ICSE Class 9 Maths Selina Solutions Chapter 4: Here we provide you with detailed ICSE Class 9 Maths Selina Chapter 4 Expansion. Our PW Subject experts have prepared these questions as per the latest syllabus prescribed by CISCE for ICSE Class 9 Maths Exam. Selina's solutions are considered very useful for ICSE class 9 maths exam preparation. For better and more strategic Preparation in the correct direction our blog will act as a pathfinder for the candidates going to appear for the ICSE Class 9 Maths Exam 2024.

ICSE Class 9 Maths Selina Solutions Chapter 4 Expansion Overview

In mathematics, the expansion of a product of sums is expressed as the sum of its products using the fact that multiplication divides addition - the reverse process of trying to write an expanded polynomial as a product is called polynomial factors. "Expand" means to remove ( ) ... but we have to do it right ( ) are called "parentheses" or "brackets". Anything inside ( ) should be treated as "package". So to say: tell everything inside the "package".

Physics Wallah ICSE Class 9 Maths Selina Solutions Chapter 4 Expansion is prepared by PW subject experts for clear understanding. By using these ICSE Class 9 Maths Selina Solutions Chapter 4 created by experts, students can improve their math skills, deepen their understanding of the subject and achieve better results in their exams.

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ICSE Class 9 Maths Selina Solutions Chapter 4 PDF

ICSE Class 9 Maths Selina Solutions Chapter 4 Expansion

Here we have provided ICSE Class 9 Maths Selina Solutions Chapter 3 Expansion for the ease of students so that they can prepare better for their ICSE Class 9 Maths Exam.

ICSE Class 9 Maths Selina Solutions Chapter 4 Exercise 4(A)

1. Find the square of:

(i) 2a + b

(ii) 3a + 7b

(iii) 3a – 4b

(iv) 3a/2b – 2b/3a

Solution:

Using the identities,

$(a + b)^2 = a^2 + b^2 + 2ab$ and

$(a - b)^2 = a^2 + b^2 - 2ab$

(i) $(2a + b)^2 = (2a)^2 + b^2 + 2(2a)(b)$

$= 4a^2 + b^2 + 4ab$

(ii) $(3a + 7b)^2 = (3a)^2 + (7b)^2 + 2(3a)(7b)$

$= 9a^2 + 49b^2 + 42ab$

(iii) $(3a - 4b)^2 = (3a)^2 + (4b)^2 - 2(3a)(4b)$

$= 9a^2 + 16b^2 - 24ab$

(iv) $(3a/2b - 2b/3a)^2 = (3a/2b)^2 + (2b/3a)^2 - 2(3a/2b)(2b/3a)$

$= 9a^2/4b^2 + 4b^2/9a^2 - 2$

2. Use identities to evaluate:

(i) $(101)^2$

(ii) $(502)^2$

(iii) $(97)^2$

(iv) $(998)^2$

Solution:

Using the identities,

$(a + b)^2 = a^2 + b^2 + 2ab$ and

$(a – b)^2 = a^2 + b^2 – 2ab$

(i) $(101)^2 = (100 + 1)^2$

$= 100^2 + 1^2 + 2×100×1$

$= 10000 + 1 + 200$

$= 10201$

(ii) $(502)^2 = (500 + 2)^2$

$= 500^2 + 2^2 + 2×500×2$

$= 250000 + 4 + 2000$

$= 252004$

(iii) $(97)^2 = (100 – 3)^2$

$= 100^2 + 3^2 – 2×100×3$

$= 10000 + 9 – 600$

$= 9409$

(iv) $(998)^2 = (1000 – 2)^2$

$= 1000^2 + 2^2 – 2×1000×2$

$= 100000 + 4 – 4000$

$= 996004$

3. Evalute:

(i) $(7x/8 + 4y/5)^2$

(ii) $(2x/7 – 7y/4)^2$

Solution:

Using the identities,

$(a + b)^2 = a^2 + b^2 + 2ab$ and

$(a - b)^2 = a^2 + b^2 - 2ab$

(i) We have, $(7x/8 + 4y/5)^2$

$= (7x/8)^2 + (4y/5)^2 + 2 \times (7x/8) \times (4y/5)$

$= 49x^2/64 + 16y^2/25 + 7xy/5$

(ii) We have, $(2x/7 - 7y/4)^2$

$= (2x/7)^2 + (7y/4)^2 - 2 \times (2x/7) \times (7y/4)$

$= 4x^2/49 + 49y^2/16 - xy$

4. Evaluate:

(i) $(a/2b + 2b/a)^2 - (a/2b - 2b/a)^2 - 4$

(ii) $(4a + 3b)^2 - (4a - 3b)^2 + 48ab$

Solution:

Using the identities,

$(a + b)^2 = a^2 + b^2 + 2ab$ and

$(a - b)^2 = a^2 + b^2 - 2ab$

(i) Given expression, $\left(\frac{a}{2b} + \frac{2b}{a}\right)^2 - \left(\frac{a}{2b} - \frac{2b}{a}\right)^2 - 4$

On expanding the term using the identity, we have

$(a/2b + 2b/a)^2 = (a/2b)^2 + (2b/a)^2 + 2 \times (a/2b) \times (2b/a)$

$= a^2/4b + 4b^2/a^2 + 2$

Next, expanding the second term using the identity, we have

$(a/2b - 2b/a)^2 = (a/2b)^2 + (2b/a)^2 - 2 \times (a/2b) \times (2b/a)$

$= a^2/4b + 4b^2/a^2 - 2$

Now, using these results in the given expression

$(a/2b + 2b/a)^2 - (a/2b - 2b/a)^2 - 4 = (a^2/4b + 4b^2/a^2 + 2) - (a^2/4b + 4b^2/a^2 - 2) - 4$

$= a^2/4b + 4b^2/a^2 + 2 – a^2/4b – 4b^2/a^2 + 2 – 4$

$= 0$

(ii) Given expression, $(4a + 3b)^2 – (4a – 3b)^2 + 48ab$

On expanding the term using the identity, we have

$(4a + 3b)^2 = (4a)^2 + (3b)^2 + 2 \times (4a) \times (3b)$

$= 16a^2 + 9b^2 + 24ab$

Next, expanding the second term using the identity, we have

$(4a – 3b)^2 = (4a)^2 + (3b)^2 – 2 \times (4a) \times (3b)$

$= 16a^2 + 9b^2 – 24ab$

Now, using these results in the given expression

$(4a + 3b)^2 – (4a – 3b)^2 + 48ab = (16a^2 + 9b^2 + 24ab) – (16a^2 + 9b^2 – 24ab) + 48ab$

$= 16a^2 + 9b^2 + 24ab – 16a^2 – 9b^2 + 24ab + 48ab$

$= 96ab$

5. If a + b = 7 and ab = 10; find a – b.

Solution:

Using the identities,

$(a + b)^2 = a^2 + b^2 + 2ab$ and

$(a – b)^2 = a^2 + b^2 – 2ab$

Rewriting the above equation as

$(a – b)^2 = a^2 + b^2 + 2ab – 4ab$

$= (a + b)^2 – 4ab$ … (i)

We have,

$a + b = 7$ and $ab = 10$

So, using these in equation (i), we get

$(a - b)^2 = (7)^2 - (4 \times 10)$

$= 49 - 40$

$= 9$

Then,

$(a - b) = \sqrt{9}$

$= \pm 3$

6. If a – b = 7 and ab = 18; find a + b.

Solution:

Using the identities,

$(a - b)^2 = a^2 + b^2 - 2ab$ and

$(a + b)^2 = a^2 + b^2 + 2ab$

Rewriting the above equation as

$(a + b)^2 = a^2 + b^2 - 2ab + 4ab$

$= (a - b)^2 + 4ab \dots (i)$

We have,

$a - b = 7$ and $ab = 18$

So, using these in equation (i), we get

$(a + b)^2 = (7)^2 + (4 \times 18)$

$= 49 + 72$

$= 121$

Then,

$(a + b) = \sqrt{121}$

$= \pm 11$

7. If x + y = 7/2 and xy = 5/2; find:

(i) $x - y$

(ii) $x^2 - y^2$

Solution:

Using the identities,

$(x + y)^2 = x^2 + y^2 + 2xy$ and

$(x - y)^2 = x^2 + y^2 - 2xy$

Rewriting the above equation as

$(x - b)^2 = x^2 + y^2 + 2xy - 4xy$

$= (x + y)^2 - 4xy$ … (1)

We have,

$x + y = 7/2$ and $xy = 5/2$

So, using these in equation (1), we get

$(x - y)^2 = (7/2)^2 - (4 \times 5/2)$

$= 49/4 - 10$

$= 9/4$

Then,

$(x - y) = \sqrt{(9/4)}$

$= \pm 3/2$ … (2)

(ii) We know that,

$x^2 - y^2 = (x + y)(x - y)$

Substituting values in RHS using given and (2), we get

$x^2 - y^2 = (7/2)(\pm 3/2)$

$= \pm 21/4$

8. If $a - b = 0.9$ and $ab = 0.36$; find:

(i) $a + b$

(ii) $a^2 - b^2$

Solution:

Using the identities,

$(a - b)^2 = a^2 + b^2 - 2ab$ and

$(a + b)^2 = a^2 + b^2 + 2ab$

Rewriting the above equation as

$(a + b)^2 = a^2 + b^2 - 2ab + 4ab$

$= (a - b)^2 + 4ab$ … (1)

We have,

$a - b = 0.9$ and $ab = 0.36$

So, using these in equation (1), we get

$(a + b)^2 = (0.9)^2 + (4 \times 0.36)$

$= 0.81 + 1.44$

$= 2.25$

Then,

$(a + b) = \sqrt{2.25}$

$= \pm 1.5$ … (2)

(ii) We know that,

$a^2 - b^2 = (a + b)(a - b)$

Substituting values in RHS using given and (2), we get

$a^2 - b^2 = (\pm 1.5)(0.9)$

$= \pm 1.35$

9. If $a - b = 4$ and $a + b = 6$; find

(i) $a^2 + b^2$

(ii) ab

Solution:

Given, $a - b = 4$ and $a + b = 6$

We know that,

$(a - b)^2 = a^2 + b^2 - 2ab$ and

$(a + b)^2 = a^2 + b^2 + 2ab$

Now, rewriting the above equation as

$(a + b)^2 = a^2 + b^2 - 2ab + 4ab$

$\Rightarrow (a + b)^2 = (a - b)^2 + 4ab$

Substituting the values in the above equation, we get

$(6)^2 = (4)^2 + 4ab$

$36 = 16 + 4ab$

$4ab = 36 - 16$

$4ab = 20$

$ab = 20/4$

(ii) Thus, $ab = 5$

Now, in the identity: $(a + b)^2 = a^2 + b^2 + 2ab$

$(a + b)^2 = (a^2 + b^2) + 2ab$

Let's substitute the values of the known terms,

$(6)^2 = (a^2 + b^2) + 2 \times (5)$

$36 = (a^2 + b^2) + 10$

$a^2 + b^2 = 36 - 10$

(i) Thus, $a^2 + b^2 = 26$

10. If a + 1/a = 6 and a ≠ 0 find:

(i) a – 1/a

(ii) $a^2 - 1/a^2$

Solution:

Using the identities,

$(a - b)^2 = a^2 + b^2 - 2ab$ and

$(a + b)^2 = a^2 + b^2 + 2ab$

(i) Now,

$(a + 1/a)^2 = a^2 + 1/a^2 + (2 \times a \times 1/a)$

$= a^2 + 1/a^2 + 2$ … (1)

Substituting the value of (a + 1/a) in the equation (1), we get

$6^2 = a^2 + 1/a^2 + 2$

$36 = a^2 + 1/a^2 + 2$

$a^2 + 1/a^2 = 36 - 2 = 34$ … (2)

Similarly,

$(a - 1/a)^2 = a^2 + 1/a^2 - (2 \times a \times 1/a)$

$= (a^2 + 1/a^2) - 2$

= 34 – 2 … [From (2)]

= 32

$\Rightarrow (a - 1/a)^2 = 32$

$a - 1/a = \pm\sqrt{32}$

$= \pm 4\sqrt{2}$ … (3)

Thus, $a - 1/a = \pm 4\sqrt{2}$

(ii) We know that,

$a^2 – 1/a^2 = (a – 1/a) (a + 1/a)$

Using the given and (3) in the above equation,

$a^2 – 1/a^2 = (\pm 4\sqrt{2}) (6)$

$= \pm 24\sqrt{2}$

Thus, $a^2 – 1/a^2 = \pm 24\sqrt{2}$

11. If a – 1/a = 8 and a ≠ 0, find:

(i) a + 1/a

(ii) $a^2 – 1/a^2$

Solution:

Using the identities,

$(a – b)^2 = a^2 + b^2 – 2ab$ and

$(a + b)^2 = a^2 + b^2 + 2ab$

(i) Now,

$(a – 1/a)^2 = a^2 + 1/a^2 – (2 \times a \times 1/a)$

$= a^2 + 1/a^2 – 2$ … (1)

Substituting the value of (a + 1/a) in the equation (1), we get

$8^2 = a^2 + 1/a^2 – 2$

$64 = a^2 + 1/a^2 – 2$

$a^2 + 1/a^2 = 64 + 2 = 66$ … (2)

Similarly,

$(a + 1/a)^2 = a^2 + 1/a^2 + (2 \times a \times 1/a)$

$= (a^2 + 1/a^2) + 2$

$= 66 + 2$ … [From (2)]

= 68

⇒ $(a + 1/a)^2 = 68$

$a + 1/a = \sqrt{68}$

$= \pm 2\sqrt{17}$ … (3)

Thus, $a + 1/a = \pm 2\sqrt{17}$

(ii) We know that,

$a^2 – 1/a^2 = (a – 1/a) (a + 1/a)$

Using the given and (3) in the above equation,

$a^2 – 1/a^2 = (8) (\pm 2\sqrt{17})$

$= \pm 16\sqrt{17}$

Thus, $a^2 – 1/a^2 = \pm 16\sqrt{17}$

12. If $a^2 – 3a + 1 = 0$, and $a \neq 0$; find:

(i) $a + 1/a$

(ii) $a^2 + 1/a^2$

Solution:

(i) Given equation,

$a^2 – 3a + 1 = 0$

$a^2 + 1 = 3a$

$(a^2 + 1)/a = 3$

⇒ $a + 1/a = 3$ … (1)

(ii) We know that,

$(a + b)^2 = a^2 + b^2 + 2ab$

Now,

$(a + 1/a)^2 = a^2 + 1/a^2 + 2(a)(1/a)$

$= a^2 + 1/a^2 + 2$

Using (1) in the above equation, we get

$(3)^2 = a^2 + 1/a^2 + 2$

$9 = a^2 + 1/a^2 + 2$

$a^2 + 1/a^2 = 9 – 2$

Thus, $a^2 + 1/a^2 = 7$

13. If $a^2 – 5a – 1 = 0$ and $a \neq 0$; find:

(i) $a – 1/a$

(ii) $a + 1/a$

(iii) $a^2 – 1/a^2$

Solution:

(i) Given, $a^2 – 5a – 1 = 0$

Rewriting the equation, we get

$a^2 – 1 = 5a$

$(a^2 – 1)/a = 5$

Hence, $a – 1/a = 5$ … (1)

(ii) We know that,

$(a + 1/a)^2 = a^2 + 1/a^2 + 2$

Manipulating the above as,

$(a + 1/a)^2 = a^2 + 1/a^2 – 2 + 4$

$(a + 1/a)^2 = (a – 1/a)^2 + 4$

Now, using (1) in the above

$(a + 1/a)^2 = (5)^2 + 4$

$(a + 1/a)^2 = 25 + 4 = 29$

Hence, $a + 1/a = \pm\sqrt{29}$ … (2)

(iii) We know that,

$a^2 – 1/a^2 = (a + 1/a)(a – 1/a)$

Now, using (1) and (2) in the above equation, we get

$a^2 – 1/a^2 = (5) \times (\pm\sqrt{29})$

Hence, $a^2 – 1/a^2 = \pm 5\sqrt{29}$

14. If 3a + 4b = 16 and ab = 4; find the value of $9a^2 + 16b^2$.

Solution:

Given, $3a + 4b = 16$ and $ab = 4$

Required to find: value of $9a^2 + 16b^2$

We know that,

$(a + b)^2 = a^2 + b^2 + 2ab$

Now, the square of $(3a + 4b)$ will be

$(3a + 4b)^2 = (3a)^2 + (4b)^2 + 2 \times (3a) \times (4b)$

$= 9a^2 + 4b^2 + 24ab$

And, given $3a + 4b = 16$

So, by squaring on both the sides

$(3a + 4b)^2 = 16^2$

$9a^2 + 4b^2 + 24ab = 256$

$9a^2 + 4b^2 + 24(4) = 256$ [Given ab = 4]

$9a^2 + 4b^2 = 256 – 96$

$\Rightarrow 9a^2 + 4b^2 = 160$

15. The number a is 2 more than the number b. If the sum of the squares of a and b is 34, then find the product of a and b.

Solution:

Given, a is 2 more than b

⇒ $a = b + 2$

And, sum of squares of a and b is 34

⇒ $a^2 + b^2 = 34$

Let's replace $a = (b + 2)$ in the above equation and solve for b

Then,

$(b + 2)^2 + b^2 = 34$

$2b^2 + 4b - 30 = 0$

$b^2 + 2b - 15 = 0$

$(b + 5)(b - 3) = 0$

So,

$b = -5$ or 3

Now,

For $b = -5$, $a = -5 + 2 = -3$

For $b = 3$, $a = 3 + 2 = 5$

Thus, the product of a and b is 15 in both cases.

16. The difference between two positive numbers is 5 and the sum of their squares is 73. Find the product of these numbers.

Solution:

Let's assume the two positive numbers as a and b

Given, the difference between them is 5 and the sum of their squares is 73

So, we have

$a - b = 5$ … (i) and

$a^2 + b^2 = 73$ … (ii)

On squaring (i) on both sides, we get

$(a – b)^2 = 5^2$

$(a^2 + b^2) – 2ab = 25$

$73 – 2ab = 25$ … [Using (ii), given]

So,

$2ab = 73 – 25 = 48$

$ab = 24$

Therefore, the product of numbers is 24.

Exercise 4(B)

1. Find the cube of:

(i) 3a – 2b

(ii) 5a + 3b

(iii) 2a + 1/2a

(iv) 3a – 1/a (a ≠ 0)

Solution:

Using the identities,

$(a + b)^3 = a^3 + 3ab (a + b) + b^3$ and

$(a – b)^3 = a^3 – 3ab (a – b) + b^3$

(i) $(3a – 2b)^3 = (3a)^3 – 3 \times 3a \times 2b (3a – 2b) – (2b)^3$

$= 27a^3 – 18ab (3a – 2b) – 8b^3$

$= 27a^3 – 54a^2b + 36ab^2 – 8b^3$

(ii) $(5a + 3b)^3 = (5a)^3 + 3 \times 5a \times 3b (5a + 3b) + (3b)^3$

$= 125a^3 + 45ab (5a + 3b) + 27b^3$

$= 125a^3 + 225a^2b + 135ab^2 + 27b^3$

(iii) $(2a + 1/2a)^3 = (2a)^3 + 3 \times 2a \times 1/2a\ (2a + 1/2a) + (1/2a)^3$

$= 8a^3 + 3\ (2a + 1/2a) + 1/8a^3$

$= 8a^3 + 6a + 3/2a + 1/8a^3$

(iv) $(3a – 1/a)^3 = (3a)^3 – 3 \times 3a \times 1/a\ (3a – 1/a) – (1/a)^3$

$= 27a^3 – 9\ (3a – 1/a) – 1/a^3$

$= 27a^3 – 27a + 9a – 1/a^3$

2. If $a^2 + 1/a^2 = 47$ and $a \neq 0$ find:

(i) $a + 1/a$

(ii) $a^3 + 1/a^3$

Solution:

(i) Given, $a^2 + 1/a^2 = 47$

We know that,

$(a + 1/a)^2 = a^2 + 1/a^2 + 2 \times a \times 1/a$

$= (a^2 + 1/a^2) + 2$

$= 47 + 2$

$= 49$

So,

$a + 1/a = \sqrt{49}$

$= \pm 7$ … (1)

(ii) Using the identity

$(a + b)^3 = a^3 + 3ab\ (a + b) + b^3$

Now,

$(a + 1/a)^3 = a^3 + 1/a^3 + 3(a + 1/a)$

$a^3 + 1/a^3 = (a + 1/a)^3 – 3(a + 1/a)$

$= (\pm7)^3 – 3(\pm7)$ … [From (1)]

$= \pm343 – \pm21$

Hence, $a^3 + 1/a^3 = \pm322$

3. If $a^2 + 1/a^2 = 18$; $a \neq 0$ find:

(i) $a – 1/a$

(ii) $a^3 – 1/a^3$

Solution:

(i) Given, $a^2 + 1/a^2 = 18$

Using the identity $(a + b)^2 = a^2 + b^2 + 2ab$

Now,

$(a – 1/a)^2 = a^2 + 1/a^2 – 2(a)(1/a)$

$= (a^2 + 1/a^2) – 2$

$= 18 – 2$

$= 16$

Hence,

$a – 1/a = \sqrt{16}$

$= \pm4$ … (1)

(ii) Using the identity,

$(a – b)^3 = a^3 – 3ab (a – b) + b^3$

Now,

$(a – 1/a)^3 = a^3 – 3a(1/a) (a – 1/a) + (1/a)^3$

$= a^3 – 3 (a – 1/a) + 1/a^3$

$a^3 + 1/a^3 = (a – 1/a)^3 + 3 (a – 1/a)$

$= (\pm 4)^3 + 3(\pm 4)$

$= \pm 64 \pm 12$

Hence,

$a^3 + 1/a^3 = \pm 76$

4. If a + 1/a = p and a ≠ 0; then show that:

$a^3 + 1/a^3 = p (p^2 – 3)$

Solution:

Given, a + 1/a = p … (1)

Now, cubing on both sides

$(a + 1/a)^3 = p^3$

$a^3 + 1/a^3 + 3(a + 1/a) = p^3$

$a^3 + 1/a^3 = p^3 – 3(a + 1/a)$

$= p^3 – 3(p)$ [From (1)]

$= p (p^2 – 3)$

– Hence proved

5. If a + 2b = 5; then show that:

$a^3 + 8b^3 + 30ab = 125$.

Solution:

Given, a + 2b = 5

Let’s cube it on both sides,

$(a + 2b)^3 = 5^3$

$a^3 + 3(a)(2b)(a + 2b) + (2b)^3 = 125$

$a^3 + 6ab(a + 2b) + 8b^3 = 125$

$a^3 + 8b^3 = 125 – 6ab (a + 2b)$

$= 125 – 6ab (5)$ … [Given]

$= 125 – 30ab$

So,

$a^3 + 8b^3 + 30ab = 125$

– Hence showed

6. If $(a + 1/a)^2 = 3$ and $a \neq 0$, then show: $a^3 + 1/a^3 = 0$.

Solution:

Given, $(a + 1/a)^2 = 3$

$\Rightarrow a + 1/a = \pm\sqrt{3}$ … (1)

We know the identity,

$(a + 1/a)^3 = a^3 + 1/a^3 + 3(a + 1/a)$

$a^3 + 1/a^3 = (a + 1/a)^3 – 3(a + 1/a)$

$= (\pm\sqrt{3})^3 – 3(\pm\sqrt{3})$

$= \pm3\sqrt{3} – (\pm3\sqrt{3})$

$= 0$

Thus, $a^3 + 1/a^3 = 0$

7. If $a + 2b + c = 0$; then show that:

$a^3 + 8b^3 + c^3 = 6abc$

Solution:

We have, $a + 2b + c = 0$

$a + 2b = -c$

Now, on cubing it on both sides we get

$(a + 2b)^3 = (-c)^3$

$a^3 + (2b)^3 + 3(a)(2b)(a + 2b) = -c^3$

$a^3 + 8b^3 + 6ab\ (a + 2b) = -c^3$

$a^3 + 8b^3 + 6ab\ (-c) = -c^3$

$a^3 + 8b^3 – 6abc = -c^3$

Hence,

$a^3 + 8b^3 + c^3 = 6abc$

8. Use property to evaluate:

(i) $13^3 + (-8)^3 + (-5)^3$

(ii) $7^3 + 3^3 + (-10)^3$

(iii) $9^3 – 5^3 – 4^3$

(iv) $38^3 + (-26)^3 + (-12)^3$

Solution:

The property is if $a + b + c = 0$ then

$a^3 + b^3 + c^3 = 3abc$

Now,

(i) $a = 13$, $b = -8$ and $c = -5$

$\Rightarrow 13^3 + (-8)^3 + (-5)^3 = 3(13)(-8)(-5)$ … [Since, $13 + (-8) + (-5) = 0$]

$= 1560$

(ii) $a = 7$, $b = 3$, $c = -10$

$\Rightarrow 7^3 + 3^3 + (-10)^3 = 3(7)(3)(-10)$ … [Since, $7 + 3 + (-10) = 0$]

$= -630$

(iii)$a = 9$, $b = -5$, $c = -4$

$\Rightarrow 9^3 – 5^3 – 4^3 = 9^3 + (-5)^3 + (-4)^3$ … [Since, $9 + (-5) + (-4) = 0$]

= 3(9) (-5) (-4) = 540

(iv) a = 38, b = -26, c = -12

⇒ $38^3 + (-26)^3 + (-12)^3 = 3(38)(-26)(-12)$ … [Since, 38 + (-26) + (-12) = 0]

= 35568

9. If a ≠ 0 and a – 1/a = 3; find:

(i) $a^2 + 1/a^2$

(ii) $a^3 - 1/a^3$

Solution:

(i) We have, a – 1/a = 3

On squaring on both sides, we get

$(a - 1/a)^2 = 3^2$

$a^2 + 1/a^2 - 2 = 9$

$a^2 + 1/a^2 = 9 + 2$

Hence,

$a^2 + 1/a^2 = 11$

(ii) We have, a – 1/a = 3

On cubing on both sides, we get

$(a - 1/a)^3 = 3^3$

$a^2 - 1/a^3 - 3(a - 1/a) = 27$

$a^2 - 1/a^3 = 27 + 3(a - 1/a)$

= 27 + 3(3)

= 27 + 9

Hence,

$a^3 – 1/a^3 = 36$

10. If a ≠ 0 and a – 1/a = 4; find:

(i) $a^2 + 1/a^2$

(ii) $a^4 + 1/a^4$

(iii) $a^3 – 1/a^3$

Solution:

(i) We have, a – 1/a = 4 … (a)

On squaring it on both sides, we get

$(a – 1/a)^2 = 4^2$

$a^2 + 1/a^2 – 2(a)(1/a) = 16$

$a^2 + 1/a^2 – 2 = 16$

$a^2 + 1/a^2 = 16 + 2 = 18$ … (1)

Hence, $a^2 + 1/a^2 = 18$

(ii) Now, we know that

$a^4 + 1/a^4 = (a^2 + 1/a^2)^2 – 2$

$= 18^2 – 2$ … [From (1)]

= 324 – 2

Hence, $a^4 + 1/a^4 = 322$

(iii) On cubing (i) on both sides, we get

$(a – 1/a)^3 = 4^3$

$a^3 – 1/a^3 – 3(a – 1/a) = 64$

$a^3 – 1/a^3 = 64 + 3(a – 1/a)$

$= 64 + 3(4)$ … [Given]

$= 64 + 12$

Hence, $a^3 – 1/a^3 = 76$

11. If $x \neq 0$ and $x + 1/x = 2$; then show that:

$x^2 + 1/x^2 = x^3 + 1/x^3 = x^4 + 1/x^4$

Solution:

We have, $x + 1/x = 2$

We know that,

$(x + 1/x)^2 = x^2 + 1/x^2 + 2$

$(2)^2 = x^2 + 1/x^2 + 2$

$x^2 + 1/x^2 = 4 – 2$

$= 2$ … (i)

Next, calculating

$(x + 1/x)^3 = x^3 + 1/x^3 + 3(x + 1/x)$

$(2)^3 = x^3 + 1/x^3 + 3(2)$

$x^3 + 1/x^3 = 2^3 – 3(2)$

$= 8 – 6$

$= 2$ … (ii)

Next, we know that

$x^4 + 1/x^4 = (x^2 + 1/x^2) – 2$

$= 2^2 – 2$ … [From (i)]

$= 4 – 2$

$= 2$ … (iii)

Therefore, from (i), (ii) and (iii) we have

$x^2 + 1/x^2 = x^3 + 1/x^3 = x^4 + 1/x^4$

12. If $2x – 3y = 10$ and $xy = 16$; find the value of $8x^3 – 27y^3$.

Solution:

Given,

$2x – 3y = 10$ … (i) and

$xy = 16$ … (ii)

Now, on cubing (i) on both sides

$(2x – 3y)^3 = 10^3$

$(2x)^3 – 3(2x)(3y) (2x – 3y) – (3y)^3 = 1000$ []

$8x^3 – 18(xy) (2x – 3y) – 27y^3 = 1000$

$8x^3 – 18 \times 16 \times 10 – 27y^3 = 1000$

$8x^3 – 2880 – 27y^3 = 1000$

$8x^3 – 27y^3 = 1000 + 2880$

$8x^3 – 27y^3 = 3880$

13. Expand:

(i) $(3x + 5y + 2z) (3x – 5y + 2z)$

(ii) $(3x – 5y – 2z) (3x – 5y + 2z)$

Solution:

(i) We have, $(3x + 5y + 2z) (3x – 5y + 2z)$

$= \{(3x + 2z) + (5y)\} \{(3x + 2z) – (5y)\}$ … [By grouping]

$= (3x + 2z)^2 – (5y)^2$ … [As $(a + b) (a – b) = a^2 – b^2$]

$= 9x^2 + 4z^2 + (2 \times 3x \times 2z) – 25y^2$

$= 9x^2 + 4z^2 + 12xz – 25y^2$

$= 9x^2 + 4z^2 – 25y^2 + 12xz$

(ii) We have, $(3x – 5y – 2z) (3x – 5y + 2z)$

$= \{(3x – 5y) – (2z)\} \{(3x – 5y) + (2z)\}$ … [By grouping]

$= (3x – 5y)^2 – (2z)^2$ … [As $(a + b) (a – b) = a^2 – b^2$]

$= 9x^2 + 25y^2 – 2 \times 3x \times 5y – 4z^2$

$= 9x^2 + 25y^2 – 30xy – 4z^2$

$= 9x^2 +25y^2 – 4z^2 – 30xy$

14. The sum of two numbers is 9 and their product is 20. Find the sum of their

(i) Squares (ii) Cubes

Solution:

Given, the sum of two numbers is 9 and their product is 20

Let's assume the numbers to 'a' and 'b'

So, we have

$a + b = 9$ … (1) and

$ab = 20$ … (2)

Now,

On squaring (1) on both sides gives, we get

$(a + b)^2 = 9^2$

$a^2 + b^2 + 2ab = 81$

$a^2 + b^2 + 2(20) = 81$ … [From (2)]

$a^2 + b^2 + 40 = 81$

$a^2 + b^2 = 81 – 40 = 41$

(i) Hence, the sum of their squares is 41

Next,

On cubing (1) on both sides, we get

$(a + b)^3 = 9^3$

$a^3 + b^3 + 3ab (a + b) = 729$

$a^3 + b^3 + 3 \times (20) \times (9) = 729$ … [From (1) and (2)]

$a^3 + b^3 = 729 – 540 = 189$

(ii) Hence, the sum of their cubes is 189.

15. Two positive numbers x and y are such that x > y. If the difference of these numbers is 5 and their product is 24, find:

(i) Sum of these numbers

(ii) Difference of their cubes

(iii) Sum of their cubes.

Solution:

Given $x – y = 5$ and $xy = 24$ $(x>y)$

$(x + y)^2 = (x – y)^2 + 4xy = 25 + 96 = 121$

So, $x + y = 11$; sum of these numbers is 11.

Cubing on both sides gives

$(x – y)^3 = 5^3$

$x^3 – y^3 – 3xy(x – y) = 125$

$x^3 – y^3 – 72(5) = 125$

$x^3 – y^3 = 125 + 360 = 485$

So, difference of their cubes is 485.

Cubing both sides, we get

$(x + y)^3 = 11^3$

$x^3 + y^3 + 3xy(x + y) = 1331$

$x^3 + y^3 = 1331 – 72(11) = 1331 – 792 = 539$

So, sum of their cubes is 539.

16. If $4x^2 + y^2 = a$ and $xy = b$, find the value of $2x + y$.

Solution:

Given, $xy = b$ … (i) and $4x^2 + y^2 = a$ … (ii)

Now,

$(2x + y)^2 = (2x)^2 + 4xy + y^2$

$= (4x^2 + y^2) + 4xy$

$= a + 4b$ … [Using (i) and (ii)]

Hence,

$2x + y = \pm\sqrt{(a + 4b)}$

Exercise 4(C)

1. Expand:

(i) $(x + 8)(x + 10)$

(ii) $(x + 8)(x – 10)$

(iii) $(x – 8)(x + 10)$

(iv) $(x – 8)(x – 10)$

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

(i) We have, $(x + 8)(x + 10)$

$= x^2 + (8 + 10)x + 8 \times 10$

$= x^2 + 18x + 80$

(ii) We have, $(x + 8)(x – 10)$

$= x^2 + (8 – 10)x + 8 \times (-10)$

$= x^2 – 2x – 80$

(iii) (ii) We have, $(x – 8)(x + 10)$

$= x^2 + (-8 + 10)x + (-8) \times 10$

$= x^2 + 2x – 80$

(iv) We have, $(x – 8)(x – 10)$

$= x^2 + (-8 – 10)x + (-8) \times (-10)$

$= x^2 – 18x + 80$

2. Expand:

(i) $(2x – 1/x)(3x + 2/x)$

(ii) $(3a + 2/b)(2a – 3/b)$

Solution:

(i) We have, $(2x – 1/x)(3x + 2/x)$

$= (2x)(3x) + (2x)(2/x) – (1/x)(3x) – (1/x)(2/x)$

$= 6x^2 + 4 – 3 – 2/x^2$

$= 6x^2 + 1 – 2/x^2$

(ii) We have, $(3a + 2/b)(2a – 3/b)$

$= (3a)(2a) – (3a)(3/b) + (2/b)(2a) – (2/b)(3/b)$

$= 6a^2 – 9a/b + 4a/b – 6/b^2$

$= 6a^2 – 5a/b – 6/b^2$

3. Expand:

(i) $(x + y – z)^2$

(ii) $(x - 2y + 2)^2$

(iii) $(5a - 3b + c)^2$

(iv) $(5x - 3y - 2)^2$

(v) $(x - 1/x + 5)^2$

Solution:

(i) $(x + y - z)^2 = x^2 + y^2 + z^2 + 2(x)(y) - 2(y)(z) - 2(z)(x)$

$= x^2 + y^2 + z^2 + 2xy - 2yz - 2zx$

(ii) $(x - 2y + 2)^2 = x^2 + (-2y)^2 + 2^2 + 2(x)(-2y) + 2(-2y)(2) + 2(2)(x)$

$= x^2 + 4y^2 + 4 - 4xy - 8y + 4x$

(iii) $(5a - 3b + c)^2 = (5a)^2 + (-3b)^2 + c^2 + 2(5a)(-3b) + 2(-3b)(c) + 2(c)(5a)$

$= 25a^2 + 9b^2 + c^2 - 30ab - 6bc + 10ac$

(iv) $(5x - 3y - 2)^2 = (5x)^2 + (-3y)^2 + (-2)^2 + 2(5x)(-3y) + 2(-3y)(-2) + 2(-2)(5x)$

$= 25x^2 + 9y^2 + 4 - 30xy + 12y - 20x$

(v) $(x - 1/x + 5)^2 = (x)^2 + (-1/x)^2 + (5)^2 + 2(x)(-1/x) + 2(-1/x)(5) + 2(5)(x)$

$= x^2 + 1/x^2 + 25 - 2 - 10/x + 10x$

$= x^2 + 1/x^2 + 23 - 10/x + 10x$

4. If $a + b + c = 12$ and $a^2 + b^2 + c^2 = 50$; find $ab + bc + ca$.

Solution:

Given, $a + b + c = 12$ and $a^2 + b^2 + c^2 = 50$

We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$12^2 = 50 + 2(ab + bc + ca)$

$144 = 50 + 2(ab + bc + ca)$

$ab + bc + ca = (144 – 50)/ 2$

$= 94/2$

Thus,

$ab + bc + ca = 47$

5. If $a^2 + b^2 + c^2 = 35$ and $ab + bc + ca = 23$; find $a + b + c$.

Solution:

Given, $a^2 + b^2 + c^2 = 35$ and $ab + bc + ca = 23$

We know that,

$(a + b + c)^2 = (a^2 + b^2 + c^2) + 2(ab + bc + ca)$

$(a + b + c)^2 = 35 + 2(23)$

$(a + b + c)^2 = 35 + 46$

$(a + b + c)^2 = 81$

$(a + b + c) = \pm\sqrt{81}$

Thus,

$a + b + c = \pm 9$

6. If $a + b + c = p$ and $ab + bc + ca = q$; find $a^2 + b^2 + c^2$.

Solution:

Given, $a + b + c = p$ and $ab + bc + ca = q$

We know that,

$(a + b + c)^2 = (a^2 + b^2 + c^2) + 2(ab + bc + ca)$

$(p)^2 = (a^2 + b^2 + c^2) + 2(q)$

$\Rightarrow a^2 + b^2 + c^2 = p^2 – 2q$

7. If $a^2 + b^2 + c^2 = 50$ and $ab + bc + ca = 47$, find $a + b + c$.

Solution:

Given, $a^2 + b^2 + c^2 = 50$ and $ab + bc + ca = 47$

We know that,

$(a + b + c)^2 = (a^2 + b^2 + c^2) + 2(ab + bc + ca)$

$(a + b + c)^2 = 50 + 2(47)$

$(a + b + c)^2 = 50 + 94$

$= 144$

$\Rightarrow (a + b + c) = \sqrt{144}$

Thus,

$a + b + c = \pm 12$

8. If $x + y – z = 4$ and $x^2 + y^2 + z^2 = 30$, then find the value of $xy – yz – zx$.

Solution:

Given, $x + y – z = 4$ and $x^2 + y^2 + z^2 = 30$

We know that,

$(x + y – z)^2 = x^2 + y^2 + z^2 + 2(xy – yz – zx)$

$4^2 = 30 + 2(xy – yz – zx)$

$16 – 30 = 2(ab + bc + ca)$

$xy – yz – zx = -14/ 2$

Thus,

$xy – yz – zx = -7$

Exercise 4(D)

1. If $x + 2y + 3z = 0$ and $x^3 + 4y^3 + 9z^3 = 18xyz$; evaluate:

$$\frac{(x+2y)^2}{xy}+\frac{(2y+3z)^2}{yz}+\frac{(3z+x)^2}{zx}$$

Solution:

Given, $x^3 + 4y^3 + 9z^3 = 18xyz$ and $x + 2y + 3z = 0$

So,

$x + 2y = -3z$, $2y + 3z = -x$ and $3z + x = -2y$

Now,

$$\frac{(x+2y)^2}{xy}+\frac{(2y+3z)^2}{yz}+\frac{(3x+x)^2}{zx}=\frac{(-3z)^2}{xy}+\frac{(-x)^2}{yz}+\frac{(-2y)^2}{zx}$$

$$=\frac{9z^2}{xy}+\frac{x^2}{yz}+\frac{4y^2}{zx}$$

$$=\frac{x^3+4y^3+9z^3}{xyz}$$

Given that $x^3 + 4y^3 + 9z^3 = 18xyz$

$$\therefore \frac{(x+2y)^2}{xy}+\frac{(2y+3z)^2}{yz}+\frac{(3z+x)^2}{zx}=\frac{18xyz}{xyz}=18$$

2. If a + 1/a = m and a ≠ 0; find in terms of 'm'; the value of:

(i) a – 1/a

(ii) $a^2 – 1/a^2$

Solution:

(i) Given, $a + 1/a = m$

On squaring on both sides, we get

$(a + 1/a)^2 = m^2$

$a^2 + 1/a^2 + 2 = m^2$

$a^2 + 1/a^2 = m^2 – 2$ … (1)

Now, consider the expansion

$(a – 1/a)^2 = a^2 + 1/a^2 – 2$

$= m^2 – 2 – 2$ … [From (1)]

$= m^2 – 4$

So,

$(a – 1/a) = \pm\sqrt{(m^2 – 4)}$ … (2)

(ii) We know that,

$a^2 – 1/a^2 = (a – 1/a) (a + 1/a)$

$= m [\pm\sqrt{(m^2 – 4)}]$

$= \pm m\sqrt{(m^2 – 4)}$

3. In the expansion of $(2x^2 – 8) (x – 4)^2$; find the value of

(i) coefficient of x^3

(ii) coefficient of x^2

(iii) constant term

Solution:

We have, $(2x^2 – 8) (x – 4)^2$

$= (2x^2 – 8) (x^2 – 2 × 4 × x + 4^2)$

$= (2x^2 – 8) (x^2 – 8x + 16)$

$= 2x^2(x^2 – 8x + 16) – 8(x^2 – 8x + 16)$

$= 4x^4 – 16x^3 + 32x^2 – 8x^2 + 64x – 128$

$= 4x^4 – 16x^3 + 24x^2 + 64x – 128$

Now,

(i) coefficient of x^3 = -16

(ii) coefficient of x^2 = 24

(iii) constant term = -128

4. If x > 0 and $x^2 + 1/9x^2 = 25/36$. Find: $x^3 + 1/27x^3$

Solution:

Given, $x^2 + 1/9x^2 = 25/36$ … (1)

Now, consider the expansion

$(x + 1/3x)^2 = x^2 + (1/3x)^2 + (2 × x × 1/3x)$

$= (x^2 + 1/9x^2) + 2/3$

= 25/36 + 2/3 … [From (1)]

= 49/36

So,

$(x + 1/3x) = ±\sqrt{(49/36)}$

= ±7/6 … (2)

Now, consider the expansion

$(x + 1/3x)^3 = x^3 + (1/3x)^3 + 3(x + 1/3x)$

$(7/6)^3 = x^3 + (1/3x)^3 + 3(7/6)$ …… [From (2)]

$343/216 = x^3 + 1/27x^3 + 21/6$

$x^3 + 1/27x^3 = 343/216 – 21/6$

$= (343 – 252)/216$

$= 91/216$

Thus, $x^3 + 1/27x^3 = 91/216$

5. If $2(x^2 + 1) = 5x$, find:

(i) $x – 1/x$

(ii) $x^3 – 1/x^3$

Solution:

(i) Given, $2(x^2 + 1) = 5x$

$x^2 + 1 = 5x/2$

On dividing by x on both sides, we have

$(x^2 + 1)/x = 5/2$

$\Rightarrow (x + 1/x) = 5/2$ … (1)

Now, consider the expansion of $(x + 1/x)^2$

$(x + 1/x)^2 = x^2 + 1/x^2 + 2$

$(5/2)^2 = x^2 + 1/x^2 + 2$ … [From (1)]

$x^2 + 1/x^2 = 25/4 – 2$

$= (25 – 8)/4$

$= 17/4$ … (2)

Now,

$(x – 1/x)^2 = x^2 + 1/x^2 – 2$

$= 17/4 – 2$ … [From (2)]

$= (17 – 8)/4$

$= 9/4$

So,

$x - 1/x = \sqrt{9/4}$

Thus,

(i) $x - 1/x = \pm 3/2$ … (3)

Next, we know that

$(x^3 - 1/x^3) = (x - 1/x)^3 + 3(x - 1/x)$

$= (\pm 3/2)^3 + 3(\pm 3/2)$ … [From (3)]

$= \pm\ 27/8 \pm 9/2$

$= \pm\ (27 + 36)/8$

$= \pm\ 63/8$

(ii) Thus, $x^3 - 1/x^3 = \pm 63/8$

6. If $a^2 + b^2 = 34$ and $ab = 12$; find:

(i) $3(a + b)^2 + 5(a - b)^2$

(ii) $7(a - b)^2 - 2(a + b)^2$

Solution:

We have, $a^2 + b^2 = 34$ and $ab = 12$

We know that,

$(a + b)^2 = (a^2 + b^2) + 2ab$

$= 34 + 2 \times 12$

$= 34 + 24$

$= 58$

Also, we know that

$(a - b)^2 = (a^2 + b^2) - 2ab$

$= 34 - 2 \times 12$

$= 34 - 24$

$= 10$

(i) $3(a + b)^2 + 5(a - b)^2$

$= 3 \times 58 + 5 \times 10$

$= 174 + 50$

$= 224$

(ii) $7(a - b)^2 - 2(a + b)^2$

$= 7 \times 10 - 2 \times 58$

$= 70 - 116$

$= -46$

7. If $3x - 4/x = 4$ and $x \neq 0$; find: $27x^3 - 64/x^3$.

Solution:

Given, $3x - 4/x = 4$

Now, let's consider the expansion of $(3x - 4/x)^3$

$(3x - 4/x)^3 = 27x^3 - 64/x^3 - 3 \times 3x \times 4/x(3x - 4/x)$

$(4)^3 = 27x^3 - 64/x^3 - 36(3x - 4/x)$

$64 = 27x^3 - 64/x^3 - 36(4)$

$64 = 27x^3 - 64/x^3 - 144$

$27x^3 - 64/x^3 = 144 + 64$

Hence,

$27x^3 - 64/x^3 = 208$

8. If $x^2 + 1/x^2 = 7$ and $x \neq 0$; find the value of: $7x^3 + 8x - 7/x^3 - 8/x$.

Solution:

Given, $x^2 + 1/x^2 = 7$

On subtracting 2 from both sides, we get

$x^2 + 1/x^2 - 2 = 7 - 2$

$(x - 1/x)^2 = 5$

$x - 1/x = \pm\sqrt{5}$ … (1)

Now, consider

$(x - 1/x)^3 = x^3 - 1/x^3 - 3(x - 1/x)$

$(\pm\sqrt{5})^3 = x^3 - 1/x^3 - 3(\pm\sqrt{5})$

$x^3 - 1/x^3 = (\pm\sqrt{5})^3 + 3(\pm\sqrt{5})$ … (2)

Taking,

$7x^3 + 8x - 7/x^3 - 8/x$

$= 7x^3 - 7/x^3 + 8x - 8/x$

$= 7(x^3 - 1/x^3) + 8(x - 1/x)$

$= 7[(\pm\sqrt{5})^3 + 3(\pm\sqrt{5})] + 8(\pm\sqrt{5})$

$= \pm 35\sqrt{5} \pm 21\sqrt{5} \pm 8\sqrt{5}$

$= \pm 64\sqrt{5}$

9. If $x = 1/(x - 5)$ and $x \neq 5$, find $x^2 - 1/x^2$.

Solution:

Given, $x = 1/(x - 5)$

By cross multiplying, we have

$x (x - 5) = 1$

$x^2 - 5x = 1$

$x^2 - 1 = 5x$

Dividing both sides by x,

$(x^2 - 1)/x = 5$

$(x - 1/x) = 5$ … (1)

Now,

$(x - 1/x)^2 = 5^2$

$x^2 + 1/x^2 - 2 = 25$

$x^2 + 1/x^2 = 25 + 2$

$= 27$ … (2)

Considering the expansion $(x + 1/x)^2$

$(x + 1/x)^2 = x^2 + 1/x^2 + 2$

$(x + 1/x)^2 = 27 + 2$ … [From (1)]

$(x + 1/x)^2 = 29$

$x + 1/x = \pm\sqrt{29}$ … (3)

We know that,

$x^2 - 1/x^2 = (x + 1/x)(x - 1/x)$

$= (\pm\sqrt{29})(5)$ … [From (3)]

$= \pm 5\sqrt{29}$

10. If $x = 1/(5 - x)$ and $x \neq 5$; find $x^3 + 1/x^3$.

Solution:

Given, $x = 1/(5 - x)$

By cross multiplying, we have

$x (5 – x) = 1$

$x^2 – 5x = -1$

$x^2 + 1 = 5x$

Dividing both sides by x,

$(x^2 + 1)/x = 5$

$x + 1/x = 5$ … (1)

Now,

$(x + 1/x)^3 = x^3 + 1/x^3 + 3(x + 1/x)$

$x^3 + 1/x^3 = (x + 1/x)^3 – 3(x + 1/x)$

$= 5^3 – 3(5)$

$= 125 – 15$

$= 110$

Thus, $x^3 + 1/x^3 = 110$

11. If 3a + 5b + 4c = 0,

Show that: $27a^3 + 125b^3 + 64c^3 = 180abc$

Solution:

Given, $3a + 5b + 4c = 0$

$\Rightarrow 3a + 5b = -4c$

On cubing on both sides, we have

$(3a + 5b)^3 = (-4c)^3$

$(3a)^3 + (5b)^3 + 3 \times 3a \times 5b\ (3a + 5b) = -64c^3$

$27a^3 + 125b^3 + 45ab\ (-4c) = -64c^3$

$27a^3 + 125b^3 – 180abc = -64c^3$

$27a^3 + 125b^3 + 64c^3 = 180abc$

– Hence Proved.

12. The sum of two numbers is 7 and the sum of their cubes is 133, find the sum of their square.

Solution:

Let's assume a and b to be the two numbers

So, $a + b = 7$ and $a^3 + b^3 = 133$

We know that,

$(a + b)^3 = a^3 + b^3 + 3ab (a + b)$

$(7)^3 = 133 + 3ab (7)$

$343 = 133 + 21ab$

$21ab = 343 – 133$

$= 210$

$\Rightarrow ab = 21$

Now,

$a^2 + b^2 = (a + b)^2 – 2ab$

$= 7^2 – 2 \times 10$

$= 49 – 20$

$= 29$

13. In each of the following, find the value of 'a':

(i) $4x^2 + ax + 9 = (2x + 3)^2$

(ii) $4x^2 + ax + 9 = (2x – 3)^2$

(iii) $9x^2 + (7a – 5)x + 25 = (3x + 5)^2$

Solution:

(i) $4x^2 + ax + 9 = (2x + 3)^2 = 4x^2 + 12x + 9$

On comparing coefficients of x terms, we get

$ax = 12x$

So,

$a = 12$

(ii) $4x^2 + ax + 9 = (2x – 3)^2 = 4x^2 + 12x + 9$

On comparing coefficients of x terms, we get

$ax = -12x$

So,

$a = -12$

(iii) $9x^2 + (7a – 5)x + 25 = (3x + 5)^2 = 9x^2 + 30x + 25$

On comparing coefficients of x terms, we get

$(7a – 5)x = 30x$

$7a – 5 = 30$

$7a = 35$

$\Rightarrow a = 5$

14. If $(x^2 + 1)/x = 3\ 1/3$ and $x > 1$; find

(i) $x – 1/x$

(ii) $x^3 – 1/x^3$

Solution:

Given,

$(x^2 + 1)/x = 3\ 1/3 = 10/3$

$x + 1/x = 10/3$

On squaring on both sides, we get

$(x + 1/x)^2 = (10/3)^2$

$x^2 + 1/x^2 + 2 = 100/9$

$x^2 + 1/x^2 = 100/9 – 2$

$= (100 – 18)/9$

$= 82/9$

Now,

$(x – 1/x)^2 = x^2 + 1/x^2 – 2$

$= 82/9 – 2$

$= (82 – 18)/9$

$= 64/9$

$x – 1/x = \sqrt{(64/9)}$

$= \pm 8/3$

On cubing both sides, we get

$(x – 1/x)^3 = (8/3)^3$

$x^3 – 1/x^3 – 3(x – 1/x) = 512/27$

$x^3 – 1/x^3 = 3(x – 1/x) + 512/27$

$= 3(8/3) + 512/27$

$= 24/3 + 512/27$

$= (216 + 512)/27$

$= 728/27$

Therefore, $x^3 – 1/x^3 = 728/27$

15. The difference between two positive numbers is 4 and the difference between their cubes is 316.

Find:

(i) Their product

(ii) The sum of their squares

Solution:

Given, difference between two positive numbers is 4

And, the difference between their cubes is 316

Let's assume the positive numbers to be a and b

So,

$a - b = 4$

$a^3 - b^3 = 316$

On cubing both sides, we have

$(a - b)^3 = 64$

$a^3 - b^3 - 3ab(a - b) = 64$

Also,

Given: $a^3 - b^3 = 316$

So,

$316 - 64 = 3ab(4)$

$252 = 12ab$

So,

$ab = 21$

Thus, the product of numbers is 21

Now,

On squaring both sides, we get

$(a - b)^2 = 16$

$a^2 + b^2 - 2ab = 16$

$a^2 + b^2 = 16 + 42 = 58$

Thus, sum of their squares is 58.

Exercise 4(E)

1. Simplify:

(i) (x + 6)(x + 4)(x – 2)

(ii) (x – 6)(x – 4)(x + 2)

(iii) (x – 6)(x – 4)(x – 2)

(iv) (x + 6)(x – 4)(x – 2)

Solution:

Using identity:

$(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$

(i) We have, $(x + 6)(x + 4)(x – 2)$

$= x^3 + (6 + 4 – 2)x^2 + [6 \times 4 + 4 \times (-2) + (-2) \times 6]x + 6 \times 4 \times (-2)$

$= x^3 + 8x^2 + (24 – 8 – 12)x – 48$

$= x^3 + 8x^2 + 4x – 48$

(ii) We have, $(x – 6)(x – 4)(x + 2)$

$= x^3 + (-6 – 4 + 2)x^2 + [-6 \times (-4) + (-4) \times 2 + 2 \times (-6)]x + (-6) \times (-4) \times 2$

$= x^3 – 8x^2 + (24 – 8 – 12)x + 48$

$= x^3 – 8x^2 + 4x + 48$

(iii) We have, $(x – 6)(x – 4)(x – 2)$

$= x^3 + (-6 – 4 – 2)x^2 + [-6 \times (-4) + (-4) \times (-2) + (-2) \times (-6)]x + (-6) \times (-4) \times (-2)$

$= x^3 – 12x^2 + (24 + 8 + 12)x – 48$

$= x^3 – 12x^2 + 44x – 48$

(iv) We have, $(x + 6)(x – 4)(x – 2)$

$= x^3 + (6 – 4 – 2)x^2 + [6 \times (-4) + (-4) \times (-2) + (-2) \times 6]x + 6 \times (-4) \times (-2)$

$= x^3 - 0x^2 + (-24 + 8 - 12)x + 48$

$= x^3 - 28x + 48$

2. Simply using following identity:

$(a \pm b)(a^2 \mp ab + b^2) = a^3 \pm b^3$

(i) $(2x + 3y)(4x^2 - 6xy + 9y^2)$

(ii) $(3x - 5/x)(9x^2 + 15 + 25/x^2)$

(iii) $(a/3 - 3b)(a^2 + ab + 9b^2)$

Solution:

(i) We have, $(2x + 3y)(4x^2 - 6xy + 9y^2)$

$= (2x + 3y)[(2x)^2 - (2x)(3y) + (3y)^2]$

$= (2x)^3 + (3y)^3$

$= 8x^3 + 27y^3$

(ii) We have, $(3x - 5/x)(9x^2 + 15 + 25/x^2)$

$= (3x - 5/x)[(3x)^2 + (3x)(5/x) + (5/x)^2]$

$= (3x)^3 + (5/x)^3$

$= 27x^3 + 125/x^3$

(iii) We have, $(a/3 - 3b)(a^2/9 + ab + 9b^2)$

$= (a/3 - 3b)[(a/3)^2 + (a/3)(3b) + (3b)^2]$

$= (a/3)^3 - (3b)^3$

$= a^3/27 - 27b^3$

3. Using suitable identity, evaluate

(i) $(104)^3$

(ii) $(97)^3$

Solution:

Using identity: $(a \pm b)^3 = a^3 \pm b^3 \pm 3ab(a \pm b)$

(i) $(104)^3 = (100 + 4)^3$

$= (100)^3 + (4)^3 + 3 \times 100 \times 4(100 + 4)$

$= 1000000 + 64 + 1200 \times 104$

$= 1000000 + 64 + 124800$

$= 1124864$

(ii) $(97)^3 = (100 - 3)^3$

$= (100)^3 - (3)^3 - 3 \times 100 \times 3(100 - 3)$

$= 1000000 - 27 - 900 \times 97$

$= 1000000 - 27 - 87300$

$= 912673$

4. Simply:

$$\frac{(x^2 - y^2)^3 + (y^2 - z^2)^3 + (z^2 - x^2)^3}{(x - y)^3 + (y - z)^3 + (z - x)^3}$$

Solution:

We know that,

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

Now, if

$(x^2 - y^2) + (y^2 - z^2) + (z^2 - x^2) = 0$

Then, we have

$(x^2 - y^2)^3 + (y^2 - z^2)^3 + (z^2 - x^2)^3 = 3(x^2 - y^2)(y^2 - z^2)(z^2 - x^2)$ … (1)

Similarly, if

$x - y + y - z + z - x = 0$

Then,

$(x - y)^3 + (y - z)^3 + (z - x)^3 = 3(x - y)(y - z)(z - x)$ … (2)

Now,

$$\frac{(x^2 - y^2)^3 + (y^2 - z^2)^3 + (z^2 - x^2)^3}{(x - y)^3 + (y - z)^3 + (z - x)^3}$$

$$= \frac{3(x^2 - y^2)(y^2 - z^2)(z^2 - x^2)}{3(x - y)(y - z)(z - x)} \quad \text{.....[From (1) and (2)]}$$

$= (x + y)(y + z)(z + x)$

5. Evaluate:

Evaluate :

(i) $\dfrac{0.8 \times 0.8 \times 0.8 + 0.5 \times 0.5 \times 0.5}{0.8 \times 0.8 - 0.8 \times 0.5 + 0.5 \times 0.5}$

(ii) $\dfrac{1.2 \times 1.2 + 1.2 \times 0.3 + 0.3 \times 0.3}{1.2 \times 1.2 \times 1.2 - 0.3 \times 0.3 \times 0.3}$

Solution:

(i) We have,

$$\frac{0.8 \times 0.8 \times 0.8 + 0.5 \times 0.5 \times 0.5}{0.8 \times 0.8 - 0.8 \times 0.5 + 0.5 \times .5}$$

Let's substitute 0.8 = a and 0.5 = b

So, now the given expression becomes

$$\frac{a \times a \times a + b \times b \times b}{a \times a - a \times b + b \times b}$$

$$= \frac{a^3 + b^3}{a^2 - ab + b^2}$$

$$= \frac{(a + b)(a^2 - ab + b^2)}{a^2 - ab + b^2}$$

= a + b

= 0.8 + 0.5

= 1.3

(ii) We have,

$$\frac{1.2 \times 1.2 + 1.2 \times 0.3 + 0.3 \times 0.3}{1.2 \times 1.2 \times 1.2 - 0.3 \times 0.3 \times 0.3}$$

Let's substitute 1.2 = a and 0.3 = b

So, now the given expression becomes

$$\frac{a \times a + a + b + b \times b}{a \times a \times a - b \times b \times b}$$

$$= \frac{a^2 + ab + b^2}{a^3 - b^3}$$

$$= \frac{a^2 + ab + b^2}{(a - b)(a^2 + ab + b^2)}$$

$$= \frac{1}{a - b}$$

$$= \frac{1}{1.2 - 0.3}$$

$$= \frac{1}{0.9}$$

$$= \frac{10}{9}$$

$$= 1\frac{1}{9}$$

$$\frac{1.2 \times 1.2 + 1.2 \times 0.3 + 0.3 \times 0.3}{1.2 \times 1.2 \times 1.2 - 0.3 \times 0.3 \times 0.3}$$

Let's substitute 1.2 = a and 0.3 = b

So, now the given expression becomes

$$\frac{a \times a + a + b + b \times b}{a \times a \times a - b \times b \times b}$$

$$= \frac{a^2 + ab + b^2}{a^3 - b^3}$$

$$= \frac{a^2 + ab + b^2}{(a - b)(a^2 + ab + b^2)}$$

$$= \frac{1}{a - b}$$

$$= \frac{1}{1.2 - 0.3}$$

$$= \frac{1}{0.9}$$

$$= \frac{10}{9}$$

$$= 1\frac{1}{9}$$

6. If a – 2b + 3c = 0; state the value of $a^3 - 8b^3 + 27c^3$.

Solution:

Given, a – 2b + 3c = 0

Then,

$a^3 - 8b^3 + 27c^3 = a^3 + (-2b)^3 + (3c)^3 = 3(a)(-2b)(3c)$

= -18abc

7. If x + 5y = 10; find the value of $x^3 + 125y^3 + 150xy - 1000$.

Solution:

Given, $x + 5y = 10$

On cubing both sides, we get

$(x + 5y)^3 = 10^3$

$x^3 + (5y)^3 + 3(x)(5y)(x + 5y) = 1000$

$x^3 + (5y)^3 + 3(x)(5y)(10) = 1000$

$x^3 + (5y)^3 + 150xy = 1000$

Thus,

$x^3 + (5y)^3 + 150xy – 1000 = 0$

8. If $x = 3 + 2\sqrt{2}$, find:

(i) $1/x$

(ii) $x – 1/x$

(iii) $(x – 1/x)^3$

(iv) $x^3 – 1/x^3$

Solution:

We have, $x = 3 + 2\sqrt{2}$

(i) $1/x = 1/(3 + 2\sqrt{2})$

$= (3 – 2\sqrt{2})/ [(3 + 2\sqrt{2}) \times (3 – 2\sqrt{2})]$

$= (3 – 2\sqrt{2})/ [3^2 – (2\sqrt{2})^2]$

$= (3 – 2\sqrt{2})/ (9 – 8)$

$= 3 – 2\sqrt{2}$

(ii) $x – 1/x = (3 + 2\sqrt{2}) – (3 – 2\sqrt{2})$ … [From (i)]

$= (3 + 2\sqrt{2} – 3 + 2\sqrt{2})$

$= 4\sqrt{2}$

(iii) $(x - 1/x)^3 = (4\sqrt{2})^3$ … [From (ii)]

$= (64 \times 2\sqrt{2})$

$= 128\sqrt{2}$

(iv) $(x^3 - 1/x^3) = (x - 1/x)^3 - 3(x - 1/x)$

$= 128\sqrt{2} - 3(4\sqrt{2})$ … [From (iii) and (ii)]

$= 128\sqrt{2} - 12\sqrt{2}$

9. If $a + b = 11$ and $a^2 + b^2 = 65$; find $a^3 + b^3$.

Solution:

Given, $a + b = 11$ and $a^2 + b^2 = 65$

Now, we know that

$(a + b)^2 = a^2 + b^2 + 2ab$

$(11)^2 = 65 + 2ab$

$121 = 65 + 2ab$

$2ab = 121 - 65$

$ab = (121 - 65)/2$

$= 56/2$

$= 28$

Considering the expansion $(a^3 + b^3)$

$(a^3 + b^3) = (a + b)(a^2 + b^2 - ab)$

$= (11)(65 - 28)$

$= 11 \times 37$

$= 407$

Thus, $a^3 + b^3 = 407$

10. Prove that:

$x^2 + y^2 + z^2 – xy – yz – zx$ is always positive.

Solution:

We have, $x^2 + y^2 + z^2 – xy – yz – zx$

$= 2(x^2 + y^2 + z^2 – xy – yz – zx)$

$= 2x^2 + 2y^2 + 2z^2 – 2xy – 2yz – 2zx$

$= x^2 + x^2 + y^2 + y^2 + z^2 + z^2 – 2xy – 2yz – 2zx$

$= (x^2 + y^2 – 2xy) + (z^2 + x^2 – 2zx) + (y^2 + z^2 – 2yz)$

$= (x – y)^2 + (z – x)^2 + (y – z)^2$

As the square of any number is positive, the given equation is always positive.

11. Find:

(i) (a + b)(a + b)

(ii) (a + b)(a + b)(a + b)

(iii) (a – b)(a – b)(a – b) by using the result of part (ii)

Solution:

(i) We have, $(a + b)(a + b)$

$= (a + b)^2$

$= a \times a + a \times b + b \times a + b \times b$

$= a^2 + ab + ab + b^2$

$= a^2 + b^2 + 2ab$

(ii) We have, $(a + b)(a + b)(a + b)$

$= (a \times a + a \times b + b \times a + b \times b)(a + b)$

$= (a^2 + ab + ab + b^2)(a + b)$

$= (a^2 + b^2 + 2ab)(a + b)$

$= a^2 \times a + a^2 \times b + b^2 \times a + b^2 \times b + 2ab \times a + 2ab \times b$

$= a^3 + a^2 b + ab^2 + b^3 + 2a^2b + 2ab^2$

$= a^3 + b^3 + 3a^2b + 3ab^2$

(iii) We have, (a – b)(a – b)(a – b)

In result (ii), replacing b by -b, we get (a – b)(a – b)(a – b)

$= a^3 + (-b)^3 + 3a^2(-b) + 3a(-b)^2$

$= a^3 - b^3 - 3a^2b + 3ab^2$

Benefits of ICSE Class 9 Maths Selina Solutions Chapter 4 Expansion

1. Helps to plan a preparation strategy

Attempting ICSE Class 9 Maths Selina Solutions Chapter 4 is the best strategy to prepare for the exam. Candidates can plan study table for all subjects and give more time to weak sections. By this candidates can attempt all the questions during the exam. And at the closing time, they have to perfect everything they have learned.

2. Preparation Level Overview

One of the best advantages of ICSE Class 9 Maths Selina Solutions Chapter 4 Expansion is that it helps candidates know where they stand in terms of their preparation level. This helps candidates know where they are lagging behind. Candidates can learn from their mistakes and follow the test exam solutions and strategies.

3. Performance analysis

No one can learn anything unless they analyze their performance. Candidates can know their weak and strong points only by analyzing them. Analyzing the ICSE Class 9 Maths solution will make the candidates aware that they will not repeat their mistakes in the exam hall. Analyzing the solutions helps the candidate to get an idea of where they stand in the competition to take the ICSE Class 9 Maths Exam.

5. Time planning aids

A candidate cannot afford to deal with a question that takes time. Some questions can be difficult and some can be solved at a glance. ICSE Class 9 Maths Selina Solutions help candidates to develop such skills. Candidates will implement the question solving pattern with right approach which will help them to crack the exam on time. Solving most of the questions in less time, they give the remaining time to the more difficult parts.

6. Improve Accuracy and Speed

It helps candidates to improve accuracy and speed. The more they solve different questions, the better they explain their concepts and tricks. Accuracy and speed are important in the selection process. This will help them clear the upcoming ICSE class 9 Maths Exam.

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