Electromagnetic Field Theory

Published By:

Physics Wallah

ISBN: 978-93-94342-39-2

Mobile App: Physics Wallah (Available on Play Store)

Website: www.pw.live

Email: support@pw.live

Rights

All rights will be reserved by Publisher. No part of this book may be used or reproduced in any manner whatsoever without the written permission from author or publisher.

In the interest of student's community:

Circulation of soft copy of Book(s) in PDF or other equivalent format(s) through any social media channels, emails, etc. or any other channels through mobiles, laptops or desktop is a criminal offence. Anybody circulating, downloading, storing, soft copy of the book on his device(s) is in breach of Copyright Act. Further Photocopying of this book or any of its material is also illegal. Do not download or forward in case you come across any such soft copy material.

Disclaimer

A team of PW experts and faculties with an understanding of the subject has worked hard for the books.

While the author and publisher have used their best efforts in preparing these books. The content has been checked for accuracy. As the book is intended for educational purposes, the author shall not be responsible for any errors contained in the book.

The publication is designed to provide accurate and authoritative information with regard to the subject matter covered.

This book and the individual contribution contained in it are protected under copyright by the publisher.

(This Module shall only be Used for Educational Purpose.)

ELECTROMAGNETIC FIELD THEORY

INDEX

1 CO-ORDINATE SYSTEM

1.1. Introduction

1. **Vector :** It has magnitude and direction. In addition, it follows Vector Law of Addition.
   e.g. :- Electric field, Magnetic Field, Force etc.

2. **Scalar :** It has magnitude and no direction. It does not follow Vector Law of Addition.
   eg :– Current, Distance, Potential etc.

3. **Tensor :** It has magnitude and direction. It does not follow Vector Law of Addition. It shows different values in different directions at the same point.
   e.g.:– Conductivity, Resistivity, Refractive Index.

4. **Unit Vector:** It is the vector which has unit magnitude and directed along increasing direction of parameters.

1.2. Equation of line in 3-dimentional

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}=constant$$

Or

$$\frac{x-x_2}{x_2-x_1}=\frac{y-y_2}{y_2-y_1}=\frac{z-z_2}{z_2-z_1}=constant$$

Equation of Plane In 3-Dimentional.

$$ax+by+cz=d$$

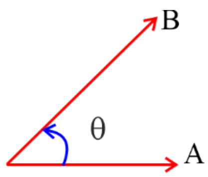
Eg. $$3x+4y=5$$

Equation of line in two-dimensional. However, equation of plane in three-dimensional.

1. X = Constant.
   (a) A plane is parallel to Y and Z-axis.
   (b) Y and Z-axis is tangential component.
   (c) X axis is normal component.
2. Y = Constant.
   (a) A plane is parallel to X and Z-axis.
   (b) X and Z axis is tangential component.
   (c) Y axis is normal component

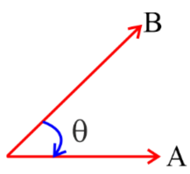
3. Z = Constant
   (a) A plane is parallel to X and Y-axis.
   (b) X and Y-axis is tangential components.
   (c) Z axis is normal components.

1.2.1. Cross Product



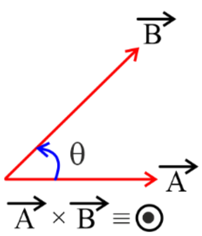
$\vec{A}\times\vec{B}\equiv AB\sin\theta\odot$

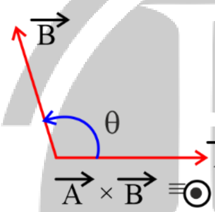
$\odot\equiv$ Outward direction (Anticlockwise)

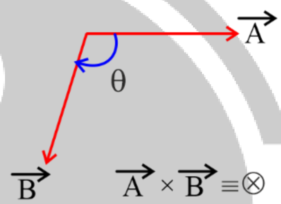


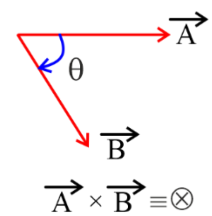
$\vec{B}\times\vec{A}\equiv AB\sin\theta\otimes$

$\otimes\equiv$ Inward direction







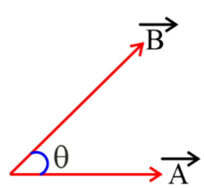


$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

1.2.2. Dot Product

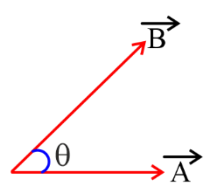
$$\vec{A}\bullet\vec{B}=a_1b_1+a_2b_2+a_3b_3$$

1. Projection of B along Vector $\vec{A}$



A||ar = B cos θ

2. Projection of vector $\vec{B}$ along Vector $\vec{A}$.



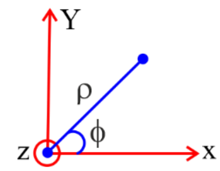
$\vec{A}$||ar = B cos θ $\hat{A}$

3. Projection of vector $\vec{B}$ perpendicular to Vector $\vec{A}$.

$\vec{A}_{\perp r}=\vec{B}-(B\cos\theta)\hat{A}$.

Point Conversion

1. Cartesian to cylindrical

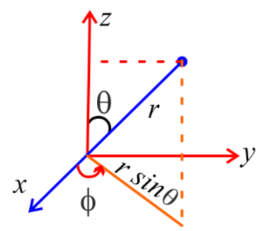


$$\rho = \sqrt{X^2 + Y^2}, \quad \phi = \tan^{-1}\left(\frac{Y}{X}\right), \quad Z = Z$$

2. Cylindrical to Cartesian

$$X = \rho\cos\phi, \; Y = \rho\sin\phi, \; \mathrm{Z} = \mathrm{Z}$$

3. Cartesian to spherical



$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

4. Spherical to Cartesian

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

5. Cylindrical to Spherical

$$r = \sqrt{\rho^2 + z^2}, \; \theta = \cos^{-1}\left(\frac{z}{\sqrt{\rho^2 + z^2}}\right), \phi = \phi$$

6. Spherical to Cylindrical

$$\rho = r\sin\theta, \; \phi = \phi, \; z = r\cos\theta.$$

Unit vector Conversion

1. Cartesian to Cylindrical

$$\begin{bmatrix} \hat{a}_\rho \\ \hat{a}_\phi \\ \hat{a}_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix}$$

2. Cylindrical to Cartesian

$$\begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{a}_\rho \\ \hat{a}_\phi \\ \hat{a}_z \end{bmatrix}$$

3. Cartesian to Spherical

$$\begin{bmatrix} \hat{a}_r \\ \hat{a}_\theta \\ \hat{a}_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix}$$

4. Spherical to Cartesian

$$\begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} \hat{a}_r \\ \hat{a}_\theta \\ \hat{a}_\phi \end{bmatrix}$$

5. Spherical to Cylindrical.

$$\begin{bmatrix} \hat{a}_\rho \\ \hat{a}_\phi \\ \hat{a}_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} \hat{a}_r \\ \hat{a}_\theta \\ \hat{a}_\phi \end{bmatrix}$$

6. Cylindrical to Spherical

$$\begin{bmatrix} \hat{a}_r \\ \hat{a}_\theta \\ \hat{a}_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{a}_\rho \\ \hat{a}_\phi \\ \hat{a}_z \end{bmatrix}$$

1.3. Differential Length, Area and Volume

Parameter			Coefficient		
u	v	w	h_1	h_2	h_3
x	y	z	1	1	1
ρ	ϕ	z	1	ρ	1
r	θ	ϕ	1	r	$r\sin\theta$

1. Differential Length

It is a vector quantity and directed along tangential direction.

In general form.

$$\vec{dl} = h_1 du\hat{a}_u + h_2 dv\hat{a}_v + h_3 dw\hat{a}_w$$

(a) Cartesian co-ordinate system.

$$\vec{dl} = dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z$$

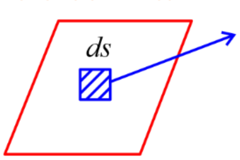
(b) Cylindrical Co-ordinate system.

$$\vec{dl} = d\rho\hat{a}_\rho + \rho d\phi\hat{a}_\phi + dz\hat{a}_z$$

(c) Spherical co-ordinate system.

$$\vec{dl} = dr\hat{a}_r + rd\theta\hat{a}_\theta + r\sin\theta d\phi\hat{a}_\phi$$

2. Differential Area



$$d\vec{s} = ds\hat{a}_n$$

$$\hat{a}_n = unit\ normal\ to\ the\ surface$$

It is a vector quantity and directed normal to the surface.
In general Form

$$d\vec{S} = h_2h_3dvdw\hat{a}_u + h_1h_3dudw\hat{a}_v + h_1h_2dudv\hat{a}_v$$

(a) Cartesian co-ordinate system.

$$d\vec{S} = dydx\hat{a}_x + dxdz\hat{a}_y + dxdy\hat{a}_z$$

(b) Cylindrical co-ordinate system.

$$d\vec{S} = \rho d\phi dz\hat{a}_\rho + d\rho dz\hat{a}_\phi + \rho d\rho d\phi\hat{a}_z$$

(c) Spherical co-ordinate system3.

$$d\vec{S} = r^2\sin\theta d\theta d\phi\hat{a}_r + r\sin\theta drd\phi\hat{a}_\theta + rdrd\theta\hat{a}_\phi$$

3. Differential Volume

In general form

$$dv = h_1h_2h_3dudvdw$$

(a) Cartesian co-ordinate system :–

$$dv = dxdydz$$

(b) Cylindrical co-ordinate system :–

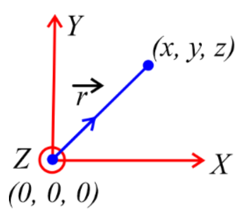
$$dv = \rho d\rho d\phi dz$$

(c) Spherical co-ordinate system.

$$dv = r^2\sin\theta drd\theta d\phi$$

1.3.1. Position Vector

(a) Cartesian co-ordinate system



$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

(b) Cylindrical Co-ordinate system.

$\vec{r} = \rho\hat{a}_\rho + z\hat{a}_z$ → 'Z' is an axis

$\vec{r} = \rho\hat{a}_\rho + y\hat{a}_y$ → 'Y' is an axis

$\vec{r} = \rho\hat{a}_\rho + x\hat{a}_x$ → 'X' is an axis

(c) Spherical Co-ordinate system :

$$\vec{r} = r\hat{a}_r$$

(d) Position vector in 2D

$$\vec{\rho} = \rho\hat{a}_\rho = (x_2 - x_1)\hat{a}_x + (y_2 - y_1)\hat{a}_y \rightarrow 'Z' axis$$

$$\vec{\rho} = \rho\hat{a}_\rho = (y_2 - y_1)\hat{a}_y + (z_2 - z_1)\hat{a}_z \rightarrow 'X' axis$$

$$\vec{\rho} = \rho\hat{a}_\rho = (x_2 - x_1)\hat{a}_x + (z_2 - z_1)\hat{a}_z \rightarrow 'Y' axis$$

❑❑❑

2 Vector Calculus

2.1. Introduction

I. Differential Form

1. Gradient of scalar function
2. Divergence.
3. Curl.
4. Laplacian operator.

II. Integral Form

1. Open line integration
   (a) Path independent
   (b) Path dependent
2. Closed Line integration.
3. Closed Surface integration.
4. Volume integration.

Gradient of scalar function 'f': –

1. **Definition:** It is a vector quantity which gives maximum rate of change of scalar function 'f' and is directed normal to surface 'f' or scalar function 'f'.

$$\vec{\nabla} f = \left.\frac{df}{dl}\right|_{max} \hat{a}_n$$

2. **Formulae:**

$$\vec{\nabla} f = \frac{1}{h_1}\frac{\partial f}{\partial u}\hat{a}_u + \frac{1}{h_2}\frac{\partial f}{\partial v}\hat{a}_v + \frac{1}{h_3}\frac{\partial f}{\partial w}\hat{a}_w$$

(a) Cartesian Co-ordinate system.

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{a}_x + \frac{\partial f}{\partial y}\hat{a}_y + \frac{\partial f}{\partial z}\hat{a}_z$$

(b) Cylindrical Co-ordinate system.

$$\vec{\nabla} f = \frac{\partial f}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{a}_\phi + \frac{\partial f}{\partial z}\hat{a}_z$$

(c) Spherical Co-ordinate system.

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{a}_\phi$$

3. Physical Significance:

(a) Maximum rate of change of scalar function ‘f’ will be given by gradient of scalar function ‘f’.

$$\left.\frac{df}{dl}\right|_{max} = \left|\vec{\nabla} f\right|$$

(b) To find directional derivate.

$$D.D = \vec{\nabla} f.\hat{A}$$

(c) It gives unit normal on surface.

$$\hat{a}_n = \frac{\vec{\nabla} f}{\left|\vec{\nabla} f\right|}$$

(d) To find angle between the surfaces.

(e) $\vec{A} = \vec{\nabla} f$

$$'f' \equiv Scalar\ function\ of\ vector \vec{A}$$

4. Properties of Gradient

(a) $\vec{\nabla}(f+g) = \vec{\nabla} f + \vec{\nabla} g$

(b) $\vec{\nabla}(fg) = f\vec{\nabla} g + g\vec{\nabla} f$

(c) $\vec{\nabla}\left(\frac{f}{g}\right) = \frac{g\vec{\nabla} f - f\vec{\nabla} g}{g^2}$

5. Application:

(a) $\vec{\nabla} r = \hat{a}_r$

(b) $\vec{\nabla}\left(\frac{1}{r}\right) = \frac{-\vec{r}}{r^3}$

(c) $\vec{\nabla}\left(r^n\right) = \left(nr^{n-2}\right)\vec{r}$

(d) $\vec{\nabla}(lnr) = \frac{\vec{r}}{r^2}$

(e) $\vec{\nabla}\left(r^2 + lnr\right) = \left(2r + \frac{1}{r}\right)\hat{a}_r$

(f) $\vec{\nabla}\left(r^2 lnr\right) = r(2 + \ln r)\hat{a}_r$

(h) $\vec{\nabla}\left(\frac{lnr}{r^2}\right) = \left(\frac{r - 2rlnr}{r^4}\right)\hat{a}_r$

Divergence:

1. Definition: It gives total outward flux per unit volume.

$$\vec{\nabla}.\vec{A} = \frac{\oiint \vec{A}.d\vec{S}}{\lim_{\Delta v \to 0} \Delta v}$$

(a) $\vec{\nabla}.\vec{A} \equiv Divergence\ at\ a\ point.$

(b) $\oiint \vec{A} \bullet d\vec{S} \equiv Divergence\ in\ a\ range.$

(c) $\oiint \vec{A} \bullet d\vec{S} = \int_{v} (\vec{\nabla} \bullet \vec{A}) dv \Rightarrow$ *Divergence theorem.*

2. Formulae:

$$\vec{\nabla} \bullet \vec{A} = \frac{1}{h_1 h_2 h_3} \left(\frac{\partial (h_2 h_3 A_u)}{\partial u} + \frac{\partial (h_1 h_3 A_v)}{\partial v} + \frac{\partial (h_1 h_2 A_w)}{\partial w} \right)$$

(a) Cartesian Co-ordinate System

$$\vec{\nabla} \bullet \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

(b) Cylindrical Co-ordinate System.

$$\vec{\nabla} \bullet \vec{A} = \frac{1}{\rho} \left(\frac{\partial (\rho A_\rho)}{\partial \rho} + \frac{\partial A_\phi}{\partial \phi} + \frac{\partial (\rho A_z)}{\partial z} \right)$$

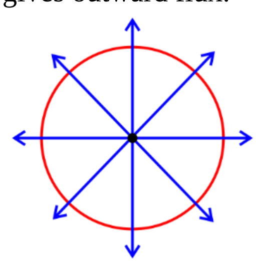
(c) Spherical Co-ordinate system.

$$\vec{\nabla} \bullet \vec{A} = \frac{1}{r^2 \sin\theta} \left(\frac{\partial (r^2 \sin\theta A_r)}{\partial r} + \frac{\partial (r \sin\theta A_\theta)}{\partial \theta} + \frac{\partial (r A_\phi)}{\partial \phi} \right)$$

3. Physical Significance:

It gives outward flux.

(a)



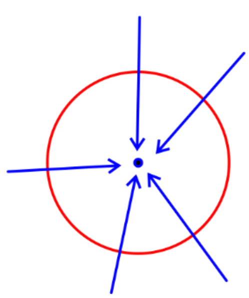
Outward Flux ≠ 0

Inward Flux = 0

Net Flux ≠ 0

$\vec{\nabla} \bullet \vec{A} > 0$

(b)



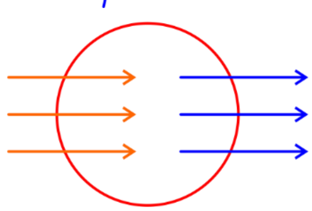
Outward Flux = 0

Inward Flux ≠ 0

Net Flux ≠ 0

$\vec{\nabla} \bullet \vec{A} < 0$

(c)

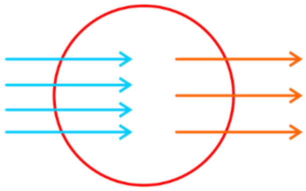


Outward Flux

= Inward Flux

Net Flux = 0

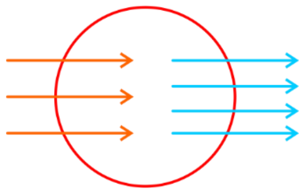
$\vec{\nabla} \bullet \vec{A} = 0$

(d) 

Outward Flux < Inward Flux

Net Flux ≠ 0

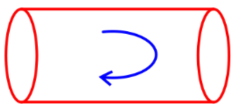
$\vec{\nabla}\cdot\vec{A} < 0$

(e) 

Outward Flux > Inward Flux

Net Flux ≠ 0

$\vec{\nabla}\cdot\vec{A} > 0$

(f) 

Outward Flux = 0
Inward Flux = 0
Net Flux = 0

$\vec{\nabla}\cdot\vec{A} = 0$

4. Properties

(a) $\vec{\nabla}\cdot\left(\vec{A}+\vec{B}\right)=\vec{\nabla}\cdot\vec{A}+\vec{\nabla}\cdot\vec{B}$

(b) $\vec{\nabla}\cdot\left(f\,\vec{A}\right)=f\left(\vec{\nabla}\cdot\vec{A}\right)+\vec{A}\cdot\left(\vec{\nabla} f\right)$

(c) $\vec{\nabla}\cdot\left(\frac{\vec{A}}{f}\right)=\frac{f\left(\vec{\nabla}\cdot\vec{A}\right)-\vec{A}\cdot\left(\vec{\nabla} f\right)}{f^2}$

5. Application

(a) $\vec{\nabla}\cdot\vec{r}=3$

(b) $\vec{\nabla}\cdot\left(r^n\hat{a}_r\right)=\left(n+2\right)r^{n-1}$

Curl:

1. Definition:

$$\vec{\nabla}\times\vec{A}=\frac{\oint\vec{A}\cdot\vec{dl}}{\lim\limits_{\Delta S\to 0}\Delta S}\hat{n}$$

Curl gives total Motive Force due to $\vec{A}$ per unit area. And it is directed normal to the rotatory plane.

(a) $\vec{\nabla}\times\vec{A}$ = Curl at a point

(b) $\oint\vec{A}\cdot\vec{dl}$ = Curl in a range

(c) $\oint\vec{A}\cdot\vec{dl} = \iint\left(\vec{\nabla}\times\vec{A}\right)\cdot d\vec{S} \Longrightarrow$ Stoke's theorem

2. Formulae:

$$\vec{\nabla}\times\vec{A}=\frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\hat{a}_u & h_2\hat{a}_v & h_3\hat{a}_w \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_1A_u & h_2A_v & h_3A_w \end{vmatrix}$$

(a) Cartesian Co-ordinate system

$$\vec{\nabla}\times\vec{A}=\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

(b) Cylindrical Co-ordinate System.

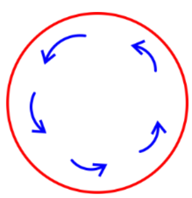
$$\vec{\nabla}\times\vec{A}=\frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \dfrac{\partial}{\partial \rho} & \dfrac{\partial}{\partial \phi} & \dfrac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

(c) Spherical Co-ordinate system

$$\vec{\nabla}\times\vec{A}=\frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\hat{a}_\phi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

3. Physical Significance: – It gives rotation.

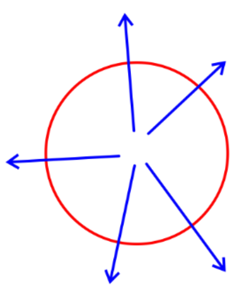
(a)



$\vec{\nabla}\times\vec{A}\neq\vec{0}$

$\vec{\nabla}\bullet\vec{A}=0$

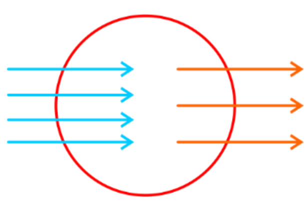
(b)



$\vec{\nabla}\times\vec{A}=\vec{0}$

$\vec{\nabla}\bullet\vec{A}\neq 0$

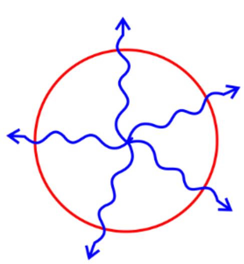
(c)



$\vec{\nabla}\times\vec{A}=\vec{0}$

$\vec{\nabla}\bullet\vec{A}\neq 0$

(d)



$\vec{\nabla}\times\vec{A}\neq\vec{0}$

$\vec{\nabla}\bullet\vec{A}\neq 0$

4. Properties:

(a) $\vec{\nabla}\times\left(\vec{A}+\vec{B}\right)=\vec{\nabla}\times\vec{A}+\vec{\nabla}\times\vec{B}$

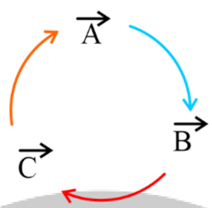
(b) $\vec{\nabla}\times\left(f\,\vec{A}\right)=f\left(\vec{\nabla}\times\vec{A}\right)-\vec{A}\times\left(\vec{\nabla}f\right)$

(c) $\vec{\nabla}\times\left(\dfrac{\vec{A}}{f}\right)=\dfrac{f\left(\vec{\nabla}\times\vec{A}\right)+\vec{A}\times\left(\vec{\nabla}f\right)}{f^{2}}$

Mixed Product:

1. Scalar Product:

(a) $\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)=\vec{B}\cdot\left(\vec{C}\times\vec{A}\right)=\vec{C}\cdot\left(\vec{A}\times\vec{B}\right)$



$$\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

(b) $\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)=0\Rightarrow\left[\vec{A}\vec{B}\vec{C}\right]=0$

Then $\vec{A},\vec{B}\;\&\;\vec{C}$ are independent vectors and they do not lie in a single plane.

(c) Volume of parallelepiped

$\left|\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)\right|$

2. Vector Product:

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right)=\left(\vec{A}\cdot\vec{C}\right)\vec{B}-\left(\vec{A}\cdot\vec{B}\right)\vec{C}$$

3. Mixed product with Del operator.

(a) $\vec{\nabla}\left(\vec{\nabla}\,f\right)=Doesnot\;\;exist$

(b) $\vec{\nabla}\cdot\left(\vec{\nabla}\,f\right)=\nabla^{2}f$

(c) $\vec{\nabla}\times\left(\vec{\nabla}\,f\right)=\vec{0}$

(d) $\vec{\nabla}\left(\vec{\nabla}\cdot\vec{A}\right)=exist$

(e) $\vec{\nabla}\cdot\left(\vec{\nabla}\cdot\vec{A}\right)=does\;\;not\;\;exist$

(f) $\vec{\nabla}\times\left(\vec{\nabla}\cdot\vec{A}\right)=does\;\;not\;\;exist$

(g) $\vec{\nabla}\left(\vec{\nabla}\times\vec{A}\right)=does\;\;not\;\;exist$

(h) $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{A}\right)=\vec{\nabla}\left(\vec{\nabla}\cdot\vec{A}\right)-\nabla^{2}\vec{A}$

(i) $\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{A}\right)=0$

Some Important Identities

1. $\vec{\nabla}\left(f+g\right)=\vec{\nabla}f+\vec{\nabla}g$

2. $\vec{\nabla}\left(fg\right)=f\vec{\nabla}g+g\vec{\nabla}f$

3. $\vec{\nabla}\left(\frac{f}{g}\right)=\frac{g\vec{\nabla}f-f\vec{\nabla}g}{g^2}$
4. $\vec{\nabla}\bullet\left(\vec{A}+\vec{B}\right)=\vec{\nabla}\bullet\vec{A}+\vec{\nabla}\bullet\vec{B}$
5. $\vec{\nabla}\bullet\left(f\vec{A}\right)=f\vec{\nabla}\bullet\vec{A}+\vec{A}\bullet\vec{\nabla}f$
6. $\vec{\nabla}\left(\frac{\vec{A}}{f}\right)=\frac{f\left(\vec{\nabla}\bullet\vec{A}\right)-\vec{A}\bullet\vec{\nabla}f}{f^2}$
7. $\vec{\nabla}\bullet\left(\vec{A}\times\vec{B}\right)=\left(\vec{\nabla}\times\vec{A}\right)\bullet\vec{B}-\left(\vec{\nabla}\times\vec{B}\right)\bullet\vec{A}$
8. $\vec{\nabla}\times\left(\vec{A}+\vec{B}\right)=\vec{\nabla}\times\vec{A}+\vec{\nabla}\times\vec{B}$
9. $\vec{\nabla}\times\left(f\vec{A}\right)=f\left(\vec{\nabla}\times\vec{A}\right)-\vec{A}\times\left(\vec{\nabla}f\right)$
10. $\vec{\nabla}\times\left(\frac{\vec{A}}{f}\right)=\frac{f\left(\vec{\nabla}\times\vec{A}\right)+\vec{A}\times\left(\vec{\nabla}f\right)}{f^2}$
11. $\vec{\nabla}\bullet\left(\vec{\nabla}f\right)=\nabla^2 f$
12. $\vec{\nabla}\times\left(\vec{\nabla}f\right)=\vec{0}$
13. $\vec{\nabla}\bullet\left(\vec{\nabla}\times\vec{A}\right)=0$
14. $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{A}\right)=\vec{\nabla}\left(\vec{\nabla}\bullet\vec{A}\right)-\nabla^2\vec{A}$

Laplacian Operator:

1. Laplacian operator with scalar function 'f'

$$\nabla^2 f=\vec{\nabla}\bullet\left(\vec{\nabla}f\right)$$

$$=\frac{1}{h_1h_2h_3}\left(\frac{\partial\left(\frac{h_2h_3}{h_1}\frac{\partial f}{\partial u}\right)}{\partial u}+\frac{\partial\left(\frac{h_1h_3}{h_2}\frac{\partial f}{\partial v}\right)}{\partial v}+\frac{\partial\left(\frac{h_1h_2}{h_3}\frac{\partial f}{\partial w}\right)}{\partial w}\right)$$

(a) Cartesian co-ordinate system

$$\nabla^2 f=\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 f}{\partial z^2}$$

(b) Cylindrical co-ordinate system.

$$\nabla^2 f=\frac{\partial^2 f}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial f}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2 f}{\partial\phi^2}+\frac{\partial^2 f}{\partial z^2}$$

(c) Spherical co-ordinate system :–

$$\nabla^2 f=\frac{\partial^2 f}{\partial r^2}+\frac{2}{r}\frac{\partial f}{\partial r}+\frac{1}{r^2}\frac{\partial^2 f}{\partial\theta^2}+\frac{\cot\theta}{r^2}\frac{\partial f}{\partial r}+\frac{1}{(r\sin\theta)^2}\frac{\partial^2 f}{\partial\phi^2}$$

Note: When scalar function '*f*' is function of 'r' only. Then, $\nabla^2 f(r) = \frac{1}{r}\frac{df(r)}{dr} + \frac{d^2 f(r)}{dr^2}$

(d) (i) $\nabla^2 r^n = n(n+1)r^{n-2}$ (ii) $\nabla^2 r = \frac{2}{r}$

(iii) $\nabla^2\left(\frac{1}{r}\right) = 0$ (iv) $\nabla^2 lnr = \frac{1}{r^2}$

2. Laplacian operator with vector $\vec{A}$

$$\nabla^2 \vec{A} = \frac{1}{h_1^2}\frac{\partial^2 \vec{A}}{\partial u^2} + \frac{1}{h_2^2}\frac{\partial^2 \vec{A}}{\partial V^2} + \frac{1}{h_3^2}\frac{\partial^2 \vec{A}}{\partial w^2}$$

(a) Cartesian co-ordinate system.

$$\nabla^2 \vec{A} = \frac{\partial^2 \vec{A}}{\partial x^2} + \frac{\partial^2 \vec{A}}{\partial y^2} + \frac{\partial^2 \vec{A}}{\partial z^2}$$

(b) Cylindrical co-ordinate system

$$\nabla^2 \vec{A} = \frac{\partial^2 \vec{A}}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 \vec{A}}{\partial \phi^2} + \frac{\partial^2 \vec{A}}{\partial z^2}$$

(c) Spherical co-ordinate system.

$$\nabla^2 \vec{A} = \frac{\partial^2 \vec{A}}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 \vec{A}}{\partial \theta^2} + \frac{1}{(r\sin\theta)^2}\frac{\partial^2 \vec{A}}{\partial \phi^2}$$

Some Important points:

(a) $\vec{\nabla} r = \hat{a}r$

(b) $\vec{\nabla} \cdot \vec{r} = 3$

(c) $\vec{\nabla} \times \vec{r} = \vec{0}$

(d) $\nabla^2 r = \frac{2}{r}$

(e) $\nabla^2 \vec{r} = \vec{0}$

(f) $\vec{\nabla} f(r) = \frac{df(r)}{dr}\hat{a}r$

(g) $\nabla^2 f(r) = \frac{2}{r}\frac{df(r)}{dr} + \frac{d^2 f(r)}{dr^2}$

(h) $\vec{\nabla} \cdot \vec{A} = \vec{0}$ then $\vec{A}$ is solenoidal and diversion less.

(i) $\vec{\nabla} \times \vec{A} = \vec{0}$ then $\vec{A}$ is irrational, conservative and path independent vector.

(j) $\nabla^2 \phi = 0$ (Laplacian Equation).

2.3. Path dependent open Line Integral.

1. $\int \vec{P} \cdot \vec{dl}$

Condition for path dependent open Line Integral is given as.

$$\vec{\nabla} \times \vec{P} \neq \vec{0}$$

Then the above integrals will be solved using parameterization process.

Example:

The Line Integral of the Vector Field

$\vec{F} = 5xz\hat{i} + (3x^2 + 2y)j + x^2 zk$ along a path from (0, 0, 0) to (1, 1, 1) parameterized by (t, t^2, t) is …...

Solution:

$x = t \Rightarrow dx = dt$

$y = t^2 \Rightarrow dy = 2tdt$

$z = t \Rightarrow dz = dt$

$$\int \vec{F} \cdot \vec{dl} = \int 5xzdx + \int (3x^2 + 2y)dy + \int x^2 zdz$$

$$= \int_0^1 5t^2 dt + \int_0^1 (3t^2 + 2t^2)(2tdt) + \int_0^1 t^3 dt$$

$$= \frac{5}{3} + \frac{10}{4} + \frac{1}{4} = \frac{20+33}{12} = \frac{53}{12} = 4.41$$

2. $\int \vec{P} \cdot \vec{dl}$

Condition for path independent open Line Integral is given by $\vec{\nabla} \times \vec{P} = \vec{0}$

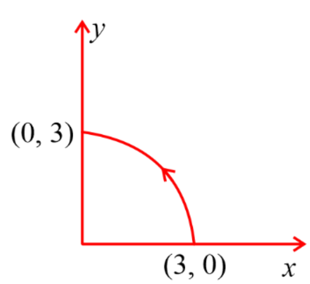
So, open line integral is performed through straight line.

Question:

As shown in the figure, C is the arc from the point (3, 0) to the point (0, 3) on the circle $x^2 + y^2 = 9$. The value of the integral.

$$\int_C (y^2 + 2yx)dx + (2xy + x^2)dy$$

is …….



Solution:

Method I: - Basic Method

$$x^2 + y^2 = 9$$

$$x = \pm\sqrt{9 - y^2}, \; y = \pm\sqrt{9 - x^2}$$

Since, the curve lies in 1st quadrant of xy plane. Hence,

$$x = \sqrt{9-y^2}\ ,\ y = \sqrt{9-x^2}$$

$$I = \int_3^0 (9-x^2)+2\left[\left(\sqrt{9-x^2}\right)x\right]dx + \int_0^3\left[2\left(\sqrt{9-y^2}\right)y+(9-y^2)\right]dy = 0$$

Method II: - Parameterization Process.

$$x^2 + y^2 = 9 \ \Rightarrow x = 3\cos\theta,\ y = 3\sin\theta$$

$$x = 3 \text{ to } 0 \ \Rightarrow \theta = 0 \text{ to } \frac{\pi}{2}$$

$$y = 0 \text{ to } 3 \ \Rightarrow \theta = 0 \text{ to } \frac{\pi}{2}$$

$$I = \int_3^0 (y^2+2yx)dx + \int_0^3 (2xy+x^2)dy$$

$$= 3\int_0^{\frac{\pi}{2}} (9\sin^2\theta + 18\sin\theta\cos\theta)(-\sin\theta d\theta) + 3\int_0^{\frac{\pi}{2}} (9\cos^2\theta + 18\sin\theta\cos\theta)(\cos\theta d\theta) = 0$$

Method III: - To check path independent or dependent.

$$\int (y^2+2yx)dx + (2xy+x^2)dy$$

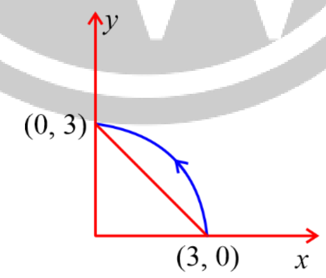
$$= \int \left((y^2+2yx)a_x + (x^2+2xy)a_y\right)\bullet\left(dx a_x + dy a_y\right)$$

$$= \int \vec{P}\bullet\vec{dl}$$

$$\vec{V}\times\vec{P} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2+2xy & 2xy+x^2 & 0 \end{vmatrix}$$

$$= (0-0)\,a_x - (0-0)\,a_y + (2x + 2y - 2y - 2x)\,a_z$$

$$= 0\,a_x + 0\,a_y + 0\,a_z = \vec{0}$$



So, the open line integral will be through straight line.

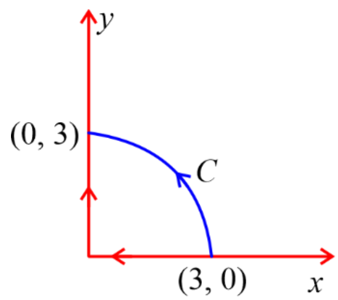
$$\frac{x-3}{3-0} = \frac{y-0}{0-3} \Rightarrow x = -y + 3 \Rightarrow y = 3 - x$$

$$I = \int (y^2+2xy)dx + (2xy+x^2)dy$$

$$\int_3^0 \left[(3-x)^2 + 2x(3-x)\right]dx + \int_0^3 \left[2(3-y)y + (3-y)^2\right]dy = 0$$

Method IV:

Open line integral is not performed through curve '*C*' but through along *x*-axis and then along *y*-axis.



Along x-axis, $y = 0$, $x = 3$ to 0, $dx \neq 0$, $dy = 0$

$$I_1 = \int \left[(0)^2 + 2x \times 0\right] dx + \int \left[2x(0) + x^2\right](0) = 0$$

Along y-axis, $x = 0$, $y = 0$ to 3, $dx = 0$, $dy \neq 0$

$$I_2 = \int_0^3 \left[2y(0) + y^2\right](0) + \int_3^0 \left[(0)^2 + 2(0)y\right] dy = 0$$

$\therefore$ $$I = I_1 + I_2 = 0 + 0 = 0$$

3. $\oint \vec{A} \cdot \vec{dl}$ = closed line integral

To solve above integral, we use closed line integral or open surface integral.

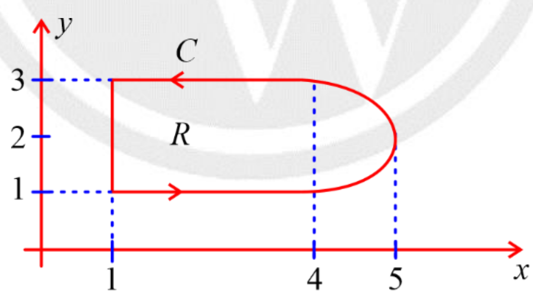
$$\oint \vec{A} \cdot \vec{dl} = \iint \left(\vec{\nabla} \times \vec{A}\right) \cdot \vec{ds}$$

Question:

Consider the line integral

$$\int_C (xdy - ydx)$$

The integral being taken in a counter clock-wise direction over the closed curve 'C' that forms the boundary of the region 'R' shown in the figure below. The region 'R' is the area enclosed by the union of a 2 × 3 rectangle and a semicircle of radius 1. The line integral evaluates to –



Solution:

Method I: Using closed line integral

Path 1. $x = 1$ to 4, $y = 1$, $dx \neq 0$, $dy = 0$

$$I_1 = \int_1^4 -y\,dx\Big|_{y=1} = -3$$

Path 2. Along semicircle centre = (4, 2)

$$(x - 4)^2 + (y - 2)^2 = (1)^2$$

$$x = \cos\phi + 4,\ y = \sin\phi + 2$$

$$y = 1 \text{ to } 3 \Rightarrow 2 + \sin\phi = 1 \text{ to } 3$$

$$\sin\phi = -1 \text{ to } 1 \Rightarrow \phi = -\frac{\pi}{2} \text{ to } \frac{\pi}{2}$$

$$x = 4 + \cos\phi \Rightarrow dx = -\sin\phi\, d\phi$$

$y = 2 + \sin\phi \Rightarrow dy = \cos\phi d\phi$

$$I_2 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (4 + \cos\phi)(\cos\phi d\phi) - \int (2 + \sin\phi)(-\sin\phi d\phi)$$

$$I_2 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4\cos\phi d\phi + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\sin\phi d\phi = 8 + \pi$$

Path 3.

$x = 4$ to 1, $y = 3$, $dx \neq 0$, $dy \neq 0$

$$I_3 = \int_4^1 -y\,dx\Big|_{y=3} = 3 \times 3 = 9$$

Path 4.

$x = 1$, $y = 3$ to 1, $dx = 0$, $dy \neq 0$

$$I_4 = \int_3^1 x\,dy\Big|_{x=1} = -2$$

$\therefore$ $I = I_1 + I_2 + I_3 + I_4$

$= -3 + (\pi + 8) + 9 - 2 = \pi + 12$

Method 2: - Using Stoke's theorem or Green's theorem

$$\oint \vec{A} \cdot \vec{dl} = \iint \left(\vec{\nabla} \times \vec{A}\right) \cdot \vec{ds}$$

$$\oint (xdy - ydx) = \oint (-ya_x + xa_y) \cdot (dxa_x + dya_y)$$

$$\vec{A} = -ya_x + xa_y$$

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & x & 0 \end{vmatrix}$$

$= [1 - (-1)]\,a_z = 2\,a_z$

Since, the given curve line in XY plane. Hence differential area will be written as

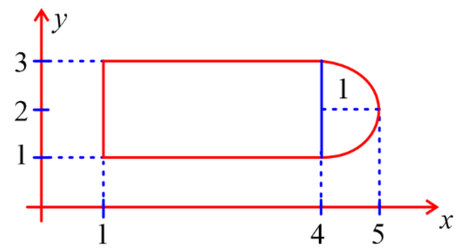
$$d\vec{s} = dx\,dy\,a_z\Big|_{z=0}$$

$$\oint (-y\,a_x + x\,a_y) \cdot (dx\,a_x + dy\,a_y)$$

$$= \iint 2a_z \cdot dx\,dy\,a_z = 2\iint dx\,dy$$

$= 2$ (Area of Curve)

$= 2$ (Area of rectangle + Area of semicircle)



$$=2\left(2\times3+\frac{\pi(1)^2}{2}\right)=\pi+12$$

Note: As we have seen that open surface integral is simpler method than closed line integral. So, we generally use Stoke's or Green theorem to solve closed line integral.

4. Closed Surface Integral or volume integrals.

$$\oiint \vec{A}\bullet d\vec{s}=\int_{\upsilon}\left(\vec{\nabla}\bullet\vec{A}\right)d\upsilon$$

$\Rightarrow$ Divergence theorem

Volume integral is easier than closed surface integral.

Question:

Consider a closed surface S surrounding a volume V. If $\vec{r}$ is the position vector of a point inside S, with n the unit normal on 'S', the value of the integral

$$\oiint 5\vec{r}\bullet nds \text{ is …….}$$

(A) 3 V

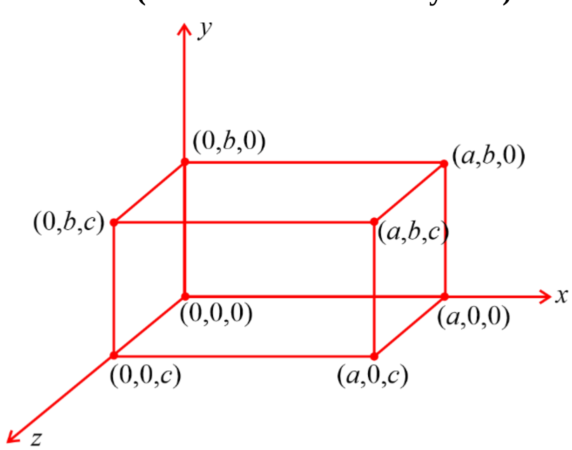
(B) 5 V

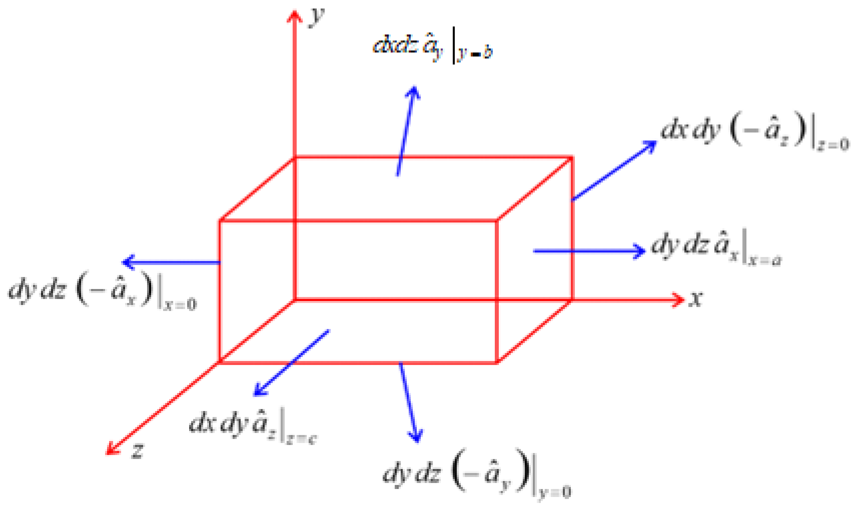
(C) 10 V

(D) 15 V

Solution:

Method I: - Assuming closed as a cuboid (cartesian co-ordinate system).





$$\oiint 5\vec{r}.d\vec{s}$$

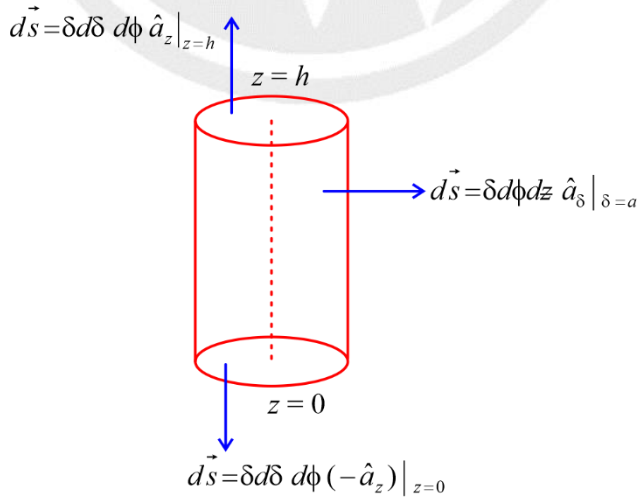
$$= \iint 5(x\hat{i}+y\hat{j}+z\hat{k}).dy\,dz\,a_x\Big|_{x=a} + 5\iint (x\hat{i}+y\hat{j}+z\hat{k}).dy\,dz\left(-a_x\right)\Big|_{x=0} + 5\iint (x\hat{i}+y\hat{j}+z\hat{k}).dx\,dz\,a_y\Big|_{y=b}$$

$$+ 5\iint (x\hat{i}+y\hat{j}+z\hat{k}).dx\,dz\left(-a_y\right)\Big|_{y=0} + 5\iint (x\hat{i}+y\hat{j}+z\hat{k}).dx\,dy\,a_z\Big|_{z=c} + 5\iint (x\hat{i}+y\hat{j}+z\hat{k}).dx\,dy\left(-a_z\right)\Big|_{z=0}$$

$$= 5\int_0^b\int_0^c x\,dy\,dz\Big|_{x=a} - 5\int_0^b\int_0^c x\,dy\,dz\Big|_{x=0} + 5\int_0^a\int_0^c y\,dx\,dz\Big|_{y=b} - 5\int_0^a\int_0^c y\,dx\,dz\Big|_{y=0} + 5\int_0^a\int_0^b z\,dx\,dy\Big|_{z=c} - 5\int_0^a\int_0^b z\,dx\,dy\Big|_{z=0}$$

$$= 5abc - 0 + 5abc - 0 + 5abc - 0 = 15abc = 15V$$

Method II: - Assuming closed surface as a cylinder (cylindrical co-ordinate system).



$$\oiint 5\vec{r}\bullet d\vec{s}$$

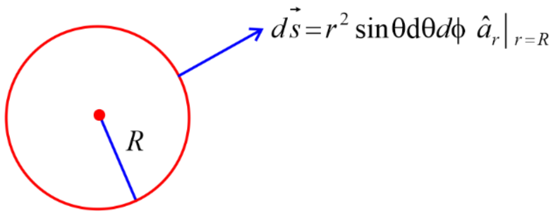
$$= 5\iint(\rho a_\rho + z a_z)\bullet \rho\, d\phi\, dz\, a_\rho\Big|_{\rho=a} + 5\iint(\rho a_\rho + z a_z)\bullet(-\rho\, d\rho\, d\phi\, a_z)\Big|_{z=0} + 5\iint(\rho a_\rho + z a_z)\bullet(\rho\, d\rho\, d\phi\, a_z)\Big|_{z=h}$$

$$= 5\int_0^{2\pi}\int_0^h \rho^2 d\phi\, dz\Big|_{\rho=a} + 5\int_0^a\int_0^{2\pi} \rho z\, d\rho\, d\phi\Big|_{z=h} - 5\int_0^a\int_0^{2\pi} \rho z\, d\rho\, d\phi\Big|_{z=0}$$

$= 5\, a^2 2\pi h + 5h\, \frac{a^2}{2} 2\pi - 0$

$= 10\pi a^2 h + 5\pi a^2 h = 15\pi a^2 h = 15V$

Method III: - Assuming closed surface as a sphere (Spherical co-ordinate system).



$$\oiint 5\vec{r} \bullet d\vec{s} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} 5(r a_r) \bullet (r^2 \sin\theta\, d\theta\, d\phi\, a_r)\Big|_{r=R}$$

$$= 5R^3 (2)\ (2\pi) = 20\pi R^3$$

$$= 15\left(\frac{4\pi R^3}{3}\right)$$

$$= 15V$$

Method IV: - Consider a cuboid as volume.

$$I = \oiint 5\vec{r} \bullet d\vec{s} = \int_V \left(\vec{\nabla} \bullet 5\vec{r}\right) dv$$

$$\vec{r} = x\hat{i} + y\,j + zk$$

$$\vec{\nabla} \bullet \vec{r} = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial x} + \frac{\partial z}{\partial z}$$

$$= 1 + 1 + 1 = 3$$

$$I = 5\int_V \left(\vec{\nabla} \bullet \vec{r}\right) dv$$

$$= 5 \times 3 \int_v dv$$

$$= 15 \int_0^a \int_0^b \int_0^c dx\, dy\, dz$$

$$= 15abc$$

Method V: - consider a cylindrical as volume.

$$\vec{r} = \rho a_\rho + z a_z$$

$$\vec{\nabla}.\vec{r} = \frac{1}{\rho}\left(\frac{\partial(\rho \bullet \rho)}{\partial \rho} + \frac{\partial(0)}{\partial \phi} + \frac{\partial(\rho \bullet z)}{\partial z}\right)$$

$$= \frac{1}{\rho}(2\rho + 0 + \rho) = 3$$

$$I = \int_v \left(\vec{\nabla} \bullet 5\vec{r}\right) dv$$

$$= 5\int_v \left(\vec{\nabla} \bullet \vec{r}\right) dv$$

$$= 5 \times 3 \int_0^a \int_0^{2\pi} \int_0^h \rho d\rho d\phi dz$$

$$= 15\ (\pi a^2 h)$$

$$= 15V$$

Method VI: - Consider a spherical as volume.

$$\vec{r} = r a_r\ ,\ \vec{\nabla} \bullet \vec{r} = \frac{1}{r^2 \sin\theta}\left[\frac{\partial(r^2 \sin\theta \bullet r)}{\partial r} + 0 + 0\right] = 3$$

$$I = \int_v \vec{\nabla} \bullet (5\vec{r})\, dv$$

$$= 15 \int_0^R \int_0^\pi \int_0^{2\pi} r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= 15\left(\frac{4\pi R^3}{3}\right)$$

$$= 15V$$

Note:

(a) In all three-co-ordinate system, closed surface integration in spherical co-ordinate is easier.

(b) Volume integral is easier than closed surface integration.

❑❑❑

3 ELECTROSTATIC

3.1. Basic Terms

(1) Electric Field Intensity $(\vec{E})$

(a) It is physical quantity.

(b) It depends upon electric force per unit charge. (Test charge is positive and tends to zero.)

$$\boxed{\vec{E} = \frac{\vec{F}}{\lim_{q\to 0} q}}$$

(c) $F = qE \Rightarrow MLT^{-2} = (AT)\,[E]$

$$[E] = \frac{MLT^{-2}}{AT} = MLA^{-1}T^{-3}$$

(d) $F = qE \Rightarrow N = C\,[E] \Rightarrow E \equiv \frac{N}{C}$

(2) Magnetic Flux density $(\vec{B})$

(a) It is physical quantity.

(b) It is measured by Lorentz's Force.

$$\vec{F}_m = q(\vec{V} \times \vec{B})$$

$$B \equiv \frac{F_m}{qV} = \frac{MLT^{-2}}{(AT)(LT^{-1})} = MA^{-1}T^{-2}$$

(c) $B \equiv \frac{F_m}{qV} = \frac{N \bullet S}{C \bullet m}$, Tesla, Gauss, $\frac{Weber}{m^2}$

(d) 1 Gauss = 10^{-4} T/ $\frac{Weber}{m^2}$ / $\frac{N.S}{C.m}$

(3) $(\vec{D}) \equiv$ Electric Flux density or Displacement density

(a) It is non-measurable quantity.

(b) It is theoretical quantity.

(c) $\vec{D} = \in \vec{E} \equiv$ Electrical Property.

(d) Gauss Law: $\oiint \vec{D} \bullet d\vec{s} = Q_{enc}$

(e) Electric flux density x Area = Charge

$\Rightarrow$ Electric flux density = $\dfrac{C}{m^2}$

$$[D] \equiv \frac{C}{m^2}$$

(f) $[D] \equiv \dfrac{AT}{L^2} = AL^{-2}T$

(4) $(\vec{H}) \equiv$ Magnetic Field Intensity

(a) It is non-measurable quantity.

(b) It is theoretical quantity.

(c) $\vec{B} = \mu \vec{H} \equiv$ Magnetic Property

(d) $\oint \vec{H} . d\vec{l} = I_{enc}$

$\Rightarrow H \equiv \dfrac{A}{m}$, $[H] = AL^{-1}$

(5) **(a)** $\in \equiv$ Electric Permittivity.

$\in_o \equiv$ Absolute Permittivity.

$\equiv 8.85 \times 10^{-12}$ F/m

$\equiv \dfrac{1}{36\pi} \times 10^{-9}$ F/m

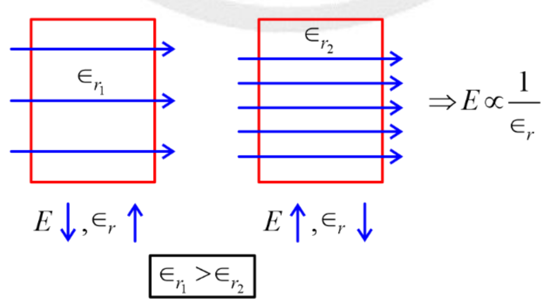
$\in_r \equiv$ Relative permittivity.

$\in_r > 1 \rightarrow$ Dielectric

$\in_r = 1 \rightarrow$ Free space or Vacuum or Air

$\in_r < 1 \rightarrow$ Metamaterial

(b) **Dielectric:** Dielectric is a type of insulator in which it has dipoles.



(c) $F = \dfrac{Q_1 Q_2}{4\pi \in r^2} \Rightarrow MLT^{-2} = \dfrac{(AT)^2}{[\in](L^2)}$

$[\in] = M^{-1} L^{-3} A^2 T^4$

(6) **(a)** $\mu \equiv$ Magnetic Permittivity.

$\mu_o \equiv$ Absolute Permittivity.

$\equiv 4\pi \times 10^{-7}$ H/m

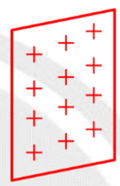
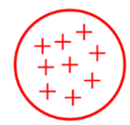
μ_r = Relative permittivity.

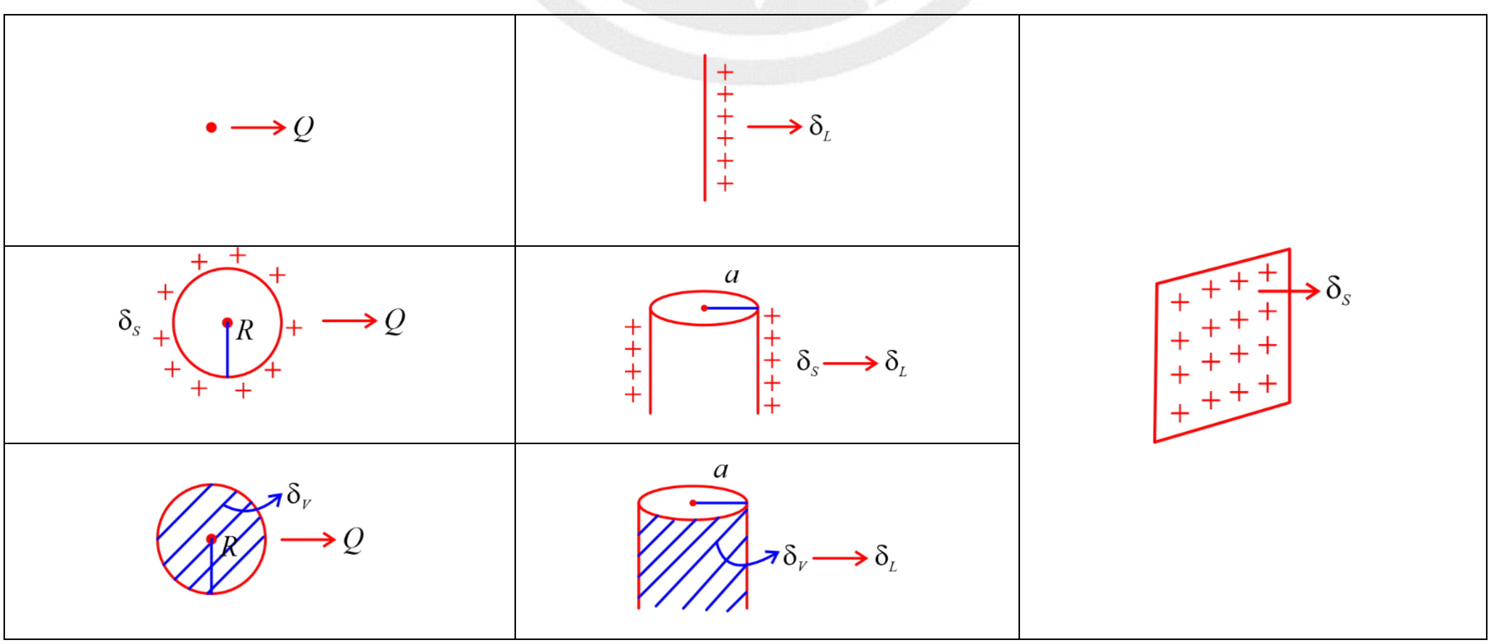
$\mu_0\mu_r \Rightarrow \mu_r > 1 \equiv$ Magnetic material

$\mu_r = 1 \equiv$ Non-magnetic material

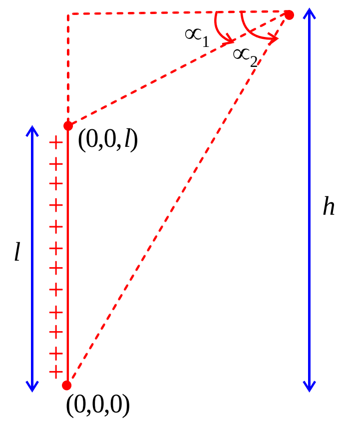
$$F = \frac{\mu(I_1 I_2)l}{4\pi r} \Rightarrow [\mu] = \frac{MLT^{-2}.L}{L(A)^2} = MLA^{-2}T^{-2}$$

3.2. C harge Distribution

1. Point	2. Line	3. Surface	4. Volume
Charge density	Charge density	Charge density	Charge density
			
Q	ρ_L, λ	ρ_S, σ	ρ_V, ρ
C	C/m	C/m^2	C/m^3
	$Q = \int \rho_L dl$	$Q = \iint \rho_S ds$	$Q = \iiint \rho_V dv$
$\vec{E} = \frac{Q}{4\pi \in r} a_r$	$\vec{E} = \int \frac{\rho_L dl}{4\pi \in r^2} a_r$	$\vec{E} = \iint \frac{\rho_S ds}{4\pi \in r^2} a_r$	$\vec{E} = \iint \frac{\rho_V dv}{4\pi \in r^2} a_r$



3.3. Electric Field and Potential due to finite length Line Charge



$$\tan \alpha_1 = \frac{h-\ell}{\rho}$$

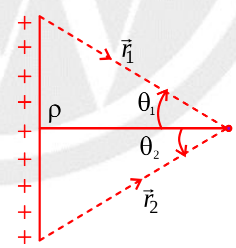
$$\tan \alpha_2 = \frac{h}{\rho}$$

$$\vec{E} = \frac{\rho_L}{4\pi r\varepsilon\rho}\left[(\sin\alpha_2 - \sin\alpha_1)a_\rho + (\cos\alpha_1 - \cos\alpha_2)a_z\right]$$

$$V = \frac{\rho_L}{4\pi\,\varepsilon}\int_{\alpha_1}^{\alpha_2} \sec\alpha \, d\alpha$$

$$= \frac{\rho_L}{4\pi\varepsilon}\log\left|\frac{\sec\alpha_2 + \tan\alpha_2}{\sec\alpha_1 + \tan\alpha_1}\right|$$

Example:



$$\alpha_1 = -\theta_1,\ \alpha_2 = -\theta_2$$

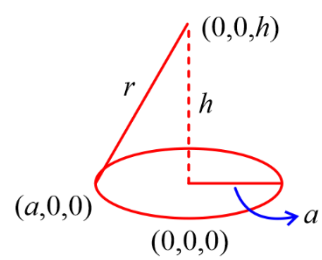
$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon\rho}\left[(\sin\theta_2 + \sin\theta_1)a_\rho + (\cos\theta_1 - \cos\theta_2)a_z\right]$$

3.4. Electric Field and Potential due to Ring

$$Q = \rho_L\,(2\pi a)$$

$$\vec{E} = \frac{Qh}{4\pi\in(a^2+h^2)^{3/2}}a_z$$

$$V = \frac{Q}{4\pi\in(a^2+h^2)^{1/2}}$$



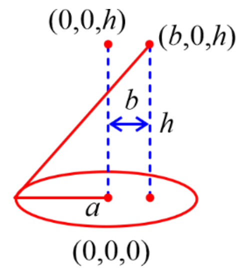
(a) **Electric Field** and **Potential** at the centre of the ring.

$$\vec{F} = \vec{0},\ V = \frac{Q}{4\pi\in a}$$

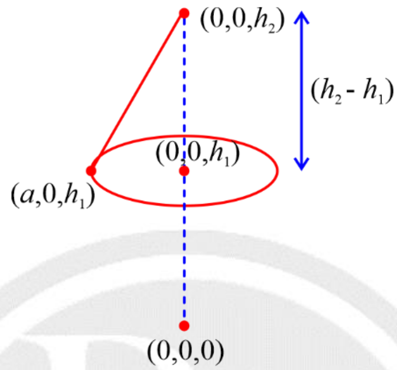
(b)

$$\vec{E}=\frac{Qh}{4\pi\in\left[(a-b)^2+h^2\right]^{3/2}}a_z$$

$$V=\frac{Q}{4\pi\in\left[(a-b)^2+h^2\right]^{1/2}}$$



(c)



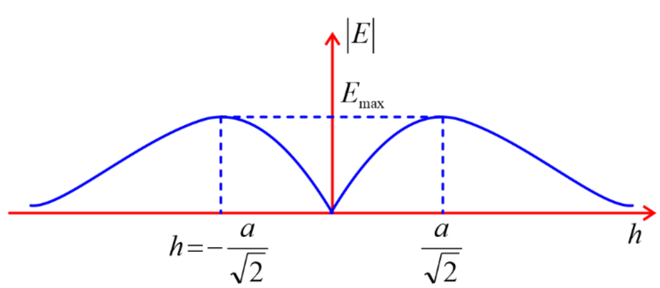
$$\vec{E}=\frac{Q(h_2-h_1)}{4\pi\in\left[a^2+(h_2-h_1)^2\right]^{3/2}}a_z$$

$$V=\frac{Q}{4\pi\in\left[a^2+(h_2-h_1)^2\right]^{1/2}}$$

(d) $V=\frac{Q}{4\pi\in(a^2+z^2)^{1/2}}$ ⇒ Potential due to ring

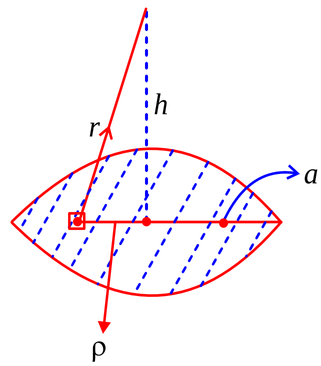
$$\vec{E}=-\vec{\nabla}V=-\frac{dV}{dz}a_z=\frac{Qz}{4\pi\varepsilon(a^2+z^2)^{3/2}}a_z$$

(e)



$$E_{max.}=\frac{Q\left(\frac{a}{\sqrt{2}}\right)}{4\pi\in\left[a^2+\left(\frac{a}{\sqrt{2}}\right)^2\right]^{3/2}}=\frac{Q}{6\sqrt{3}\pi\in a^2}$$

3.5. Electric Field and Potential due to disc



(a) $Q = \rho_s (\pi a^2)$

(b) $\vec{E} = \frac{\rho_S}{2\in}\left(1 - \frac{h}{\sqrt{a^2+h^2}}\right)a_z$

(c) $V = \frac{\rho_S}{2\in}\left(\sqrt{a^2+h^2} - h\right)$

(d) $\vec{E} = \frac{Q}{2\pi\in a^2}\left[1 - \frac{h}{\sqrt{a^2+h^2}}\right]a_z$

(e) $V = \frac{Q}{2\pi\in a^2}\left(\sqrt{a^2+h^2} - h\right)$

(f) At the centre of disc ($h = 0$)

$$\vec{E} = \frac{\rho_S}{2\in}a_z = \frac{Q}{2\pi\in a^2}a_z$$

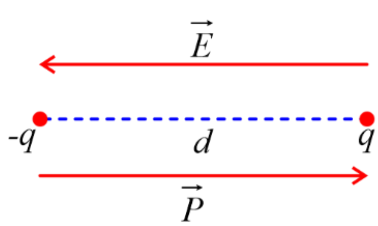
$$V = \frac{\rho_S a}{2\in} = \frac{Q}{2\pi\in a}$$

(g) $V = \frac{\rho_S}{2\in}\left[\sqrt{a^2+z^2} - z\right]$

$$\vec{E} = -\vec{\nabla}V = -\frac{dV}{dz}a_z\bigg|_{z=h} = \frac{\rho_S}{2\in}\left[1 - \frac{h}{\sqrt{a^2+h^2}}\right]a_z$$

3.6. Electric Field and Potential due to Dipole

(a)



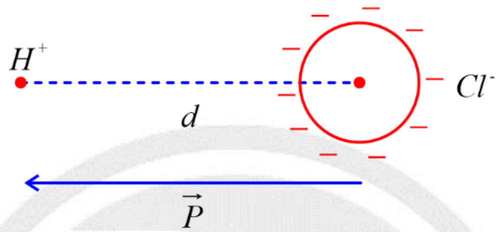
Dipole moment = qd

$\vec{P} = qd$ (-ve to +ve charge)

(b)

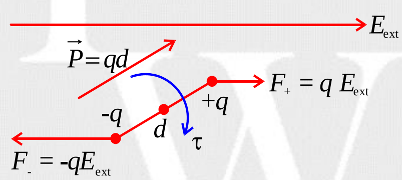
$\vec{P} = Qd$ (-ve to +ve)

(c)



$\vec{P} = ed$ (Cl to H)

(d)



$\vec{\tau} = \vec{P} \times \vec{E}_{ext.}$

Example:

If two-point charge $+Q$ & $-Q$ are placed at (-1, -2, -3) and (2, 3, 1) respectively then find the dipole moment for $Q = 1C$. And also find torque on the dipole, if it is placed in a field region. $\vec{E} = 2\hat{i} + 2j + 2k\ V/m$

Solution:

$$\vec{d} = (-1-2)\hat{i} + (-2-3)\,j + (-3-1)\,k$$

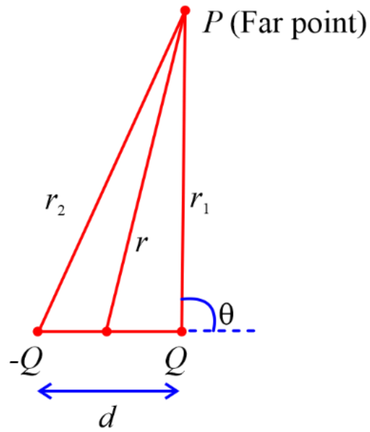
$$= -3\hat{i} - 5j - 4k$$

$$\vec{P} = Q\vec{d} = 1(-3\hat{i} - 5j - 4k) = (-3\hat{i} - 5j - 4k)\ \text{C.m}$$

$$\vec{\tau} = \vec{P} \times \vec{E}_{ext} = \begin{vmatrix} \hat{i} & j & k \\ -3 & -5 & -4 \\ 2 & 2 & 2 \end{vmatrix}$$

$$= (-2\hat{i} - 2j + 4k)\ \text{N.m}$$

(e)



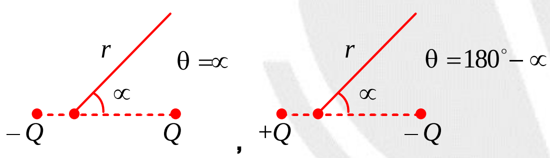
$r_2 > r > r_1 >> d$

(i) $V_P = \dfrac{Qd\cos\theta}{4\pi \in r^2} = \dfrac{P\cos\theta}{4\pi \in r^2} = \dfrac{\vec{P} \bullet a_r}{4\pi \in r^2}$

(ii) $\vec{E}_P = \dfrac{2P\cos\theta}{4\pi \in r^3} a_r + \dfrac{P\sin\theta}{4\pi \in r^3} a_\theta$

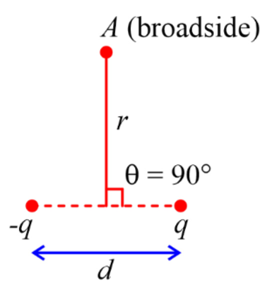
$= \dfrac{2(\vec{P} \bullet a_r)}{4\pi \in r^3} a_r + \dfrac{\left|\vec{P} \times a_r\right|}{4\pi \in r^3} a_\theta$

(iii)



(iv) $V \propto \dfrac{1}{r^2}$, $E \propto \dfrac{1}{r^3}$

(v)



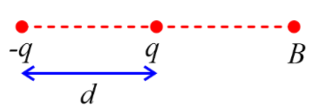
A = Broadside point

$V_A = 0$

$\vec{E}_A = \dfrac{P}{4\pi \in r^3} a_\theta$

At broadside point, Electric field is directed along elevation side.

(vi)



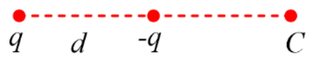
θ = 0°

B $\equiv$ End fire point, $V_B = \dfrac{P}{4\pi \in r^2}$

$\vec{E}_B = \dfrac{2P}{4\pi \in r^3} a_r$

At end fire point, Electric field is directed along radial direction.

(vii)



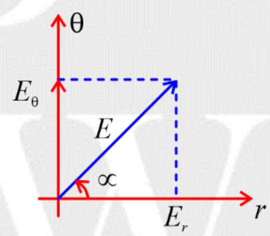
θ = 180°

C $\equiv$ End fire point, $V_C = \dfrac{-P}{4\pi \in r^2}$

$\vec{E}_C = \dfrac{-2P}{4\pi \in r^3} a_r$

(viii) $\dfrac{\text{Electric Field at broadside}}{\text{Electric Field at end fire}} = \dfrac{E_A}{E_B} = \dfrac{1}{2}$

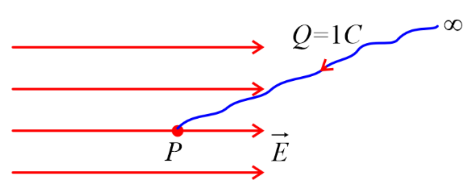
(ix) $\vec{E} = \dfrac{2P\cos\theta}{4\pi \in r^3} a_r + \dfrac{P\sin\theta}{4\pi \in r^3} a_\theta$



$$\tan\alpha = \frac{E_\theta}{E_r} = \frac{\dfrac{P\sin\theta}{4\pi \in r^3}}{\dfrac{2P\cos\theta}{4\pi \in r^3}} \Rightarrow \boxed{\tan \propto = \frac{1}{2}\tan\theta}$$

3.7. Electric Potential

The work done required to carry 1C of change from ∞ to any 'P', using any arbitrary path in the pressure of external Electric field ($\vec{E}$), with constant velocity taking reference potential at '∞'.



(a) ΔK.E = 0 (Change in kinetic energy is zero.)

ΔP.E ≠ 0 (Change in potential energy is non-zero.)

(b) Velocity is constant. Acceleration will be zero. Hence, External Force is zero.

$\vec{F}_{net} = \vec{0} \Rightarrow \vec{F}_e + \vec{F}_{ext} = \vec{0} \Rightarrow \vec{F}_{ext} = -\vec{F}_e = -Q\vec{E}$

(c) Work done = $\int_{\infty}^{F} \vec{F}_{ext} \bullet d\vec{l} = -\int_{\infty}^{P} Q\vec{E} \bullet d\vec{l}$

(d) ΔP.E = $Q(V_f - V_i) = Q(V_P - V_\infty)$

$$-\cancel{Q}\int_{\infty}^{P} \vec{E} \bullet d\vec{l} = \cancel{Q}(V_P - V_\infty)$$

$$\boxed{V_P - V_\infty = -\int_{\infty}^{P} \vec{E} \bullet d\vec{l}}$$

Summary:

(1) $V_P - V_\infty = -\int_{\infty}^{P} \vec{E} \bullet d\vec{l}$

Reference will be at infinity.

Hence, $V_\infty = 0V$

$$V_P = -\int_{\infty}^{P} \vec{E} \bullet d\vec{l}$$

(2) $V_{AB} = V_A - V_B = (V_A - V_\infty) - (V_B - V_\infty)$

$$= -\int_{\infty}^{A} \vec{E} \bullet d\vec{l} - \left(-\int_{\infty}^{B} \vec{E} \bullet d\vec{l}\right) = -\int_{B}^{A} \vec{E} \bullet d\vec{l}$$

$$V_{AB} = \underset{final}{V_A} - \underset{initial}{V_B} = -\int_{B \to initial}^{A \to final} \vec{E} \bullet d\vec{l}$$

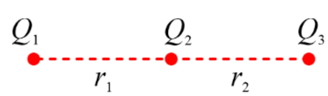
During calculation of potential difference. There is no need for reference potential.

3.8. Potential Energy

(1)

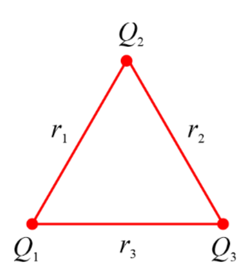
Potential Energy = $\dfrac{Q_1 Q_2}{4\pi \in r}$

(2)



Potential Energy = $\dfrac{Q_1 Q_2}{4\pi \in r_1} + \dfrac{Q_2 Q_3}{4\pi \in r_2} + \dfrac{Q_1 Q_3}{4\pi \in (r_1 + r_2)}$

(3)



Potential Energy = $\dfrac{Q_1 Q_2}{4\pi \in r_1} + \dfrac{Q_2 Q_3}{4\pi \in r_2} + \dfrac{Q_1 Q_3}{4\pi \in r_3}$

3.9. Electric Energy Density

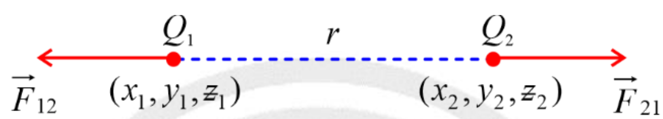
$$\mu_e = \frac{1}{2}\vec{D}.\vec{E} = \frac{1}{2}\in E^2 = \frac{D^2}{2\in}$$

$$W_e = \int_{\upsilon}\left(\frac{1}{2}\vec{D}.\vec{E}\right)dv = \int_{\upsilon}\left(\frac{1}{2}\in E^2\right)dv = \int_{\upsilon}\frac{D^2}{2\in}dv$$

μ_e = Electric Energy density

W_e = Electric Energy

3.10. Coulomb's Force



$\vec{F}_{21}$ = Force on Q_2 due to Q_1

$\vec{F}_{12}$ = Force on Q_1 due to Q_2

Assumptions:

The size of charge is very smaller than distance between two charge. Hence, Q_1 and Q_2 are being considered as point charge.

(1) $\vec{F}_{21} = \dfrac{Q_1Q_2}{4\pi\in r}a_r$

$\vec{r} = (x_2 - x_1)a_x + (y_2 - y_1)a_y + (z_2 - z_1)a_z$

$a_r = \dfrac{\vec{r}}{r}$

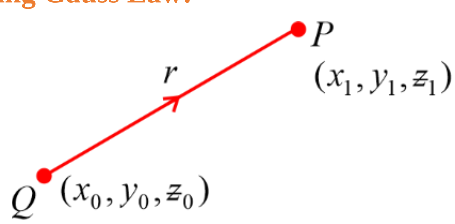
(2) $\vec{F}_{12} = \dfrac{Q_1Q_2}{4\pi\in r}a_r$

$\vec{r} = (x_2 - x_1)a_x + (y_2 - y_1)a_y + (z_1 - z_2)a_z$

$\hat{a}_r = \dfrac{\vec{r}}{r}$

3.11. Coulomb's Force

(1) Electric field due to point charge using Gauss Law: -



(a) $\vec{r}=(x_1-x_0)\hat{i}+(y_1-y_0)j+(z_1-z_0)k$

(b) Point charge is placed symmetrically.

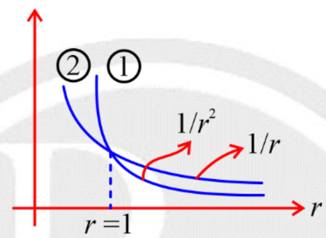
(c) Medium should be Linear, Homogeneous & Isotropic.

(d) $\vec{E}=\dfrac{Q}{4\pi\in r^2}a_r$

(e) $\vec{D}=\dfrac{Q}{4\pi r^2}a_r$

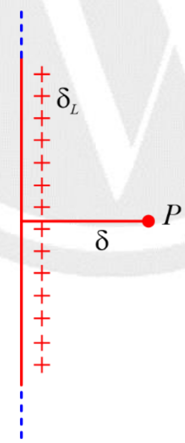
(f) $V_P-V_\infty=\dfrac{Q}{4\pi\in r}$

(g) $\vec{\nabla}\bullet\vec{D}=\rho_v\Rightarrow\rho_v=0$



$E\propto\dfrac{1}{r^2}, V\propto\dfrac{1}{r}$

(2) Electric field due to infinite length line charge:-



(a) When line charge is placed along ~~z~~ axis.

$$\vec{\rho}=(x_2-x_1)a_x+(y_2-y_1)a_y$$

(b) When line charge is placed along x-axis.

$$\vec{\rho}=(y_2-y_1)a_y+(z_2-z_1)a_z$$

(c) When line charge is placed along y-axis.

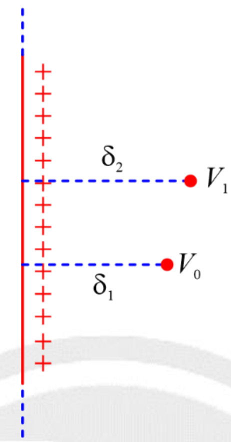
$$\vec{\rho}=(x_2-x_1)a_x+(z_2-z_1)a_z$$

(d) $\vec{D}=\dfrac{\rho_L}{2\pi\rho}a_\rho$

(e) $\vec{E}=\frac{\rho_L}{2\pi\in\rho}a_\rho$

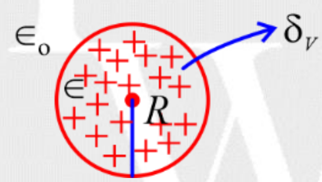
(f) $\rho_V=\vec{\nabla}.\vec{D}=0$

(g)



$$V_0-V_1=\frac{\rho_L}{2\pi\in}ln\left(\frac{\rho_2}{\rho_1}\right)$$

(3) Electric field due to solid sphere:



(a) $Q_T=\int_0^R\int_0^\pi\int_0^{2\pi}\rho_V r^2\sin\theta dr\,d\theta\,d\phi=\rho_V\left(\frac{4\pi R^3}{3}\right)$

(b) Electric field inside solid sphere. ($r\leq R$)

(i) $Q_{enc}\propto r^3$, G.S $\propto r^2$, $E=\frac{Q_{enc}}{G.S}\Rightarrow E\propto r$

(ii) $\vec{E}=\frac{\rho_v.r}{3\in}a_r=\frac{Q.r}{4\pi\in R^3}a_r$

(iii) $E(r=R^-)=\frac{\rho_v R}{3\in}=\frac{Q}{4\pi\in R^2}$

(c) Electric field outside solid sphere ($r > R$)

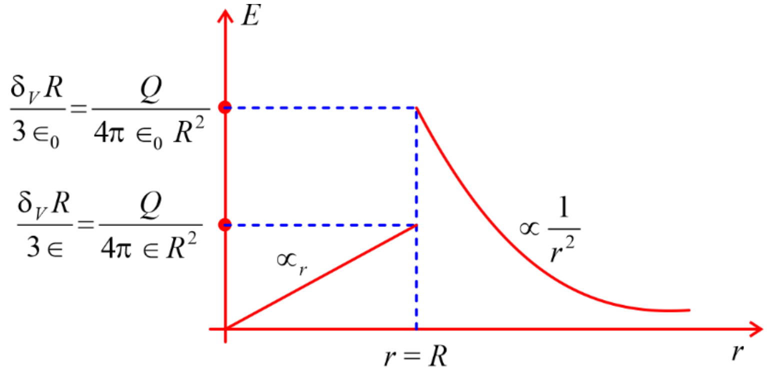
(i) Q_{enc} = Constant, G.S $\propto r^2$, $E=\frac{Q_{enc}}{G.S}$

$\therefore$ $E\propto\frac{1}{r^2}$

(ii) $\vec{E}=\frac{\rho_V R^3}{3\in_0 r^2}a_r=\frac{Q}{4\pi\in_0 r^2}a_r$

(iii) $E(r=R^+)=\frac{\rho_V R}{3\in_0}=\frac{Q}{4\pi\in_0 R^2}$

(d) Graph of Electric field intensity.



Electric field is discontinuous at $r = R$ or surface of solid sphere.

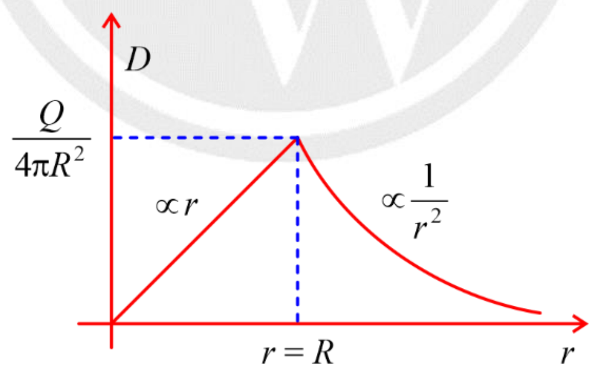
$$\frac{E(r=R^+)}{E(r-R^-)}=\frac{\text{Electric field just outside sphere}}{\text{Electric field just inside sphere}}$$

$$=\frac{\rho_V\frac{R}{3}\in_0}{\rho_V\frac{R}{3}\in}=\in_r$$

(e) Graph of Electric flux density.

$\vec{D}=\frac{\rho_V r}{3}a_r=\frac{Qr}{4\pi R^3}a_r \qquad r\le R$

$=\frac{\rho_V R^3}{3r^2}a_r=\frac{Q}{4\pi r^2}a_r \qquad r>R$



(f) Electric potential of solid sphere.

$$V_P-V_\infty=\frac{Q}{4\pi\in_0 r},\ r\ge R$$

$$V_P-V_\infty=\frac{Q}{4\pi\in_0 R}+\frac{Q}{8\pi\in R^3}(R^2-r^2),\ r\le R$$

At the centre of solid sphere r = 0

$$V_{centre}-V_\infty=\frac{Q}{4\pi\in_0 R}+\frac{Q}{8\pi\in R}$$

(g) Energy density inside or outside solid sphere.

(i) Inside Solid Sphere r < R

$$u_e = \frac{1}{2}\in E^2 = \frac{1}{2}\in\left(\frac{Q.r}{4\pi\in R^3}\right)^2 = \frac{Q^2 r^2}{32\pi^2\in R^6} = \frac{\rho_V{}^2 r^2}{18\in}$$

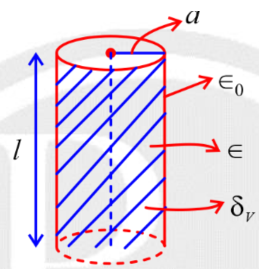
(ii) Inside Solid Sphere r > R

$$u_e = \frac{1}{2}\in_0\left(\frac{\rho_V R^3}{3\in_0 r^2}\right)^2 = \frac{\rho_V{}^2 R^6}{18\in_0 r^4}$$

(iii) Energy stored in solid sphere.

$$W = \int_{r=0}^{R}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\frac{\rho_V{}^2 r^2}{18\in}.r^2\sin\theta dr\,d\theta\,d\phi + \int_{r=R}^{\infty}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\frac{\rho_V{}^2R^6}{18\in_0 r^4}(r^2\sin\theta dr\,d\theta\,d\phi) = \frac{2\pi\rho_V{}^2R^5}{45\in} + \frac{2\pi\rho_V{}^2R^5}{9\in_0}$$

(4) Electric field due to solid cylinder:



(a) $Q_T = \int_{\rho=0}^{a}\int_{\phi=0}^{2\pi}\int_{z=0}^{\ell}\rho_V.(\rho d\rho d\phi dz) = \rho_V(\pi a^2\ell)$

(b) Since, length is infinite. Hence, line charge is defined.

$$\rho_L = \int_{\delta=0}^{a}\int_{\phi=0}^{2\pi}\rho_V(\rho d\rho d\phi) = \rho_V(\pi a^2)$$

(c) Electric field solid cylinder (ρ < a)

$Q_{enc} \propto \rho^2$, Gaussian Surface $\propto \rho$,

∴ $E \propto \rho$

$$\vec{E} = \frac{\rho_V\cdot\rho}{2\in}a_\rho = \frac{\rho_L\cdot\rho}{2\pi\in a^2}a_\rho$$

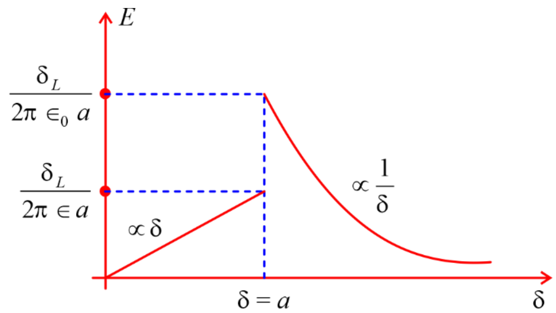
(d) Electric field outside solid sphere (ρ > a)

$Q_{enc} \equiv$ Constant, Gaussian Surface $\propto \rho$

∴ $E \propto \frac{1}{\rho}$

∴ $\vec{E} = \frac{\rho_L}{2\pi\in_0\rho}a_\rho$

(e) Graph of Electric field



Electric field is discontinuous at the surface solid cylinder.

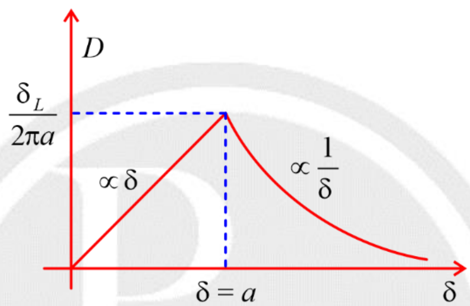
$$\frac{\text{E.F just outside the solid cylinder}}{\text{E.F just inside the solid cylinder}} = \frac{E(\rho = a^+)}{E(\rho = a^+)}$$

$$= \frac{\dfrac{\rho_L}{2\pi \in_0 a}}{\dfrac{\rho_L}{2\pi \in a}} = \in_r$$

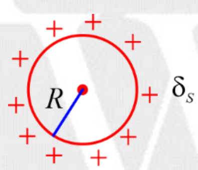
(f) Displacement Flux density

$$\vec{D} = \frac{\rho_L}{2\pi\rho} a_\rho, \qquad \rho > a$$

$$= \frac{\rho_L \cdot \rho}{2\pi a^2} a_\rho, \qquad \rho < a$$



(5) Electric field due to hollow sphere: -

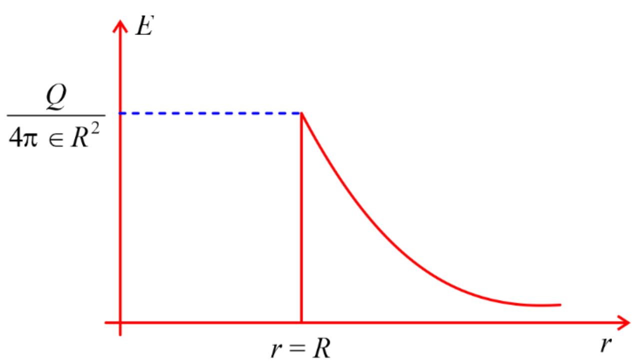


(a) $Q_T = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \rho_s (r^2 \sin\theta\, d\theta\, d\phi)|_{r=R} = \rho_S R^2 (4\pi) = \rho_S (4\pi R^2)$

(b) Electric field and potential.

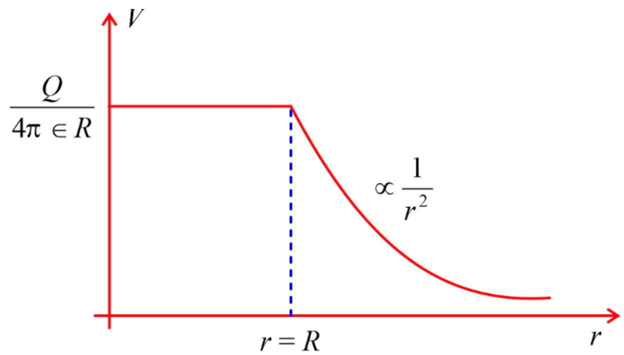
$$\vec{E} = \vec{0} \qquad r \le R$$

$$= \frac{\rho_S R^2}{\in r^2} a_r = \frac{Q}{4\pi \in r^2} a_r \qquad r > R$$

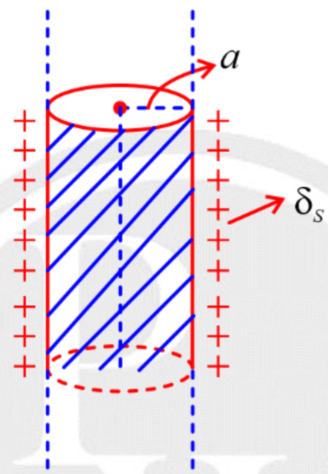


$$V = \frac{Q}{4\pi \in R} \qquad r \le R$$

$$= \frac{Q}{4\pi \in r} \qquad r \ge R$$



(6) Electric field due to hollow cylinder:



(a) $Q_T = \int_{\phi=0}^{2\pi}\int_{z=0}^{\ell} \rho_s (\rho d\phi dz)\Big|_{\rho=a}$

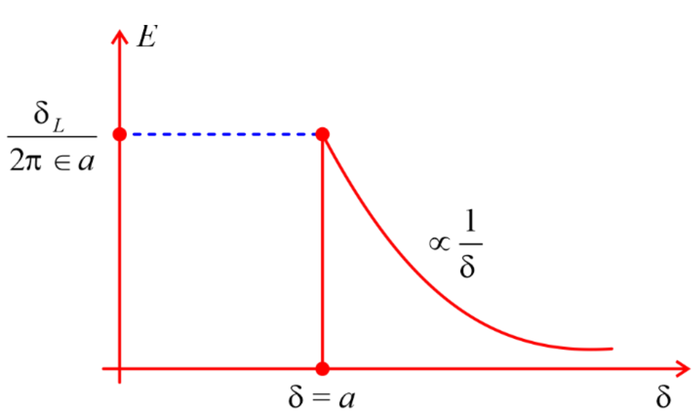
(b) Since, length is infinite. Hence line charge is defined.

$$\rho_L = \int_{\phi=0}^{2\pi} \rho_S (\rho d\phi)\Big|_{\rho=a}$$

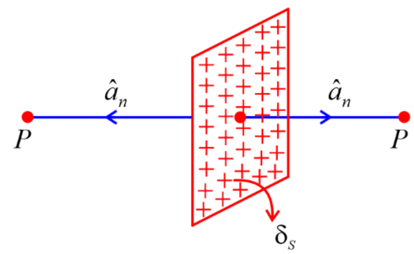
$$\boxed{\rho_L = \rho_S (2\pi a)}$$

(c) $\vec{E} = \vec{0}$ $\quad\quad \rho \leq a$

$= \dfrac{\rho_S . a}{\in \rho} a_\rho = \dfrac{\rho_L}{2\pi \in \rho} a_\rho$ $\quad\quad \rho > a$



(7) Electric field due to thin sheet:



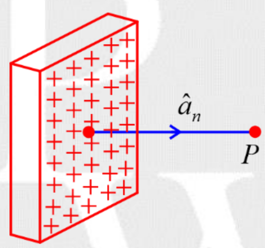
$$\vec{E}_P = \frac{\rho_S}{2\in} a_n$$

$$\vec{D}_P = \frac{\rho_S}{2} a_n$$

a_n = unit normal to the thin sheet.

$$\boxed{a_n = \frac{\vec{\nabla} f}{|\vec{\nabla} f|}}$$

(8) Electric field due to thick Sheet:



$$\vec{E}_P = \frac{\rho_S}{\in} a_n$$

$$\vec{D}_P = \rho_S a_n$$

a_n = unit normal to the thick sheet.

$$\boxed{a_n = \frac{\vec{\nabla} f}{|\vec{\nabla} f|}}$$

3.12. Equipotential Surface

- The surface at which potential is constant, is known as Equipotential surface.
- Equipotential Surface ⇒ In three dimensional.
- Equipotential Curve ⇒ In two dimensional.

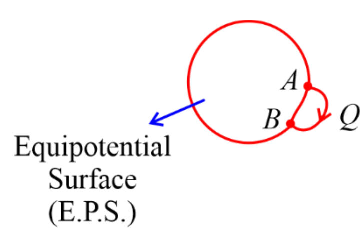
(1) Properties of Equipotential Surface

(a) Work done along tangential direction of equipotential surface is zero.

$W_{AB} = Q\,(V_B - V_A) = 0$

$(\therefore V_A = V_B)$

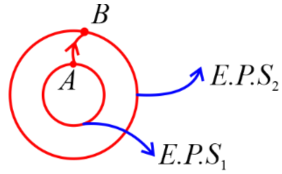
$\boxed{\therefore W_{tang.} = 0}$



(b) Electric field is zero along tangential direction. Because, work done along tangential direction of E.P.S. is zero.

$W_{tang.} = 0 \quad \Rightarrow Q\int E_t dl = 0 \quad \Rightarrow \boxed{E_t = 0}$

(c) Work done along normal direction of E.P.S. is non-zero.

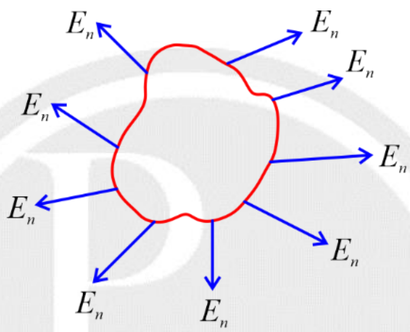


$W_{AB} = Q\,(V_A - V_B) \neq 0$

$(\because V_A \neq V_B)$

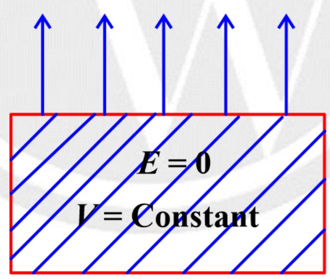
$W_{normal} \neq 0$

(d) Hence, on equipotential surface, only normal components of electric field will exist on the surface of conductor.



(e) Equipotential surface and Electric field will intersect orthogonally or at 90°.

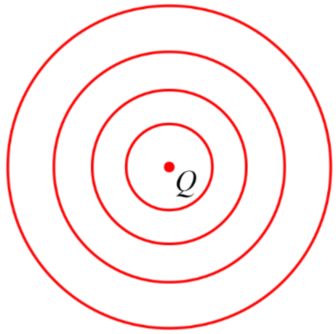
(f) **Conductor (Perfect Electric Conductor)**



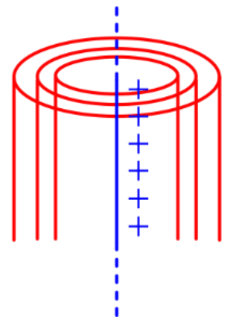
Since, Electric field inside conductor is zero. Hence, potential inside conductor is constant. So, conductor is equipotential surface. Thus, only normal component of Electric field will exist on surface of conductor.

Different types of Equipotential surface of different charge distribution.

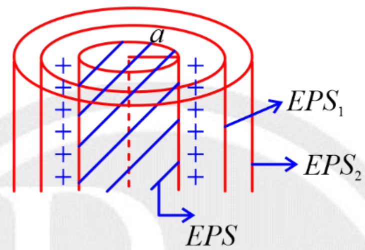
1. **Point Charge:** - E.P.S. due to point charge is concentric sphere.



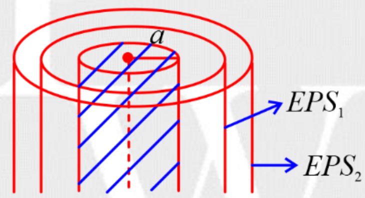
2. **Infinite length line charge: -** Co-axial cylinder



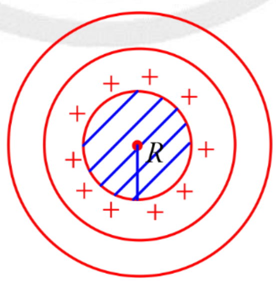
3. **Hollow Cylinder: -**
   Inside $\equiv$ Entire Cylinder is E.P.S.
   Outside $\equiv$ Co-axial Cylinder



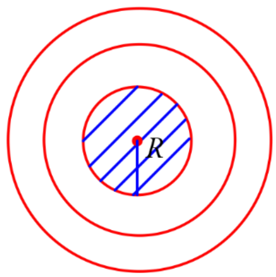
4. **Solid Cylinder: -**
   Outside $\equiv$ Co-axial Cylinder



5. **Hollow Sphere: -**
   Inside $\equiv$ Entire hollow sphere
   Outside $\equiv$ Concentric sphere

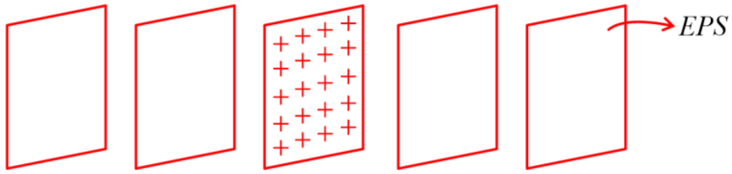


6. **Solid Sphere: -**
   Outside $\equiv$ Concentric sphere

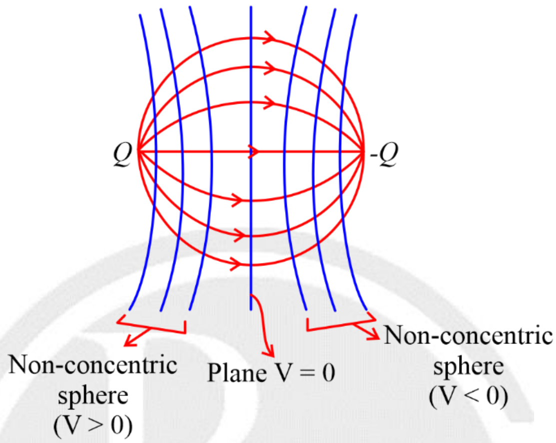


7. Thin Sheet: -

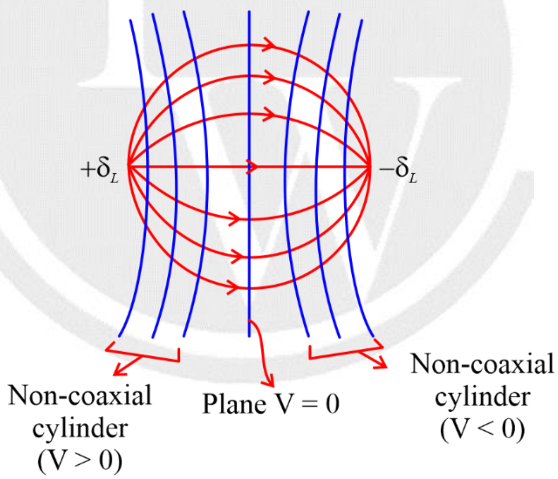
Planes parallel to sheet



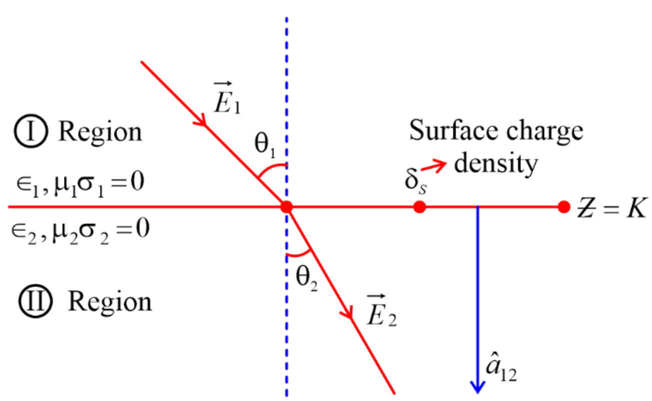
8. Point charge dipole: -



9. Line charge dipole: -



3.13. Dielectric – Dielectric Boundary



- $\vec{E}_1 = \vec{E}_{t_1} + \vec{E}_{n_1}$
- $\vec{E}_2 = \vec{E}_{t_2} + \vec{E}_{n_2}$
- $E_{t_1} = E_1 \sin\theta_1,\ E_{t_2} = E_2 \sin\theta_2$

  $E_{n_1} = E_1 \cos\theta_1,\ E_{n_2} = E_2 \cos\theta_2$
- Z = K ⇒ Boundary or Interface.
- a_{12} = unit vector which is directed 1st medium to 2nd medium.
- $\oint\!\!\oint \vec{D} \cdot d\vec{s} = Q_{enc} \quad \Rightarrow (\vec{D}_2 - \vec{D}_1) \cdot a_{12} = \rho_S$
- $\oint \vec{E} \cdot \vec{dl} = 0 \quad \Rightarrow \vec{E}_{t_1} = \vec{E}_{t_2}$
- $\dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{\epsilon_1}{\epsilon_2}$ when $\rho_s = 0$
- When ϵ_1 (denser) > ϵ_2 (rarer)

  Then θ_1 (away from normal) > θ_2 (to words the normal)
- Electric field is directed away from the normal in denser medium and is directed towards the normal in rarer medium.

Case I: - $\boldsymbol{\rho_S}$ = 0 (Charge free boundary)

(1) $D_{n2} = D_{n1}$

Normal component of Electric flux density is continuous across boundary or interface.

(2) $E_{n1} = E_{n2}$

Normal component of Electric field intensity is discontinuous across boundary or interface.

(3) $E_{t2} = E_{t1}$

Tangential component of Electric field intensity is continuous across boundary or interface.

(4) $D_{t2} \neq D_{t1}$

Tangential component of Electric flux density is discontinuous across boundary or interface.

Case II: - $\boldsymbol{\rho_S}$ = 0 (Charged boundary)

(1) $D_{n2} \neq D_{n1}$

Normal component of Electric flux density is discontinuous across boundary or interface.

(2) $E_{n1} \neq E_{n2}$ or $E_{n1} = E_{n2}$

Normal component of Electric field intensity may be continuous or discontinuous across boundary or interface.

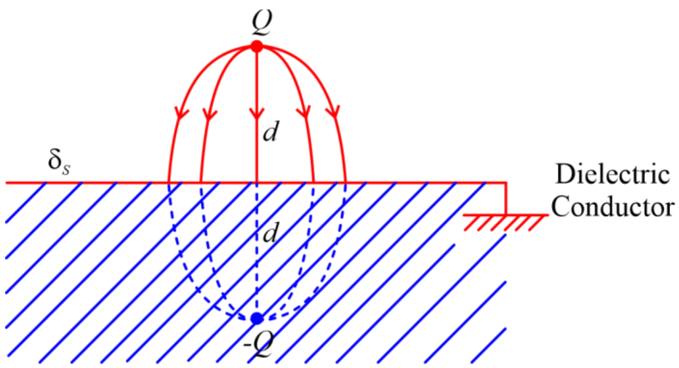
(3) $E_{t2} = E_{t1}$

Tangential component of Electric field intensity is continuous across boundary or interface.

(4) $D_{t2} \neq D_{t1}$

Tangential component of Electric flux density is discontinuous across boundary or interface.

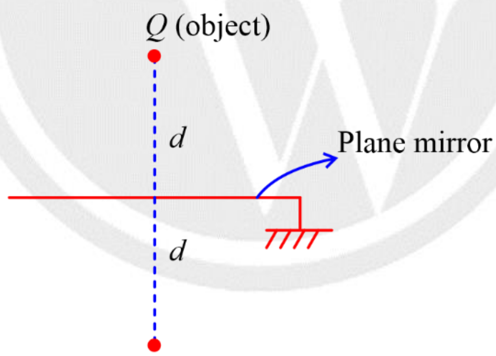
3.14. Dielectric – Conductor Boundary



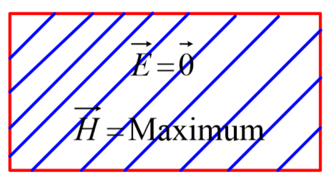
- As conductor is equipotential surface, then electric field will exist normal to the surface of conductor.
- Since, magnitude of electric field is non-uniform at every point of conductor. Hence, surface charge (ρ_s) induced in non-uniform on the surface of conductor.
  Since, surface charge is non-uniformly distributed on the surface of conductor. Hence, centre of charge is introduced to make mathematical calculation easy. The centre of charge is known as image charge. Image charge actually does not exist inside the conductor.

How to calculate image charge and location of charge.

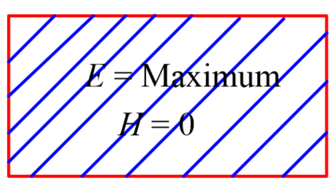
- Dielectric – conductor interface is considered as plane mirror.
- Charge is considered as object.



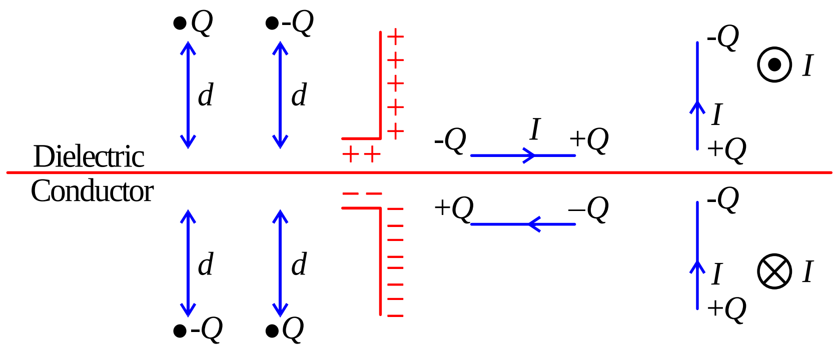
- PEC (Perfect Electric Conductor)



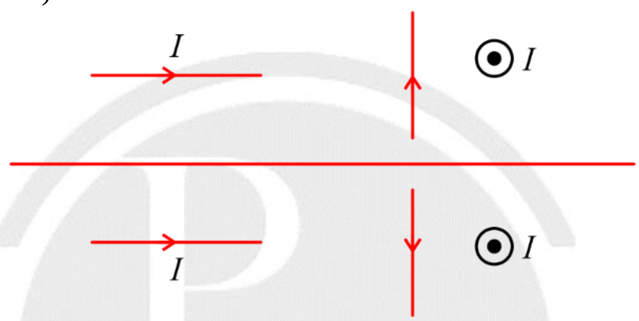
- PMC (Perfect Magnetic Conductor)

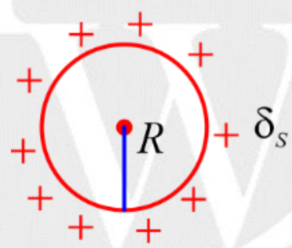


- PEC (Perfect Electric Conductor)



- PMC (Perfect Magnetic Conductor)



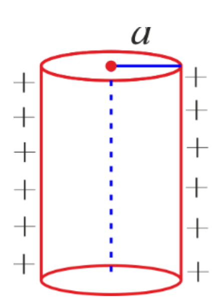
- 

$$\vec{E} = \frac{\rho_S R^2}{\in r^2}$$

At $r = R$ (surface)

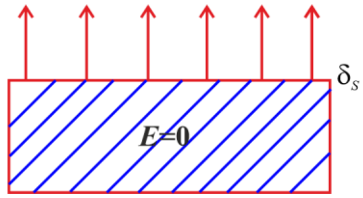
$$\vec{E} = \frac{\rho_S}{\in} a_r$$

$$\vec{D} = \in \vec{E} = \rho_S a_r$$

- 

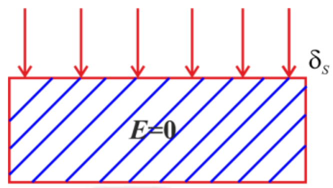
$$\vec{D}(\rho = a) = \rho_S a_\rho$$

(I) When Electric field is pointing away from the conductor.



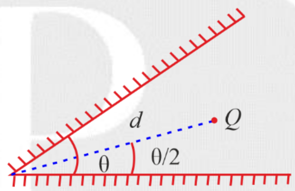
$$\rho_S = |\vec{D}|$$

(II) When Electric field id pointing towards the conductor.



$$-\rho_S = |\vec{D}|$$

When conductor are placed at angle 'θ'.



Assumption:

(a) Charge should be placed at symmetrical.

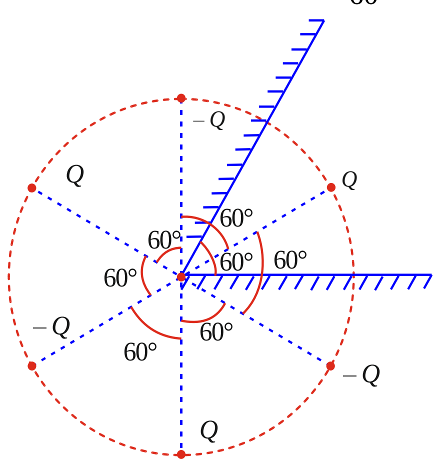
(b) $\frac{180^\circ}{\theta}$ = integer = n

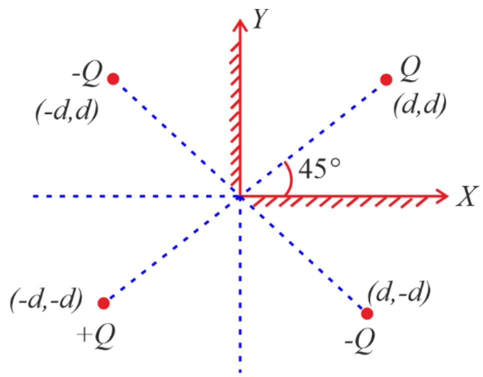
Then, Number of image = $\frac{360^\circ}{\theta} - 1$

Number of image + object = $\frac{360^\circ}{\theta}$

To find the location of image.

It θ = 60°, Number of image = $\frac{360^\circ}{60^\circ} - 1 = 5$





- θ = 60°, Q

  Number of images = $\frac{360^\circ}{60^\circ} - 1 = 5$ → 4 (– Q), 3 (+ Q)

- θ = 45°, -Θ

  Number of images = $\frac{360^\circ}{45^\circ} - 1 = 7$ → 4 (+Q), 3 (-Q)

Question:

A point charge 'Q' is situated at a distance '*l*' from the centre of a grounded conducting sphere of radius 'R' (d > R). the value of the image charge is 'q' at a distance 'b' from the centre. The quantities 'q' and 'b' will be respectively.

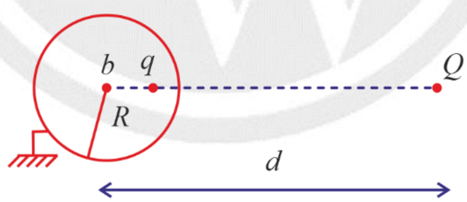
(a) $\left(-\frac{R}{d}\right)Q$ and $\frac{R^2}{d}$

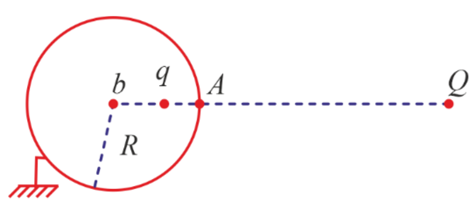
(c) $\left(\frac{R^2}{d}\right)Q$ and $\frac{R^2}{d}$

(b) $\left(-\frac{R^2}{d}\right)Q$ and $\frac{d^2}{R}$

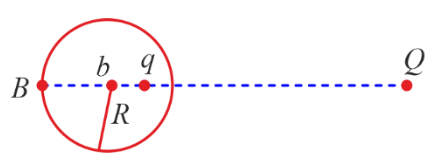
(d) $\left(-\frac{R}{d}\right)Q$ and $\frac{d^2}{R}$

Solution:





$$V_A = \frac{q}{4\pi \in (R-b)} + \frac{Q}{4\pi \in (d-R)} = 0 \Rightarrow q = -\frac{Q(R-b)}{(d-R)} \quad \ldots(i)$$



$$V_B = \frac{q}{4\pi \in (R+b)} + \frac{Q}{4\pi \in (d+R)} = 0 \Rightarrow q = -\frac{Q(R+b)}{(d+R)} \quad \text{...(ii)}$$

From (i) & (ii)

$$-\frac{Q(R-b)}{(d-R)} = -\frac{Q(R+b)}{(d+R)} \Rightarrow \frac{R-b}{d-R} = \frac{(R+b)}{(d+R)}$$

$\Rightarrow$ $(R - b)(d + R) = (d - R)(R + b)$

$\Rightarrow$ $dR - bd + R^2 - bR = dR - R^2 + db - bR$

$\Rightarrow$ $2bd = 2R^2 \Rightarrow \boxed{b = \frac{R^2}{d}}$

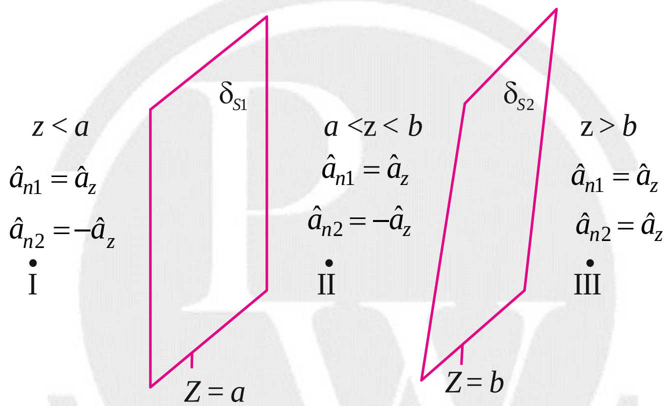
$\therefore$ $q = -Q\left(\frac{R - \frac{R^2}{d}}{d-R}\right) = -\frac{QR}{d}$

❑❑❑

4 CAPACITOR

4.1. Introduction

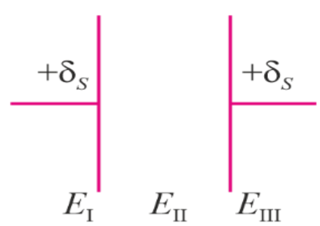
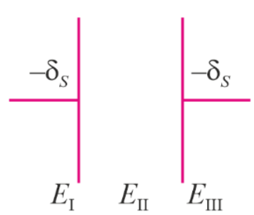
1.

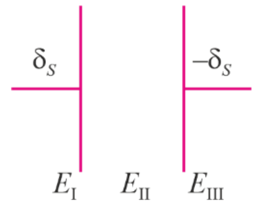


$$\vec{E}_{\text{I}} = \left(\frac{\delta_{S1} + \delta_{S2}}{2\in}\right)(-\hat{a}_z)$$

$$\vec{E}_{\text{II}} = \left(\frac{\delta_{S1} - \delta_{S2}}{2\in}\right)\hat{a}_z$$

$$\vec{E}_{\text{III}} = \left(\frac{\delta_{S1} + \delta_{S2}}{2\in}\right)\hat{a}_z$$

(a)		$\vec{E}_{\text{I}} = \frac{-\delta_S}{\in}\hat{a}_z$ $\vec{E}_{\text{II}} = \vec{0}$ $\vec{E}_{\text{III}} = \frac{\delta_S}{\in}\hat{a}_z$
(b)		$\vec{E}_{\text{I}} = \frac{\delta_S}{\in}\hat{a}_z$ $\vec{E}_{\text{II}} = \vec{0}$ $\vec{E}_{\text{III}} = \frac{-\delta_S}{\in}\hat{a}_z$

(c)		$\vec{E}_I = \vec{0}$ $\vec{E}_{II} = \frac{\delta_S}{\in}\hat{a}_z$ $\vec{E}_{III} = \vec{0}$

2. Capacitor:

It is passive device in which it store Electric Field inside it.

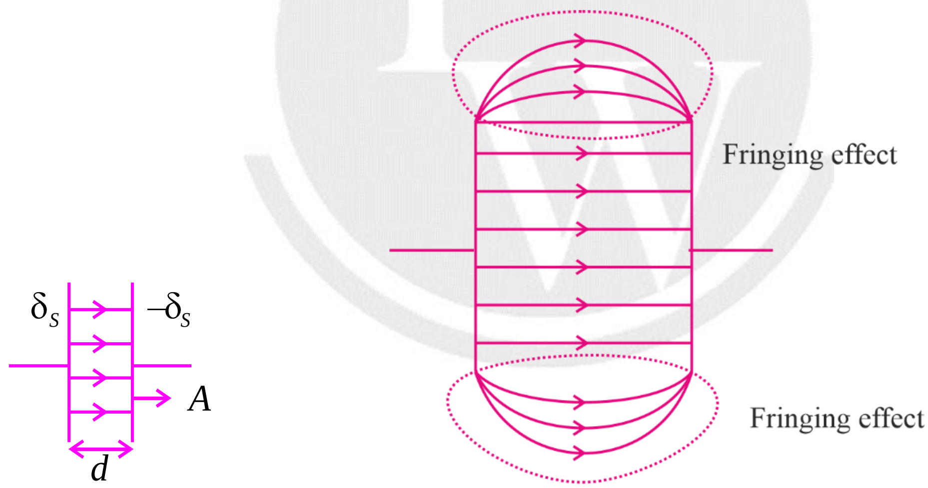
- Electric Field outside capacitor is zero
- Electric Field inside capacitor is non- zero.

3. Capacitance

It is an ability to store Electric Field inside it.

$$C = \frac{Q}{V} \Rightarrow \frac{C}{\text{volt}} = \text{Farad}$$

4. Fringing Effect:



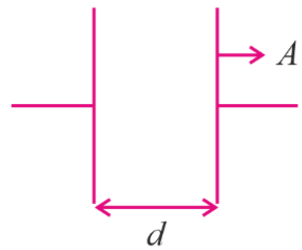
(a) (b)

When $\frac{A}{d} >> 1 \Rightarrow$ No fringing effect.

$\frac{A}{d} << 1 \Rightarrow$ Fringing effect.

- During Fringing effect, Electric Field will exist outside the capacitor. So, capacitor will show non- linear effect.
- When Fringing Effect $\Rightarrow C \neq \frac{A\epsilon}{d}$
- When no Fringing Effect $\Rightarrow C = \frac{A\epsilon}{d}$

5. Parallel Plate Capacitor



(a) $Q = \delta_S A$

(b) $A = l\,W$

(c) $E = \frac{\delta_S}{\in} = \frac{Q}{A\in}$

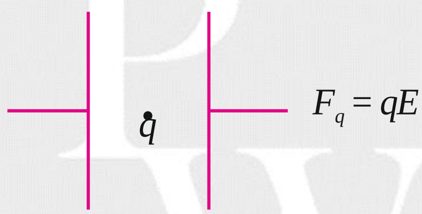
(d) $V = Ed$

(e) $C = \frac{A\in}{d}$

(f) $D = \delta s$

(g) $\psi = Q \qquad (\psi = \text{Flux})$

(h) F_q = Force on charge 'q' placed inside capacitor



(i) F = Force between the plate of capacitor

$$F = \frac{1}{2}QE = \frac{Q^2}{2A\in} = \frac{\delta_S^2 A}{2\in}$$

$$F \propto Q^2, F \propto A, F \propto \frac{1}{\in_r}$$

(j) W = Energy stored inside capacitor

$$W = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$$

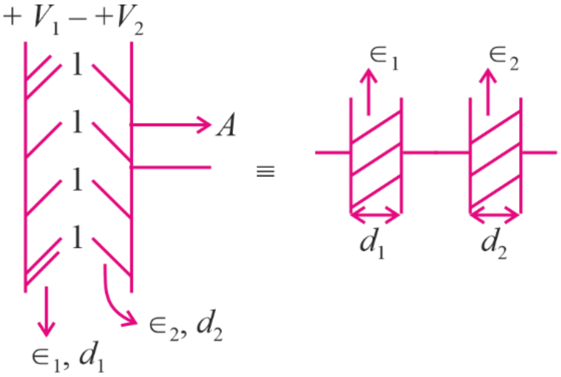
(k) μ_e = Electric energy density.

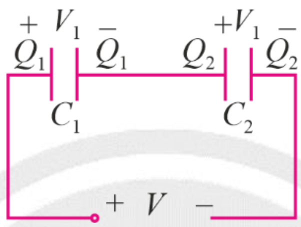
$$\mu_e = \frac{1}{2}\in E^2 = \frac{\delta_s^2}{2\in} = \frac{Q^2}{2A^2\in}$$

(l) $F = \frac{1}{2}QE = \frac{Q^2}{2A\in}$ Pressure $= \frac{F}{A} = \frac{Q^2}{2A^2\in} = \mu_e$

Hence, pressure & electric energy density both are same.

6. Series Combination of Parallel Plate Capacitor



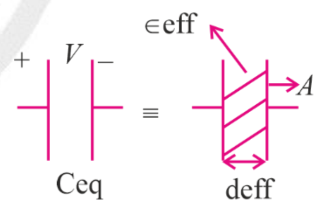


$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{A\epsilon_0\epsilon_{eff}}{d_1}$$

(a) $Q_1 = Q_2 = Q$

(b) $C_1 = \frac{A\epsilon_0\epsilon_1}{d_1},\ C_2 = \frac{A\epsilon_0\epsilon_2}{d_2}$

$$C_{eq} = \frac{A\epsilon_0\epsilon_1\epsilon_2}{d_1\epsilon_2 + d_2\epsilon_1} = \frac{A\epsilon_0}{\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2}}$$



$$C_{eq} = \frac{A\epsilon_0\epsilon_{eff}}{d_{eff}}$$

$$\therefore \frac{d_{eff}}{\epsilon_{eff}} = \frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2}$$

(c) $\frac{C_1}{C_2} = \left(\frac{\epsilon_1}{\epsilon_2}\right)\left(\frac{d_2}{d_1}\right)$

(d) $\frac{V_1}{V_2} = \left(\frac{\epsilon_2}{\epsilon_1}\right)\left(\frac{d_1}{d_2}\right)$

(e) $\frac{\delta_{S1}}{\delta_{S2}} = 1 \Rightarrow \boxed{\delta_{S2} = \delta_{S1}}$

(f) $\frac{E_1}{E_2} = \left(\frac{\epsilon_2}{\epsilon_1}\right)$

(g) $\frac{D_1}{D_2} = 1 \Rightarrow D_1 = D_2$

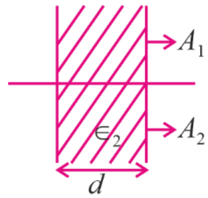
(h) $\frac{\psi_1}{\psi_2} = 1 \Rightarrow \psi_1 = \psi_2$

(i) $\frac{F_1}{F_2} = \left(\frac{\epsilon_2}{\epsilon_1}\right)$

(j) $\frac{W_1}{W_2} = \left(\frac{\epsilon_2}{\epsilon_1}\right)\left(\frac{d_1}{d_2}\right)$

(k) $\frac{u_1}{u_2} = \left(\frac{\epsilon_2}{\epsilon_1}\right)$

7. Parallel combination of Parallel Plate Capacitor.



$$C_1 = \frac{A_1 \in_0 \in_1}{d}$$

$$C_2 = \frac{A_2 \in_0 \in_2}{d}$$

$$C_{eq} = \frac{(A_1 \in_1 + A_2 \in_2) \in_0}{d}$$

(a) $\frac{C_1}{C_2} = \frac{A_1 \in_1}{A_2 \in_2}$

(b) $V_1 = V_2 = V$

(c) $\frac{Q_1}{Q_2} = \frac{A_1 \in_1}{A_2 \in_2}$

(d) $\frac{\delta_{S1}}{\delta_{S2}} = \frac{\in_1}{\in_2}$

(e) $E_1 = E_2$

(f) $\frac{D_1}{D_2} = \frac{\in_1}{\in_2}$

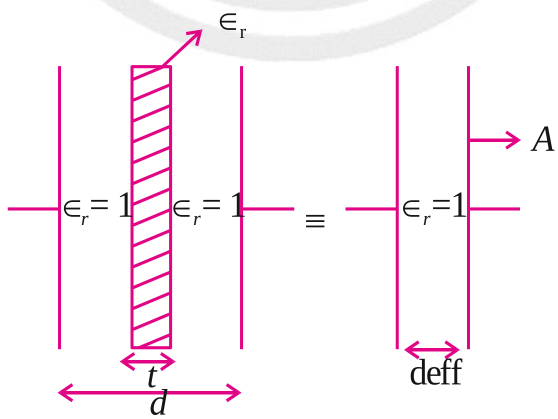
(g) $\frac{\psi_1}{\psi_2} = \frac{A_1 \in_1}{A_2 \in_2}$

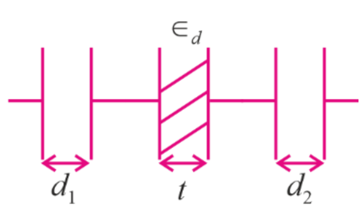
(h) $\frac{F_1}{F_2} = \frac{A_1 \in_1}{A_2 \in_2}$

(i) $\frac{W_1}{W_2} = \frac{A_1 \in_1}{A_2 \in_2}$

(j) $\frac{u_1}{u_2} = \frac{\in_1}{\in_2}$

8. When dielectric is placed between Parallel Plate Capacitor





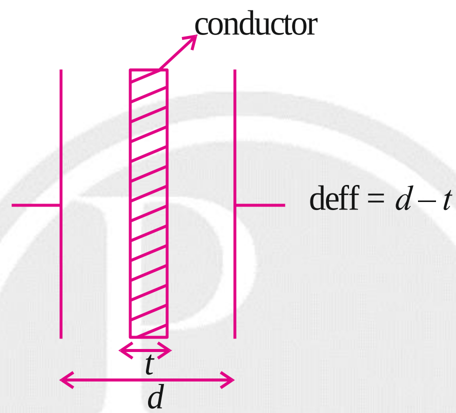
$$C_1 = \frac{A \in_0}{d_1} \qquad C_2 = \frac{A \in_0 \in_r}{t} \qquad C = \frac{A \in_0}{d_{eff}}$$

$$\frac{1}{C_{eq}} = \frac{d_2}{A \in_0} + \frac{t}{A \in_0 \in_r} + \frac{d_1}{A \in_0}$$

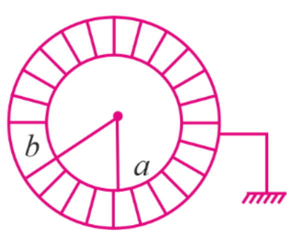
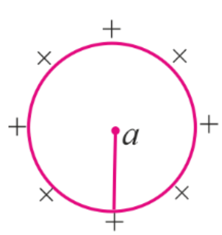
$$\Rightarrow \qquad \frac{d_{eff}}{A \in_0} = \frac{d_2}{A \in_0} + \frac{t}{A \in_0 \in_r} + \frac{d_1}{A \in_0}$$

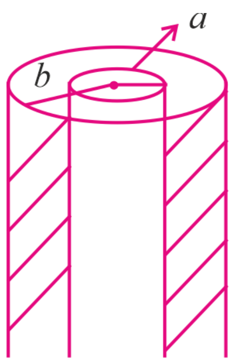
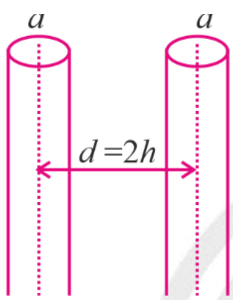
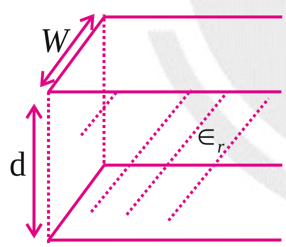
$$d_{eff} = d_1 + d_2 + \frac{t}{\in_r} = (d - t) + \frac{t}{\in_r}$$

9. When conductor is placed between Parallel Plate Capacitor

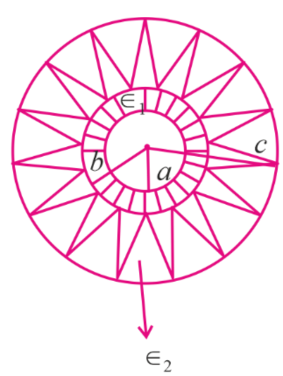


10. Different types of Capacitor

	Capacitor	Name	Formula
1.		Parallel Plate Capacitor	$C = \frac{A \in}{d}$
2.		Spherical Capacitor	$C = \frac{4\pi \in}{\frac{1}{a} - \frac{1}{b}}$
3.		Isolated Spherical Capacitor	$C = 4\pi\epsilon a$

4.		Co-axial Capacitor	$C=\dfrac{2\pi\in}{ln(b/a)}$ Capacitance per unit length
5.		Twin Wire Capacitor	$C'=\dfrac{\pi\in}{ln\left(\dfrac{h}{a}+\sqrt{\left(\dfrac{h}{a}\right)^2-1}\right)}$ $=\dfrac{\pi\in}{\cosh^{-1}\left(\dfrac{h}{a}\right)}$ for $h >> a$ $C'=\dfrac{\pi\in}{ln\left(\dfrac{d}{a}\right)}$
6.		Semi-Infinite Parallel Plate Capacitor	$C'=\dfrac{W\in}{d}$

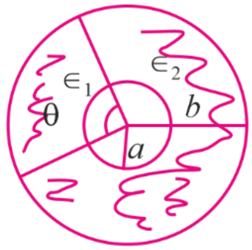
11. Series combination of spherical capacitor



$$C_1=\frac{4\pi\in_0\in_1}{\left(\frac{1}{b}-\frac{1}{c}\right)} \qquad C_2=\frac{4\pi\in_0\in_2}{\left(\frac{1}{b}-\frac{1}{c}\right)} \qquad C_{eq}=\frac{C_1C_2}{C_1+C_2}$$

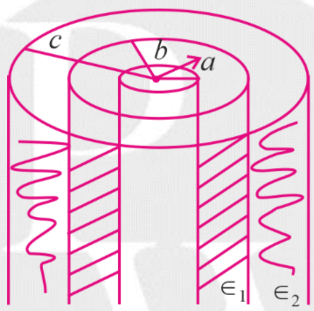
12. Parallel combination of Spherical Capacitor.

(θ = should be in radian)



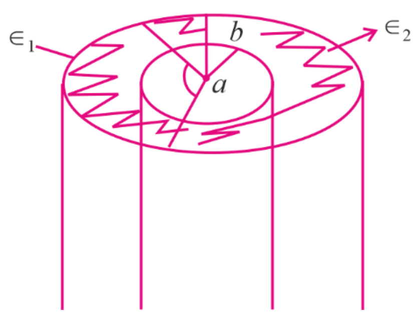
$$C_1 = \frac{2\theta \in_0 \in_1}{\left(\frac{1}{a} - \frac{1}{b}\right)} \qquad C_2 = \frac{2(2\pi - \theta) \in_0 \in_1}{\left(\frac{1}{a} - \frac{1}{b}\right)}, \qquad C_{eq} = C_1 + C_2$$

13. Series combination of Coaxial Capacitor



$$C_1 = \frac{2\pi \in_0 \in_1}{ln\left(\frac{b}{a}\right)} \qquad C_2 = \frac{2\pi \in_0 \in_1}{ln\left(\frac{c}{b}\right)} \qquad Ceq = \frac{C_1 C_2}{C_1 + C_2}$$

14. Parallel combination of Coaxial Capacitor.

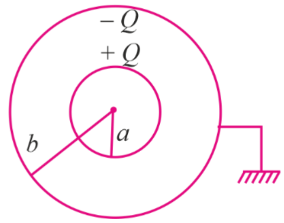


$$C_1 = \frac{\theta(\in_0 \in_1)}{ln(b/a)}$$

$$C_2 = \frac{(2\pi - \theta) \in_0 \in_1}{ln(b/a)}$$

$C_{eq} = C_1 + C_2$

15. Energy stored in Spherical Capacitance.



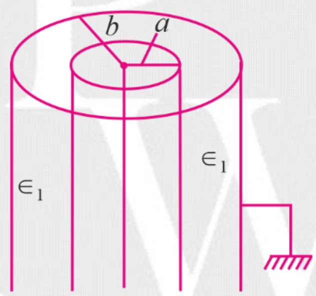
$$\vec{E} = \frac{Q}{4\pi \in r^2}\hat{a}r \quad a \le r \le b$$

$$\mu_e = \frac{1}{2}\in E^2 = \frac{1}{2}\in\left(\frac{Q}{4\pi\in r^2}\right)^2 = \frac{Q}{32\pi^2 \in r^4}$$

$$W = \int_{r=a}^{b}\int_{\theta=0}^{2\pi}\int_{\phi=0}^{\pi}\frac{Q^2}{32\pi^2\in r^4}r^2 \sin\theta\, dr\, d\theta\, d\phi = \frac{Q^2}{8\pi\in}\left(-\frac{1}{r}\right)_a^b = \frac{Q^2}{8\pi\in}\left(\frac{1}{a}-\frac{1}{b}\right) = \frac{Q^2}{2C}$$

where $C = \dfrac{4\pi\in}{\left(\dfrac{1}{a}-\dfrac{1}{b}\right)}$

16. Energy density inside Co-axial Capacitor

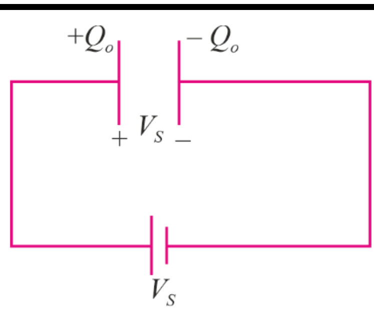
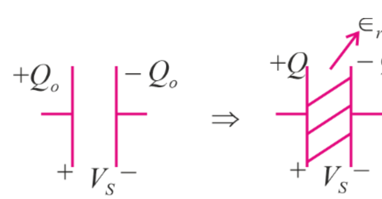


$$u_e = \frac{1}{2}\in\left(\frac{\delta_L}{2\pi\in\delta}\right)^2$$

$$W = \left(\frac{\delta_L^2 l}{4\pi\in}\right) ln\left(\frac{b}{a}\right), \quad Q = \delta_L l, \quad C = \frac{2\pi\in l}{ln\left(\frac{b}{a}\right)}, \quad W = \frac{Q^2}{2C}$$

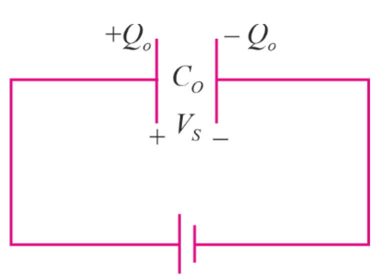
Where l = Length Co-Axial Capacitor.

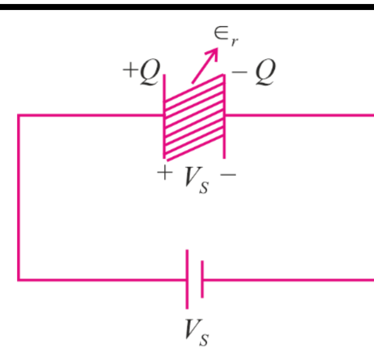
17. Inserting dielectric keeping charge on the plates of capacitor constant.

a.	$C_0 = \dfrac{A\in_0}{d}$	1.	$C = C_0 \in_r$
b.	$V = V_S = \dfrac{Q_0}{C_0}$	2.	$Q = Q_0 = CV$
c.	$Q = Q_0 = C_0 V_S$	3.	$V = \dfrac{V_S}{\in_r}$
d.	$\delta_{SO} = \dfrac{Q_0}{A}$	4.	$\delta_S = \delta_{So}$
e.	$E_o = \dfrac{\delta_{So}}{\in_0} = \dfrac{V_s}{d}$	5.	$E = \dfrac{E_0}{\in_r}$
f.	$D_0 = \delta_{s0}$	6.	$D = D_0$
g.	$\psi_o = Q_o$	7.	$\psi_o = Q_0$
h.	$F_o = \dfrac{1}{2} Q_o E_o$	8.	$F = \dfrac{F_0}{\in_r}$
i.	$W_o = \dfrac{{Q_o}^2}{2C_o}$	9.	$W = \dfrac{W_o}{\in_r}$
j.	$u_o = \dfrac{1}{2} \in_o E_o^2$	1	$u = \dfrac{u_0}{\in_r}$

18. Inserting dielectric inside capacitance keeping potential drop across it constant.





a.	$C_o = \dfrac{A\in_o}{d}$	a.	$C = C_o \in_r$
b.	$V = V_S$	b.	$V = V_S$

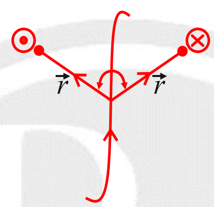
c.	$Q_o = C_o V_S$	c.	$Q = Q_o \in_r$
d.	$\delta_{So} = \dfrac{Q_o}{A}$	d.	$\delta_S = \delta_{So} \in_r$
e.	$E_o = \dfrac{\delta_{So}}{\in_0} = \dfrac{V_s}{d}$	e.	$E = E_o$
f.	$D_o = \delta_{so}$	f.	$D = D_o \in_r$
g.	$\psi_o = Q_o$	g.	$\psi_o = \psi_o \in_r$
h.	$F_o = \dfrac{1}{2} Q_o E_o$	h.	$F = F_o \in_r$
i.	$W_o = \dfrac{1}{2} Q_o Vs$	i.	$W = W_o \in_r$
j.	$u_o = \dfrac{1}{2} \in_o E_o^2$	j.	$u = u_o \in_r$

❑❑❑

5 MAGNETOSTATICS

5.1. MAGNETOSTATS

(a) Biot-Savart Law



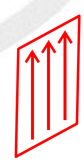
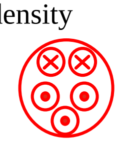
$$d\vec{H} = \frac{I(\vec{dl} \times \hat{a}_r)}{4\pi r^2}$$

$I\vec{dl}$ ≡ Elemental current

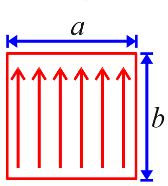
$\hat{a}_r$ ≡ source to observing point

$\hat{a}_H = \hat{a}_I \times \hat{a}_r$

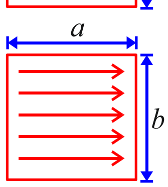
(b) Current Distribution

(a) Point current density or Elemental current $I \uparrow \updownarrow dl$	(b) Line current density 	(c) Sheet current or surface current 	(d) Volume current density 
$I\vec{dl}$	I	$\vec{k}, \vec{J}_s, \vec{J}$ (A/m)	$\vec{J}, \vec{J}_v$ (A/m^2)

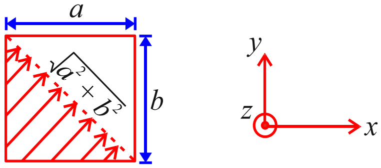
(c) Surface current

(i) 

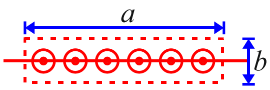
$$K = \frac{I}{a}$$

(ii) 

$$K = \frac{I}{b}$$

(iii) 

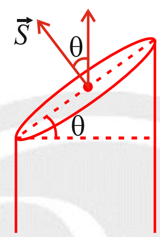
$K = \dfrac{I}{\sqrt{a^2 + b^2}}$

(iv) 

$K = \dfrac{I}{a}$

(v) $I_T = \int (\vec{K} \times \hat{a}_n) \bullet \vec{dl}$ or $\int \vec{K} \bullet (\vec{dl} \times \hat{a}_n)$

(d) Volume Current Density

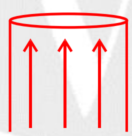


$$J = \frac{\text{Total Current}}{\text{Cross-sectional Area}}$$

$$J = \frac{I}{\cos\theta}, \quad I = J\, S \cos\theta$$

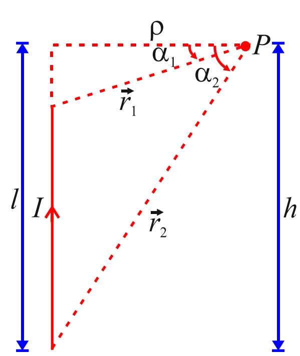
$$I = \vec{J} \bullet \vec{S}$$

$$I = \iint \vec{J} \bullet d\vec{s}$$



$$I_T = \oiint \vec{J} \bullet d\vec{s}$$

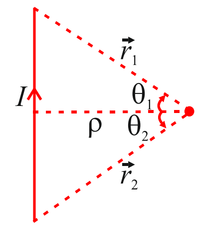
(e) Magnetic Field due to Finite Length Wire



$$\vec{H} = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)(\hat{a}_I \times \hat{a}_r)$$

$\hat{a}_r$ = Direction of unit vector from source to observing point

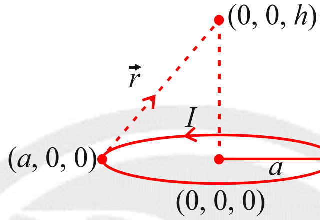
$\hat{a}_I$ = Direction of current



$$\vec{H} = \frac{I}{4\pi\rho}(\sin\theta_2 + \sin\theta_1)(\hat{a}_I \times \hat{a}_r)$$

$\hat{a}_r$ = Direction of unit vector from source point to observing point

(f) Magnetic Field due to Ring Current

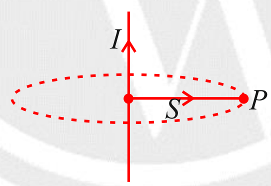


$$\vec{H} = \frac{Ia^2\hat{a}_z}{2(a^2+h^2)^{3/2}}$$

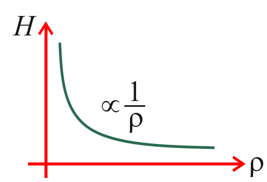
At centre of ring ($h = 0$)

$$\vec{H} = \frac{I}{2a}\hat{a}_z$$

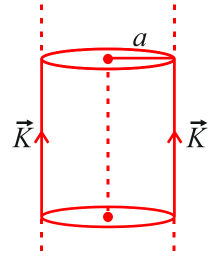
(g) Magnetic Field due to Infinite Length Wire.



$$\vec{H} = \frac{I}{2\pi\rho}(\hat{a}_I \times \hat{a}_r)$$



(h) Hollow Cylinder

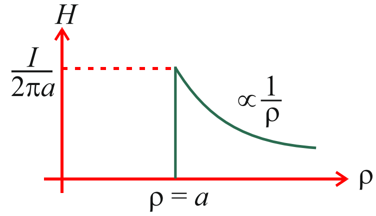


$I = K$ (cross-sectional length)

$I = K(2\pi a)$

$\vec{H} = \vec{0} \qquad \rho \le a$

$= \frac{ka}{\rho}(\hat{a}_I \times \hat{a}_r) = \frac{I}{2\pi\rho}(\hat{a}_I \times \hat{a}_r) \qquad \rho > a$



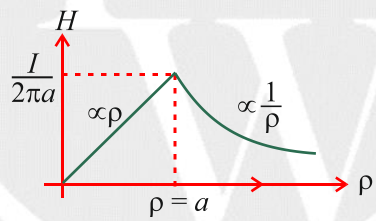
For $\rho > a$, hollow cylinder behaves like a line current.

(i) Solid cylinder

(i) $I_T = \int_{\rho=0}^{a}\int_{\phi=0}^{2\pi} J(\rho d\rho d\phi) = J(\pi a^2)$

(ii) $\vec{H} = \frac{J\rho}{2}(\hat{a}_I \times \hat{a}_r) = \frac{I\rho}{2\pi a^2}(\hat{a}_I \times \hat{a}_r) \qquad \rho \le a$

$= \frac{Ja^2}{2\rho}(\hat{a}_I \times \hat{a}_r) = \frac{I}{2\pi\rho}(\hat{a}_I \times \hat{a}_r) \qquad \rho \ge a$



Outside it behaves like a line current.

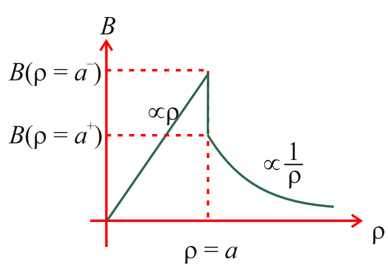
(iii) $\vec{B} = \mu\vec{H} = \frac{\mu I\rho}{2\pi a^2} \qquad \rho < a$

$= \mu_0\vec{H} = \frac{\mu_0 I}{2\pi\rho} \qquad \rho > a$

$B(\rho = a^-)$ = Magnetic flux density just inside solid cylinder.

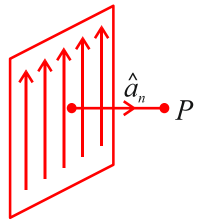
$B(\rho = a^+)$ = Magnetic flux density just outside solid cylinder.

$$\frac{B(\rho = a^-)}{B(\rho = a^+)} = \frac{\mu}{\mu_0} = \mu_r > 1$$



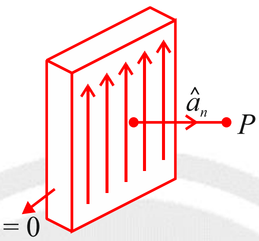
Magnetic Flux Density is discontinuous at the surface of solid cylinder ($\rho = a$).

(j) Thin Sheet



$$\vec{H} = \frac{\vec{K} \times \hat{a}_n}{2}$$

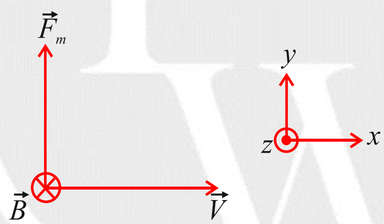
(k) Thick sheet



$$\vec{H} = \vec{K} \times \hat{a}_n$$

(l) Circular Motion in Magnetic field

(i)



$$\vec{B} = B_0(-\hat{a}_z)$$

$$\vec{V} = V_0(+\hat{a}_x)$$

$$\vec{F}_m = q(\vec{V} \times \vec{B})$$

$$\vec{F}_m = q\left(V_0\hat{a}_x \times B_0(-\hat{a}_z)\right) = qV_0B_0\hat{a}_y$$

(ii) $\frac{mV_0^2}{R} = qV_0B_0 \Rightarrow R = \frac{mV_0}{qB_0}$ (radius of circular motion)

(iii) Angular frequency = $\omega = \frac{V_0}{R} = \frac{qB_0}{m}$

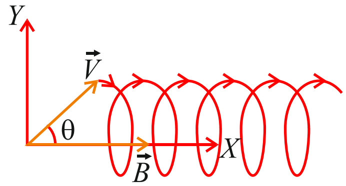
$$f = \frac{\omega}{2\pi} = \frac{qB_0}{2\pi m}$$

(iv) Time period of circular motion

$$T = \frac{1}{f} = \frac{2\pi m}{qB_0}$$

(i) Elliptical Motion in Magnetic Field

(i)



$$\vec{B} = B_0 \hat{a}_x$$

$$\vec{V} = V_0 \cos\theta \hat{a}_x + V_0 \sin\theta \hat{a}_y$$

$$\vec{F}m = q(\vec{V} \times \vec{B}) = q\left((V_0 \cos\theta \hat{a}_x + V_0 \sin\theta \hat{a}_y) \times (B_0 \hat{a}_x)\right)$$

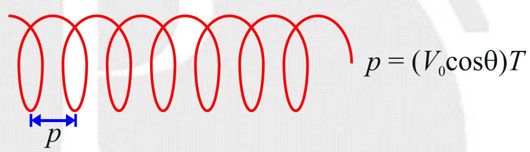
$$= qB_0 V_0 \sin\theta(-\hat{a}_z)$$

$$V \longrightarrow \underset{\substack{\downarrow \\ \text{Linear Motion}}}{V\cos\theta} + \underset{\substack{\downarrow \\ \text{Circular Motion}}}{V\sin\theta} \Rightarrow \text{Helical motion}$$

(ii) $$\frac{mV_0^2 \sin^2\theta}{R} = qB_0 V_0 \sin\theta$$

$$R = \frac{mV_0 \sin\theta}{qB_0}$$

(iii) $$\omega = \frac{V_0 \sin\theta}{R} = \frac{qB_0}{m},\ f = \frac{qB_0}{2\pi m}$$



(iv) $$T = \frac{2\pi m}{qB_0}$$

$$p = \frac{2\pi m V_0 \cos\theta}{qB_0}$$

Note: θ = Angle between $\vec{B}$ and $\vec{V}$.

$\theta = 0° \Rightarrow$ Linear motion

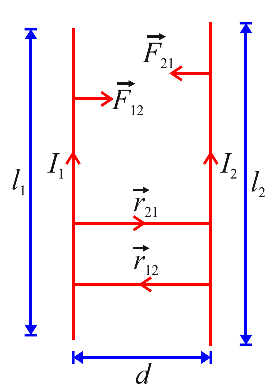
$\theta = 90° \Rightarrow$ circular motion

$0° < \theta < 90° \Rightarrow$ Elliptical motion

$V_{||ar}$ = Linear motion

$V_{\perp r}$ = Circular motion

(j) Magnetic Force



$\vec{F}_{21}$ = Force on wire (2) due to wire (1).

$\vec{F}_{12}$ = Force on wire (1) due to wire (2).

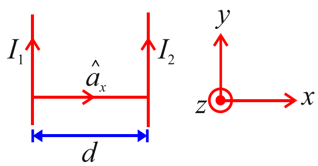
$\vec{r}_{21}$ = Position vector which directed from wire (1) towards wire (2).

$\vec{r}_{12}$ = Position vector which directed from wire (2) towards wire (1).

$$\vec{F}_{21} = \frac{\mu I_1 I_2}{2\pi d} l_2 \left(\hat{a}_{I_2}\left(\hat{a}_{I_1} \times \hat{a}_{r_{21}}\right)\right)$$

$$\vec{F}_{12} = \frac{\mu I_1 I_2}{2\pi d} l_1 \left(\hat{a}_{I_1}\left(\hat{a}_{I_2} \times \hat{a}_{r_{12}}\right)\right)$$

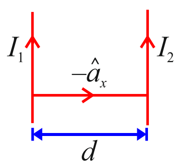
(i)



$$\vec{F}_{21} = \frac{\mu I_1 I_2 l_2}{d}\left(\hat{a}_y\left(\hat{a}_y \times \hat{a}_x\right)\right)$$

$$= \frac{\mu I_1 I_2 l_2}{d}(-\hat{a}_x)$$

(ii)



$$\vec{F}_{12} = \frac{\mu I_1 I_2 l_1}{d}\left(\hat{a}_y \times \left(\hat{a}_y \times -\hat{a}_x\right)\right)$$

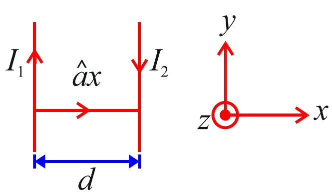
$$= \frac{\mu I_1 I_2 l_2}{d}(\hat{a}_x)$$

Hence, force per unit length.

$$\frac{\vec{F}_{21}}{l_2} = \vec{F}'_{21}, \frac{\vec{F}_{12}}{l_1} = \vec{F}'_{12}$$

$\vec{F}'_{21} = -\vec{F}'_{12}$ (Equal and opposite hence it follows Newton's third law.)

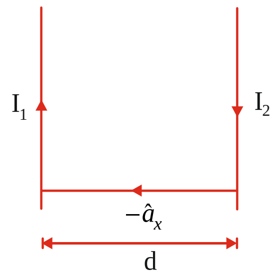
(iii)



$$\vec{F}_{21} = \frac{\mu I_1 I_2 l_2}{2\pi d}\left(\left(-\hat{a}_y\right) \times \left(\hat{a}_y \times \hat{a}_x\right)\right)$$

$$\vec{F}'_{21} = \frac{\vec{F}_{21}}{l_2} = \frac{\mu I_1 l_2}{2\pi d}(\hat{a}_x)$$

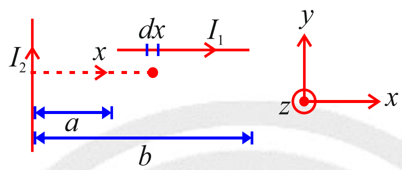
(iv)



$$\vec{F}_{12} = \frac{\mu I_1 I_2 l_1}{2\pi d}\left(\hat{a}_y \times \left(-\hat{a}_y \times \left(-\hat{a}_x\right)\right)\right)$$

$$\vec{F}'_{12} = \frac{\vec{F}_2}{l_1} = \frac{\mu I_1 I_2}{2\pi d}\left(-\hat{a}_x\right)$$

(v)

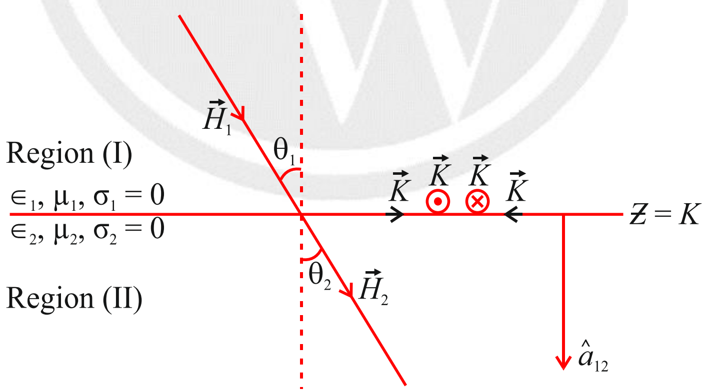


$$d\vec{F}_{12} = \frac{\mu I_1 I_2}{2\pi x} dx\left(\hat{a}_x \times \left(\hat{a}_y \times \hat{a}_x\right)\right)$$

$$\vec{F}_{12} = \int_a^b \frac{\mu I_1 I_2}{2\pi x} dx\left(\hat{a}_y\right) = \frac{\mu I_1 I_2}{2\pi} \ln\left(\frac{b}{a}\right)\left(\hat{a}_y\right)$$

$$\vec{F}_{21} = -\vec{F}_{12} = \frac{\mu I_1 I_2}{2\pi} \ln\left(\frac{b}{a}\right)\left(-\hat{a}_y\right)$$

(k) Magnetic - Magnetic Boundary



1. K = Surface current (A/m).
2. $\vec{H}_1 = \vec{H}_{t_1} + \vec{H}_{n_1}$
3. $\vec{H}_2 = \vec{H}_{t_2} + \vec{H}_{n_2}$
4. $H_{t_1} = H_1 \sin\theta_1,\ H_{t_2} = H_2 \sin\theta_2$

   $H_{n_1} = H_1 \cos\theta_1,\ H_{n_2} = H_2 \cos\theta_2$
5. $Z = K \Rightarrow$ boundary or Interface.

6. $\hat{a}_{12}$ = Unit vector which is directed to 1st medium to 2nd medium.

7. $\oiint \vec{B} \cdot d\vec{s} = 0 \Rightarrow \left(\vec{B}_2 - \vec{B}_1\right) \cdot \hat{a}_{12} = 0 \Rightarrow \vec{B}_{n_2} = \vec{B}_{n_1}$

8. $\oint \vec{H} \cdot \vec{dl} = I_{enc} \Rightarrow \vec{H}_{t_2} - \vec{H}_{t_1} = \vec{K} \times \hat{a}_{12}$ or $\vec{K} = \hat{a}_{12} \times \left(\vec{H}_{t_2} - \vec{H}_{t_1}\right)$

9. $\dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{\mu_1}{\mu_2}$ when $K = 0$ (surface current)

10. When μ_1 (denser) > μ_2 (rarer)

    then θ_1 (away from normal) > θ_2 (towards the normal)

11. Magnetic field is directed away from the normal in denser medium and directed towards the normal in rarer medium.

Case I : $K = 0$ (Current free boundary)

1. $B_{n_2} = B_{n_1}$

   Normal component of magnetic flux density is continuous across boundary or interface.

2. $H_{n_2} \neq H_{n_1}$

   Normal component of magnetic field intensity is discontinuous across boundary or interface.

3. $H_{t_2} = H_{t_1}$

   Tangential component of magnetic field intensity is continuous across boundary or interface.

4. $B_{t_2} \neq B_{t_1}$

   Tangent component of magnetic flux density is discontinuous across boundary or interface.

Case II: $K \neq 0$ (Surface current $\neq 0$)

1. $B_{n_2} = B_{n_1}$

   Normal component of magnetic flux density is continuous across boundary or interface.

2. $H_{n_2} \neq H_{n_1}$

   No component of magnetic field intensity is discontinuous across boundary or interface.

3. $H_{t_2} \neq H_{t_1}$

   Tangential component of magnetic field intensity is discontinuous across boundary or interface.

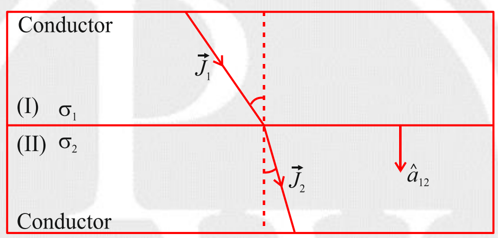
4. $B_{t_1} = B_{t_2}$ or $B_{t_1} \neq B_{t_2}$

Tangential component of magnetic flux density may be continuous or discontinuous across boundary or interface.

I.	Dielectric – Dielectric Boundary	II.	Magnetic – Magnetic Boundary
1.	$\vec{E}_{t_1} + \vec{E}_{n_1} = \vec{E}_1$	1.	$\vec{H}_1 = \vec{H}_{t_1} + \vec{H}_{n_1}$
2.	$\vec{E}_{t_2} + \vec{E}_{n_2} = \vec{E}_2$	2.	$\vec{H}_2 = \vec{H}_{t_2} + \vec{H}_{n_2}$
3.	$D_{n_2} - D_{n_1} = \rho s$	3.	$B_{n_2} = B_{n_1}$

4.	$\vec{E}_{t_2} = \vec{E}_{t_1}$	4.	$\vec{H}_{t_2} - \vec{H}_{t_1} = \vec{K} \times \hat{a}_{12}$ or $K = \hat{a}_{12} \times \left(\vec{H}_{t_2} = \vec{H}_{t_1}\right)$
5.	$\left(\vec{D}_2 - \vec{D}_1\right) \cdot \hat{a}_{12} = \rho_s$	5.	$\left(\vec{B}_2 - \vec{B}_1\right) \cdot \hat{a}_{12} = 0$
6.	$\rho_s = 0$	6.	$K = 0$
	a. $D_{n_2} = D_{n_1}$		a. $B_{n_2} = B_{n_1}$
	b. $E_{t_2} = E_{t_1}$		b. $H_{t_2} = H_{t_1}$
	c. $\vec{E}_2 = \dfrac{E_1 \vec{E}_n}{E_2} + \vec{E}_{t_1}$		c. $H_2 = \dfrac{\mu_1 H n_1}{\mu_1} + H t_1$
	d. $\dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{\epsilon_1}{\epsilon_2}$		d. $\dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{\mu_1}{\mu_2}$
	e. Denser Medium $\Rightarrow$ away from the normal		e. Denser Medium $\Rightarrow$ away from the normal
	f. Rarer Medium $\Rightarrow$ towards the normal		f. Rarer Medium $\Rightarrow$ towards the normal

(l) Conductor – Conductor Boundary



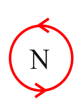
(i) $\oiint \vec{J} \cdot d\vec{s} = 0$ (KCL)

$$\Rightarrow \vec{J}_{n_2} = \vec{J}_{n_1}, \left(\vec{J}_2 - \vec{J}_1\right) \cdot \hat{a}_{12} = 0$$

(ii) $\vec{E}_{t_2} = \vec{E}_{t_1} \Rightarrow \dfrac{\vec{J}_{t_2}}{\sigma_2} = \dfrac{\vec{J}_{t_1}}{\sigma_1}$

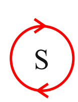
$$\therefore \quad \vec{J}_2 = \vec{J}_{n_1} + \frac{\sigma_2 \vec{J}_{t_1}}{\sigma_1}$$

(m) Magnetic Dipole

(a) 

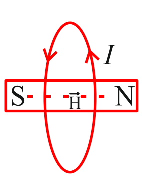
I ≡ Anticlockwise
⊙ ≡ Outward
⊗ ≡ Inward

H ≡ ⊙
North pole ≡ ⊙
South pole ≡ ⊗



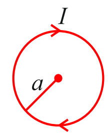
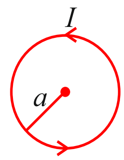
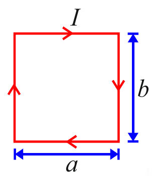
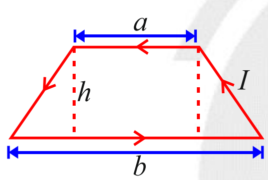
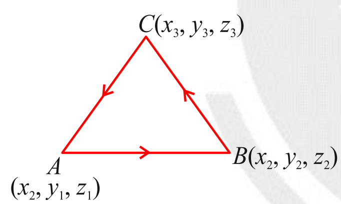
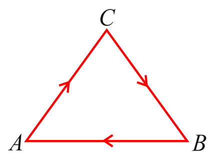
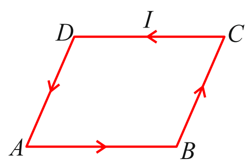
I ≡ Clockwise
H ≡ ⊗

North pole≡ ⊗
South pole ≡ ⊙

(b) 

(c) Magnetic dipole moment ($\vec{M}$)

$\vec{M}$ = (Current) · (Area) and (directed normal to the surface)

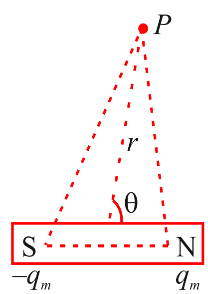
	$\vec{M} = I(\pi a^2)\otimes$
	$\vec{M} = I(\pi a^2)\odot$
	$\vec{M} = I(ab)\otimes$
	$\vec{M} = \frac{1}{2}((a+b)h)\odot$
	$\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$ $\overrightarrow{AC} = (x_3 - x_1)\hat{i} + (y_3 - y_1)\hat{j} + (z_3 - z_1)\hat{k}$ $\vec{M} = I\left(\frac{1}{2}\left(\overrightarrow{AB} \times \overrightarrow{AC}\right)\right)$
	$\vec{M} = \frac{1}{2}\left(I\left(\overrightarrow{AC} \times \overrightarrow{AB}\right)\right)$
	$\vec{M} = I\left(\overrightarrow{AB} \times \overrightarrow{AD}\right)$

$\vec{M} = NI\vec{S}$, N = Number of turns.

I = Current flowing through loop.

S = Area of loop

(d) $\vec{B} = \frac{\mu_0 M}{4\pi r^3}\left(2\cos\theta \hat{a}_r + \sin\theta \hat{a}_\theta\right)$



6 MAXWELL EQUATION

6.1. Type of Medium

(A) Free space/air.

- Volume charge density (ρ_v) = 0
- Conductivity (σ) = 0 (perfect insulator)
- Dipole moment (P) ≠ 0
- Displacement current density (J_d) ≠ 0
- $\epsilon = \epsilon_o = \frac{1}{36\pi} \times 10^{-9} F/m = 8.85 \times 10^{-12} F/m$
- $\mu = \mu_o = 4\pi \times 10^{-7} H/m$
- $\epsilon_r = 1$, $\mu_r = 1$

(B) Lossless Dielectric/perfect dielectric: -

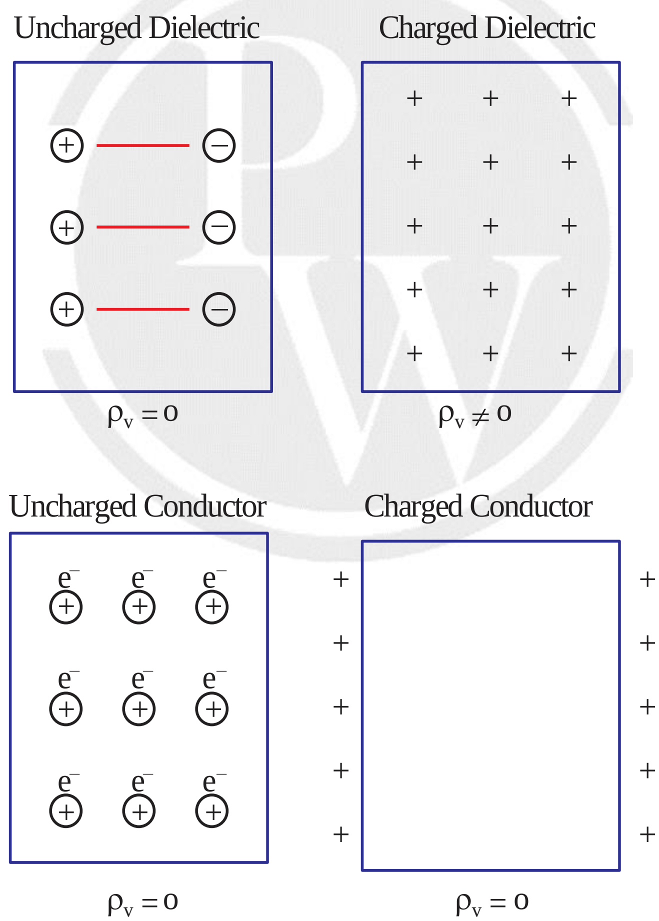
- Dielectric is a type of insulator in which it has dipole.
- Perfect dielectric means perfect insulator.
- $\rho_v = 0$,
- Conductivity of dielctric (σ_d) = 0
- Dipole moment (P) ≠ 0
- Displacement current (J_d) ≠ 0
- $\epsilon = \epsilon_0 \epsilon_r$
- $\mu = \mu_o \mu_r$
- In general $\epsilon_r > 1$ and $\mu_r = 1$ (Non-magnetic).

(C) Lossy Dielectric/Imperfect dielectric: -

- $\rho_v = 0$
- $\sigma_d \neq 0$ (It gives conductor loss)
- $\epsilon = \epsilon_0 \epsilon_r$

- $\mu = \mu_o \mu_r$
- In general $\epsilon_r > 1$ and $\mu_r = 1$ (Non-magnetic).
- Dipole moment (P) $\neq 0$
- $J_d \neq 0$ (due to dipoles)
- Conduction current density (J_c) $\neq 0$

  → due to conductivity of lossy dielectric
- $\frac{\sigma_d}{\omega \in} < \frac{1}{100} \Rightarrow$ Low loss dielectric
- $\frac{1}{100} < \frac{\sigma_d}{\omega \in} < 100 \Rightarrow$ Medium-loss dielectric
- $\frac{\sigma_d}{\omega \in} > 100 \Rightarrow$ *High*-loss dielectric



but $\rho_s \neq 0$ (surface charge density).

(D) Conductor:

- $\rho_v = 0$, $\sigma_c \neq 0$ (Very high)
- σ_c = conductivity of conductor.

- Conductor is very high loss dielectric.
- Perfect conductor: - $\sigma_c = \infty$ (Very-very high)
  eg: - Gold, Silver etc.
- Good conductor: - σ_c = very high
  eg: - Brass, conductor etc.
- Poor conductor: - σ_c = high
  eg: - Aluminium

6.2. Types of Conductors

(A) Perfect Electric Conductor (PEC)

$$E = 0, H = \text{maximum}$$

(B) Perfect Magnetic Conductor (PMC)

$$E = \text{maximum}, H = 0$$

(C) Super Conductor

$$E = 0, H = 0$$

6.3 Properties of Medium

ϵ, μ & σ are known as consecutive property.

(A) **Linear**: If consecutive property of any medium does not depends upon strength of field, then that medium is linear.

$\sigma, \mu, \epsilon \neq f(E, H) \rightarrow$ Linear Medium $\sigma, \mu, \epsilon = f(E, H) \rightarrow$ Non-Linear Medium

$\epsilon_r = 5 \rightarrow$ Linear $\qquad \epsilon_r = \frac{E}{10} + \frac{E^2}{1023} + \frac{H^3}{20319} + \ldots.. \Rightarrow$ Non-Linear

(B) **Homogeneous Medium**: If consecutive property of any medium does not depends on the point in space, them that medium is homogeneous.

$\epsilon_r = 5 \rightarrow$ Homogeneous

$\epsilon_r = 10x(x + 9) \rightarrow$ Inhomogeneous

$\epsilon_r = f(10f + 1) \rightarrow$ Inhomogeneous

$\epsilon_r, \mu_r, \sigma = f(x, y, z, \delta, \phi, z\ r, \theta, \phi) \rightarrow$ Inhomogeneous

$\epsilon_r, \mu_r, \sigma \neq f(x, y, z, \delta, \phi, z\ r, \theta, \phi) \rightarrow$ Inhomogeneous

(C) **Isotropic Medium**: If consecutive property of any medium does not depends upon direction, them that medium is isotropic.

Case I: Isotropic Medium

$$\vec{D} = \epsilon \vec{E}, \quad \epsilon_r = 3$$

$$\vec{D} = 3\epsilon_o \vec{E}, \Rightarrow \left(D_x\hat{i} + D_y\hat{j} + D_z\hat{k}\right) = 3\epsilon_o\left(E_x\hat{i} + E_y\hat{j} + E_z\hat{k}\right)$$

$$D_x = 3\epsilon_o E_x, D_y = 3\epsilon_0 E_y, D_z = 3\epsilon_o E_z$$

Since Er is same in all direction. Hence medium is isotropic.

$$\begin{bmatrix} Dx \\ Dy \\ Dz \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} E_o \begin{bmatrix} Ex \\ Ey \\ Ez \end{bmatrix}$$

↳ Scalar Matrix ⟶ Isotropic Medium

Case II: Uni-isotropic Medium

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \epsilon_o \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

↳ Diagonal Matrix

$D_x = 3\epsilon_o E_x$, $D_y = 4\epsilon_o E_y$, $D_z = 5\epsilon_o E_z$

Case III: Anisotropic Medium

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \epsilon_o \begin{bmatrix} 2 & 3 & 4 \\ 5 & 7 & 9 \\ 6 & 3 & 1 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

$D_x = 2\epsilon_o E_x + 3\epsilon_o E_y + 4\epsilon_o E_z$

$D_y = 5\epsilon_o E_x + 7\epsilon_o E_y + 9\epsilon_o E_z$

$D_z = 6\epsilon_o E_x + 3\epsilon_o E_y + \epsilon_o E_z$

Note: All homogeneous are isotropic and all isotropic are homogeneous.

$H \rightleftharpoons I$

(D) Non-Dispersive: - If consecutive property of any medium does not depend upon frequency then that medium is non-dispersive.

$\sigma = 2$ ⟶ Non-dispersive

$\sigma = 5\omega\epsilon$ ⟶ dispersive

6.4. Electric Gauss Law

Total electric flux through closed surface is equal to algebraic sum of charges enclosed.

$$\oiint \vec{D} \cdot \vec{ds} = Q_{enc}$$

(a) $\vec{\nabla} \cdot \vec{D} = \rho_v$ ⟶ Point Form of Gauss law

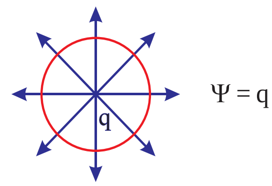
(b) $\oiint \vec{D} \cdot \vec{ds} = \iiint (\vec{\nabla} \cdot \vec{D}) dv = \iiint \rho_v dv$

Gauss theorem or divergence theorem

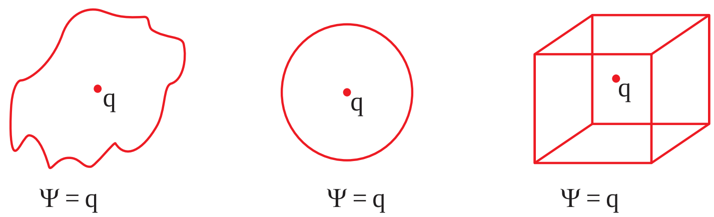
(c) $\oiint \vec{D} \cdot \vec{ds} \equiv$ Total Flux through Closed Surface.

(d) $\oiint \vec{E} \cdot \vec{ds} \equiv$ Total number of electric field lines.

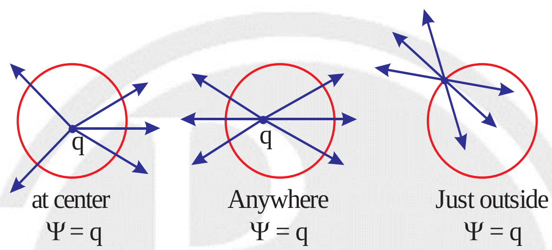
(e) Gaussian surface must be closed.



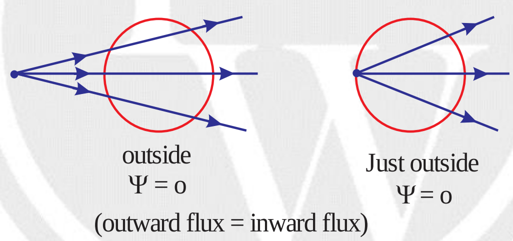
(f) Gaussian surface may be irregular/regular.



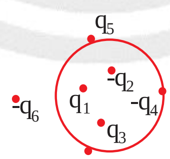
(g) The charge will be placed anywhere inside the Gaussian surface.



(h) When charge is placed outside the Gaussian surface.



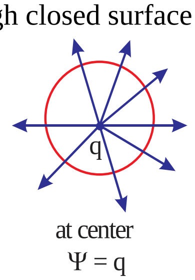
(i) $Q_{enc} \equiv$ algebraic sum of the charges enclosed.



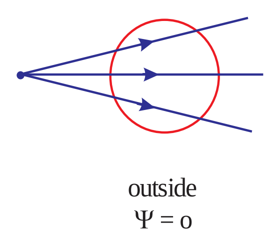
$Q_{enc} = q_1 - q_2 + q_3 - q_4$

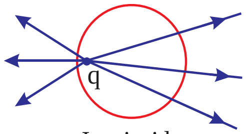
(j) Total flux through closed surface

(i)

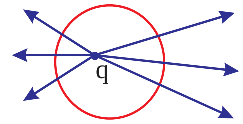


(ii)

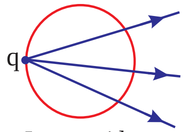


(iii) 

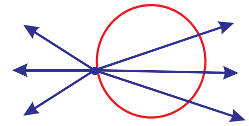
Just inside
$\Psi = q$

(iv) 

Anywhere inside
$\Psi = q$

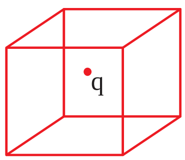
(v) 

Just outside
$\Psi = 0$

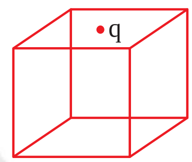
(vi) 

at the Surface
$\Psi = \frac{q}{2}$

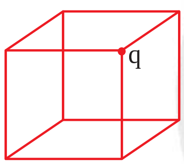
(k) Total flux through cube, when charge is placed at

(i) 

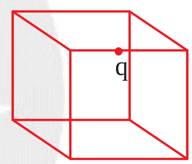
$\Psi = q$
(Flux through one cube).

(ii) 

$\Psi = \frac{q}{2}$
(Flux through one cube).

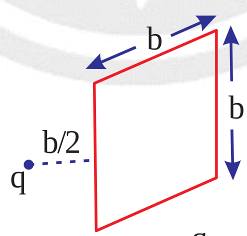
(iii) 

$\Psi = \frac{q}{8}$
(Flux through one cube).

(iv) 

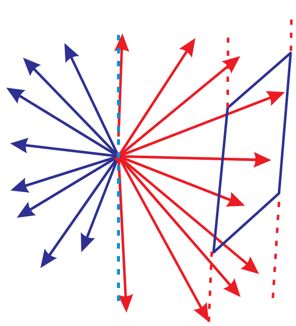
$\Psi = \frac{q}{4}$
(Flux through one cube).

(l) Flux through the plane



$\Psi = \frac{q}{6}$

(m) Flux through infinite length plane



$\Psi = \frac{q}{2}$

6.5. Magnetic Gauss Law

Total magnetic flux through any closed surface is zero.

$$\oint\!\!\oint \vec{B}\bullet d\vec{s} = 0$$

(a) Point form $\Rightarrow \vec{\nabla}\bullet\vec{B} = 0$

(b) Integral form $\Rightarrow \vec{B}\bullet d\vec{s} = 0$

(c) $\oint\!\!\oint \vec{B}.d\vec{s} = 0$, $\vec{\nabla}.\vec{B} = 0$ →
- Divergericeless
- Solenoidal
- Magnetic monopole does not exist
- Originating and terminating point are not defined

6.6. Electric Field Conservative/KVL.

$$\oint \vec{E}.\vec{dl} = 0,\ \vec{\nabla}\times\vec{E} = \vec{0}$$

Work done in a closed path is zero.

$\oint \vec{E}.\vec{dl} = 0$, $\vec{\nabla}\times\vec{E} = \vec{0}$ →
- Irrotational
- Path independent
- Conservative
- KVL
- Does not exist in a closed loop

6.7. Ampeare Circuital Law

Total magneto motive force in a loop is equal to algebraic sum of current enclose.

$$\oint \vec{H}.\vec{dl} = I_{enc}, \vec{\nabla}\times\vec{H} = \vec{J}.$$

(a) If loop is closed in anticlockwise or counter clockwise direction, then

(i) Outward current is taken as positive.

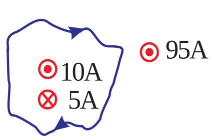
(ii) Inward current is taken as negative.

(b) If loop is closed in clockwise direction then

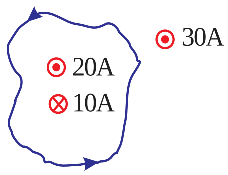
(i) Inward current is taken as negative.

(ii) Outward current is taken as positive.

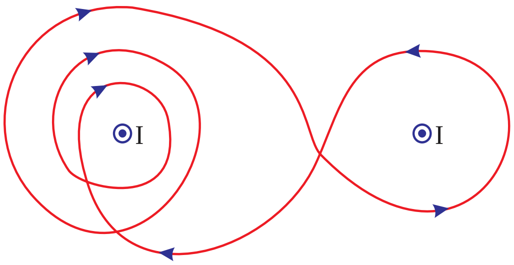
(c)



$$\oint \vec{H}.\vec{dl} = Ienc$$
$$= -10 + 5 = 5A$$

(d) 

$$\oint \vec{H}.\vec{dl} = I_{enc}$$
$$= -20 - 10 = 10A$$

(e) 

$$\oint \vec{H}.\vec{dl} = I_{enc} = -3I - I = -2I$$

6.8. Maxwell Equation in Statics

Integral Form	**Point Form**
(a) $\oint \vec{D}.\vec{dl} = Q_{enc}$	$\vec{\nabla} \times \vec{D} = \rho_v$

Electric Gauss Law

(b) $\oint \vec{E}.\vec{dl} = 0$	$\vec{\nabla} \times \vec{E} = \vec{0}$

Electric field conservative

(c) $\oiint \vec{B}.d\vec{s} = 0$	$\vec{\nabla} \times \vec{B} = 0$

Magnetic Gauss Law

(d) $\oint \vec{H}.\vec{dl} = I_{enc}$	$\vec{\nabla} \times \vec{H} = \vec{J}$

Ampeare Circuital Law

6.9. Maxwell Equation in Ideal and Practical Medium

		Ideal	**Practical**
(a)	$\oiint \vec{D}.\vec{ds} = Q_{enc}$	✓	✓
(b)	$\vec{\nabla} \bullet \vec{D} = \rho_v$	✓	×
(c)	$\oiint \vec{E} \bullet \vec{ds} = \frac{Qenc}{\epsilon}$	✓	×
(d)	$\vec{\nabla} \times \vec{E} = \frac{\rho_v}{\epsilon}$	✓	×

(e)	$\oint \vec{E} \cdot \vec{dl} = 0$	✓	✓
(f)	$\vec{\nabla} \times \vec{E} = \vec{0}$	✓	✓
(g)	$\oint \vec{D} \cdot \vec{dl} = 0$	✓	×
(h)	$\vec{\nabla} \times \vec{D} = \vec{0}$	✓	×
(i)	$\oiint \vec{B} \cdot \vec{ds} = 0$	✓	✓
(j)	$\vec{\nabla} \cdot \vec{B} = 0$	✓	✓
(k)	$\oiint \vec{H} \cdot \vec{ds} = 0$	✓	×
(l)	$\vec{\nabla} \cdot \vec{H} = 0$	✓	×
(m)	$\oint \vec{H} \cdot \vec{dl} = I_{enc}$	✓	✓
(n)	$\vec{\nabla} \times \vec{H} = \vec{J}$	✓	✓
(o)	$\oint \vec{B} \cdot \vec{dl} = \mu I_{enc}$	✓	×
(p)	$\vec{\nabla} \times \vec{B} = \mu \vec{J}$	✓	×

- **Ideal Medium**: Linear, Homogeneous, and Isotropic
- **Practical Medium**: Non-linear, Inhomogeneous and an isotropic.

6.10. Maxwell Equation in Different types of Medium

(a) Charge Free Medium

$$Q = 0,\ \rho_L = 0,\ \rho_s = 0,\ \rho_v = 0$$

- $\oiint \vec{D} \cdot \vec{ds} = 0$ $\quad \vec{\nabla} \cdot \vec{D} = 0$
- $\oint \vec{E} \cdot \vec{dl} = 0$ $\quad \vec{\nabla} \times \vec{E} = \vec{0}$
- $\oiint \vec{B} \cdot \vec{ds} = 0$ $\quad \vec{\nabla} \cdot \vec{B} = 0$
- $\oint \vec{H} \cdot \vec{dl} = I_{enc}$ $\quad \vec{\nabla} \times \vec{H} = \vec{J}$

(b) Current Free Medium

$$I = 0, K = 0, J = 0$$

- $\oiint \vec{D} \cdot \vec{ds} = Q_{enc}$ $\quad \vec{\nabla} \cdot \vec{D} = \rho_V$
- $\oint \vec{E} . \vec{dl} = 0$ $\quad \vec{\nabla} \times \vec{E} = \vec{0}$
- $\oiint \vec{B} \cdot \vec{ds} = 0$ $\quad \vec{\nabla} \cdot \vec{B} = 0$
- $\oint \vec{H} . \vec{dl} = 0$ $\quad \vec{\nabla} \times \vec{H} = \vec{0}$

(c) Source Free Medium

$$Q = 0, \rho_L = 0, \rho_S = 0, \rho_V = 0$$

$$I = 0, K = 0, J = 0$$

- $\oiint \vec{D} \cdot \vec{ds} = 0$ $\quad \vec{\nabla} \cdot \vec{D} = 0$
- $\oint \vec{E} \cdot \vec{dl} = 0$ $\quad \vec{\nabla} \times \vec{E} = \vec{0}$
- $\oiint \vec{B} \cdot \vec{ds} = 0$ $\quad \vec{\nabla} \cdot \vec{B} = 0$
- $\oint \vec{H} \cdot \vec{dl} = 0$ $\quad \vec{\nabla} \times \vec{H} = \vec{0}$

6.11. To Find Unknowns of $\vec{E}$, $\vec{H}$ & V.

(a) $\vec{\nabla} \cdot \vec{B} = 0 \rightarrow$ Unknown of $\vec{B}$

(b) $\vec{\nabla} \times \vec{E} = \vec{0} \rightarrow$ Unknown of $\vec{E}$

(c) $\nabla^2 V = 0 \rightarrow$ Unknown of V

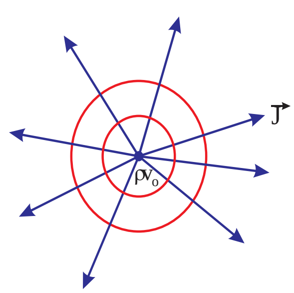
(d) $\rho_V = \vec{\nabla} \cdot \vec{D}$

(e) $\vec{J} = \vec{\nabla} \times \vec{H}$

6.12. Continuity Equation/KCL

- It gives flow of charge in a medium.
- Continuity Equation in point form.

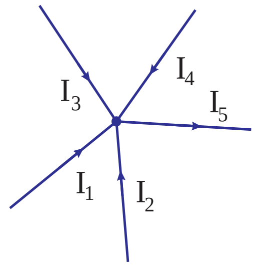
$$\vec{\nabla} \cdot \vec{J} = \frac{-\partial \rho_V}{\partial t}$$



- Continuity Equation in integral form

$$\oiint \vec{J} \bullet d\vec{s} = -\iiint_{\vartheta} \left(\frac{\partial \rho_v}{\partial t}\right) dv$$

- KCL

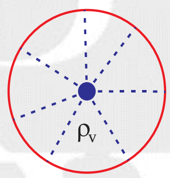


At node

$$\sum I_i = 0$$

$$\Rightarrow \vec{\nabla}.\vec{J} = 0$$
$$\mapsto \text{KCL}$$
$$\Rightarrow \oiint \vec{J}.d\vec{s} = 0$$

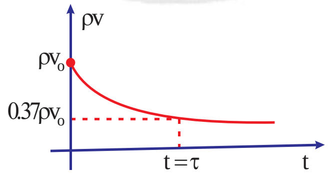
- Equation of flow of charge in a medium



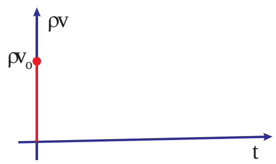
(a) $\rho_V = \rho_{vo} e^{-\frac{\sigma}{\varepsilon}t}$ (b) $\rho_S = \rho_{So} e^{-\frac{\sigma}{\varepsilon}t}$

(c) $\rho_L = \rho_{Lo} e^{-\frac{\sigma t}{\varepsilon}}$ (d) $Q = Q_o e^{-\frac{\sigma t}{\varepsilon}}$

Where $\frac{\varepsilon}{\sigma} = \tau$ (Relaxation time constant)



- Flow of charge in conductor.
  σ is very high
  So, τ (Relaxation time constant) = 0
  Hence, $\rho_V = \rho_{vo} e^{-\frac{\sigma}{\varepsilon}t} \simeq 0$

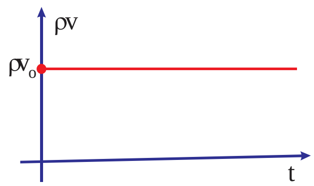


Very fast flow of charge in a conductor. So, charge does not reside inside conductor.

- Flow of charge inside insulator.
  $\sigma_d = 0$ (very low)

So, τ (Relaxation time constant) = ∞

$$\rho_V = \rho_{Vo} e^{-\frac{\sigma}{\varepsilon}t} = \rho_{Vo}$$



There is no flow of charge inside perfect insulator.

6.13. Laplace's Equation/Poisson Equation

(a) Poisson Equation in point form

(i) For Ideal Medium

$$\nabla^2 V = \frac{-\delta_V}{\varepsilon}$$

(ii) For Practical Medium

$$\vec{\nabla}.\left(\varepsilon\left(\vec{\nabla}V\right)\right) = -\delta_V$$

$$\varepsilon\left(\nabla^2 V\right) + \left(\vec{\nabla}_V\right).\left(\vec{\nabla}_\varepsilon\right) = -\delta_V$$

(b) Laplace Equation

$\delta_V \rightarrow$ Free space/air/vacuum/uncharged/ dielectric/conductor/singular charge distribution/cavity/source free medium charge free medium.

$\nabla^2 V = 0 \Rightarrow$ Laplacian Equation

6.14. Difference between Laplacian & Poisson Equation

Laplace's Equation		Poisson Equation	
(a)	$\nabla^2 V = 0$	(i)	$\nabla^2 V = \frac{-\rho_V}{\varepsilon}$
(b)	$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$	(ii)	$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{-\rho_V}{\varepsilon}$
(c)	Complementary function	(iii)	Complementary function + Particular integral
(d)	Unique solution and follow uniqueness theorem	(iv)	Does not follow uniqueness theorem
(e)	Linear Equation	(v)	Non- Linear Equation
(f)	Homogeneous Equation	(vi)	Non- Homogeneous Equation

6.15. Magnetic Force

$$\vec{F}_m = I\left(\vec{l} \times \vec{B}\right)$$

$\vec{l} \equiv$ length of wire and directed along current direction.

6.16. Magnetic Energy Density

$$\mu_m = \frac{1}{2}\vec{B}.\vec{H} = \frac{1}{2}\mu H^2 = \frac{B^2}{2\mu}$$

$$\mu_m = \frac{1}{2}\vec{J}.\vec{A} \text{ where } \vec{A} \equiv \text{Magnetic vector potential}$$

6.17. Faraday's Law

(a) Faraday's 1st Law (Qualitative Analysis):

When magnetic field lines cuts conductor, then an electromotive force will be developed, which is known as induced electromotive force.

(b) Faraday's 2nd Law (Quantitative Analysis): -

The induced electromotive force is directly proportional to time rate of charge of flux.

$$e \,\alpha\, \frac{d\psi m}{dt} \Rightarrow \text{Faraday's Law}$$

$$e = -\frac{d\psi m}{dt}$$

(−) → Lenz's Law

$\frac{d\psi m}{dt}$ → Faraday's Law

(C) Faradays and Lenz's Law in integral form

$$\oint \vec{E} \bullet d\vec{l} = \iint -\frac{\partial \vec{B}}{\partial t} \bullet d\vec{s}$$

(d) Faraday's and Lenz's Law in point form

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

(e)

- ψ_m = Magnetic Flux

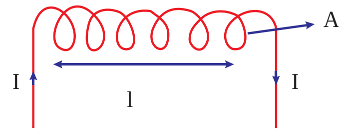
  N = Number of turns

  l = length of solenoid

  ϕ_m = Magnetic flux due to one turn

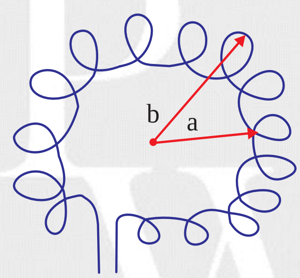
  A = cross-sectional area of solenoid

(f)



- l = (N-1)d
- $H = \frac{NI}{l}$
- $B = \mu H = \frac{\mu NI}{l}$
- $\phi_m = BA = \frac{\mu NIA}{l}$
- $\psi_m = LI$ (Weber)
- $e = \frac{-d\psi_m}{dt} = \frac{-\mu N^2 A}{l}\left(\frac{di}{dt}\right) = -L\frac{dI}{dt}$
- $V = -e = +L\frac{dI}{dt}$

(g) Toroid

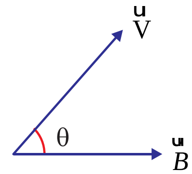


- $r = \frac{a+b}{2}$
- $l = 2\pi r$
- $H = \frac{NI}{2\pi r}$
- $B = \mu H = \frac{\mu NI}{2\pi r}$
- $\phi_m = \left(\frac{\mu NIA}{2\pi r}\right)$
- $\psi_m = N\phi_m = \left(\frac{\mu N^2 A}{2\pi r}\right) I = LI$
- $V = -e = +\frac{d\psi_m}{dt} = L\frac{dI}{dt}$
- $L = \frac{\mu N^2 A}{2\pi r}$

Lorentz's Force

- $\vec{F}_{net} = q(\vec{V} \times \vec{B}) + q\vec{E}_{in}$

- $\vec{E}_{in} = \vec{B} \times \vec{V}$
- $e = BVL \sin\theta$



$$e = (\vec{V} \times \vec{B}).\vec{l}$$

6.18. Modified Ampere's Circuital Law:

(a) Conduction current: - It is due to flow of electron. It is due to conductivity.

- Since, conductivity of conductor is very high. Hence, conduction current is very high.
- Since, conductivity of dielectric is very low. Hence, conduction current is very low. (Leakage Current)

(b) Displacement current: - It is due to rotation of dipoles.

- Since, number of dipoles inside conductor is very low. Hence, displacement current is very low.
- Since, number of dipoles inside dielectric is very high. Hence, displacement current is very high.

(c)

Conductor	Dielectric
$I_c \gg I_d$	$I_c \ll I_d$

(d) Ampeare's circuit law is modified by maxwell

$$\oint \vec{H} \bullet \vec{dl} = I_{enc} \Rightarrow \text{In statics}$$

$$\oint \vec{H} \bullet \vec{dl} = I_c + I_d \Rightarrow \text{In time varrying}$$

(e) Modified Ampeare's circuital low in integral low

$$\oint \vec{H} \bullet \vec{dl} = I_c + I_d$$

(f) Modified Ampeare's circuital law in point form

$$\vec{\nabla} \times \vec{H} = \vec{J_c} + \vec{J_d}$$

(g) $\vec{J_c}$ **= Conduction current density =** $\sigma\vec{E}$

$$\vec{J_d} = \frac{\partial \vec{D}}{\partial t} = \varepsilon \frac{\partial \vec{E}}{\partial t} = \text{Displacement current density}$$

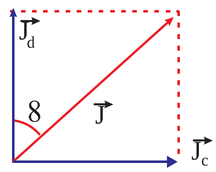
(h) $I_c = \iint \vec{J}_c . d\vec{s}$

$I_d = \frac{\partial \psi_e}{dt} \equiv$ Time rate of change of electric flux per unit time.

(i)

$J = \sqrt{J_c^2 + J_d^2}$

$\tan\delta = \frac{J_c}{J_d}$



$I = I_c + I_d$

(j) $\vec{\nabla} \times \vec{H} = \vec{J}_c + \vec{J}_d = \sigma\vec{E} + \varepsilon\frac{\partial \vec{E}}{\partial t} = (\sigma + jw\varepsilon)\vec{E}$

(k) $\oint \vec{H} \bullet \vec{dl} = \iint \left(\sigma\vec{E} + \varepsilon\frac{\partial \vec{E}}{\partial t} \right) \bullet d\vec{s} = \iint (\sigma + j\omega\varepsilon)\vec{E} \bullet d\vec{s}$

6.19. Maxwell's Equation in time Varying Fields

(a) $\oiint \vec{D} \bullet d\vec{s} = Q_{enc}$ (i) $\vec{\nabla} \bullet \vec{D} = \rho_V$

Electric Gauss Law

(b) $\oiint \vec{B} \bullet d\vec{s} = 0$ (ii) $\vec{\nabla} \bullet \vec{B} = 0$

Magnetic Gauss Law

(c) $\oint \vec{E} \bullet \vec{dl} = -\iint \frac{\partial \vec{B}}{\partial t} \bullet d\vec{s}$ (iii) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -jw\mu\vec{H}$

Faraday's and Lenz's Law

(d) $\oint H \bullet \vec{dl} = I_c + I_d$ (iv) $\vec{\nabla} \times \vec{H} = \sigma\vec{E} + \varepsilon\frac{\partial \vec{E}}{\partial t} = (\sigma + jw\varepsilon)\vec{E}$

Modified Ampeare Circuital

6.20. Magnetic Vector Potential and Magnetic scaler Potential

(a) Magnetic scalar potential: -

$\vec{J} = \vec{0} \Rightarrow$ Volume current density = 0

$\vec{\nabla} \times \vec{H} = \vec{0} \Rightarrow \vec{H} = -\vec{\nabla}V_m$

$\boxed{V_m = -\int \vec{H} . \vec{dl}} \rightarrow$ Magnetic scalar potential

(b) Magnetic Vector Potential

- $\vec{B} = \vec{\nabla} \times \vec{A}$ ($\vec{A} \equiv$ Magnetic Vector Potential)
- $\vec{A} = \frac{\mu}{4\pi} \int \frac{I\,\vec{dl}}{R}$
- $\vec{A} = \frac{\mu}{4\pi} \iint \frac{\vec{k}ds}{R}$
- $\vec{A} = \frac{\mu}{4\pi} \iiint \frac{\vec{J}dv}{R}$
- $\oint \vec{A} \bullet \vec{dl} = \iint (\vec{\nabla} \times \vec{A}) \bullet d\vec{s} = \iint \vec{B}.d\vec{s} = \psi_m = \text{Magentic flux}$
- $\vec{\nabla}(\vec{\nabla} \bullet \vec{A}) - \nabla^2 \vec{A} = \mu \vec{J}$

(a) In statics $\boxed{\vec{\nabla} \bullet \vec{A} = 0}$ $\therefore \nabla^2 \vec{A} = -\mu \vec{J}$

(b) In time varying $\left(\vec{\nabla}(\vec{\nabla} \bullet \vec{A})\right) = \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2}$

$$\nabla^2 \vec{A} - \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu \vec{J}$$

❑❑❑