



GOVERNMENT OF KARNATAKA

DEPARTMENT OF SCHOOL EDUCATION (PRE-UNIVERSITY)

18TH CROSS, MALLESHWARAM, BENGALURU – 560 012

CHAPTER-WISE MULTIPLE-CHOICE QUESTIONS FOR COMPETITIVE EXAM

SUBJECT: PHYSICS

NAME OF THE CHAPTER: I PUC – SYSTEM OF PARTICLES AND ROTATIONAL MOTION

SYNOPSIS

1: Vector Product

➤ If $\vec{A}$ and $\vec{B}$ are two vectors making an angle of θ between them, then their vector product or *cross product* is given by

$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$$

where $\hat{n}$ is the unit vector perpendicular to the plane containing both $\vec{A}$ and $\vec{B}$ and is given by **right hand clasp rule**.

This *cross product* is maximum when

$$\theta = 90^\circ \Rightarrow |\vec{A} \times \vec{B}| = AB,$$

minimum when

$$\theta = 0 \text{ or } 180^\circ \Rightarrow \vec{A} \times \vec{B} = \mathbf{0}$$

➤ The *vector product* between two vectors is NOT *commutative* : $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$.

➤ The vector product is *distributive* : $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$.

➤ If $\hat{i}, \hat{j}$ and $\hat{k}$ are the unit vectors along x, y and z axes respectively, then $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \mathbf{0}$

$$\hat{i} \times \hat{j} = \hat{k}; \hat{j} \times \hat{k} = \hat{i} \text{ and } \hat{k} \times \hat{i} = \hat{j}.$$

➤ If A_x, A_y and A_z are the x, y and z components of vector $\vec{A}$ respectively and if B_x, B_y and B_z are the x, y and

z components of vector $\vec{B}$ respectively then

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}.$$

EXAMPLE 1

If $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$ (in SI units) and $\vec{B} = \hat{i} + 2\hat{j} + 5\hat{k}$ (in SI units), then the magnitude of cross product $\vec{A} \times \vec{B}$ is

- (A) $\sqrt{179}$ SI units (B) $\sqrt{33}$ SI units (C) $\sqrt{10}$ SI units (D) $\sqrt{30}$ SI units

Ans (A)

Solution

We take

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 2 & 5 \end{vmatrix}$$

$$= [(1 \times 5) - (2 \times -1)]\hat{i} - [(2 \times 5) - (1 \times -1)]\hat{j} + [(2 \times 2) - (1 \times 1)]\hat{k}$$

Solving, we get

$$\vec{A} \times \vec{B} = 7\hat{i} - 11\hat{j} + 3\hat{k}$$

The magnitude of this is

$$|\vec{A} \times \vec{B}| = \sqrt{7^2 + (-11)^2 + 3^2} = \sqrt{179} \text{ SI units}$$

EXAMPLE 2

In the previous question, the angle between $\vec{A}$ and $\vec{B}$ is nearly

- (A) 10° (B) 25° (C) 60° (D) 85°

Ans (D)

Solution

The magnitudes of vectors $\vec{A}$ and $\vec{B}$ are : $A = \sqrt{2^2 + 1^2 + (-1)^2} = \sqrt{6} \text{ SI units}$

and : $B = \sqrt{1^2 + 2^2 + (5)^2} = \sqrt{30} \text{ SI units}$

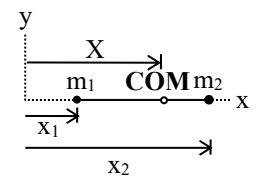
Hence from : $|\vec{A} \times \vec{B}| = AB \sin \theta \Rightarrow \sin \theta = \frac{|\vec{A} \times \vec{B}|}{AB}$

$$\Rightarrow \sin \theta = \frac{\sqrt{179}}{\sqrt{6} \times \sqrt{30}} \Rightarrow \theta \approx 85^\circ$$

2: Center of Mass

➤ If m_1 and m_2 are the masses of two particles lying on the x axis at co ordinates x_1 and x_2 respectively, then their center of mass (COM) co ordinate is given by

$$X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}.$$



➤ If $m_1 = m_2 = m$, then the COM co ordinate is given by $X = \frac{x_1 + x_2}{2}$.

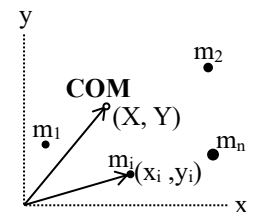
➤ If $m_2 \gg m_1$, then X is very close to x_2 : i.e., $X \approx x_2$.

➤ In general for particles of masses $m_1, m_2, m_3, \dots, m_n$ etc fixed on the x axis at co ordinates $x_1, x_2, \dots, x_n$ then the COM co ordinate is given by

$$X = \frac{m_1 x_1 + m_2 x_2 + \dots + m_n x_n}{m_1 + m_2 + \dots + m_n} = \sum_{i=1}^n \frac{m_i x_i}{m_i}.$$

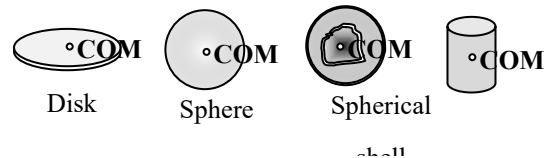
➤ For mass distribution in a plane (say, x – y plane) then the center of mass x and y co ordinates are

$$X = \sum_{i=1}^n \frac{m_i x_i}{m_i} \text{ and } Y = \sum_{i=1}^n \frac{m_i y_i}{m_i}$$



➤ For symmetric mass distributions, the center of mass is at their *geometric centers*.

➤ For example: the COM of a uniform circular disk, uniform sphere, a uniform spherical shell, a uniform cylinder etc is at their respective geometric centers.

**EXAMPLE 3**

Three point objects of masses 1 kg, 2 kg and m are located at $x = 1 \text{ m}$, $x = 2 \text{ m}$ and $x = 5 \text{ m}$ respectively. If the center of mass of the three objects is at $x = 3 \text{ m}$, the value of m is

- (A) 0.1 kg (B) 1 kg (C) 3 kg (D) 10 kg

Ans (B)

Solution

The COM co ordinate is given by : $X = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} \Rightarrow 3 = \frac{(1)(1) + (2)(2) + m(5)}{1 + 2 + m}$

Solving for m , we get : $9 + m = 5 + 5m \Rightarrow m = 1 \text{ kg}$

EXAMPLE 4

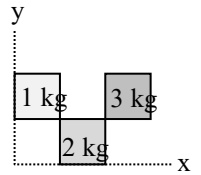
Three uniform squares of sides 2 m each have masses as indicated in the figures. The x and y co ordinates of the center of mass of the three squares are respectively

(A) (2.3 m, 3.7 m)

(B) (3.7 m, 2.3 m)

(C) (2.3 m, 0)

(D) (0, 2.3 m)



Ans (B)

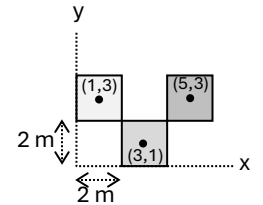
Solution

The x and y co ordinates of the three

squares are given by

$$: (x_1, y_1) = (1 \text{ m}, 3 \text{ m}), (x_2, y_2) = (3 \text{ m}, 1 \text{ m}) \text{ and}$$

$$(x_3, y_3) = (5 \text{ m}, 3 \text{ m}),$$



The COM x co ordinate is given by : $X = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$

$$\Rightarrow 3 = \frac{(1)(1) + (2)(2) + (3)(5)}{1 + 2 + 3} = 3.7 \text{ m}$$

and the COM y co ordinate is given by: $Y = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} \Rightarrow 3 = \frac{(1)(3) + (2)(1) + (3)(3)}{1 + 2 + 3} = 2.3 \text{ m}$

3: Motion of the Center of Mass

➤ If m_1 and m_2 are the masses of the two particles having velocities $\vec{v}_1$ and $\vec{v}_2$ respectively, their center of mass velocity is given by

$$\vec{V} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}.$$

➤ In general we write

$$\vec{V} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots + m_n \vec{v}_n}{m_1 + m_2 + \dots + m_n} = \sum_{i=1}^n \frac{m_i \vec{v}_i}{m_i}.$$

➤ The total momentum of the system of particle is

$$\vec{P} = M\vec{V}$$

where

$M = \sum_{i=1}^n m_i$ is the total mass of all the particles of a system.

➤ If m_1 and m_2 are the masses of the two particles having velocities $\vec{a}_1$ and $\vec{a}_2$ respectively, their center of mass acceleration is given by

$$\vec{A} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2}{m_1 + m_2}.$$

➤ In general we write

$$\vec{A} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots + m_n \vec{a}_n}{m_1 + m_2 + \dots + m_n} = \sum_{i=1}^n \frac{m_i \vec{a}_i}{m_i}.$$

➤ The total external force on the system of particle is

$$\vec{F}_{ext} = M\vec{A}$$

where

$M = \sum_{i=1}^n m_i$ is the total mass of all the particles of a system.

EXAMPLE 5

Two particles having masses m and $2m$ are moving with speeds of 2 m/s and 1 m/s along + x direction and - x direction respectively. At that instant, they are acted upon only by their mutual repulsion force. The center of mass velocity and center of mass acceleration of the two particles are respectively

(A) $0, -\hat{i} + 2\hat{j} \text{ m/s}^2$

(B) $0, 0$

(C) $-\hat{i} \text{ m/s}, 0$

(D) $\hat{i} \frac{m}{s} \text{ and } 2\hat{j} \text{ m/s}^2$

Ans (B)

Solution

The velocities of m kg and $2m$ kg are : $\vec{v}_1 = 2\hat{i}$ and $\vec{v}_2 = -\hat{i}$ in meter per second.

Hence the COM velocity is : $\vec{V} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \frac{m(2\hat{i}) + 2m(-\hat{i})}{m + 2m} = 0$

Since : the force between them is mutual repulsion,

there exists only : internal force.

the : external force is zero $\Rightarrow \vec{F}_{ext} = 0$

From : $\vec{F}_{ext} = M\vec{A} \Rightarrow \vec{A} = \frac{\vec{F}_{ext}}{m + 2m} = \frac{0}{3m} = 0$

EXAMPLE 6

Two particles of masses M and $1.5 M$ are moving along $+x$ axis at speeds u and $2u$ respectively. A third particle is moving with a velocity v_3 along the x axis. The total momentum of the system is $3Mu$, then the velocity v_3 is

(A) $0.5u$ along the $+x$ axis

(B) u along $-x$ axis

(C) $0.5u$ along $-x$ axis

(D) u along $+x$ axis

Ans (C)

Solution

From the expression : $\vec{P} = M\vec{V} \Rightarrow P_x = MV_x \Rightarrow P_x = M \times \frac{m_1v_{1x} + m_2v_{2x} + m_3v_{3x}}{M}$

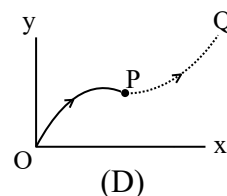
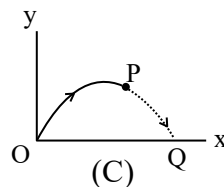
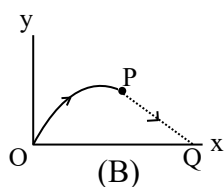
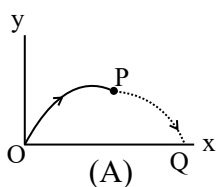
Or : $P_x = m_1v_{1x} + m_2v_{2x} + m_3v_{3x} \Rightarrow 3Mu = Mu + 1.5M \times 2u + 2Mv_3$

Solving for v_3 : $v_3 = -0.5u$

Hence : $v_3 = 0.5$ m/s along $-x$ axis.

EXAMPLE 7

A grenade is fired from the ground at certain initial velocity and at a certain initial angle with the horizontal. It explodes at point P into several fragments. The path of the center of mass after the explosion is correctly shown in which of the following four figures?



Ans (C)

Solution

Before the explosion (OP), the only force acting on the COM of the grenade : is gravitational force.

After the explosion (PQ), the only force acting on the COM : is also gravitational force.

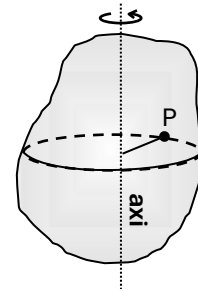
If before the explosion, the COM moved along the : path OP.

Hence after the explosion also, the COM must move along : path PQ as in choice (3)

4: Rigid Body and Rigid Body Kinematics

➤ A body whose two particles will maintain fixed separation when forces are applied on it is called a *rigid body*. Eg: a fan's blade, a flywheel, a CD disk, a roller cylinder, shot put sphere etc.

➤ When a *rigid body* rotates, its particles along a specific line always remain stationary. This line is called axis of rotation.



5: Angular Position, Angular displacement, Average Angular velocity and Angular Velocity

➤ The angle between the line joining the particle of the rigid body to the axis and the reference line OX is called the *angular position* (θ).

➤ The change of angular position is called *angular displacement*.

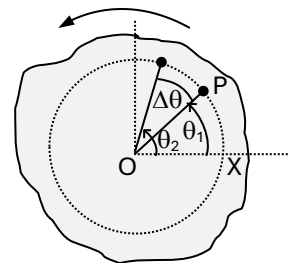
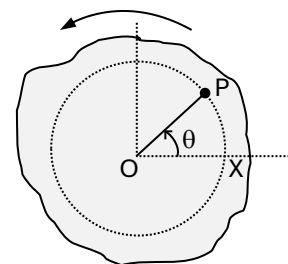
Therefore $\Delta\theta = \theta_2 - \theta_1$ (measured in *radians*)

➤ The ratio of angular displacement to the time taken for that displacement is called *average angular velocity*.

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$$

➤ The rate of change of angular position is called *angular velocity*.

$$\omega = \frac{d\theta}{dt}$$



➤ The direction of the angular velocity is given by the right hand clasp rule as shown:

➤ The relation between the distance travelled and angular displacement is $s = r\theta$.

➤ The speed of the particle in rotational motion and angular speed are related by $v = r\omega$.

➤ In terms of vectors, we can write $\vec{v} = \vec{\omega} \times \vec{r}$.

EXAMPLE 8

The angle made by the line joining the axis to a particle of the rotating fan varies with time according to $\theta = 2t^3$ where θ is in radian and t is in second. The average angular velocity between $t = 0$ to $t = 2$ s and the angular velocity of the particle at $t = 2$ s are respectively (expressed in SI units)

- (A) 8, 8 (B) 32, 8 (C) 8, 32 (D) 32, 32

Ans (C)

Solution

The angular positions at $t = 0$ and $t = 2$ s are : $\theta_1 = 2(0)^3 = 0$ and $\theta_2 = 2(2)^3 = 16$ rad.

Hence the average angular velocity is $\bar{\omega} = \frac{\Delta\theta}{\Delta t} = \frac{16-0}{2-0} = 8 \text{ rad/s}$.

The instantaneous angular velocity : $\omega = \frac{d\theta}{dt} \Rightarrow \omega = \frac{d(2t^3)}{dt} = 6t^2 \Rightarrow \omega(2) = 6(2)^2 = 24 \text{ rad/s}$

6: Average Angular Acceleration and Angular Acceleration

➤ If the angular velocity change is $\Delta\omega$ occurring in time interval Δt , the *average angular acceleration* is given by $\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$.

➤ The *angular acceleration* is given by the rate of change of angular velocity: $\alpha = \frac{d\omega}{dt}$.

➤ The angular acceleration and the tangential acceleration are related by $a = r\alpha$.

EXAMPLE 9

The angular velocity of the blade of a ceiling fan changes with time according to $\omega = 4t^2$. Where ω is in rad/s and t is in second. The average acceleration in the first 2 s and angular acceleration at $t = 1$ s are respectively (in SI units)

- (A) 8, 8 (B) 32, 8 (C) 8, 32 (D) 32, 32

Ans (A)

Solution

The angular velocities at $t = 0$ and $t = 2$ s are : $\omega_1 = 4(0)^2 = 0$ and $\omega_2 = 4(2)^2 = 16$ rad/s.

Hence the average angular acceleration is
$$: \bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{16-0}{2-0} = 8 \text{ rad/s}^2$$

and the angular acceleration is
$$: \alpha = \frac{d\omega}{dt} = \frac{d(4t^2)}{dt} = 8t \Rightarrow \alpha(1) = 8(1) = 8 \text{ rad/s}^2$$

7: Rotation with Uniform Angular Acceleration

➤ If ω_0 is the initial angular velocity, ω is the angular velocity of a rigid body after a time t and α is the uniform angular acceleration, then

$$\omega = \omega_0 + \alpha t.$$

If θ is the angular displacement in this time t , then

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2.$$

Also we may write

$$\omega^2 = \omega_0^2 + 2\alpha\theta.$$

The average angular velocity is

$$\bar{\omega} = \frac{\omega_0 + \omega}{2}.$$

EXAMPLE 10

A flywheel of radius R , starting from rest rotates about a fixed axis passing through its center at a constant angular acceleration α . The total acceleration of a point on its rim after a time t is

- (A) $a_t = R\sqrt{(\alpha^2 t^4 + 1)}$ (B) $a_t = R\alpha\sqrt{(\alpha^2 t^4 + 1)}$
 (C) $a_t = \frac{\alpha\sqrt{(\alpha^2 t^4 + 1)}}{R}$ (D) $a_t = R\alpha\sqrt{(\alpha^2 t^2 + 1)}$

Ans (B)

Solution

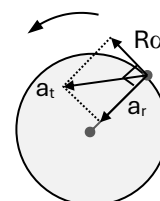
The angular speed after a time t is
$$: \omega = \omega_0 + \alpha t \Rightarrow \omega = 0 + \alpha t \Rightarrow \omega = \alpha t$$

Hence the radial accⁿ of a point on the rim is : $a_r = \omega^2 R = \alpha^2 R t^2$

The tangential acceleration of the point is : a_{tan}

Thus the total acceleration is
$$: a_t = \sqrt{a_r^2 + a_{tan}^2} = \sqrt{(\alpha^2 R t^2)^2 + (R\alpha)^2}$$

Or
$$: a_t = R\alpha\sqrt{(\alpha^2 t^2 + 1)}$$



EXAMPLE 11

A wheel rotates about a fixed axle at a uniform angular acceleration of 2 rad/s^2 starting from rest. If its radius is 0.1 m , the distance moved by a point on its edge in the first 2 s is

- (A) 10 m (B) 20 m (C) 10 cm (D) 20 cm

Ans (D)

Solution

The angle rotated by the point on the edge is : $\theta = \omega_0 t + \frac{1}{2} \alpha t^2 \Rightarrow \theta = (0)(2) + \frac{1}{2} (2)(2)^2 = 2 \text{ rad}$.

The distance moved by the point in the said time : $s = r\theta \Rightarrow s = (0.1)(2) = 0.2 \text{ m} = 20 \text{ cm}$.

EXAMPLE 12

A cylinder is rotating about its central axis at a constant angular speed of 3 rad/s . five rotations later, it uniformly comes to stop. If its radius is r , the tangential acceleration of a point on it at a distance $r/2$ from the axis is

- (A) $\frac{40r}{9\pi}$ (B) $\frac{9r}{40}$ (C) $\frac{9r}{40\pi}$ (D) $\frac{9r}{20}$

Ans (C)

Solution

The angular speed after angle rotated is θ is : $\omega^2 = \omega_0^2 + 2\alpha\theta \Rightarrow \alpha = \frac{\omega^2 - \omega_0^2}{2\theta} \Rightarrow \alpha = \frac{0^2 - 3^2}{2(5 \times 2\pi)} = -\frac{9}{20\pi}$

Hence the tangential accⁿ of the said point is : $|a_t| = \frac{r}{2} \alpha = \frac{9r}{40\pi}$

8: Moment of Inertia

➤ If m is the mass of a particle in rotation about a fixed axis, r is its perpendicular distance from the axis

of rotation, then moment of inertia (see fig a) (MI) is given by

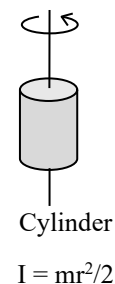
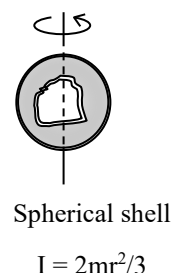
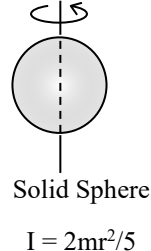
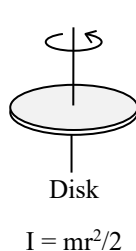
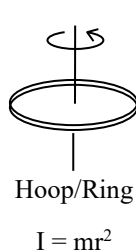
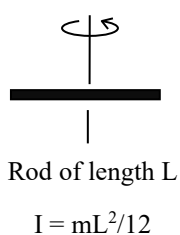
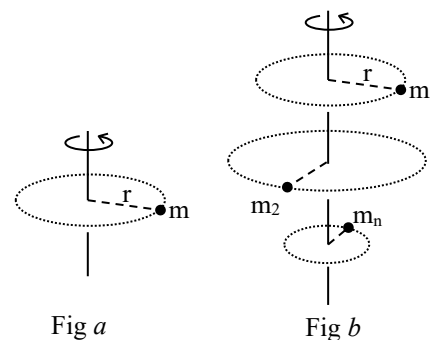
$$I = mr^2 \quad (\text{measured in kg.m}^2).$$

➤ If $m_1, m_2, \dots, m_n$ are the masses of particles having perpendicular distances from axis of rotations $r_1, r_2, \dots, r_n$ respectively, then the total MI of the system of particles is

$$I = m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2 = \sum_{i=1}^n m_i r_i^2$$

➤ For continuous mass distributions, $I = \int r^2 dm$.

➤ MI of certain uniform, symmetrical objects about the axes shown:



➤ **Parallel axis theorem** (NOT REQUIRED FOR KCET): If I_0 is the MI of the body about an axis passing through the COM, I is the MI about any other parallel axis such that the two axes are at perpendicular distance h away from each other [Fig (a)] then

$$I = I_0 + Mh^2$$

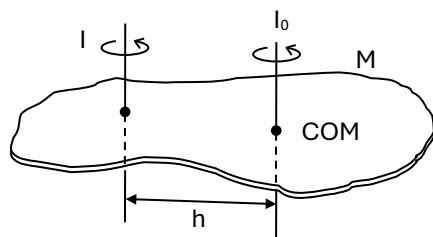


Fig (a)

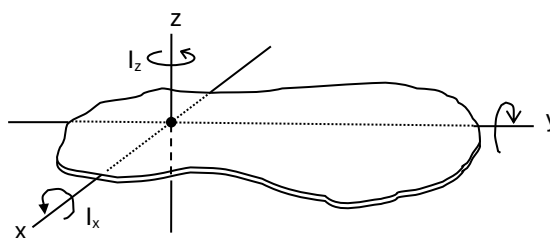


Fig (b)

➤ **Perpendicular axis theorem** (NOT REQUIRED FOR KCET): If I_x , I_y and I_z are the MIs of a plane lamina about x , y and z axes respectively (See [Fig (b)]), then $I_z = I_x + I_y$.

➤ The radius of gyration of a rigid body of mass M , having MI about a fixed axis as I is given by $k = \sqrt{\frac{I}{M}}$.

➤ For example, for rod about central axis: $k_{rod} = \sqrt{\frac{\frac{1}{12}ML^2}{M}} = \frac{L}{2\sqrt{3}}$ and for hollow sphere about its central axis,

$$k_{sph} = \sqrt{\frac{\frac{2}{5}MR^2}{M}} = \sqrt{\frac{2}{5}}R \text{ etc.}$$

EXAMPLE 13

Two particles of masses 1 kg and 2 kg are connected to a vertical massless rod by two identical but perpendicular thin, light rods each of length 1 m. The three rods are mutually perpendicular to each other. The moment of inertia (in kg.m^2) of the two particles about the vertical rod is

(A) 1

(B) 2

(C) 3

(D) 4

Ans (C)

Solution

The MI of the system is

$$: I = m_1 r_1^2 + m_2 r_2^2 = (1)(1)^2 + (2)(1)^2 = 3 \text{ kgm}^2$$

EXAMPLE 14

The MI of a uniform hollow sphere of mass M and radius R about an axis passing through its surface (as shown) is

(A) $(2/3)MR^2$

(B) $(3/7)MR^2$

(C) $(5/3)MR^2$

(D) $(5/9)MR^2$

Ans (C)

Solution

The MI of the hollow sphere about COM axis

$$: I_0 = \frac{2}{3}MR^2$$

and

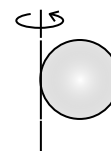
$$: h = R$$

From

$$: \text{parallel axis theorem, } I = I_0 + mh^2$$

we can write

$$: I = (2/3)MR^2 + R^2 = (5/3)MR^2$$



9: Torque and Moment of Force

- If a force $\vec{F}$ acts at a point having a position vector $\vec{r}$, then the torque of the force is given by $\vec{\tau} = \vec{r} \times \vec{F}$.
- The magnitude of this torque is called the *moment of force* which is given by $\tau = rF \sin \theta$.
- This torque is **zero**, when $\theta = 0$ or 180° , and is maximum $\tau_{\max} = rF$ when $\theta = 90^\circ$.
- The direction of the torque is given by the *right hand clasp rule*.
- If a particle is moving along the circular path of radius R , due to a tangential force of F , then the torque on it is given by $\tau = FR$.
- If α is the angular acceleration of the particle, then $\tau = I\alpha$.

Where I is the MI of the particle about the axis of rotation.

- If ω is the angular speed of a particle of mass m rotating about a fixed axis in radius R , then the kinetic energy (KE) is given by $K_{\text{rot}} = \frac{1}{2} I\omega^2$.
- The rate of change of KE is called the power $P = \frac{dK_{\text{rot}}}{dt} = \tau\omega$.

EXAMPLE 15

A force of $(2\hat{i} + \hat{j})N$ acts on a particle having a position vector of $(\hat{j} - 3\hat{k})m$. The magnitude of the torque on the particle is

- (A) 2 Nm (B) 4 Nm (C) 5 Nm (D) 7 Nm

Ans (D)

Solution

The torque acting on the particle is given by : $\vec{\tau} = \vec{r} \times \vec{F}$

So that
$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 0 \\ 0 & 1 & -3 \end{vmatrix}$$

Solving, it we get
$$\vec{\tau} = (1 \times 3 - 1 \times 0)\hat{i} - (2 \times -3 - 0 \times 0)\hat{j} + (2 \times 1 - 0 \times 1)\hat{k}$$

or
$$\vec{\tau} = 3\hat{i} + 6\hat{j} + 2\hat{k} \Rightarrow |\vec{\tau}| = \sqrt{3^2 + 6^2 + 2^2} = 7Nm$$

EXAMPLE 16

A uniform wheel of mass m fixed at its axle is initially rotating at an angular velocity of ω_0 . It is uniformly brought to rest by applying a tangential retarding force of F at its edge in time t . The radius of the wheel is

- (A) $\frac{2Ft}{m\omega_0}$ (B) $\frac{Ft}{2m\omega_0}$ (C) $\frac{2Ft}{3m\omega_0}$ (D) $\frac{Ft}{m\omega_0}$

Ans (A)

Solution

$\vec{R}$ The torque of the force is
$$\tau = RF$$

Also the angular acceleration magnitude is
$$|\alpha| = \left| \frac{\omega - \omega_0}{t} \right| \Rightarrow |\alpha| = \left| \frac{0 - \omega_0}{t} \right| = \frac{\omega_0}{t}$$

Since
$$\tau = I|\alpha| = \frac{1}{2}mR^2|\alpha| = \frac{1}{2}mR^2 \frac{\omega_0}{t}$$

Finally
$$RF = \frac{1}{2}mR^2 \frac{\omega_0}{t} \Rightarrow R = \frac{2Ft}{m\omega_0}$$

EXAMPLE 16

A fan starting from rest accelerates uniformly at 2 rad/s^2 . If its moment of inertial about the axis of rotation is 0.4 kg.m^2 , the average power consumed in 10 s is

- (A) 16 W (B) 10 W (C) 8 W (D) 4 W

Ans (C)

Solution

The angular velocity gained is : $\omega = \omega_0 + \alpha t \Rightarrow \omega = 0 + (2)(10) = 20 \text{ rad/s}$

The total work done is the change in rotational KE : $W = \Delta K = \frac{1}{2} I\omega^2 - \frac{1}{2} I\omega_0^2$
 $\Rightarrow W = \frac{1}{2} (0.4)(20)^2 - \frac{1}{2} (0.4)(0)^2$

Or : $W = 80 \text{ J.}$

The average power : $\bar{P} = \frac{W}{t} = \frac{80}{10} = 8 \text{ W}$

10: Couple and Moment of Couple

➤ If a body is acted upon by two equal but opposite forces at its ends, producing rotation about its center, these forces constitute a *couple*.

➤ The moment of couple is the product of any one of the forces of the couple and the perpendicular distance between them.

$$\text{Moment of Couple} = F \times 2D.$$

10: Angular Momentum of a Particle

➤ If a particle having mass m moving with a velocity $\vec{v}$ located at point has a position vector $\vec{r}$, then the angular momentum of the particle is given by

$$\vec{l} = \vec{r} \times m\vec{v}$$

➤ The magnitude of this *angular momentum* is given by $l = mvr \sin \theta$.

➤ This *angular momentum* is **zero**, when $\theta = 0$ or 180° , and is maximum $l_{\max} = mvr$ when $\theta = 90^\circ$.

➤ The direction of the angular momentum is given by the *right hand clasp rule*.

➤ When a torque $\vec{\tau}$ acts on the particle its angular momentum changes at a rate of $\frac{d\vec{l}}{dt}$ given by

$$\vec{\tau} = \frac{d\vec{l}}{dt}.$$

EXAMPLE 17

A particle of mass m moving in a circle of radius r undergoes a constant angular acceleration of α . If it starts from rest, the angular momentum of the particle after a time t is

- (A) $0.5 \text{ } m r^2 \alpha t$ (B) $m r^2 \alpha t$ (C) $m r \alpha^2 t$ (D) $m r \alpha / t$

Ans (2)

Solution

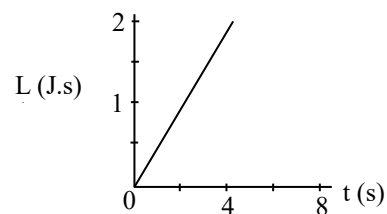
The angular velocity after time t : $\omega = \omega_0 + \alpha t \Rightarrow \omega = \alpha t$

Hence linear velocity after time t : $v = r\omega = r\alpha t$

Angular momentum : $l = m v r \sin 90^\circ = m (r\alpha t)r = m r^2 \alpha t$

EXAMPLE 18

The angular momentum of a particle about a certain axis varies with time according to the graph shown. The torque acting on the particle about the axis is



- (A) 0.5 Nm (B) 1 Nm
(C) 2 Nm (D) 4 Nm

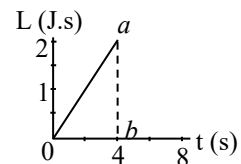
Ans (A)

Solution

The torque and angular momentum are related by : $\tau = \frac{dl}{dt} \Rightarrow \tau = \text{slope of } l \text{ vs } t \text{ graph}$

Hence

$$: \tau = \frac{ab}{ob} = \frac{2-0}{4-0} \Rightarrow \tau = 0.5 \text{ Nm}$$

**12: Angular Momentum of a Rigid Body about a Fixed Axis**

➤ If a rigid body having moment of inertia I rotating about a fixed axis with angular velocity $\vec{\omega}$ then the angular momentum of the rigid body is given by $\vec{L} = I\vec{\omega}$

➤ When a torque $\vec{\tau}$ acts on the rigid body its angular momentum changes at a rate of $\frac{d\vec{L}}{dt}$ given by

$$\vec{\tau} = \frac{d\vec{L}}{dt}.$$

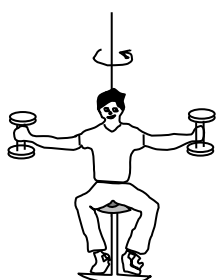
13: Law of Conservation of Angular Momentum

➤ If the total external torque acting on a system of rotating rigid bodies is zero, the angular momentum of the system is constant or conserved.

➤ That is if $\vec{\tau} = 0 \Rightarrow \frac{\Delta \vec{L}}{\Delta t} = 0 \Rightarrow \vec{L} = \text{constant} \Rightarrow L_i = L_f \Rightarrow I_i \omega_i = I_f \omega_f$

➤ Examples of Law of Conservation of Angular Momentum:

(1) Skater rotating about her foot, (2) a person in a rotating stool and (3) a spring board diver.



I large ω small



I small ω large



I large ω small



I small ω large

EXAMPLE 19.

A skater holding his arms outstretched has an MI of 4 kg.m^2 . He is spinning about his toes at an angular speed of 2 rad/s . He pulls his arms close to his chest. Now he spins at an angular speed of 3 rad/s . His new moment of inertia (in kg.m^2) about the same axis is

- (A) 3 (B) 2 (C) $8/3$ (D) $3/8$

Ans (B)

Solution

By conservation of angular momentum $: I_f \omega_f = I_i \omega_i \Rightarrow I_f = I_i \frac{\omega_i}{\omega_f} \Rightarrow I_f = (4) \frac{(2)}{(3)} = \frac{8}{3} \text{ kg.m}^2$

EXAMPLE 20

A spherical star having radius of R is spinning about its axis at an angular speed of ω_1 . Now, its radius suddenly shrinks to R/4. The new angular speed of spinning of the star is

- (A) $2\omega_1$ (B) $4\omega_1$ (C) $8\omega_1$ (D) $16\omega_1$

Ans (D)

Solution

By conservation of angular momentum $: I_f \omega_f = I_i \omega_i \Rightarrow \omega_f = \frac{I_i}{I_f} \omega_i \Rightarrow \omega_f = \frac{\frac{2}{5}mR^2}{\frac{2}{5}m(\frac{R}{4})^2} \omega_1 = 16\omega_1$

14: Equilibrium of a Rigid Body

➤ When a rigid body is in equilibrium, the total force on it is zero

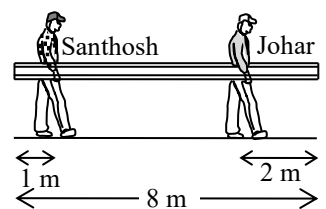
$$\vec{F} = 0 \Rightarrow F_x = 0, F_y = 0 \text{ and } F_z = 0.$$

➤ When a rigid body is in equilibrium, the total torque about any axis is zero $\vec{\tau} = 0$.

EXAMPLE 21

A uniform beam of length 8 m and weight 500 N is carried by two workers, Santhosh and Johar, as shown in the figure. The force that Santhosh exerts on the beam (in N) is

- (A) 100 (B) 200
(C) 300 (D) 400



Ans (B)

Solution

The beam is in : equilibrium

Hence the net force acting on it $: F = 0 \Rightarrow F_1 + F_2 - 500 = 0$

$$\Rightarrow F_1 + F_2 = 500 \text{ ..(1)}$$

The net torque acting about

any point is zero $: \tau \text{ about c is zero}$

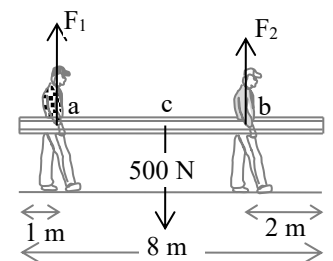
or

$$: - F_1 \times ac + F_2 \times cb = 0$$

$$\Rightarrow - F_1 \times 3 + F_2 \times 2 = 0 \Rightarrow F_2 = 1.5 F_1 \text{(2)}$$

Hence from Eqs (1) and (2),

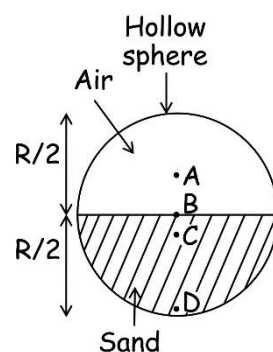
$$: 2.5 F_1 = 500 \Rightarrow F_1 = 200 \text{ N}$$



PRACTICE QUESTIONS - SYSTEM OF PARTICLES AND ROTATIONAL MOTION

1. Which of the following points is the likely position of the centre of mass of the system shown in Figure?

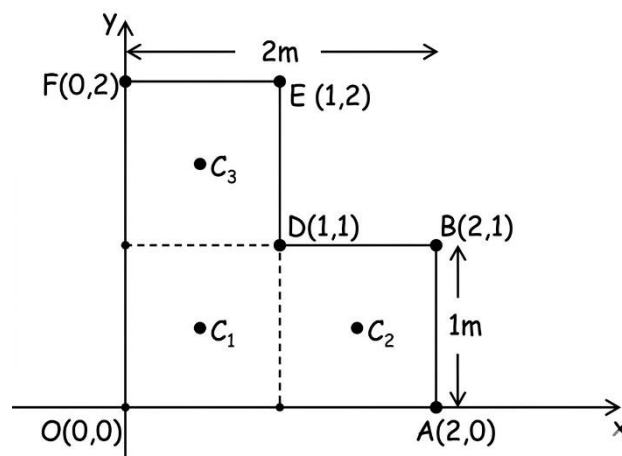
(A) B
(B) D
(C) A
(D) C



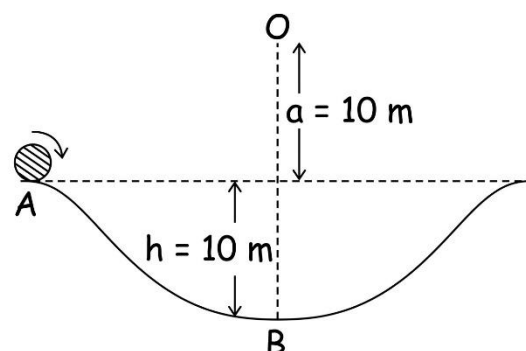
2. Two balls are thrown simultaneously in air. The acceleration of the centre of mass of the two balls while in air
- (A) depends on the direction of the motion of the balls
(B) depends on the masses of the two balls
(C) depends on the speeds of the two balls
(D) is equal to g .
3. Two blocks of masses 10 kg and 30 kg are placed along a vertical line. The first block is raised through a height of 7 cm. By what distance should the second mass be moved to raise the centre of mass by 1 cm?
- (A) 1 cm upward (B) 1 cm downward (C) 2 cm upward (D) 2 cm downward
4. The density of a non-uniform rod of length 1 m is given by $\rho(x) = a(1 + bx^2)$ where a and b are constants and $0 < x \leq 1$. The centre of mass of the rod will be at
- (A) $\frac{3(2+b)}{4(3+b)}$ (B) $\frac{4(2+b)}{3(3+b)}$ (C) $\frac{3(3+b)}{4(2+b)}$ (D) $\frac{4(3+b)}{3(2+b)}$
5. Two-point masses of 0.3 kg and 0.7 kg are fixed at the ends of a rod of length 1.4 m and of negligible mass. The position of centre of mass of the system from 0.3 kg mass is
- (A) 0.98 m (B) 0.70 m (C) 0.42 m (D) 1 m
6. Two objects of mass 1 kg and 2 kg are connected by a light string that goes over a frictionless light pulley. The acceleration of the centre of mass of the blocks is
- (A) $\frac{g}{3}$ downward (B) $\frac{g}{9}$ downward (C) $\frac{g}{9}$ upward (D) $\frac{g}{3}$ upward
7. A ball is dropped off a high tower while, simultaneously, an identical ball is launched directly upward into the air. The center of mass of the two balls
- (A) stays in the same place
(B) initially rises, then falls, but begins to fall before the cannonball launched into the air starts falling
(C) initially rises, then falls, but begins to fall at the same time as the cannonball launched upward begins to fall
(D) Initially rises, then falls, but begins to fall *after* the cannonball launched upward begins to fall.

8. The coordinates of centre of mass of a uniform L-shaped lamina (a thin flat plate) of mass 3 kg with dimensions as shown are

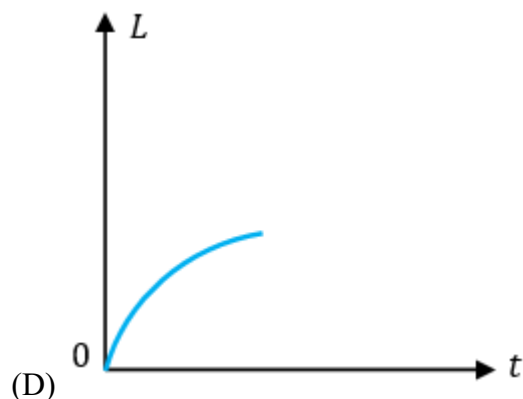
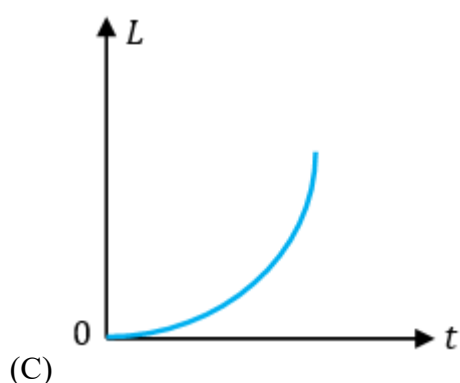
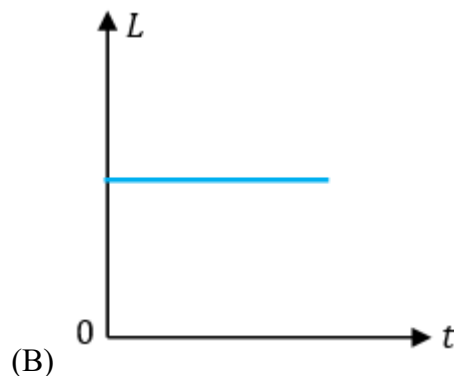
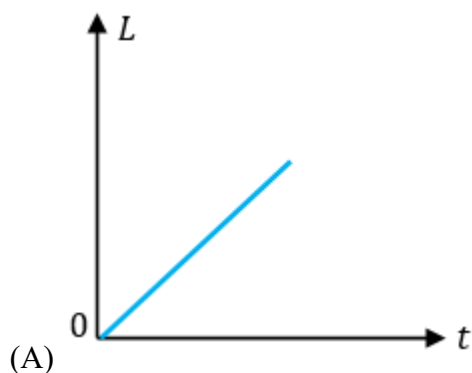
(A) $(\frac{5}{6} m, \frac{5}{6} m)$
(B) $(\frac{5}{3} m, \frac{5}{6} m)$
(C) $(\frac{1}{2} m, \frac{1}{2} m)$
(D) $(\frac{1}{3} m, \frac{1}{3} m)$



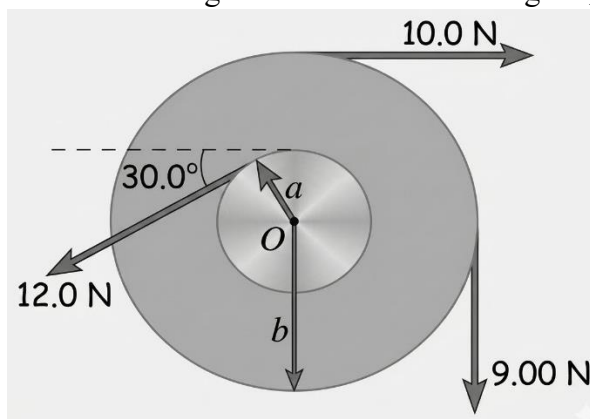
9. From a uniform disk of radius R , a circular hole of radius $R/2$ is cut out. The centre of the hole is at $R/2$ from the centre of the original disc. The centre of mass of the resulting flat body is at
 (A) at $R/2$ from the center of original disc opposite to the center of cut portion
 (B) at $R/6$ from the center of original disc opposite to the center of cut portion
 (C) at $R/3$ from the center of original disc opposite to the center of cut portion
 (D) at the centre of original disc
10. Two particles of equal mass have velocities $4\hat{i} \text{ ms}^{-1}$ and $4\hat{j} \text{ ms}^{-1}$. First particle has an acceleration $(5\hat{i} + 5\hat{j}) \text{ ms}^{-2}$ and acceleration of second particle is zero. The trajectory of centre of mass of the system is:
 (A) a straight line (B) a parabola (C) a circle (D) an ellipse
11. A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. The mass of the metre stick is
 (A) 33 g (B) 66 g (C) 20 g (D) 40 g
12. A body is in pure rotation. The linear speed v of a particle, the distance r of the particle from the axis and the angular velocity ω of the body are related as: $\omega = \frac{v}{r}$. Thus
 (A) $\omega \propto \frac{1}{r}$ (B) $\omega \propto r$
 (C) ω is independent of r (D) $\omega = 0$
13. A rigid body rotates with constant angular velocity $\vec{\omega} = \omega\hat{k}$. Four particles are located at: P (3,4,0), Q (4,3,0), R (0,5,0) and S (0,0,5). Which particle has zero linear velocity?
 (A) P (B) Q (C) R (D) S
14. A particle moves with a constant velocity parallel to the X-axis. Its angular momentum with respect to the origin
 (A) is zero (B) remains constant (C) goes on increasing (D) goes on decreasing
15. Which of the following is incorrect for a particle moving on a straight line with a uniform velocity.
 (A) Its angular momentum is always zero
 (B) Its angular momentum is zero about a point on the straight line
 (C) Its angular momentum is not zero about a point away from the straight line
 (D) Its angular momentum about any given point remains constant.
16. Angular momentum of a single particle in uniform circular motion about the centre of the circle
 (A) changes in magnitude but remains same in the direction
 (B) remains same in magnitude and direction
 (C) remains same in magnitude but changes in the direction
 (D) is zero
17. A particle of mass 100 g is projected at time $t = 0$ from a point P with a speed 20 m/s at an angle of 30° to the horizontal. The magnitude of the angular momentum of the particle about the point P at time $t = 1 \text{ s}$ will be
 (A) $10\sqrt{3} \text{ Js}$ (B) 20 Js (C) $5\sqrt{3} \text{ Js}$ (D) zero
18. A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point A, as shown in the figure. The point A is at height h from point B. The particle slides along the frictionless surface. When the particle reaches point B, its angular momentum about O will be _____ J s.
 (A) 8 (B) 3
 (C) 2 (D) 6



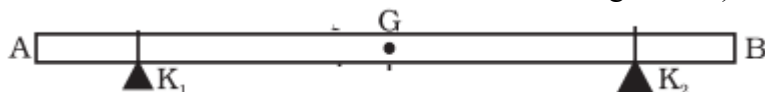
19. An object is projected with a certain speed at an angle θ ($\neq 90^\circ$) above the horizontal. The magnitude of angular momentum of the object about the point of projection during its flight as a function of time is correctly shown in



20. The net torque on the wheel in figure about the axle through O, is: ($a = 10\text{cm}$ and $b = 25\text{cm}$)

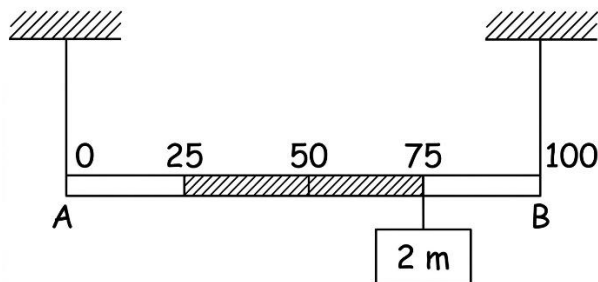


- (A) 3.55 Nm out of the page (B) 3.5 Nm into the page
(C) 12 Nm out of the page (D) zero
21. Consider the following statements:
Statement I: If the effort arm is longer than the load arm, less effort is required to lift the load.
Statement II: This follows from the principle of moments.
(A) Both statements are true.
(B) Statement I is true, but Statement II is false.
(C) Statement I is false, but Statement II is true.
(D) Both statements are false.
22. A metal bar 70 cm long and 4.00 kg in mass supported on two knife-edges placed 10 cm from each end. A 6.00 kg weight is suspended at 30 cm from one end. The reactions at the knife-edge K_1 is: (Assume the bar to be of uniform cross section and homogeneous.)

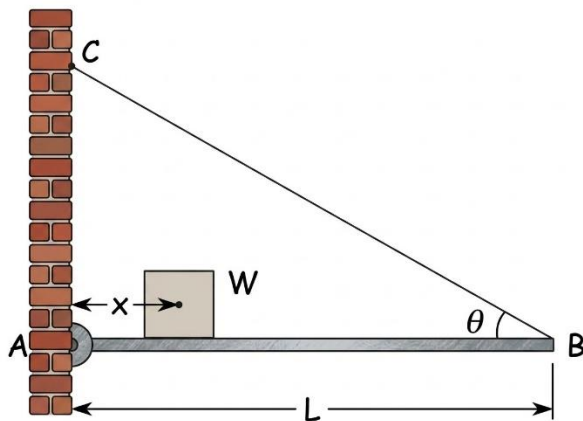


- (A) 56 N (B) 50 N (C) 44 N (D) 25 N

23. Shown in the above figure, a rigid and uniform 1 m long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass m and has another weight of mass 2 m hung at a distance of 75 cm from A. The tension in the string at A is



- (A) 1 mg (B) 2 mg (C) 0.5 mg (D) 0.75 mg
24. A table has a heavy circular top of radius 1 m and mass 20 kg placed on four light (considered massless) legs placed symmetrically on its circumference. The maximum mass that can be kept anywhere on the table without toppling it is close to
- (A) 20 kg (B) 34 kg (C) 47 kg (D) 59 kg
25. In Figure, suppose the length L of the uniform bar is 3.00 m and its weight is 200 N. Also, let the block's weight $W = 300\text{ N}$ and the angle $\theta = 30^\circ$. The wire can withstand a maximum tension of 500 N. The maximum possible distance x before the wire breaks will be



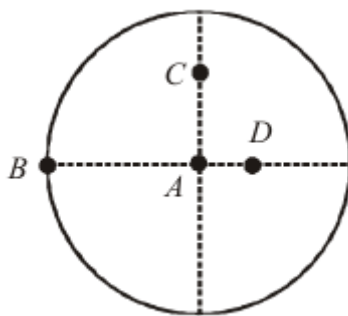
- (A) 2 m (B) 1.5 m (C) 1.25 m (D) 1 m
26. Two uniform discs have equal mass M . The radius of disc A is twice that of disc B. If both rotate about their central axes with the same angular speed, the ratio of their rotational kinetic energies is
- (A) 1 : 1 (B) 2 : 1 (C) 4 : 1 (D) 8 : 1
27. A uniform disc of mass M and radius R rotates about an axis passing through its centre and perpendicular to its plane. Another identical disc rotates with the same angular speed about one of its diameters. The ratio of their rotational kinetic energies is
- (A) 1:1 (B) 2:1 (C) 4:1 (D) 1:2
28. A disc has moment of inertia I about an axis perpendicular to its plane through its centre. The moment of inertia about any diameter through the centre is
- (A) I (B) $\frac{I}{2}$ (C) $2I$ (D) $\frac{I}{4}$
29. A thin ring and a solid disc have equal mass and equal radius. They are acted upon by equal torques about their respective central axes. The ratio of their angular accelerations is
- (A) 1 : 2 (B) 2 : 1 (C) 1 : 1 (D) 4 : 1

30. The moment of inertia of a body depends upon: (1) total mass (2) distribution of mass (3) position of axis. Choose the correct option.
 (A) Only 1 (B) Only 1 and 2 (C) Only 2 and 3 (D) All three
31. A rod of length L rotates about one end. If its length is doubled while keeping the mass constant, its moment of inertia becomes
 (A) unchanged (B) doubled (C) four times (D) eight times
32. Statement I: A hollow cylinder rolls down an incline more slowly than a solid cylinder.
 Statement II: A hollow cylinder has a larger moment of inertia.
 Choose the correct option
 (A) Both statements I and II are true and Statement II is the correct explanation to the Statement I.
 (B) Both statements I and II are true but Statement II is not the correct explanation to the Statement I.
 (C) Statement I is false
 (D) Both Statement I and II are false
33. Two bodies have the same mass but different distributions of mass about their respective axes of rotation. Choose the correct option
 (A) Both must have the same moment of inertia.
 (B) The body with mass distributed farther from the axis has a greater moment of inertia.
 (C) The body with smaller radius always has a greater moment of inertia.
 (D) Moment of inertia depends only on the angular velocity.
34. A particle of mass 2 kg moves in a circle of radius 0.5 m. Its moment of inertia about the centre is
 (A) 0.25 kgm² (B) 0.5 kgm² (C) 1 kgm² (D) 2 kgm²
35. A body is free to rotate about two different parallel axes. Which quantity remains unchanged?
 (A) Moment of inertia (B) Radius of gyration
 (C) Mass (D) Angular acceleration
36. Two bodies have equal masses but moments of inertia I_1 and I_2 ($I_2 > I_1$). If equal rotational kinetic energies are given to them, then
 (A) $\omega_1 = \omega_2$ (B) $\omega_1 > \omega_2$ (C) $\omega_1 < \omega_2$ (D) Cannot be determined
37. A uniform disc is rotating about its central axis perpendicular to its plane with angular speed ω . If it is made to rotate about one of its diameters while keeping its rotational kinetic energy unchanged, the new angular speed will be
 (A) ω (B) $\sqrt{2} \omega$ (C) 2ω (D) $\frac{\omega}{\sqrt{2}}$
38. The radius of gyration of a body is doubled while its mass remains unchanged. Its moment of inertia becomes
 (A) Double (B) Four times (C) Eight times (D) Unchanged
39. The moment of inertia of a body is 18 kg m². If its mass is 2 kg, its radius of gyration is
 (A) 2 m (B) 3 m (C) 4 m (D) 9 m

40. A flywheel rotates with constant angular velocity. Its angular momentum is doubled without changing its shape. Which quantity must also double?
- (A) Radius (B) Mass
(C) Moment of inertia (D) Angular acceleration
41. A solid cylinder of mass 4 kg and radius 0.20 m rotates about its axis with an angular speed of 20 rad s^{-1} . Its rotational kinetic energy is
- (A) 8 J (B) 16 J (C) 32 J (D) 64 J
42. A rigid body rotating about a fixed axis has rotational kinetic energy K . If its angular speed becomes three times while its moment of inertia remains unchanged, the rotational kinetic energy becomes
- (A) $3K$ (B) $6K$ (C) $9K$ (D) $27K$
43. Which of the following quantities is the rotational analogue of linear momentum?
- (A) Torque (B) Angular displacement
(C) Angular momentum (D) Angular velocity
44. For a body rotating about a fixed axis, torque is the rotational analogue of
- (A) Mass (B) Force (C) Momentum (D) Work
45. Two identical discs rotate with the same angular speed. Disc A has twice the moment of inertia of Disc B. The ratio of their angular momenta is
- (A) 1:2 (B) 2:1 (C) 1:1 (D) 4:1
46. A uniform disc is rotating about its central axis with angular speed ω . Another identical disc is rotating about one of its diameters with angular speed 2ω .
- Which of the following quantities is the same for the two discs?
- (A) Angular momentum (B) Rotational kinetic energy
(C) Angular acceleration (D) Moment of inertia
47. A skater rotating with outstretched arms suddenly pulls her arms inward. Which quantity definitely increases?
- (A) Moment of inertia (B) Angular momentum
(C) Angular speed (D) External torque
48. A body rotates under the action of a constant torque. Which of the following quantities remains constant?
- (A) Angular velocity (B) Angular momentum
(C) Angular acceleration (D) Rotational kinetic energy
49. A rotating body has moment of inertia 5 kg m^2 and Angular velocity 4 rad s^{-1} . Its angular momentum is
- (A) $10 \text{ kg m}^2/\text{s}$ (B) $15 \text{ kg m}^2/\text{s}$ (C) $20 \text{ kg m}^2/\text{s}$ (D) $25 \text{ kg m}^2/\text{s}$
50. A constant torque acts on a rigid body about a fixed axis. Which graph correctly represents angular velocity versus time?
- (A) Horizontal straight line (B) Straight line with positive slope
(C) Parabola opening upward (D) Hyperbola

51. A uniform disc and a uniform ring have the same mass and radius. They are released from rest under identical torques about their respective symmetry axes. Which statement is correct?
- (A) Both acquire the same angular speed after the same time.
 (B) The ring acquires greater angular speed.
 (C) The disc acquires greater angular speed.
 (D) Their angular speeds cannot be compared.
52. A constant torque of 10 N m acts on a body having moment of inertia 5 kg m^2 . The angular acceleration produced is
- (A) 0.5 rad s^{-2} (B) 2 rad s^{-2} (C) 5 rad s^{-2} (D) 50 rad s^{-2}
53. A rigid body rotating with angular speed ω possesses rotational kinetic energy K . If its angular momentum is doubled while the moment of inertia remains unchanged, the new rotational kinetic energy becomes
- (A) $2K$ (B) $4K$ (C) $8K$ (D) $16K$
54. A wheel has moment of inertia 8 kg m^2 and angular acceleration 3 rad s^{-2} . The torque acting on it is
- (A) 11 Nm (B) 16 Nm (C) 24 Nm (D) 32 Nm
55. A machine delivers a constant torque of 150 N m to maintain a shaft rotating at 40 rad s^{-1} . The power supplied is
- (A) 3 kW (B) 4 kW (C) 6 kW (D) 8 kW
56. Three rotating bodies have equal angular momentum. Their moments of inertia satisfy $I_1 < I_2 < I_3$. Which statement is correct?
- (A) $\omega_1 = \omega_2 = \omega_3$ (B) $\omega_1 > \omega_2 > \omega_3$
 (C) $\omega_1 < \omega_2 < \omega_3$ (D) Their rotational kinetic energies are equal.
57. A rotating body has moment of inertia 4 kg m^2 and angular velocity 5 rad s^{-1} . If no external torque acts on it and its moment of inertia becomes 2 kg m^2 , its new angular velocity is
- (A) 5 rad s^{-1} (B) 7.5 rad s^{-1} (C) 10 rad s^{-1} (D) 20 rad s^{-1}
58. Which of the following changes when a rotating skater pulls her arms inward?
- (1) Angular speed (2) Moment of inertia (3) Angular momentum
- (A) 1 only (B) 1 and 2 only
 (C) 2 and 3 only (D) 1, 2 and 3
59. A body rotates with angular momentum L . If both its moment of inertia and angular velocity become half their original values simultaneously, the new angular momentum is
- (A) L (B) $\frac{L}{2}$ (C) $\frac{L}{4}$ (D) $2L$
60. A rotating body has angular momentum L and rotational kinetic energy K . If its angular velocity is doubled while its moment of inertia remains unchanged, the ratio of the new kinetic energy to the new angular momentum is
- (A) $\frac{K}{L}$ (B) $2\frac{K}{L}$ (C) $\frac{1}{2}\frac{K}{L}$ (D) $4\frac{K}{L}$

61. A flywheel has moment of inertia 2 kg m^2 . A constant torque of 8 N acts on it for 5 s , starting from rest. Its final angular velocity is
 (A) 10 rad s^{-1} (B) 15 rad s^{-1} (C) 20 rad s^{-1} (D) 25 rad s^{-1}
62. A uniform solid cylinder and a uniform ring have equal masses M and equal radii R . They are initially at rest. Equal constant torques are applied for the same duration t .
 The ratio of their rotational kinetic energies after time t is
 (A) $1 : 1$ (B) $2 : 1$ (C) $1 : 2$ (D) $4 : 1$
63. A constant torque acts on a body. During the fourth second its angular displacement is 70 rad . If it started from rest, its angular acceleration is
 (A) 10 rad s^{-2} (B) 16 rad s^{-2} (C) 20 rad s^{-2} (D) 24 rad s^{-2}
64. A wheel having a moment of inertia of 2 kg m^2 about its vertical axis rotates at 60 rpm . The constant torque required to bring the wheel to rest in 1 minute is:
 (A) $\frac{\pi}{18} \text{ N m}$ (B) $\frac{2\pi}{15} \text{ N m}$ (C) $\frac{\pi}{12} \text{ N m}$ (D) $\frac{\pi}{15} \text{ N m}$
65. The moment of inertia of a uniform circular disc is maximum about an axis perpendicular to the disc and passing through



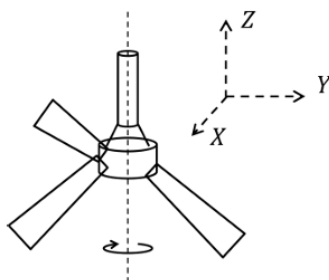
- (A) B (B) C (C) D (D) A

KEY ANSWERS

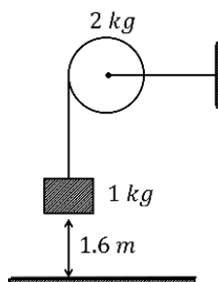
1	2	3	4	5	6	7	8	9	10	11	12	13
D	D	B	A	A	B	B	A	B	A	B	C	D
14	15	16	17	18	19	20	21	22	23	24	25	26
B	A	B	C	D	C	B	A	A	A	C	B	C
27	28	29	30	31	32	33	34	35	36	37	38	39
B	B	B	D	C	A	B	B	C	C	B	B	B
40	41	42	43	44	45	46	47	48	49	50	51	52
C	B	C	C	B	B	A	C	C	C	B	C	B
53	54	55	56	57	58	59	60	61	62	63	64	65
B	C	C	B	C	B	C	B	C	B	C	A	B

PREVIOUS YEAR QUESTIONS (2020 ONWARDS)

1. The angular speed of a motor wheel is increased from 1200 rpm to 3120 rpm in 16 seconds. The angular acceleration of the motor wheel is: (2022)
 (A) $6\pi \text{ rad/s}^2$ (B) $2\pi \text{ rad/s}^2$ (C) $8\pi \text{ rad/s}^2$ (D) $4\pi \text{ rad/s}^2$
2. The centre of mass of an extended body on the surface of the earth and its centre of gravity: (2022)
 (A) can never be at the same point
 (B) are always at the same point for any size of the body
 (C) centre of mass coincides with the centre of gravity of a body if the size of the body is negligible as compared to the size (or radius) of the earth
 (D) are always at the same point only for spherical bodies.
3. A ceiling fan is rotating around a fixed axle as shown. The direction of angular velocity is along _____ direction. (2024)



- (A) $+\hat{j}$ (B) $-\hat{j}$ (C) $+\hat{k}$ (D) $-\hat{k}$
4. A body of mass 1 kg is suspended by a weightless string which passes over a frictionless pulley of mass 2 kg as shown in the figure. The mass is released from a height of 1.6 m from the ground. With what velocity does it strike the ground?



- (A) 16 ms^{-1} (B) 8 ms^{-1} (C) $4\sqrt{2} \text{ ms}^{-1}$ (D) 4 ms^{-1}
5. Three particles of mass 1 kg, 2 kg and 3 kg are placed at the vertices A, B and C respectively of any equilateral triangle ABC of side 1 m. The centre of mass of the system from vertex A (located at origin) is:
 (A) (0,0) (B) $\left(\frac{7}{12}, \frac{3\sqrt{3}}{12}\right)$ (C) $\left(\frac{9}{12}, \frac{3\sqrt{3}}{12}\right)$ (D) $\left(\frac{7}{12}, \frac{6+3\sqrt{3}}{12}\right)$
 6. If the Earth were to suddenly contract to half of its present radius, what would be the duration of the day?
 (A) 24 hours (B) 12 hours (C) 6 hours (D) 3 hours [2026]
 7. The angular momentum of a moving body remains constant if: [2026]
 (A) net external force is applied (B) net external force is zero
 (C) net external torque is zero (D) net external torque is applied

KEY ANSWERS

1	2	3	4	5	6	7
D	C	D	D	B	C	C