Defence

Agniveer Vayu (Group XY) Official Model Paper

Mathematics

Q1 What is the nature of relation R, if R is defined as $R = \{(x, y) : 2x + y = 41, x, y \in N\}$?

(A) reflexive
(B) symmetric
(C) transitive
(D) None of these

Q2 $\cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ =$

(A) $\frac{1}{2}$
(B) $\frac{3}{2}$
(C) 3
(D) 0

Q3 $\sec^{-1}\left[\frac{x^2+1}{x^2-1}\right] = ?$

(A) $2\tan^{-1} x$
(B) $2x^2$
(C) $2\cot^{-1} x$
(D) x^2

Q4 Find the foci of hyperbola $9x^2 - 16y^2 = 144$.

(A) $(0, \pm 5)$
(B) $(\pm 5, 0)$
(C) $(\pm 5, 1)$
(D) $(5, \pm 1)$

Q5 Find the nature of the triangle whose vertices are A(12,8), B(-2,6) & C(6,0).

(A) Isosceles Right angle triangle
(B) Equilateral triangle
(C) Scalene triangle
(D) None of these

Q6 For every point P(x, y, z) on the xy - plane,

(A) x = 0
(B) y = 0
(C) z = 0
(D) None of these

Q7 Find the conjugate of $(6 + 5i)^2$.

(A) 60 + 11i
(B) 11 - 60i
(C) 11+ 60i
(D) 60 - 11i

Q8 C(n, r) + 2C(n, r - 1) + C(n, r - 2) =

(A) C(n+1, r)
(B) C(n+2, r)
(C) C(n+2, r-1)
(D) C(n+1, r-1)

Q9 If n^{th} term of a G.P. is 2^n then find the sum of its first 6 terms.

(A) 126
(B) 124
(C) 190
(D) 154

Q10 Find the coefficient of x^2 in the expansion of $\left(3x - \frac{1}{x}\right)^6$.

(A) 405
(B) 7290
(C) 2430
(D) 1215

Q11 $\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}^2 =$

(A) $a^2b^2c^2$
(B) $4a^2b^2c^2$
(C) $\frac{1}{4}a^2b^2c^2$
(D) $(a+b+c)^2$

Q12 If $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, then what is the value of A^{-1} ? (where I represents Indentity Matrix)

(A) A
(B) $-A$
(C) I
(D) $-I$

Q13 If ω is the cube root of unity, then $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} =$

(A) 1
(B) ω
(C) ω^2
(D) 0

Q14 $\lim_{x \to 0} \frac{\sin(2+x) - \sin(2-x)}{x} = ?$

(A) $\frac{1}{2}\cos 2$
(B) 1

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(C) $2\cos 2$ (D) 9

Q15 $\frac{d}{dx}\left\{\tan^{-1}(\sec x+\tan x)\right\}=?$

(A) $-\frac{1}{2}$ (B) 1

(C) -1 (D) $\frac{1}{2}$

Q16 Find $\frac{d^2y}{dx^2}$, if $\sqrt{x+y}+\sqrt{y-x}=c$.

(A) $\frac{2}{c}$ (B) $-\frac{2}{c^2}$

(C) $\frac{2}{c^2}$ (D) $\frac{4}{c^2}$

Q17 An edge of a cube is increasing at the rate of 3 cm/sec. Find the rate at which does the volume increase (in cm^3 / sec) if the edge of the cube is 10 cm .

(A) 900 (B) 725

(C) 700 (D) 825

Q18 If $s=t^3-4t^2+5$ describes the motion of a particle, then its velocity (in unit /sec) when the acceleration vanishes, is

(A) $\frac{16}{9}$ (B) $-\frac{32}{3}$

(C) $\frac{4}{3}$ (D) $\frac{-16}{3}$

Q19 Find the standard deviation of 8,12,13,15,22.

(A) 3.54 (B) 3.72

(C) 4.21 (D) 4.6

Q20 If a coin is tossed thrice, find the probability of getting one or two heads.

(A) $\frac{4}{5}$ (B) $\frac{5}{8}$

(C) $\frac{3}{4}$ (D) $\frac{6}{7}$

Q21 If the points

$A\left(60\hat{i}+3\hat{j}\right), B\left(40\hat{i}-8\hat{j}\right)$ and are

$C\left(a\hat{i}-52\hat{j}\right)$

collinear, then a is equal to

(A) 40 (B) -40

(C) 20 (D) -20

Q22

$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sin^2 x dx=?$

(A) 1 (B) $\frac{\pi}{3}-\frac{\sqrt{3}}{4}$

(C) $\frac{\pi}{2}-\frac{1}{4}$ (D) 0

Q23 $\int \frac{\cos 2x}{\cos^2 x \cdot \sin^2 x} dx=?$

(A) -cotx - tanx + c

(B) cotx - tanx + c

(C) cotx + tanx +c

(D) tanx - cotx + c

Q24 Find the solution of the differential equation $\frac{dy}{dx}=e^{x+y}+x^2e^y$.

(A) $e^x-e^y+\frac{y^3}{3}=c$ (B) $e^x+e^y+\frac{x^3}{3}=c$

(C) $e^x+e^{-y}+\frac{x^3}{3}=c$ (D) $e^x+e^{-y}+\frac{y^3}{3}=c$

Q25 Find the area of the region (in sq.units) bounded by the curve $y^2=2y-x$ and y-axis.

(A) $\frac{8}{3}$ (B) $\frac{4}{3}$

(C) $\frac{5}{3}$ (D) $\frac{2}{3}$

Answer Key

Q1	(D)
Q2	(A)
Q3	(C)
Q4	(B)
Q5	(A)
Q6	(C)
Q7	(B)
Q8	(B)
Q9	(A)
Q10	(D)
Q11	(B)
Q12	(A)
Q13	(D)
Q14	(C)
Q15	(D)
Q16	(C)
Q17	(A)
Q18	(D)
Q19	(D)
Q20	(C)
Q21	(B)
Q22	(B)
Q23	(A)
Q24	(C)
Q25	(B)

Defence

Hints & Solutions

Q1 Text Solution:

Given relation What is the nature of relation R, if R is defined as

$R=\{(x, y): 2x+y=41, x, y \in N\}$?

Reflexivity:

Let x belongs to N. For Reflexive, $(x, x) \in R$

$\Rightarrow 2x+x=41$

$\Rightarrow x=\frac{41}{3} \notin N$

$\Rightarrow (x, x) \notin R$

Hence, it is not reflexive relation

Symmetry:

Let $(x, y) \in R$. Then, $2x+y=41$ Which is not equal to $2y+x=41$ i.e. $(y, x) \notin R$

So, R is not symmetric relation.

Transitivity:

Let (x, y) and $(y, z) \in R$

$\Rightarrow 2x+y=41$ and $2y+z=41$

It does not mean that $2x+z=41$

i.e. $(x, z) \notin R$

Thus, R is not transitive relation.

Q2 Text Solution:

$\cos 24^{\circ}+\cos 55^{\circ}+\cos 125^{\circ}+\cos 204^{\circ}+\cos 300^{\circ}$

$=\cos 24^{\circ}+\cos 55^{\circ}+\cos (180-55)^{\circ}+\cos (180+24)^{\circ}+\cos (360-60)^{\circ}$

$=\cos 24^{\circ}+\cos 55^{\circ}-\cos 55^{\circ}-\cos 24^{\circ}+\cos 60^{\circ}$

$=\cos 60^{\circ}$

$=\frac{1}{2}$

Q3 Text Solution:

Let $y=\sec^{-1}\left[\frac{x^2+1}{x^2-1}\right]$, Let $x=cot\theta \Rightarrow \theta=cot^{-1}x$

$\Rightarrow y=\sec^{-1}\left(\frac{\cot^2 \theta+1}{\cot^2 \theta-1}\right)$

$=\sec^{-1}\left(\frac{\frac{1}{\tan^2 \theta}+1}{\frac{1}{\tan^2 \theta}-1}\right)$

$=\sec^{-1}\left(\frac{1+\tan^2 \theta}{1-\tan^2 \theta}\right)$

$=\sec^{-1}\left(\frac{1}{\cos 2\theta}\right) \quad \left[\because \cos 2\theta=\left(\frac{1-\tan^2 \theta}{1+\tan^2 \theta}\right)\right]$

$=\sec^{-1}(sec2\theta)$

$\Rightarrow y=2\theta=2cot^{-1}x$

Q4 Text Solution:

Given hyperbola $9x^2-16y^2=144$

$\Rightarrow \frac{x^2}{4^2}-\frac{y^2}{3^2}=1$

$a=4$ and $b=3$.

We know $e=\sqrt{1+\frac{b^2}{a^2}}$

$\Rightarrow e=\sqrt{1+\frac{9}{16}}$

$\Rightarrow e=\frac{5}{4}$

$\text{foci}=(\pm ae, 0) \Rightarrow \left(\pm 4 \times \frac{5}{4}, 0\right)$

$\Rightarrow \text{foci}=(\pm 5, 0)$

Q5 Text Solution:

Given a Triangle whose vertices are $A(12,8), B(-2,6), C(6,0)$

Distance between the two points $A(12,8)$ and $B(-2,6)=$

$\sqrt{(-2-12)^2+(6-8)^2}=\sqrt{196+4}$

$=\sqrt{200}$

Distance between the two points $B(-2,6)$ and $C(6,0)=$

$\sqrt{(6+2)^2+(0-6)^2}=\sqrt{64+36}=\sqrt{100}$

$=10$

Distance between the two points $C(6,0)$ and $A(12,8)=$

$\sqrt{(12-6)^2+(8-0)^2}=\sqrt{36+64}$

$=\sqrt{100}=10$

Since, $(AB)^2 = (BC)^2 + (CA)^2$
Hence, the triangle is Right Angled Triangle and also its two sides are equal.
Hence, it is Isosceles Right Angled Triangle.

Q6 Text Solution:

We know, for the point is in x-y plane, its z-component is 0 .
Hence, z = 0 in x-y plane.

Q7 Text Solution:

Given complex number $z = (6 + 5i)^2$

$$(6+5i)^2 = 6^2 + (5i)^2 + 2(6)(5i)$$
$$\Rightarrow (6+5\mathrm{i})^2 = 36 - 25 + 60\mathrm{i}$$
$$\Rightarrow (6+5\mathrm{i})^2 = 11 + 60\mathrm{i}$$
$$\Rightarrow \mathrm{z} = 11 + 60\mathrm{i}$$

Hence, $\bar{z} = 11 - 60\mathrm{i}$

Q8 Text Solution:

$$\mathrm{C(n,r)} + 2\mathrm{C(n,r-1)} + \mathrm{C(n,r-2)}$$
$$= {}^nC_r + 2\,{}^nC_{r-1} + {}^nC_{r-2}$$
$$= {}^nC_r + {}^nC_{r-1} + {}^nC_{r-1} + {}^nC_{r-2}$$
$$= {}^{n+1}C_r + {}^{n+1}C_{r-1}$$
$$= {}^{n+2}C_r$$

Q9 Text Solution:

Given nth term of GP, $a_n = 2^n$,
Here first term $a = 2^1 = 2$ and common ratio $r = 2$

$$\Rightarrow \text{Sum of} n \text{terms} = \frac{a(r^n-1)}{(r-1)}$$
$$\Rightarrow S_n = \frac{2(2^6-1)}{2-1}$$
$$= 2(64-1)$$
$$= 2(63) = 126$$

Q10 Text Solution:

Given Binomial Expansion, $\left(3x - \frac{1}{x}\right)^6$

We know, General Term

$$T_{r+1} = {}^6C_r\,(3x)^{6-r}\left(-\frac{1}{x}\right)^r$$
$$\Rightarrow T_{r+1} = (-1)^r\,{}^6C_r\,(3)^{6-r}\,x^{6-2r} \quad \text{......(i)}$$

Hence for $x^2 \Rightarrow 6 - 2r = 2 \Rightarrow r = 2$

Putting r=2 in (i), we get

$$T_{2+1} = (-1)^2\,{}^6C_2\,\left(3\right)^{6-2} x^2$$
$$\Rightarrow T_3 = \frac{6\times5}{2}\left(3\right)^4 x^2$$
$$\Rightarrow T_3 = 1215\,x^2$$

Q11 Text Solution:

Let $\Delta = \begin{vmatrix} 0 & \mathrm{c} & \mathrm{b} \\ \mathrm{c} & 0 & \mathrm{a} \\ \mathrm{b} & \mathrm{a} & 0 \end{vmatrix}$

$$= 0 - c(0 - ab) + b(ca - 0)$$
$$= abc + abc$$
$$= 2abc$$
$$\Rightarrow \Delta^2 = (2abc)^2$$
$$\Rightarrow \begin{vmatrix} 0 & \mathrm{c} & \mathrm{b} \\ \mathrm{c} & 0 & \mathrm{a} \\ \mathrm{b} & \mathrm{a} & 0 \end{vmatrix}^2 = 4a^2b^2c^2$$

Q12 Text Solution:

Given, $\mathrm{A} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow |A| = -1$

Let C_{ij} be the cofactor of a_{ij} element of matrix A,

$$\Rightarrow \mathrm{adj}\,(\mathrm{A}) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$$
$$\Rightarrow \mathrm{adj}\,(A) = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

We know, $\mathrm{A}^{-1} = \frac{\mathrm{adj}\,A}{|A|}$

$$\Rightarrow A^{-1} = \frac{1}{-1}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$
$$\Rightarrow A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \mathrm{A}$$

Q13 Text Solution:

We know, $1+\omega+\omega^2=0$, where ω is complex cube root of unity,

$$\Delta=\begin{vmatrix}1 & \omega & \omega^2\\ \omega & \omega^2 & 1\\ \omega^2 & 1 & \omega\end{vmatrix}$$

$C_1 \rightarrow C_1+C_2+C_3$

$$\Rightarrow \Delta=\begin{vmatrix}1+\omega+\omega^2 & \omega & \omega^2\\ 1+\omega+\omega^2 & \omega^2 & 1\\ 1+\omega+\omega^2 & 1 & \omega\end{vmatrix}$$

$$\Rightarrow \Delta=\begin{vmatrix}0 & \omega & \omega^2\\ 0 & \omega^2 & 1\\ 0 & 1 & \omega\end{vmatrix}=0$$

Q14 Text Solution:

$\lim_{x\to 0}\frac{\sin(2+x)-\sin(2-x)}{x}$

$\Rightarrow$

$$\lim_{x\to 0}\frac{2\cos\left(\frac{(2+x)+(2-x)}{2}\right)\sin\left(\frac{(2+x)-(2-x)}{2}\right)}{x}$$

$$\Rightarrow \lim_{x\to 0} 2\cos(2)\frac{\sin(x)}{x}$$

$$\Rightarrow 2\cos(2)\lim_{x\to 0}\frac{\sin x}{x}$$

$$\Rightarrow 2\cos(2)\times 1$$

$$\Rightarrow 2\cos 2$$

Q15 Text Solution:

$$\text{Let } y=\tan^{-1}[\sec x+\tan x]$$

$$\Rightarrow y=\tan^{-1}\left[\frac{1+\sin x}{\cos x}\right]$$

$$\Rightarrow y=\tan^{-1}\left[\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\right]$$

$$\Rightarrow y=\tan^{-1}\left[\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right]=\frac{\pi}{4}+\frac{x}{2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2}$$

Q16 Text Solution:

Given $\sqrt{x+y}+\sqrt{y-x}=c\ldots\ldots(1)$

By Rationalizing LHS, we get

$$\Rightarrow \frac{(\sqrt{x+y}+\sqrt{y-x})\times(\sqrt{x+y}-\sqrt{y-x})}{(\sqrt{x+y}-\sqrt{y-x})}=c$$

$$\Rightarrow \frac{(x+y)-(y-x)}{\sqrt{x+y}-\sqrt{y-x}}=c$$

$$\Rightarrow \frac{2x}{\sqrt{x+y}-\sqrt{y-x}}=c$$

$$\Rightarrow \sqrt{x+y}-\sqrt{y-x}=\frac{2x}{c}\ldots(2)$$

Eq. (1) and (2) Adding, we get

$$2\sqrt{x+y}=c+\frac{2x}{c}$$

$$\Rightarrow 4(x+y)=c^2+\frac{4x^2}{c^2}+4x$$

$$\Rightarrow 4y=c^2+\frac{4x^2}{c^2}$$

$$\Rightarrow 4\frac{dy}{dx}=4\cdot\frac{2x}{c^2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2x}{c^2}$$

$$\Rightarrow \frac{d^2y}{dx^2}=\frac{2}{c^2}$$

Q17 Text Solution:

Let x be the side of the cube and V be the volume of the cube,

Given, $\frac{dx}{dt}=3\frac{cm}{\sec}$ and to find $\frac{dV}{dt}$, when $x=10\,\text{cm}$

$$V=x^3$$

$$\frac{dV}{dt}=3x^2\frac{dx}{dt}$$

$$\frac{dV}{dt}=3(10)^2(3)$$

$$\frac{dV}{dt}=900\frac{\text{cm}^3}{\sec}$$

Q18 Text Solution:

Given $s=t^3-4t^2+5$

Hence, the velocity of the particle,

$$v=\frac{ds}{dt}$$

$$v=3t^2-8t$$

When acceleration(a) of particle vanishes, means a=0

$a=\frac{dv}{dt}$
$a=6t-8$
$\Rightarrow 6t-8=0$
$\Rightarrow t=\frac{4}{3}sec$
Velocity at $t=\frac{4}{3}$sec.
$v=3\left(\frac{4}{3}\right)^2-8\left(\frac{4}{3}\right)$
$=\frac{16}{3}-\frac{32}{3}$
$v=-\frac{16}{3}$ unit /sec

Q19 Text Solution:

Given data 8,12,13,15,22
$\Rightarrow \bar{x}=\frac{8+12+13+15+22}{5}=14$
Variance $=\frac{\sum(x_i-\bar{x})^2}{n}$
Variance $=\frac{(-6)^2+(-2)^2+(-1)^2+(1)^2+(8)^2}{5}$
$=\frac{36+4+1+1+64}{5}$
$=\frac{106}{5}$
$=21.2$
$S.D=\sqrt{21.2}=4.6$

Q20 Text Solution:

A coin is tossed thrice, Hence its sample space is
$S=\{HHH, HHT, HTH, THH, THT, TTH,$ HTT, TTT $\}$
Probability of getting one or two heads:
$A=\{HHT, HTH, THH, THT, TTH, HTT\}$
$P(A)=\frac{6}{8}$
$=\frac{3}{4}$

Q21 Text Solution:

Given three position vectors,
A $\left(60\hat{i}+3\hat{j}\right)$, B $\left(40\hat{i}-8\hat{j}\right)$ and
C $\left(a\hat{i}-52\hat{j}\right)$
$\therefore \overrightarrow{AB}=\vec{B}-\vec{A}$
$=\left(40\hat{i}-8\hat{j}\right)-\left(60\hat{i}+3\hat{j}\right)=-20\hat{i}-11\hat{j}$
$\overrightarrow{BC}=\vec{C}-\vec{B}$
$=\left(a\hat{i}-52\hat{j}\right)-\left(40\hat{i}-8\hat{j}\right)=\left(a-40\right)\hat{i}-44\hat{j}$
Since all position vectors are collinear, hence
$\overrightarrow{AB}=\lambda\,\overrightarrow{BC}$
$\Rightarrow -20\hat{i}-11\hat{j}=\lambda\left((A-40)\hat{i}-44\hat{j}\right)$
$\Rightarrow -11=-44\lambda$ and $-20=(a-40)\lambda$
$\Rightarrow \lambda=\frac{1}{4}$
Now,
$-20=(a-40)\lambda$
$\Rightarrow -20=(a-40)\frac{1}{4}$
$\Rightarrow a=-40$

Q22 Text Solution:

Let $I=\int_{-\pi/3}^{\pi/3}\sin^2 x dx$
$=2\int_0^{\pi/3}\sin^2 x dx$
$=2\int_0^{\pi/3}\frac{1-\cos 2x}{2}$
$=\int_0^{\pi/3}1dx-\int_0^{\pi/3}\cos 2x dx$
$=[x]_0^{\pi/3}-\left[\frac{\sin 2x}{2}\right]_0^{\pi/3}$
$=\frac{\pi}{3}-\frac{1}{2}\sin\frac{2\pi}{3}$
$=\frac{\pi}{3}-\frac{1}{2}\sin\left(\pi-\frac{\pi}{3}\right)$
$=\frac{\pi}{3}-\frac{\sqrt{3}}{4}$

Q23 Text Solution:

$\int \frac{\cos 2x}{\cos^2 x \cdot \sin^2 x} dx$

$= \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \cdot \sin^2 x} dx$

$= \int \left(\frac{1}{\sin^2 x} - \frac{1}{\cos^2 x}\right) dx$

$= \int \frac{1}{\sin^2 x} dx - \int \frac{1}{\cos^2 x} dx$

$= \int \operatorname{cosec}^2 x dx - \int \sec^2 x dx$

$= -\cot x - \tan x + C$

Q24 Text Solution:

Given differential equation, $\frac{\mathrm{dy}}{\mathrm{dx}} = \mathrm{e}^{\mathrm{x+y}} + \mathrm{x}^2\mathrm{e}^{\mathrm{y}}$

$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = \mathrm{e}^{\mathrm{x}} \cdot \mathrm{e}^{\mathrm{y}} + \mathrm{x}^2\mathrm{e}^{\mathrm{y}}$

$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = \mathrm{e}^{\mathrm{y}}\left(\mathrm{e}^{\mathrm{x}} + \mathrm{x}^2\right)$

Using Variable Separable and Integrating, we get

$\Rightarrow \int \frac{dy}{e^y} = \int \left(e^x + x^2\right) dx$

$\Rightarrow \int e^{-y} dy = \int \left(e^x + x^2\right) dx$

$\Rightarrow -e^{-y} = e^x + \frac{x^3}{3} + c$

$\Rightarrow \mathrm{e}^{\mathrm{x}} + \mathrm{e}^{-\mathrm{y}} + \frac{\mathrm{x}^3}{3} = c$

Q25 Text Solution:

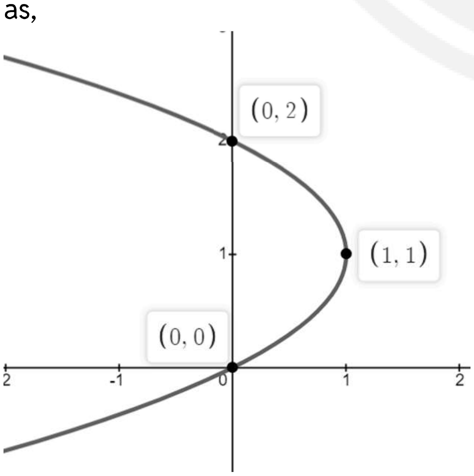
Given parabola $y^2 = 2y - x$ and y-axis

$y^2 - 2y = -x$

$\Rightarrow y^2 - 2y + 1 = -x + 1$

$\Rightarrow (y-1)^2 = -(x-1)$

Graph of $(y-1)^2 = -(x-1)$ can be drawn as,



Hence Required Area$= \int_0^2 x dy$

$= \int_0^2 \left(2y - y^2\right) dy$

$= \left[y^2 - \frac{y^3}{3}\right]_0^2 = 4 - \frac{8}{3} = \frac{4}{3}$