MATHS

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SQUARES AND SQUARE ROOTS

Topics To Be Covered

1. All Topics in One Short

BASIC INTRODUCTION

- Square: $n \times n \Rightarrow n^2$

$1^2 = 1 \times 1 = 1$ $\quad 3^2 = 3 \times 3 = 9$. $\quad 72^2 = 72 \times 72$

$2^2 = 2 \times 2 = 4$ $\quad 4^2 = 4 \times 4 = 16$. $\quad 89^2 = 89 \times 89$.

- Perfect Square Or Square Numbers:

$1^2 = 1$ $\quad 3^2 = 9$ $\quad 5^2 = 25$

$2^2 = 4$ $\quad 4^2 = 16$

- Is 42 a Perfect Square?

$6^2 = 36$

$7^2 = 49$

$\therefore 42$

SQUARE OF A NUMBER

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$

$6^2 = 36$

$7^2 = 49$

$8^2 = 64$

$9^2 = 81$

$10^2 = 100$

$11^2 = 121$

$12^2 = 144$

$13^2 = 169$

$14^2 = 196$

$15^2 = 225$

$16^2 = 256$

$17^2 = 289$

$18^2 = 324$

$19^2 = 361$

PROPERTIES OF PERFECT SQUARE

PROPERTY 1

- A Number ending with 2, 3, 7 or 8 is never a Perfect Square.

Question

The following numbers are obviously not perfect squares. Give reason.

(i) 1057 → ending with 7.

(ii) 64000

(iii) 23453 → 3

(iv) 89722 → 2

(v) 7928 → 8

(vi) 222000

→ There can never be an odd no. of zeroes in perfect Square.

PROPERTIES OF PERFECT SQUARE

PROPERTY 2

- If a number has 1 or 9 in the units place, then it's square ends in 1.
- If a number has 2 or 8 in the units place, then it's square ends in 4.
- If a number has 3 or 7 in the units place, then it's square ends in 9.
- If a number has 4 or 6 in the units place, then it's square ends in 6.
- If a number has 5 in the units place, then it's square ends in 5.

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$

$9^2 = 81$

$8^2 = 64$

$7^2 = 49$

$6^2 = 36$

$10^2 = 100$.

Question

What will be the unit digit of the squares of the following numbers?

(i) 1234 = 6

$4^2 = 16$

(ii) 272 = 4

$55^2 = 5$

(iii) 26387 = 9

$7^2 = 49$

$(100)^2 = 0$

(iv) 12796 = 6

(v) 52698 = 4

(vi) 3853 = 9

PROPERTIES OF PERFECT SQUARE

$(7000)^2 = 49000000$

PROPERTY 3

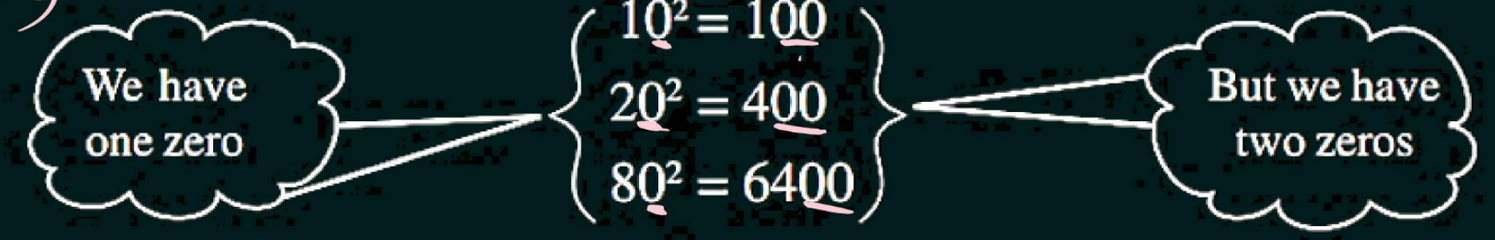
- A Number ending with an odd numbers of zeros is never a Perfect Square.

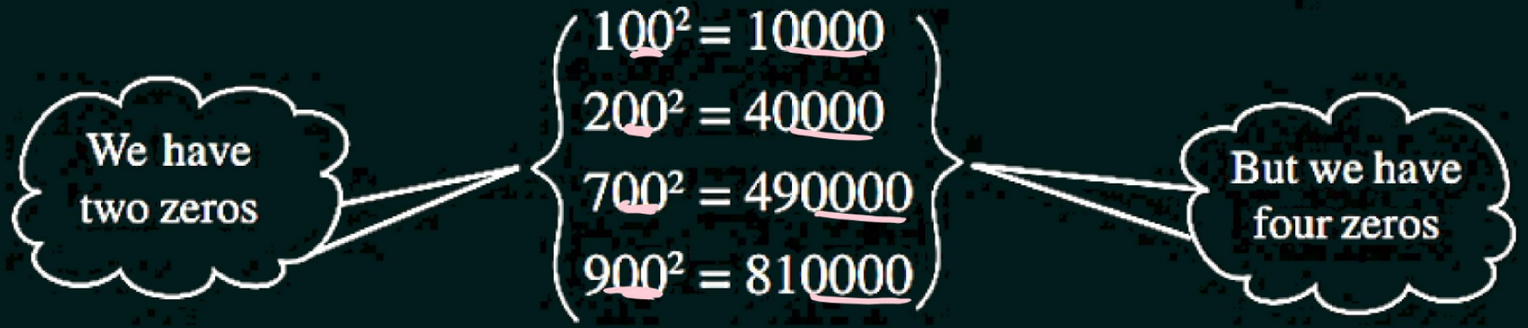
$(840000)^2$

$4 \times 2 = 8$

No. of zeroes = n

After squaring = 2n





PROPERTIES OF PERFECT SQUARE

PROPERTY 4

- The square of an even number is always an even number

Multiple of 2.

$2^2 = 4$

$16^2 = 256$

- The square of an odd number is always an odd number

$7^2 = 49$ $(29)^2 = 841$

Question

The squares of which of the following would be odd numbers?

(i) 431 → odd

(ii) 2826 → even

(iii) 7779 → odd

(iv) 82004 → even.

PROPERTIES OF PERFECT SQUARE

PROPERTY 5

* * * * *

- Every Perfect Square number can be expressed as sum of consecutive Odd Numbers Starting from 1

consecutive = লাগাতার

$1 = 1 = 1^2$

$1+3 = 4 = 2^2$

$1+3+5 = 9 = 3^2$

$1+3+5+7 = 16 = 4^2$

$1+3+5+7+9 = 25 = 5^2$

$1+3+5+7+9+11 = 36 = 6^2$

$\vdots$

$1+3+5+\cdots+n = n^2$.

Question

Express 121 as the sum of 11 odd numbers.

$1+3+5+7+9+11+13+15+17+19+21$

Question

Without adding, find the sum.

(i) $1 + 3 + 5 + 7 + 9 = 25$

(ii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 10^2 = 100$

(iii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 = 144$

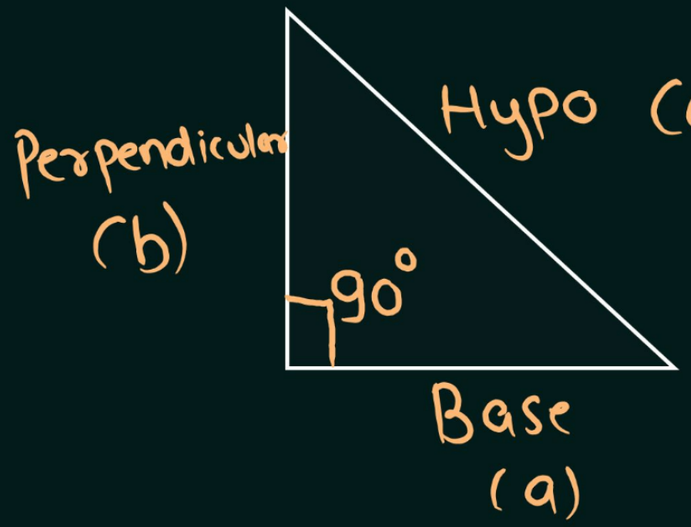
PROPERTIES OF PERFECT SQUARE

PROPERTY 6

- Three natural numbers a, b and c are said to form a pythagorean triplet if $a^2 + b^2 = c^2$

Note

For every natural number $m > 1$ we have $2m, m^2 - 1, m^2 + 1$ as Pythagorean triplet



$\Rightarrow c^2 = a^2 + b^2$

$5^2 = 3^2 + 4^2$

$25 = 9 + 16$

$25 = 25$

3, 4, 5

$m = 2$

$2m = 2 \times 2 = 4$

$m^2 - 1 = 2^2 - 1 = 3$

$m^2 + 1 = 2^2 + 1 = 5$

(3, 4, 5)

$m = 5$

$2 \times 5 = 10$

$5^2 - 1 = 24$

$5^2 + 1 = 26$

(10, 24, 26)

Question

Write a Pythagorean triplet whose one member is.

(i) 6 $\Rightarrow$ $2m = 6$

$m = 3$

$m^2 - 1 = 3^2 - 1 = 8$

$m^2 + 1 = 3^2 + 1 = 10$

6, 8, 10

(ii) 14 $\rightarrow$ $2m = 14$

$m = 7$

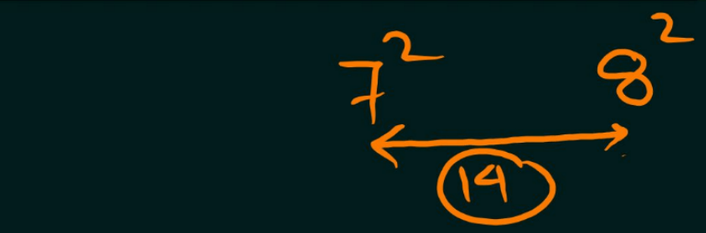
$m^2 - 1 = 7^2 - 1 = 48$

$m^2 + 1 = 7^2 + 1 = 50$

14, 48, 50

PROPERTIES OF PERFECT SQUARE

PROPERTY 7



- Between two consecutive square number n^2 and $(n+1)^2$, there are $2n$ non perfect square numbers.

Question

How many numbers lie between squares of the following numbers?

(i) 12^2 and 13^2 $= 2 \times 12 = 24$

(ii) 25^2 and 26^2 $= 2 \times 25 = 50$

(iii) 99^2 and 100^2 $= 2 \times 99 = 198$

PROPERTIES OF PERFECT SQUARE

PROPERTY 8

1 (+2) 3 (+3) 6 (+4) 10 (+5) 15 (+6) 21 (+7) 28 (+8) 36 (+9) 45

- If we combine two consecutive triangular numbers, we get a square number

1, 3 → $4 = 2^2$

3, 6 → $9 = 3^2$

6, 10 → $16 = 4^2$

10, 15 → $25 = 5^2$

15, 21 → $36 = 6^2$

21, 28 → $49 = 7^2$

28, 36 → 64

36, 45 → 81

PROPERTIES OF PERFECT SQUARE

PROPERTY 09

- we can express the square of any odd number as the sum of two consecutive positive integers.
- For any odd number n, Two Consecutive Positive integers are $\frac{n^2-1}{2}, \frac{n^2+1}{2}$

$\frac{120}{2}, \frac{122}{2}$

$60, 61$

$11^2 = 60 + 61 = 121 = (11)^2$.

$\frac{11^2-1}{2}, \frac{11^2+1}{2}$

$\frac{121-1}{2}, \frac{121+1}{2}$

Question

Express the following as the sum of two consecutive integers.

(i) $21^2 = \frac{21^2-1}{2}, \frac{21^2+1}{2}$

$= \frac{441-1}{2}, \frac{441+1}{2} \Rightarrow \frac{440}{2}, \frac{442}{2} \Rightarrow 220, 221$

$21^2 = 220 + 221$

$\Rightarrow 441 = (21)^2$

(ii) 19^2

Halwa Question - 1.

PROPERTIES OF PERFECT SQUARE

PROPERTY 10

Product of two consecutive even or odd natural numbers.

$(a + 1) \times (a - 1) = a^2 - 1.$

$13 \times 11 = 12^2 - 1 = 143$

$46 \times 44 = 2025 - 1 = 2024$

$15 \times 17 = 16^2 - 1 = 255$

$19 \times 21 = 20^2 - 1$
$= 400 - 1$
$= 399$

48×50
$= 49^2 - 1$
$= 2401 - 1$
$= 2400$

SOME PATTERNS IN SQUARE NUMBERS

PATTERN 1

The squares of numbers; 1, 11, 111 ... etc.

$1^2 = 1$

$11^2 = 121$

$111^2 = 12321$

$1111^2 = 1234321$

$11111^2 = 123454321$

$(1111111)^2 = 1234567654321$

SOME PATTERNS IN SQUARE NUMBERS

PATTERN 2

The squares of numbers; 11, 101, 1001, 10001 ... etc.

$11^2 = 121$

$101^2 = 10201$

$1001^2 = 1002001$

$10001^2 = 100020001$

$(10000001)^2$

10000000200000001

FINDING A SQUARE OF A NUMBER

Find the square of the following numbers without actual multiplication.

(i) $(39)^2 = (40-1)$

$= 40^2 - 2 \times 40 \times 1 + 1^2$

$= 1600 - 80 + 1$

$= 1520 + 1$

$= 1521$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

FINDING A SQUARE OF A NUMBER

Find the square of the following numbers without actual multiplication.

(ii) $(42)^2 = (40+2)^2$

$= (40)^2 + 2(40)(1) + 2^2$

$= 1600 + 80 + 4$

$= \underline{1684}$

FINDING A SQUARE OF A NUMBER

Find the squares of the following numbers containing 5 in unit's place.

(i) 45 = 2025

4×5

(ii) 95 = 9025

9×10

(iii) 105 = 11025

10×11

$(35)^2$

1225

$(75)^2$

5625

$(85)^2$

7225

Halwa question-2

$\Downarrow$

$(125)^2$

SQUARE ROOT

Radical Sign

$\sqrt[2]{1024}$

$(1024)^{1/2}$

$(32^2)^{1/2}$ $\{(a^m)^n = a^{m \times n}\}$

$32^{2 \times \frac{1}{2}}$

32

$\sqrt{2601}$

SQUARE ROOT THROUGH REPEATED SUBTRACTION

$1+3+5+7=16$

① $16-1=15$

② $15-3=12$

③ $12-5=7$

④ $7-7=0$

$\sqrt{16}=4$

$\sqrt{81}$

$81-1=80$

$80-3=77$

$77-5=72$

$72-7=65$

$65-9=56$

$56-11=45$

$45-13=32$

$32-15=17$

$17-17=0$

$\sqrt{81}=9$

SQUARE ROOT THROUGH PRIME FACTORISATION

2, 3, 5, 7, 11, 13, 17 . . .

- Is 225 a Perfect Square ? If so , Find the number whose Square is 225

$\sqrt{3\times3\times5\times5} = 3\times5$

$\sqrt{225} = 15$

$\Rightarrow \sqrt{3^2\times5^2}$

$= \sqrt{(3\times5)^2}$

$= \left[(3\times5)^2\right]^{1/2}$

$= 15$

3	225
3	75
5	25
5	5
	1

SQUARE ROOT THROUGH PRIME FACTORISATION

- Is 1024 a Perfect Square ? If so, Find the number whose Square is 1024.

$$\sqrt{1024} = \sqrt{2\times2\times2\times2\times2\times2\times2\times2\times2\times2}$$
$$= 2\times2\times2\times2\times2$$
$$= 32$$

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

SQUARE ROOT THROUGH PRIME FACTORISATION

- Is 17424 a Perfect Square ? If so, Find the number whose Square is 17424.

$$\sqrt{17424} = \sqrt{2\times2\times2\times2\times3\times3\times11\times11}$$

$$= 2\times2\times3\times11$$

$$= 4\times3\times11$$

$$= 12\times11$$

$$= 132$$

2	17424
2	8712
2	4356
2	2178
3	1089
3	363
11	121
11	11
	1

Question

Show 5808 is not a Perfect Square.

$$\sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 11 \times 11}$$

$$= 2 \times 2 \times 11 \sqrt{3}$$

$$= 44\sqrt{3}$$

2	5808
2	2904
2	1452
2	726
3	363
11	121
11	11
	1

Question

By what least number should 4732 be multiplied to get a Perfect Square Number?

7

2	4732^2
2	2366
7	1183
13	169
13	13

$\Rightarrow \sqrt{2 \times 2 \times \boxed{7} \times 13 \times 13 \times \boxed{7}}$

Question

By what least number should 2028 be ~~multiplied~~ divide. to get a Perfect Square Number?

2	2028
2	1014
3	507
13	169
13	13
	1

$$\sqrt{2 \times 2 \times \underset{3}{\boxed{3}} \times 13 \times 13}$$

Question

The students of Class VIII of a school donated RS 2401 in all, for Prime Minister's National Relief Fund. Each student donated as many rupees as the number of students in the class. Find the number of students in the class.

$$\sqrt{2401} = \sqrt{7 \times 7 \times 7 \times 7}$$

$$= 7 \times 7$$

$$= 49$$

Square Root Through Long Division Method

- Find The square root of 676 by Long Division Method.

$\sqrt{676} = 26$

$$\begin{array}{r} 3 \\ 46 \\ \times \quad 6 \\ \hline 276 \end{array}$$

$$\begin{array}{r|l} & 2\boxed{6} \\ \hline 2 & \overline{6}\ \overline{7}\ 6 \\ +2 & -4 \\ \hline 4\boxed{6} & 276 \\ & -276 \\ \hline & 000 \end{array}$$

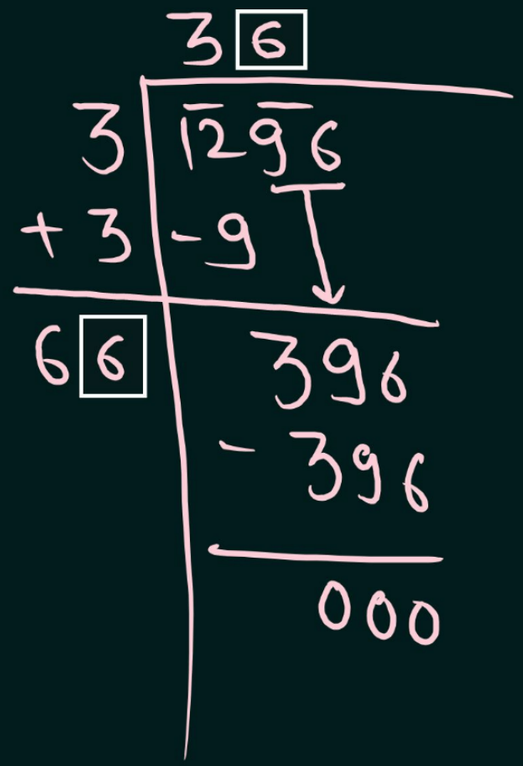
Square Root Through Long Division Method

- Find The square root of 1296 by Long Division Method.

$\sqrt{1296} = 36$

3
66
X 6
396

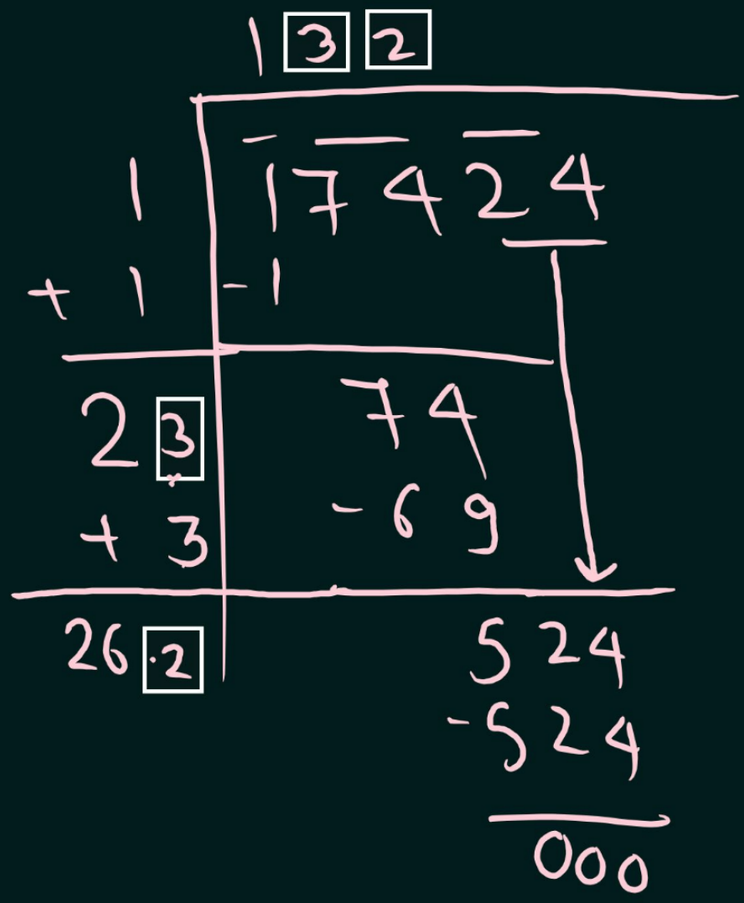
1
64
X 4
256



Square Root Through Long Division Method

- Find The square root of ~~7921~~ 17429 by Long Division Method.

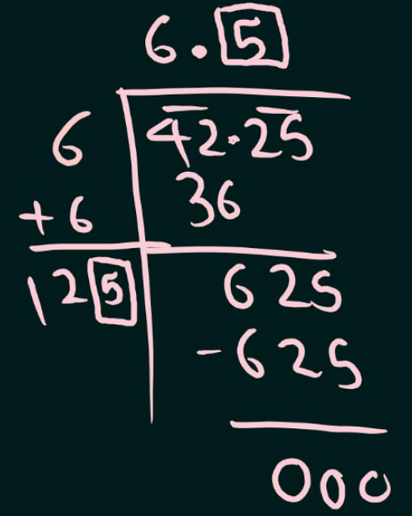
$$\sqrt{17424} = 132$$



Square Root Of Decimal Numbers

- Find The square root of 42.25 by Long Division Method.

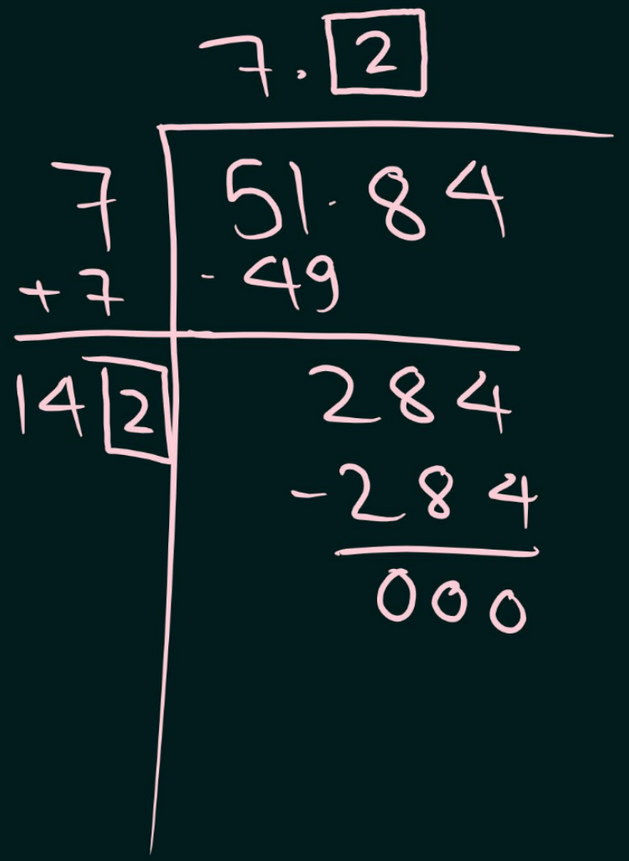
$\sqrt{42.25} = 6.5$



Square Root Of Decimal Numbers

- Find The square root of 51.84 by Long Division Method.

$$\sqrt{51.84} = 7.2$$



Thank You