



# JEE Mains (Dropper)

## Sample Paper - III

**DURATION : 180 Minutes**

**M. MARKS : 300**

### General Instructions:

1. Immediately fill in the particulars on this page of the test booklet.
2. The test is of **3 hours** duration.
3. The test booklet consists of **90** questions (**75 to attempt**). The maximum marks are **300**.
4. There are three subjects in the question paper, Subject I, II and III consisting of Section-I (**Physics**), Section-II (**Chemistry**), Section-III (**Mathematics**), and having **30 questions** in each part.
5. There will be a total of **20 MCQs** and **10 Numerical** Value Based Questions (**attempt any 5**).
6. Each correct answer will give 4 marks while 1 Marks will be deducted for a wrong response.
7. No student is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. inside the examination room/hall.
8. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. However, the candidates are allowed to take away this Test Booklet with them.
9. **Do not fold or make any stray mark on the Answer Sheet (OMR).**

Name of the Student (In CAPITALS): \_\_\_\_\_

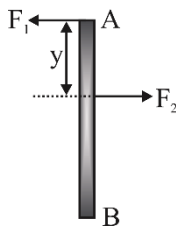
Roll Number: \_\_\_\_\_

Candidate's Signature: \_\_\_\_\_

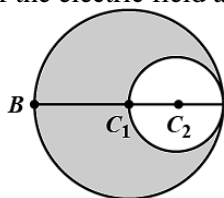
## Section-I (PHYSICS)

### [Section – A]

1. A thin uniform rod AB of mass  $m = 1.0 \text{ kg}$  moves translationally with acceleration  $a = 2.0 \text{ m/s}^2$  due to two antiparallel forces  $F_1$  and  $F_2$  as shown in figure. The distance between the points at which these forces are applied is equal to  $y = 20 \text{ cm}$ . Besides, it is known that  $F_2 = 5.0 \text{ N}$ . Find the length of the rod.

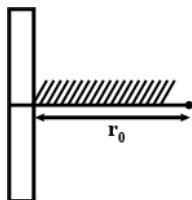


- (1) 1 m (2) 2 m  
(3) 3 m (4) 4 m
2. A positively charged sphere of radius  $r_0$  carries a volume charge density  $\rho$  (Figure). A spherical cavity of radius  $r_0/2$  is then scooped out and left empty, as shown. What is the direction and magnitude of the electric field at point B?



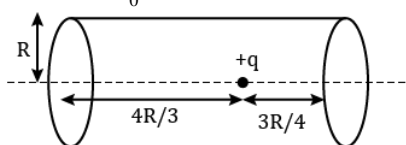
- (1)  $\frac{17\rho r_0}{54\epsilon_0}$  left (2)  $\frac{\rho r_0}{6\epsilon_0}$  left  
(3)  $\frac{17\rho r_0}{54\epsilon_0}$  right (4)  $\frac{\rho r_0}{6\epsilon_0}$  right

3. A positive point charge is released from rest at a distance  $r_0$  from a positive line charge with uniform density. The speed ( $v$ ) of the point charge, as a function of instantaneous distance  $r$  from line charge, is proportional to



- (1)  $v \propto \left(\frac{v}{r_0}\right)$  (2)  $v \propto e^{+\frac{r}{r_0}}$   
(3)  $v \propto \ln\left(\frac{r}{r_0}\right)$  (4)  $v \propto \sqrt{\ln\left(\frac{r}{r_0}\right)}$

4. A point charge  $+q$  is placed on the axis of a closed cylinder of radius  $R$  and height  $\frac{25R}{12}$  as shown. If electric flux coming out from the curved surface of cylinder is  $\frac{xq}{10\epsilon_0}$ , then calculate  $x$ .



- (1)  $\frac{5q}{10\epsilon_0}$  (2)  $\frac{6q}{10\epsilon_0}$   
(3)  $\frac{7q}{10\epsilon_0}$  (4)  $\frac{8q}{10\epsilon_0}$

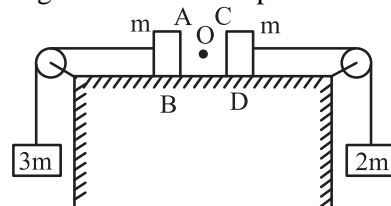
5. A projectile is fired from the surface of earth of radius  $R$  with a velocity  $kv_e$  (where  $v_e$  is the escape velocity from surface of earth and  $k < 1$ ). Neglecting air resistance, the maximum height of rise from the centre of earth is

- (1)  $\frac{R}{k^2 - 1}$  (2)  $k^2 R$   
(3)  $\frac{R}{1 - k^2}$  (4)  $kR$

6. A satellite of mass  $m$  revolves around the earth of radius  $R$  at a height  $x$  from its surface. If  $g$  is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is

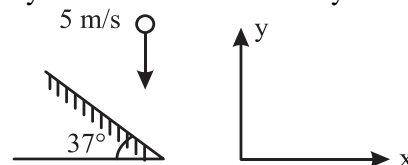
- (1)  $\sqrt{gx}$  (2)  $\sqrt{\frac{gR}{R-x}}$   
(3)  $\sqrt{\frac{gR^2}{R-x}}$  (4)  $\sqrt{\frac{gR^2}{R+x}}$

7. Two blocks each of masses  $m$  lie on a smooth table. They are attached to two other masses as shown in figure. The pulleys and strings are light. An object  $O$  is kept at rest on the table. The sides  $AB$  and  $CD$  of the two blocks are made reflecting. The acceleration of two images formed in those two reflecting surfaces with respect to each other is



- (1)  $\frac{5g}{6}$  (2)  $\frac{5g}{3}$   
(3)  $\frac{17g}{12}$  (4)  $\frac{17g}{6}$

8. An object  $O$  is just about to strike a perfectly reflecting inclined plane of inclination  $37^\circ$ . Its velocity is  $5 \text{ m/s}$ . Find the velocity of its image.



- (1)  $3\hat{i} + 4\hat{j}$  (2)  $4\hat{i} + 3\hat{j}$   
(3)  $4.8\hat{i} + 1.4\hat{j}$  (4)  $1.4\hat{i} + 4.8\hat{j}$

9. A plane mirror is moving with velocity  $4\hat{i} + 4\hat{j} + 8\hat{k}$ . A point object in front of the mirror moves with a velocity  $3\hat{i} + 4\hat{j} + 5\hat{k}$ . Here,  $\hat{k}$  is along the normal to the plane mirror and facing towards the object. The velocity of the image is

- (1)  $-3\hat{i} - 4\hat{j} + 5\hat{k}$  (2)  $3\hat{i} + 4\hat{j} + 11\hat{k}$   
(3)  $-4\hat{i} + 5\hat{j} + 11\hat{k}$  (4)  $7\hat{i} + 9\hat{j} + 3\hat{k}$

10. A thin prism of glass is placed in air and water respectively. If  $n_g = \frac{3}{2}$  and  $n_w = \frac{4}{3}$ , then the ratio of deviation produced by the prism for a small angle of incidence when placed in air and water separately is:

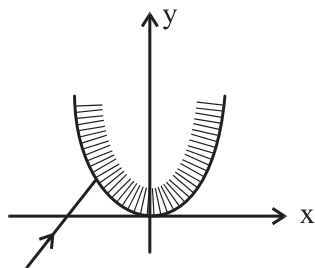
(1) 9 : 8 (2) 4 : 3  
(3) 3 : 4 (4) 4 : 1

11. Let the  $zx$ -plane be the boundary between two transparent media. Medium 1 in  $z \leq 0$  has a refractive index of  $\sqrt{2}$  and medium 2 with  $z > 0$  has a refractive index of  $\sqrt{3}$ . A ray of light in medium 1 given by the vector  $A = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$  is incident on the plane of separation. The angle of refraction in medium 2 is

(1)  $45^\circ$  (2)  $60^\circ$   
(3)  $75^\circ$  (4)  $30^\circ$

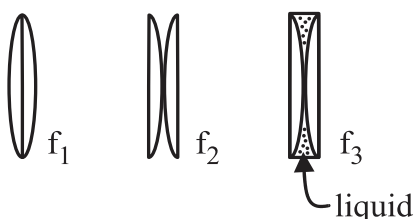
12. You are given a parabolic mirror whose inner surface is silvered so that its outer surface behaves like a mirror. The equation of the curve formed by its intersection with  $x$ - $y$  plane is given by  $y = \frac{x^2}{4}$ .

A ray travelling in  $x$ - $y$  plane along line  $y = x + 3$ , hits the mirror in second quadrant and gets reflected. The unit vector in the direction of reflected ray is:



(1)  $\frac{1}{\sqrt{2}}(-\hat{i} - \hat{j})$  (2)  $\frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$   
(3)  $\frac{1}{\sqrt{2}}(-\hat{i} + \hat{j})$  (4) None of these

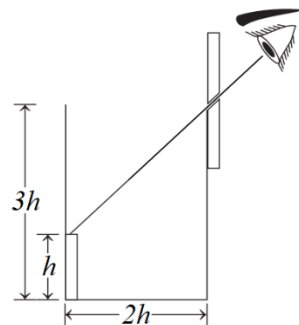
13. The following figure shows different arrangements of two identical pieces of thin plano-convex lenses. The refractive index of the liquid used is less than that of the glass. The effective focal lengths in the three cases are related as:



(1)  $f_1 = f_2 > f_3$  (2)  $f_1 \neq f_2 \neq f_3$   
(3)  $f_1 = f_2 < f_3$  (4)  $f_1 = f_2 = f_3$

14. An observer can see through a pin-hole the top end of a thin rod of height  $h$ , placed as shown in the figure. The beaker height is  $3h$  and its radius  $h$ . When the beaker is filled with a liquid up to a height  $2h$ , he can see the lower end of the rod. Then the refractive index of the liquid is

(1)  $\frac{5}{2}$   
(2)  $\sqrt{\frac{5}{2}}$   
(3)  $\sqrt{\frac{3}{2}}$   
(4)  $\frac{3}{2}$



15. The excitation energy of a hydrogen like ion to its first excited state is 40.8 eV. The energy needed to remove the electron from the ion in the ground state is

(1) 54.4 eV  
(2) 62.6 eV  
(3) 72.6 eV  
(4) 58.6 eV

16. An electron in a hydrogen atom makes a transition from first excited state to ground state. The equivalent current due to circulating electron

(1) increases 4 times  
(2) decreases 4 times  
(3) increases 8 times  
(4) decreases 8 times

17. If  $E_n$  and  $L_n$  denote the total energy and the angular momentum of an electron in the  $n^{\text{th}}$  orbit of Bohr atom, then

(1)  $E_n \propto L_n$  (2)  $E_n \propto \frac{1}{L_n}$   
(3)  $E_n \propto L_n^2$  (4)  $E_n \propto \frac{1}{L_n^2}$

18. A moving hydrogen atom makes an head-on collision with a stationary hydrogen atom. Before collision, both atoms are in ground state and after collision they move together. The minimum value of the kinetic energy of the moving hydrogen atom, such that one of the atoms reaches one of the excitation state is

(1) 20.4 eV  
(2) 10.2 eV  
(3) 54.4 eV  
(4) 13.6 eV

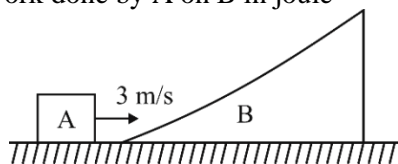
19. Consider the following nuclear reaction,  
 $X^{200} \longrightarrow A^{110} + B^{90} + \text{Energy}$   
 If the binding energy per nucleon for  $X$ ,  $A$  and  $B$  are 7.4 MeV, 8.2 MeV and 8.2 MeV respectively, the energy released will be  
 (1) 90 MeV (2) 110 MeV  
 (3) 200 MeV (4) 160 MeV

20. A radioactive isotope is being produced at a constant rate  $A$ . The isotope has a half-life  $T$ . Initially, there are no nuclei, after a time  $t \gg T$ , the number of nuclei becomes constant. The value of this constant is

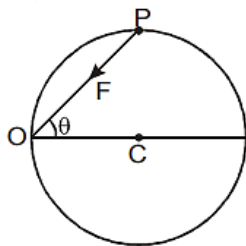
- (1)  $AT$  (2)  $\frac{A}{T} \ln 2$   
 (3)  $AT \ln 2$  (4)  $\frac{AT}{\ln 2}$

[Section – B]

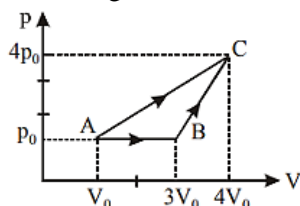
21. In the figure shown A is of mass 1 kg and B of mass 2 kg. A moves with velocity 3 m/s and rises on B which is initially at rest. All the surfaces are smooth. By the time A reaches the highest point on B, find the work done by A on B in joule



22. A particle P is moving on a circle under the action of only one force, which always acts towards a fixed point O lying on the circumference. Find ratio of  $\frac{d^2\theta}{dt^2}$  to  $\left(\frac{d\theta}{dt}\right)^2$  at the moment when  $\theta = 45^\circ$ . (C is centre of circle)

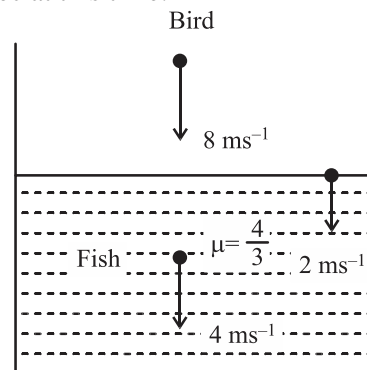


23. A certain quantity of ideal gas takes up 56J of heat in the process AB and 360 J in the process AC. What is the number of degrees of freedom of the gas.

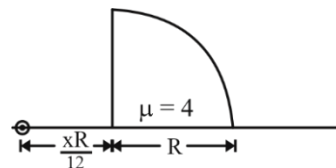


24. A solid sphere of radius  $R$  has a charge  $Q$  distributed in its volume with a charge density  $\rho = kr^a$ , where  $k$  and  $a$  are constants and  $r$  is the distance from its centre. If the electric field at  $r = \frac{R}{2}$  is  $\frac{1}{8}$  times that at  $r = R$ , find the value of  $a$

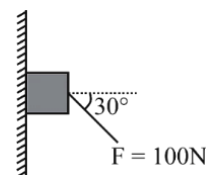
25. Speed of bird with respect to fish in  $\text{ms}^{-1}$  at given instant, if water surface is moving downward 2 m/sec at this time:



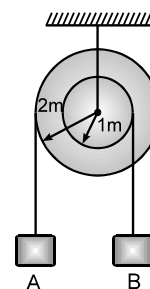
26. Find the value of  $x$  so that image of object formed at infinite from a quarter cylinder of refractive index  $\mu = 4$



27. A force of 100 N is applied on a block of mass 3 kg as shown in figure. The coefficient of friction between the wall and the surface of the block is  $1/4$ . Calculate frictional force in nearest integer acting on the block.



28. In the pulley system shown in the figure, if radii of the bigger and smaller pulley are 2 m and 1 m respectively and the acceleration of block A is  $5 \text{ m/s}^2$  in the downward direction, then the acceleration of block B will be  $\frac{N}{10}$ , then find N:



29. A block of mass 2kg is pulled by a constant power 100W is placed on a rough horizontal plane. The frictional coefficient between block and surface is 1. Find the maximum velocity of block.

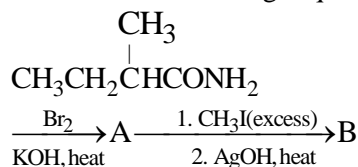
30. A particle is projected from the ground at an angle of  $30^\circ$  with the horizontal with an initial speed of 20 m/sec. At what time will the velocity vector of the projectile be perpendicular to the initial velocity (in seconds)?

## Section-II (CHEMISTRY)

### [Section - A]

- 31.** The molar conductances of HCl, NaCl and  $\text{CH}_3\text{COONa}$  are 426, 126 and  $91 \Omega^{-1} \text{cm}^2 \text{mol}^{-1}$  respectively. The molar conductance for  $\text{CH}_3\text{COOH}$  is:
- (1)  $561 \Omega^{-1} \text{cm}^2 \text{mol}^{-1}$
  - (2)  $391 \Omega^{-1} \text{cm}^2 \text{mol}^{-1}$
  - (3)  $261 \Omega^{-1} \text{cm}^2 \text{mol}^{-1}$
  - (4)  $612 \Omega^{-1} \text{cm}^2 \text{mol}^{-1}$
- 32.** The molar freezing point constant for water is  $1.86^\circ\text{C}/\text{mole}$ . If 342 gm of cane sugar ( $\text{C}_{12}\text{H}_{22}\text{O}_{11}$ ) is dissolved in 1000 gram of water, the solution will freeze at:
- (1)  $-1.86^\circ\text{C}$
  - (2)  $1.86^\circ\text{C}$
  - (3)  $-3.92^\circ\text{C}$
  - (4)  $2.42^\circ\text{C}$
- 33.** An alloy of copper, silver and gold is found to have copper constituting the C.C.P. lattice. If silver atoms occupy the edge centres and gold is present at body centre, the alloy has a formula:
- (1)  $\text{Cu}_4\text{Ag}_2\text{Au}$
  - (2)  $\text{Cu}_4\text{Ag}_4\text{Au}$
  - (3)  $\text{Cu}_4\text{Ag}_3\text{Au}$
  - (4)  $\text{CuAgAu}$
- 34.** Which of following processes best describes the purification of muddy water by addition of alum:
- (1) Absorption
  - (2) Adsorption
  - (3) Dialysis
  - (4) Co-agulation
- 35.** For a chemical reaction  $\text{A} \rightarrow \text{products}$ , the rate of reaction doubles when the concentration of A is increased by 4 times. The order of reaction is:
- (1) 4
  - (2) 0
  - (3)  $1/2$
  - (4) 1
- 36.** In Lassaingne's test for detection of nitrogen, the blue colour is due to the compound:
- (1)  $\text{Na}_4[\text{Fe}(\text{CN})_6]$
  - (2)  $\text{NaCN}$
  - (3)  $\text{Na}_3[\text{Fe}(\text{CN})_6]_4$
  - (4)  $\text{Fe}_4[\text{Fe}(\text{CN})_6]_3$
- 37.** In Bohr's model, atomic radius of the first orbit is  $y$ , the radius of the 3<sup>rd</sup> orbit, is:
- (1)  $y/3$
  - (2)  $y$
  - (3)  $3y$
  - (4)  $9y$
- 38.** Which of the following set contains species having same angle around the central atom:
- (1)  $\text{SF}_4$ ,  $\text{CH}_4$ ,  $\text{NH}_3$
  - (2)  $\text{NF}_3$ ,  $\text{BCl}_3$ ,  $\text{NH}_3$
  - (3)  $\text{BF}_3$ ,  $\text{NF}_3$ ,  $\text{AlCl}_3$
  - (4)  $\text{BF}_3$ ,  $\text{BCl}_3$ ,  $\text{BBr}_3$
- 39.** Which one of the following has the regular tetrahedral structure? (Atomic no.: B = 5, S = 16, Ni = 28, Xe = 54)
- (1)  $\text{BF}_4^-$
  - (2)  $\text{SF}_4$
  - (3)  $\text{XeF}_4$
  - (4)  $[\text{Ni}(\text{CN})_4]^{2-}$
- 40.** From  $\text{B}_2\text{H}_6$ , all the following can be prepared except
- (1)  $\text{B}_2\text{O}_3$
  - (2)  $\text{H}_3\text{BO}_3$
  - (3)  $\text{B}_2(\text{CH}_3)_6$
  - (4)  $\text{NaBH}_4$
- 41.** The non existence of  $\text{PbI}_4$  is due to:
- (1) Small size of  $\text{Pb}^{4+}$  ions and large size  $\text{I}^-$  ions
  - (2) Highly oxidising power of  $\text{Pb}^{4+}$  ions
  - (3) Highly reducing power of  $\text{I}^-$  ions
  - (4) Both (2) and (3)
- 42.** Match List-I with List-II and select the correct answer using codes given below the lists:
- | List-I                                    | List-II                |
|---|------------------------|
| Metal ions                                | Magnetic moment (B.M.) |
| (1) $\text{Cr}^{3+}$                      | i. $\sqrt{35}$         |
| (2) $\text{Fe}^{2+}$                      | ii. $\sqrt{30}$        |
| (3) $\text{Ni}^{2+}$                      | iii. $\sqrt{24}$       |
| (4) $\text{Mn}^{2+}$                      | iv. $\sqrt{15}$        |
|   | v. $\sqrt{8}$          |
| (1) (1) – i, (2) – iii, (3) – v, (4) – iv |                        |
| (2) (1) – ii, (2) – iii, (3) – v, (4) – i |                        |
| (3) (1) – iv, (2) – iii, (3) – v, (4) – i |                        |
| (4) (1) – iv, (2) – v, (3) – iii, (4) – i |                        |
- 43.** In the smelting of  $\text{Cu}_2\text{S}$ , the flux is:
- (1)  $\text{CaSiO}_3$
  - (2)  $\text{FeO}$
  - (3)  $\text{FeSiO}_3$
  - (4)  $\text{SiO}_2$
- 44.** Extraction of silver from  $\text{Ag}_2\text{S}$  by the use of sodium cyanide is an example of:
- (1) roasting
  - (2) hydrometallurgy
  - (3) electrometallurgy
  - (4) smelting
- 45.** 2-Butyne and 1-Butyne show resemblance in all except:
- (1) Both decolourise alkaline  $\text{KMnO}_4$
  - (2) Both turn bromine water colourless
  - (3) Both undergo addition reaction
  - (4) Both form white precipitate with Tollen's reagent
- 46.**  $\text{CH}_3\text{CCl}_3 \xrightarrow[\text{hydrolysis}]{\text{alkaline}} \text{A} \xrightarrow[\text{AgNO}_3]{\text{NH}_4\text{OH}} \text{B} \xrightarrow[\text{CCl}_4]{\text{Br}_2} \text{C}$
- A and C in the above sequence are respectively:
- (1) Acetic acid, ethyl bromide
  - (2) Acetic acid, ethyl chloride
  - (3) Acetic acid, methyl bromide
  - (4) Acetic acid, methyl chloride

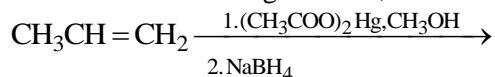
47. Consider the following sequence of reactions:



The major product (B) is:

- (1)  $\text{CH}_3\text{CH}_2\text{CH}=\text{CH}_2$
- (2)  $\begin{array}{c} \text{CH}_3 \\ | \\ \text{CH}_3\text{CH}_2\text{CHCON}(\text{CH}_3)_2 \end{array}$
- (3)  $\text{CH}_3\text{CH}=\text{CHCH}_3$
- (4)  $\begin{array}{c} \text{CH}_3 \\ | \\ \text{CH}_3\text{CH}_2\text{CHN}(\text{CH}_3)_2 \end{array}$
48. Simplest amino acid is:
- (1) Lysine                      (2) Glycine
- (3) Leucine                      (4) Alanine

49. Consider the following reaction,

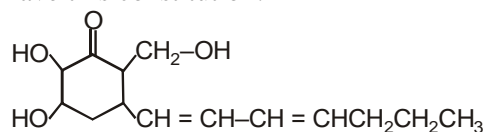


The product formed in the reaction is:

- (1)  $\begin{array}{c} \text{CH}_3 \quad \text{O} \\ | \quad || \\ \text{CH}_3\text{CH}-\text{O}-\text{C}-\text{CH}_3 \end{array}$
- (2)  $\begin{array}{c} \text{CH}_3 \\ | \\ \text{CH}_3-\text{CH}-\text{OCH}_3 \end{array}$
- (3)  $\begin{array}{c} \text{CH}_3\text{CH}-\text{CH}_2 \\ | \quad | \\ \text{OCH}_3 \quad \text{OCOCH}_3 \end{array}$
- (4)  $\begin{array}{c} \text{CH}_3 \quad \text{H} \quad \text{CH}_2 \\ | \quad | \quad | \\ \text{CH}_3\text{COO} \quad \text{OCOCH}_3 \end{array}$
50. The general order of reactivity of carbonyl compounds for nucleophilic addition reactions is:
- (1)  $\text{H}_2\text{C}=\text{O} > \text{RCHO} > \text{ArCHO} > \text{R}_2\text{C}=\text{O} > \text{Ar}_2\text{C}=\text{O}$
- (2)  $\text{ArCHO} > \text{Ar}_2\text{C}=\text{O} > \text{RCHO} > \text{R}_2\text{C}=\text{O} > \text{H}_2\text{C}=\text{O}$
- (3)  $\text{Ar}_2\text{C}=\text{O} > \text{R}_2\text{C}=\text{O} > \text{ArCHO} > \text{RCHO} > \text{H}_2\text{C}=\text{O}$
- (4)  $\text{H}_2\text{C}=\text{O} > \text{R}_2\text{C}=\text{O} > \text{Ar}_2\text{C}=\text{O} > \text{RCHO} > \text{ArCHO}$

## [Section - B]

51. How many atoms are there in a unit cell of Mg which forms hexagonal crystals, there being a face-centred atom in each end of the unit cell and 3 completely enclosed atoms within the unit cell:
52. If the rate of reaction becomes 2 times for every  $10^\circ\text{C}$  rise in temperature, by what factor the rate of reaction increases when temperature is increased from  $30^\circ\text{C}$  to  $80^\circ\text{C}$ :
53. How many spherical nodes are present in a 4s orbital in hydrogen atom:
54. Equivalent weight of oxidising agent will be:  
 $2\text{H}_2 + \text{O}_2 \rightarrow 2\text{H}_2\text{O}$
55. The oxidation states of Fe atom in compound A, B, C respectively are x, y, z. Then sum of x, y, z is:  
 $[\text{Fe}(\text{H}_2\text{O})_5\text{NO}]\text{SO}_4, \text{K}_3[\text{Fe}(\text{CN})_6], [\text{Fe}(\text{CO})_5]$
56. A naturally occurring substance has the constitution shown. How many stereoisomers may have this constitution:



57. The complex  $\text{CoCl}_3 \cdot 5\text{NH}_3$  in aqueous solution ionizes to give a total number of ions equal to:
58. Find the number of electrons shared in the formation of nitrogen molecule.
59. How many aldols will be formed by  $\text{CH}_3\text{CHO}$  and  $\text{CH}_3-\text{CH}_2-\text{CHO}$ :
60. Maximum Number of atoms lying in same plane in  $\text{SF}_4$  will be:

## Section-III (MATHEMATICS)

### [Section - A]

61. If the sum of the series  
 $20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + \dots$  upto  $n^{\text{th}}$  term is 448  
 and the  $n^{\text{th}}$  term is negative, then:
- (1)  $n^{\text{th}}$  term is  $-4\frac{2}{5}$       (2)  $n = 41$
- (3)  $n^{\text{th}}$  term is  $-4$       (4)  $n = 60$
62. If  $z_1, z_2$  are complex numbers such that  
 $\text{Re}(z_1) = |z_1 - 1|, \text{Re}(z_2) = |z_2 - 1|$  and  
 $\arg(z_1 - z_2) = \frac{\pi}{6}$ , then  $\text{Im}(z_1 + z_2)$  is equal to:
- (1)  $2\sqrt{3}$                       (2)  $\frac{\sqrt{3}}{2}$
- (3)  $\frac{1}{\sqrt{3}}$                       (4)  $\frac{2}{\sqrt{3}}$

63. Let  $a, b, c \in \mathbb{R}$  be such that  $a^2 + b^2 + c^2 = 1$ . If  $a \cos \theta = b \cos \left( \theta + \frac{2\pi}{3} \right) = c \cos \left( \theta + \frac{4\pi}{3} \right)$ , where  $\theta = \frac{\pi}{9}$ , then the angle between the vectors  $\hat{a}\hat{i} + b\hat{j} + c\hat{k}$  and  $b\hat{i} + c\hat{j} + a\hat{k}$  is:
- (1)  $\frac{\pi}{9}$  (2)  $\frac{2\pi}{3}$   
(3) 0 (4)  $\frac{\pi}{2}$
64. Let the latus rectum of the parabola  $y^2 = 4x$  be the common chord to the circles  $C_1$  and  $C_2$  each of them having radius  $2\sqrt{5}$ . Then, the distance between the centres of the circles  $C_1$  and  $C_2$  is:
- (1)  $4\sqrt{5}$  (2)  $8\sqrt{5}$   
(3) 8 (4) 12
65. Let  $R_1$  and  $R_2$  be two relations defined as follows:  
 $R_1 = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 \in \mathbb{Q}\}$  and  
 $R_2 = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 \notin \mathbb{Q}\}$ , where  $\mathbb{Q}$  is the set of all rational numbers. Then:
- (1) Neither  $R_1$  nor  $R_2$  is transitive.  
(2)  $R_1$  is transitive but  $R_2$  is not transitive.  
(3)  $R_1$  and  $R_2$  are both transitive  
(4)  $R_2$  is transitive but  $R_1$  is not transitive.
66. If the value of integral  $\int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}} dx$  is  $\frac{k}{6}$ , then  $k$  is equal to:
- (1)  $2\sqrt{3} + \pi$  (2)  $3\sqrt{2} - \pi$   
(3)  $2\sqrt{3} - \pi$  (4)  $3\sqrt{2} + \pi$
67. If  $x^3 dy + xy dx = x^2 dy + 2y dx$ ;  $y(2) = e$  and  $x > 1$ , then  $y(x)$  is equal to
- (1)  $\frac{\sqrt{e}}{2}$  (2)  $\frac{1}{2} + \sqrt{e}$   
(3)  $\frac{3}{2}\sqrt{e}$  (4)  $\frac{3}{2} + \sqrt{e}$
68. The probability that a randomly chosen 5 - digit number is made from exactly two digits is:
- (1)  $\frac{134}{10^4}$  (2)  $\frac{121}{10^4}$   
(3)  $\frac{150}{10^4}$  (4)  $\frac{135}{10^4}$
69. If the surface area of a cube is increasing at a rate of  $3.6 \text{ cm}^2/\text{sec}$ , retaining its shape; then the rate of change of its volume (in  $\text{cm}^3/\text{sec}$ ), when the length of a side of the cube is 10 cm, is:
- (1) 9 (2) 18  
(3) 10 (4) 20
70. The set of all real values of  $\lambda$ , for which the quadratic equations,  $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$  always have exactly one root in the interval  $(0, 1)$  is
- (1) (0, 2) (2) (2, 4]  
(3)  $(-3, -1)$  (4) (1, 3]
71. If  $\Delta ABC$  have vertices  $A(-1, 7)$ ,  $B(-7, 1)$  and  $C(5, -5)$ , then its orthocenter has coordinates:
- (1)  $\left(\frac{3}{5}, -\frac{3}{5}\right)$  (2)  $(-3, 3)$   
(3)  $(3, -3)$  (4)  $\left(-\frac{3}{5}, \frac{3}{5}\right)$
72. Let  $p, q, r$  be three statements such that the truth value of  $(p \wedge q) \rightarrow (\sim q \vee r)$  is  $F$ . Then the truth values of  $p, q, r$  are respectively:
- (1)  $T, F, T$  (2)  $T, T, F$   
(3)  $F, T, F$  (4)  $T, T, T$
73. The plane which bisects the line joining the points  $(4, -2, 3)$  and  $(2, 4, -1)$  at right angles also passes through the point:
- (1)  $(0, -1, 1)$  (2)  $(4, 0, 1)$   
(3)  $(4, 0, -1)$  (4)  $(0, 1, -1)$
74. Let  $e_1$  and  $e_2$  be the eccentricities of the ellipse  $\frac{x^2}{25} + \frac{y^2}{b^2} = 1 (b < 5)$  and the hyperbola,  $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$  respectively satisfying  $e_1 e_2 = 1$ . If  $\alpha$  and  $\beta$  are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair  $(\alpha, \beta)$  is equal to
- (1) (8, 10) (2)  $\left(\frac{24}{5}, 10\right)$   
(3)  $\left(\frac{20}{3}, 12\right)$  (4) (8, 12)
75. Let  $A$  be a  $3 \times 3$  matrix such that  $\text{adj} A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$  and  $B = \text{adj}(\text{adj} A)$ . If  $|A| = \lambda$  and  $\left| (B^{-1})^T \right| = \mu$ , then the ordered pair,  $(|\lambda|, \mu)$  is equal to:
- (1)  $\left(9, \frac{1}{9}\right)$  (2) (3, 81)  
(3)  $\left(9, \frac{1}{81}\right)$  (4)  $\left(3, \frac{1}{81}\right)$
76. If  $\int \sin^{-1} \left( \sqrt{\frac{x}{1+x}} \right) dx = A(x) \tan^{-1}(\sqrt{x}) + B(x) + C$ , where  $C$  is a constant of integration, then the ordered pair  $(A(x), B(x))$  can be
- (1)  $(x-1, -\sqrt{x})$  (2)  $(x-1, \sqrt{x})$   
(3)  $(x+1, -\sqrt{x})$  (4)  $(x+1, \sqrt{x})$



77. Let  $x_i (1 \leq i \leq 10)$  be ten observations of a random variable  $X$ . If  $\sum_{i=1}^{10} (x_i - p) = 3$  and  $\sum_{i=1}^{10} (x_i - p)^2 = 9$  where  $0 \neq p \in R$ , then the standard deviation of these observations is

- (1)  $\sqrt{\frac{3}{5}}$  (2)  $\frac{4}{5}$   
 (3)  $\frac{7}{10}$  (4)  $\frac{9}{10}$

78. If the term independent of  $x$  in the expansion of  $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$  is  $k$ , then  $18k$  is equal to:

- (1) 9  
 (2) 5  
 (3) 7  
 (4) 11

79. Suppose  $f(x)$  is a polynomial of degree four, having critical points at  $-1, 0, 1$ . If  $T = \{x \in R \mid f(x) = f(0)\}$ , then the sum of squares of all the elements of  $T$  is:

- (1) 8  
 (2) 6  
 (3) 2  
 (4) 4

80.  $\lim_{x \rightarrow a} \frac{(a+2x)^{\frac{1}{3}} - (3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}} - (4x)^{\frac{1}{3}}} (a \neq 0)$  is equal to:

- (1)  $\left(\frac{2}{3}\right)^{\frac{4}{3}}$  (2)  $\left(\frac{2}{9}\right)^{\frac{4}{3}}$   
 (3)  $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$  (4)  $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$

### [Section – B]

81. If the tangent to the curve,  $y = e^x$  at a point  $(c, e^c)$  and the normal to the parabola,  $y^2 = 4x$  at the point  $(1, 2)$  intersect at the same point on the  $x$ -axis, then the value of  $c$  is.....
82. The total number of 3-digit numbers, whose sum of digits is 10, is

83. Let  $S$  be the set of all integer solutions,  $(x, y, z)$ , of the system of equation  $x - 2y + 5z = 0$   
 $-2x + 4y + z = 0$   
 $-7x + 14y + 9z = 0$   
 Such that  $15 \leq x^2 + y^2 + z^2 \leq 150$ . Then the number of elements in the set  $S$  is equal to.....

84. Let a plane  $P$  contain two lines  $\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j}), \lambda \in R$  and  $\vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k}), \mu \in R$ . If  $Q(\alpha, \beta, \gamma)$  is the foot of the perpendicular drawn from the point  $M(1, 0, 1)$  to  $P$ , then  $3(\alpha + \beta + \gamma)$  equals\_\_\_\_\_

85. If  $m$  arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4<sup>th</sup> A.M. is equal to 2<sup>nd</sup> G.M, then  $m$  is equal to\_

86. Let  $f: [-1, 1] \rightarrow R$  be defined as  $f(x) = ax^2 + bx + c$  for all  $x \in [-1, 1]$ , where  $a, b, c \in R$  such that  $f(-1) = 2, f'(-1) = 1$  and for  $x \in (-1, 1)$  the maximum value of  $f''(x)$  is  $\frac{1}{2}$ . If  $f(x) \leq \alpha \ x \in [-1, 1]$ , then the least value of  $\alpha$  is equal to

87. Let the normal at all the points on a given curve pass through a fixed point  $(a, b)$ . If the curve passes through  $(3, -3)$  and  $(4, -2\sqrt{2})$ , and given that  $a - 2\sqrt{2}b = 3$ , then  $(a^2 + b^2 + ab)$  is equal to

88. In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at  $(0, 5\sqrt{3})$ , then the length of its latus rectum is

89. If the function  $f$  given by  $f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$ , for some  $a \in R$  is increasing in  $(0, 1]$  and decreasing in  $[1, 5)$ , then a root of the equation,  $\frac{f(x) - 14}{(x-1)^2} = 0 (x \neq 1)$  is

90. Let  $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$  and  $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$  be two  $2 \times 1$  matrices with real entries such that  $A = XB$ , where  $X = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$ , and  $k \in R$ . If  $(a_1^2 + a_2^2) = \frac{2}{3}(b_1^2 + b_2^2)$  and  $(k^2 + 1)b_2^2 \neq -2b_1b_2$  then the value of  $k$  is

