Manzil JEE 2025

Mathematics

DPP: 6

Sequence and Series

Q1 Given that x, y, z are positive reals such that $xyz = 32$. The minimum value of $x^2 + 4xy + 4y^2 + 2z^2$ is equal to :

(A) 64 (B) 256

(C) 96 (D) 216

Q2 If G be the GM between x and y, then the value of $\frac{1}{G^2-x^2} + \frac{1}{G^2-y^2}$ is equal to

(A) G^2

(B) $\frac{2}{\mathrm{G}^2}$

(C) $\frac{1}{\mathrm{G}^2}$

(D) $3\mathrm{G}^2$

Q3 The minimum value of the quantity $\frac{(a^2+3a+1)(b^2+3b+1)(c^2+3c+1)}{abc}$, where $a, b, c \in R^+$, is

(A) $\frac{11^3}{2^3}$

(B) 125

(C) 25

(D) 27

Q4 If $\frac{1+3+5+\ldots \text{ upto } n \text{ terms}}{4+7+10+\ldots \text{ upto } n \text{ terms}} = \frac{20}{7\log_{10} x}$ and $\mathrm{n} = \log_{10} \mathrm{x} + \log_{10} \mathrm{x}^{\frac{1}{2}} + \log_{10} \mathrm{x}^{\frac{1}{4}} + \log_{10} \mathrm{x}^{\frac{1}{8}} + \ldots\ldots + \infty$ then x is equal to

(A) 10^3 (B) 10^5

(C) 10^6 (D) 10^7

Q5 If $x_1, x_2, \ldots, x_{20}$ are in H.P. and $x_1, 2, x_{20}$ are in G.P., then $\sum_{r=1}^{19} x_r x_{r+1} =$

(A) 76 (B) 80

(C) 84 (D) none of these

Q6 A software company sets up m number of computer systems to finish an assignment in 17 days. If 4 computer systems crashed on the start of the second day, 4 more computer systems crashed on the start of the third day and so on, then it took 8 more days to finish the assignment. The value of m is equal to :

(A) 180 (B) 150

(C) 160 (D) 125

Q7 If $\mathrm{S} = 1^2 + 3^2 + 5^2 + \ldots\ldots + (99)^2$ then the value of the sum $2^2 + 4^2 + 6^2 + \ldots\ldots + (100)^2$ is

(A) $\mathrm{S} + 2550$

(B) $2\ \mathrm{S}$

(C) $4\ \mathrm{S}$

(D) $\mathrm{S} + 5050$

Q8 Let x_n, y_n, z_n, w_n denotes nth terms of four different arithmetic progressions with positive terms. If $x_4 + y_4 + z_4 + w_4 = 8$ and $x_{10} + y_{10} + z_{10} + w_{10} = 20$. then maximum value of $x_{20} \cdot y_{20} \cdot z_{20} \cdot w_{20}$ is

(A) 10^4 (B) 10^6

(C) 10^8 (D) 10^{10}

Q9 Three numbers are in $\mathrm{A.P.}$ such that their sum is 24 and sum of their squares is 200. The numbers are

(A) $2, 8, 14$ (B) $4, 8, 12$

(C) $6, 8, 10$ (D) $5, 8, 11$

Q10 Direction :

$1, \log_y x, \log_z y, -15 \log_x z$ be the first four terms of an AP with common difference d. All the terms of an AP being real. The value of $\left(x^2 y^2\right)$ is

(A) 0 (B) 1
(C) 2 (D) 3

Q11 If the first and the nth terms of a GP are a and b respectively and if P is the product of the first n terms, then P^2 is equal to

(A) ab
(B) $(ab)^{n/2}$
(C) $(ab)^n$
(D) None of these

Q12 If $0 < \theta, \phi < \frac{\pi}{2}, x = \sum_{n=0}^{\infty} \cos^{2n} \theta, y =$ and $\sum_{n=0}^{\infty} \sin^{2n} \phi$
$z = \sum_{n=0}^{\infty} \cos^{2n} \theta \cdot \sin^{2n} \phi$ then

(A) $xyz = 4$
(B) $xy - z = (x + y)z$
(C) $xy + yz + zx = z$
(D) $xy + z = (x + y)z$

Q13 Sum the following series to n terms:
$\frac{1}{1 \cdot 6} + \frac{1}{6 \cdot 11} + \frac{1}{11 \cdot 14} + \frac{1}{14 \cdot 19} + \ldots$
$+ \frac{1}{(5n-4)(5n+1)}$

(A) $\frac{n}{5n+1}$ (B) $\frac{n}{2n+1}$
(C) $\frac{2n}{5n+1}$ (D) $\frac{3n}{5n+1}$

Q14 If the common difference of an A. P. is 1 and 6^{th} term is $\frac{(4 \cos^2 \alpha + 1)}{\cos^2 \alpha}$, then the first term is, where $\alpha \neq (2n+1)\frac{\pi}{2}$

(A) $\sec^2 \alpha$
(B) $\tan^2 \alpha$
(C) $\cot^2 \alpha$
(D) $\text{cosec}^2 \alpha$

Q15 Sequence $\{t_n\}$ of positive terms is a G.P. If $t_6, 2, 5, t_{14}$ form another G.P. in that order, then the product $t_1 t_2 t_3 \ldots\ldots t_{18} t_{19}$ is equal to :

(A) 10^9
(B) 10^{10}
(C) $10^{17/2}$
(D) $10^{19/2}$

Q16 If sum of series $S = (\sqrt{2} + 1) + 1 + (\sqrt{2} - 1) + \ldots \infty$ is given as $\frac{a + b\sqrt{2}}{c}$, (where $a, b, c \in \mathrm{N}$) then the value of $\frac{a+b}{c}$ is

(A) -1 (B) $\frac{2}{7}$
(C) $\frac{7}{2}$ (D) 7

Q17 The sum of all positive integers n for which $\frac{1^3 + 2^3 + \ldots\ldots + (2n)^3}{1^2 + 2^2 + \ldots\ldots + n^2}$ is also an integer is.

(A) 8 (B) 9
(C) 15 (D) infinite

Q18 If $a_1, a_2, a_3, \ldots\ldots$ are in $\mathrm{A.P.}$ such that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$, then $a_1 + a_2 + a_3 + \ldots\ldots + a_{23} + a_{24}$ is equal to

(A) 909 (B) 75
(C) 750 (D) 900

Q19 If m is the AM of two distinct real numbers l and n $(l, n > 1)$ and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals

(A) $4l^2 m^2 n^2$
(B) $4\,\mathrm{lm}^2\, n$
(C) $4l^2 mn$
(D) $4\mathrm{lmn}^2$

Q20 The sum of the series $\frac{x}{1-x^2} + \frac{x^2}{1-x^4} + \frac{x^4}{1-x^8} + \cdots$, to infinite terms, if $|x| < 1$, is

(A) $\frac{x}{1-x}$
(B) $\frac{1}{1-x}$

(C) $\frac{1+x}{1-x}$

(D) 1

Q21 If $2, h_1, h_2, \ldots\ldots, h_{20}, 6$ are in harmonic progression and $2, a_1, a_2, \ldots\ldots, a_{20}, 6$ are in arithmetic progression, then the value of $a_3 h_{18}$ is equal to

(A) 6 (B) 12

(C) 3 (D) 9

Q22 If $(1 \cdot 5)^{30} = k$, then the value of $\sum_{n=2}^{29}(1 \cdot 5)^n$, is :

(A) $2k - 3$

(B) $k + 1$

(C) $2k + 7$

(D) $2k - \frac{9}{2}$

Q23 $\sum_{j=1}^{n}\sum_{i=1}^{n} i \cdot j =$

(A) $\sum n$

(B) $\sum n^2$

(C) $\sum n^3$

(D) $\sum (n+1)$

Q24 The sum of the series upto 10 terms

$\left(1\frac{2}{3}\right)^2 + \left(2\frac{1}{3}\right)^2 + 3^2 + \left(3\frac{2}{3}\right)^2 + \ldots\ldots$ is

(A) $\frac{1390}{9}$

(B) $\frac{1790}{9}$

(C) $\frac{1990}{9}$

(D) $\frac{2290}{9}$

Q25 Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series $1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 +$ If $B - 2A = 100\lambda$, then λ is equal to:

(A) 464 (B) 496

(C) 232 (D) 248

Answer Key

Q1	(C)
Q2	(C)
Q3	(B)
Q4	(B)
Q5	(A)
Q6	(B)
Q7	(D)
Q8	(A)
Q9	(C)
Q10	(B)
Q11	(C)
Q12	(D)
Q13	(A)
Q14	(B)
Q15	(D)
Q16	(C)
Q17	(A)
Q18	(D)
Q19	(C)
Q20	(B)
Q21	(B)
Q22	(D)
Q23	(C)
Q24	(D)
Q25	(D)