CBSE Class 11 Maths Notes Chapter 9: CBSE Class 11 Maths, Chapter 9: Sequences and Series, you'll learn about ordered sets of numbers and how to add them up.

In this chapter we will learn about different kinds of sequences, like ones where each number increases by the same amount (called arithmetic sequences) or ones where each number is multiplied by the same amount (geometric sequences).

You'll also learn about formulas to find specific terms in a sequence and how to add up a bunch of numbers in a series. With easy-to-understand notes and examples, this chapter helps you get the hang of sequences and series, setting you up for more advanced math later on.

CBSE Class 11 Maths Notes Chapter 9 Sequences and Series PDF

You can find the PDF for CBSE Class 11 Maths Notes Chapter 9 Sequences and Series by clicking on the link provided below.

Whether you need to review concepts, prepare for exams, or practice, this PDF has useful explanations to help you understand the subject better.

CBSE Class 11 Maths Notes Chapter 9 Sequences and Series PDF

Sequence and Series Class 11 Concepts

In Chapter 9 of Class 11 Mathematics, titled "Sequences and Series," the following topics and subtopics are covered:

1. Introduction
2. Sequences
3. Series
4. Arithmetic Progression (A.P.)
5. Arithmetic mean
6. Geometric Progression (G.P.)
7. The general term of a G.P.
8. Sum to n terms of a G.P.
9. Geometric Mean (G.M.)
10. Relationship Between A.M. and G.M.
11. Sum to n Terms of Special Series

CBSE Class 11 Maths Notes Chapter 9 Sequences and Series

The solutions of CBSE Class 11 Maths Notes Chapter 9: Sequences and Series are provided below. This chapter covers fundamental concepts such as arithmetic and geometric sequences, as well as their corresponding series.

Sequences

In mathematics, any function with its domain as a set of natural numbers is termed a sequence. Specifically, a real sequence is one where the range consists of a subset of real numbers. Sequences can be represented as a list of elements, such as a1, a2, a3, ..., an. When these elements are summed up, it forms what is known as a series.

For example, if a1, a2, a3, ..., an represents a sequence, then the sum of these terms, denoted as a1+a2+a3+...+an, is considered a series. Progression occurs when the terms of a sequence follow a particular pattern, such as in Arithmetic Progressions (A.P.) or Geometric Progressions (G.P.). However, it's essential to note that not all sequences necessarily adhere to a distinct pattern.

Series

- A series is the sum of the terms of a sequence. It is formed by adding up all the terms in a sequence.
- Students learn how to find the sum of a series and understand the notation used to represent series, such as sigma notation.

Arithmetic Progression (A.P.)

An arithmetic progression (AP) is a sequence of numbers where each successive term is obtained by adding a fixed number, known as the common difference, to the preceding term. If this common difference is positive, the AP is termed an increasing AP, while if it's negative, it is termed a decreasing AP.

The common difference, typically denoted by 'd', plays a crucial role in AP calculations. In an AP, let 'a' represent the first term. The nth term (tn) of an AP is given by tn = a + (n - 1)d, where 'd' is the common difference.

The sum of the first N terms of an AP, denoted by Sn, is calculated using the formula Sn = n/2 * [a + l], where 'l' represents the last term of the AP sequence. These formulas provide a convenient way to find specific terms and sums of terms in arithmetic progressions.

Properties of an AP

- Increasing, Decreasing, Multiplying and dividing each term of an AP by a non-zero constant results into an AP.
- 3 numbers in an AP: $a-d, a, a+d$
- 4 numbers in an AP: $a-3d, a-d, a+d, a+3d$
- 5 numbers in an AP: $a-2d, a-d, a, a+d, a+2d$
- 6 numbers in an AP: $a-5d, a-3d, a-d, a+d, a+3d, a+5d$
- An AP can have zero, positive or negative common difference.
- The sum of the two terms of an AP equidistant from the beginning & end is constant and equal to the sum of first & last terms.
- Any term of an AP (except the first) is equal to half the sum of terms which are equidistant from it. $\Rightarrow an=12(an-k+an+k), k<n$
- $tr=Sr-Sr-1$
- If three numbers are in AP : a, b, c are in AP $\Rightarrow 2b=a+c$
- Nth term of an AP is a linear expression in n: $An+B$ where A is the common difference of an AP.

Geometric Progression (GP)

A geometric progression (GP) is a sequence where each term is obtained by multiplying the preceding term by a fixed number, known as the common ratio.

The first term of a GP is nonzero. The common ratio (r) can be found by dividing any term by its consecutive preceding term. In a GP with the first term 'a' and common ratio 'r', the sequence is represented as a, ar, ar^2, ar^3, ar^4, and so on. The nth term (tn) of a GP is given by tn = ar^(n-1).

The sum of the first N terms of a GP, denoted by Sn, is calculated using the formula Sn = a(1 - r^n)/(1 - r), where 'r' is not equal to 1.

For an infinite GP where |r| < 1, the sum (S∞) can be found as S∞ = a/(1 - r). Several properties characterize GPs, such as the product of terms equidistant from the beginning and the end being constant, and the relationship between consecutive terms forming a GP. If three numbers are in GP (a, b, c), then b^2 = ac. These properties make GPs versatile and applicable in various mathematical contexts.

Means

Arithmetic mean

- The arithmetic mean, also known as the average, is the sum of all the terms in a sequence divided by the total number of terms.
- Students learn how to calculate the arithmetic mean of a sequence and its significance in the context of sequences and series.

n-Arithmetic Means Between Two Numbers

When two numbers, 'a' and 'b', are given and 'a', 'a1', 'a2', 'a3', ..., 'an', 'b' form an arithmetic progression (AP), then 'a1', 'a2', 'a3', ..., 'an' represent 'n' arithmetic means (AMs) between 'a' and 'b'. To find these arithmetic means, we use the formula A1 = a + d, A2 = a + 2d, ..., An = a + nd, where 'd' is the common difference between consecutive terms, calculated as 'b - a(n + 1)'.

It is important to note that the sum of 'n' AMs inserted between 'a' and 'b' equals 'n' times a single AM between 'a' and 'b'. This property aids in finding the sum of AMs within an arithmetic progression and highlights the relationship between consecutive terms in the sequence.

Geometric Mean

Geometric Mean between Two Numbers

In a geometric progression (GP) where 'a', 'b', and 'c' are in GP, 'b' is termed the geometric mean (GM) between 'a' and 'c'. Mathematically, this relationship is represented by the equation *b*2=*ac*b2=ac, or alternatively, *b*=*ac*b=ac, given that 'a' and 'c' are positive.

n-Geometric Means between Two Numbers

When 'a' and 'b' are two numbers, and 'a', 'G1', 'G2', 'G3', ..., 'Gn', 'b' form a geometric progression (GP), then 'G1', 'G2', 'G3', ..., 'Gn' represent 'n' geometric means (GMs) between 'a' and 'b'. These GMs are calculated using the formula *G*1=*ar*,*G*2=*ar*2,...,*Gn*=*arn*−1G1=ar,G2=ar2,...,Gn=arn−1, where 'r' is calculated as (*ba*)1*n*+1(ab)n+11.

An important property to note here is that the product of 'n' GMs inserted between 'a' and 'b' is equal to the nth power of a single GM between 'a' and 'b', denoted as ∏*r*=1*nGr*=*Gn*∏r=1nGr=Gn.

Arithmetic, Geometric, and Harmonic Means between Two Given Numbers

Let 'A', 'G', and 'H' be the arithmetic, geometric, and harmonic mean between two integer numbers 'a' and 'b'. These means are calculated as *A*=*a*+*b*2A=2a+b, *G*=*ab*G=ab, and *H*=2*aba*+*b*H=a+b2ab respectively.

These means satisfy the properties $A \ge G \ge H$, $G^2 = AH$, indicating that they form a GP. Furthermore, the quadratic equation $x^2 - 2Ax + G^2 = 0$ has 'a' and 'b' as its roots.

Properties of Arithmetic & Geometric Means between Two Quantities

If 'A' and 'G' are arithmetic and geometric means between 'a' and 'b', then the quadratic equation $x^2 - 2Ax + G^2 = 0$ has 'a' and 'b' as its roots.

If 'A' and 'G' are AM and GM between two numbers 'a' and 'b', then $a = A + \sqrt{A^2 - G^2}$ and $b = A - \sqrt{A^2 - G^2}$. These properties are crucial in understanding the relationships between different types of means and their applications in various mathematical contexts.

Sigma Notations

Theorems

(i)

$$\sum_{r=1}^{n} (a_r + b_r) = \sum_{r=1}^{n} a_r + \sum_{r=1}^{n} b_r$$

(ii)

$$\sum_{r=1}^{n} k a_r = k \sum_{r=1}^{n} a_r$$

(iii)

$$\sum_{r=1}^{n} k = nk$$

Sum of n Terms of Some Special Sequences

Sum of first n natural numbers

$$\sum_{k=1}^{n} k = 1 + 2 + 3 + \ldots\ldots. + n = \frac{n(n+1)}{2}$$

Sum of squares of first n natural numbers

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \ldots\ldots. + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Sum of cubes of first n natural numbers

$$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + 3^3 + \ldots\ldots. + n^3 = \left[\frac{n(n+1)}{2}\right]^2 = \left[\sum_{k=1}^{n} k\right]^2$$

Arithmetico-Geometric series

An arithmetic-geometric progression (A.G.P.) is a sequence where each term is the product of the terms of an arithmetic progression (AP) and a geometric progression (GP). This means that each term in the sequence can be expressed as the product of corresponding terms from an AP and a GP.

$AP: 1, 3, 5, \ldots\ldots$ and $GP; 1, x, x^2, \ldots\ldots$

$AGP: 1, 3x, 5x^2, \ldots\ldots \Rightarrow AGP: 1, 3x, 5x^2, \ldots\ldots$

Sum of n terms of an Arithmetico-Geometric Series

$$S_n = a + (a+d)r + (a+2d)r^2 + \ldots\ldots + [a+(n-1)d]r^{n-1}$$

$$S_n = \frac{a}{1-r} + \frac{dr(1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]r^n}{1-r}, r \neq 1$$

Sum to Infinity

If $|r|<1$ \& $n\to\infty$, then $\lim_{n\to\infty}=0. S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$.

Harmonic Progression (HP)

A sequence, reciprocal of whose terms forms an AP is called HP.

If the sequence $a_1, a_2, a_3, \ldots\ldots\ldots\ldots, a_n$ is an HP, then

$\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots\ldots, \frac{1}{a_n}$ is an AP or vice versa. There is no formula for the sum of the n terms of an HP. For HP with first terms is a and second term is b, then nth term is $t_n = \frac{ab}{b+(n-1)(a-b)}$ If a, b, c are in $HP \Rightarrow b = \frac{2ac}{a+c}$ or $\frac{a}{c} = \frac{a-b}{b-c}$.

Harmonic Mean

If a,b,ca,b,c are in HP then, bb is the HM between a&c$a\&c$

b=2aca+c⇒b=2aca+c