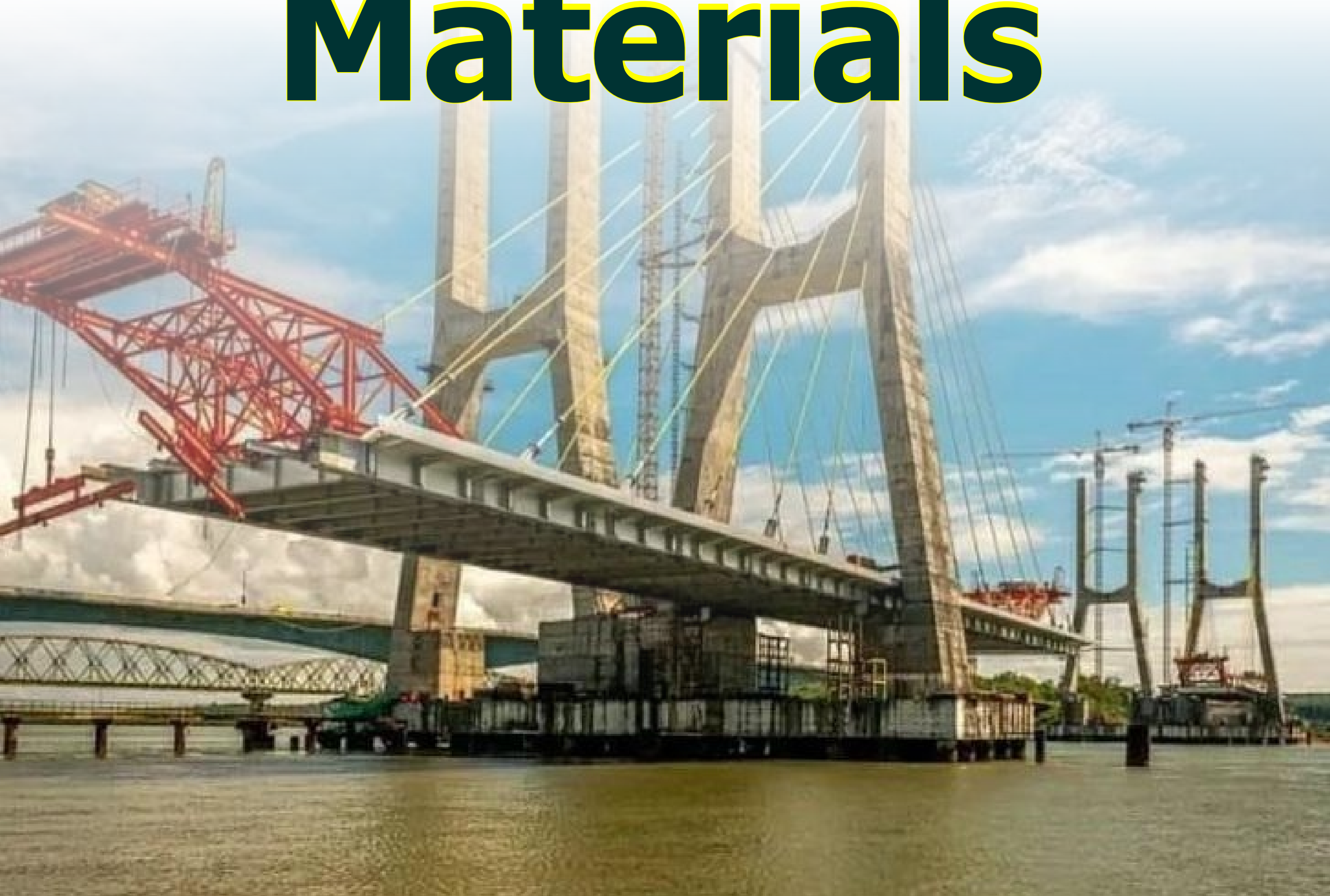
Strength of Materials



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The publication is designed to provide accurate and authoritative information with regard to the subject matter covered.

(This Module shall only be Used for Educational Purpose.)

Strength of Materials

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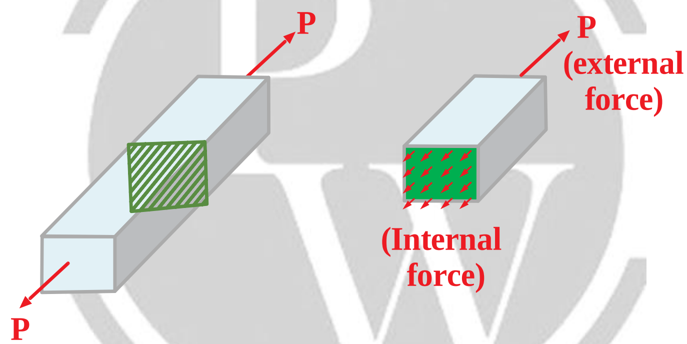
1 INTRODUCTION & PROPERTIES OF MATERIAL

1.1 Introduction to stress

STRESS

- It is the measure of internal resistance of the body against external load.
- It is defined as internal force per unit area at a given point on any plane.
- SI unit is Pa (N/m^2).



$$\sum F_x = 0$$

Fig. 1.1 Stress in a bar subjected to axial force

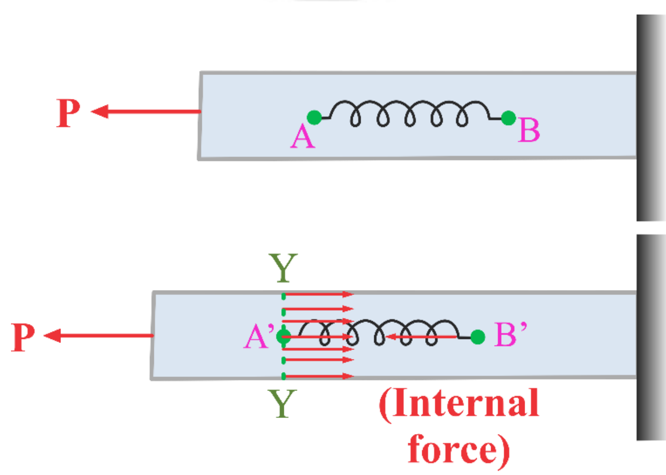


Fig. 1.2 Internal resisting force due to external load

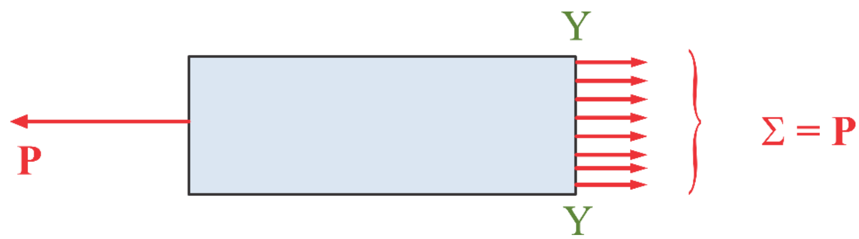


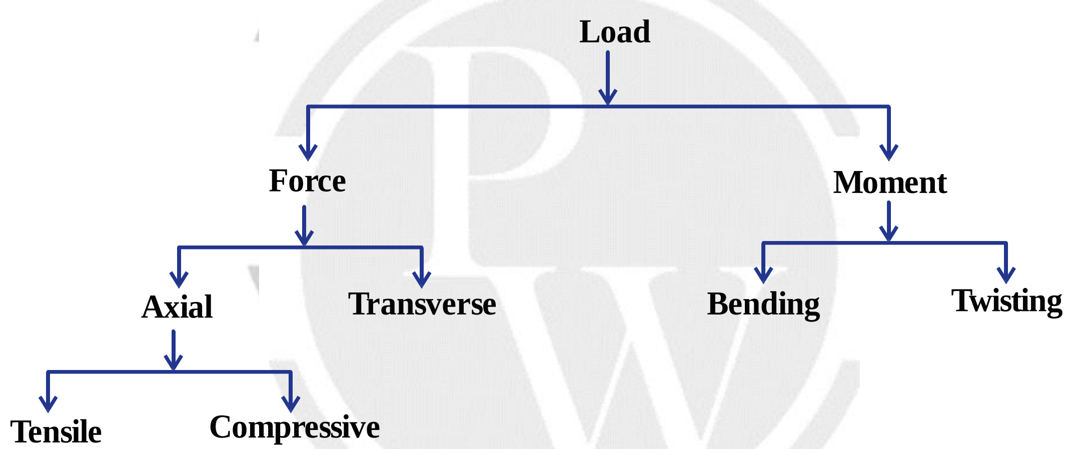
Fig. 1.3 Cut section of the bar to represent internal resisting force

$$\sum F_x = 0$$

$$Stress = \frac{P}{A} \qquad \frac{N}{m^2} = Pa$$

$$\frac{N}{mm^2} = \frac{N}{10^{-6}m^2} = 10^6\frac{N}{m^2} = MPa$$

1.2 Types of Loads



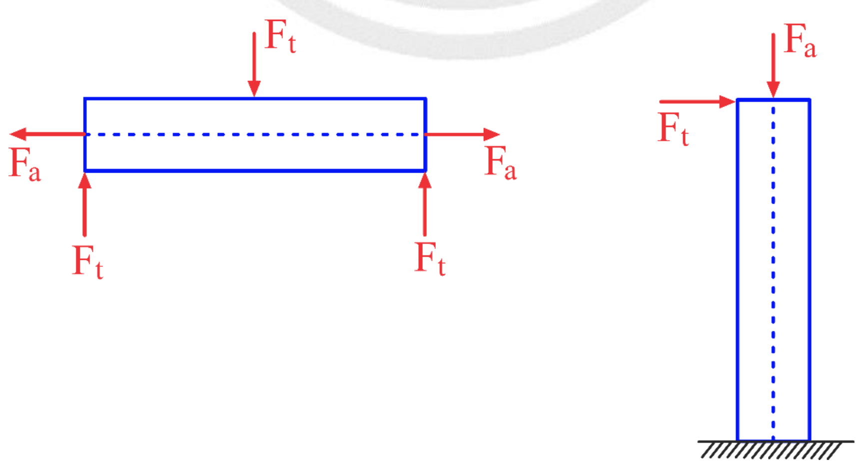


Fig. 1.4 Axial and Transverse force representation

- $F_a \rightarrow$ Axial force
- $F_t \rightarrow$ Transverse force

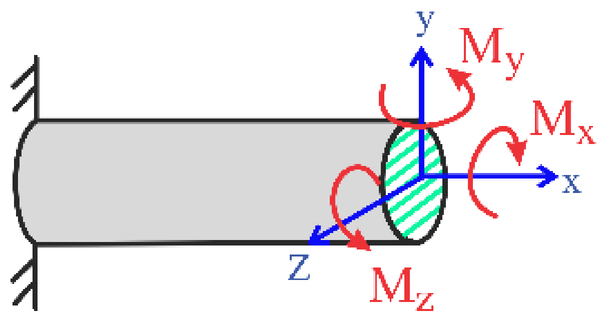
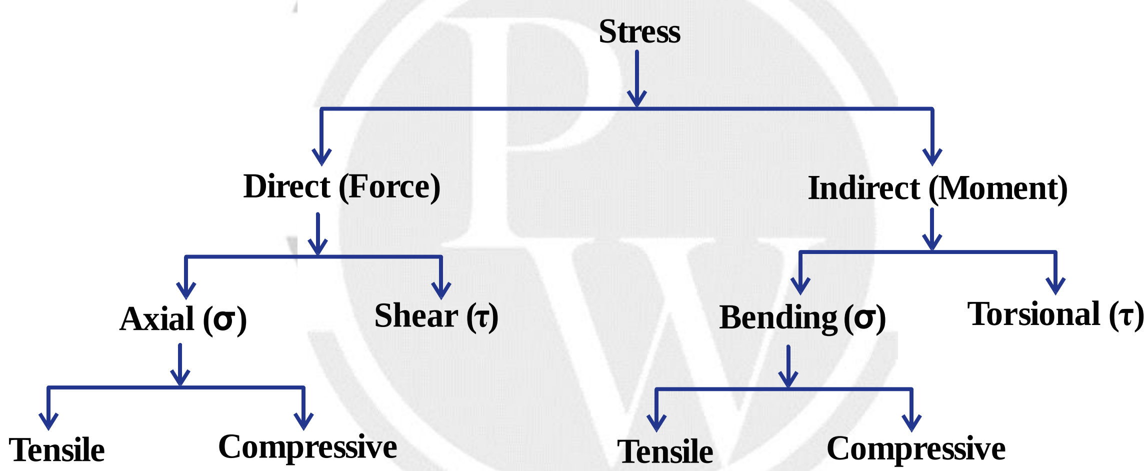


Fig. 1.5 Bending and Twisting Couples

- x ⟶ Normal to Plane
- y/z ⟶ Parallel to Plane
- M_x ⟶ Twisting/Torsional moment/Torque
- M_y/M_z ⟶ Bending moment

1.3 Types of stresses



1.3.1 Direct stress

- Direct stresses are developed due to external force directly acting on the plane.

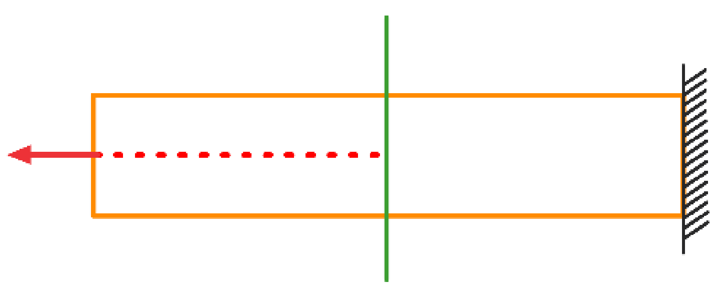


Fig. 1.6 Bar subjected to direct axial stress

1.3.2 Indirect stress

- Indirect stresses are developed due to moments, when external force is not passing through the centroid of the plane.

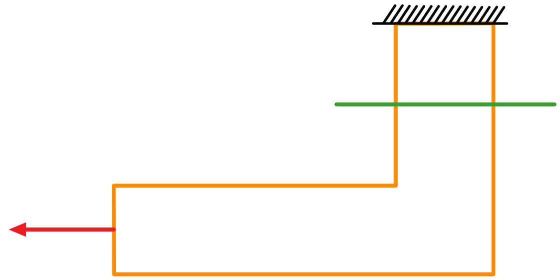


Fig. 1.7 Bar subjected to indirect stress

1.3.3 Normal & Shear Stress

- Normal stress (σ) is developed when the internal forces are acting normal (perpendicular) to the plane and shear stress (τ) is developed when the internal forces are acting parallel to the plane.
- If the internal force is acting at some angle to the plane, both normal stress and shear stress are developed on the plane

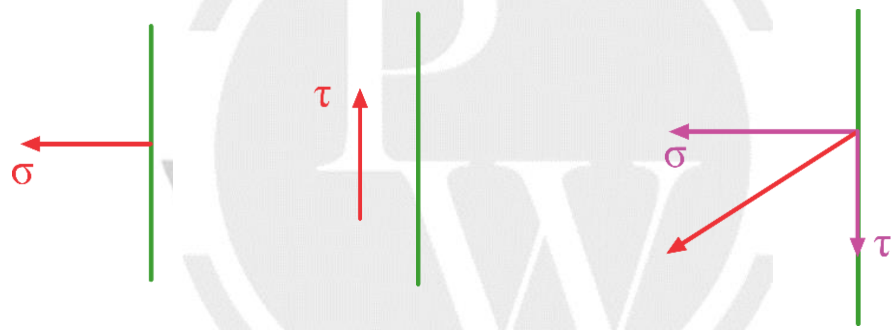
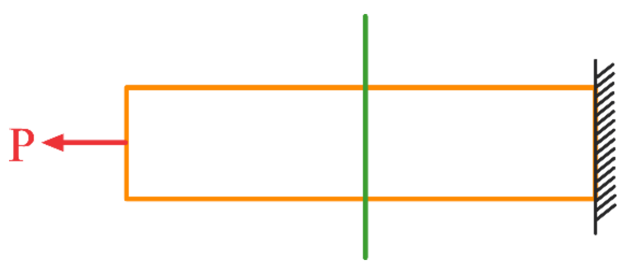


Fig. 1.8 Normal and shear stress representation

1.3.4 Direct Axial Stress



$$\sigma = \frac{P}{A}$$

Fig. 1.9 Direct axial stress

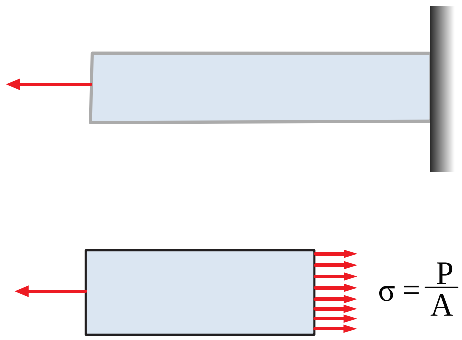


Fig. 1.10 Direct axial tensile stress

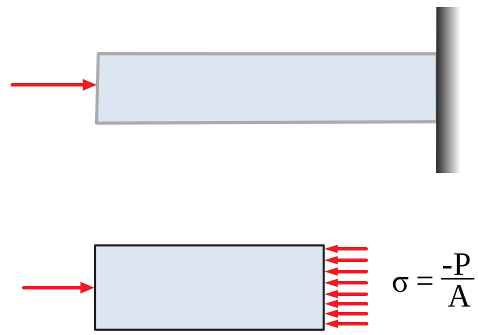


Fig. 1.11 Direct axial compressive stress

1.3.5 Direct Shear Stress

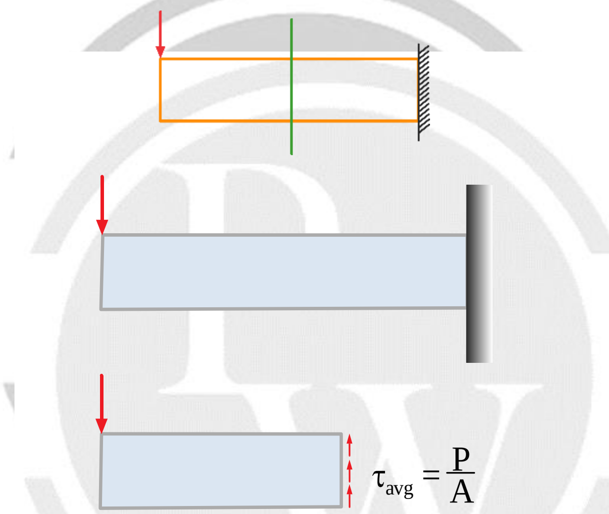


Fig. 1.12 Direct Shear stress

1.3.6 Bending Stress

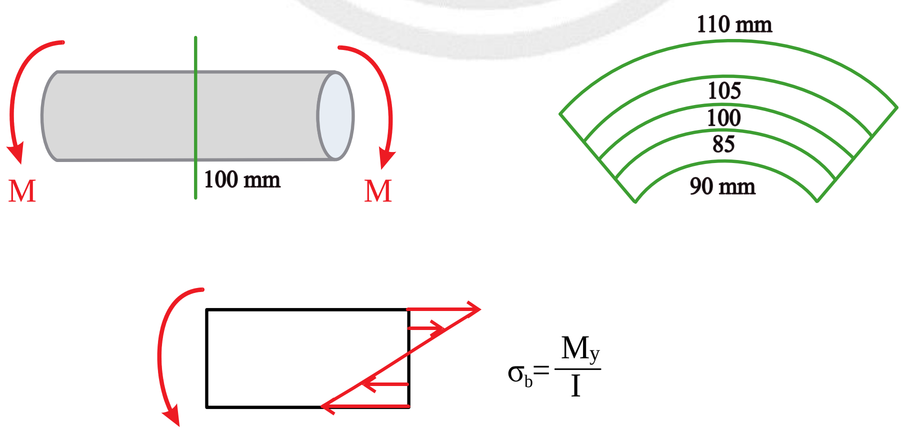
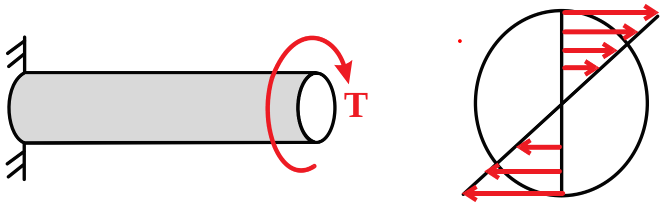


Fig. 1.13 Bending stress

1.3.7 Torsional Stress



$$\tau = \frac{Tr}{J}$$

Fig. 1.14 Torsional stress

1.4 Stress analysis under general loading

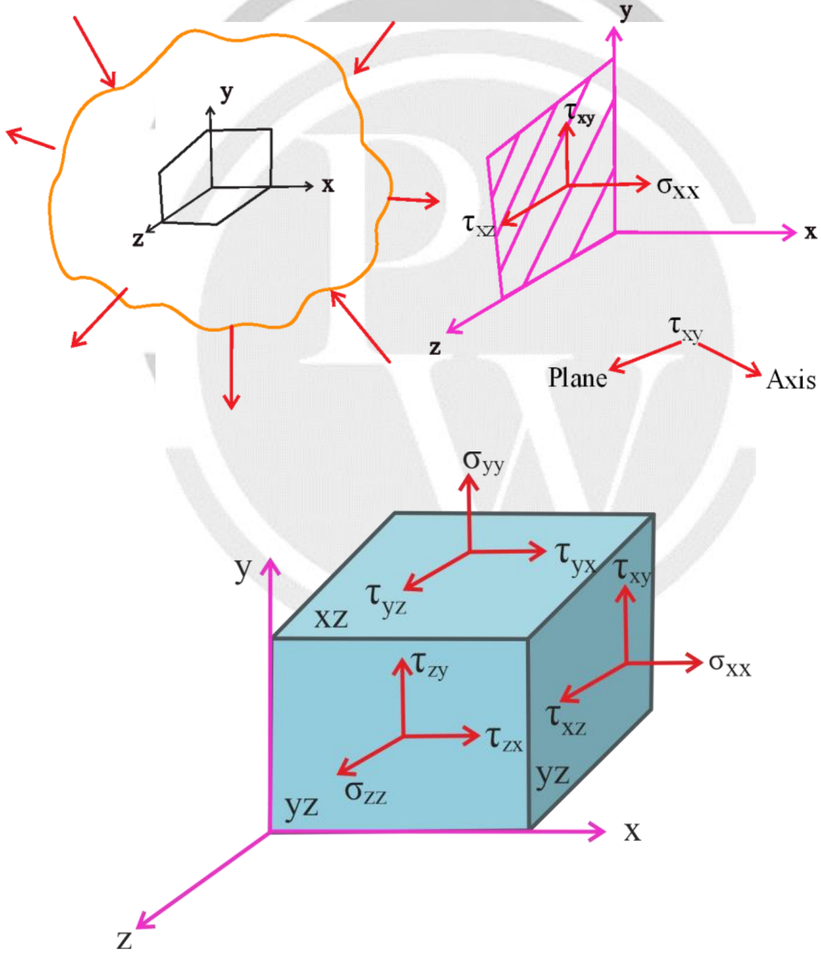
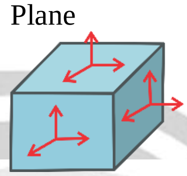


Fig. 1.15 Triaxial state of stress at a point

$$\begin{matrix} & & x & y & z \\ (yz) & x & \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ (xz) & y & \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ (xy) & z & \tau_{zx} & \tau_{zy} & \tau_{zz} \end{matrix} \qquad \left.\begin{matrix} \tau_{xy} = \tau_{yx} \\ \tau_{xz} = \tau_{zx} \\ \tau_{yz} = \tau_{zy} \end{matrix}\right\} \text{Complimentary Shear Stress}$$

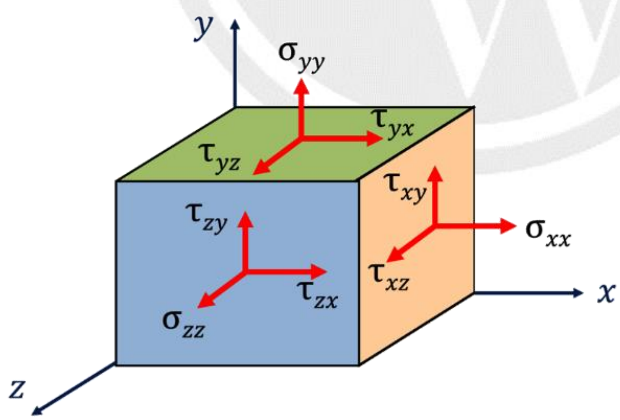
(Symmetric matrix)

Scalar	Vector	Tensor
• Magnitude	• Magnitude • Direction $\rightarrow \begin{Bmatrix} 100 \\ 0 \\ 0 \end{Bmatrix} \begin{matrix} i \\ j \\ k \end{matrix}$	• Magnitude • Direction • Plane 

1.5 Tensor

- Tensors are quantities which are characterized by magnitude, direction and plane.
- Examples – Stress, Strain and Moment of Inertia

1.5.1 Stress Tensor



$$\begin{matrix} & x & y & z \\ x & \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ y & \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ z & \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{matrix}$$

Fig. 1.16 Stress Tensor at a point

- 6 Independent Stress components
- 3 Dependent Stress components

1.6 Complimentary shear stress

- If there is a shear stress on one plane, there must be an equal and opposite shear stress on the perpendicular plane known as complimentary shear stress.

$\Sigma Fy = 0$

$\Sigma Fx = 0$

$\Sigma M = 0$

$(\tau_{xy} \times bc) \times a = (\tau_{yx} \times ab) \times c$

$(\tau_{xy} \times abc) = \tau_{yx} \times abc$

$\tau_{xy} = \tau_{yx}$

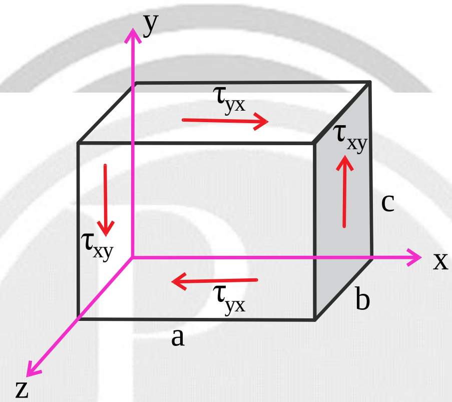
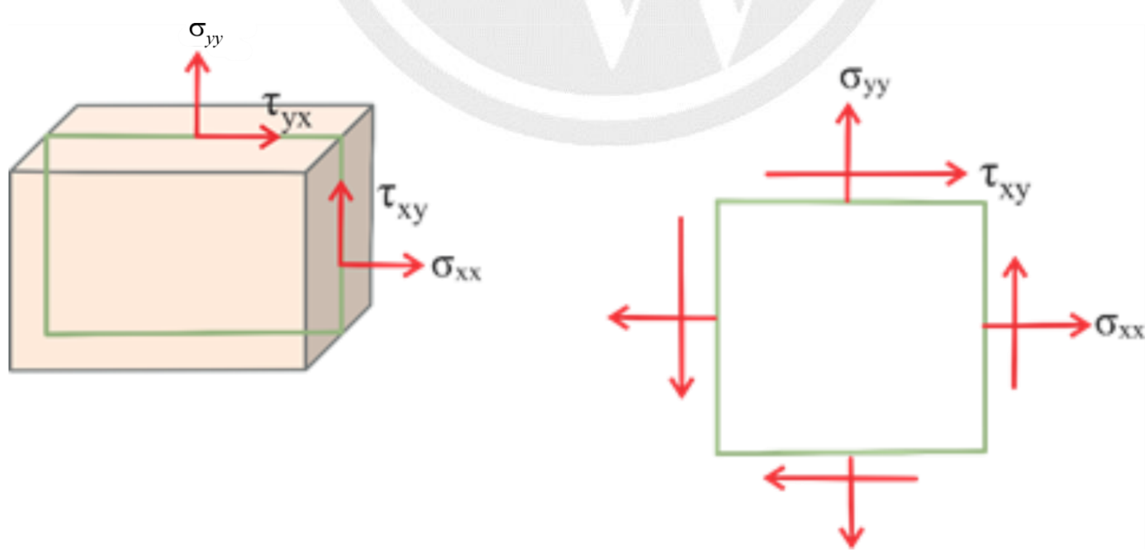


Fig. 1.17 Cross shear or complimentary shear

1.7 Bi-axial stress (Plane Stress)



$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix}$$

Fig. 1.18 Bi-axial state of stress or Plane stress condition

- Examples of Plane Stress

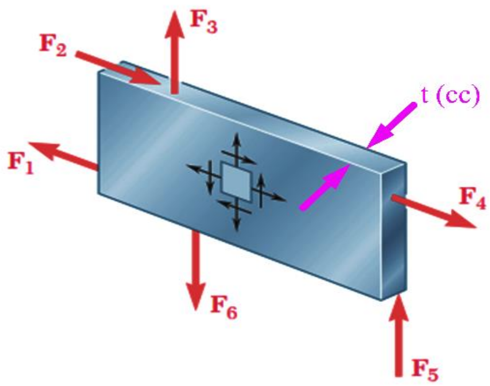


Fig. 1.19. Thin plate subjected to forces acting in the midplane of the plate

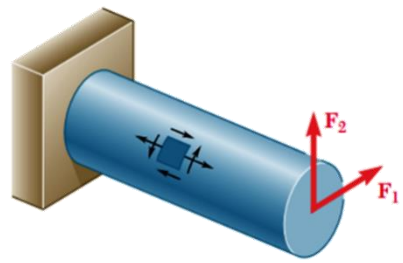
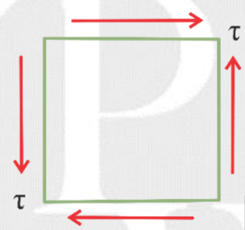


Fig. 1.20. On the free surface of a structural element or machine

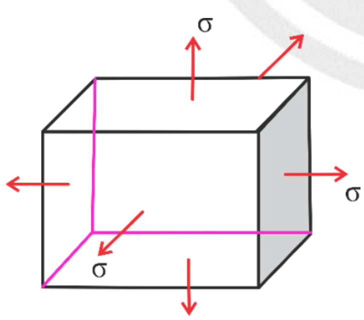
1.8 Pure Shear Stress



$$\begin{bmatrix} 0 & \tau \\ \tau & 0 \end{bmatrix}$$

Fig. 1.21 Pure shear state of stress

1.9 Hydrostatic Stress

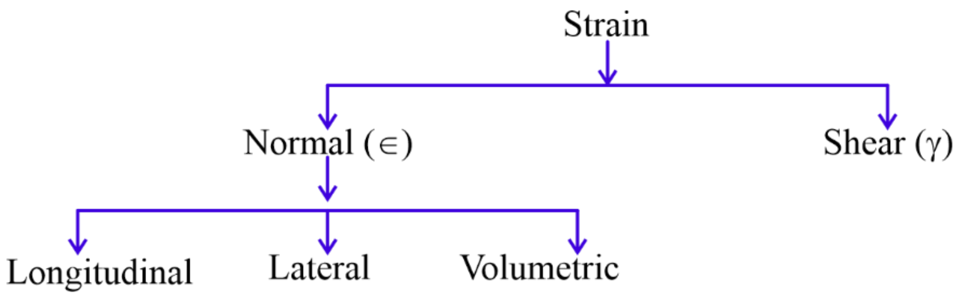


$$\begin{bmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{bmatrix}$$

$$\sigma_x = \sigma_y = \sigma_z = \sigma$$

Fig. 1.22 Hydrostatic state of stress

1.10 Types of strain



1. 11 Normal Strain (ϵ)

- It is the measure of change in size.
- It is defined as change in a dimension per unit original dimension.

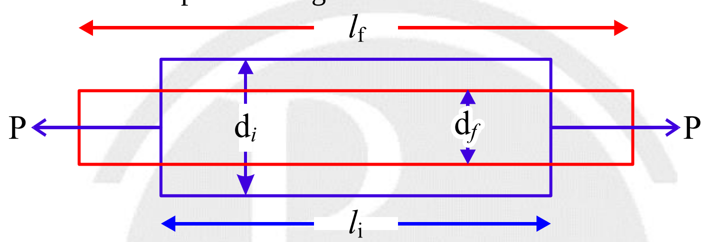


Fig.1.23 Bar Subjected to pure axial loading

$$\varepsilon_{long} = \varepsilon_x = \frac{\Delta l}{l}$$

$$\varepsilon_{lateral} = \varepsilon_y = \varepsilon_z = \frac{\Delta d}{d}$$

$$\varepsilon_{vol} = \frac{\Delta v}{v} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

1.12 Shear Strain (γ)

- It is the measure of change in shape.
- It is a defined as change in angle between two mutually perpendicular planes.

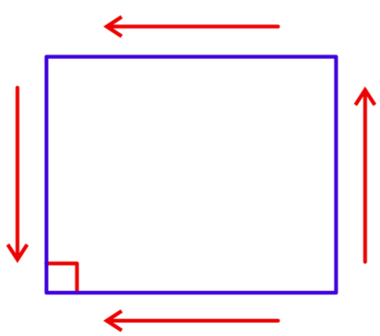


Fig. 1.24 Pure shear state of stress

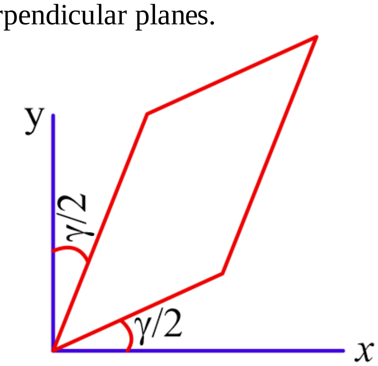


Fig. 1.25 Distorted member due to shear load

1.12.1 Strain Tensor:

$\sigma \rightarrow \varepsilon$

$\tau \rightarrow \frac{\gamma}{2}$

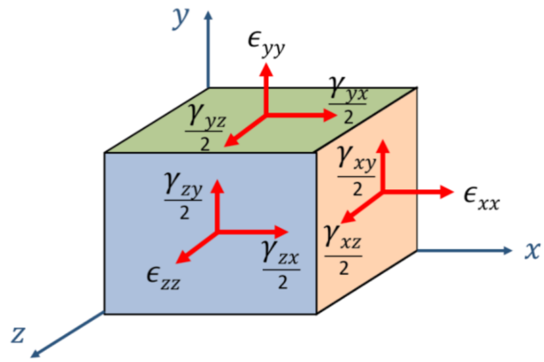


Fig.1.26 Triaxial state of strain at a point

1.13 Types of Material

Types of Material

- Homogeneous
- Non homogenous

- Isotropic
- Anisotropic
- Orthotropic

1.13.1 Homogenous Material:

Material properties are same at all points in the same direction.

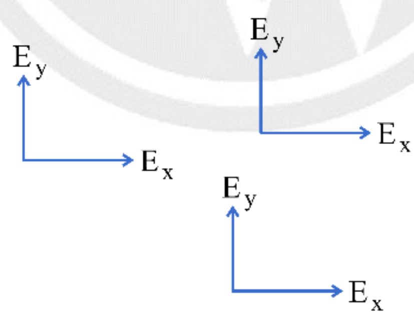


Fig.1.27 Homogeneous Material

1.13.2 Non - Homogenous Material:

Material properties are different at all points in the same direction.

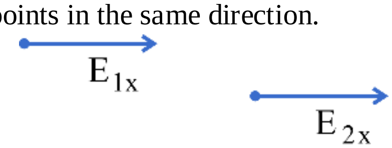


Fig.1.28 Non-Homogeneous Material

1.13.3 Isotropic Material:

Material properties are same in every direction at a point.

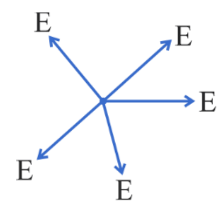


Fig.1.29 Isotropic Material

1.13.4 Anisotropic Material:

Material properties are different in every direction at a point.

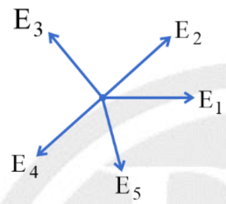


Fig.1.30 Anisotropic Material

1.13.5 Orthotropic Material:

Material properties are different in mutually perpendicular directions at a point.

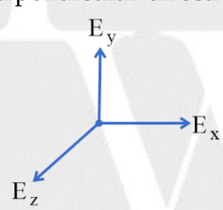
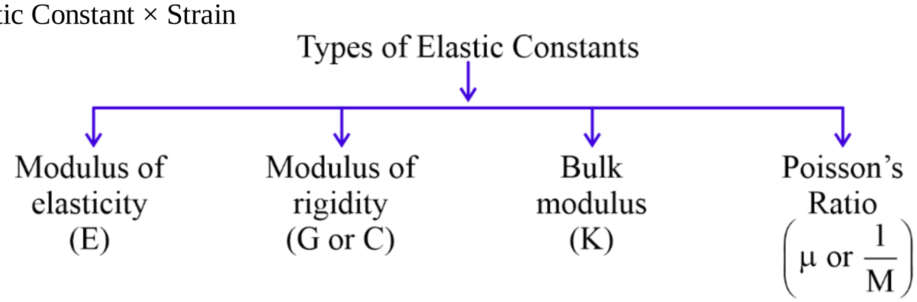


Fig.1.31 Orthotropic Material

1.14 Elastic Constants

- Elastic constants are material properties.
- They are the relation between the stress and strain.
- The magnitude of strain under external load, depends on elastic constants of the material.
  Stress = Elastic Constant × Strain



1.14.1 Modulus of Elasticity/Young's Modulus (E):

Ratio of normal stress and normal (longitudinal) strain.

$$E = \frac{\sigma}{\varepsilon} \qquad 1\ GP_a = 10^9 P_a = 10^3 MP_a$$

$$E_{steel} = 200GP_a$$

$$E_{Cu} = 100GP_a$$

$$E_{Al} = 70GP_a$$

1.14.2 Modulus of Rigidity/Shear Modulus (C or G):

Ratio of shear stress and shear strain.

$$G = \frac{\tau}{\gamma}$$

1.14.3 Bulk Modulus (K):

Ratio of hydrostatic stress and volumetric strain.

$$K = \frac{\sigma}{\varepsilon_v}$$

1.14.4 Poisson's Ratio (1/m):

Ratio of magnitude of lateral strain and longitudinal strain.

$\mu = 0$ to 0.5

$\mu = 0 \rightarrow$ Cork

$\mu = 0.5 \rightarrow$ Rubber

$\mu = 0.25$ to $0.33 \rightarrow$ Metals

$$\mu = -\frac{\varepsilon_{lateral}}{\varepsilon_{long}}$$

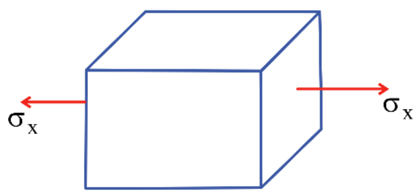
1.15 Relation between Elastic Constants

(a) $E = 2G(1 + \mu)$

(b) $E = 3K(1 - 2\mu)$

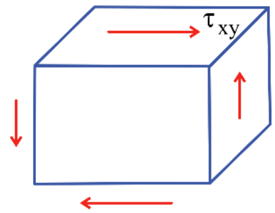
1.16 Hooke's Law

- Stress is directly proportional to corresponding strain within proportional limit.
- Constants of proportionality are the elastic constants.



$$\sigma_x \propto \varepsilon_x$$
$$\sigma_x = E.\varepsilon_x$$

Fig. 1.32 Member subjected to axial load



$$\tau_{xy} \propto \gamma_{xy}$$
$$\tau_{xy} = G.\gamma_{xy}$$

Fig. 1.33 Member subjected to shear load

Generalised Hooke's Law

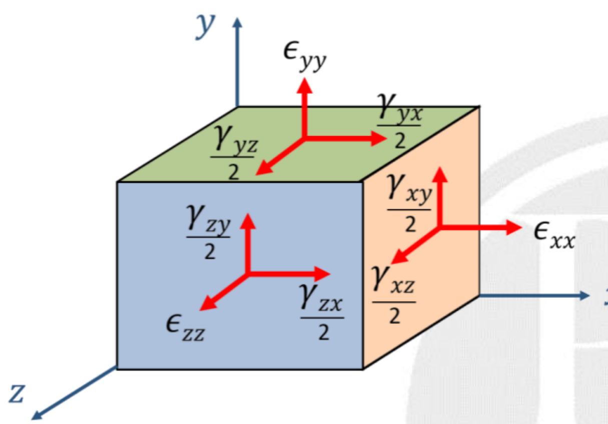


Fig. 1.34 Triaxial state of strain at a point

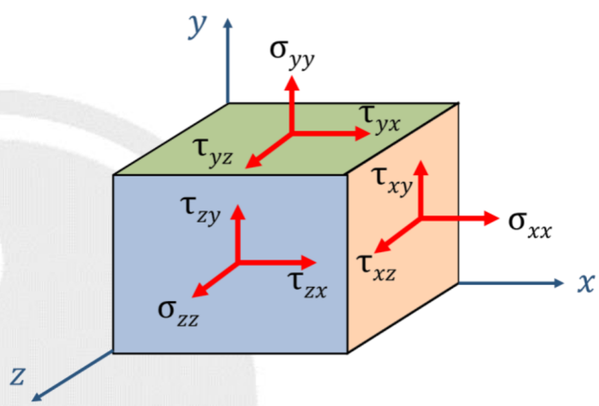


Fig. 1.35 Triaxial state of stress at a point

$$\varepsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E} \qquad \gamma_{xy} = \frac{\tau_{xy}}{G}$$
$$\varepsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E} \qquad \gamma_{xz} = \frac{\tau_{xz}}{G}$$
$$\varepsilon_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} \qquad \gamma_{yz} = \frac{\tau_{yz}}{G}$$

Minimum number of Independent Elastic Constants

Material	Triaxial Stress	Biaxial Stress
Isotropic	2	2
Orthotropic	9	4
Anisotropic	21	6

1.17 Stress – Strain Curve for Ductile Materials

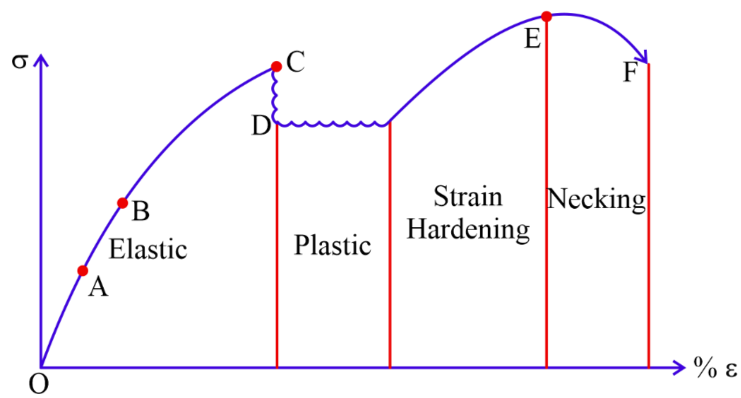


Fig. 1.36 Engineering stress vs Engineering strain diagram for ductile material

1.18 Stress – Strain Curve for Brittle Materials

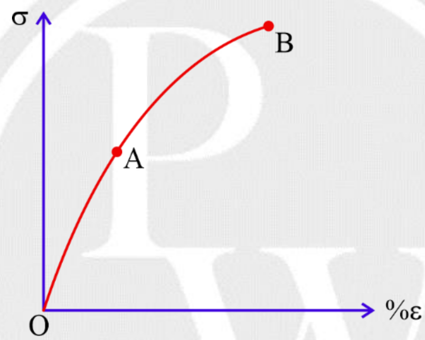


Fig. 1.37 Stress vs Strain diagram for Brittle Material

1.18.1 Strength:

Maximum magnitude of stress that the material can sustain without failure.

(i) Yield Strength: Maximum stress that the material can sustain without yielding.

(ii) Ultimate Strength: Maximum stress that the material can sustain without fracture.

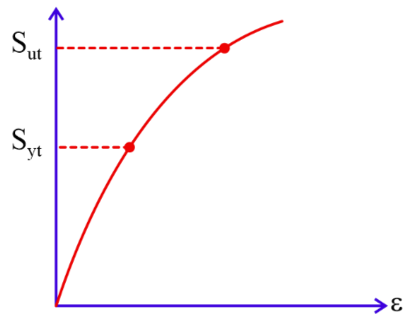


Fig. 1.38 Engineering stress vs Engineering strain diagram

Ductile: $S_{yt} = S_{yc} > S_{ys}$

Brittle: $S_{uc} > S_{us} > S_{ut}$

Ductility:

Ability of a material to deform plastically.

% change in length $= \frac{\Delta l}{l} \times 100\%$

% change in area $= \frac{\Delta A}{A} \times 100\%$

Resilience:

Ability of a material to absorb strain energy without permanent deformation.

Modulus of resilience = S.E./Volume

$= \frac{1}{2}\sigma \times \varepsilon = \frac{1}{2} \times \sigma \times \frac{\sigma}{E}$

$= \frac{\sigma^2}{2E}$

$= \frac{{S_{yt}}^2}{2E}$

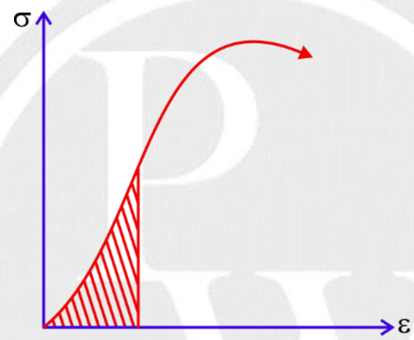


Fig. 1.39 Modulus of Resilience

Toughness:

Ability of a material to absorb strain energy without fracture.

Modulus of toughness = Toughness / Volume

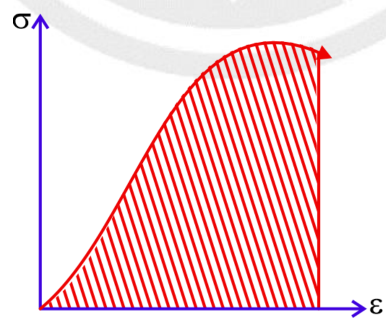


Fig. 1.40 Modulus of Toughness

1.19 True Stress - Strain

$\sigma_E = \frac{P}{A_0}$ $\sigma_T = \frac{P}{A_f}$

$\varepsilon_E = \frac{\Delta l}{l_0}$ $\varepsilon_T = l_n \frac{l_f}{l_0} = l_n \frac{A_0}{A_f}$

- $\varepsilon_T = \ell_n(1 + \varepsilon_E)$
- $\sigma_T = \sigma_E(1 + \varepsilon_E)$

Here σ_E = Engineering stress, σ_T = True stress, ε_E = Engineering strain, ε_T = True strain

1.20 Power Law

$\sigma_T = k\varepsilon_T{}^n$

k → strength coefficient

n → Strain hardening exponent (0 to 1)

At ultimate point

$$\varepsilon_T = n$$

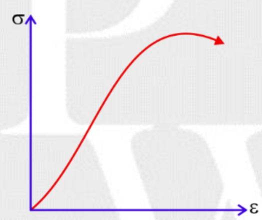


Fig. 1.41 Stress vs strain diagram

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2 AXIALLY LOADED MEMBERS

2.1 Axially Loaded Members

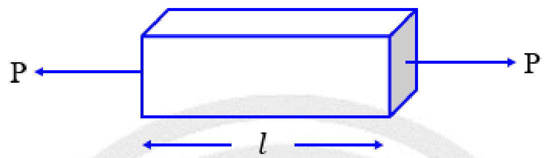
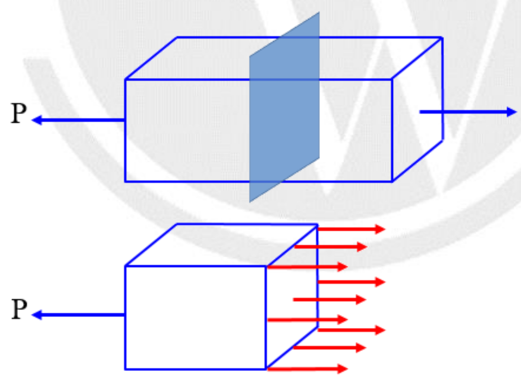


Fig.2.1 Bar subjected to axial load

Assumptions

- Material of the bar is homogeneous and isotropic.
- Bar is of constant cross-sectional area.
- Axial load passes through the centroid of the cross section.
- Stresses are within proportional limit.



$$\sigma = \frac{P}{A}$$

Fig.2.2 Stress representation on the plane of cross section

Elongation of bar

$$\Delta l = \frac{Pl}{AE} \qquad \Delta l = \int_0^l \frac{Pdx}{AE}$$

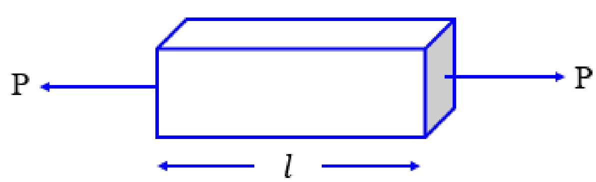


Fig.2.3 Bar subjected to Pure Axial load

Calculation of Internal Force P

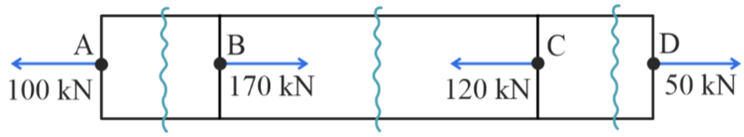


Fig.2.4 Bar subjected to variable axial loads

$$P_{AB} = 100\,kN$$
$$P_{BC} = 100 - 170 = -70kN$$
$$P_{CD} = 50kN$$

2.2 Elongation of Prismatic Bar due to self-weight

$$\Delta l = \frac{\gamma l^2}{2E}$$

or

$$\Delta l = \frac{Wl}{2AE}$$

Here,

W = self-weight

= γAL

γ = weight/volume 'or' weight density

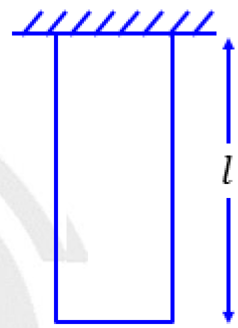


Fig.2.5 Prismatic Bar under self-weight

2.3 Elongation of Conical Bar due to self-weight

$$\Delta l = \frac{\gamma l^2}{6E}$$

or

$$\Delta l = \frac{Wl}{2AE}$$

Here W = self-weight $= \frac{\gamma Al}{3}$

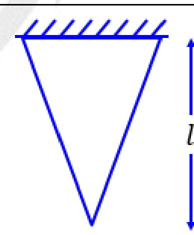


Fig.2.6 Conical Bar under self-weight

2.4 Elongation of Circular Tapered Bar

$$\Delta l = \frac{Pl}{\frac{\pi}{4} d_1 . d_2 . E}$$

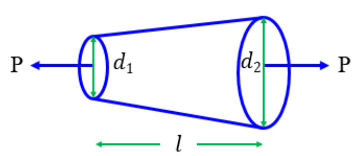


Fig.2.7 Elongation of circular tapered bar under axial load

2.5 Elongation of Rectangular Tapered Bar

$$\Delta l = \frac{Pl}{\dfrac{(b_2 - b_1)}{\ln\left(\dfrac{b_2}{b_1}\right)}.t.E}$$

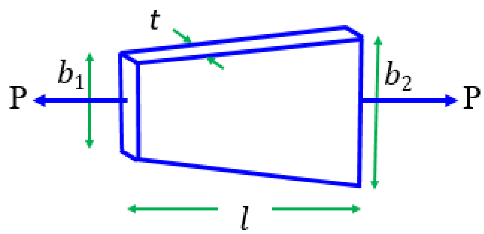


Fig.2.8 Elongation of rectangular tapered bar under axial load

2.6 Statically Indeterminate Bars

$$\Sigma F_x = 0$$

$$R_A = 100 + R_C$$

Compatibility Equation:

$$\Delta l = 0$$

$$\Delta l_{AB} + \Delta l_{BC} = 0$$

$$\frac{\mathrm{R_A} \times l_{\mathrm{AB}}}{\mathrm{A_{AB}} \times \mathrm{E_{AB}}} + \frac{\mathrm{R_C} \times l_{\mathrm{BC}}}{\mathrm{A_{BC}} \times \mathrm{E_{BC}}}$$

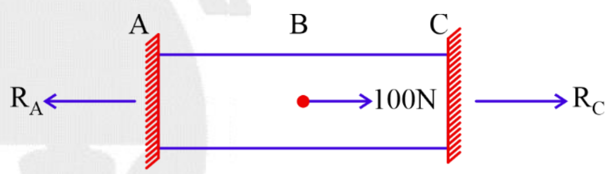


Fig.2.9 Bars fixed at both the ends, under axial load

2.7 Thermal Stress

$$\Delta l_{\text{thermal}} = l \propto \Delta T$$

$\propto \rightarrow$ coefficient of thermal expansion $(/°C)$

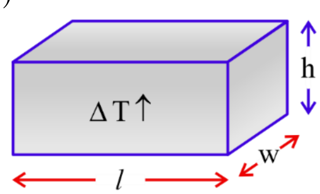


Fig.2.10 Free expansion of rectangular block

2.7.1 Thermal Stress in Bars fixed in one direction:

$$\Delta l_{total} = 0$$

$$\Delta l_{thermal} + \Delta l_{mech} = 0$$

$$(l \propto \Delta T) - \left(\frac{Pl}{AE}\right) = 0$$

$$\sigma = \frac{P}{A} = E \propto \Delta T$$

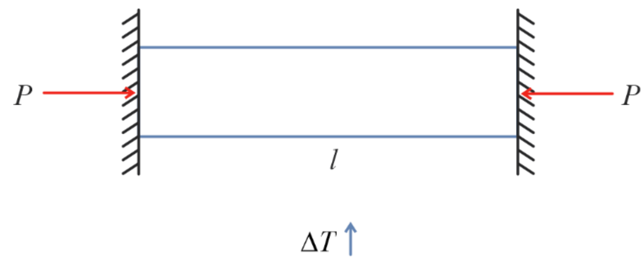


Fig.2.11 Thermal Stress in Bars fixed in one direction

2.7.2 Thermal Stress in Bars fixed in two directions:

$$\sigma_x = \sigma_y = \frac{E \propto \Delta T}{(1-v)}$$

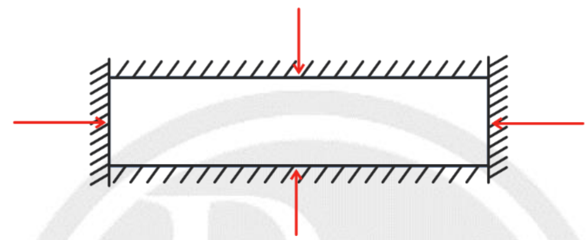
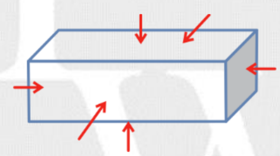


Fig.2.12 Thermal Stress in Bars fixed in two directions

2.7.3 Thermal Stress in Bars fixed in all directions:



$$\sigma_x = \sigma_y = \sigma_z = \frac{E \propto \Delta T}{(1-2v)}$$

Fig.2.13 Thermal Stress in Bars fixed in all directions

2.7.4 Thermal Stress in a Bars when there is a gap/yielding of supports

Fig.2.14 Thermal Stress in a Bars when there is a gap/yielding of supports

$$\Delta l_{total} = \delta$$

$$\Delta l_{thermal} + \Delta_{mech} = \delta$$

$$(l \propto \Delta T) - \left(\frac{Pl}{AE}\right) = \delta$$

2.7.5 Thermal Stress in a tapered bar fixed in one direction

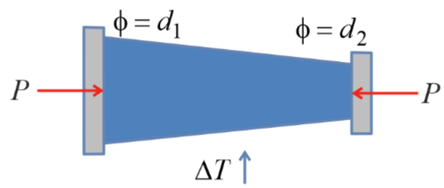


Fig.2.15 Thermal Stress in tapered bar fixed in one direction

$$\Delta l_{total} = 0$$

$$\Delta l_{thermal} + \Delta l_{mech} = 0$$

$$(l \propto \Delta T) - \left(\frac{Pl}{\frac{\pi}{4} d_1 d_2 E} \right) = 0$$

2.7.6 Thermal Stress in a compound bar

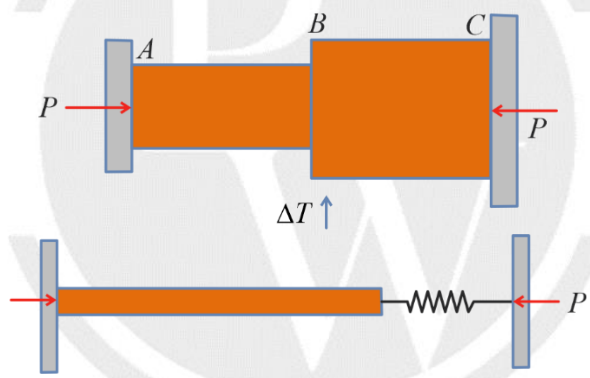


Fig.2.16 Thermal Stress in a compound bar

$$\Delta l_{mech} = \frac{Pl}{AE} + \frac{P}{k}$$

2.7.7 Thermal Stress in Composite Bar

$$\propto_{Al} > \propto_S$$

$$(\Delta l_{total})_S = (\Delta l_{total})_{Al}$$

$$(\Delta \propto \Delta T)_S + \left(\frac{Pl}{AE}\right)_S = (l \propto \Delta T)_{Al} - \left(\frac{Pl}{AE}\right)_{Al}$$

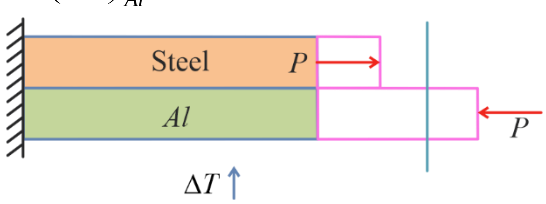


Fig.2.17 Thermal Stress in a composite bar

2.8 Strain Energy due to Axial Load

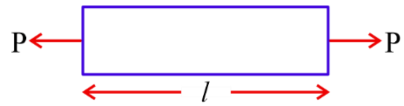


Fig.2.18 Strain Energy due to Axial Load

$$U = \frac{P^2 l}{2AE}$$

2.9 Axial Impact Load

$$\sigma_s = \frac{W}{A} \qquad \Delta l_s = \frac{Wl}{AE}$$

$$\sigma_I = \sigma_s \times IF$$

$$\Delta l_I = \Delta l_s \times IF$$

$$IF = 1 + \sqrt{1 + \frac{2h}{\Delta l_s}}$$

For suddenly applied load ($h \to 0$)

IF = 2

Here,

IF = Impact factor

σ_s = Stress due to static load

Δl_s = Elongation due to static load

σ_I = Stress due to impact load

Δl_I = Elongation due to impact load

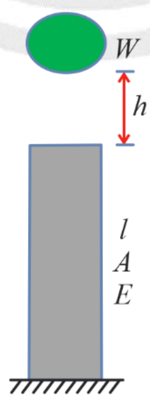


Fig.2.19 Member subjected to Impact axial load

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3 TORSION IN CIRCULAR SHAFTS

3.1 Torsion Equation

Assumptions

- Material of the shaft is homogeneous and isotropic.
- Stresses are within proportional limit.
- All the transverse sections remain plane and undistorted after twisting. In other words, the diameter of the shaft remains straight after twisting.

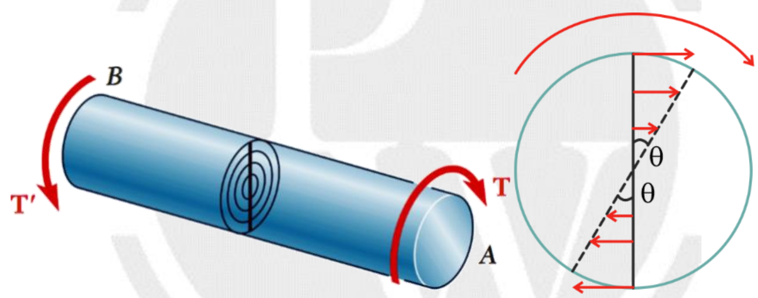

Fig.3.1 Shaft Subjected to pure torsion

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l}$$

J = Polar moment of inertia

r = radial distance from axis

θ = angle of twist (radians)

3.1.1 Maximum Shear Stress

$$\frac{T}{J} = \frac{\tau_{max}}{r_{max}}$$

$$\tau_{max} = \frac{T.r_{max}}{J} = \frac{T \times d/2}{\pi/32\, d^4}$$

$$\tau_{\max} = \frac{16T}{\pi d^3} \qquad \text{[For solid shaft]}$$

$$\tau_{\max} = \frac{16T}{\pi d^3\left(1-K^4\right)} \qquad \left(K = \frac{d_1}{d_0}\right) \rightarrow \text{[For hollow shaft]}$$

3.1.2 Angle of Twist

$$\frac{T}{J} = \frac{G\theta}{l}$$

$$\theta = \frac{Tl}{GJ} \qquad \text{(in radians)}$$

3.1.3 Polar Section Modulus

- It is the measure of the strength (maximum applicable torque) of shaft.
- It depends on the shape and size of the cross section.

$$\frac{T}{J} = \frac{\tau_{\max}}{r_{\max}}$$

$$T \propto \frac{J}{r_{\max}}$$

$$\frac{J}{r_{\max}} = Z_P$$

$Zp \uparrow \rightarrow T_R \uparrow \rightarrow \tau_{\max} \downarrow \rightarrow$ chances of failure $\downarrow$

For same area of cross-section

$$\left(z_p\right)_H > \left(z_p\right)_S$$

$$T_H > T_S$$

3.1.4 Torsional rigidity (GJ)

It is the measure of the resistance to deformation under twisting moment.

3.1.5 Torsional stiffness (*q*)

It is the magnitude of torque required for unit angle of twist.

$$q = \frac{T}{\theta} = \frac{GJ}{l}$$

3.1.6 Internal Torque

$$T_{AB} = 100\, Nm$$

$$T_{BC} = 100 - 240 = -140\, Nm$$

$$T_{CD} = 220\, Nm$$

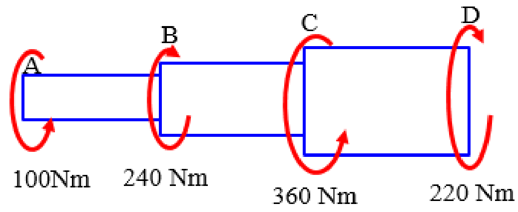

Fig.3.2 Shaft Subjected to variable twisting moments

3.2 Statically Indeterminate Shaft

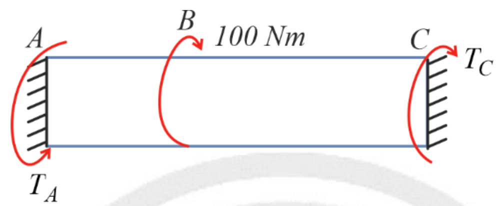

Fig.3.3 Shaft fixed at both the ends and subjected to twisting moments

$\Sigma M = 0$

$T_A = 100 + T_C$

Compatibility eq^n

$\theta_{AC} = 0$

$\theta_{AB} + \theta_{BC} = 0$

$$\frac{T_A \times l_{AB}}{G_{AB} \times J_{AB}} + \frac{T_C \times l_{BC}}{G_{BC} \times J_{BC}} = 0$$

3.3 Composite Shaft

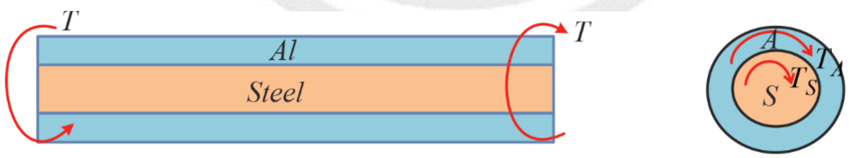

Fig.3.4 Composite shaft subjected to twisting moment

$\Sigma M = 0$

$T_S + T_A = T$

Compatibility equation:

$\theta_S = \theta_A$

$$\frac{T_S \times l}{G_S \times J_S} = \frac{T_A \times l}{G_A \times J_A}$$

3.4 Strain Energy due to Torsion

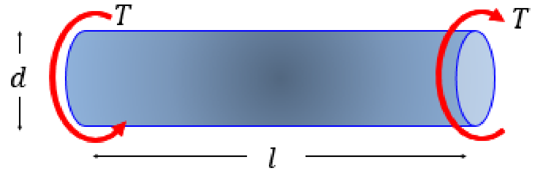

Fig.3.5 Strain Energy due to Torsion

$$U = \frac{T^2 l}{2GJ}$$

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4 SHEAR FORCE & BENDING MOMENT

4.1 Beams

Beams are Structural member used to support transverse loads.

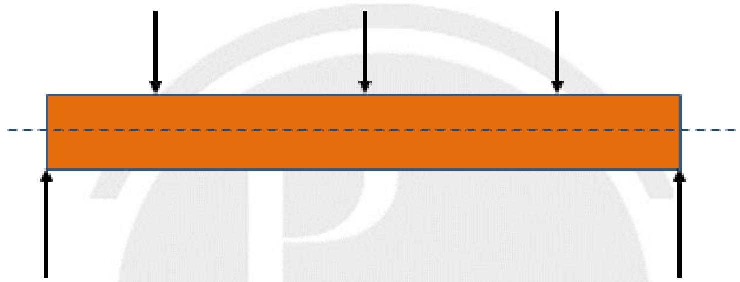

Fig.4.1 Beam subjected to transverse shear load

4.1.1 Types of Beams

Types of Beams

Statically Determinate

1. Cantilever Beam
2. Simply Supported Beam
3. Overhang Beam

Statically Indeterminate

1. Propped Cantilever Beam
2. Continuous Beam
3. Fixed (Built-in) Beam

(A) Cantilever Beam

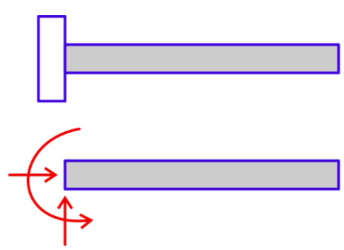

Fig.4.2 Cantilever Beam

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum M = 0$$

(B) Simply Supported Beam

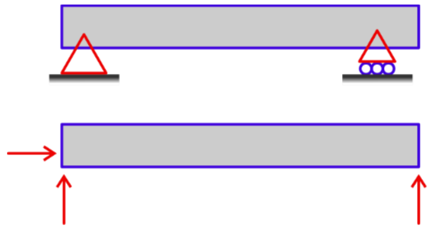

Fig.4.3 Simply Supported Beam

(C) Overhanging Beam

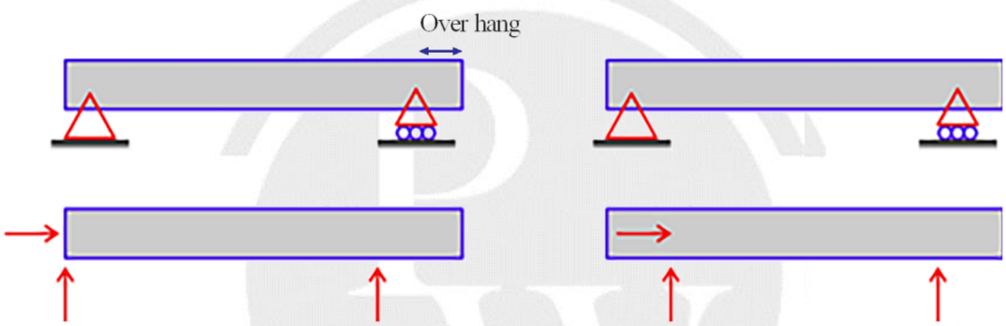

Fig.4.4 Overhanging Beam

(D) Propped Cantilever Beam

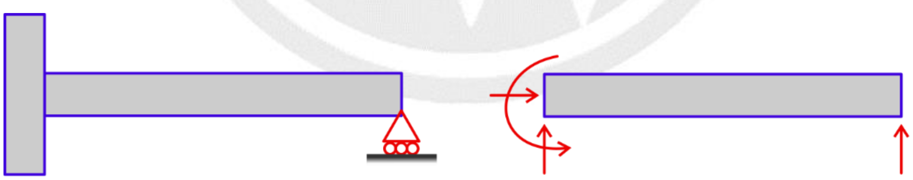

Fig.4.5 Propped Cantilever Beam

(E) Continuous Beam

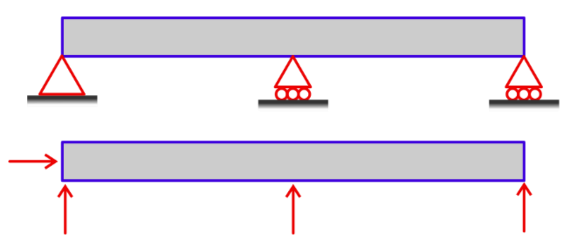

Fig.4.6 Continuous Beam

(F) Fixed Beam

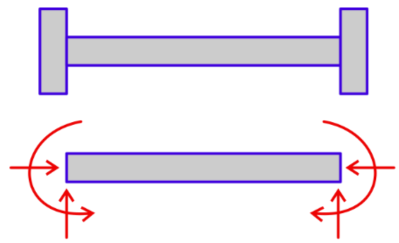

Fig.4.7 Fixed Beam

(G) Beam with Internal Hinge

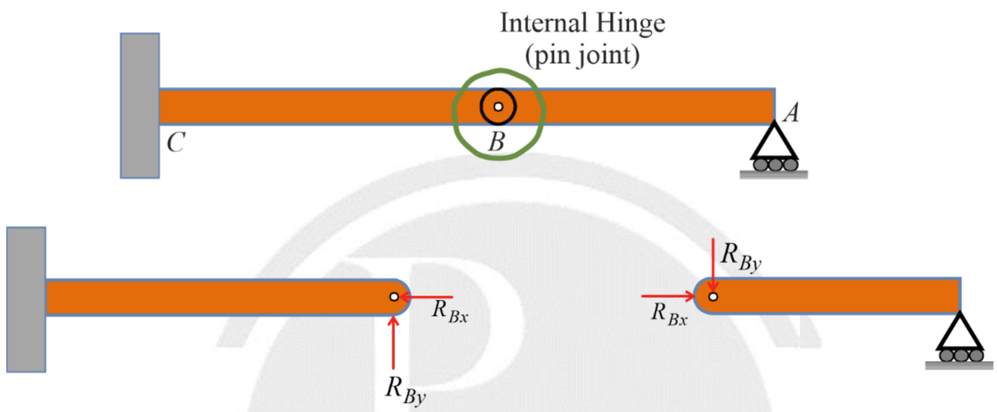

Fig.4.8 Beam with Internal Hinge

(H) Distributed loads

Load intensity = w (KN/m)

Total load = Area under the loading diagram

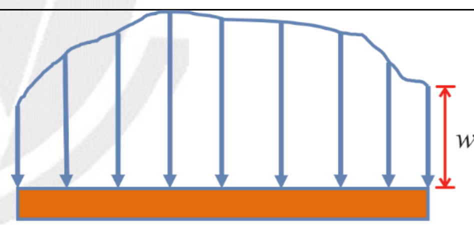

Fig.4.9 Beam subjected to distributed loads

(I) Uniformly Distributed load

Total load = wl

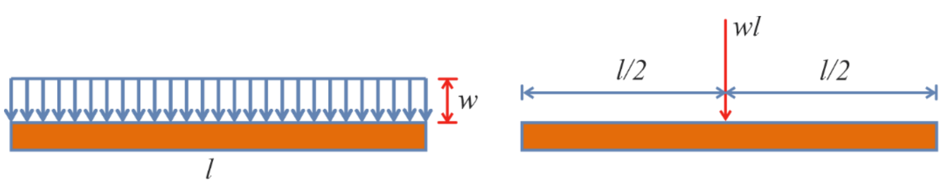

Fig.4.10 Beam subjected to uniformly distributed load

(J) Uniformly Varying Load

Total load $= \frac{1}{2}.w.l$

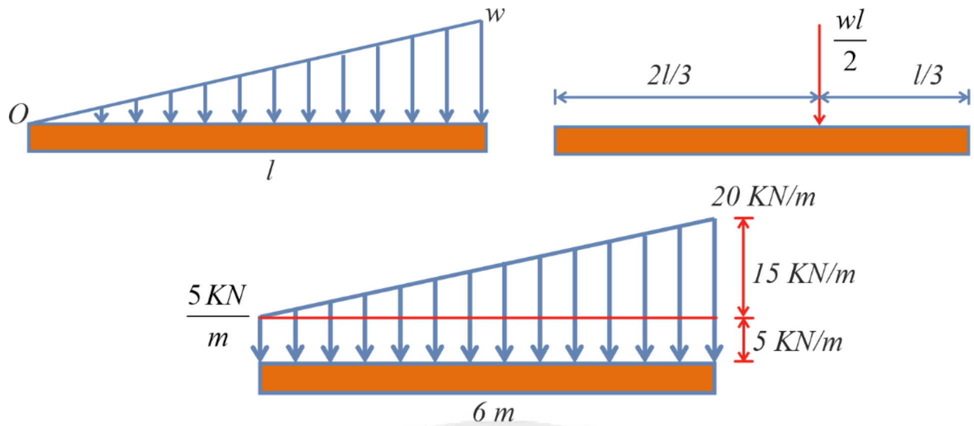

Fig.4.11 Beam subjected to uniformly varying load

Total load $= 6\times5+\frac{1}{2}\times6\times15$

= 30 + 45

= 75 KN

4.2 Shear Force

- Shear force is the transverse internal force at a section.
- It is equal to the sum of total transverse force either on the left or right side of the section.

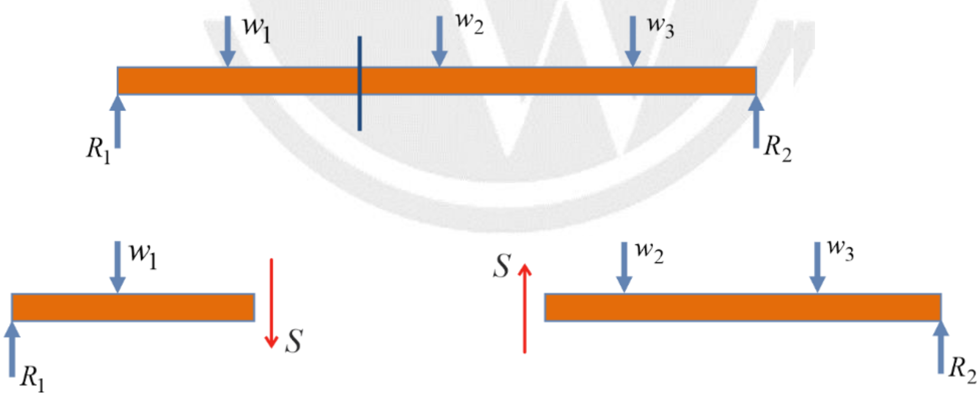

Fig.4.12 Beam subjected to transverse shear loads

Sign Convention

Fig.4.13 Sign convention of shear force

4.3 Bending Moment

- Bending moment is the internal moment at a section.
- It is equal to the sum of moment of all the forces either on the left or right side of the section.

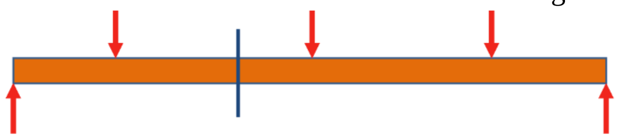

Fig.4.14 Bending moment due to transverse shear loads

Sign Convention:

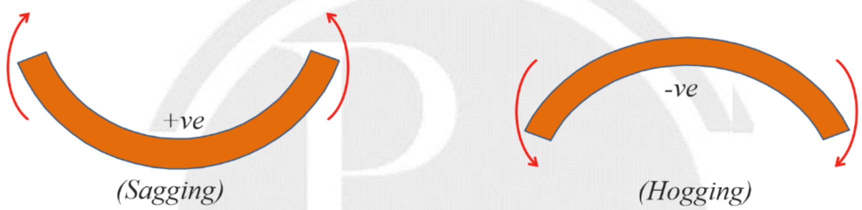

Fig.4.15 Bending moment sign convention

4.4 Relation Between Load Intensity(w), Shear Force (F) and Bending Moment (M)

(a) $\frac{ds}{dx} = w$ $\qquad\qquad$ $\int_A^B dM = \int_A^B S dx$

(b) $\frac{dM}{dx} = S$ $\qquad\qquad$ $M_B - M_A =$ Area of SFD between A and B

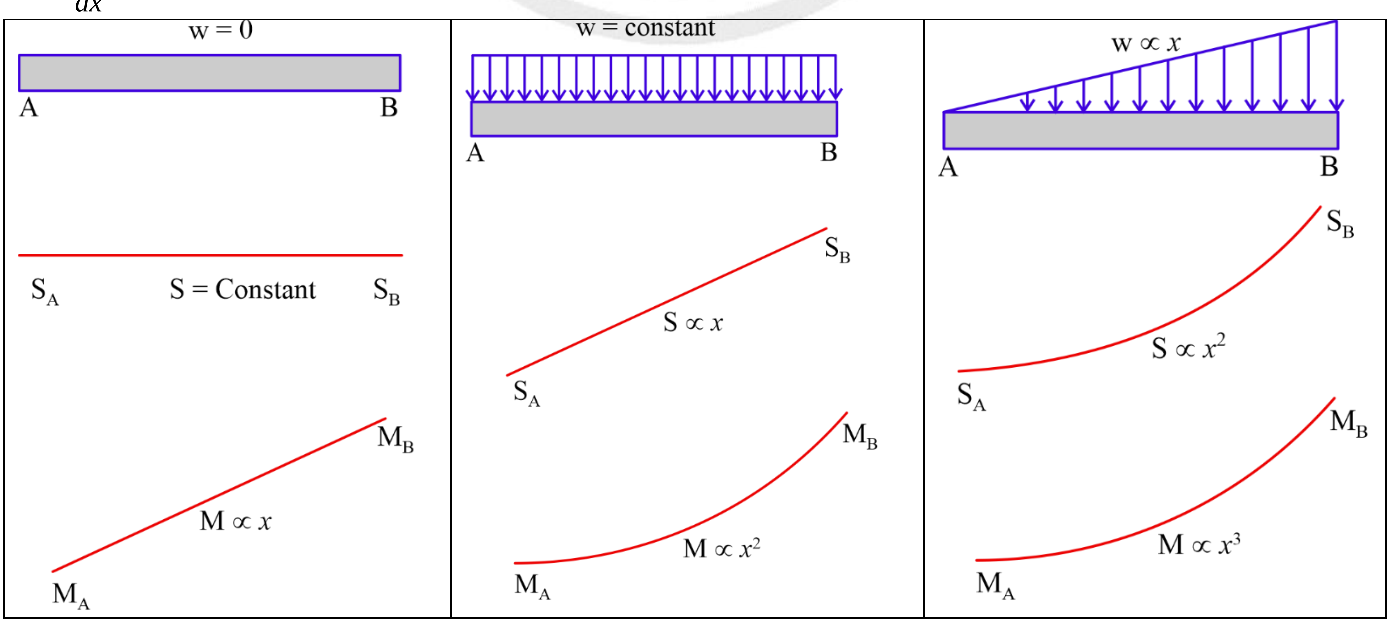

Fig.4.16 Relation Between Load Intensity, Shear Force and Bending Moment

4.4.1 Sudden Change in Shear Force

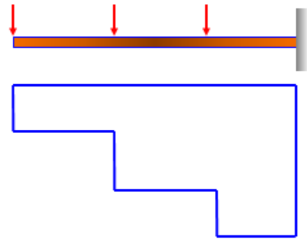

Fig.4.17 Shear force diagram

4.4.2 Sudden Change in Bending Moment

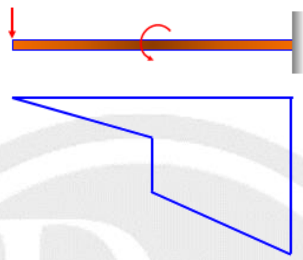

Fig.4.18 Bending Moment Diagram

4.4.3 Point of Maximum Bending Moment

Bending moment is maximum at a section if

- The sign of shear force changes at the section

  Or

- There is a couple at that section

4.5 Point of Contra flexure

It is the point at which sign of bending moment changes and the curvature of beam changes from sagging to hogging or hogging to sagging.

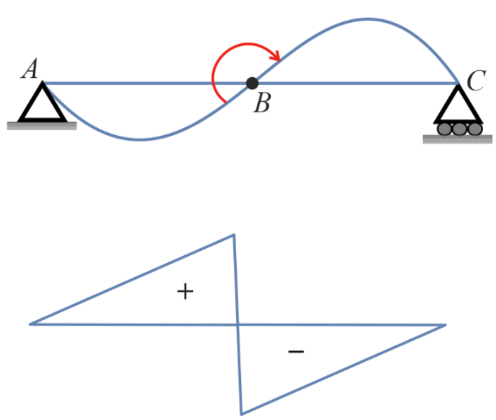

Fig.4.19 BMD Representing Point of Contra Flexure

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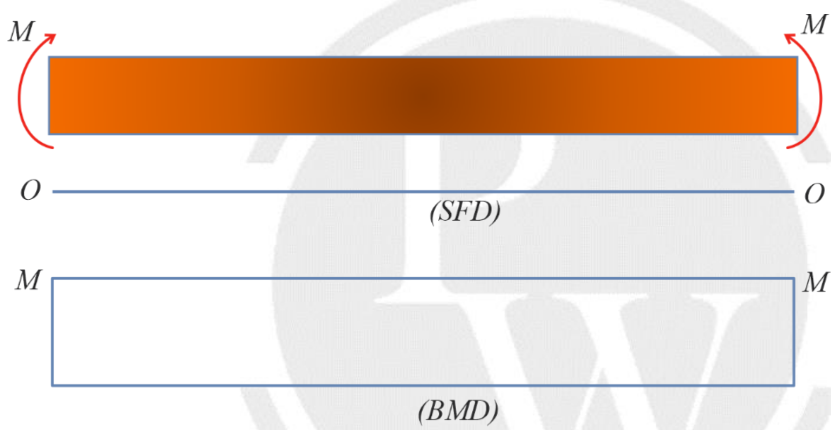
5 BENDING STRESS IN BEAMS

5.1 Bending Stress in Beams

5.1.1 Pure bending

- A beam is under pure bending when it is subjected to constant bending moment.
- Pure bending occurs when shear force is zero.



$$\frac{dM}{dx} = S = O$$

Fig.5.1 Beam subjected to pure bending

5.1.2 Euler – Bernoulli's Beam Theory

(A) Assumptions

- Material of the beam is homogeneous and isotropic.
- Young`s modulus in tension and compression is same.
- Stresses are within proportional limit.
- The beam is under pure bending.
- All the transverse sections remain plane after bending.
- Beam is initially straight and bends into a circular arc.
- Cross section of the beam is symmetric about the plane of loading.
- All the transverse sections remain plane after bending.

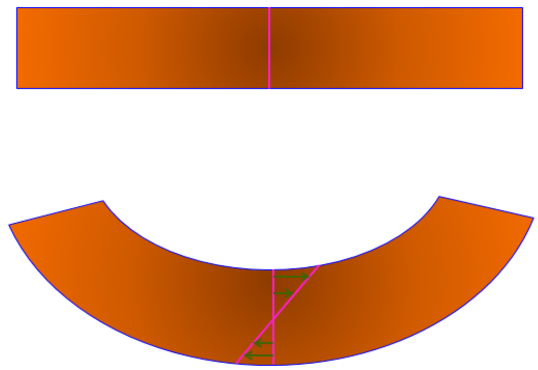


Fig.5.2 Beam subjected to sagging bending moment

- Cross section of the beam is symmetric about the plane of loading.

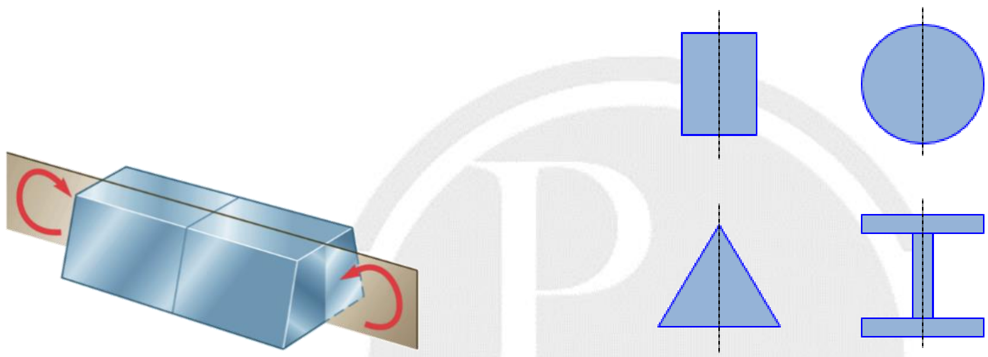


Fig.5.3 Cross section of beam under bending

(B) Neutral Layer

Undeformed longitudinal layer.

Fig.5.4 Undeformed neutral axis during bending

(C) Neutral axis

- Axis about which beam bends.
- Intersection of neutral layer with the cross section.

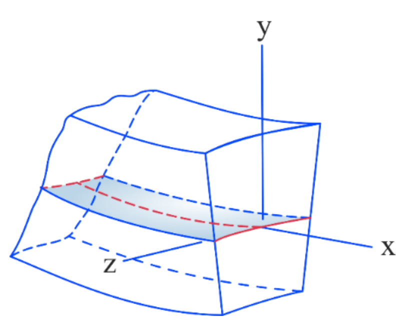


Fig.5.5 Neutral Surface and Neutral axis

Neutral axis passes through the centroid of the section, if

- The material is homogenous.
- There is no plastic deformation.

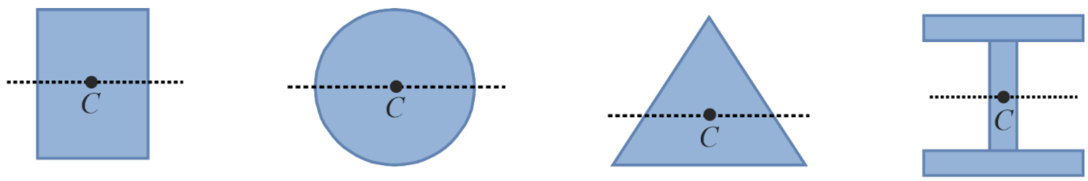


Fig.5.6 Various cross sections of beams

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$$

y → vertical distance from N.A.

I → MOI about NA

R → Radius of curvature of Neutral fiber

$\sigma \propto y$

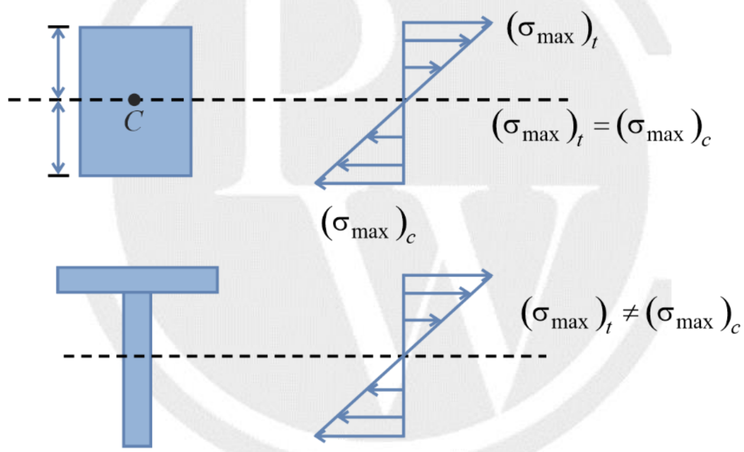


Fig.5.7 Bending stress distribution

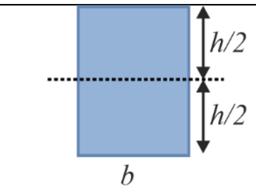
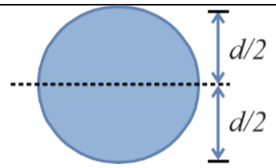
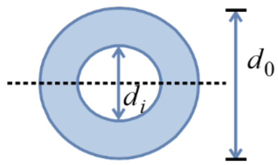
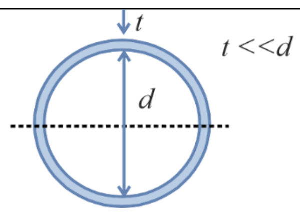
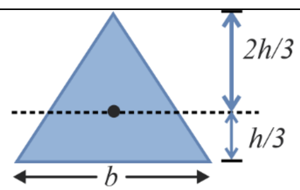
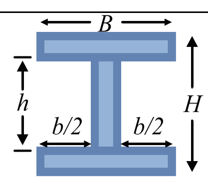
(D) Section Modulus

- It is the measure of the strength of beam (maximum applicable bending moment).

$$Z = \frac{I}{y_{max}}$$

For same cross-sectional area

$$Z_I > Z_L > Z_0$$

Cross Section	Y_{max}	I_{NA}	Z
	$\frac{h}{2}$	$\frac{bh^3}{12}$	$\frac{bh^2}{6}$
	$\frac{d}{2}$	$\frac{\pi}{64}d^4$	$\frac{\pi}{32}d^3$
	$\frac{d_0}{2}$	$\frac{\pi}{64}\left(d_0^4 - d_i^4\right)$	$\frac{\pi}{32}d_0^3\left(1-K^4\right)$ $K = \frac{d_i}{d_0}$
	$\frac{d}{2}$	$\frac{\pi d^3 t}{8}$	$\frac{\pi d^2 t}{4}$
	$\frac{2h}{3}$	$\frac{bh^3}{36}$	$\frac{bh^2}{24}$
	$\frac{H}{2}$	$\frac{BH^3}{12} - \frac{bh^3}{12}$	

- **Flexural rigidity (EI)**

  It is used in the design of beam based on rigidity criteria

- **Flexural stiffness**

  $\frac{EI}{l}$

5.2 Combined Axial load and Bending Moment

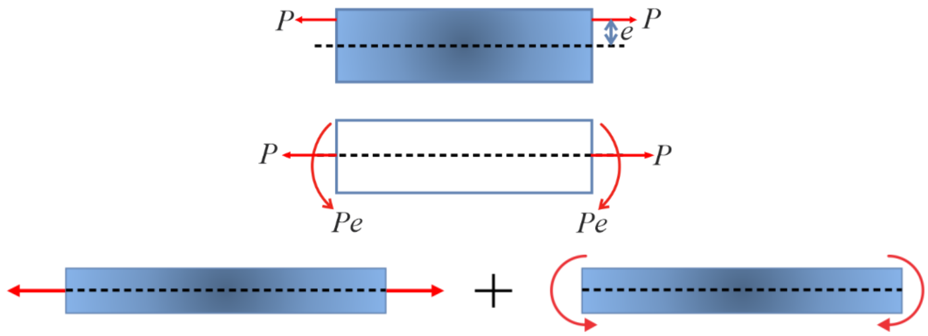


Fig.5.8 Beam subjected to combined axial load and bending moment

$$(\sigma_{max})_t = \frac{P}{A} + \frac{My_{max}}{I}$$

$$(\sigma_{max})_C = \frac{P}{A} - \frac{My_{max}}{I}$$

5.3 Beam of Uniform Strength

- Beam of uniform strength is a beam subjected to same maximum bending stress throughout the length.
- Beam of uniform strength has varying cross section.

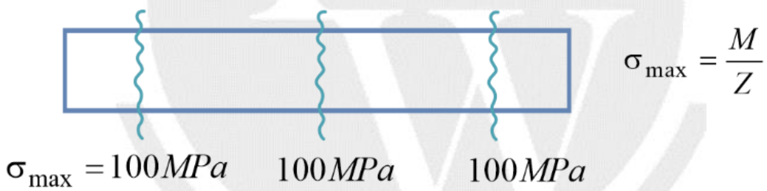


Fig.5.9 Beam of Uniform Strength

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6 SHEAR STRESS IN BEAMS

6.1 Shear Stress in Beams

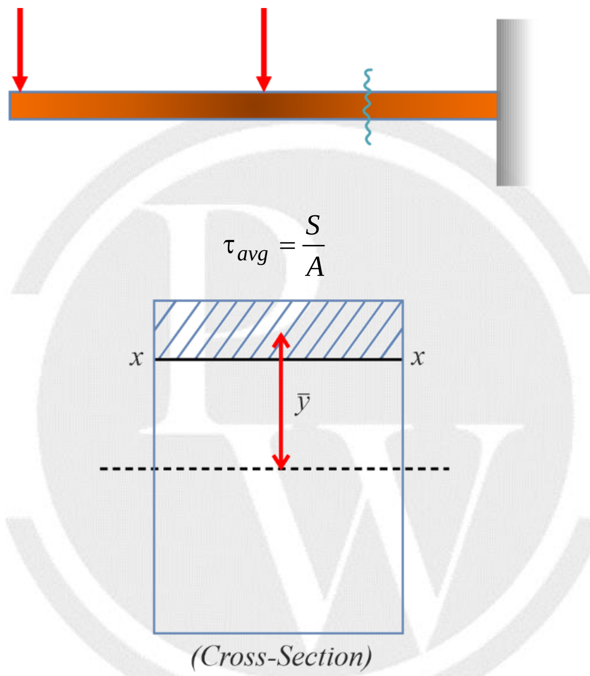


Fig.6.1 Shear stress in Beams

$$\tau = \frac{s(A\bar{y})}{Ib}$$

b → width of layer X-X

$(A\bar{y})$ → First moment of area above / below x-x about NA.

6.1.1 Rectangular Section

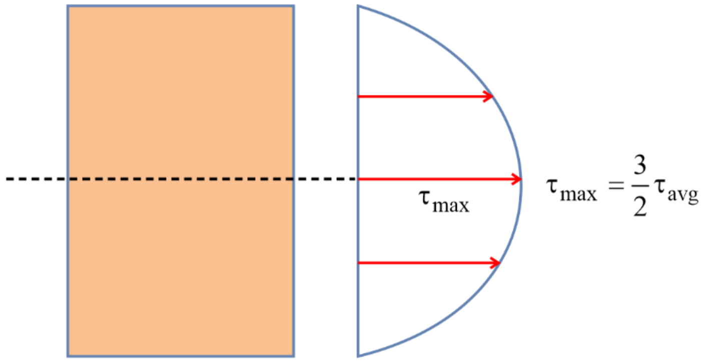


Fig.6.2 Shear stress distribution for rectangular cross section

6.1.2 Circular Section

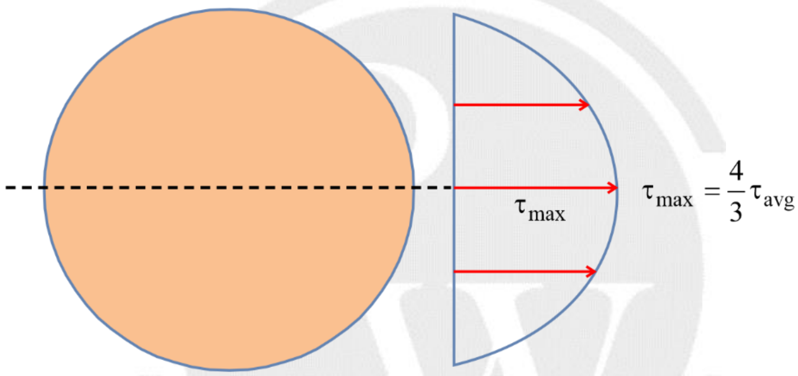


Fig.6.3 Shear stress distribution for circular cross section

6.1.3 Triangular Section

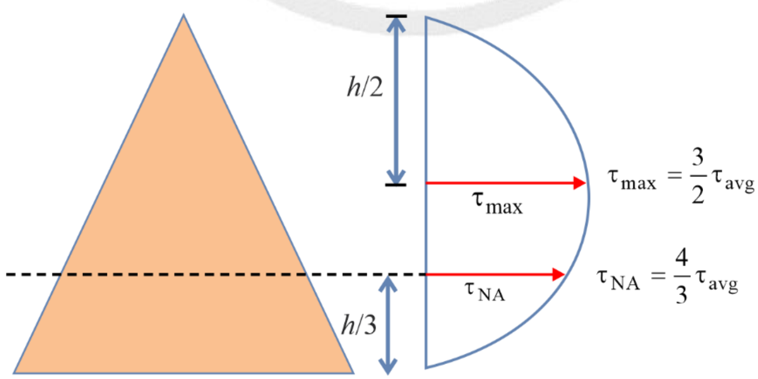


Fig.6.4 Shear stress distribution for triangular cross section

6.1.4 Diamond Section

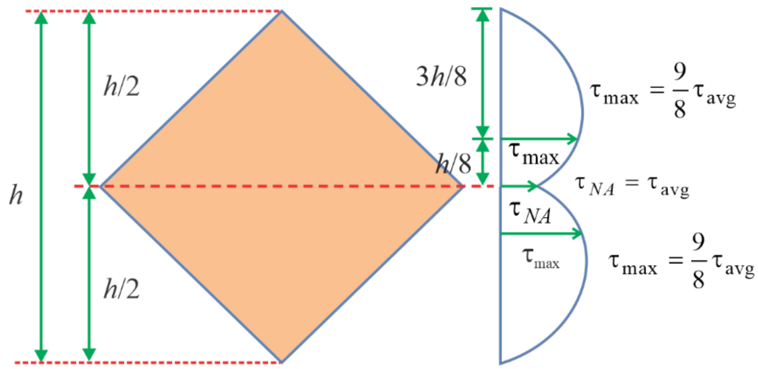


Fig.6.5 Shear stress distribution for diamond cross section

6.1.5 I Section

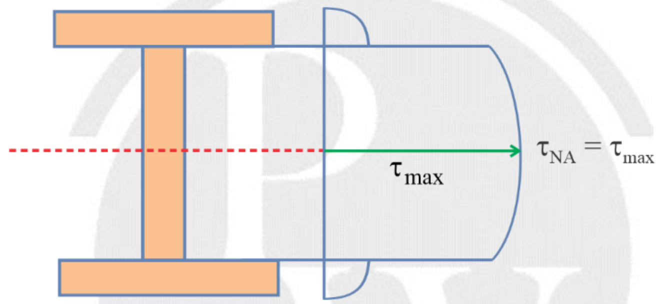


Fig.6.6 Shear stress distribution for I section beam

6.1.6 T Section

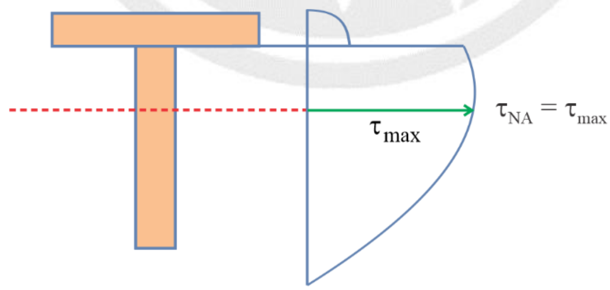
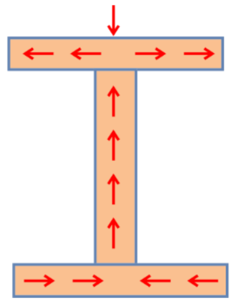


Fig.6.7 Shear stress distribution for T section beam

6.2 Shear Flow

In thin walled members (I section, T section) shear flow is the shear force per unit length.



$$q = \frac{SA\bar{y}}{I}$$

Fig.6.8 Shear flow in I section

6.3 Shear Centre

It is the point on the beam section at which the transverse load can be applied without causing twisting.

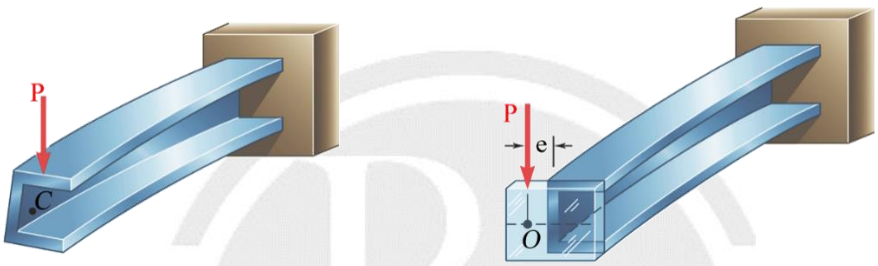
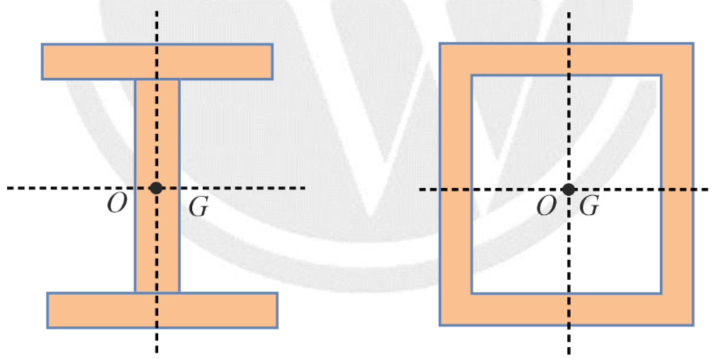


Fig.6.9 Shear center for thin-walled sections

(A) Sections with two axes of symmetry



$G \rightarrow$ *centroid*

$O \rightarrow$ *shear center*

Fig.6.10 Shear center when there are two axes of symmetry

(B) Sections with one axis of symmetry

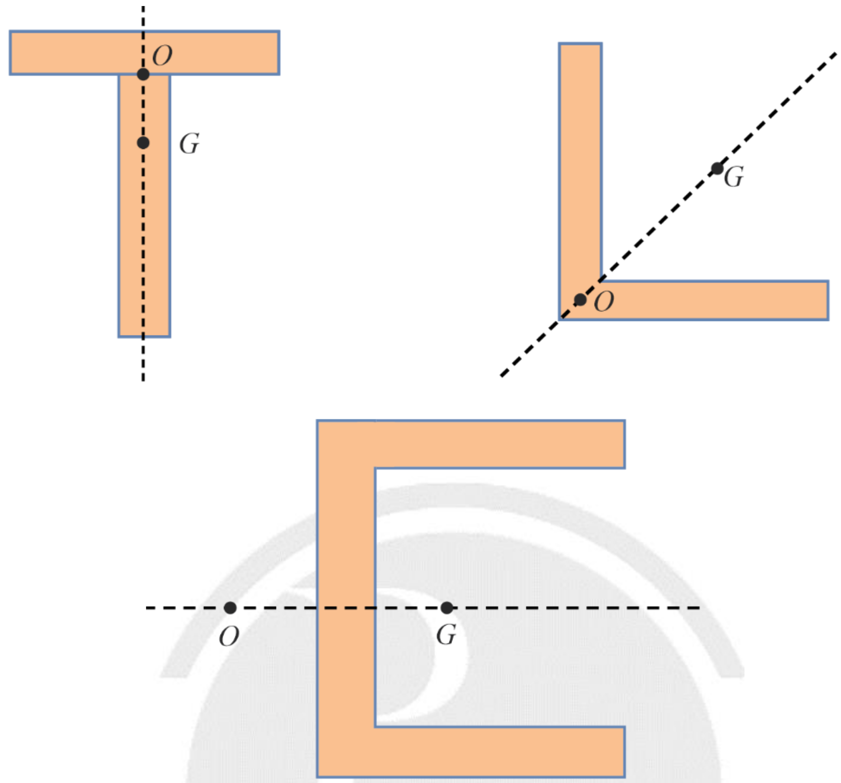


Fig.6.11 Shear center when there is one axes of symmetry

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7 DEFLECTION OF BEAMS

7.1 Deflection of Beams

Deflection represents linear deviation of a point and the slope represents the angular deviation of the point on the longitudinal axis of the beam

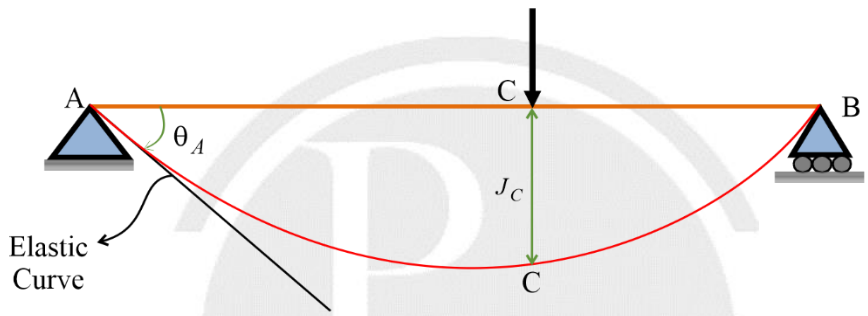


Fig.7.1 Deflection of Beam

Methods to find slope and deflection of beams

- Double Integration and Macaulay's method
- Moment – Area method
- Strain Energy method

7.2 Double Integration Method

$$EI\frac{d^2y}{dx^2} = M_x$$

$$EI\frac{dy}{dx} = \int M_x + C_1 \qquad \ldots\ldots(1)$$

$$EIy = \int\int M_x + C_1x + C_2 \qquad \ldots\ldots.(2)$$

From equation 1 and 2 slope and deflection can be determined at any location of the beam.

7.3 Macaulay's Method or Modified double integration method

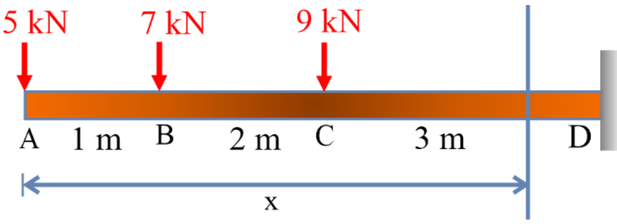


Fig.7.2 Macaulay's method for beam subjected to multiple loads

$$EI\frac{d^2y}{dx^2} = -5x - 7(x-1) - 9(x-3)$$

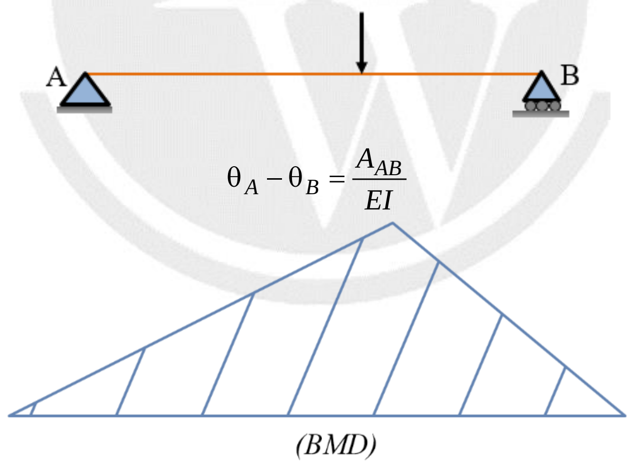
$$EI\frac{dy}{dx} = \frac{-5x^2}{2} - \frac{7(x-1)^2}{2} - \frac{9(x-3)^2}{2} + C_1$$

This method is preferred for simply supported beam unsymmetric loading and cantilever beam subjected to multiple loads. Here the terms within the bracket are known as Macaulay function and they are integrated as whole.

7.4 Moment – Area Method

7.4.1 Mohr's 1st Theorem

The change in slope between any two points A and B on the elastic curve is equal to the area of the bending moment diagram between A and B divided by EI.



$$\theta_A - \theta_B = \frac{A_{AB}}{EI}$$

Fig.7.3 Slope calculation from Mohr's 1st Theorem

7.4.2 Mohr's 2nd Theorem

The vertical deviation of any point A on the elastic curve from the tangent of a point B on the elastic curve is equal to the first moment of area of bending moment diagram between A and B about point A divided by EI.

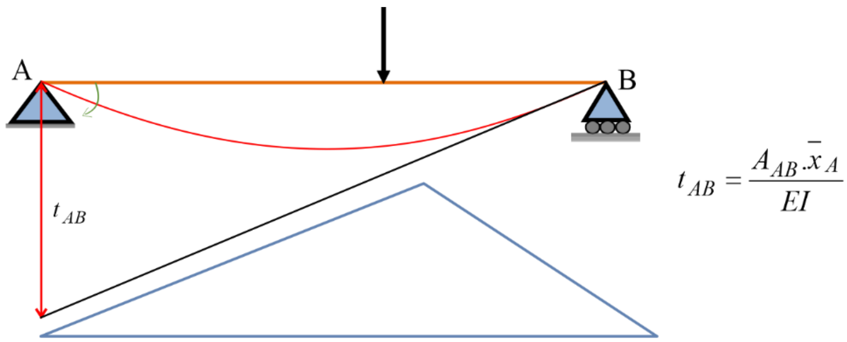


Fig.7.4 Deflection calculation from Mohr's 2nd Theorem

7.5 Strain Energy due to Bending

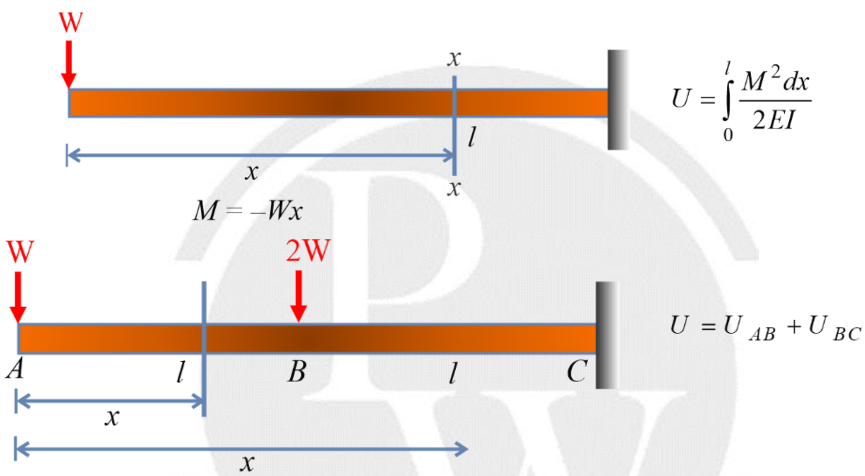


Fig.7.5 Strain energy due to bending

$M_{AB} = -Wx$ ($x = 0$ to l)

$M_{BC} = -Wx - 2W(x - l)$ ($x = l$ to $2l$)

7.6 Castigliano's Theorem

The partial derivative of the total strain energy in a structure with respect to any force at a point is equal to the deflection at that point in the direction of the force.

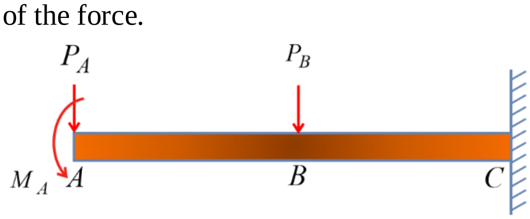


Fig.7.6 Castigliano's Theorem

U = Total S.E.

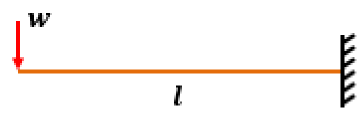
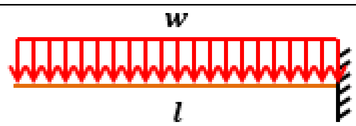
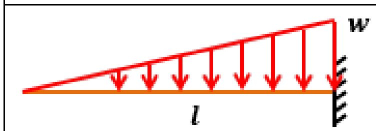
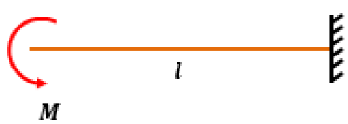
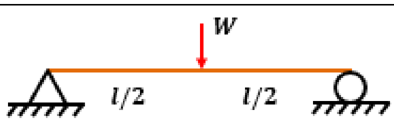
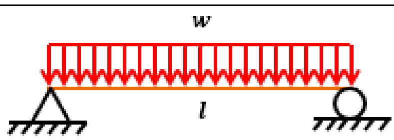
$$\frac{\partial U}{\partial P_A} = y_A$$

$$\frac{\partial U}{\partial P_B} = y_B$$

The partial derivative of the total strain energy in a structure with respect to a moment at a point is equal to the slope at that point.

$$\frac{\partial U}{\partial M_A} = \theta_A$$

7.7 Slope and Deflection of standard case

Loading	θ_{max}	y_{max}
	$\frac{Wl^2}{2EI}$	$\frac{Wl^2}{3EI}$
	$\frac{Wl^3}{6EI}$	$\frac{Wl^4}{8EI}$
	$\frac{Wl^3}{24EI}$	$\frac{Wl^4}{30EI}$
	$\frac{Ml}{EI}$	$\frac{Ml^2}{2EI}$
	$\frac{Wl^2}{16EI}$	$\frac{Wl^3}{48EI}$
	$\frac{Wl^3}{24EI}$	$\frac{5Wl^4}{384EI}$

7.8 Principle of Superposition

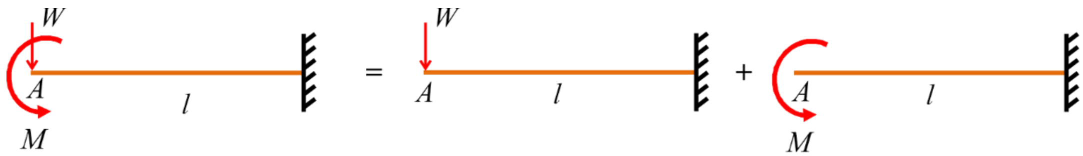


Fig.7.7 Principle of Superposition

$$\theta_A = \frac{Wl^2}{2EI} + \frac{Ml}{EI}$$

$$y_A = \frac{Wl^3}{3EI} + \frac{Ml^2}{2EI}$$

7.9 Maxwell's Reciprocal Theorem

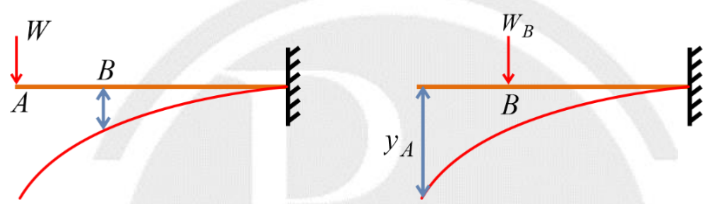


Fig.7.8 Maxwell's Reciprocal Theorem

$W_A.y_A = W_B.y_B$

Special Case in Cantilever Beams

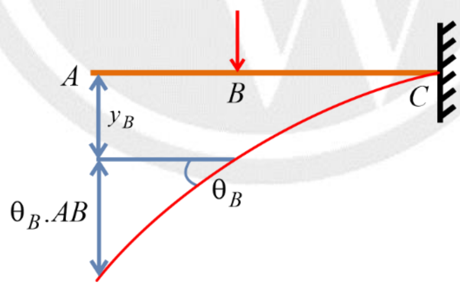
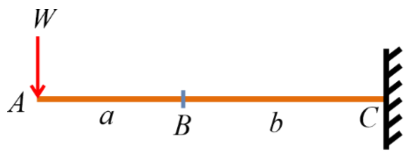


Fig.7.9 Elastic curve becomes straight line (AB)

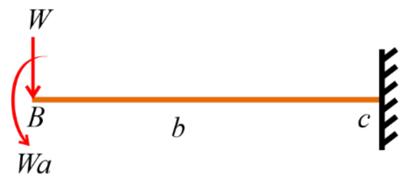
$\theta_A = \theta_B$ (Since elastic curve becomes straight line)

$y_A = y_B + \theta_B.AB$ (This equation is valid only when elastic curve becomes straight line)



$\theta_B = ?$

$y_B = ?$



7.10 Statically Indeterminate Beams

(1) In this case, deflection at A is zero.

Compatibility equation

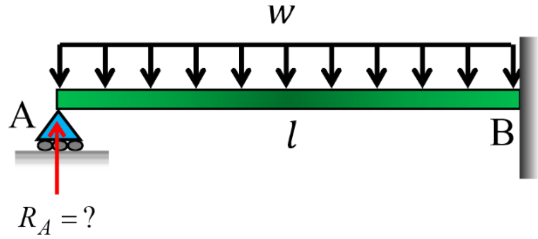


Fig.7.10 Propped cantilever beam reaction calculation

$y_A = 0$

$\downarrow y_A$ due to w + $\uparrow y_A$ due to R_A = 0

$$\frac{wl^4}{8EI} - \frac{R_A l^3}{3EI} = 0$$

$$R_A = \frac{3wl}{8}$$

(2) In this case deflection at point A in the beam is equal to the deflection in the spring

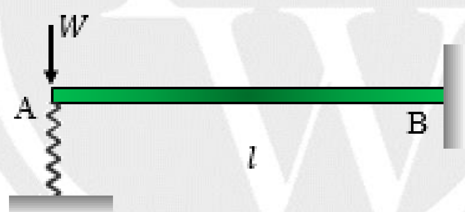


Fig.7.11 Spring support at one end of cantilever beam

$y_A = y_{spring}$

$\downarrow y_A$ due to w + $\uparrow y_A$ due to R_S = $\downarrow y_{spring}$

$$\frac{wl^3}{3EI} - \frac{R_S l^3}{3EI} = \frac{R_S}{K}$$

$R_S = ?$

FBD

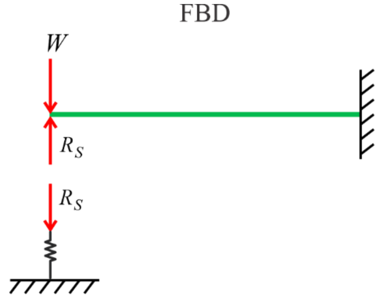


Fig.7.12 Free body diagram

(3) In this case, deflection at point B and C will be same

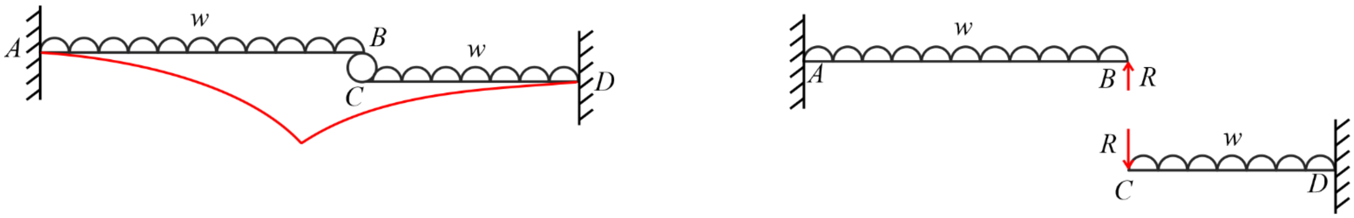


Fig.7.13 Two cantilever beam attached at their free ends

$y_B = y_C$

$\theta_B \neq \theta_C$

$y_B = y_C$

$\downarrow y_b$ due to w + $\uparrow y_B$ due to R

$= \downarrow y_c$ to w + $\downarrow y_c$ due to R

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8 COMPLEX STRESS

8.1 Complex Stress

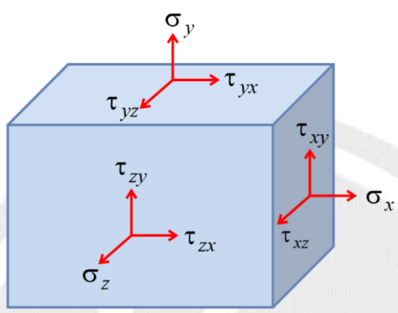

Fig.8.1 Point is subjected to triaxial state of stress

8.2 Plane Stress

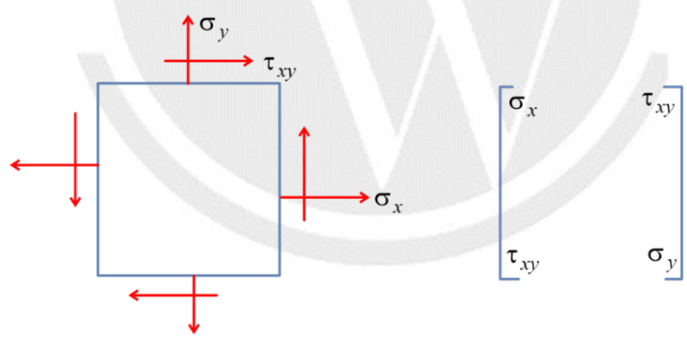

Fig.8.2 Point is subjected to biaxial state of stress

8.3 Stresses on Oblique Planes

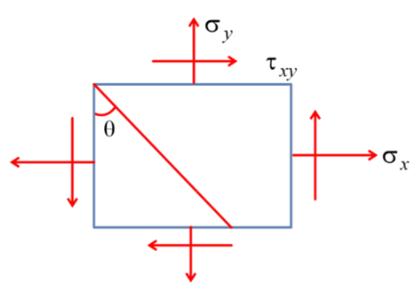

Fig.8.3 Stresses on oblique planes

$$\sigma_\theta = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_\theta = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Sign Convention

(a) σ:

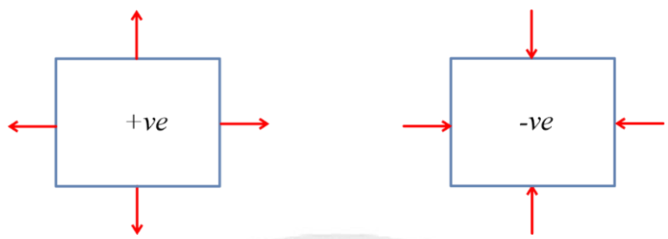

Fig.8.4 Normal stress sign convention

(b) τ:

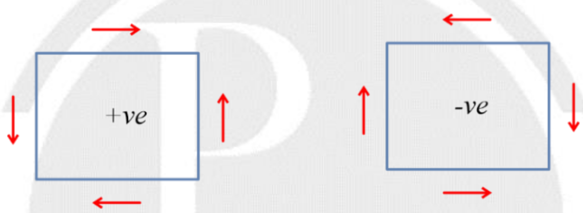

Fig.8.5 Shear stress sign convention

(c) θ:

Fig.8.6 Sign convention for θ (location of oblique plane)

8.4 Mohr's Circle

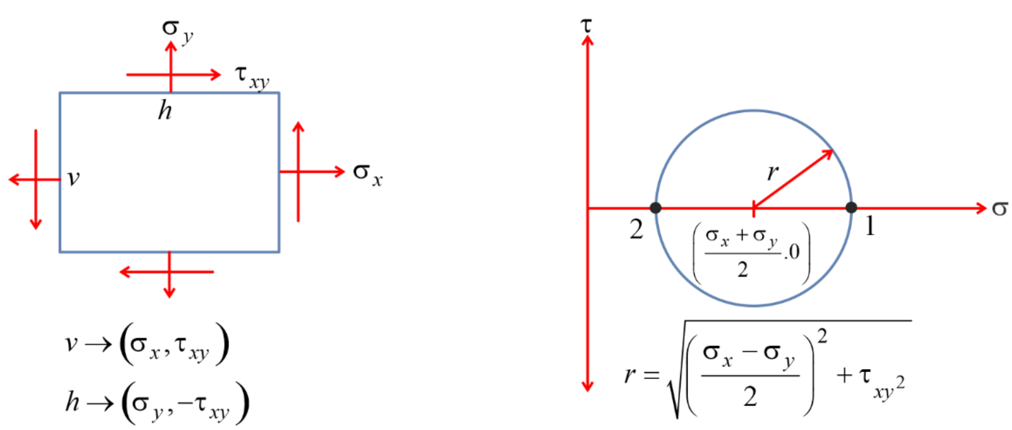

Fig.8.7 Mohr's circle for biaxial state of stress

8.5 Principal Planes

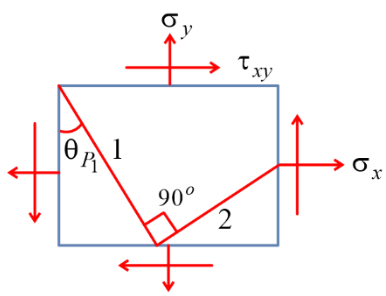

Fig.8.8 Location of Principal Planes

$$\theta_{P_1} = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

$$\theta_{P_2} = \theta_{P_1} + 90^o$$

8.6 Principal Stresses

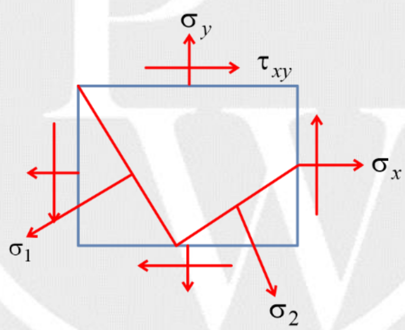

Fig.8.9 Principal stresses in complex state of stress

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}}$$

$$\sigma_1 + \sigma_2 = \sigma_x + \sigma_y$$

$$\sigma_1 . \sigma_2 = \sigma_x . \sigma_y - \tau_{xy^2}$$

8.7 Maximum Shear Stress

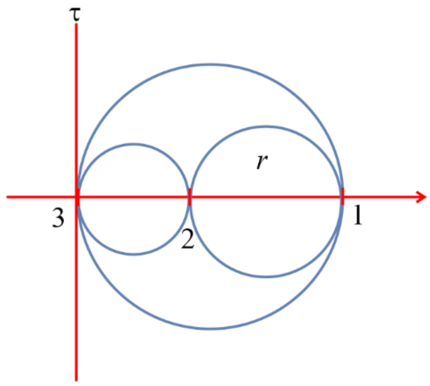

Fig.8.10 Mohr's circle for triaxial state of stress

$$(\tau_{max})_{\text{in-plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

or

$$\left(\frac{\sigma_1 - \sigma_2}{2}\right)$$

$$\tau_{max} = \max{}^m \text{ of } \left|\frac{\sigma_1 - \sigma_2}{2}\right|, \left|\frac{\sigma_2 - \sigma_3}{2}\right|, \left|\frac{\sigma_3 - \sigma_1}{2}\right|$$

8.8 Combined Bending & Twisting

$$\sigma_{max} = \frac{16}{\pi d^3}\left[M + \sqrt{M^2 + T^2}\right]$$

$$\tau_{max} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

$$M_{eq} = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]$$

$$T_{eq} = \sqrt{M^2 + T^2}$$

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9 COMPLEX STRAIN

9.1 Complex Strain

Strain analysis is similar to the stress analysis, just replace normal stress by normal strain and shear stress by half of the shear strain

$$\sigma \rightarrow \varepsilon$$

$$\tau \rightarrow \frac{\gamma}{2}$$

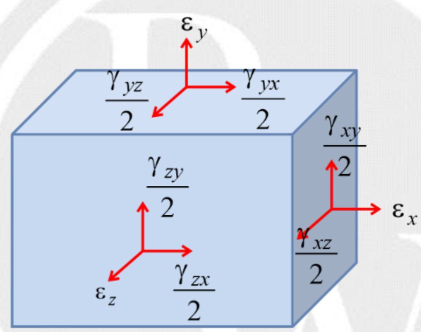
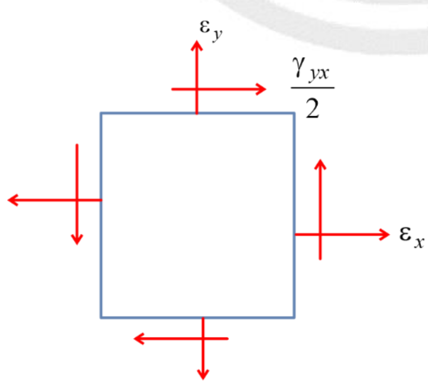


Fig.9.1 Point is subjected to triaxial state of strain

9.2 Plane Strain



$$\begin{bmatrix} \varepsilon_x & \frac{\gamma_{xy}}{2} \\ \frac{\gamma_{yx}}{2} & \varepsilon_y \end{bmatrix}$$

Fig.9.2 Point is subjected to biaxial state of strain

9.3 Strains on Oblique Planes

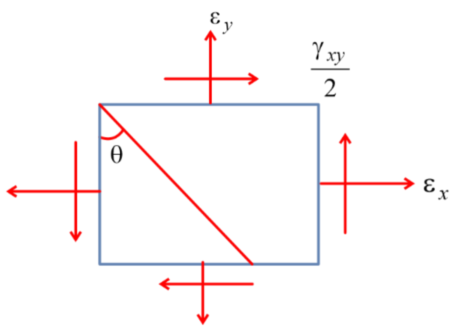


Fig.9.3 Strains on oblique planes

$$\varepsilon_\theta = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_\theta}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

9.4 Mohr's Circle

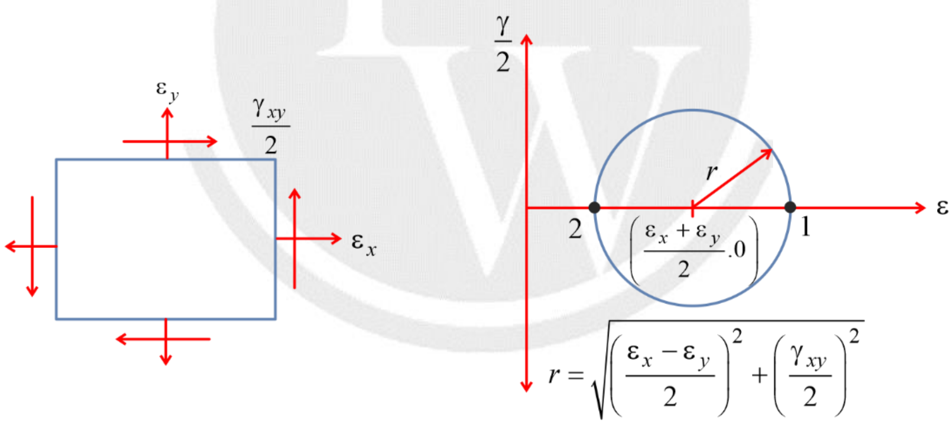


Fig.9.4 Mohr's circle for biaxial state of strain

9.5 Principal Planes

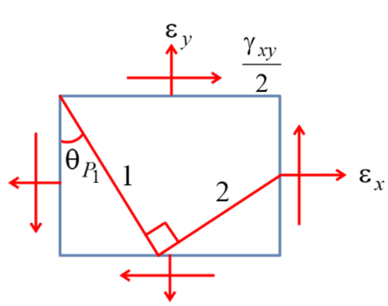


Fig.9.5 Location of Principal Planes

$$\theta_{P_1} = \frac{1}{2}\tan^{-1}\left(\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}\right)$$

$$\theta_{P_2} = \theta_{P_1} + 90^o$$

9.6 Principal Strains

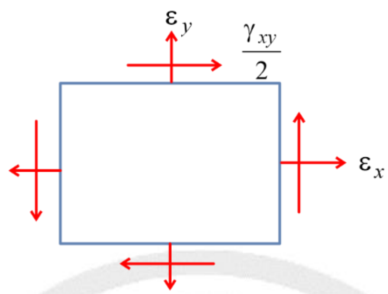


Fig.9.6 Principal strain for biaxial state of strain

$$\varepsilon_{1,2} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right) + \left(\frac{\gamma_{xy}}{2}\right)}$$

$$\varepsilon_1 + \varepsilon_2 = \varepsilon_x + \varepsilon_y$$

$$\varepsilon_1 . \varepsilon_2 = \varepsilon_x . \varepsilon_y - \left(\frac{\gamma_{xy}}{2}\right)^2$$

9.7 Maximum Shear Strain

$$(\gamma_{\max})_{in-plane} = (\varepsilon_1 - \varepsilon_2)$$

$$\gamma_{\max} = \max^m of\ |\varepsilon_1 - \varepsilon_2|, |\varepsilon_2 - \varepsilon_3|, |\varepsilon_3 - \varepsilon_1|$$

9.8 Strain Rosette

Combination of three strain gauges arranged in three different directions.

Fig.9.7 Strain rosette

$$\varepsilon_A = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cos 2\theta_A + \frac{\gamma_{xy}}{2}\sin 2\theta_A$$

$$\varepsilon_b = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cos 2\theta_b + \frac{\gamma_{xy}}{2}\sin 2\theta_B$$

$$\varepsilon_C = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cos 2\theta_C + \frac{\gamma_{xy}}{2}\sin 2\theta_C$$

9.8.1 Rectangular Strain Rosette

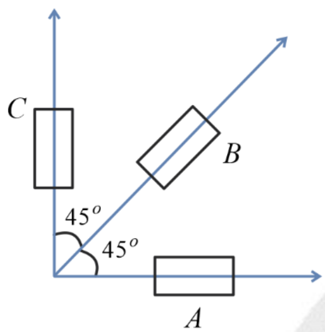


Fig.9.8 Rectangular strain rosette

$\theta_A = 0^o$

$\theta_B = 45^o$

$\theta_C = 90^o$

$\varepsilon_x = \varepsilon_A$

$\varepsilon_y = \varepsilon_C$

$\gamma_{xy} = 2\varepsilon_B - (\varepsilon_A + \varepsilon_C)$

9.8.2 Delta Strain Rosette

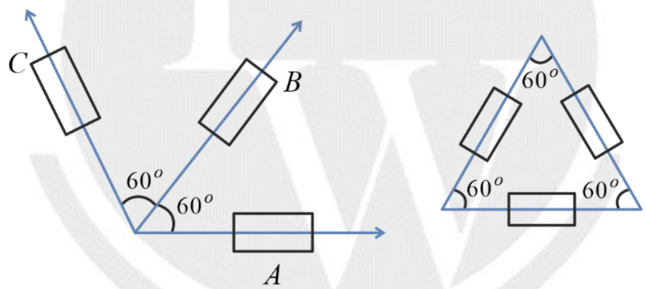


Fig.9.9 Delta strain rosette

9.8.3 Star Strain Rosette

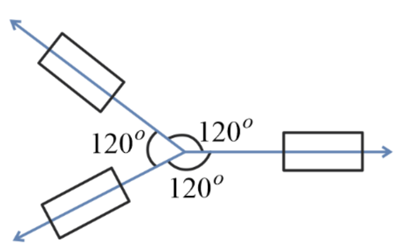


Fig.9.10 Star strain rosette

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10 PRESSURE VESSELS

10.1 Pressure Vessels

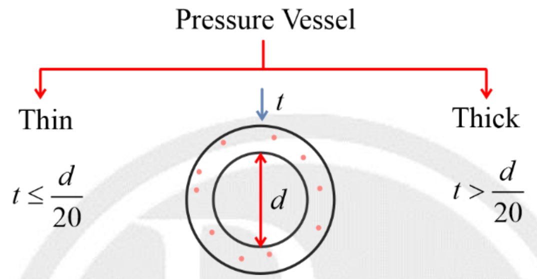


Fig.10.1 Pressure vessel

10.2 Thin Cylinder

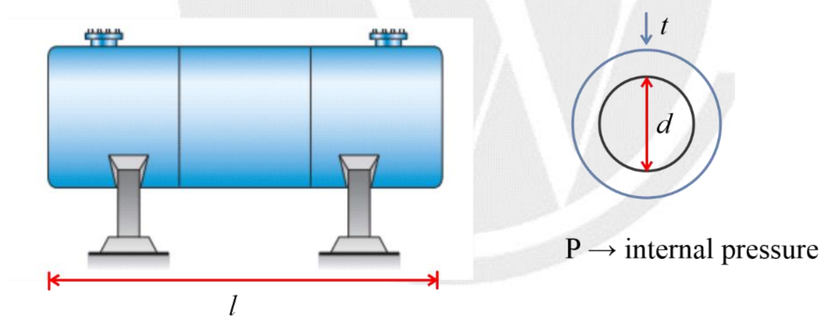


Fig.10.2 Thin Cylindrical Pressure Vessel

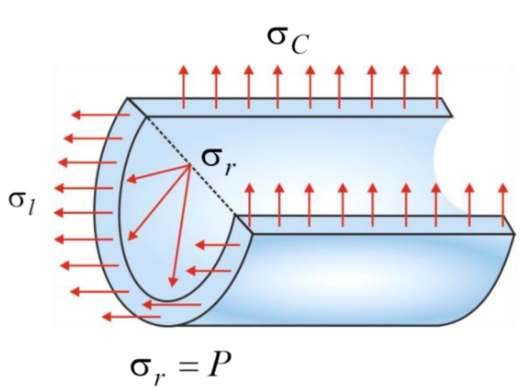


Fig.10.3 Various stresses on thin cylindrical pressure vessel

10.2.1 Longitudinal Stress

$$\sigma_l = \frac{Pd}{4t}$$

10.2.2 Circumferential/Hoop Stress

$$\sigma_C = \frac{Pd}{2t}$$

$$\sigma_1 = \sigma_c, \sigma_2 = \sigma_l$$

$$\sigma_{\max} = \frac{Pd}{2t}$$

$$(\tau_{\max})_{in\ plane} = \frac{Pd}{8t}$$

$$\tau_{\max} = \frac{Pd}{4t}$$

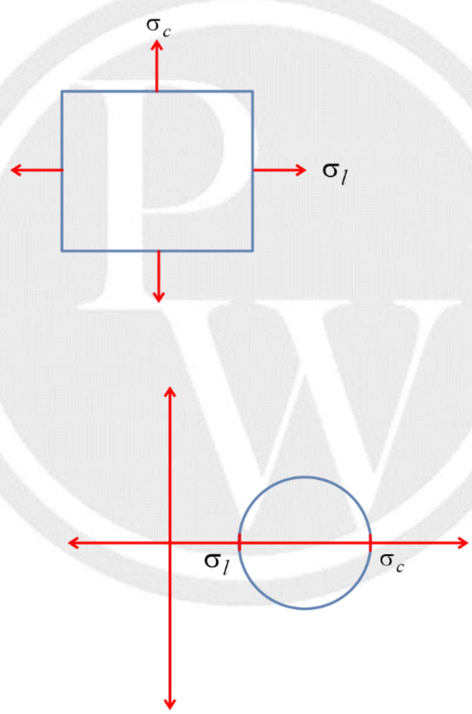


Fig.10.4 Mohr's circle for biaxial state of stress of thin cylindrical pressure vessel subjected to internal pressure

10.2.3 Longitudinal Strain

$$\varepsilon_l = \frac{\sigma_l}{E} - \nu\frac{\sigma_c}{E}$$

$$\varepsilon_l = \frac{Pd}{4tE}(1-2v) = \frac{\Delta l}{l}$$

10.2.4 Circumferential/Hoop Strain

$$\varepsilon_c = \frac{\sigma_c}{E} - \nu\frac{\sigma_l}{E}$$

$$\varepsilon_c = \frac{Pd}{4tE}(2-\nu) = \frac{\Delta d}{d}$$

10.2.5 Volumetric Strain

$$\varepsilon_v = \varepsilon_l + 2\varepsilon_c$$

$$\varepsilon_v = \frac{Pd}{4tE}(5-4\nu) = \frac{\Delta v}{v}$$

10.3 Thin Sphere

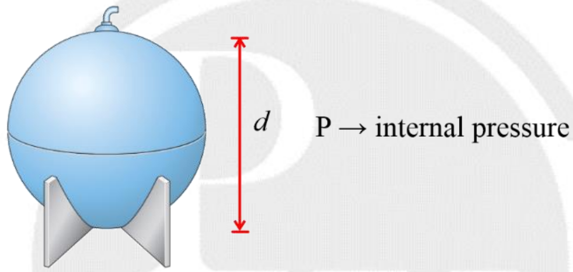


Fig.10.5 Thin spherical pressure vessel

10.3.1 Circumferential/Hoop Stress

$$\sigma_c = \frac{Pd}{4t}$$

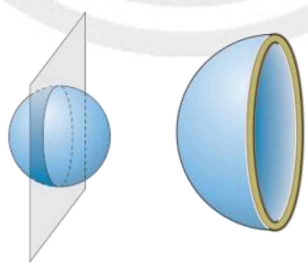
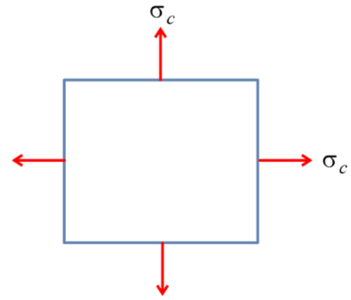


Fig.10.6 Cross sectional view of thin spherical pressure vessel



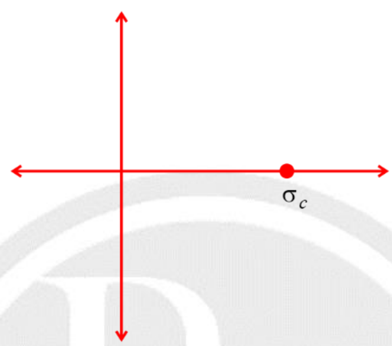


Fig.10.7 Mohr's circle for biaxial state of stress of thin spherical pressure vessel subjected to internal pressure

$$\sigma_1 = \sigma_2 = \sigma_c$$

$$\sigma_{max} = \frac{Pd}{4t}$$

$$\left(\tau_{max}\right)_{in\ plane} = 0$$

$$\tau_{max} = \frac{Pd}{8t}$$

10.3.2 Circumferential/Hoop Strain

$$\varepsilon_c = \frac{\sigma_c}{E} - v\frac{\sigma_c}{E}$$

$$\varepsilon_c = \frac{Pd}{4tE}(1-v) = \frac{\Delta d}{d}$$

10.3.2 Volumetric Strain

$$\varepsilon_v = 3\varepsilon_c$$

$$\varepsilon_v = \frac{3Pd}{4tE}(1-v) = \frac{\Delta v}{v}$$

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11 COLUMNS

11.1 Columns

Column is a structural member used to support axial compressive loads.

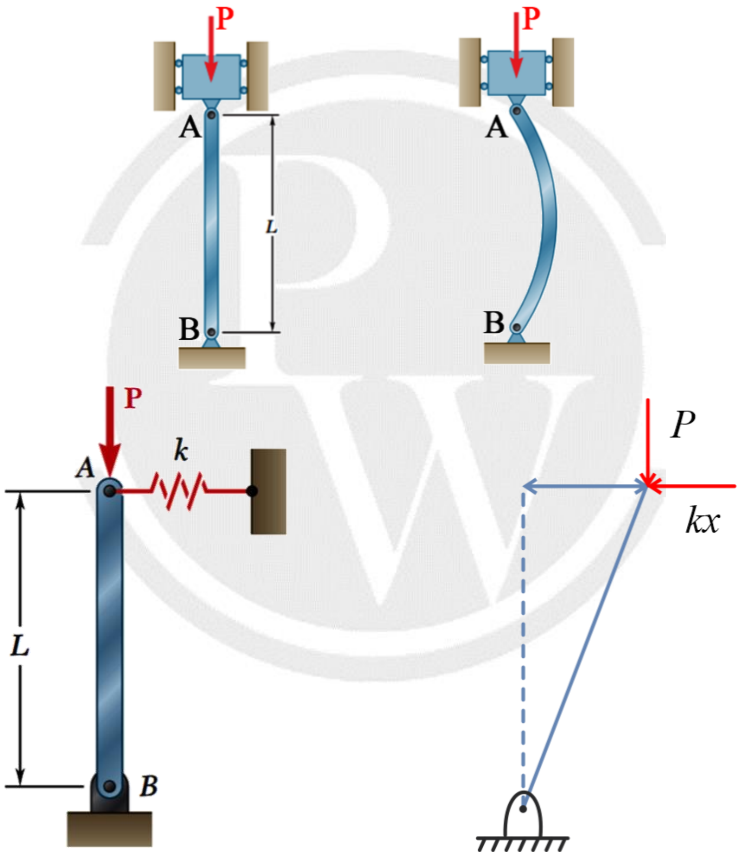

Fig.11.1 Columns

If $Px < kx.l$ – Stable
If $Px > kx.l$ – Unstable
If $Px = kx.l$ – Critical
$P_{cr} = k.l$

11.2 Euler's theory of Buckling

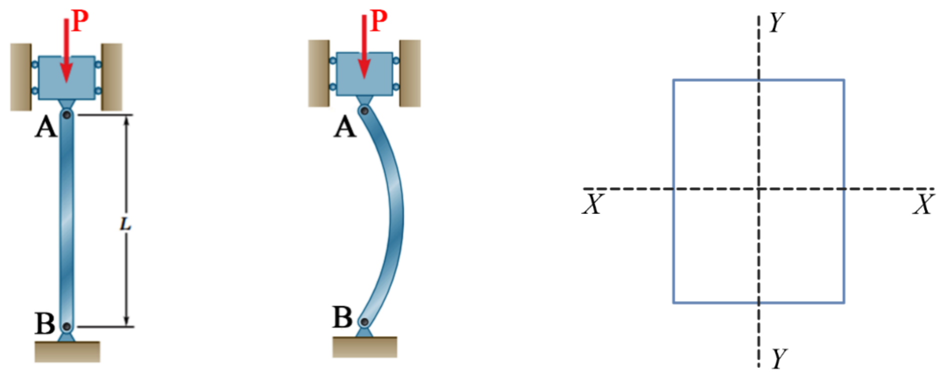

Fig.11.2 Euler's theory of Buckling for column

$$P_{cr} = \frac{\pi^2 EI_{min}}{l_e^2}$$

$I_{min} = min^m$ of I_X, I_Y

$l_e \rightarrow$ effective length

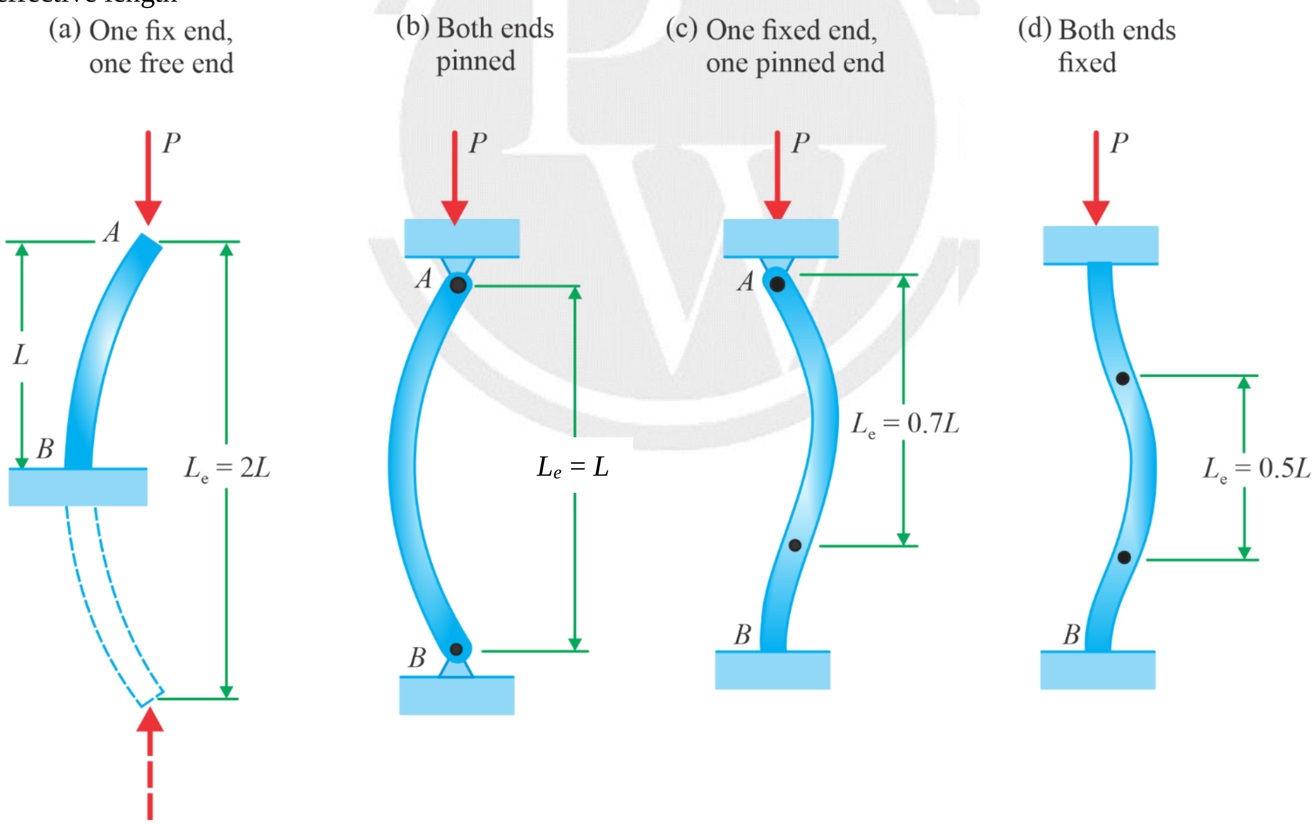

Fig.11.3 Various end conditions for column

$$P_{cr} \propto \frac{1}{\ell_c^2}$$

End Conditions		Effective length l_e
(1)	One end fixed, other free	2L
(2)	Both ends hinged	L
(3)	One end fixed, other hinged	$\frac{l}{\sqrt{2}}$
(4)	Both ends fixed	$\frac{l}{2}$

11.3 Limitation of Euler's theory of Buckling

$$\lambda \geq \pi\sqrt{\frac{E}{\sigma_c}}$$

$$\lambda = \frac{l_e}{k_{\min}} = \text{Slenderness ratio}$$

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