ (from eqn. (i))

Thus, the point given in option (B) only satisfies the eq. $y = 2x$

Q11 Text Solution:

We have,

$\log_e\left(\frac{dy}{dx}\right) = 3x + 4y \Rightarrow \frac{dy}{dx} = e^{3x+4y}$

$\Rightarrow \frac{dy}{dx} = e^{3x} \cdot e^{4y} \Rightarrow e^{-4y}dy = e^{3x}dx$

$\Rightarrow \int e^{-4y}dy = \int e^{3x}dx \Rightarrow -\frac{e^{-4y}}{4} = \frac{e^{3x}}{3} + C$

.....(i)

Since, it passes through $(0, 0)$, we get

$\frac{e^{-0}}{-4} = \frac{e^0}{3} + C \Rightarrow C = -\frac{1}{4} - \frac{1}{3} = -\frac{7}{12}$

$\therefore \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$ (from (i))

$\Rightarrow -3e^{-4y} = 4e^{3x} - 7$

$\Rightarrow 4e^{3x} + 3e^{-4y} - 7 = 0$

Q12 Text Solution:

Since, $\sum P(X) = 1$

$\therefore k + 2k + 3k + 3k + k = 1$

$\Rightarrow 10k = 1 \Rightarrow k = 1/10$

Now, $P(X > 2) = 3k + 3k + k = 7k = 7/10$

Q13 Text Solution:

Given differential equation is,

$\frac{dy}{dx} = \frac{2(x+1)y}{x^2+2x+2}, y > 0$

$\Rightarrow \frac{dy}{y} = \frac{2(x+1)}{x^2+2x+2}dx$

$\Rightarrow \int \frac{dy}{y} = \int \frac{2(x+1)}{x^2+2x+2}dx$

$\Rightarrow \log y = \log\left(x^2 + 2x + 2\right) + \log C$

$\Rightarrow y = C\left(x^2 + 2x + 2\right)$ .....(i)

Since, it passes through (0,4), we get

$4 = \mathrm{C}(2) \Rightarrow \mathrm{C} = 2$

$\Rightarrow \mathrm{y} = 2\left(\mathrm{x}^2 + 2\mathrm{x} + 2\right)$ (using (i))

which is the required equation of the curve

Since, eq. (ii) passes through $\mathrm{P}(-1, \mathrm{k})$ and $\mathrm{Q}(\lambda, 10)$, we get

$\mathrm{k} = 2$ and $10 = 2\left(\lambda^2 + 2\lambda + 2\right)$

$\Rightarrow \mathrm{k} = 2$ and $\lambda^2 + 2\lambda + 2 = 5$

$\Rightarrow \mathrm{k} = 2$ and $\lambda^2 + 2\lambda - 3 = 0$

$\Rightarrow \mathrm{k} = 2$ and $(\lambda + 3)(\lambda - 1) = 0$

$\Rightarrow \mathrm{k} = 2$ and $\lambda = -3, 1$

Thus, $\mathrm{P}(-1, 2)$ and $\mathrm{Q}(1, 10)$ or $\mathrm{Q}(-3, 10)$

When $\mathrm{P}(-1, 2)$ and $\mathrm{Q}(1, 10)$, we have

$(\mathrm{PQ})^2 = (1+1)^2 + (10-2)^2 = 68$

When $\mathrm{P}(-1, 2)$ and $\mathrm{Q}(-3, 10)$, we have

$(\mathrm{PQ})^2 = (-3+1)^2 + (10-2)^2 = 68$

Q14 Text Solution:

Let $\mathrm{I} = \int \frac{\mathrm{dx}}{\sqrt{\mathrm{x}} - \sqrt{\mathrm{x}-1}}$

$= \int \frac{\sqrt{x}+\sqrt{x-1}}{(\sqrt{x})^2 - (\sqrt{x-1})^2} dx$

$= \int \frac{\sqrt{x}+\sqrt{x-1}}{x-x+1} dx$

$= \int (\sqrt{x} + \sqrt{x-1}) dx$

$= \frac{x^{3/2}}{3/2} + \frac{(x-1)^{3/2}}{3/2} + C$

On comparing with the given equation, we get

$\lambda = 2/3$

Therefore, option(D) is the correct option.

Q15 Text Solution:

We have,

$Max.\, Z = ax + by$

Since, Z is maximum at both points $(30, 0)$ and $(20, 30)$.

So, $Z(30, 0) = Z(20, 30)$

$\Rightarrow 30\mathrm{a} = 20\mathrm{a} + 30\ \mathrm{b}$

$\Rightarrow 10\mathrm{a} = 30\ \mathrm{b}$

$\Rightarrow \mathrm{a} = 3\ \mathrm{b}$

Therefore,option(B) is the correct option.

Q16 Text Solution:

The relation $\mathrm{R}b \Leftrightarrow \mathrm{a} \leq \mathrm{b}$ is not an equivalence relation as

$\mathrm{a} \leq \mathrm{b}$ then $b \not\leq a$

So, if (a, b) $\in \mathrm{R}$ then $(\mathrm{b}, \mathrm{a}) \notin \mathrm{R}$

Thus, the above relation is not symmetric.

Hence, option (C) is not an equivalence relation.

Q17 Text Solution:

We have, $f(x) = \frac{\mathrm{x}}{\mathrm{x}+1}$

$\because f'\left(x\right) = \frac{1}{(x+1)^2} > 0$

$\therefore$ f is one-one.

Let $y \in$ co-dom(f)

$\Rightarrow \mathrm{y} = \frac{\mathrm{x}}{\mathrm{x}+1}$

$xy + y = x$

$x - xy = y$

$x\left(1 - y\right) = y$

$\Rightarrow \mathrm{x} = \frac{\mathrm{y}}{1-\mathrm{y}} \in \mathrm{dom}\left(\mathrm{f}\right)$

So, for every y in codomain of f there exist a x in domain of f

$\therefore$ f is onto

Therefore,option(A) is the correct option.

Q18 Text Solution:

We have,

$X + Y = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$ ...(i)

$X - Y = \begin{bmatrix} 0 & 3 \\ 7 & 9 \end{bmatrix}$ ...(ii)

$2X = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ 7 & 9 \end{bmatrix}$ [from (i) +(ii)]

$\Rightarrow 2\mathrm{X} = \begin{bmatrix} 2 & 4 \\ 10 & 14 \end{bmatrix}$

$\Rightarrow X = \begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix} - \mathrm{Y} = \begin{bmatrix} 0 & 3 \\ 7 & 9 \end{bmatrix}$

[from eqn. (ii)]

$\Rightarrow \mathrm{Y} = \begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & -2 \end{bmatrix}$

Therefore, option(C) is the correct option.

Q19 Text Solution:

We have,

$(A-B)^2 = A^2 - 2AB + B^2$
$\Rightarrow (\mathrm{A}-\mathrm{B})(\mathrm{A}-\mathrm{B}) = \mathrm{A}^2 - 2\mathrm{AB} + \mathrm{B}^2$
$\Rightarrow \mathrm{A}^2 - \mathrm{AB} - \mathrm{BA} + \mathrm{B}^2 = \mathrm{A}^2 - 2\mathrm{AB} + \mathrm{B}^2$
$\Rightarrow \mathrm{AB} = \mathrm{BA}$
$\Rightarrow \mathrm{A}^2\,\mathrm{B} = A(AB) = A(BA) = (AB)A$
$= (BA)A = B(\mathrm{AA}) = \mathrm{BA}^2$
$\Rightarrow \mathrm{A}^2\,(\mathrm{A}^2\,\mathrm{B}) = \mathrm{A}^2\,(\mathrm{BA}^2)$
$\Rightarrow \mathrm{A}^4\,\mathrm{B} = \mathrm{A}^2\,\mathrm{B}\,\mathrm{A}^2 = \mathrm{BA}^2\;\mathrm{A}^2 = \mathrm{BA}^4$
$\Rightarrow (\mathrm{A}^4\,\mathrm{B})\mathrm{B}^2 = (\mathrm{BA}^4)\mathrm{B}^2$
$\Rightarrow \mathrm{A}^4\,\mathrm{B}^3 = \mathrm{B}(\mathrm{A}^2\mathrm{B})^2$
$\Rightarrow \mathrm{A}^4\,\mathrm{B}^3 = \mathrm{B}(\mathrm{BA}^2)^2$
$\Rightarrow \mathrm{A}^4\,\mathrm{B}^3 = \mathrm{B}\,(\mathrm{B}^2\mathrm{A}^4)$
$\Rightarrow \mathrm{A}^4\,\mathrm{B}^3 = \mathrm{B}^3\mathrm{A}^4$
$\Rightarrow \mathrm{A}^4\,\mathrm{B}^3 - \mathrm{B}^3\mathrm{A}^4 = 0$
Hence, Option C is correct.

Q20 Text Solution:

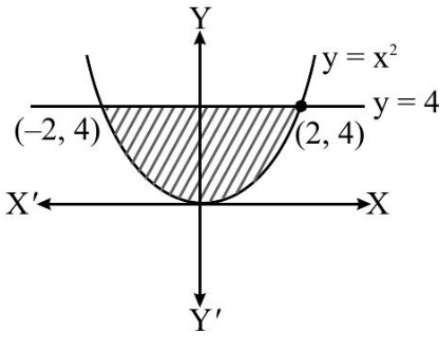
Given $\mathrm{AB} = \mathrm{O}$ (null matrix), then
$\mathrm{A} = \mathrm{O}$ and $\mathrm{B} \neq \mathrm{O}$
or, $\mathrm{A} \neq \mathrm{O}$ and $\mathrm{B} \neq \mathrm{O}$
or, $\mathrm{A} = \mathrm{O}$ and $\mathrm{B} = \mathrm{O}$
or, $\mathrm{A} \neq \mathrm{O}$ and $\mathrm{B} = \mathrm{O}$
Hence, all statements are true.
Therefore, option(D) is the correct option.

Q21 Text Solution:

Let $I = \int \frac{x^2+3x-1}{(x+1)^2}dx$
$I = \int \frac{x^2+2x+x+1-1-1}{(x+1)^2}dx$
$I = \int \frac{(x+1)^2+x-2}{(x+1)^2}dx$
$I = \int \frac{(x+1)^2+x+1-1-2}{(x+1)^2}dx$
$I = \int \frac{(x+1)^2+(x+1)-3}{(x+1)^2}dx$
$= \int \left(1 + \frac{1}{\mathrm{x}+1} - \frac{3}{(\mathrm{x}+1)^2}\right)\mathrm{dx}$
$= \int 1.\,dx + \int \frac{1}{x+1}dx - 3\int \frac{1}{(x+1)^2}dx$
$= x + \log(x+1) + \frac{3}{x+1} + C$

Q22 Text Solution:

Given, equation of curve is $y = x^2$
and equation of line is y=4



On solving equations (i) and (ii), we get points of intersections are (-2,4) and (2,4)
∴ Required area = Area of shaded region
$= 2\int_0^2 (4-x^2)dx = 2\left[\int_0^2 4dx - \int_0^2 x^2dx\right]$
$= 2\left[\left\{4x\right\}_0^2 - \left\{\frac{x^3}{3}\right\}_0^2\right] = 2\left(8 - \frac{8}{3}\right)$
$= 2\left(\frac{16}{3}\right) = \frac{32}{3}$sq. units

Q23 Text Solution:

Let $\int \frac{(\mathrm{x}^2+1)(\mathrm{x}^2+2)}{(\mathrm{x}^2+3)(\mathrm{x}^2+4)}\mathrm{dx}$
Put $x^2 = t$, we have $\frac{(t+1)(t+2)}{(t+3)(t+4)} = \frac{t^2+3t+2}{t^2+7t+12}$
$= 1 + \frac{-4\mathrm{t}-10}{\mathrm{t}^2+7\mathrm{t}+12} = 1 - \frac{4\mathrm{t}+10}{(\mathrm{t}+3)(\mathrm{t}+4)}$
Now, applying partial fraction method, we get
$\frac{4t+10}{(t+3)(t+4)} = \frac{-2}{(t+3)} + \frac{6}{(t+4)}$
$= \frac{-2}{x^2+3} + \frac{6}{x^2+4}\left[\because t = x^2\right]$
Now, $I = \int \left[1 - \left(\frac{-2}{(x^2+3)} + \frac{6}{(x^2+4)}\right)\right]dx$
$= \int 1.\,dx + 2\int \frac{1}{x^2+3}dx - 6\int \frac{1}{x^2+4}dx$
$= \int 1.\,dx + 2\int \frac{1}{x^2+(\sqrt{3})^2}dx - 6\int \frac{1}{x^2+2^2}dx$
$= \mathrm{x} + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\mathrm{x}}{\sqrt{3}}\right) - 6 \times \frac{1}{2}\tan^{-1}\frac{\mathrm{x}}{2}$
$+ \mathrm{C}'$
$= \mathrm{x} + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\mathrm{x}}{\sqrt{3}}\right) - 3\tan^{-1}\left(\frac{\mathrm{x}}{2}\right) + \mathrm{C}'$
Now, comparing with the given equation, we get
$\mathrm{a} = 1,\ \mathrm{b} = \frac{2}{\sqrt{3}}$ and $\mathrm{c} = -3$
Thus, $\frac{\mathrm{a}+\mathrm{c}}{\mathrm{b}} = \frac{1-3}{\frac{2}{\sqrt{3}}} = -\sqrt{3}$

Q24 Text Solution:

$. L. H. L = \lim_{x\to 0^-} \frac{\sqrt{1+px}-\sqrt{1-px}}{x}$

$= \lim_{x\to 0^-} \frac{\sqrt{1+px}-\sqrt{1-px}}{x} \times \frac{\sqrt{1+px}+\sqrt{1-px}}{\sqrt{1+px}+\sqrt{1-px}}$ (on rationalizing)

$= \lim_{x\to 0^-} \frac{2\,px}{x(\sqrt{1+px}+\sqrt{1-px})} = p$

and R.H.L.

$= \lim_{x\to 0^+} f\left(x\right) = \lim_{x\to 0^+} \frac{2x+1}{x-2} = -\frac{1}{2}$

$= -\frac{1}{2}$

Since given f(x) is continuous in [-1,1]

So, L.H.L. = R.H.L.

$\Rightarrow p = -\frac{1}{2}$

Q25 Text Solution:

Given, $y = \log_e\left(\sin\left(\frac{x^2}{3}-1\right)\right)$

$\therefore \frac{dy}{dx} = \frac{1}{\sin\left(\frac{x^2}{3}-1\right)}\cos\left(\frac{x^2}{3}-1\right)\frac{2x}{3}$

$= \frac{2x}{3}\cot\left(\frac{x^2}{3}-1\right)$

Differentiating again, we get

$\frac{d^2y}{dx} = \frac{2}{3}\cot\left(\frac{x^2}{3}-1\right) - \frac{2x}{3}\,\mathrm{cosec}^2\left(\frac{x^2}{3}-1\right) . \frac{2x}{3}$

$= \frac{2}{3}\cot\left(\frac{x^2}{3}-1\right) - \left(\frac{2x}{3}\right)^2\mathrm{cosec}^2\left(\frac{x^2}{3}-1\right)$

$= \frac{2}{3}\cot\left(\frac{x^2}{3}-1\right) - \frac{4x^2}{9}\,\mathrm{cosec}^2\left(\frac{x^2}{3}-1\right)$

Q26 Text Solution:

We have,

$z = \tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$

$= \frac{\tan\left(\sin^{-1}\frac{3}{5}\right)+\tan\left(\cot^{-1}\frac{3}{2}\right)}{1-\tan\left(\sin^{-1}\frac{3}{5}\right)\tan\left(\cot^{-1}\frac{3}{2}\right)}$

$\left[\because \tan\left(A+B\right) = \frac{\tan A+\tan B}{1-\tan A\cdot\tan B}\right]$

$\sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$

$cot^{-1}\left(\frac{3}{2}\right) = \tan^{-1}\left(\frac{2}{3}\right)$

$= \frac{\tan\left(\tan^{-1}\frac{3}{4}\right)+\tan\left(\tan^{-1}\frac{2}{3}\right)}{1-\tan\left(\tan^{-1}\frac{3}{4}\right)\tan\left(\tan^{-1}\frac{2}{3}\right)}$

$= \frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}} = \frac{\frac{17}{12}}{1-\frac{6}{12}} = \frac{\frac{17}{12}}{\frac{6}{12}} = \frac{17}{6}$

$\Longrightarrow 6z = 6\times\frac{17}{6} = 17$

Q27 Text Solution:

Given, $x = \sqrt{t}, y = \sin t$

$\Rightarrow y = \sin\left(x^2\right)$

$\therefore \frac{dy}{dx} = \cos\left(x^2\right)\cdot 2x$

and

$\frac{d^2y}{dx^2} = \cos\left(x^2\right)\cdot 2 + \left(-\sin\left(x^2\right)\right)\cdot 2x\cdot 2x$

$\Rightarrow \frac{d^2y}{dx^2} = 2\cos\left(x^2\right) - 4x^2\sin\left(x^2\right)$

On comparing with the given equation, we get

$a = 2$ and $b = -4$

$\therefore a^2 + b^2 = 4 + 16 = 20$

Therefore,option(C) is the correct option.

\text{Given:} \\

x = \sqrt{t} \implies t = x^2 \\

y = \sin(t) = \sin(x^2) \\

\text{First Derivative:} \\

\frac{dy}{dx} = \frac{d}{dx}[\sin(x^2)] \\

\frac{dy}{dx} = \cos(x^2) \cdot 2x = 2x\cos(x^2) \\

\text{Second Derivative (using Product Rule):} \\

\frac{d^2y}{dx^2} = \frac{d}{dx}[2x] \cdot \cos(x^2) + 2x \cdot \frac{d}{dx}[\cos(x^2)] \\

\frac{d^2y}{dx^2} = 2\cos(x^2) + 2x[-\sin(x^2) \cdot 2x] \\

\frac{d^2y}{dx^2} = 2\cos(x^2) - 4x^2\sin(x^2) \\

\text{Comparing with } \frac{d^2y}{dx^2} = a\cos(x^2) + bx^2\sin(x^2): \\

a = 2, \quad b = -4 \\

\text{Final Calculation:} \\

a^2 + b^2 = (2)^2 + (-4)^2 = 4 + 16 = 20

Q28 Text Solution:

We have, $x\sqrt{1+y} + y\sqrt{1+x} = 0, x \neq y$

$\Rightarrow x^2\left(1+y\right) = y^2\left(1+x\right)$

$\Rightarrow x^2 + x^2y = y^2 + xy^2$

$\Rightarrow x^2 - y^2 + xy\left(x-y\right) = 0$

$\Rightarrow (x-y)(x+y) + xy(x-y) = 0$

$\Rightarrow (x-y)(x+y+xy) = 0$

$\Rightarrow x - y = 0$ or $x + y + xy = 0$

$\because x \neq y$ therefore $x + y + xy = 0$....(i)

$1 + \frac{dy}{dx} + x\frac{dy}{dx} + y = 0$

$\Rightarrow \frac{dy}{dx}\left(1+x\right)=-\left(1+y\right) \Rightarrow \frac{dy}{dx}=$....(ii)

$-\frac{(1+y)}{(1+x)}$

Now at $x=-3$, from eq. (i) we have

$-3+\mathrm{y}-3\mathrm{y}=0 \Rightarrow -2\mathrm{y}=3 \Rightarrow \mathrm{y}=-3/2$

From eq. (ii), we get

$\frac{\mathrm{dy}}{\mathrm{dx}}=-\frac{\left(1-\frac{3}{2}\right)}{1-3}=-\frac{-\frac{1}{2}}{-2}=-\frac{1}{4}$

$\therefore 2\frac{\mathrm{dy}}{\mathrm{dx}}+\mathrm{y}=2\left(-\frac{1}{4}\right)+\left(-\frac{3}{2}\right)=-2$

Q29 Text Solution:

We have, surface area of sphere, $S=4\pi r^2$

$\Rightarrow \frac{\mathrm{dS}}{\mathrm{dt}}=8\pi\mathrm{r}\frac{\mathrm{dr}}{\mathrm{dt}}=8\pi\times 20\times 0.5$

$\left[\because \mathrm{r}=20\ \mathrm{cm} \text{ and } \frac{\mathrm{dr}}{\mathrm{dt}}=0.5\ \mathrm{cm/min}\right]$

$=80\pi\,\mathrm{cm}^2/\mathrm{min}$

Therefore,option(D) is the correct option.

Q30 Text Solution:

Given equations of curves are:

$x(\theta)=2\sqrt{2}(\cos\theta+\theta\sin\theta)$....(i)

$y(\theta)=2\sqrt{2}(\sin\theta-\theta\cos\theta)$.....(ii)

Differentiating eqs. (i) and (ii) w.r.t. 'θ'

$\frac{\mathrm{dx}}{\mathrm{d}\theta}=2\sqrt{2}(-\sin\theta+\sin\theta+\theta\cos\theta)$

$\Rightarrow \frac{\mathrm{dx}}{\mathrm{d}\theta}=2\sqrt{2}\theta\cos\theta$....(iii)

and

$\frac{\mathrm{dy}}{\mathrm{d}\theta}=2\sqrt{2}\left(\cos\theta-\cos\theta+\theta\sin\theta\right)$

$\Rightarrow \frac{\mathrm{dy}}{\mathrm{d}\theta}=2\sqrt{2}\theta\sin\theta$.....(iv)

$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}}=\frac{2\sqrt{2}\theta\sin\theta}{2\sqrt{2}\theta\cos\theta}=\tan\theta$ (From eqs. (iii) and (iv))

Now, $\frac{\mathrm{dy}}{\mathrm{dx}}\Big|_{\theta=\frac{\pi}{4}}=\tan\frac{\pi}{4}=1$

Also, $x\left(\frac{\pi}{4}\right)=2\sqrt{2}\left[\cos\left(\frac{\pi}{4}\right)+\frac{\pi}{4}\sin\left(\frac{\pi}{4}\right)\right]$

$=2\sqrt{2}\left(\frac{1}{\sqrt{2}}+\frac{\pi}{4}\cdot\frac{1}{\sqrt{2}}\right)=2\left(1+\frac{\pi}{4}\right)$

and $\mathrm{y}\left(\frac{\pi}{4}\right)=2\sqrt{2}\left[\sin\left(\frac{\pi}{4}\right)-\frac{\pi}{4}\cos\left(\frac{\pi}{4}\right)\right]$

$=2\sqrt{2}\left(\frac{1}{\sqrt{2}}-\frac{\pi}{4}\cdot\frac{1}{\sqrt{2}}\right)=2\left(1-\frac{\pi}{4}\right)$

Now, the required equation of tangent is

$\mathrm{y}-\mathrm{y}\left(\frac{\pi}{4}\right)=\left(\frac{\mathrm{dy}}{\mathrm{dx}}\Big|_{\theta=\frac{\pi}{4}}\right)\left[\mathrm{x}-\mathrm{x}\left(\frac{\pi}{4}\right)\right]$

$\Rightarrow \mathrm{y}-2\left(1-\frac{\pi}{4}\right)=1\left[\mathrm{x}-2\left(1+\frac{\pi}{4}\right)\right]$

$\Rightarrow \mathrm{y}-\mathrm{x}=-\frac{\pi}{2}-\frac{\pi}{2} \Rightarrow \mathrm{x}-\mathrm{y}=\pi$

Q31 Text Solution:

(d) A.

$\vec{\mathrm{r}}\cdot\left(\hat{\mathrm{i}}-\hat{\mathrm{j}}+\hat{\mathrm{k}}\right)=2 \Rightarrow \left(x\hat{\mathrm{i}}+\mathrm{y}\hat{\mathrm{j}}+\mathrm{zk}\right)$

$\cdot\left(\hat{\mathrm{i}}-\hat{\mathrm{j}}+\hat{\mathrm{k}}\right)=2$

$\Rightarrow \mathrm{x}-\mathrm{y}+\mathrm{z}-2=0$

$\therefore$ Distance, $\mathrm{d}=\left|\frac{1+1+1-2}{\sqrt{3}}\right|=\frac{1}{\sqrt{3}}$ units

B. $\vec{\mathrm{r}}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=2$

$\Rightarrow \left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(\hat{i}+\hat{j}+\hat{k}\right)=2$

$\Rightarrow \mathrm{x}+\mathrm{y}+\mathrm{z}=2$

C.

$\because \vec{r}\cdot\left(\hat{i}+\hat{j}-\hat{k}\right)=5 \Rightarrow \left(x\hat{i}+y\hat{j}+z\hat{k}\right)$

$\cdot\left(\hat{i}+\hat{j}-\hat{k}\right)=5$

$\Rightarrow \mathrm{x}+\mathrm{y}-\mathrm{z}-5=0$

Distance $\mathrm{d}=\left|\frac{1-1-1-5}{\sqrt{3}}\right|=\left|\frac{-6}{\sqrt{3}}\right|=2\sqrt{3}$ units

D. $\vec{\mathrm{r}}\cdot\left(-\hat{\mathrm{i}}+\hat{\mathrm{j}}+\hat{\mathrm{k}}\right)=5$

$\Rightarrow \left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(-\hat{i}+\hat{j}+\hat{k}\right)=5$

$\Rightarrow -\mathrm{x}+\mathrm{y}+\mathrm{z}=5$

Q32 Text Solution:

(d) Here,

$\overrightarrow{\mathrm{OA}}=\left(\mathrm{a}-0\right)\hat{\mathrm{i}}+\left(\mathrm{a}-0\right)\hat{\mathrm{j}}+\left(\mathrm{a}-0\right)\hat{\mathrm{k}}$

$\Rightarrow \overrightarrow{OA}=a\hat{i}+a\hat{j}+a\hat{k}=a\left(\hat{i}+\hat{j}+\hat{k}\right)$

and

$\overrightarrow{\mathrm{BE}}=\left(a-0\right)\hat{i}+\left(0-a\right)\hat{j}+\left(0-a\right)\hat{k}$

$\Rightarrow \overrightarrow{BE}=a\hat{i}-a\hat{j}-a\hat{k}=a\left(\hat{i}-\hat{j}-\hat{k}\right)$

Let θ be the angle between $\overrightarrow{\mathrm{OA}}$ and $\overrightarrow{\mathrm{BE}}$, then

$\cos\theta=\left|\frac{a^2\left[\left(\hat{i}+\hat{j}+\hat{k}\right)\cdot\left(\hat{i}-\hat{j}-\hat{k}\right)\right]}{\sqrt{3}a\cdot\sqrt{3}a}\right|=\left|\frac{1-1-1}{3}\right|$

$=\frac{1}{3}$

$\therefore \theta=\cos^{-1}\left(\frac{1}{3}\right)$

Q33 Text Solution:

The given parametric equations for the path of the bullet are:

$x = 2t$....(i)

$y = t - 2$....(ii)

$z = 5 - t$....(iii)

To find the path, we eliminate the parameter 't' from equations (i), (ii), and (iii):

From (i): $t = \frac{x}{2}$

From (ii): $t = \frac{y+2}{1}$

From (iii): $t = \frac{z-5}{-1}$

By equating the values of 't', we get the symmetric equation of the path:

$\frac{x}{2} = \frac{y+2}{1} = \frac{z-5}{-1}$

This equation is in the standard form of a straight line: $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

Where **(a, b, c)** represent the **Direction Ratios** of the line.

Comparing the equations, the direction ratios are $(2,\ 1,\ -1)$.

Therefore, the path of the bullet is a straight line with direction ratios 2, 1, -1.

Correct Option: B

Q34 Text Solution:

For option (C), $\left(\frac{24}{7}, \frac{33}{7}\right)$

$\Rightarrow 8x + 6y \leq 48$

$\Rightarrow 8 \times \frac{24}{7} + 6 \times \frac{33}{7}$

$= 55.71 > 48$

Hence, it does not satisfies the constraints.

i.e., $\left(\frac{24}{7}, \frac{33}{7}\right)$ does not satisfies the given constraints.

Hence, it cannot be a corner point.

Therefore,option(C) is the correct option.

Q35 Text Solution:

Let two vectors be $\vec{a}$ and $\vec{b}$ s.t.

$\left|\vec{a}\right| = \left|\vec{b}\right| = 1$

and $\left|\vec{a} + \vec{b}\right| = 1$

$\Rightarrow \left|\vec{a} + \vec{b}\right|^2 = 1$

$\Rightarrow \left(\vec{a} + \vec{b}\right) \cdot \left(\vec{a} + \vec{b}\right) = 1$

$\Rightarrow \left|\vec{a}\right|^2 + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} + \left|\vec{b}\right|^2 = 1$

$\Rightarrow 2\vec{a} \cdot \vec{b} = -1 \ \Rightarrow \vec{a} \cdot \vec{b} = -\frac{1}{2}$

Now, $\left|\vec{a} - \vec{b}\right|^2 = \left(\vec{a} - \vec{b}\right) \cdot \left(\vec{a} - \vec{b}\right)$

$= \left|\vec{a}\right|^2 - \vec{a} \cdot \vec{b} - \vec{b} \cdot \vec{a} + \left|\vec{b}\right|^2$

$= 1 - 2\vec{a} \cdot \vec{b} + 1 = 2 - 2\left(-\frac{1}{2}\right) = 3$

$\therefore \left|\vec{a} - \vec{b}\right| = \sqrt{3}$

Q36 Text Solution:

We have,

$\hat{i} \cdot \left(\hat{j} \times \hat{k}\right) + \hat{j} \cdot \left(\hat{k} \times \hat{i}\right) + \hat{k} \cdot \left(\hat{i} \times \hat{j}\right)$

$= \hat{\mathrm{i}} \cdot \left(\hat{\mathrm{i}}\right) + \hat{\mathrm{j}} \cdot \left(\hat{\mathrm{j}}\right) + \hat{\mathrm{k}} \cdot \left(\hat{\mathrm{k}}\right)$

$= 1 + 1 + 1 = 3$

Q37 Text Solution:

We have, $\left|\vec{\mathrm{a}} - \vec{\mathrm{b}}\right| = 10$

$\Rightarrow \left|\vec{a} - \vec{b}\right|^2 = 100$

$\Rightarrow \left(\vec{a} - \vec{b}\right) \cdot \left(\vec{a} - \vec{b}\right) = 100$

$\Rightarrow \left|\vec{a}\right|^2 - \vec{a} \cdot \vec{b} - \vec{b} \cdot \vec{a} + \left|\vec{b}\right|^2 = 100$

$\Rightarrow \left(8\right)^2 - 2\vec{a} \cdot \vec{b} + \left(6\right)^2 = 100$

$\left[\because \left|\vec{a}\right| = 8 \text{ and } \left|\vec{b}\right| = 6\right]$

$\Rightarrow 64 - 2a \cdot b + 36 = 100$

$\Rightarrow 2\vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a} \perp \vec{b}$

Thus, angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{2}$.

Q38 Text Solution:

The given constraints for the Linear Programming Problem are:

1. $x + y \geq 10$

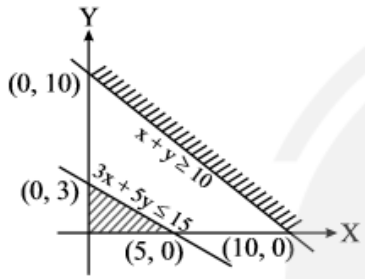
2. $3x + 5y \leq 15$
3. $x \geq 0,\ y \geq 0$

For the first constraint $x + y \geq 10$:

- If $x = 0$, then null (Point A: 0, 10)
- If $y = 0$, then $x = 10$ (Point B: 10, 0)
- This line represents the region away from the origin (0,0).

For the second constraint $3x + 5y \leq 15$:

- If $x = 0$, then $y = 3$ (Point C: 0, 3)
- If $y = 0$, then $x = 5$ (Point D: 5, 0)
- This line represents the region towards the origin (0,0).



The first region is located far from the origin (intercepts at 10), while the second region is constrained very close to the origin (intercepts at 5 and 3). There is **no** intersection or overlapping area between these two sets of inequalities.
Since there is no Feasible Region, the problem is Inconsistent.
Correct Option: B

Q39 **Text Solution:**
First ball drawn is red.
Second ball drawn is black.
So, we have remaining 4 red and 6 black balls.
Probability (Third ball drawn is red) $= \frac{4}{4+6} = \frac{2}{5}$
The initial state of the bag is as follows:
Number of Red balls = 5
Number of Black balls = 7
Total number of balls = 12
According to the given condition, three balls are drawn without replacement. The first ball is red and the second ball is black:
After drawing the first Red ball, the remaining balls are: 4 Red, 7 Black (Total = 11)
After drawing the second Black ball, the remaining balls are: 4 Red, 6 Black (Total = 10)
Now, to find the probability that the third ball is Red (R_3):

$$P\left(R_3 \middle| R_1 \cap B_2\right) = \frac{\text{Remaining Red balls}}{\text{Total remaining balls}}$$

$\Rightarrow P = \frac{4}{10} = \frac{2}{5}$
Correct Option: A

Q40 **Text Solution:**
Since, $\mathrm{P}\left(\left(\mathrm{A}' \cap \mathrm{B}'\right)'\right) = \mathrm{P}(\mathrm{A} \cup \mathrm{B})$
$\therefore \mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \frac{3}{5}$
We know that
$\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - \mathrm{P}(\mathrm{A} \cap \mathrm{B})$
Since, events are independent, then
$P(A \cap B) = \mathrm{P}(\mathrm{A}).\mathrm{P}(\mathrm{B})$
$\therefore \mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - \mathrm{P}(\mathrm{A})(\mathrm{B})$
$\Rightarrow \frac{3}{5} = \frac{1}{2} + \mathrm{P}(\mathrm{B}) - \frac{1}{2} \times \mathrm{P}(\mathrm{B})$
[given $\mathrm{P}(\mathrm{A}) = \frac{1}{2}$]
$\Rightarrow \frac{3}{5} - \frac{1}{2} = \frac{\mathrm{P}(\mathrm{B})}{2} \Rightarrow \frac{\mathrm{P}(\mathrm{B})}{2} = \frac{1}{10} \Rightarrow \mathrm{P}(\mathrm{B}) = \frac{1}{5}$

Q41. **Text Solution:**
For the line $\frac{x-x_1}{A_1} = \frac{y-y_1}{B_1} = \frac{z-z_1}{C_1}$ and plane $A_2x + \mathrm{B}_2\mathrm{y} + \mathrm{C}_2\mathrm{z} + \mathrm{D}_2 = 0$ acute angle θ is given by $\sin\theta = \left|\frac{\mathrm{A}_1\,\mathrm{A}_2 + \mathrm{B}_1\,\mathrm{B}_2 + \mathrm{C}_1\mathrm{C}_2}{\sqrt{\mathrm{A}_1^2+\mathrm{B}_1^2+\mathrm{C}_1^2}\sqrt{\mathrm{A}_2^2+\mathrm{B}_2^2+\mathrm{C}_2^2}}\right|$
$\therefore$ Require acute angle,
$\sin\theta = \left|\frac{10.2 + 2.3 - 11.6}{\sqrt{10^2+2^2+(-11)^2}\sqrt{2^2+3^2+6^2}}\right|$
$= \left|\frac{-40}{105}\right| = \frac{8}{21}$
$\therefore \theta = \sin^{-1}\left(\frac{8}{21}\right)$

Q42. **Text Solution:**
We have, equation of line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6} = \lambda$
$\therefore$ A point on line $\equiv (2\lambda - 1, 3\lambda, 6\lambda + 3)$
Substituting this point in the equation of plane, we get

$10(2\lambda-1)+2(3\lambda)-11(6\lambda+3)-3=0$
$\Rightarrow 20\lambda-10+6\lambda-66\lambda-33-3=0$
$\Rightarrow -40\lambda=46 \Rightarrow \lambda=-\frac{23}{20}$

$\therefore$ The required point of intersection is
$\left[2\times\left(\frac{-23}{20}\right)-1, 3\times\left(\frac{-23}{20}\right), 6\times\left(\frac{-23}{20}\right)+3\right]$
or, $\left(\frac{-46}{20}-1, -\frac{69}{20}, \frac{-138}{20}+3\right)$
or, $\left(\frac{-33}{10}, \frac{-69}{20}, \frac{-39}{10}\right)$

Q43. Text Solution:

Coordinate of origin $\equiv (0,0,0)$
Required distance $= \left|\frac{10\times0+2\times0-11\times0-3}{\sqrt{(10)^2+(2)^2+(-11)^2}}\right|$

$$= \left|\frac{-3}{\sqrt{225}}\right| = \frac{3}{15} = \frac{1}{5} \text{ unit}$$

Q44. Text Solution:

Let coordinates of foot of perpendicular be (α,β,γ).
So, $\frac{\alpha-0}{10}=\frac{\beta-0}{2}=\frac{\gamma-0}{-11}=\frac{-(-3)}{225}$
$\therefore \alpha=\frac{30}{225}=\frac{2}{15}$
$\beta=\frac{6}{225}=\frac{2}{75}$
and $\gamma=\frac{-33}{225}=\frac{-11}{75}$
Therefore, $(\alpha,\beta,\gamma)\equiv\left(\frac{2}{15},\frac{2}{75},\frac{-11}{75}\right)$

Q45. Text Solution:

$\because$ Coordinates x and y are zero at z-axis.
So, $10\times0+2\times0-11z-3=0 \quad \Rightarrow$
$z=\frac{-3}{11}$
$\therefore$ required point is $\left(0,0,\frac{-3}{11}\right)$

Q46. Text Solution:

Volume, $V(x)$ = length $\times$ breadth $\times$ height
$=(8-2x)\times(3-2x)\times x$
$=x\left(24+4x^2-22x\right)$
$=4x^3-22x^2+24x$

Q47. Text Solution:

We have, $f(x)=4x^3-22x^2+24x$
$\therefore f'(x)=12x^2-44x+24$

Q48. Text Solution:

For maximum value, $f'(x)=0$
$\Rightarrow 12x^2-44x+24=0$
$\Rightarrow 3x^2-11x+6=0$
$\Rightarrow 3x^2-9x-2x+6=0$
$\Rightarrow 3x(x-3)-2(x-3)=0$
$\Rightarrow (x-3)(3x-2)=0$
$\Rightarrow x=3$ or $x=\frac{2}{3}$
Now, $f''(x)=24x-44$
At $x=3, f''(3)=24\times3-44=72-44$
$=28>0$
At $x=\frac{2}{3}, f''$
$\left(\frac{2}{3}\right)=24\times\frac{2}{3}-44=16-44=-28<0$
Hence, $f(x)$ attains maximum value at $x=\frac{2}{3}$
So, volume is maximum when $x=\frac{2}{3}$

Q49. Text Solution:

Length of carton
$=8-2x=8-2\times\frac{2}{3}=8-\frac{4}{3}=\frac{20}{3}$ m

Q50. Text Solution:

We have, Volume, $V(x)=4x^3-22x^2+24x$
At $x=\frac{2}{3}$
$$V\left(\frac{2}{3}\right)=4\left(\frac{2}{3}\right)^3-22\left(\frac{2}{3}\right)^2+24\left(\frac{2}{3}\right)$$
$$=4\times\frac{8}{27}-22\times\frac{4}{9}+\frac{48}{3}$$
$$=\frac{32}{27}-\frac{88}{9}+\frac{48}{3}=\frac{200}{27} \text{ m}^3$$

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