Manzil JEE 2025

Mathematics

DPP: 5

Vectors

Q1 If $\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors then $(3\vec{a}+2\vec{b})\cdot(5\vec{a}-6\vec{b})=?$

(A) 3
(B) 5
(C) 6
(D) 12

Q2 The value of $\hat{i}\cdot(\hat{j}\times\hat{k})+\hat{j}\cdot(\hat{i}\times\hat{k})+\hat{k}\cdot(\hat{i}\times\hat{k})$ is :

(A) 0
(B) -1
(C) 1
(D) 3

Q3 If the volume of parallelopiped formed by the vectors $\hat{i}+\lambda\hat{j}+\mathrm{k}, \hat{j}+\lambda\hat{k}$ and $\lambda\hat{i}+k$ is minimum, then λ is equal to:

(A) $-\frac{1}{\sqrt{3}}$
(B) $\sqrt{3}$
(C) $-\sqrt{3}$
(D) $\frac{1}{\sqrt{3}}$

Q4 Consider three vectors $\mathbf{p}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}, \mathbf{q}=2\hat{\mathbf{i}}+4\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $\mathbf{r}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ and let s be a unit vector, then If $(p\times q)\times r=up+vq+wr$, then $(u+v+w)$ is equal to

(A) 8
(B) 2
(C) -2
(D) 4

Q5 If the two adjacent sides of two rectangles are represented by the vectors $\vec{p}=5\vec{a}-3\vec{b}; \vec{q}=-\vec{a}-2\vec{b}$ and $\vec{r}=-4\vec{a}-\vec{b}; \vec{s}=-\vec{a}+\vec{b}$ respectively, then the angle between the vectors $\vec{x}=\frac{1}{3}(\vec{p}+\vec{r}+\vec{s})$ and $\overrightarrow{\mathrm{y}}=\frac{1}{5}(\overrightarrow{\mathrm{r}}+\overrightarrow{\mathrm{s}})$

(A) is $-\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$
(B) is $\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$
(C) is $\pi-\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$
(D) cannot be evaluated

Q6 Two adjacent sides of a parallelogram $ABCD$ are given by $AB=2\hat{\mathbf{i}}+10\hat{\mathbf{j}}+11\hat{\mathbf{k}}$ and $AD=-\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2\hat{\mathbf{k}}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by

(A) $\frac{8}{9}$
(B) $\frac{\sqrt{17}}{9}$
(C) $\frac{1}{9}$
(D) $\frac{4\sqrt{5}}{9}$

Q7 If $\vec{a}$ and $\vec{b}$ are non zero, non collinear, and the linear combination $(2\mathrm{x}-\mathrm{y})\overrightarrow{\mathrm{a}}+4\overrightarrow{\mathrm{b}}=5\overrightarrow{\mathrm{a}}+(\mathrm{x}-2\mathrm{y})\overrightarrow{\mathrm{b}}$ holds for real x and y then $\mathrm{x}+\mathrm{y}$ has the value equal to

(A) -3
(B) 1
(C) 17
(D) 3

Q8 Let us define the length of a vector $a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}$ as $|a|+|b|+|c|$. This definition coincides with the usual definition of length of a vector $a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}$ if and only if

(A) $a=b=c=0$
(B) any two of a, b and c are zero

(C) any one of a, b and c is zero

(D) $a+b+c=0$

Q9 Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the three vectors having magnitudes 1, 5 and 3 respectively such that the angle between $\vec{a}$ and $\vec{b}$ is θ and $\vec{a} \times (\vec{a} \times \vec{b}) = \vec{c}$ then $\tan\theta$ is equal to

(A) 0

(B) $\frac{2}{3}$

(C) $\frac{3}{5}$

(D) $\frac{3}{4}$

Q10 For non-zero vectors $\vec{a}, \vec{b}, \vec{c}, |\vec{a} \times \vec{b} \cdot \vec{c}| = |\vec{a}||\vec{b}||\vec{c}|$ holds if and only if;

(A) $\vec{a} \cdot \vec{b} = 0, \vec{b} \cdot \vec{c} = 0$

(B) $\vec{c} \cdot \vec{a} = 0, \vec{a} \cdot \vec{b} = 0$

(C) $\vec{a} \cdot \vec{c} = 0, \vec{b} \cdot \vec{c} = 0$

(D) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$

Q11 The edges of a parallelopiped are of unit length and are parailel to non-coplanal usit vector $\hat{\mathbf{a}}, \hat{\mathbf{b}}, \hat{\mathbf{c}}$ such that $\hat{\mathbf{a}} \cdot \hat{\mathbf{b}} = \hat{\mathbf{b}} \cdot \hat{\mathbf{c}} = \hat{\mathbf{c}} \cdot \hat{\mathbf{a}} = \frac{1}{2}$. Then, the volume of the parallelopiped is

(A) $\frac{1}{\sqrt{2}}$ cu unit

(B) $\frac{1}{2\sqrt{2}}$ cu unit

(C) $\frac{\sqrt{3}}{2}$ cu unit

(D) $\frac{1}{\sqrt{3}}$ cu unit

Q12 If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$, from a reciprocal system of vectors, then

$\vec{a} . \vec{a}' + \vec{b} . \vec{b}' + \vec{c} . \vec{c}' =$

(A) 0 (B) 1

(C) 2 (D) 3

Q13 A vector whose modulus is $\sqrt{51}$ and makes the same angle with $\mathbf{a} = \frac{\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}}{3}, \mathbf{b} = \frac{-4\hat{\mathbf{i}} - 3\hat{\mathbf{k}}}{5}$ and $\mathbf{c} = \hat{\mathbf{j}}$, will be

(A) $5\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + \hat{\mathbf{k}}$

(B) $5\hat{\mathbf{i}} + \hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

(C) $5\hat{\mathbf{i}} + \hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

(D) $\pm(5\hat{\mathbf{i}} - \hat{\mathbf{j}} - 5\hat{\mathbf{k}})$

Q14 Let PQR be a triangle. The points A, B and C are on the sides QR, RP and PQ respectively such that $\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}$. Then $\frac{\text{Area}(\triangle PQR)}{\text{Area}(\triangle ABC)}$ is equal to

(A) 4 (B) 3

(C) 2 (D) $\frac{5}{2}$

Q15 Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non zero vectors such that $\vec{b} \cdot \vec{c} = 0$ and $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b} - \vec{c}}{2}$. If $\vec{d}$ be a vector such that $\vec{b} \cdot \vec{d} = \vec{a} \cdot \vec{b}$, then $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$ is equal to

(A) $\frac{3}{4}$

(B) $\frac{1}{2}$

(C) $-\frac{1}{4}$

(D) $\frac{1}{4}$

Q16 Consider $\triangle ABC$ and $\triangle A_1B_1C_1$ in such a way that $\overrightarrow{AB} = \overrightarrow{A_1B_1}$ and M, N, M_1, N_1 be the mid points of AB, BC, A_1B_1 and B_1C_1 respectively, then

(A) $\overrightarrow{MM_1} = \overrightarrow{NN_1}$

(B) $\overrightarrow{CC_1} = \overrightarrow{MM_1}$

(C) $\overrightarrow{CC_1} = \overrightarrow{NN_1}$

(D) $\overrightarrow{MM_1} = \overrightarrow{BB_1}$

Q17 Let O be the origin and $\overrightarrow{OX}, \overrightarrow{OY}, \overrightarrow{OZ}$ be three unit vectors in the directions of the sides, QR, RP, PQ respectively of a $\triangle PQR$.

$|\overrightarrow{OX} \times \overrightarrow{OY}| =$

(A) $\sin(P+Q)$

(B) $\sin(P+R)$
(C) $\sin(Q+R)$
(D) $\sin 2R$

Q18 If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\overrightarrow{0}$ then $(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=?$
(A) $\frac{1}{2}$
(B) $\frac{-1}{2}$
(C) $\frac{3}{2}$
(D) $\frac{-3}{2}$

Q19 Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\overrightarrow{a}\times(\overrightarrow{b}\times\overrightarrow{c})=\frac{\sqrt{3}}{2}(\overrightarrow{b}+\overrightarrow{c})$. If $\overrightarrow{b}$ is not parallel to $\overrightarrow{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is
(A) $\frac{2\pi}{3}$
(B) $\frac{5\pi}{6}$
(C) $\frac{3\pi}{4}$
(D) $\frac{\pi}{2}$

Q20 Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a}\times\vec{b})\times\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is
(A) $\frac{2\sqrt{2}}{3}$
(B) $\frac{-\sqrt{2}}{3}$
(C) $\frac{2}{3}$
(D) $\frac{-2\sqrt{3}}{3}$

Q21 If $|\mathbf{a}+\mathbf{b}|<|\mathbf{a}-\mathbf{b}|$, then the angle between $\mathbf{a}$ and $\mathbf{b}$ can lie in the interval.
(A) $(-\pi/2, \pi/2)$
(B) $(0, \pi)$
(C) $(\pi/2, 3\pi/2)$
(D) $(0, 2\pi)$

Q22 Let $\overrightarrow{a}=2\hat{i}+3\hat{j}+4\hat{k}$, $\overrightarrow{b}=\hat{i}-2\hat{j}+\hat{k}$ and $\overrightarrow{c}=\hat{i}+\hat{j}-\hat{k}$. If $\overrightarrow{r}\times\vec{a}=\vec{b}$ and $\vec{r}\cdot\vec{c}=3$, then the value of $|\vec{r}|$ is equal to
(A) $\sqrt{155}$
(B) $\sqrt{17}$
(C) $2\sqrt{17}$
(D) 3

Q23 A plane passing through $(1,1,1)$ cuts positive direction of coordinate axes at A, B and C, then the volume of tetrahedron $OABC$ satisfies
(A) $V\leq\frac{9}{2}$
(B) $V\geq\frac{9}{2}$
(C) $V=\frac{9}{2}$
(D) none of these

Q24 Let $\vec{a}=2\hat{i}+\lambda_1\hat{j}+3\hat{k}, \vec{b}=4\hat{i}+(3-\lambda_2)\hat{j}+6\hat{k}$ and $\overrightarrow{c}=3\hat{i}+6\hat{j}+(\lambda_3-1)\hat{k}$ be three vectors such that $\vec{b}=2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$. Then a possible value of $(\lambda_1,\lambda_2,\lambda_3)$ is:
(A) $(1,5,1)$
(B) $(1,3,1)$
(C) $\left(-\frac{1}{2},4,0\right)$
(D) $\left(\frac{1}{2},4,-2\right)$

Q25 Let $\overrightarrow{\mathbf{a}}, \overrightarrow{\mathbf{b}}, \overrightarrow{\mathbf{c}}$ be three non-coplanar vectors and $\overrightarrow{\mathbf{r}}$ be any arbitrary vector, then the expression $(\overrightarrow{\mathbf{a}}\times\overrightarrow{\mathbf{b}})\times(\overrightarrow{\mathbf{r}}\times\overrightarrow{\mathbf{c}})+(\overrightarrow{\mathbf{b}}\times\overrightarrow{\mathbf{c}})\times(\overrightarrow{\mathbf{r}}\times\overrightarrow{\mathbf{a}})+(\overrightarrow{\mathbf{c}}\times\overrightarrow{\mathbf{a}})\times(\overrightarrow{\mathbf{r}}\times\overrightarrow{\mathbf{b}})$ is always equal to :
(A) $[\overrightarrow{\mathbf{a}}\,\overrightarrow{\mathbf{b}}\,\overrightarrow{\mathbf{c}}]\overrightarrow{\mathbf{r}}$
(B) $2[\overrightarrow{\mathbf{a}}\,\overrightarrow{\mathbf{b}}\,\mathbf{c}]\overrightarrow{\mathbf{r}}$
(C) $4[\overrightarrow{\mathbf{a}}\,\overrightarrow{\mathbf{b}}\,\mathbf{c}]\overrightarrow{\mathbf{r}}$
(D) $\overrightarrow{\mathbf{0}}$

Answer Key

Q1	(A)
Q2	(A)
Q3	(D)
Q4	(B)
Q5	(B)
Q6	(B)
Q7	(B)
Q8	(B)
Q9	(D)
Q10	(D)
Q11	(A)
Q12	(D)
Q13	(D)
Q14	(B)
Q15	(D)
Q16	(D)
Q17	(A)
Q18	(D)
Q19	(B)
Q20	(A)
Q21	(C)
Q22	(A)
Q23	(B)
Q24	(C)
Q25	(B)