

**ICSE Class 9 Maths Selina Solutions Chapter 12:** Here are the Selina Solutions for the problems found in the ICSE Class 9 Maths Selina Solutions Chapter 12, Mid-Point and Its Converse. Students study the intercept theorem in detail as well as the issue of mid-point and its converse of a triangle in this chapter. By completing all of the questions in the Selina textbook, students can easily receive a perfect score on their exams.

The ICSE Class 9 Maths Selina Solutions Chapter 12 is quite simple to comprehend. All the exercise questions in the book are addressed in these solutions, which follow the ICSE or CISCE syllabus. This page contains the ICSE Class 9 Maths Selina Solutions Chapter 12 PDF, which can be viewed online or downloaded. To practice offline, students can now access and download the Selina Solutions for free.

## ICSE Class 9 Maths Selina Solutions Chapter 12 Overview

ICSE Class 9 Maths Selina Solutions Chapter 12, "Mid Point and Its Converse," offer clear explanations and step-by-step solutions to understand the concept of midpoints in geometry. This chapter focuses on properties and theorems related to midpoints of line segments.

It covers topics such as the midpoint formula, properties of line segments divided by their midpoint, and the converse theorems associated with these properties. The solutions provided are structured to help students grasp the concepts effectively, with plenty of practice exercises to reinforce learning. Overall, ICSE Class 9 Maths Selina Solutions Chapter 12 for this chapter serves as a valuable resource for students aiming to strengthen their understanding and problem-solving skills in geometry.

## ICSE Class 9 Maths Selina Solutions Chapter 12

Below we have provided ICSE Class 9 Maths Selina Solutions Chapter 12 -

**1. In triangle ABC, M is the mid-point of AB, and a straight line through M and parallel to BC cuts AC in N. Find the lengths of AN and MN if BC = 7 cm and AC = 5 cm.**

**Solution:**

The triangle is shown below:

Since M is the midpoint of AB and  $MN \parallel BC$

Then, by the mid-point theorem N is the midpoint of AC.

Therefore,

$$MN = \frac{1}{2} BC = \frac{1}{2} \times 7 = 3.5\text{cm}$$

$$\text{And, } AN = \frac{1}{2} AC = \frac{1}{2} \times 5 = 2.5\text{cm}$$

**2. Prove that the figure obtained by joining the mid-points of the adjacent sides of a rectangle is a rhombus.**

**Solution:**

The figure is shown below:

Let ABCD be a rectangle, with P, Q, R, and S representing the AB, BC, CD, and DA midpoints. The next step is to demonstrate that PQRS is a rhombus.

The two diagonals BD and AC should be drawn as seen in the figure

And,  $BD = AC$  [Since diagonals of a rectangle are equal]

Proof:

From  $\triangle ABD$  and  $\triangle BCD$ , we have

$$PS = \frac{1}{2} BD = QR \text{ and } PS \parallel BD \parallel QR$$

$$2PS = 2QR = BD \text{ and } PS \parallel QR \dots (1)$$

Similarly,

$$2PQ = 2SR = AC \text{ and } PQ \parallel SR \dots (2)$$

From (1) and (2) we get

$$PQ = QR = RS = PS$$

Therefore, PQRS is a rhombus.

– Hence proved

**3. D, E, and F are the mid-points of the sides AB, BC, and CA of an isosceles  $\triangle ABC$  in which  $AB = BC$ . Prove that  $\triangle DEF$  is also isosceles.**

**Solution:**

The figure is shown below:

Given that  $AB = AC$  and that ABC is an isosceles triangle

Since the midpoints of AB, BC, and CA are, respectively, D, E, and F

Therefore, by the mid-point theorem

$$2DE = AC \text{ and } 2EF = AB$$

$$\Rightarrow DE = EF$$

Therefore, DEF is an isosceles triangle where  $DE = EF$ .

– Hence proved

**4. The following figure shows a trapezium ABCD in which  $AB \parallel DC$ . P is the mid-point of AD and  $PR \parallel AB$ . Prove that:**

$$PR = \frac{1}{2} (AB + CD)$$

**Solution:**

Given,

P is the midway between AD and  $PR \parallel AB$  in  $\triangle ABD$ .

Q is therefore BD's midpoint. Using the midpoint theorem

Similarly, R is the midpoint of BC as  $PR \parallel CD \parallel AB$

Now, from  $\triangle ABD$

$$2PQ = AB \dots (1)$$

And, from  $\triangle BCD$

$$2QR = CD \dots (2)$$

Adding (1) and (2), we get

$$2(PQ + QR) = AB + CD$$

$$2PR = AB + CD$$

$$PR = \frac{1}{2} (AB + CD)$$

– Hence proved.

**5. The figure, given below, shows a trapezium ABCD. M and N are the mid-points of the non-parallel sides AD and BC respectively. Find:**

**(i) MN, if  $AB = 11$  cm and  $DC = 8$  cm.**

**(ii) AB, if DC = 20 cm and MN = 27 cm.**

**(iii) DC, if MN = 15 cm and AB = 23 cm.**

**Solution:**

Let's draw a diagonal AC as shown in the figure below,

(i) Given, AB = 11cm and CD = 8cm

From  $\triangle ABC$ , we have

$$ON = \frac{1}{2} AB = \frac{1}{2} \times 11 = 5.5\text{cm}$$

From  $\triangle ACD$ , we have

$$OM = \frac{1}{2} CD = \frac{1}{2} \times 8 = 4\text{cm}$$

Hence,  $MN = OM + ON$

$$= (4 + 5.5)$$

$$= 9.5\text{cm}$$

(ii) Given, CD = 20cm and MN = 27cm

From  $\triangle ACD$ , we have

$$OM = \frac{1}{2} CD = \frac{1}{2} \times 20 = 10\text{cm}$$

$$\text{Therefore, } ON = 27 - 10 = 17\text{cm}$$

Then from  $\triangle ABC$ , we have

$$AB = 2 ON$$

$$= 2 \times 17$$

$$= 34\text{cm}$$

(iii) Given, AB = 23cm and MN = 15cm

From  $\triangle ABC$ , we have

$$ON = \frac{1}{2} AB = \frac{1}{2} \times 23 = 11.5\text{cm}$$

$$\text{Therefore, } OM = 15 - 11.5 = 3.5\text{cm}$$

Then from  $\triangle ACD$ , we have

$$CD = 2 OM$$

$$= 2 \times 3.5$$

$$= 7\text{cm}$$

**6. The diagonals of a quadrilateral intersect at right angles. Prove that the figure obtained by joining the mid-points of the adjacent sides of the quadrilateral is a rectangle.**

**Solution:**

The figure is shown below:

Let ABCD be a quadrilateral, with P, Q, R, and S representing the AB, BC, CD, and DA midpoints.

At point O, the diagonals AC and BD intersect at a straight angle.

Proof that PQRS is a rectangle is necessary.

Proof:

From  $\triangle ABC$  and  $\triangle ADC$ , we have

$$2PQ = AC \text{ and } PQ \parallel AC \dots (1)$$

$$2RS = AC \text{ and } RS \parallel AC \dots (2)$$

From (1) and (2) we get,

$$PQ = RS \text{ and } PQ \parallel RS$$

Similarly,

$$PS = RQ \text{ and } PS \parallel RQ$$

Therefore, PQRS is a parallelogram.

Now as  $PQ \parallel AC$ , we have

$$\angle AOD = \angle PXO = 90^\circ \text{ [Corresponding angles]}$$

Again, as  $BD \parallel RQ$ , we have

$$\angle PXO = \angle RQX = 90^\circ \text{ [Corresponding angle]}$$

Similarly,

$$\angle QRS = \angle RSP = \angle SPQ = 90^\circ$$

Therefore, PQRS is a rectangle.

– Hence proved

**7. L and M are the mid-points of sides AB and DC respectively of parallelogram ABCD. Prove that segments DL and BM trisect diagonal AC.**

**Solution:**

The required figure is shown below:

We have,

BL = DM and BL  $\parallel$  DM and BLMD is a parallelogram

$$\Rightarrow BM \parallel DL$$

Now, in  $\triangle ABY$ , we have

L is the midpoint of AB and XL  $\parallel$  BY,

Therefore, x is the midpoint of AY

$$\Rightarrow AX = XY \dots (1)$$

Similarly for triangle CDX

$$\Rightarrow CY = XY \dots (2)$$

From (1) and (2), we get

$$AX = XY = CY \text{ and } AC = AX + XY + CY$$

Consequently, the parallelogram ABCD's diagonal AC is trisected by segments DL and BM.

**8. ABCD is a quadrilateral in which AD = BC. E, F, G, and H are the mid-points of AB, BD, CD, and AC respectively. Prove that EFGH is a rhombus.**

**Solution:**

$$\text{Given, } AD = BC \dots (1)$$

From the figure,

In  $\triangle ADC$  and  $\triangle ABD$ , we have

$$2GH = AD \text{ and } 2EF = AD,$$

$$\Rightarrow 2GH = 2EF = AD \dots (2)$$

Now, in  $\triangle BCD$  and  $\triangle ABC$ , we have

$$2GF = BC \text{ and } 2EH = BC$$

$$\Rightarrow 2GF = 2EH = BC \dots (3)$$

From (1), (2), (3) we get

$$2GH = 2EF = 2GF = 2EH$$

$$\Rightarrow GH = EF = GF = EH$$

Therefore, EFGH is a rhombus.

– Hence proved

**9. A parallelogram ABCD has P as the mid-point of DC and Q as a point of AC such that  $CQ = \frac{1}{4} AC$ . PQ produced meets BC at R.**

**Prove that:**

**(i) R is the midpoint of BC**

**(ii)  $PR = \frac{1}{2} DB$**

**Solution:**

Let's draw the diagonal BD as shown below.

At point X, the diagonal cuts AC and BD meet.

A parallelogram's diagonals are known to bisect one another.

Therefore,  $AX = CX$  and  $BX = DX$

Given,

$$CQ = \frac{1}{4} AC$$

$$CQ = \frac{1}{4} \times 2CX$$

$$CQ = \frac{1}{2} CX$$

Therefore, Q is the midpoint of CX.

(i) For  $\triangle CDX$ ,  $PQ \parallel DX$  or  $PR \parallel BD$

And in  $\triangle CBX$ , Q is the midpoint of CX and  $QR \parallel BX$

Therefore, R is the midpoint of BC

(ii) In  $\triangle BCD$ ,

As P and R are the midpoint of CD and B, we have

Thus,  $PR = \frac{1}{2} DB$

**10. D, E, and F are the mid-points of the sides AB, BC, and CA respectively of  $\triangle ABC$ . AE meets DF at O. P and Q are the mid-points of OB and OC respectively. Prove that DPQF is a parallelogram.**

**Solution:**

The required figure is shown below:

In  $\triangle ABC$  and  $\triangle OBC$ , we have

$2DE = BC$  and  $2PQ = BC$ ,

Therefore,  $DE = PQ \dots (1)$

In  $\triangle ABO$  and  $\triangle ACO$ , we have

$2PD = AO$  and  $2FQ = AO$ ,

Therefore,  $PD = FQ \dots (2)$

From (1) and (2), we get that PQFD is a parallelogram.

– Hence proved.

**11. In triangle ABC, P is the mid-point of side BC. A line through P and parallel to CA meets AB at point Q, and a line through Q and parallel to BC meets median AP at point R. Prove that:**

**(i)  $AP = 2AR$**

**(ii)  $BC = 4QR$**

**Solution:**



The required figure is shown below:

P is the midpoint of BC,  $PQ \parallel AC$ , and  $QR \parallel BC$ , as can be seen.

Consequently, Q and R are the midpoints of AB and AP, respectively.

(i) Thus,  $AP = 2AR$

(ii) Let's extend QR such that it cuts AC at S as shown in the figure.

Now, in  $\triangle PQR$  and  $\triangle ARS$ , we have

$\angle PQR = \angle ARS$  (Opposite angles)

$PR = AR$

$PQ = AS$  (Since,  $PQ = AS = \frac{1}{2} AC$ )

Thus,  $\triangle PQR \cong \triangle ARS$  by SAS congruence criterion

Therefore, by CPCT

$QR = RS$

Now,

$BC = 2QS$

$BC = 2 \times 2QR$

$BC = 4QR$

– Hence proved

**12. In trapezium ABCD, AB is parallel to DC; P and Q are the mid-points of AD and BC respectively. BP produced meets CD produced at point E. Prove that:**

**(i) Point P bisects BE,**

**(ii) PQ is parallel to AB.**

**Solution:**

The required figure is shown below:

(i) In  $\triangle PED$  and  $\triangle ABP$ , we have

$PD = AP$  [Since P is the mid-point of AD]

$$\angle DPE = \angle APB \text{ [Opposite angles]}$$

$$\angle PED = \angle PBA \text{ [Alternate angles as } AB \parallel CE]$$

$\therefore \triangle PED \cong \triangle ABP$  by ASA congruence postulate

Thus, by CPCT

$$EP = BP$$

(ii) For  $\triangle ECB$ , we have  $PQ \parallel CE$

Also,  $CE \parallel AB$

Therefore,  $PQ \parallel AB$

– Hence proved

**13. In a triangle ABC, AD is a median and E is the mid-point of median AD. A line through B and E meets AC at point F. Prove that  $AC = 3AF$ .**

**Solution:**

The required figure is shown below:

Let's draw a line  $DG \parallel BF$

Now,

In  $\triangle ADG$ , we have

$DG \parallel BF$  and E is the midpoint of AD

Therefore, F is the midpoint of AG

$$\Rightarrow AF = GF \dots (1)$$

And, in  $\triangle BCF$ , we have

$DG \parallel BF$  and D is the midpoint of BC

Therefore, G is the midpoint of CF

$$\Rightarrow GF = CF \dots (2)$$

$$AC = AF + GF + CF \text{ [From figure]}$$

$$AC = 3AF \text{ [From (1) and (2)]}$$

– Hence proved.

**14. D and F are midpoints of sides AB and AC of a triangle ABC. A line through F and parallel to AB meets BC at point E.**

**(i) Prove that BDFE is parallelogram**

**(ii) Find AB, if EF = 4.8 cm.**

**Solution:**

The required figure is shown below:

(i) Since F is the midpoint and  $EF \parallel AB$

Therefore, E is the midpoint of BC

So,  $BE = \frac{1}{2} BC$  and  $EF = \frac{1}{2} AB \dots (1)$

And,

Since D and F are the midpoint of AB and AC

Therefore,  $DE \parallel BC$

So,  $DF = \frac{1}{2} BC$  and  $DB = \frac{1}{2} AB \dots (2)$

From (1) and (2), we get

$BE = DF$  and  $BD = EF$

Hence, BDEF is a parallelogram.

(ii) Now,  $AB = 2EF$

$= 2 \times 4.8$

$= 9.6\text{cm}$

**15. In triangle ABC, AD is the median, and DE, drawn parallel to side BA, meets AC at point E. Show that BE is also a median.**

**Solution:**

In  $\triangle ABC$ , we have

AD is the median of BC

D is the mid-point of BC

Given that  $DE \parallel BA$

So, by the converse of the mid-point theorem,

DE bisects AC

$\Rightarrow$  E is the mid-point of AC

And, BE is the median of AC

Hence, BE is also a median.

**16. In  $\triangle ABC$ , E is the mid-point of the median AD, and BE produced meets side AC at point Q. Show that BE: EQ = 3:1.**

**Solution:**

Construction: Draw  $DY \parallel BQ$

In  $\triangle BCQ$  and  $\triangle DCY$ , we have

$\angle BCQ = \angle DCY$  [Common]

$\angle BQC = \angle DYC$  [Corresponding angles]

So,  $\triangle BCQ \sim \triangle DCY$  by AA similarity criterion

Thus,

$BQ/DY = BC/DC = CQ/CY$  [Corresponding sides are proportional]

$BQ/DY = 2 \dots$  (i)

Similarly,  $\triangle AEQ \sim \triangle ADY$

$EQ/DY = AE/ED = \frac{1}{2}$  [E is the mid-point of AD]

$\Rightarrow EQ/DY = \frac{1}{2} \dots$  (ii)

On dividing (i) by (ii), we get

$BQ/EQ = 4$

$BQ = 4 EQ$

$BE + EQ = 4EQ$

$$BE = 3EQ$$

Therefore,  $BE/EQ = 3/1$

**17. In the given figure, M is the mid-point of AB and DE, whereas N is the mid-point of BC and DF. Show that:  $EF = AC$ .**

**Solution:**

In  $\triangle EDF$ , we have

M is the mid-point of AB and N is the mid-point of DE

So,  $MN = \frac{1}{2} EF$  (By mid-point theorem)

$$EF = 2MN \dots (i)$$

In  $\triangle ABC$ , we have

M is the mid-point of AB and N is the mid-point of BC

$MN = \frac{1}{2} AC$  (By mid-point theorem)

$$AC = 2MN \dots (ii)$$

From (i) and (ii), we get

$$EF = AC$$

## **ICSE Class 9 Maths Selina Solutions Chapter 12 Exercise 12B**

**1. Use the following figure to find:**

(i) BC, if  $AB = 7.2$  cm.

(ii) GE, if  $FE = 4$  cm.

(iii) AE, if  $BD = 4.1$  cm

(iv) DF, if  $CG = 11$  cm.

**Solution:**

According to the equal intercept theorem, as  $CD = DE$

$$\Rightarrow AB = BC \text{ and } EF = GF$$

Thus,

$$(i) BC = AB = 7.2\text{cm}$$

$$(ii) GE = EF + GF$$

$$= 2EF$$

$$= 2 \times 4$$

$$= 8 \text{ cm}$$

Since B, D, and F are the midpoints and  $AE \parallel BF \parallel CG$

Therefore,  $AE = 2BD$  and  $CG = 2DF$

$$(iii) AE = 2BD$$

$$= 2 \times 4.1$$

$$= 8.2 \text{ cm}$$

$$(iv) DF = \frac{1}{2} CG$$

$$= \frac{1}{2} \times 11$$

$$= 5.5 \text{ cm}$$

**2. In the figure, given below,  $2AD = AB$ , P is the mid-point of AB, and Q is the mid-point of DR and  $PR \parallel BS$ . Prove that:**

**(i)  $AQ \parallel BS$**

**(ii)  $DS = 3 \text{ Rs.}$**

**Solution:**

Given,  $AD = AP = PB$  as  $2AD = AB$  and P is the midpoint of AB

(i) In  $\triangle DPR$ , we have

A and Q are the midpoints of DP and DR

Therefore,  $AQ \parallel PR$

Now, as  $PR \parallel BS$

$\therefore AQ \parallel BS$

(ii) In  $\triangle ABC$ , P is the midpoint, and  $PR \parallel BS$

Therefore, R is the midpoint of BC

Now, in  $\triangle BRS$  and  $\triangle QRC$ , we have

$$\angle BRS = \angle QRC$$

$$BR = RC$$

$$\angle RBS = \angle RCQ$$

$\therefore \triangle BRS \cong \triangle QRC$  by SAS Congruence criterion

Hence, by CPCT

$$QR = RS$$

$$\text{Thus, } DS = DQ + QR + RS$$

$$= QR + QR + RS$$

$$= 3RS$$

**3. The side AC of a triangle ABC is produced to point E so that  $CE = \frac{1}{2} AC$ . D is the mid-point of BC and ED produced meets AB at F. Lines through D and C are drawn parallel to AB which meet AC at point P and EF at point R respectively. Prove that:**

**(i)  $3DF = EF$  (ii)  $4CR = AB$ .**

**Solution:**

Let's consider the figure below:

In this case, DP is parallel to AB and D is the midpoint of BC.

Consequently,  $PD = \frac{1}{2} AB$ , and P is the midway of AC.

(i) Again, in  $\triangle AEF$  we have  $AE \parallel PD \parallel CR$  and  $AP = \frac{1}{3} AE$

$$\text{Therefore, } DF = \frac{1}{3} EF$$

$$\Rightarrow 3DF = EF$$

– Hence proved

(ii) In  $\triangle PED$ , we have  $PD \parallel CR$ , and C is the midpoint of PE

$$\text{So, } CR = \frac{1}{2} PD$$

Now,

$$PD = \frac{1}{2} AB$$

$$\frac{1}{2} PD = \frac{1}{4} AB$$

$$CR = \frac{1}{4} AB$$

$$4CR = AB$$

– Hence proved

**4. In triangle ABC, the medians BP and CQ are produced up to points M and N respectively such that BP = PM and CQ = QN. Prove that:**

**(i) M, A, and N are collinear.**

**(ii) A is the mid-point of MN.**

**Solution:**

The figure is shown below:

(i) In  $\triangle BPC$  and  $\triangle MPA$ , we have

$$\angle BPC = \angle APN \text{ [Vertically opposite angle]}$$

$$BP = MP$$

$$PC = PA$$

$\therefore \triangle BPC \cong \triangle MPA$  by SAS congruence postulate

Thus, by CPCT

$$\angle PCB = \angle PAM \dots (1)$$

$$\text{And, } BC = AM \dots (2)$$

Similarly,

Considering  $\triangle CQB$  and  $\triangle NQA$ , we have

$$\angle QBC = \angle QAN \dots (3)$$

$$\text{And, } BC = AN \dots (4)$$



Now, by angle sum property of  $\triangle ABC$

$$\angle ABC + \angle ACB + \angle BAC = 180^\circ$$

$$\Rightarrow \angle QBC + \angle PCB + \angle BAC = 180^\circ$$

$$\angle QAN + \angle PAM + \angle BAC = 180^\circ \text{ [From (1) and (3)]}$$

Therefore, it's a straight angle, and M, A, and N must be collinear.

(ii) Now, from (2) and (4) we have

$$AM = AN$$

Hence, A is the midpoint of MN.

**5. In triangle ABC, angle B is obtuse. D and E are mid-points of sides AB and BC respectively and F is a point on side AC such that EF is parallel to AB. Show that BEFD is a parallelogram.**

**Solution:**

The figure is shown below:

We have,

$EF \parallel AB$  and E is the midpoint of BC

Therefore, F is the midpoint of AC

And,

$EF = BD$  as D is the midpoint of AB

Now, as  $BE \parallel DF$

$BE = DF$  as E is the midpoint of BC.

Therefore, BEFD is a parallelogram.

**6. In a parallelogram ABCD, E, and F are mid-points of the sides AB and CD respectively. The line segments AF and BF meet the line segments ED and EC at points G and H respectively. Prove that:**

**(i) Triangles HEB and FHC are congruent**

**(ii) GEHF is a parallelogram.**

**Solution:**

The figure is shown below:

(i) In  $\triangle HEB$  and  $\triangle HCF$ , we have

$$BE = FC \text{ [Given]}$$

$$\angle EHB = \angle FHC \text{ [Vertically opposite angles]}$$

$$\angle HBE = \angle HFC \text{ [Alternate angles]}$$

$\therefore \triangle HEB \cong \triangle HCF$  by ASA congruence criterion

$$\therefore EH = CH, BH = FH$$

(ii) Similarly,  $AG = GF$  and  $EG = DG \dots (1)$

In  $\triangle ECD$ , we have

F and H are the midpoints of CD and EC respectively

$$\text{Therefore, } HF \parallel DE \text{ and } HF = \frac{1}{2} DE \dots (2)$$

From (1) and (2), we get

$$HF = EG \text{ and } HF \parallel EG$$

Similarly, we can show

$$EH = GF \text{ and } EH \parallel GF$$

Therefore, GEHF is a parallelogram.

**7. In triangle ABC, D, and E are points on side AB such that  $AD = DE = EB$ . Through D and E, lines are drawn parallel to BC which meet side AC at points F and G respectively. Through F and G, lines are drawn parallel to AB which meet side BC at points M and N respectively. Prove that:  $BM = MN = NC$ .**

**Solution:**

The figure is shown below:

In  $\triangle AEG$ , we have

D is the midpoint of AE and  $DF \parallel EG \parallel BC$

Therefore, F is the midpoint of AG

$$\Rightarrow AF = GF \dots (1)$$

Again, we have  $DF \parallel EG \parallel BC$  and  $DE = BE$

$$\text{Therefore, } GF = GC \dots (2)$$

From (1) and (2), we get

$$AF = GF = GC$$

Similarly, as  $GN \parallel FM \parallel AB$  and  $AF = GF$

$$\text{Therefore, } BM = MN = NC$$

– Hence proved.

**8. In triangle ABC; M is the mid-point of AB, N is the mid-point of AC and D is any point in base BC. Use the intercept theorem to show that MN bisects AD.**

**Solution:**

The figure is shown below

As M and N are the midpoint of AB and AC respectively and  $MN \parallel BC$

Then according to the intercept theorem, we have

$$AM = BM,$$

$$\text{And therefore, } AX = DX.$$

– Hence proved

**9. If the quadrilateral formed by joining the mid-points of the adjacent sides of quadrilateral ABCD is a rectangle, show that the diagonals AC and BD intersect at a right angle.**

**Solution:**

The figure is shown below:

Let ABCD be a quadrilateral where P, Q, R, and S are the midpoints of AB, BC, CD, and DA.

And, PQRS is a rectangle

Diagonal AC and BD intersect at point O.

Required to show: AC and BD intersect at right angle.

Proof:

As  $PQ \parallel AC$ ,

$$\Rightarrow \angle AOD = \angle PXO \text{ [Corresponding angles] ... (1)}$$

Again, as  $BD \parallel RQ$ ,

$$\Rightarrow \angle PXO = \angle RQX = 90^\circ \text{ [Corresponding angle and angle of rectangle] ... (2)}$$

From (1) and (2), we get

$$\angle AOD = 90^\circ$$

Similarly,

$$\angle AOB = \angle BOC = \angle DOC = 90^\circ$$

Therefore, diagonals AC and BD intersect at a right angle

– Hence proved.

**10. In triangle ABC; D and E are mid-points of the sides AB and AC respectively. Through E, a straight line is drawn parallel to AB to meet BC at F. Prove that BDEF is a parallelogram. If AB = 16 cm, AC = 12 cm, and BC = 18 cm, find the perimeter of the parallelogram BDEF.**

**Solution:**

The figure is shown below:

As E is the midpoint of AC and  $EF \parallel AB$ , we have

Thus, F is the midpoint of BC and

$$2DE = BC \text{ or } DE = BF$$

Again, as D and E are midpoints, we have

$$DE \parallel BF \text{ and } EF = BD$$

Hence, BDEF is a parallelogram.

Now,

$$BD = EF = \frac{1}{2} AB$$

$$= \frac{1}{2} \times 16$$

$$= 8\text{cm}$$

$$BF = DE = \frac{1}{2} BC$$

$$= \frac{1}{2} \times 18$$

$$= 9\text{cm}$$

$$\text{Therefore, perimeter of BDEF} = 2(BF + EF) = 2(9 + 8) = 34\text{cm}$$

**11. In the given figure, AD and CE are medians and  $DF \parallel CE$ . Prove that:  $FB = \frac{1}{4} AB$**

**Solution:**

Considering that  $DF \parallel CE$  and AD and CE are medians

Based on the midpoint theorem, we are aware that

When two lines are parallel and a segment begins at one side's midway, the other point meets at the other side's midpoint.

Consider  $\triangle BEC$ ,

Given that  $DF \parallel CE$  and D is the midpoint of BC

So, F must be the midpoint of BE

$$\Rightarrow FB = \frac{1}{2} BE \dots (i)$$

$$\text{But, } BE = \frac{1}{2} AB$$

On substituting the value of BE in (i), we get

$$FB = \frac{1}{4} AB$$

– Hence Proved

**12. In parallelogram ABCD, E is the mid-point of AB and AP is parallel to EC which meets DC at point O and BC produced at P.**

**Prove that:**

$$(i) BP = 2AD$$

(ii) O is the mid-point of AP.

**Solution:**

Given ABCD is parallelogram,

$\Rightarrow AD = BC$  and  $AB = CD$

(i) Now, in  $\triangle APB$

Given, EC is parallel to AP and E is the midpoint of side AB

So, by the midpoint theorem,

C is the midpoint of BP

So,  $BP = 2BC$

But,

$BC = AD$  as ABCD is a parallelogram.

Hence,  $BP = 2AD$

(ii) In  $\triangle APB$ , we have

$AB \parallel OC$  as ABCD is a parallelogram

So, by the midpoint theorem

O is the midpoint of the AP

Hence Proved.

**13. In trapezium ABCD, sides AB and DC are parallel to each other. E is the mid-point of AD and F is the mid-point of BC. Prove that:  $AB + DC = 2EF$ .**

**Solution:**

In trapezium ABCD, we have

E and F are the midpoints of sides AD and BC respectively

We know that,  $AB = GH = IJ$

From the midpoint theorem, we have

$EG = \frac{1}{2} DI$  and

$HF = \frac{1}{2} JC$

Now, consider the L.H.S, we have

$$AB + CD = AB + CJ + JI + ID$$

$$= AB + 2HF + AB + 2EG$$

$$\text{So, } AB + CD = 2(AB + HF + EG)$$

$$= 2(EG + GH + HF)$$

$$= 2EF$$

$$\therefore AB + CD = 2EF$$

– Hence Proved.

**14. In  $\triangle ABC$ , AD is the median and DE is parallel to BA, where E is a point in AC. Prove that BE is also a median.**

**Solution:**

Given, that  $\triangle ABC$  and AD are the median

So, D is the midpoint of side BC

Also, given  $DE \parallel AB$

The midway theorem states that

E must be AC's midway.

Thus, the median is always the line that connects the vertex with the opposite side's midway.

Therefore, BE is also  $\triangle ABC$ 's median.

**15. Adjacent sides of a parallelogram are equal and one of the diagonals is equal to any one of the sides of this parallelogram. Show that its diagonals are in the ratio  $\sqrt{3}:1$ .**

**Solution:**

It is a rhombus if the parallelogram's neighboring sides are equal.

At this point, a rhombus's diagonals are perpendicular to one another and bisect one another.

Assume that x and y are the diagonals' lengths.

The length of the diagonal should match the rhombus's sides.

Now, in right-angled  $\triangle BOC$

By Pythagoras theorem,

$$OB^2 + OC^2 = BC^2$$

$$(y/2)^2 + (x/2)^2 = y^2$$

$$x^2/4 = y^2 - y^2/4$$

$$x^2/4 = (4y^2 - y^2)/4$$

$$x^2/4 = 3y^2/4$$

$$x^2 = 3y^2$$

$$x^2/y^2 = 3/1$$

$$x/y = \sqrt{3}/1$$

Thus, the diagonals are in the ratio  $\sqrt{3}: 1$

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