CA FOUNDATION

MARATHON JUNE 2024

Quantitative Aptitude

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CA WALLAH

5 marks + 5 + 5

Index Numbers

Index Numbers

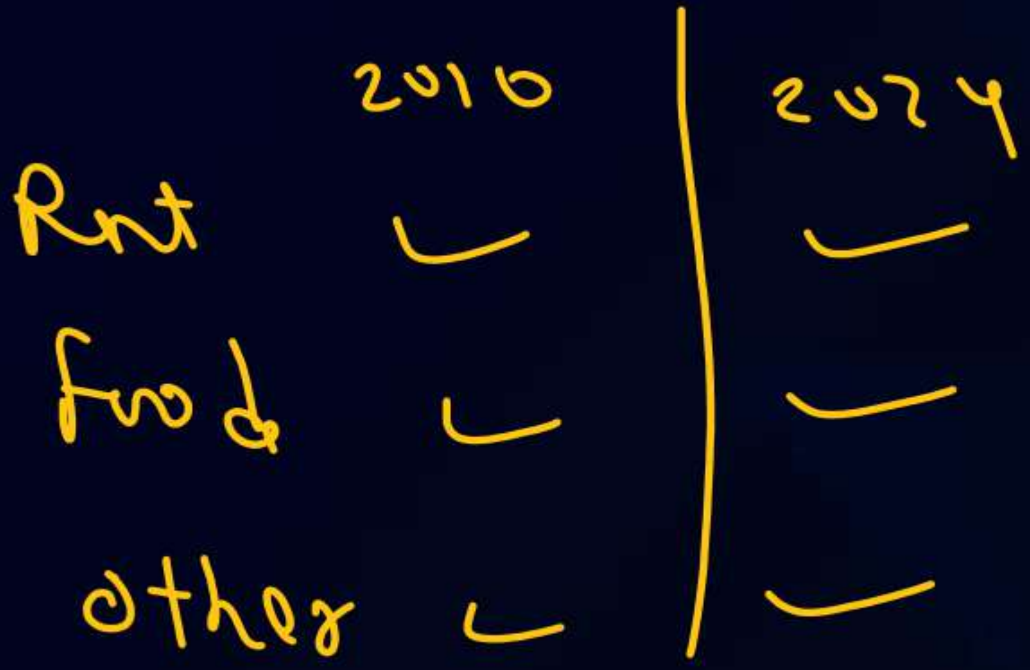
Method of evaluating changes in a variable
With respect to geographical location, time,
and other features

Index Number

- They Calculate the changes in variable

- Quantitative Expression

- Averages



Methods Of Constructing Price Index Numbers

Changes in Price

Methods
- Simple
  - Aggregating
  - Relative
- Weighted
  - Aggregative
  - Relative

Price index

⇓

P_{01}

0 = Base year

1 = current year

Simple Aggregative Method

	P_0 2010	P_1 2024
milk	18	32
Tomato	10	40
Potato	20	50
other	15	30
	63	152

$$P_{01} = \frac{\Sigma P_1}{\Sigma P_0} \times 100$$

$$P_{01} = \frac{152}{63} \times 100$$

$$= 241.26\%$$

Simple Relative Method

Simple Price Relative method

	P_0 2010	P_1 2024	$\frac{P_1}{P_0} \times 100$	$\log\left(\frac{P_1}{P_0} \times 100\right)$
A	15	30	$\frac{30}{15} \times 100 = 200$	$\log 200 = 2.3010$
B	20	50	$\frac{50}{20} \times 100 = 250$	$\log(250) = 2.3979$
C	8	12	$\frac{12}{8} \times 100 = 150$	$\log(150) = 2.1760$
D	16	64	$\frac{64}{16} \times 100 = 400$	$\log(400) = 2.6020$
			1000	9.4769

P_{01}

Arithmetic mean method ⇓

$$P_{01} = \frac{\Sigma\left(\frac{P_1}{P_0}\times 100\right)}{N}$$

$$= \frac{1000}{4}$$

$$= 250\%$$

Geometric mean method ⇓

$$P_{01} = AL\left[\frac{\Sigma \log\left(\frac{P_1}{P_0}\times 100\right)}{N}\right]$$

$$= AL\left(\frac{9.4769}{4}\right) = 234\%$$

Log x

→ $\sqrt{\ }$ 19 times

→ -1

→ $\times 227695$

AL

→ $\div 227695$

→ $+1$

→ $\boxed{X =}$ 19 times

Price relative is-

(a) $\frac{P_1}{P_0} \times 100$

(b) P

(c) P_0

(d) $\frac{P_1}{P_0}$

Weighted Aggregative Method

$$P_{01} = \frac{\Sigma p_1 q}{\Sigma p_0 q} \times 100$$

Laspeyre

$$P_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$$

Paasche

$$P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$$

Dorbish & Bowley

$$P_{01} = \frac{L + P}{2}$$

Eg $P_{01}(L) = 180$

$P_{01}(P) = 240$

find P_{01} of Dorbish & Bowley

Sol: $P_{01} = \frac{180 + 240}{2} = 210$

Fishers

$$P_{01} = \sqrt{L \times P}$$

Geometric mean

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$$

Marshall & Edgeworth

$$P_{01} = \frac{\Sigma P_1 \left(\frac{q_0+q_1}{2}\right)}{\Sigma P_0 \left(\frac{q_0+q_1}{2}\right)} \times 100$$

$$P_{01} = \frac{\Sigma P_1 q_0 + \Sigma P_1 q_1}{\Sigma P_0 q_0 + \Sigma P_0 q_1} \times 100$$

From the following data base year:

Commodity	Base year		Current year	
	Price	Quantity	Price	Quantity
A	4	3	6	2
B	5	4	6	4
C	7	2	9	2
D	2	3	1	5

Fisher's Ideal Index is

(a) 117.30 (b) 115.43

(c) 118.35 (d) 116.48 (1 mark)

Fisher

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$$

$$= \sqrt{\frac{63}{52} \times \frac{59}{52}} \times 100$$

$$= \sqrt{1.3746} \times 100$$

$$= 117.24$$

p_0	q_0	p_1	q_1	$p_1 q_0$	$p_1 q_1$	$p_0 q_0$	$p_0 q_1$
4	3	6	2	18	12	12	8
5	4	6	4	24	24	20	20
7	2	9	2	18	18	14	14
2	3	1	5	3	5	6	10
				63	59	52	52

Product	2010 Price	2010 Value	2020 Quantity	2020 Value
A	6	24	7	42
B	4	20	6	48
C	5	50	8	64

find Price index

i> Laspeyre ii> Paasche iii> fishers

Sol:

	2010 P_0	2010 q_0	2020 P_1	q_1	P_0q_0	P_0q_1	P_1q_0	P_1q_1
A	6	4	6	7	24	42	24	42
B	4	5	8	6	20	24	40	48
C	5	10	8	8	50	40	80	64
					94	106	144	154

Laspeyre

$$P_{01} = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100$$

$$= \frac{144}{94} \times 100 = 153.19$$

Paasche

$$P_{01} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \times 100$$

$$= \frac{154}{106} \times 100 = 145.28$$

fishers

$$P_{01} = \sqrt{L \times P}$$

$$= 149.18$$

Weighted Price Relative method

⇓

Arithmetic mean ⇓

$$P_{01} = \frac{\sum W_i \left(\frac{P_1}{P_0} \times 100\right)}{\sum W}$$

Geometric mean ⇓

$$P_{01} = AL\left[\frac{\sum W_i \log\left(\frac{P_1}{P_0} \times 100\right)}{\sum W}\right]$$

$$W = P_0 q_0$$

Eg

	P_0	P_1	w_i	$\frac{P_1}{P_0}\times 100$	$w_i\left(\frac{P_1}{P_0}\times 100\right)$
A	10	18	40	180	7200
B	12	28	35	233.33	8166.55
C	15	30	25	200	5000
			100		20366.55

$$P_{01} = \frac{\sum w_i\left(\frac{P_1}{P_0}\times 100\right)}{\sum w_i}$$

$$= \frac{20366.55}{100} = 203.66$$

If Fisher's index number is 160 and paasche's index number is 140 laspeyre's Index is :

(a) 187.77
(b) 182.86
(c) 183.25
(d) 186.25

(1 mark)

$f = 160$ & $P = 140$

$L = ?$

$f = \sqrt{L \times P}$

$160 = \sqrt{L \times 140}$

$(160)^2 = L \times 140$

$L = \frac{(160)^2}{140}$

$L = 182.85$

Quantity Index (Volume Index)

Quantity index numbers measure the change in the quantity or volume of goods sold, consumed or produced during a given time period

Denoted by Q_{01}

methods

- Simple
  - Aggregative
  - Relative
- Weighted
  - Aggregative
  - Relative

Simple Aggregative Quantity Index

$$Q_{01} = \frac{\Sigma Q_1}{\Sigma Q_0} \times 100$$

Simple Relative Quantity index

AM — GM

$$Q_{01} = \frac{\Sigma \left(\frac{Q_1}{Q_0} \times 100 \right)}{N}$$

$$Q_{01} = AL \left[\frac{\Sigma \log \left(\frac{Q_1}{Q_0} \times 100 \right)}{N} \right]$$

Weighted Aggregative Quantity index

Laspeyre

$$Q_{01} = \frac{\Sigma Q_1 P_0}{\Sigma Q_0 P_0} \times 100$$

or

$$\frac{\Sigma P_0 Q_1}{\Sigma P_0 Q_0}$$

Paasche

$$Q_{01} = \frac{\Sigma Q_1 P_1}{\Sigma Q_0 P_1} \times 100$$

or

$$\frac{\Sigma P_1 Q_1}{\Sigma P_1 Q_0} \times 100$$

Fishers

$$Q_{01} = \sqrt{L \times P}$$

Value Index

The value index number compares the value of a commodity in the current year, with its value in the base year

$$V_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0} \times 100$$

2013

p_0	q_0
10	15

2024

p_1	q_1
18	10

$L \Rightarrow \frac{270}{150} \times 100 = 180$

$P \Rightarrow \frac{180}{100} \times 100 = 180$

$V_{01} = \frac{180}{150} \times 100 = 120\%$

Test of Adequacy of Index Numbers

- Unit Test
- Time Reversal Test
- Factor Reversal Test
- Circular Test

1)Unit Test

A unit test in index numbers is a test that ensures that an index number formula doesn't change the value of the index number even if the units of price or quantities change

-All methods satisfy this test
Except Simple aggregative method does

	P_0	P_1	$\frac{P_1}{P_0}\times 100$
A	10	20	200
B	20	60	300
C	15	30	200
	45	110	700

Simple aggregate method

$$P_{01} = \frac{\Sigma P_1}{\Sigma P_0}\times 100$$

$$= \frac{110}{45}\times 100$$

$$= 244.44\%$$

Simple Relative

$$P_{01} = \frac{\Sigma\left(\frac{P_1}{P_0}\times 100\right)}{N}$$

$$= \frac{700}{3}$$

$$= 233.33\%$$

P_0	P_1	$\frac{P_1}{P_0}\times 100$
10	20	200
40	120	300
60	120	200
110	260	700

$$P_{01} = \frac{\Sigma P_1}{\Sigma P_0}\times 100 = \frac{260}{110}\times 100$$

$$= 236.36$$

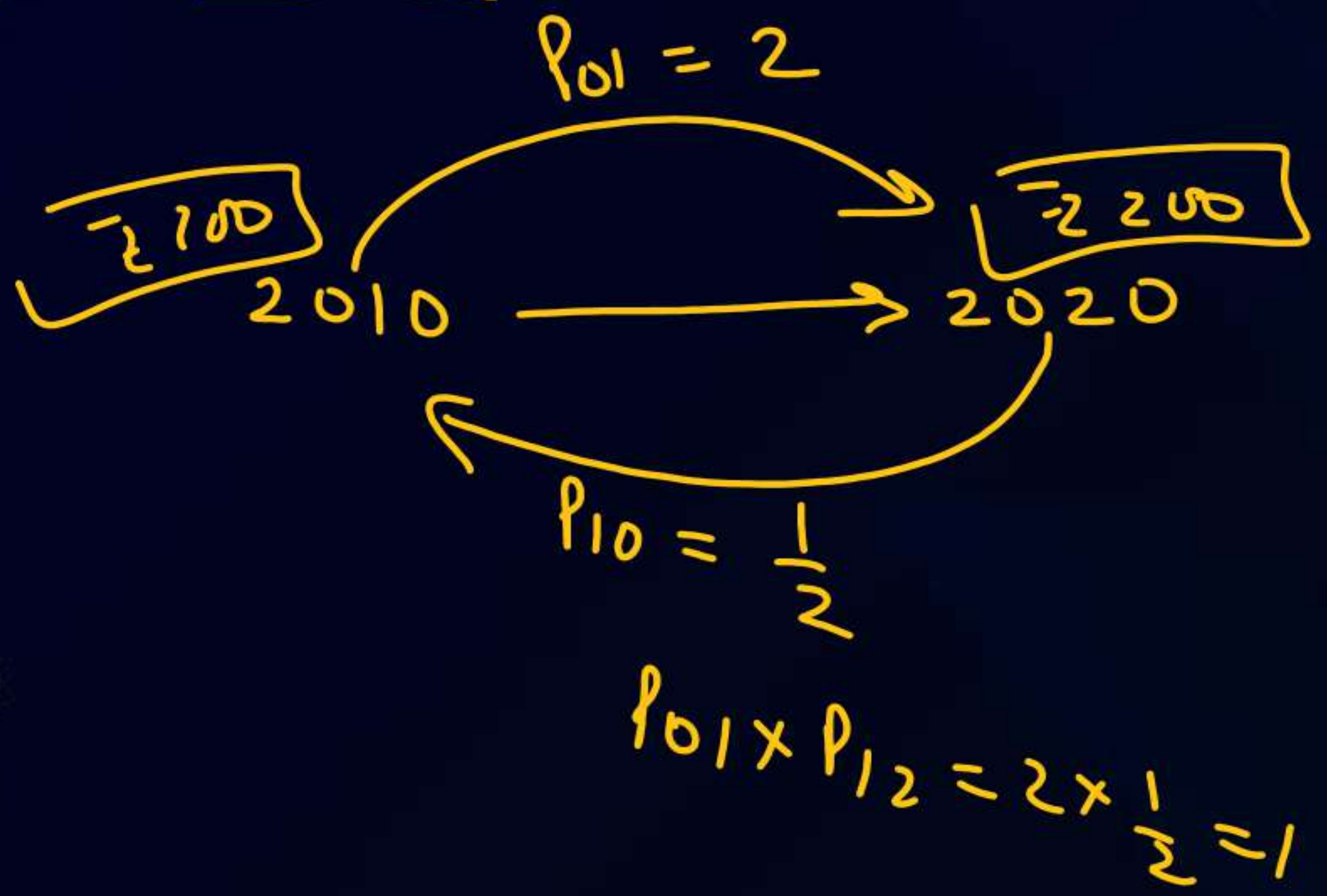
$$P_{01} = \frac{\Sigma\left(\frac{P_1}{P_0}\times 100\right)}{N} = \frac{700}{3}$$

$$= 233.33$$

2)Time Reversal Test

Time reversal test is a test to determine whether a given method will work both ways in time, forward and backward

$$P_{01} \times P_{10} = 1$$



This test is satisfied By

- Fisher's Method
- Simple Geometric Mean price Relative
- Weighted Geometric Of Price Relative
- Marshal Edgeworth

Laspeyre

$$P_{01} = \frac{\sum p_1 q_0}{\sum p_0 q_0}$$

$$P_{10} = \frac{\sum p_0 q_1}{\sum p_1 q_1}$$

Now

$$P_{01} \times P_{10} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_0 q_1}{\sum p_1 q_1}$$

$$\neq 1$$

Fisher

$$P_{01} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}}$$

$$P_{10} = \sqrt{\frac{\sum p_0 q_1}{\sum p_1 q_1} \times \frac{\sum p_0 q_0}{\sum p_1 q_0}}$$

Now

$$P_{01} \times P_{10} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1} \times \frac{\sum p_0 q_1}{\sum p_1 q_1} \times \frac{\sum p_0 q_0}{\sum p_1 q_0}}$$

$$= \sqrt{1}$$

$$= 1$$

3)Factor Reversal Test

According to this test the product of a price index and the quantity index should be equal to the corresponding value index

$$P_{01} \times Q_{01} = V_{01}$$

This test is satisfied By

- Fisher's Method

$$P_{01} \times Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

⇓ Ideal Price index

Laspeyres

$$P_{01} = \frac{\sum p_1 q_0}{\sum p_0 q_0}$$

$$Q_{01} = \frac{\sum p_0 q_1}{\sum p_0 q_0}$$

$$P_{01} \times Q_{01} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_0 q_1}{\sum p_0 q_0}$$

$$\neq \frac{\sum p_1 q_1}{\sum p_0 q_0}$$

✗

Fishers

$$P_{01} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}}$$

$$Q_{01} = \sqrt{\frac{\sum p_0 q_1}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_1 q_0}}$$

$$P_{01} \times Q_{01} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1} \times \frac{\sum p_0 q_1}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_1 q_0}}$$

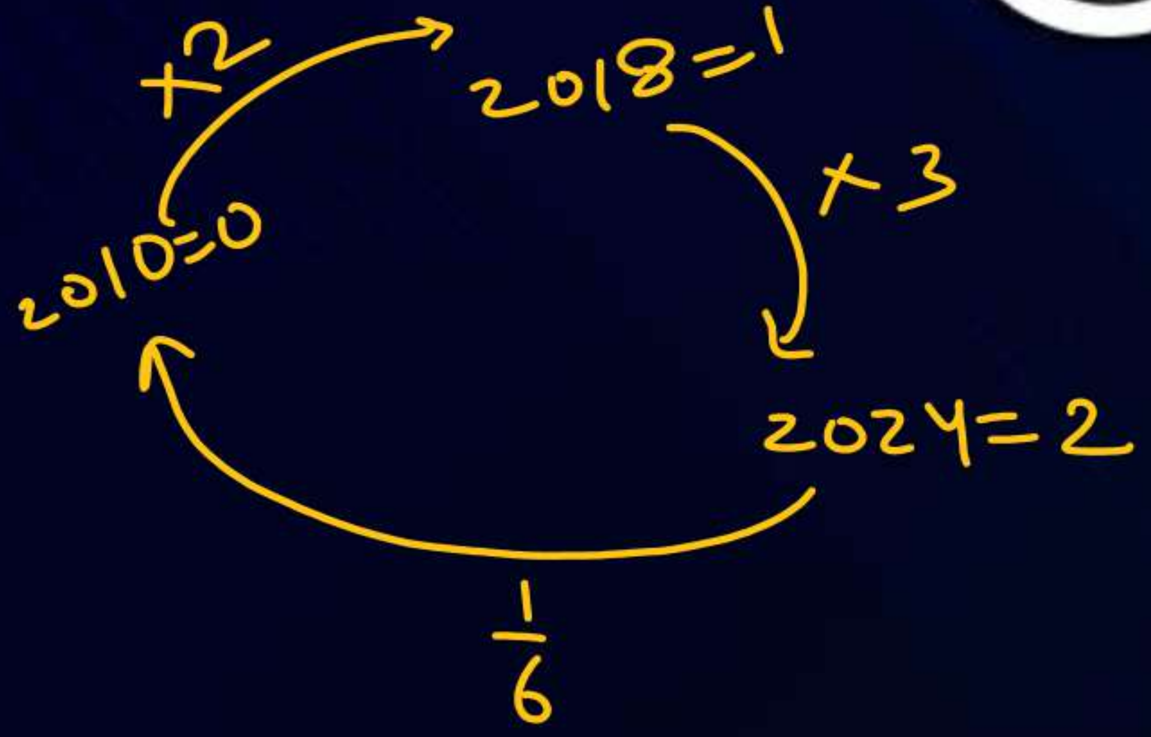
$$= \sqrt{\left(\frac{\sum p_1 q_1}{\sum p_0 q_0}\right)^2}$$

$$= \frac{\sum p_1 q_1}{\sum p_0 q_0} = V_{01}$$

4)Circular Test

This is the extension of Time reversal

$$P_{01} \times P_{12} \times P_{20} = 1$$



- Circular Test is met by simple geometric mean of price relatives &
- weighted aggregative with fixed weights.

Fisher's index does not satisfy following test.

(a) Unit Test

(b) Time Reversal Test

(c) Circular Test

(d) Factor Reversal Test

Link Relative & Chain Base Index

$$\text{Link Relative} = \frac{\text{Price of current year}}{\text{Price of previous year}} \times 100$$

$$\text{Chain Index} = \text{Chain index of previous year} \times \frac{\text{Link Relative of current year}}{100}$$

Year	Price	Link Relative	Chain index
Base ← 2010	20	100	100
2011	25	$\frac{25}{20} \times 100 = 125$	$100 \times 125\% = 125$
2012	30	$\frac{30}{25} \times 100 = 120$	$125 \times 120\% = 150$
2013	45	$\frac{45}{30} \times 100 = 150$	$150 \times 150\% = 225$
2014	72	$\frac{72}{45} \times 100 = 160$	$225 \times 160\% = 360$

Year	Link Relative	Chain index
2010	100	100
2011	120	$100 \times \frac{120}{100} = 120$
2012	130	$120 \times \frac{130}{100} = 156$
2013	110	$156 \times \frac{110}{100} = 171.6$
2014	145	$171.6 \times \frac{145}{100} = 248.82$

Base Shifting

$$\text{Shifted Price index} = \frac{\text{origin Price index}}{\text{Price index of New Base year}} \times 100$$

Year	Index (Base = 2010)	Shifted Price index (Base - 2012)
2010 (old Base year)	100	$\frac{100}{125} \times 100 = 80$
2011	110	$\frac{110}{125} \times 100 = 88$
2012 (new Base year)	125	→ 100 = 100
2013	190	$\frac{190}{125} \times 100 = 152$
2014	240	$\frac{240}{125} \times 100 = 192$
2015	300	$\frac{300}{125} \times 100 = 240$

Splicing Of Index Number

⇓

merging
of two
Different index series

Year	Index-A	Index-B
2012	100	.
2013	90	.
2014	125	100
2015	.	120
2016	.	150

spling

A to B

$\frac{100}{125} \times 100 = 80$

$\frac{100}{125} \times 90 = 72$

100

120

150

B to A

100

90

125

$\frac{125}{100} \times 120 = 150$

$\frac{125}{100} \times 150 = 187.5$

Deflation

Income = ₹1000

Year	Price Index	Income	Real Income	Basmati Rice
2010	100	₹1000	₹1000	₹100/kg
2011	120	₹1200	$\frac{1200}{120} \times 100 = 1000$	
2012	150	₹1300	$\frac{1300}{150} \times 100 = 866.66$	

Quantity $= \frac{1000}{100} = 10\text{kg}$

Income = ₹1000

Rice ⇒ ₹120/kg

Quantity ⇒ $\frac{₹1000}{₹120/kg} = 8.33\text{kg}$

10kg → 10kg

$\frac{1300}{150} = 8.66\text{kg}$

$\frac{1200}{120} = 10\text{kg}$

Deflating

$$\text{Real Wage/Deflated Value} = \frac{Current\ Value}{Price\ Index} \times 100$$

$$\text{Purchasing Power Of Money} = \frac{1}{Price\ Index} \times 100$$

Year	wages	Index	Real wages	Purchasing Power of money
2012	180	100	$\frac{180}{100} \times 100 = 180$	$\frac{1}{100} \times 100 = 1$
2013	208	120	$\frac{208}{120} \times 100 = 173.33$	$\frac{1}{120} \times 100 = 0.83$
2014	225	125	$\frac{225}{125} \times 100 = 180$	$\frac{1}{125} \times 100 = 0.8$
2015	247	140	$\frac{247}{140} \times 100 = 17642$	—
2016	316	180	$\frac{316}{180} \times 100 = 175.55$	—
2017	330	200	$\frac{330}{200} \times 100 = 165$	—

Consumer Price Index (C.P.I.)

(Cost of Living Index)

It measures how much the consumer of a particular class have to pay more/less for a certain basket of goods and services in a given period with respect to the base period

Methods

- Total Expenditure Method
- Family Budget Method

Aggregate Expenditure Method

$$C.P.I = \frac{\text{Total Exp in C.Y.}}{\text{Total Exp in B.Y.}} \times 100$$

or

$$CPI = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$$

(Laspeyre)

Family Budget Method

$$C.P.I = \frac{\Sigma W_i \left(\frac{P_1}{P_0} \times 100 \right)}{\Sigma W_i}$$

$\frac{\Sigma WI}{\Sigma W}$

weight Price Relative (A.M.)

When $W_i = P_0 q_0$

Eg

	Index	weight
food	120	30
Rent	110	50
other	115	20

find consumer price index.

Sol:

I	W	IW
120	30	3600
110	50	5500
115	20	2300
	100	11400

$$CPI = \frac{\Sigma W_i I_i}{\Sigma W_i}$$

$$= \frac{11400}{100}$$

$$CPI = 114$$

QUESTION

CA

During the certain period the C.L.I. goes up from 110 to 200 and the Salary of a worker is also raised from 330 to 500, then the real terms is

(a) Loss by ₹ 50 (b) Loss by 75 (c) Loss by ₹ 90 (d) None of these.

Index	Salary	Real Salary
110	330	$\frac{330}{110} \times 100 = 300$
200	500	$\frac{500}{200} \times 100 = 250$

Loss of ₹ 50

QUESTION

Net Monthly income of an employee was ₹ 800 in 1980. The consumer price Index number was 160 in 1980. It is rises to 200 in 1984. If he has to be rightly compensated. The additional dearness allowance to be paid to the employee is :

(a) ₹ 240 (b) ₹ 275 (c) ₹ 250 (d) None of these.

Year	Income	Index
1980	₹ 800	160
1984	x	200

$$\frac{800}{x} = \frac{160}{200} \Rightarrow x = 1000$$

$$1000 - 800 = 200$$

QUESTION

CA

With the base year 1960 the C. L. I. in 1972 stood at 250. x was getting a monthly Salary of ₹ 500 in 1960 and ₹ 750 in 1972. In 1972 to maintain his standard of living in 1960 x has to receive as extra allowances of

(a) ₹ 600/- (b) ₹ 500/- (c) ₹ 300/- (d) none of these.

Year	Index	Salary
1960	100	500
1972	250	x

$$\frac{100}{250} = \frac{500}{x} \Rightarrow x = 1250$$

$$\begin{array}{r} 1250 \\ -\ 750 \\ \hline 500 \end{array}$$

QUESTION

An Index number constructed to measure the relative change in the price of an item or a group of item is called:

(a) Quantity index number

(b) Price index number

(c) Volume index number

(d) Composite index number

(1 mark)

QUESTION

The Index number of prices for a country at a given date in 250. In comparison to the base period price the price of all commodities in the country has increase by ____________ times.

(a) 1.25 (b) 1.5

(c) 2 (d) 2.5 (1 mark)

Index

Base 100

C.Y. 250

$\frac{250}{100} = 2.5$

QUESTION

Which of the following index is computed taking the average of base year and current year?

(a) Marshall- Edgeworth's index
(b) Paasche's index
(c) Laspeyre's Index
(d) Fisher's index

(1 mark)

L

$$\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}$$

P

$$\frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}$$

F

$$\sqrt{L \times P}$$

Marshall

$$\frac{\Sigma p_1 \left(\frac{q_0+q_1}{2}\right)}{\Sigma p_0 \left(\frac{q_0+q_1}{2}\right)}$$

QUESTION

Geometric mean method used in which index number to find it out .

(a) Laspeyres (b) Paasches

(c) Fishers index Number (d) None (1 mark)

QUESTION

Index numbers are not helpful in

(a) Framing economics policies

(b) Revealing trend

(c) Forecasting

(d) Identifying errors

QUESTION

Index Numbers are expressed as

(a) Squares

(b) Ratio

(c) Percentages

(d) Combinations

QUESTION

CA

________ play a very important part in the construction of index numbers.

a) weights   b) classes   c) estimations   d) none

QUESTION

The ________ makes index numbers time-reversible.

a) A.M. b) G.M. c) H.M. d) none

Time Reversal

→ Fisher (GM)

→ Simple Relative (GM)

→ Weighted Relative (GM)

QUESTION

The ________ of group indices gives the General Index

a) H.M. b) G.M. c) A.M. d) none

	I	w_i	$I_i w_i$
food index	110	40	
Edu	115	50	
other Index	140	10	

Weighted Relative (AM)

$$\text{General} = \frac{\sum I_i w_i}{\sum w_i}$$

QUESTION

The total value of retained imports into India in 1960 was ₹ 71.5 million per month. The corresponding total for 1967 was ₹ 87.6 million per month. The index of volume of retained imports in 1967 composed with 1960 (= 100) was 62.0. The price index for retained inputs for 1967 our 1960 as base is

(a) 198.61 (b) 197.61 (c) 198.25 (d) None of these.

Year	Imports	Quantity Index	Price index
1960	71.5	100%	100%
1967	87.6	62%	?

Factor Reversal

$$P_{01} \times Q_{01} = V_{01}$$

$$P_{01} \times \frac{62}{100} = \frac{87.6}{71.5}$$

$$P_{01} = 1.9760$$

or

197.60%

$P_{01} = 162\%$

$Q_{01} = 150$

$V_{01} = ?$

$P_{01} \times Q_{01} = V_{01}$

$\frac{162}{100} \times \frac{150}{100} = V_{01}$

$V_{01} = 2.43$ or

$= 243\%$

QUESTION

CA

The consumer price Index for April 1985 was 125. The food price index was 120 and other items index was 135. The percentage of the total weight index given to food is

(a) 66.67 (b) 68.28 (c) 90.25 (d) None of these.

C.P.I = 125

food index = 120

other index = 135

	(I) Index	(W) Weight	(I)(W)
food	120	x	$120x$
other	135	$(100-x)$	$13500-135x$
		100	$13500-15x$

$$C.P.I = \frac{\Sigma WI}{\Sigma W}$$

$$125 = \frac{13500-15x}{100}$$

$$12500 = 13500 - 15x$$

$$15x = 1000$$

$$x = 66.66\%$$

QUESTION

CA

Purchasing Power of Money is

(a) Reciprocal of price index number.

(b) Equal to price index number.

(c) Unequal to price index number.

(d) None of these.

QUESTION

If the price index for the year, say 1960 be 110.3 and the price index for the year, say 1950 be 98.4, then the purchasing power of money (Rupees) of 1950 in 1960 is

(a) ₹ 1.12 (b) ₹ 1.25 (c) ₹ 1.37 (d) None of these.

Year	Price
1950	98.4
1960	110.3

Price index of 1950 if 1960 is the base $= \frac{98.4}{110.3} \times 100 = 89.2112$

Purchasing power of money

$= \frac{1}{89.2112} \times 100$

$= 1.12$

QUESTION

CA

When the cost of Tobacco was increased by 50%, a certain hardened smoker, who maintained his formal scale of consumption, said that the rise had increased his cost of living by 5%. Before the change in price, the percentage of his cost of living was due to buying Tobacco is

(a) 15% (b) 8% (c) 10% (d) None of these.

Tobacco x

Other y

Cost of Living 100

$x + y = 100$

New Price of Tobacco $= x + x \times \frac{50}{100}$

$= 1.5x$

New cost of Living $= 105$

So $1.5x + y = 105$

$1.5x + y = 105$

$x + y = 100$

$0.5x = 5$

$x = \frac{50}{5}$

$x = 10$


THANK YOU

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$$Equation$$

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Probability

The Chances Of Occurrence of an Event

Subjective Probability -Dependent on personal judgement and experience

Objective Probability- Chances are based on recorded data, facts or Collected data

Subjective probability may be used in

A. Accountancy

B. *Mathematics*

C. Statistics

D. Management

Random Experiment

⇓

An experiment which has more than one possible outcomes & exact result can not be predicted

Eg A Dice is thrown once

$S = \{1, 2, 3, 4, 5, 6\}$

$6^1 = 6$

$6^2 = 36$

$6^3 = 216$

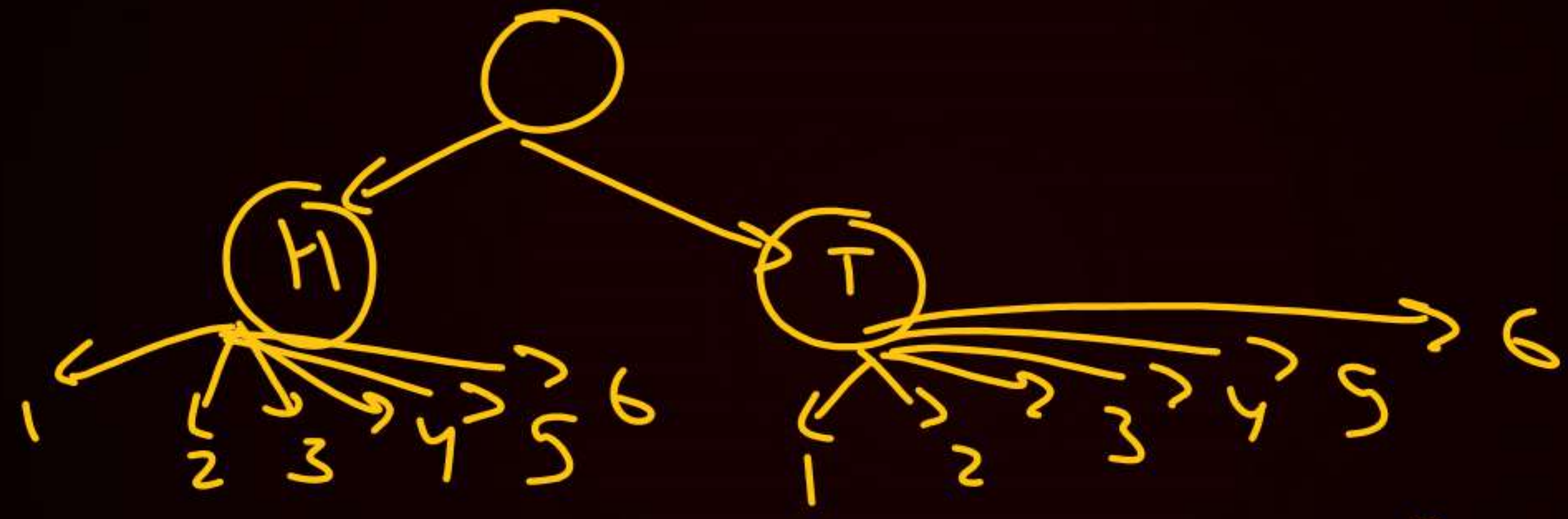
$6^n = ?$

Eg A dice is thrown twice

$$S = \begin{Bmatrix} (11) & (12) & (13) & (14) & (15) & (16) \\ (21) & (22) & (23) & (24) & (25) & (26) \\ (31) & (32) & (33) & (34) & (35) & (36) \\ (41) & (42) & (43) & (44) & (45) & (46) \\ (51) & (52) & (53) & (54) & (55) & (56) \\ (61) & (62) & (63) & (64) & (65) & (66) \end{Bmatrix}$$

Q A coin is tossed & then a dice is thrown

Sol:



$$S = \begin{Bmatrix} H1, H2, H3, H4, H5, H6 \\ T1, T2, T3, T4, T5, T6 \end{Bmatrix}$$

Events

Event ⇒ It is a subset of sample space

eg A coin is tossed once.

Sample = {H, T}

<u>Subsets</u>

$E_1 = \{\}$ $E_3 = \{T\}$

$E_2 = \{H\}$ $E_4 = \{H, T\}$

eg A dice is thrown once

Sample = {1, 2, 3, 4, 5, 6}

Events

E_1: Even numbers = {2, 4, 6}

E_2: odd numbers = {1, 3, 5}

E_3: Prime numbers = {2, 3, 5}

E_4: even Prime no = {2}

Simple events
(Elementary event)

⇓

An event which has exactly one element

eg $E_1 = \{H\}$

eg $E_2 = \{5\}$

eg $E_3 = \{(HT)\}$

Compound event
(Composite event)

⇓

An event is composite event if it contains more than one element

eg $E_1 = \{H, T\}$

eg $E_2 = \{1, 3, 5\}$

Empty event (Impossible event)

⇓

which does not contain any element

$P(\text{Impossible event}) = 0$

Q A dice is thrown twice

E_1: Sum of numbers on two Dice is 12 $= \{(6 6)\}$ = Simple event

E_2: Sum of numbers on two Dice is 10 $= \{(6 4), (5 5), (4 6)\}$ = compound event

E_3: Sum of numbers on two Dice is 13 $= \{\ \} = \phi$

Sure event

⇓

An event which contain all elements of sample space

E = sample space

$$P(E) = 1$$

Eg A dice is thrown once

$S = \{1, 2, 3, 4, 5, 6\}$

E_1: Number on dice is less than 10

$= \{1, 2, 3, 4, 5, 6\}$

CA WALLAH

$$0 \le P(E) \le 1$$

$P(E) = 0$

Impossible event

$P(E) = 1$

Sure event

Compliment of an event

⇓

Non occurrent of an event

Denoted by $\bar{A}$ or A'

$$P(A) + P(\text{Not } A) = 1$$

$$P(\text{Not } A) = 1 - P(A)$$

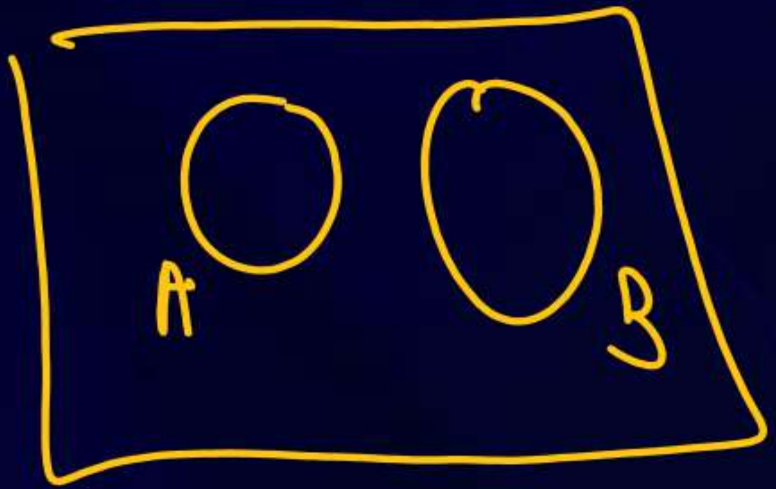
$$P(\bar{A}) = 1 - P(A)$$

Mutually exclusive events (Incompatible events)

⇓

two or more events are mutually exclusive if only one event can be selected at one time

& selection of one event results rejection of other events.



$A \cap B = \phi$

$P(A \cap B) = 0$

Q A dice is thrown once

i) E_1 = odd numbers = $\{1, 3, 5\}$

E_2 = even numbers = $\{2, 4, 6\}$

$E_1 \cap E_2 = \phi$ M. exclusive events

ii) E_1: Numbers less than 5 = $\{1, 2, 3, 4\}$

E_2: Prime numbers = $\{2, 3, 5\}$

$E_1 \cap E_2 = \{2, 3\}$

Not mutually exclusive.

Mutually Exhaustive Events

Two or more events are m. exhaustive if their union makes sample space.

$$E_1 \cup E_2 = S$$

$$P(E_1) + P(E_2) = 1$$

Eg A dice is thrown once

Sample $= \{1, 2, 3, 4, 5, 6\}$

E_1 = even numbers $= \{2, 4, 6\}$

E_2 = odd numbers $= \{1, 3, 5\}$

$E_1 \cup E_2 = \{2, 4, 6, 1, 3, 5\}$

$E_1 \cup E_2 = S$

E_1 & E_2 are m. exhaustive.

CA WALLAH

Q. A coin is tossed two times.

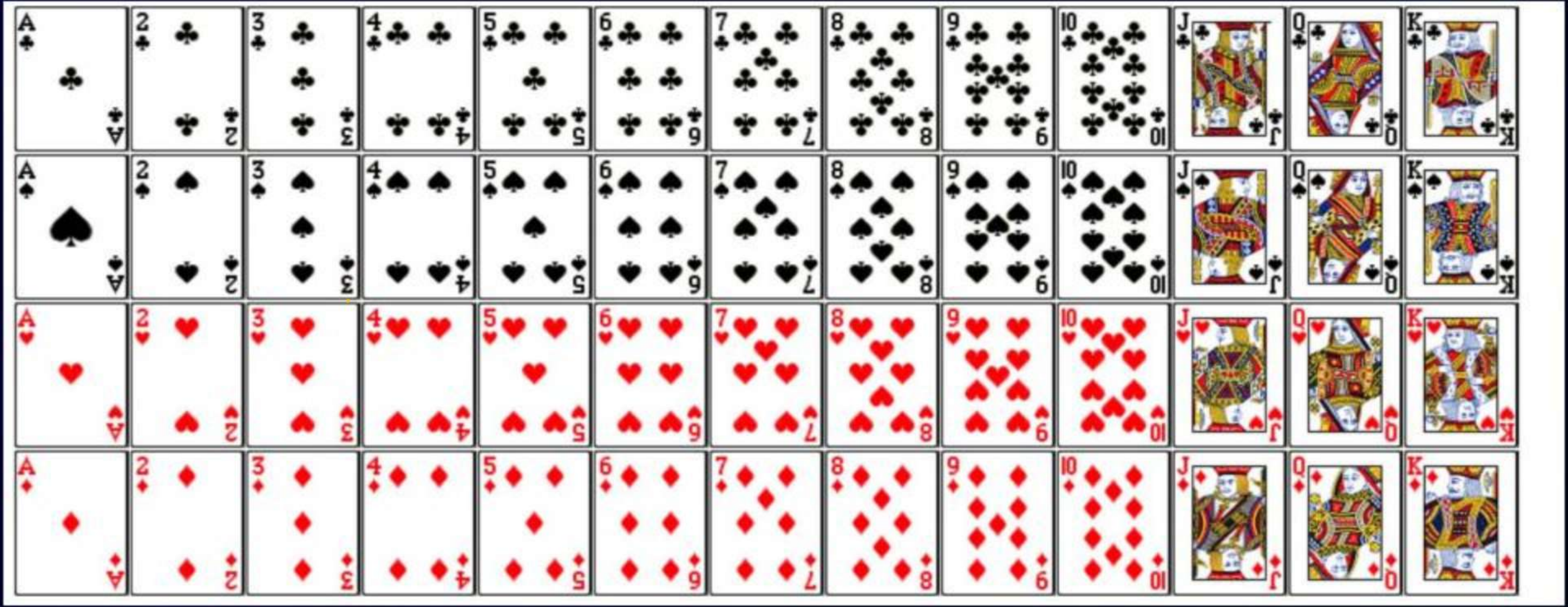
$S = \{HH, HT, TH, TT\}$

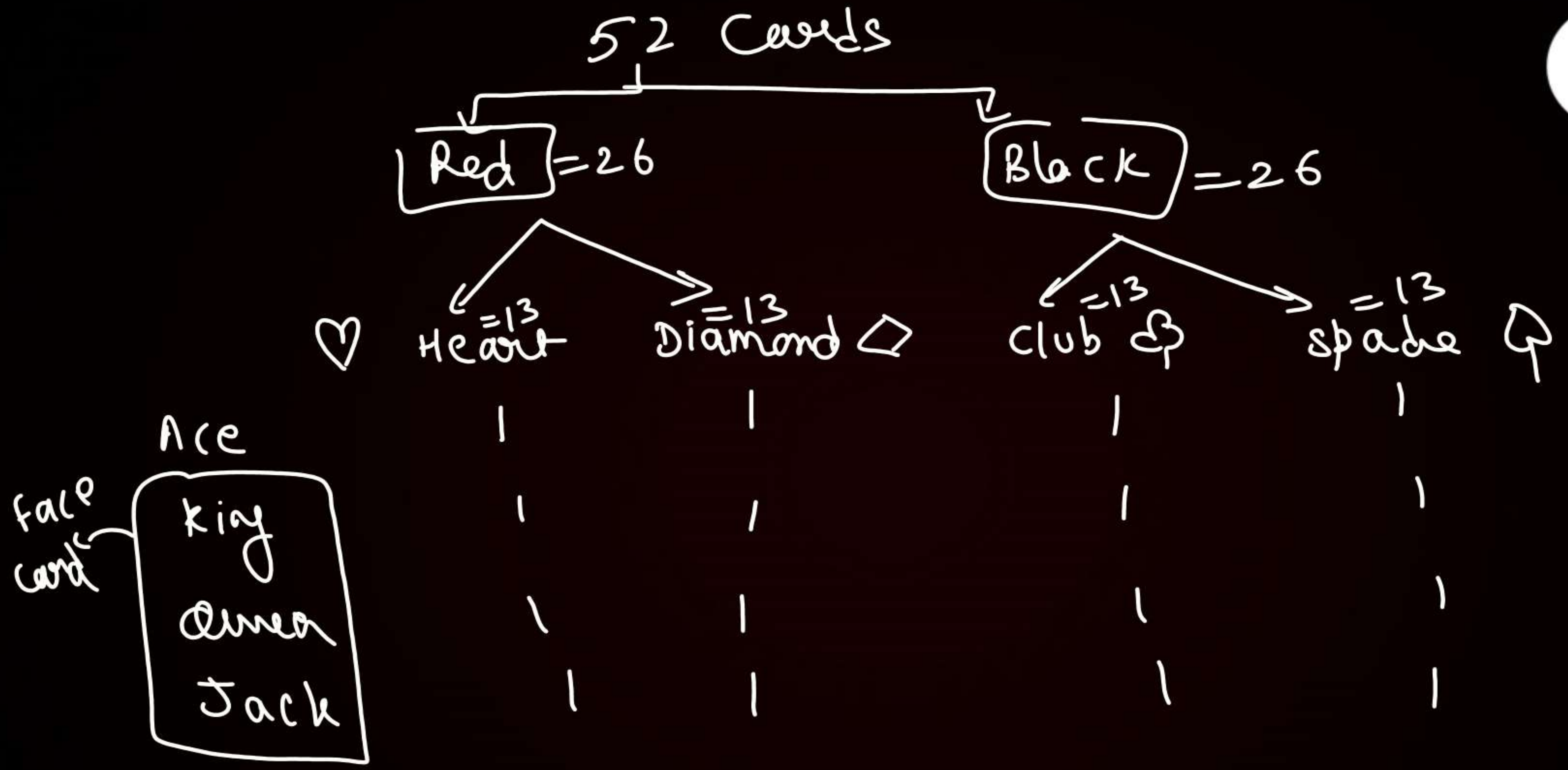
E_1: Tail on only one coin $= \{HT, TH\}$

E_2: No tails $= \{HH\}$

$E_1 \cup E_2 = \{HT, TH, HH\} \neq S$

E_1 & E_2 are not m. Exhaustive.




52 Cards
Red = 26
Black = 26
Heart = 13
Diamond = 13
Club = 13
Spade = 13
Ace
Face card
King
Queen
Jack

Equally Likely outcomes

When probability of each possible outcome is equal to prob. of other outcomes.

eg. Coin is tossed once

$S = \{H, T\}$

$P(H) = \frac{1}{2}$

$P(T) = \frac{1}{2}$

eg. A dice is thrown once

$S = \{1, 2, 3, 4, 5, 6\}$

$P(1) = \frac{1}{6}$ | $P(3) = \frac{1}{6}$ | $P(5) = \frac{1}{6}$

$P(2) = \frac{1}{6}$ | $P(4) = \frac{1}{6}$ | $P(6) = \frac{1}{6}$

Classical Definition Of Probability
(Priori Definition)

$$P(E) = \frac{\text{Total no. of favorable outcomes}}{\text{Total no. of possible outcomes}}$$

Q. A coin is tossed 2 times , Find the probability of following

a> Exactly One head

b> Two tails

c> Atleast One tail

Sol: $S = \{HH, HT, TH, TT\}$

a> P(Exactly one Head)

$= P(HT, TH)$

$= \frac{2}{4} = \frac{1}{2}$

ii> P(two tails)

$= P(TT)$

$= \frac{1}{4}$

iii> P(Atleast one tail)

$= \frac{3}{4}$

Q. A coin is tossed 3 times, Find the probability of following

a) Exactly One head

b) Atleast Two tails

c) Atmost two heads

Sol: S = { HHH, HHT, HTH, HTT, THH, THT, TTH, TTT }

a) P(Exactly one Head)

= P(HTT, THT, TTH)

$= \frac{3}{8}$

b) P(Atleast two tails)

= P(2 Tails) + P(3 tails)

= P(HTT, THT, TTH, TTT)

$= 4/8 = 1/2$

c) P(Atmost two head)

= P(2H) + P(1H) + P(0H)

= P(HHT, HTH, THH) + P(HTT, THT, TTH) + P(TTT)

$= 7/8$

Q. A dice is tossed 2 times , Find the probability of following

a) 6 exactly once

b) Doublet

c) Sum is 5

d) Sum is Atleast 10

Sol: i) $P(6 \text{ exactly once})$

$= \frac{10}{36}$

ii) $P(\text{Doublet})$

$= P[(11)(22)\cdots(66)]$

$= 6/36 = 1/6$

iii) $P(\text{Sum is } 5)$

$= P[(14)(23)(32)(41)]$

$= \frac{4}{36} = \frac{1}{9}$

iv) $P(\text{sum is atleast } 10)$

$= P(\text{Sum is } 10) + P(\text{Sum } 11) + P(\text{Sum } 12)$

$= P(46, 55, 64) + P(65, 56) + P(66)$

$= \frac{6}{36} = \frac{1}{6}$

Q. A Card is drawn from a Pack of 52 Cards Find the probability of following

i) Face Card

ii) King of spade

iii) Red Queen

iv) Black Ace

i) $\frac{12}{52} = \frac{3}{13}$

ii) $\frac{1}{52}$

iii) $\frac{2}{52} = \frac{1}{26}$

iv) $\frac{2}{52} = \frac{1}{26}$

Probability Of 53 Mondays in A non Leap Year

→ 365 Days

A. 1/7

B. 1/365

C. 53/365

D. None

$$7 \,\overline{)\,365\,}\, 52$$
35
15
14
1

365 Days = 52 weeks + 1 Day

Possible Days
{ Sunday, Monday, Tuesday, wednesd, Thursd, friday, Saturday }

1/7

Probability Of 53 Mondays in A Leap Year

= 366 Days

A. 1/7

B. 2/7

C. 2/366

D. 53/365

$7 \overline{)366}$ (52
35
16
14
2

366 Days
= 52 weeks + 2 Days.

(Sunday & monday)
(Monday & Tuesday)
(Tuesday & wednes)
(wednes & Thursday)
(Thurs & friday)
(friday & Saturd)
(Saturday & Sunday)

$\frac{2}{7}$

The following data relate to the distribution of wages of a group of workers:

Wages in Rs:	50-60	60-70	70-80	80-90	90-100	100-110	110-120
No. of workers:	15	23	36	42	17	12	5

If a worker is selected at random from the entire group of workers, what is the probability that

(a) his wage would be less than Rs 50? → $0/150 = 0$

(b) his wage would be less than Rs 80? → $74/150 = \frac{37}{75}$

(c) his wage would be more than Rs 100? → $17/150$

(d) his wages would be between Rs 70 and Rs 100? → $\frac{95}{150} = \frac{19}{30}$

Total workers = 150

Introduction Of Set Theory In Probability

Sample = S

$$P(A) = \frac{n(A)}{n(S)}$$

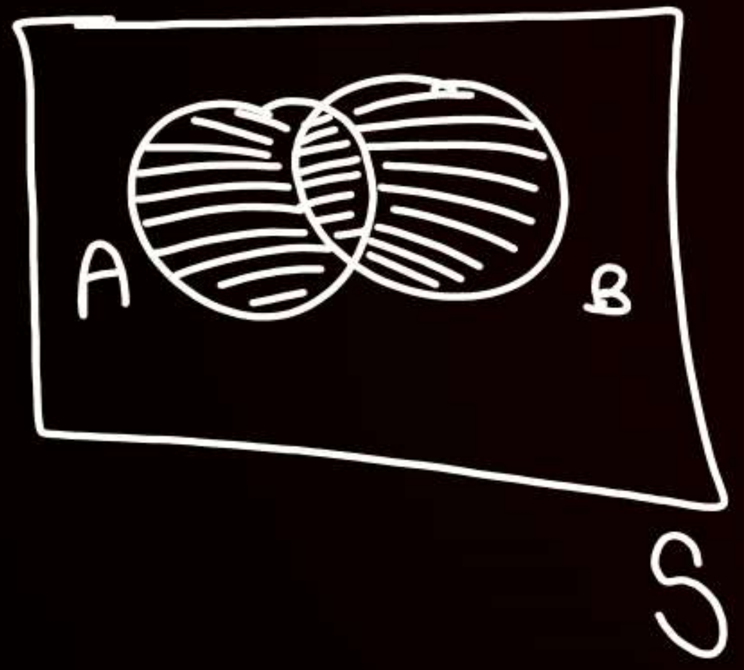
Eg

$$P(A) = \frac{2}{6} = \frac{1}{2}$$

$P(A \text{ or } B)$

$= P(A \cup B)$

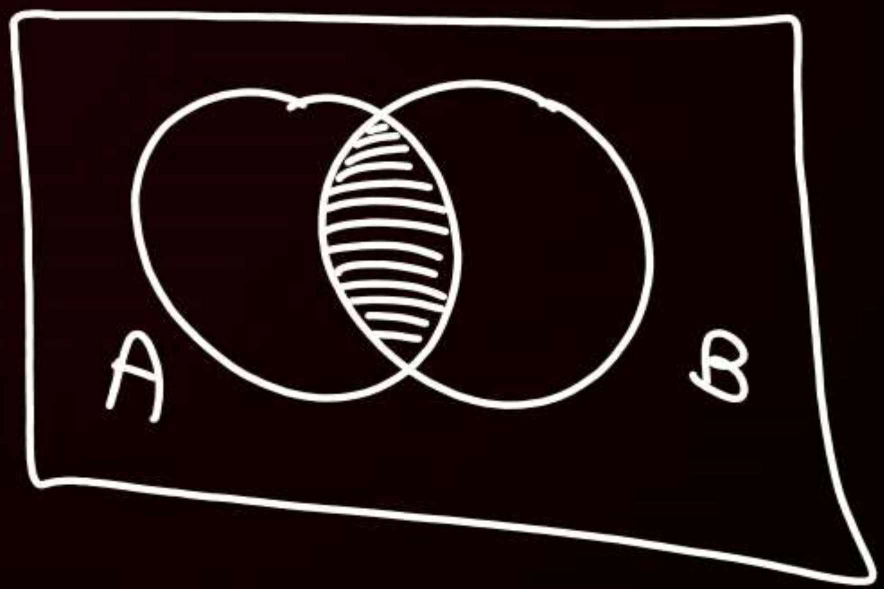
$= P(A) + P(B) - P(A \cap B)$



$P(A \,\&\, B)$

$= P(A \cap B)$

$= P(A) + P(B) - P(A \cup B)$



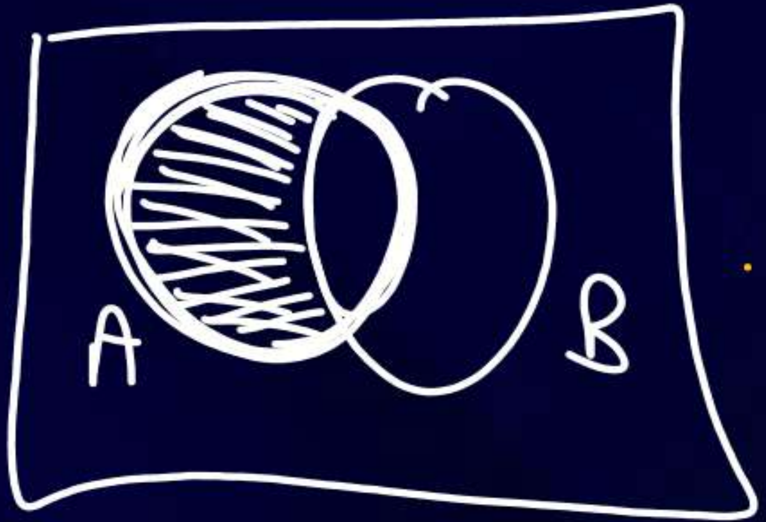
$P(\text{only } A)$

$= P(A - B)$

$= P(A \text{ but not } B)$

$= P(A \cap \overline{B})$

$= P(A) - P(A \cap B)$



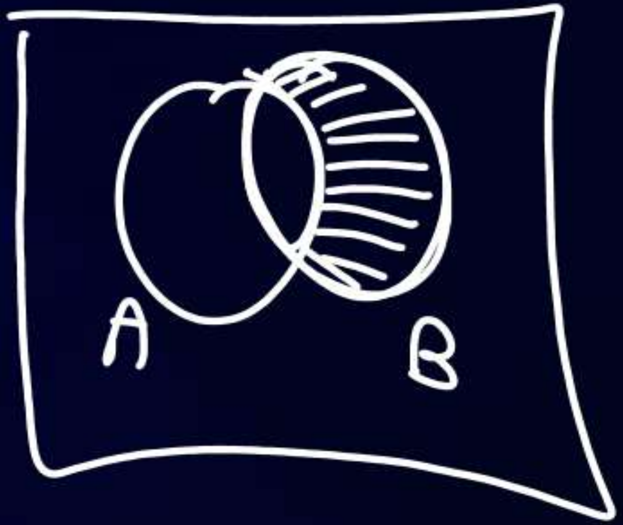
$P(\text{only } B)$

$= P(B - A)$

$= P(B \text{ but not } A)$

$= P(B \cap \overline{A})$

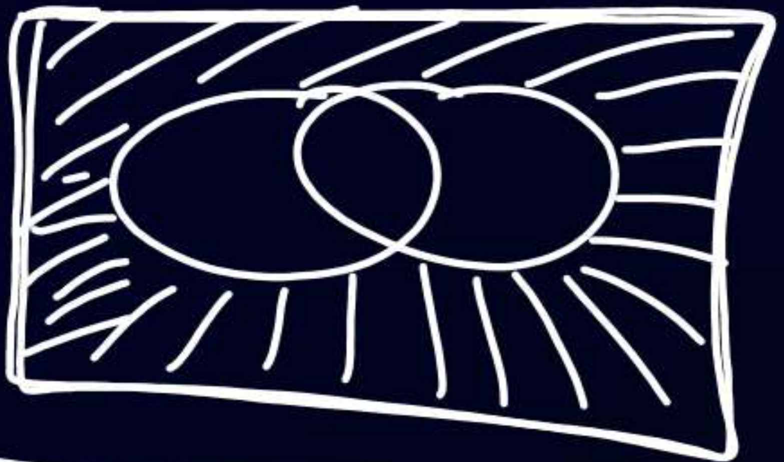
$= P(B) - P(A \cap B)$



$P(\text{Neither } A \text{ Nor } B)$

$= P(\overline{A} \cap \overline{B})$

$= 1 - P(A \cup B)$



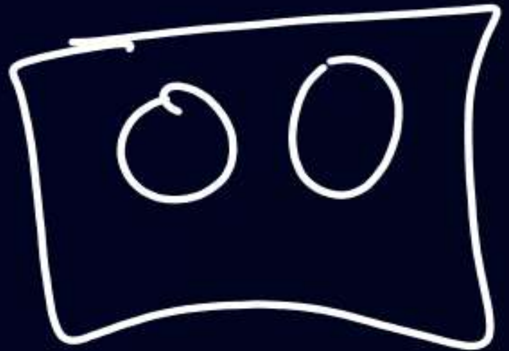
$P(\text{Not } A \text{ or Not } B)$

$= P(\overline{A} \cup \overline{B})$

$= 1 - P(A \cap B)$

Given P (A)= $\frac{3}{5}$ and P (B) = $\frac{1}{5}$. Find P(A or B), if A and Bare mutually exclusive events

$\Rightarrow P(A \cap B) = 0$



(A.) 4/5

(B.) 3/5

(C.) 2/5

(D.) None

$P(A \text{ or } B)$
$= P(A \cup B)$
$= P(A) + P(B) - P(A \cap B)$
$= \frac{3}{5} + \frac{1}{5} - 0 = \frac{4}{5}$

A and B are two mutually exclusive events of an experiment. If P ('not A') = 0.65, P(A ∪ B) = 0.65 and P(B) = p, find the value of p

A. 0.35

B. 0.30

C. 0.65

D. None

$P(A \cap B) = 0$

$P(\bar{A}) = 0.65$

$P(A \cup B) = 0.65$

$P(B) = p$

Now $P(A) = 1 - P(\bar{A})$

$= 1 - 0.65$

$P(A) = 0.35$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.65 = 0.35 + p - 0$

$0.65 - 0.35 = p$

$0.30 = p$

The probability of two events A and B are 0.25 and 0.50 respectively. The probability of their simultaneous occurrence is 0.14. Find the probability that neither A nor B occurs.

A. 0.39

B. 0.61

C. 0.75

D. None

$P(A) = 0.25$

$P(B) = 0.50$

$P(A \cap B) = 0.14$

Now

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.25 + 0.50 - 0.14$

$= 0.61$

$P(\text{Neither A Nor B})$

$= P(\bar{A} \cap \bar{B})$

$= 1 - P(A \cup B)$

$= 1 - 0.61$

$= 0.39$

One number is chosen from numbers 1 to 200. Find the probability that it is divisible by 4 or 6?

A. 67/200

B. 83/200

C. 55/200

D. None

A = No is Divisible by 4 = 4, 8, 12, ..., 200 $= \frac{200}{4} = 50$

B = No is Divisible by 6 = 6, 12, 18, ... $= \frac{200}{6} = 33.33$

$A \cap B$ = No is Div by 4 & 6 = 12, 24, 36, ... $= \frac{200}{12} = 16.66$

$P(4 \text{ or } 6)$

$= P(A \text{ or } B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{50 + 33 - 16}{200} = \frac{67}{200}$

Three events A, B and C are mutually exclusive, exhaustive and equally likely. What is the probability of the complementary event of A?

A. 1/3

B. 2/3

C. 3/4

D. None

$A \cap B = \phi$

$B \cap C = \phi$

$A \cap C = \phi$

A, B & C are m. exhaustive.

$P(A) + P(B) + P(C) = 1$

$x + x + x = 1$

$3x = 1$

$x = 1/3$

P(compliment of A)

$= P(\text{Not } A)$

$= 1 - P(A)$

$= 1 - x$

$= 1 - \frac{1}{3}$

$= \frac{2}{3}$

The probability that an Accountant's job applicant has a B. Com. Degree is 0.85, that he is a CA is 0.30 and that he is both B. Com. and CA is 0.25 out of 500 applicants, how many would be B. Com. or CA?

(A.) 0.90 ✗

(B.) 450

(C.) 0.45 ✗

(D.) 900 ✗

$P(Bcom) = 0.85$

$P(CA) = 0.30$

$P(Bcom \cap CA) = 0.25$

Now $P(Bcom \text{ or } CA)$

$= P(Bcom \cup CA)$

$= P(Bcom) + P(CA) - P(Bcom \cap CA)$

$= 0.85 + 0.30 - 0.25$

$= 0.90$

500×0.90

$= 500 \times \frac{90}{100}$

$= 450$

If P(A–B) = 1/5, P(A) = 1/3 and P (B) = 1/2, what is the probability that out of the two events A and B, only B would occur?

A. 1/15

B. 2/15

C. 7/15

D. None

$P(A-B) = \frac{1}{5}$

$P(A) = \frac{1}{3}$

$P(B) = \frac{1}{2}$

$P(\text{only } B) = ?$

Now

$P(A-B) = P(A) - P(A \cap B)$

$\frac{1}{5} = \frac{1}{3} - P(A \cap B)$

$P(A \cap B) = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$

Now

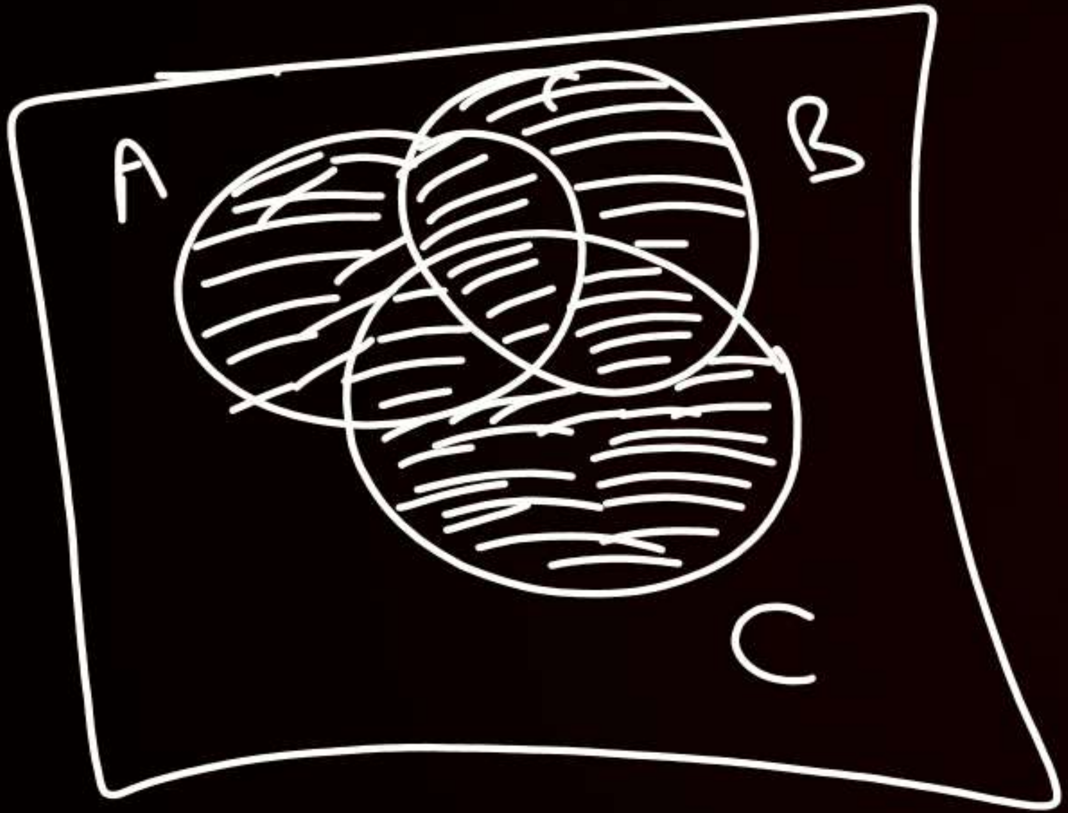
$P(\text{only } B)$

$= P(B) - P(A \cap B)$

$= \frac{1}{2} - \frac{2}{15}$

$= \frac{15-4}{30} = \frac{11}{30}$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$



P(Atlest one of them) = $P(A \cup B \cup C)$

P(Neither A nor B & Nor C) = $1 - P(A \cup B \cup C)$

There are three persons A, B and C having different ages. The probability that A survives another 5 years is 0.80, B survives another 5 years is 0.60 and C survives another 5 years is 0.50. The probabilities that A and B survive another 5 years is 0.46, B and C survive another 5 years is 0.32 and A and C survive another 5 years 0.48. The probability that all these three persons survive another 5 years is 0.26. Find the probability that at least one of them survives another 5 years.

(A) 0.90

(B) 450

(C) 0.45

(D) 900

$P(A) = 0.80$

$P(B) = 0.60$

$P(C) = 0.50$

$P(A \cap B) = 0.46$

$P(B \cap C) = 0.32$

$P(A \cap C) = 0.48$

$P(A \cap B \cap C) = 0.26$

$P(A \cup B \cup C)$

$= A + B + C - A \cap B - B \cap C - A \cap C + A \cap B \cap C$

$= 0.80 + 0.60 + 0.50 - 0.46 - 0.32 - 0.48 + 0.26$

$= 0.9$

QUESTION

A bag Contain 3 red and 2 black Balls .

One ball is drawn at random

Find the probability of Red Ball

3 Red
2 Black
———
5

$\underbrace{R_1, R_2, R_3}, B_1, B_2$

$P(\text{Red ball}) = \frac{3}{5}$

QUESTION

A bag Contain 3 red and 2 black Balls.

Two balls are drawn at random without replacement

Find the probability of

i) two Red balls

ii) One Red and One Black

iii) Two Black

$S = \{R_1R_2, R_1R_3, R_2R_3, R_1B_1, R_1B_2, R_2B_1, R_2B_2, R_3B_1, R_3B_2, B_1B_2\}$

Red = 3 $(R_1 R_2 R_3)$

Black = 2 $(B_1 B_2)$

5

i) $P(2 \text{ Red}) = \frac{3}{10}$

ii) $P(1 \text{ Red} \,\&\, 1 \text{ Black}) = \frac{6}{10}$

iii) $P(2 \text{ Black}) = \frac{1}{10}$

CA WALLAH

Red = 3

Black = 2

5

whenever 2 or more elements are drawn without Replacement

⇓

we have to use combination

i> P(2 Red balls)

$$= \frac{^3C_2}{^5C_2}$$

$$= \frac{\frac{3!}{2!\,1!}}{\frac{5!}{2!\,3!}}$$

$$= \frac{3}{10}$$

$$^nC_r = \frac{n!}{r!\,(n-r)!}$$

ii> P(1 Red & 1 Black)

$$= \frac{^3C_1 \times {^2C_1}}{^5C_2} = \frac{3 \times 2}{10} = \frac{6}{10}$$

iii> P(2 Black)

$$= \frac{^2C_2}{^5C_2} = \frac{1}{10}$$

QUESTION

A bag Contain 3Red,2Black and 4 White Balls .

4 Balls are drawn at random

Find the probability of

i)2 Red,1 White and 1 Black

ii)1 Red

3 - R
Non Red [2 - B, 4 - W]
9 Balls

i> P(2R, 1W & 1B)

$$= \frac{{}^3C_2 \times {}^4C_1 \times {}^2C_1}{{}^9C_4} = \frac{3 \times 4 \times 2}{126} = \frac{24}{126}$$

ii> P(1 Red & 3 Non Red)

$$= \frac{{}^3C_1 \times {}^6C_3}{{}^9C_4}$$

QUESTION

CA

A committee of 7 members is to be formed from a group comprising 8 gentlemen and 5 ladies. What is the probability that the committee would comprise 2 ladies,

A. 140/429

B. 130/429

C. 135/429

D. None

8-M & 5-W → 7 committee

$P(2W \& 5M)$

$$= \frac{{}^5C_2 \times {}^8C_5}{{}^{13}C_7}$$

$$= \frac{\frac{5!}{2!\,3!} \times \frac{8!}{5!\,3!}}{\frac{13!}{7!\,6!}}$$

$$= \frac{10 \times 56}{1716}$$

$$= \frac{10 \times 14}{429} = \frac{140}{429}$$

Conditional Probability

$P(A)$ = Prob of event A = Unconditional.

$P(A/B)$ = Probability of Event A when event B has already occurred

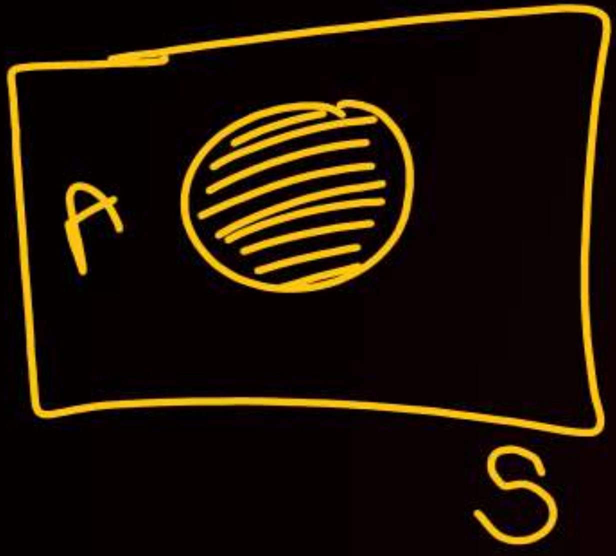
or

$P(A/B)$ = Probability of Event A when Event B is given

or

$P(A/B)$ = Prob. of Event A when event 'B' is used as a new sample space.

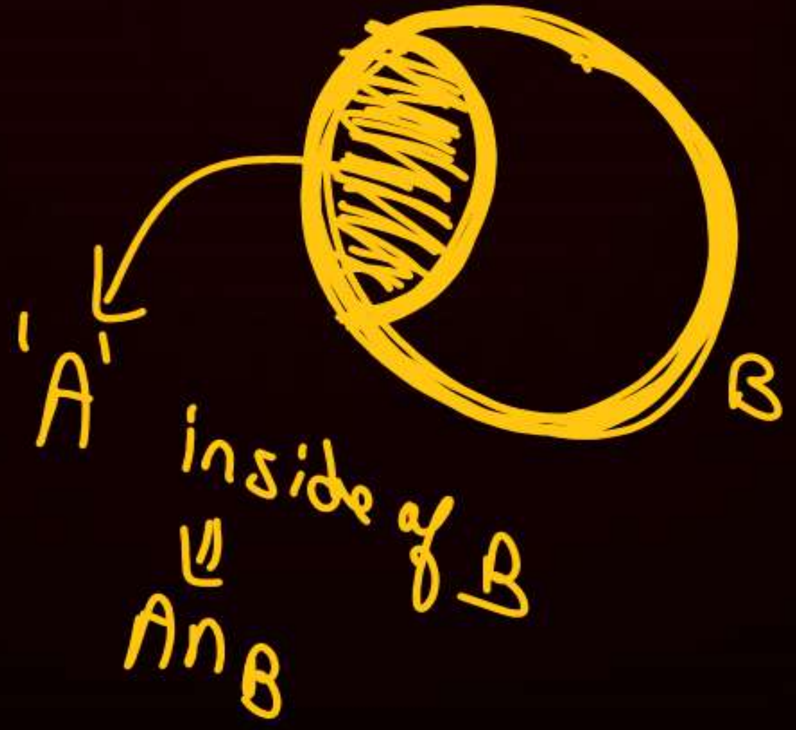
$$P(A) = \frac{n(A)}{n(S)}$$



$P(A/B)$ → A की Prob. बताओ

→ अगर B नया sample space है।

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$



$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(P/Q) = \frac{P(P \cap Q)}{P(Q)}$$

$$P(M/N) = \frac{P(M \cap N)}{P(N)}$$

Eg $P(A) = 0.4$

$P(B) = 0.6$

$P(A \cap B) = 0.2$

find i) $P(A/B) = ?$

ii) $P(B/A) = ?$

Sol: i)

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.6} = \frac{1}{3}$$

ii)

$$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{0.2}{0.4} = \frac{1}{2}$$

QUESTION

CA

Evaluate $P(A \cup B)$ if $2P(A) = P(B) = \frac{5}{13}$ & $P\left(\frac{A}{B}\right) = \frac{2}{5}$

a) 11/26
b) 11/27
c) 11/29
d) 11/40

$2P(A) = \frac{5}{13}$ | $P(B) = \frac{5}{13}$

$P(A) = \frac{5}{26}$

Now $P(A/B) = \frac{2}{5}$

$\frac{P(A \cap B)}{P(B)} = \frac{2}{5}$

$P(A \cap B) = \frac{2}{5} \times P(B)$

$P(A \cap B) = \frac{2}{5} \times \frac{5}{13} = \frac{2}{13}$

$P(A \cup B)$
$= P(A) + P(B) - P(A \cap B)$
$= \frac{5}{26} + \frac{5}{13} - \frac{2}{13}$
$= \frac{5 + 10 - 4}{26} = \frac{11}{26}$

QUESTION

CA

$P(A) = 0.4\,, P(B) = 0.8\,, P\left(\frac{B}{A}\right) = 0.6$ *Then* Find $P\left(\frac{\bar{B}}{\bar{A}}\right)$

a) 1/15
b) 2/15
c) 14/15
d) None

$P(A) = 0.4$

$P(B) = 0.8$

$P(B/A) = 0.6$

$P(B/A) = \frac{P(A \cap B)}{P(A)}$

$0.6 \times 0.4 = P(A \cap B)$

$P(A \cap B) = 0.24$

$P(A \cup B)$

$= A + B - A \cap B$

$= 0.4 + 0.8 - 0.24$

$= 1.20 - 0.24$

$= 0.96$

$P\left(\frac{\bar{B}}{\bar{A}}\right)$

$= \frac{P(\bar{A} \cap \bar{B})}{P(\bar{A})}$

$= \frac{1 - P(A \cup B)}{1 - P(A)}$

$= \frac{1 - 0.96}{1 - 0.4}$

$= \frac{0.04}{0.60} = \frac{1}{15}$

QUESTION

CA

A coin is tossed two times , find the probability of getting two tails if there is Atleast one tail

a) 1/4

b) 1/4

c) 3/4

d) None

S = {HH, HT, TH, TT}

A: Two tails = {TT}

B: Atleast one tail = {HT, TH, TT}

$A \cap B = \{(TT)\}$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1}{3}$$

QUESTION

A family has two children , Find the probability of two boys if the elder child is a boy

a) 1/2
b) 1/3
c) 1/4
d) None

$S = \{BB, BG, GG, GB\}$

A = Two Boys $= \{BB\}$

B = Elder child is a Boy $= \{BB, BG\}$

$A \cap B = \{BB\}$

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{1}{2}$

Joint Probability (Compound Probability Theorem)

$$P(AB) = P(A) \times P(B/A)$$

$$P(BC) = P(B) \times P(C/B)$$

$$P(ABC) = P(A) \times P(B/A) \times P(C/AB)$$

$$P(ABCD) = P(A) \times P(B/A) \times P(C/AB) \times P(D/ABC)$$

Eg

Bag

5 - Good Mango | 3 - Bad Mango

2 mangoes are drawn one by one without Replacement. Find the Prob

i) P(2 Good)

ii) P(2 Bad)

iii) P(first Good & second Bad)

Sol:

(1)

5 - G
3 - B
8

$P(GG) = \frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$

$P(BB) = \frac{3}{8} \times \frac{2}{7} = \frac{6}{56}$

$P(GB) = \frac{5}{8} \times \frac{3}{7}$

G - 4
B - 3
7

QUESTION

A bag contains 10 white balls & 15 black balls. Two balls are drawn in succession without replacement. Find the probability that the first ball is white & the second ball is black.

a) 1/3

b) 1/4

c) 5/7

d) None

10-W → 9

15-B → 15

25 → 24

2 balls without Replacement

$$P(WB) = \frac{10}{25} \times \frac{15}{24} = \frac{1}{4}$$

QUESTION

A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.

a)0.48

b) 0.54

c)0.32

d)0.75

12 - Good
3 = Bad
15

3 oranges are drawn without Replacement

P(Approval for sale)
= P(G G G)

$$= \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13}$$

Bag

4 - Black
6 - Red

10

2 balls are drawn
with Replacement

$P(BB) = \frac{4}{10} \times \frac{4}{10}$

$P(RR) = \frac{6}{10} \times \frac{6}{10}$

Without Replacement

$P(BB) = \frac{4}{10} \times \frac{3}{9}$

$P(RR) = \frac{6}{10} \times \frac{5}{9}$

Independent Events

$$P\left(\frac{A}{B}\right) = P(A)$$

$P\left(\frac{A}{B}\right)$ ⇓ Prob. of A when event B has already occurred

$P(A)$ ⇓ Prob. of A

⇓ When occurrence of one event does not affect the probability of other

A & B are independent

→ $P(A/B) = P(A)$

→ $P(B/A) = P(B)$

Now $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$P(A) = \dfrac{P(A \cap B)}{P(B)}$

$P(A) \times P(B) = P(A \cap B)$

$P(A \cap B) = P(A)\,P(B)$

$P(\bar{A} \cap \bar{B}) = P(\bar{A})\,P(\bar{B})$

$P(A \cap \bar{B}) = P(A) \cdot P(\bar{B})$

$P(\bar{A} \cap B) = P(\bar{A})\,P(B)$

Eg $P(A) = 0.4$

$P(B) = 0.3$

A & B are independent

$P(A \cup B) = ?$

Sol:

$$P(A \cap B) = P(A) \times P(B)$$
$$= 0.4 \times 0.3$$
$$= 0.12$$

$$P(A \cup B)$$
$$= P(A) + P(B) - P(A \cap B)$$
$$= 0.4 + 0.3 - 0.12$$
$$= 0.70 - 0.12$$
$$= 0.58$$

QUESTION

The probability that a husband & wife will be alive 20 years from now are 0.8 & 0.9 respectively. Find the probability that in 20 years both of them will be alive

$P(H) = 0.8$

$P(W) = 0.9$

a) 1

b) 1.7

c) 0.28

d) 0.72

$P(H \cap W) = P(H) \times P(W)$

$= 0.8 \times 0.9$

$P(HW) = 0.72$

QUESTION

Probability of solving a specific problem independently by A & B are $\frac{1}{2}$ & $\frac{1}{3}$ respectively. If both try to solve the problem independently, then find the probability that the problem is solved

a) 2/3

b) 1/3

c) 1/4

d) None

$P(A) = \frac{1}{2}$ | $P(\bar{A}) = 1 - \frac{1}{2} = \frac{1}{2}$

$P(B) = \frac{1}{3}$ | $P(\bar{B}) = 1 - \frac{1}{3} = \frac{2}{3}$

P(Prob. is solved)

$= P(A\bar{B}) + P(\bar{A}B) + P(AB)$

$= \left(\frac{1}{2}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{3}\right) = \frac{4}{6} = \frac{2}{3}$

Or P(Prob. is solved)

$= 1 -$ P(Prob not solved)

$= 1 - P(\bar{A}\,\bar{B})$

$= 1 - \frac{1}{2} \times \frac{2}{3}$

$= 1 - \frac{1}{3} = \frac{2}{3}$

QUESTION

A problem in mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$.

What is the probability that the problem is solved?

a) 1/4

b) 3/4

c) 2/5

d) None

$P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(C) = \frac{1}{4}$

$P(\bar{A}) = 1 - \frac{1}{2} = \frac{1}{2}$

$P(\bar{B}) = 1 - \frac{1}{3} = \frac{2}{3}$

$P(\bar{C}) = 1 - \frac{1}{4} = \frac{3}{4}$

P(Prob is solved)

$= 1 - P(\text{Prob is not solved})$

$= 1 - P(\bar{A}\bar{B}\bar{C})$

$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$

$= 1 - \frac{1}{4} = \frac{3}{4}$

Odds in Favour of A

$= \text{Ratio}$

$= A : \bar{A}$

Odds against A

$= \text{Ratio}$

$= \bar{A} : A$

Rahul

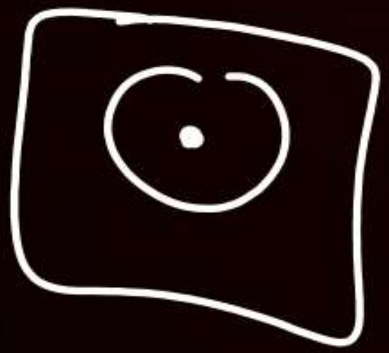
||

Gun

10 Fires

7 Hit

3 Don't Hit



odds in favour of Rahul hitting target $= 7 : 3$

odds against Rahul hitting the target $= 3 : 7$

Odds in Favour of A solving a problem is 2 : 3

2 → Solve ✓

3 → Don't solve ✗

$P(A) = \frac{2}{5}$

$P(\bar{A}) = \frac{3}{5}$

Odds against B solving a problem is 3 : 7

3 → Not solving ✗

7 → Solving ✓

$P(B) = \frac{7}{10}$

$P(\bar{B}) = \frac{3}{10}$

QUESTION

CA

The odds in favour of an event is 2 : 3 and the odds against another event is 3 : 7. Find the probability that only one of the two events occurs.

a) 26/31

b) 27/50

c) 28/51

d) None

$P(A) = \frac{2}{5}$ | $P(B) = \frac{7}{10}$

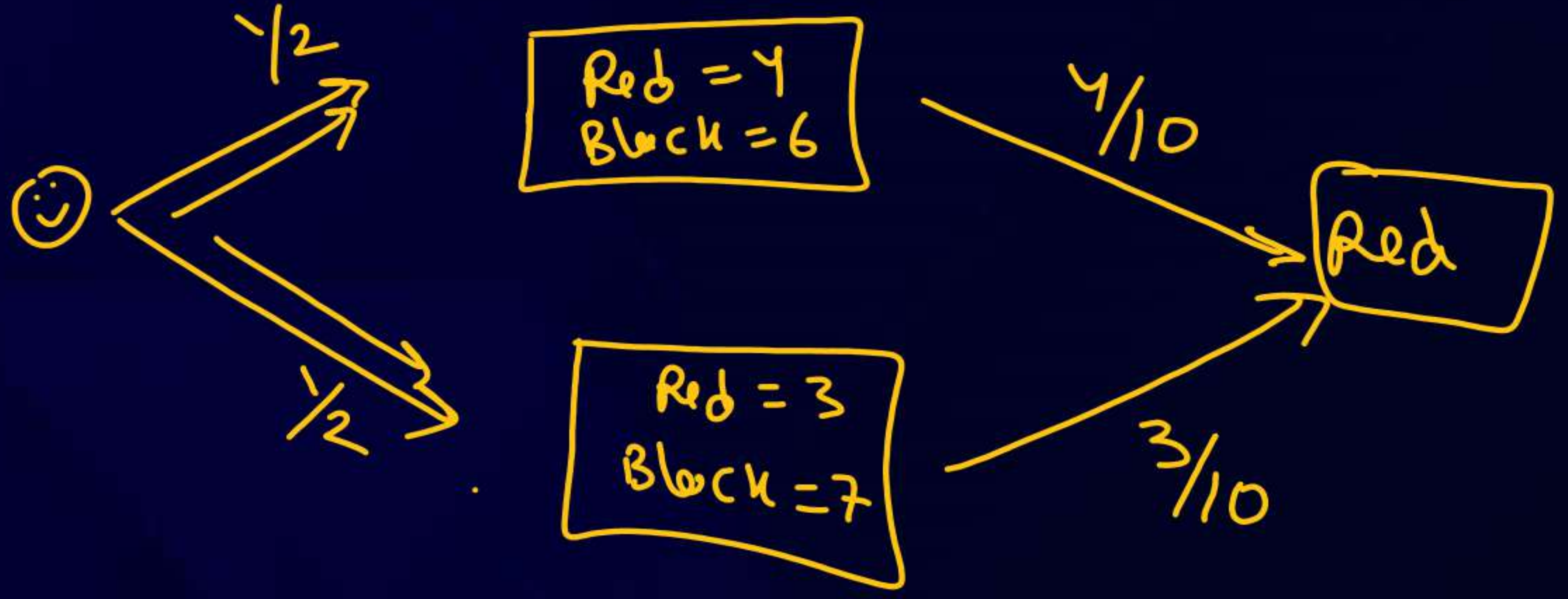
$P(\bar{A}) = \frac{3}{5}$ | $P(\bar{B}) = \frac{3}{10}$

P(only one event occur)

$= P(A\bar{B}) + P(B\bar{A})$

$= \frac{2}{5} \times \frac{3}{10} + \frac{7}{10} \times \frac{3}{5} = \frac{6+21}{50} = \frac{27}{50}$

Total Probability Theorem



$$P(Red) = \left(\frac{1}{2}\right)\left(\frac{4}{10}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{10}\right)$$

$$= \frac{4}{20} + \frac{3}{20} = \frac{7}{20}$$

QUESTION

CA

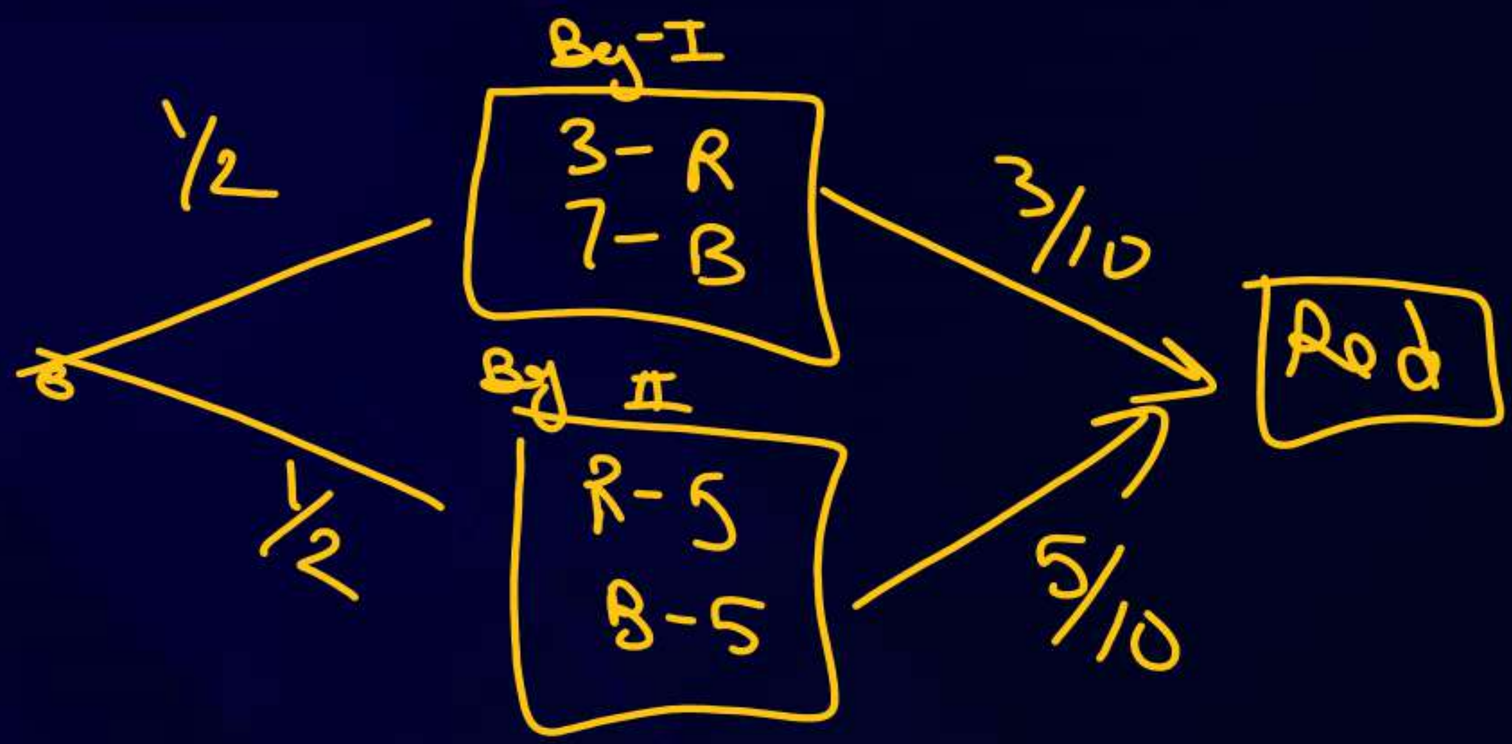
Bag 1 contains 3 red and 7 black balls ,another Bag 2 Contants 5 Red and 5 Black Balls , One Bag Is chosen at random and then a ball is drawn from it , Find the probability that it will be red in color

a) 2/5

b) 7/10

c) 4/5

d) None



$P(\text{Red})$

$= \frac{1}{2} \times \frac{3}{10} + \frac{1}{2} \times \frac{5}{10}$

$= \frac{8}{20} = \frac{4}{10} = \frac{2}{5}$

There are 3 boxes with the following composition :

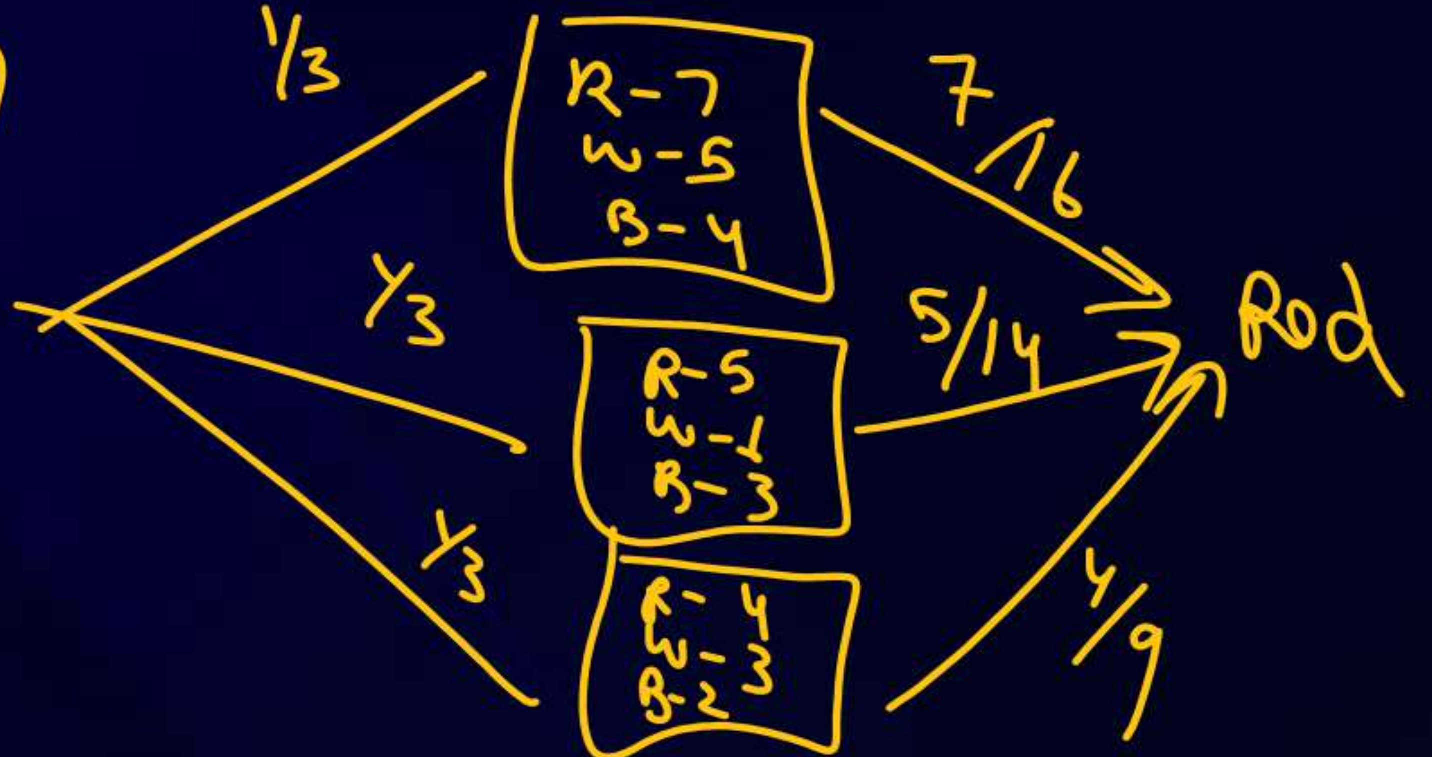
Box I : 7 Red + 5 White + 4 Blue balls

Box II : 5 Red + 6 White + 3 Blue balls

Box III : 4 Red + 3 White + 2 Blue balls

One of the boxes is selected at random and a ball is drawn from it. What is the probability that the drawn ball is red?

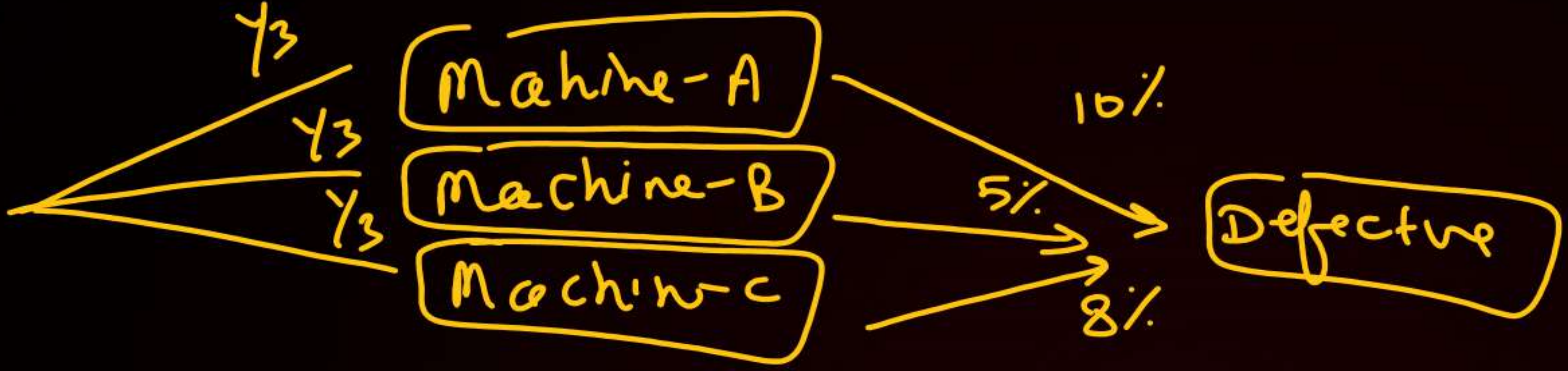
a) 1249/3024
b) 1280/3024
c) 1179/3024
d) None



P(Red)

$$= \frac{1}{3}\times\frac{7}{16} + \frac{1}{3}\times\frac{5}{14} + \frac{1}{3}\times\frac{4}{9}$$

$$= 0.4130$$



P(Defective output)

$$= \frac{1}{3}\left(\frac{10}{100}\right) + \frac{1}{3}\left(\frac{5}{100}\right) + \frac{1}{3}\left(\frac{8}{100}\right)$$

$$= \frac{10+5+8}{300} = \frac{23}{300}$$

P(Non Defective)

$$= 1 - \frac{23}{300}$$

$$= \frac{277}{300}$$

Probability Distribution

Class- 12th

marks: 5, 1, 3, 4, 5, 2, 1, 1, 3, 2, 2, 4, 5, 2, 1

Frequency Distribution.

variable (marks) = x_i	f_i
1	4
2	4
3	2
4	2
5	3
	15

Probability Distribution.

Random Variable ⇓ Descrete Variable

marks (x_i)	p_i
1	4/15
2	4/15
3	2/15
4	2/15
5	3/15
	1

Probability Distribution

Distribution of total probability on the Basis of a Random variable

Random variable = X_i

Descrete — Probability mass function

x_i	p_i
1	0.6
2	0.3
3	0.1
	1

Continous — Probability Density function.

CI	p_i
0-2	0.4
2-4	0.5
4-6	0.1
	1

x_i	p_i	$p_i x_i$	$p_i x_i^2$
	1		

$E(x) = \Sigma p x$

$E(x^2) = \Sigma p x^2$

$E(x^3) = \Sigma p x^3$

$E(x-\mu) = \Sigma p(x-\mu)$

Expectation (mean)

$= E(x)$

$\mu = \Sigma p_i x_i$

Variance

$= \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2$

or

$\Sigma p_i (x_i - \mu)^2$

$= E(x_i - \mu)^2$

$S.D = \sqrt{\text{variance}}$

CA WALLAH

Q

x_i	p_i	$p_i x_i$	$p_i x_i^2$	$p_i (x_i - \mu)^2$
1	0.4	0.4	0.4	0.196
2	0.5	1.0	2.0	0.045
3	0.1	0.3	0.9	0.169
	1	1.7	3.3	0.41

find mean & variance.

Sol: μ = mean
= Exp.
= $E(x)$
= $\Sigma p_i x_i$
= 1.7

Variance
$= \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2$
$= 3.3 - (1.7)^2$
$= 0.41$

$S.D = \sqrt{\text{variance}}$
$= \sqrt{0.40}$
$= 0.63$

$E(x) = \sum p_i x_i$

$E(K) = K$

$E(x+y) = E(x) + E(y)$

$E(Kx) = K\,E(x)$

x & y are independent

$E(xy) = E(x) \times E(y)$

$y = a + b x$

mean of y = a + b(mean of x)

S.D of y = |b| × (S.D of x)

Variance of y = b^2 × (Var. of x)

g

$E(x) = 2$
(mean)

& variance of $x = V(x) = 3$

$y_i = 5 + \boxed{6} x_i$

New Expectation

$= 5 + 6(2)$
$= 17$

New variance

$= 6^2 \times 3$
$= 36 \times 3$
$= 108$

If $V(x) = 3.04$

find $V(3x-4) = ?$

Sol:

$$V(x) = 3.04$$

$$V(3x-4) = (3)^2 \times 3.04$$

$$= 27.36$$

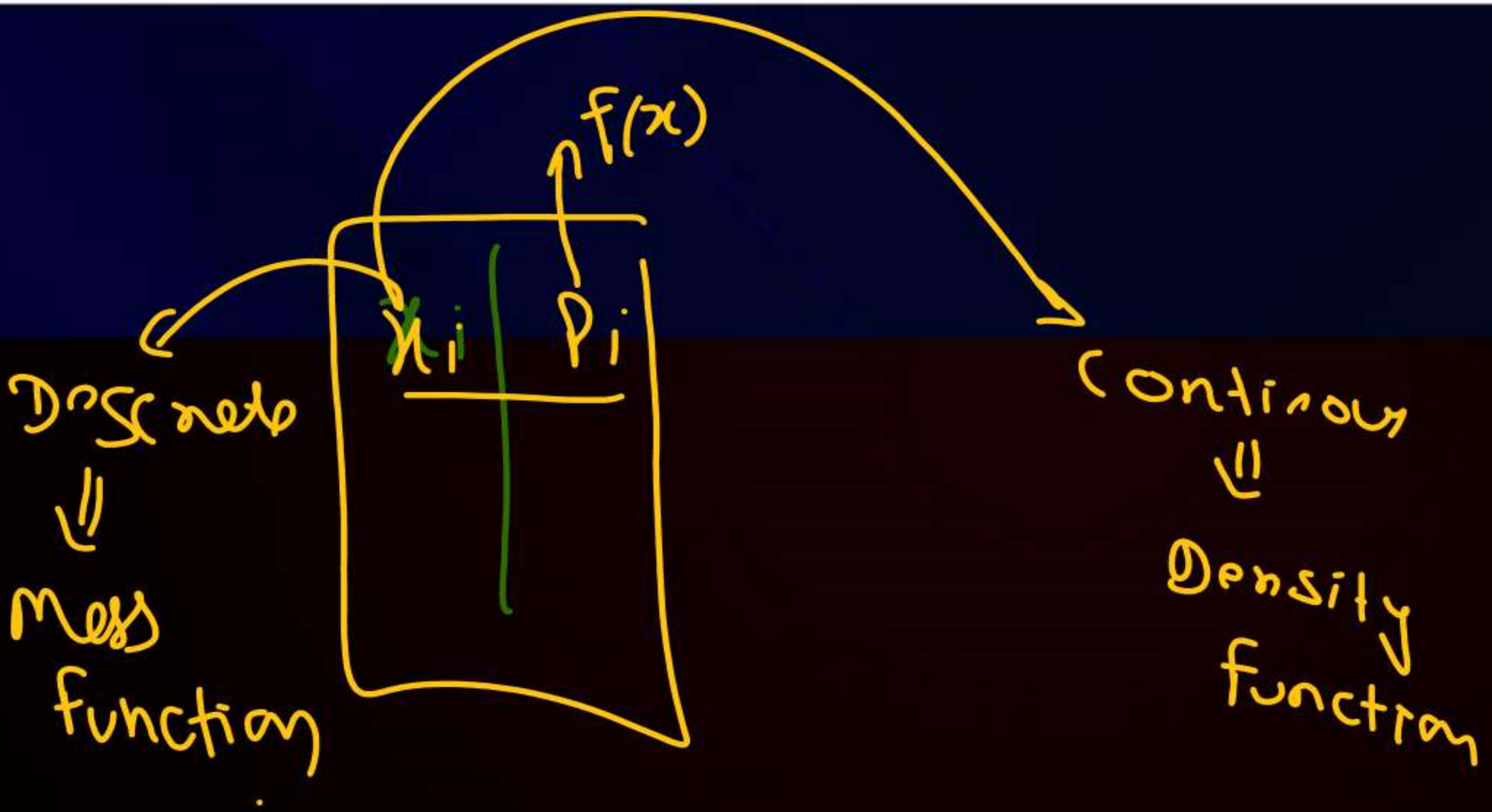
When X is a continuous function f(x) is called

a) probability mass function

b) probability density function

c) both

d) none



QUESTION

CA

If a random variable X has the following probability distribution, then the expected value of X is:

X	-1	-2	0	1	2
F(X)	1/3	1/6	1/5	1/6	1/3

(a) 3/2
(b) 1/2
(c) 1/6
(d) 1/5

(1 mark)

x_i	p_i	$p_i x_i$
-1	1/3	-1/3
-2	1/6	-2/6
0	1/5	0
1	1/6	1/6
2	1/3	2/3

Expected value of x

$= \text{mean}$

$= \Sigma p_i x_i$

$= -\frac{1}{3} - \frac{2}{6} + 0 + \frac{1}{6} + \frac{2}{3}$

$= \frac{-2-2+1+4}{6}$

$= \frac{1}{6}$

QUESTION

CA

A number is selected at random from the first 100 natural numbers. What is the probability that it would be a multiple of 3 or 7?

(a) $\frac{33}{100}$

(b) $\frac{4}{100}$

(c) $\frac{21}{100}$

(d) $\frac{43}{100}$

(1 mark)

$P(3) + P(7) - P(3 \& 7)$

$= \frac{100}{3} + \frac{100}{7} - \frac{100}{21}$

$= 33 + 14 - 4$

$= 43$

$\frac{43}{100}$

QUESTION

If $P(A \cap B) = \frac{1}{3}$, $P(A \cup B) = \frac{5}{6}$, $P(B) = \frac{1}{2}$, then $P(\bar{A})$ is:

(a) $\frac{2}{3}$

(b) $\frac{1}{3}$

(c) $\frac{1}{4}$

(d) $\frac{3}{4}$

$P(\bar{B}) = \frac{1}{2}$

$P(B) = 1 - P(\bar{B})$
$= 1 - \frac{1}{2}$
$= \frac{1}{2}$

$P(A \cup B) = A + B - A \cap B$

$\frac{5}{6} = A + \frac{1}{2} - \frac{1}{3}$

$\frac{5}{6} - \frac{1}{2} + \frac{1}{3} = A$

$A = \frac{5-3+2}{6} = \frac{4}{6} = \frac{2}{3}$

$P(A) = \frac{2}{3}$

$P(\text{Not } A) = 1 - \frac{2}{3}$
$= \frac{1}{3}$

QUESTION

Four persons are chosen at random frame a group of 3 men, 2 women and 4 children. The probability that exactly 2 of them are children is?

(a) 10/21

(b) 1/12

(c) 1/5

(d) 1/9

(1 mark)

3-m, 2w & 4c → 4

(2-children 2-non children)

$$= \frac{{}^4C_2 \times {}^5C_2}{{}^9C_4} = \frac{6 \times 10}{126} = \frac{30}{63} = \frac{10}{21}$$

QUESTION

CA

For a probability distribution, probability is given by, $P(X_i) = \frac{X_i}{k}, X_i = 1, 2, \ldots\ldots 9$. The value of k is

(a) 55
(b) 9
(c) 45
(d) 81

(1 mark)

$P(x_i) = \frac{x_i}{k}$

$P(1) = \frac{1}{k}$

$P(2) = \frac{2}{k}$

x_i	p_i
1	1/k
2	2/k
3	3/k
4	4/k
5	5/k
6	6/k
7	7/k
8	8/k
9	9/k
	1

$\frac{45}{k} = 1$

$k = 45$

QUESTION

Assume that the probability for rain on a day is 0.4. An umbrella salesman can earn ₹ 400 per day in case of rain on that day and will lose ₹ 100 per day if there is no rain. The expected earnings in (in ₹) per day of the salesman is

(a) 400
(b) 200
(c) 100
(d) 0

(1 mark)

	p_i	x_i	$p_i x_i$
Rain	0.4	₹400	160
No Rain	0.6	₹(−100)	−60
			100

Expectation

$= E(x)$

$= \Sigma p_i x_i$

$= ₹100$

RELATIVE FREQUENCY DEFINITION OF PROBABILITY

Let us consider a random experiment repeated a very good number of times, say n, under an identical set of conditions. We next assume that an event A occurs f_A times. Then the limiting value of the ratio of f_A to n as n tends to infinity is defined as the probability of A.

i.e. $$P(A) = \lim_{n \to \infty} \frac{F_A}{n}$$

This statistical definition is applicable if the above limit exists and tends to a finite value.

According to the statistical definition of probability, the probability of an event A is the

(a) limiting value of the ratio of the no. of times the event A occurs to the number of times the experiment is repeated

(b) the ratio of the frequency of the occurrences of A to the total frequency

(c) the ratio of the frequency of the occurrences of A to the non-occurrence of A

(d) the ratio of the favourable elementary events to A to the total number of elementary events.

QUESTION

CA

A, B and C are three mutually exclusive and exhaustive events such that P (A) = 2 P (B) = 3P(C). What is P (B)?

(a) 6/11 (b) 3/11

(c) 1/6 (d) 1/3

$A \cap B \cap C = \phi$

$P(A) = 2P(B) = 3P(C) = k$

$P(A) + P(B) + P(C) = 1$

$\frac{k}{1} + \frac{k}{2} + \frac{k}{3} = 1$

$\Rightarrow \frac{6k + 3k + 2k}{6} = 1 \Rightarrow 11k = 6$

$\Rightarrow k = \frac{6}{11}$

$P(A) = k$ | $2P(B) = k$, $P(B) = \frac{k}{2}$ | $3P(C) = k$, $P(C) = \frac{k}{3}$

$P(B) = \frac{\left(\frac{6}{11}\right)}{2} = \frac{3}{11}$


THANK YOU

Theoretical Distribution

Distribution of total probability (i.e. 1) on the basis of some Random variables (Discrete or continuous) makes a probability distribution.

⇓

This Probability Distribution is known as Theoretical Distribution

Age (x_i)	f_i	p_i
15 yr.	40	0.20
16 yr	60	0.30
17 yr	30	0.15
18 yr	70	0.35
	200	1

weight	p_i
40-45 kg	0.20
45-50 kg	0.40
50-55 kg	0.05
55-60 kg	0.35
	1

Probability Distribution

- Descrete
  - ⇓ Probability mass function
    - Binomial Distribution
    - Poisson Distribution
- continous
  - ⇓ Probability Density function.
    - ⇓ Normal Distribution.

Binomial Distribution

⇒ If an experiment is repeated 'n' times

$n \Rightarrow$ No. of trials

⇒ Each trial has two possible outcomes

Success & Failure

Probability of success $= p$

Probability of failure $= q$

$$\boxed{p + q = 1}$$

$$q = 1 - p$$

⇒ In each trial the probability of success & failure will be same.

X ⇒ Random variable

Probability mass function

$f(x) = P(X=r) = {}^{n}C_{r}\, p^{r}\, q^{n-r}$ for all $r = 0, 1, 2, 3, \ldots, n$

$${}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$n = 10$

$P(X=4) = {}^{10}C_4 \, p^4 \, q^6$

$n = 20$

$P(X=12) = {}^{20}C_{12} \, p^{12} \, q^8$

$n = 8$

$P(X=1) = {}^{8}C_1 \, p^1 \, q^7$

minimum success = 0

maximum success = n

$P(X=0) + P(X=1) + P(X=2) + \cdots + P(X=n) = 1$

Eg $n = 6$

$p = 0.7$

$q = 0.3$

find Prob of 2 success.

Sol:

$P(X=2) = {}^6C_2\, p^2\, q^4$

$= \frac{6!}{2!\,4!}(0.7)^2(0.3)^4$

$= 15(0.49)(0.0081)$

$= 0.0595$

$$n = 4$$

$$P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) = 1$$

P (Atleast one success)

$$= P(X \geq 1)$$

$$= P(X=1) + P(X=2) + P(X=3) + \cdots + P(X=n)$$

$$= 1 - P(X=0)$$

P (Atleast two success)

$$= P(X \geq 2)$$

$$= P(X=2) + P(X=3) + \cdots + P(X=n)$$

$$= 1 - P(X=0) - P(X=1)$$

$X \Rightarrow$ Random variable

Binomial Distribution

$X \sim B(n, p)$

n & p are parameters

" Binomial Dist. is biparametric"

Eg $X \sim B(10, \frac{1}{3})$

find $P(X=1)$

Sol:

$n = 10$

$p = \frac{1}{3}$

$q = 1 - p$

$= 1 - \frac{1}{3}$

$q = \frac{2}{3}$

Now

$P(X=1) = {}^{10}C_1 \, p^1 q^9$

$= {}^{10}C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^9$

$= 10 \times \frac{1}{3} \times \left(\frac{2}{3}\right)^9$

$= 0.0867$

Binomial Distribution

Mean $(\mu) = np$

Variance $(\sigma^2) = npq$

mean > variance

S.D. $(\sigma) = \sqrt{npq}$

Q $X \sim B(6, \frac{1}{4})$

find mean & S.D.

Sol: $n = 6$ $\quad p = \frac{1}{4}$

$q = 1 - p$

$= 1 - \frac{1}{4}$

$q = \frac{3}{4}$

Now

mean

$= np$

$= 6 \times \frac{1}{4}$

$= 1.5$

S.D

$= \sqrt{npq}$

$= \sqrt{6 \times \frac{1}{4} \times \frac{3}{4}} = 1.0606$

Mode of Binomial Distribution

→ first calculate the value of $(n+1)p$

Case-1 If $(n+1)p$ is integer then there are two mode

first mode $= (n+1)p$

second mode $= (n+1)p - 1$

Case-2 If $(n+1)p$ is non integer then only one mode

mode $= [(n+1)p]$ ⇓ Integral part

Q $n = 7$

$p = \frac{1}{4}$

$q = \frac{3}{4}$

mode = ?

Sol: $(n+1)p$

$= (7+1) \times \frac{1}{4}$

$= 8 \times \frac{1}{4}$

$= 2$

Integer

2 & 1 are two modes

Q $n = 8$

$p = \frac{1}{3}$

$q = \frac{2}{3}$

find mode = ?

Sol: $(n+1)p$

$= (8+1)\left(\frac{1}{3}\right)$

$= 9 \times \frac{1}{3}$

$= 3$ Integer

3 & 2 are two modes.

Q $n = 11$

$p = \frac{1}{5}$

$q = 4/5$

find mode

Sol: $(n+1)p$

$= (11+1) \times \frac{1}{5}$

$= 2.4$

mode $= [2.4] = 2$

Additive Rule

$X \sim B(n_1, p)$

$Y \sim B(n_2, p)$

$X+Y \sim B(n_1+n_2, p)$

Eg $X \sim B(5, \frac{1}{2})$

$Y \sim B(12, \frac{1}{2})$

$X+Y \sim B(17, \frac{1}{2})$

maximum variance $= \frac{n}{4}$

Variance will be maximum when $p = q = \frac{1}{2}$

QUESTION

An experiment succeeds twice as many times as it fails. Find the chance that in 6 trials, there will be atleast 5 success

a) 37/729
b)256/729
c)87/729
d)None

$p = 2q$

$p + q = 1$

$2q + q = 1$

$3q = 1$

$q = \frac{1}{3}$

$p = \frac{2}{3}$

$n = 6$

P(Atleast 5 success)

$= P(X \geq 5)$

$= P(X=5) + P(X=6)$

$= {}^6C_5\, p^5 q^1 + {}^6C_6\, p^6 q^0$

$= 6\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right) + (1)\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^0$

$= \frac{192}{729} + \frac{64}{729} = \frac{256}{729}$

QUESTION

CA

An experiment succeeds thrice as many times as it fails . Find the chance that in 5 trials, there will be no success at all

a)1/1024

b)2/429

c)5/512

d)None

$p = 3q$

$n = 5$

$P(X = 0)$

$= {}^5C_0 \, p^0 q^5$

QUESTION

CA

If 15 dates are selected at random, what is the probability of getting two Sundays?

a) 0.29

b) 0.30

c) 0.31

d) 0.34

$n = 15$

$p = \text{Sunday} = \frac{1}{7}$

$q = 1 - \frac{1}{7} = \frac{6}{7}$

$P(X=2) = {}^{15}C_2\, p^2\, q^{13}$

$= \frac{15!}{2!\,13!} \times \left(\frac{1}{7}\right)^2 \times \left(\frac{6}{7}\right)^{13}$

$= 105 \times \frac{1}{49} \times \left(\frac{6}{7}\right)^{13}$

$= 0.2888$

QUESTION

The incidence of occupational disease in an industry is such that the workmen have a 10% chance of suffering from it. What is the probability that out of 5 workmen, 3 or more will contract the disease?

a) 0.0086

b) 0.0092

c) 0.0045

d) 0.0096

$n = 5$

$p = \text{Disease} = \frac{10}{100} = \frac{1}{10} = 0.1$

$q = 1 - 0.1 = 0.9$

$P(X \geq 3)$

$= P(X=3) + P(X=4) + P(X=5)$

$= {}^5C_3\, p^3 q^2 + {}^5C_4\, p^4 q^1 + {}^5C_5\, p^5 q^0$

$= 10(0.1)^3(0.9)^2 + 5(0.1)^4(0.9)^1 + (1)(0.1)^5(1)$

$= 0.0081 + 0.00045 + 0.00001$

$= 0.00856$

QUESTION

Find the probability of a success for the binomial distribution satisfying the following relation 4 P (x = 4) = P (x = 2) and having the parameter n as six.

$p = ?$

a) 1/3

b) 1/4

c) 1/5

d) 1/6

$4\,P(x=4) = P(x=2)$ & $n = 6$

$4 \times {}^6C_4\, p^4 q^2 = {}^6C_2\, p^2 q^4$

$4 \times 15 \times p^4 q^2 = 15\, p^2 q^4$

$4p^2 = q^2$

$4p^2 = (1-p)^2$

$4p^2 = 1 + p^2 - 2p$

$3p^2 + 2p - 1 = 0$

$3p^2 + 3p - p - 1 = 0$

$3p(p+1) - 1(p+1) = 0$

$(p+1)(3p-1) = 0$

$p+1 = 0$ | $3p-1 = 0$

$p = -1$ ✗ | $p = 1/3$

QUESTION

Find the probability of success in a binomial distribution for which mean and standard deviation are 6 and 2 respectively.

a) 1/3

b) 2/3

c) 1/4

d) 1/5

mean = 6

$np = 6$

& SD = 2

$\sqrt{npq} = 2$

$npq = 4$

$6q = 4$

$q = \frac{4}{6} = \frac{2}{3}$

$p = 1 - q$

$= 1 - \frac{2}{3}$

$p = \frac{1}{3}$

QUESTION

What is the mode of the binomial distribution for which mean and SD are 10 and $\sqrt{5}$ respectively.

a) 10

b) 12

c) 9

d) None

mean = 10 & SD = $\sqrt{5}$

$np = 10$

$\sqrt{npq} = \sqrt{5}$

$npq = 5$

$10q = 5$

$q = 0.5$

$p = 0.5$

Now

$np = 10$

$n(0.5) = 10$

$n = 20$

$(n+1)p$

$= (20+1)(0.5)$

$= 10.5$ (Non integer)

mode $= [10.5]$

$= 10$

Poisson Distribution

$n \Rightarrow$ Large number

$p \Rightarrow$ very small

$q \Rightarrow$ very close to 1

$$\text{mean} = \mu = m = np$$

(m ⇓ Average)

$$P(x = r) = \frac{e^{-m}\, m^{r}}{r!}$$

for $r = 0, 1, 2, 3 \ldots \infty$

$e = 2.7183$

$$P(X=2) = \frac{e^{-m} m^2}{2!}$$

$$P(X=4) = \frac{e^{-m} m^4}{4!}$$

$$P(X=0) = \frac{e^{-m} m^0}{0!}$$

$$P(X=10) = \frac{e^{-m} m^{10}}{10!}$$

Eg $m = 2$

find $f(x = 3)$

Sol:

$$f(x=3) = \frac{e^{-m} m^3}{3!}$$

$$= \frac{e^{-2} (2)^3}{3 \times 2 \times 1}$$

$$= \frac{0.1353 \times 8}{6}$$

$$= 0.1804$$

$$e^{-2} = (2.7183)^{-2}$$
$$= 0.1353$$

Poisson Distribution

Eg

$n = 1000$

$p = \frac{1}{100}$

find $P(X = 5)$

Sol:

$m = np$

$= 1000 \times \frac{1}{100}$

$m = 10$

Now

$$P(X=5) = \frac{e^{-m} (m)^5}{5!}$$

$$= \frac{(2.7183)^{-10} (10)^5}{120}$$

$$= 0.0378$$

mean = m

Variance = m

S.D. = $\sqrt{m}$

$$\text{mode} = \begin{cases} m \ \& \ m-1 & \text{if } m \text{ is integer} \\ [m] & \text{if } m \text{ is non integer} \end{cases}$$

[m] integral part

eg m = 6

mode = 6 & 5

eg m = 7.2

mode = [7.2] = 7

Binomial

$X \sim B(n, P)$

Poisson

$X \sim P(m)$

Uniparametric.

Additive Property

$X \sim P(m_1)$

$Y \sim P(m_2)$

$X + Y \sim P(m_1 + m_2)$

QUESTION

If the probability that a person suffers a bad reaction from an injection of a given serum is 0.001 . Determine the probability out of 2000 individual exactly 3 person suffer from a bad reaction

a)0.1804

b)0.1504

c)0.1213

d)None

p = Reaction = 0.001

$N = 2000$

$m = Np$

$= 2000 \times 0.001$

$= 2$

$P(X = 3)$

$= \frac{e^{-m} m^3}{3!}$

$= \frac{e^{-2} \cdot (2)^3}{6}$

$= 0.1804$

QUESTION

CA

A company has two cars which it hires out during the day . The numbers of cars demanded with mean 1.5 . Find the percentage of days on which only one car was in demand is equal to

a) 23.26

b) 33.47

c) 44.62

d) 46.40

$m = 1.5$

$P(X = 1)$

$= \frac{e^{-m} m^1}{1!} = e^{-1.5}(1.5)$

$= 0.2231 \times 1.5$

$= 0.3346$

$= \frac{1.5}{e^{1.5}}$

$= \frac{1.5}{e^{3/2}} = \frac{1.5}{(e^{\frac{1}{2}})^3}$

$= 0.3346$

or

33.46%

$$e^{1.5} = e^{1+0.5}$$
$$= e^{1} \cdot e^{0.5}$$
$$= e^{1}\, e^{\frac{1}{2}}$$
$$= (2.7183)\sqrt{2.7183}$$
$$= 4.4817$$

#

$$e^{x} = AL\,(x \times 0.4343)$$

$$e^{-1.5} = AL\,(-1.5 \times 0.4343)$$
$$= 0.2231$$

QUESTION

Find the mean and standard deviation of x where x is a Poisson variate satisfying the condition P (x = 2) = P (x = 3).

$$e^{-m}\frac{m^2}{2!} = e^{-m}\frac{m^3}{3!}$$

$$\frac{m \cdot m}{2} = \frac{m\,m\,m}{6}$$

$$\frac{6}{2} = m \Rightarrow \boxed{m = 3}$$

mean = 3

variance = 3

S.D = $\sqrt{3}$ = 1.732

QUESTION

CA

X is a Poisson variate satisfying the following relation

$P(X = 2) = 9P(X = 4) + 90P(X = 6)$. What is the SD X?

a) 1

b) 2

c) 3

d) 4

$P(X=2) = 9P(X=4) + 90P(X=6)$

$\frac{e^{-m} m^2}{2!} = 9\frac{e^{-m} m^4}{4!} + 90\frac{e^{-m} m^6}{6!}$

$\frac{e^{-m} m^2}{2} = \frac{9e^{-m} m^4}{24} + \frac{90}{720} e^{-m} m^6$

$\frac{1}{2} = \frac{3}{8}m^2 + \frac{1}{8}m^4$

$\frac{1}{2} = \frac{3m^2 + m^4}{8}$

$4 = m^4 + 3m^2$

$m^4 + 3m^2 - 4 = 0$

$m^4 + 4m^2 - m^2 - 4 = 0$

$m^2(m^2+4) - 1(m^2+4) = 0$

$(m^2-1)(m^2+4) = 0$

$m^2 - 1 = 0$ | $m^2 + 4 = 0$

$m^2 = 1$ | $m^2 = -4$ ✗

$m = 1$

QUESTION

If 2 per cent of electric bulbs manufactured by a company are known to be defectives, what is the probability that a sample of 150 electric bulbs taken from the production process of that company would contain

a) exactly one defective bulb?

b) more than 2 defective bulbs?

$P = \text{Defective} = \frac{2}{100}$

$n = 150$

$m = np = 150 \times \frac{2}{100} = 3$

$P(X=1)$

$= \frac{e^{-m} m^1}{1!}$

$= e^{-3}(3)$

$= (2.7183)^{-3} \times 3$

$= 0.1493$

$P(X > 2)$

$= P(X=3) + P(X=4) + P(X=5) + \dots \infty$

$= 1 - P(X=0) - P(X=1) - P(X=2)$

$= 1 - \frac{e^{-m} m^0}{0!} - \frac{e^{-m} m^1}{1!} - \frac{e^{-m} m^2}{2!}$

$= 1 - e^{-m}\left(1 + m + \frac{m^2}{2}\right)$

QUESTION

CA

Between 9 and 10 AM, the average number of phone calls per minute coming into the switchboard of a company is 4. Find the probability that during one particular minute, there will be

a) no phone calls

b) at most 3 phone calls

(given $e^{-4} = 0.018316$)

$m = 4$

$P(X=0) = \frac{e^{-m} m^0}{0!} = \frac{e^{-4} \times 1}{1} = 0.018316$

$P(X \le 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= \frac{e^{-m} m^0}{0!} + \frac{e^{-m} m^1}{1!} + \frac{e^{-m} m^2}{2!} + \frac{e^{-m} m^3}{3!}$

$= e^{-m}\left[1 + m + \frac{m^2}{2} + \frac{m^3}{6}\right]$

$= e^{-4}\left[1 + 4 + 8 + 10.6666\right]$

$= 0.4334$