GATE WALLAH

ESE-2024

MAIN EXAM DETAILED SOLUTION

MECHANICAL ENGINEERING

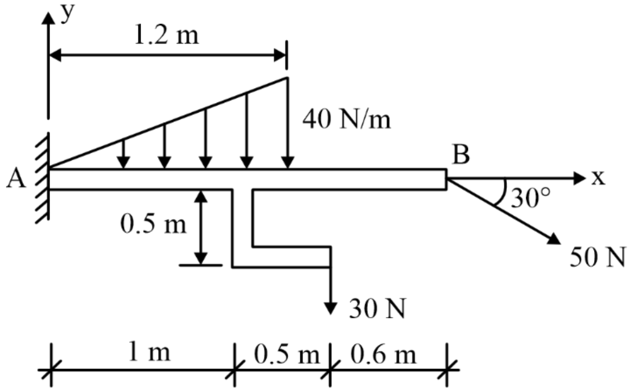
PAPER-II

EXAM DATE - 23 JUNE 2024

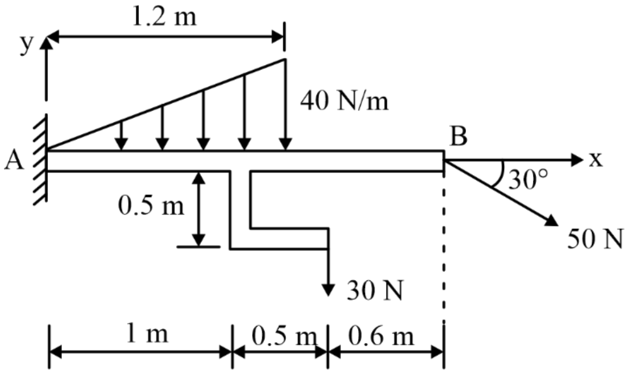
2 : 30 PM to 5 : 30 PM

SECTION-A

Q.1. **(a)** Find the support reaction at the fixed end A of the loaded beam:



Sol. **1.** **(a)**



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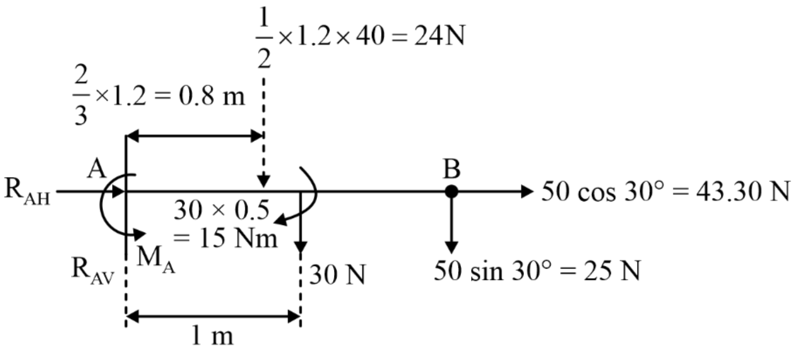


For static equilibrium

$\Sigma F_H = 0$

$R_{AH} + 43.30 = 0$

$R_{AH} = -43.30$ N

Negative sign shows our assumed direction is wrong. Change the direction of R_{AH}

$\Sigma F_V = 0$

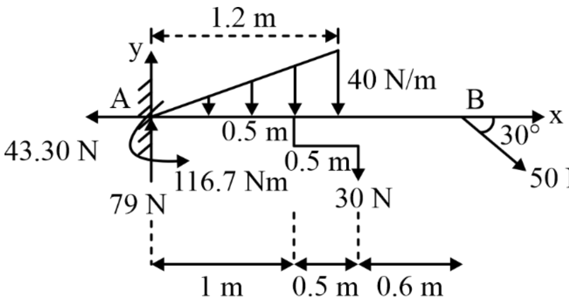
$R_{AV} - 30 - 25 - 24 = 0$

$R_{AV} = 79$ N

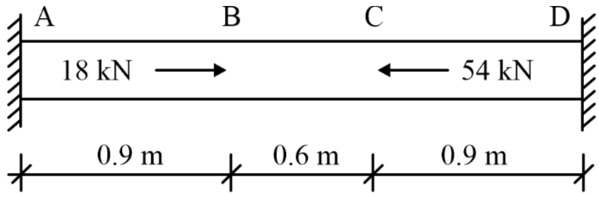
$\Sigma M_A = 0$

$-M_A + 15 + 24 \times 0.8 + 30 \times 1 + 25 \times 2.1 = 0$

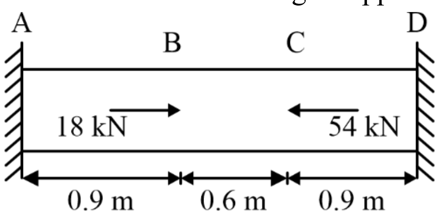
$M_A = 116.7$ Nm



(b) The straight bar AD of uniform cross-section is attached to the rigid end supports. Find the force acting on any cross-section in the regions AB, BC and CD:

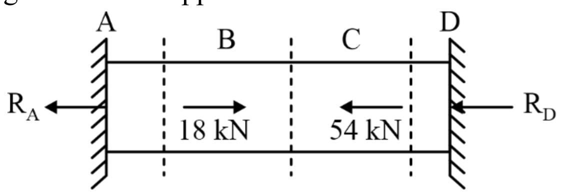


Sol. 1. (b) Uniform cross-section rigid support



To get force in any cross-section, we will use method of section.

Applying reaction at support A and D.



Internal force calculation.

$P_{AB} = R_A$ OR $18 - 54 - R_D = -36 - R_D$

$P_{BC} = R_A - 18$ OR $-54 - R_D$

$P_{CD} = R_A - 18 + 54$ OR $-R_D$

$= R_A + 36$

Since it is fixed from both ends

$\delta_{CD} = 0$ (compatibility equation)

$\delta_{AB} + \delta_{BC} + \delta_{CD} = 0$

$$\left(\frac{PL}{AE}\right)_{AB} + \left(\frac{PL}{AE}\right)_{BC} + \left(\frac{PL}{AE}\right)_{CD} = 0$$

$R_A \times 0.9 + (R_A - 18)0.6 + (R_A + 36) \times 0.9 = 0$

$[AE_{AB} = AE_{BC} = AE_{CD}]$

$2.4\ R_A = -21.6$ kN

$R_A = -9$ kN

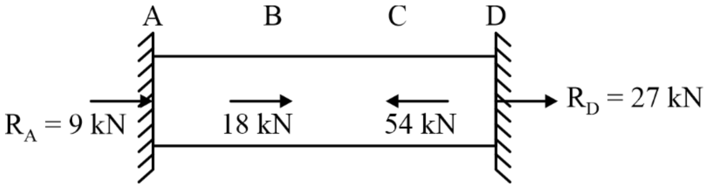
$\Sigma F_H = 0$ (Static equilibrium equation)

$R_A - 18 + 54 + R_D = 0$

$-9 - 18 + 54 = -R_D$

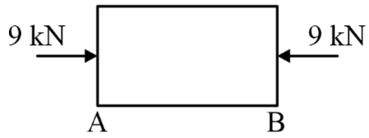
$R_D = -27$ kN

Since R_A and R_D is negative, our assumed direction is wrong.



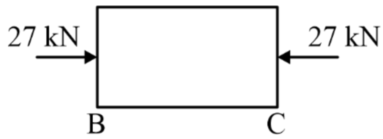
Internal force is AB

$P_{AB} = -9$ kN



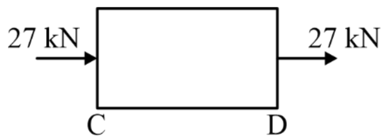
AB member is under compression

$P_{BC} = 9 - 18 = -27$ kN



BC member is under compression

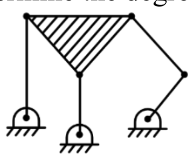
$P_{CD} = 27$ kN



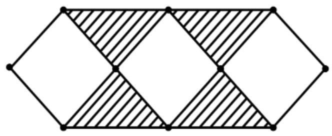
CD member is under tension.

(c) (i) Determine the degrees of freedom of the following planar linkages/ kinematic chains:

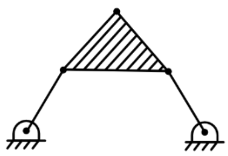
(1)



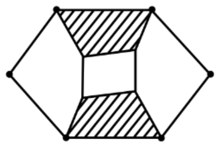
(2)



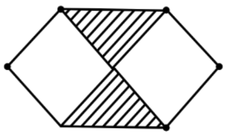
(3)



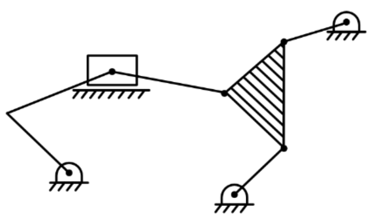
(4)



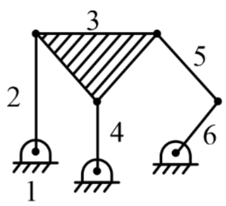
(5)



(6)

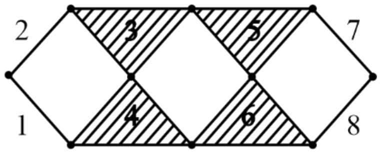


Sol. **(c)** **(1)**



l = 6
j = 7
h = 0
f = 3(l – 1) – 2j – h
= 15 – 14 – 0
$\boxed{F = 1}$

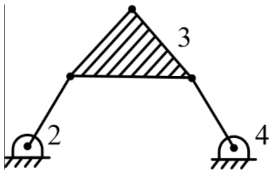
(2)



l = 8
j = 10
h = 0

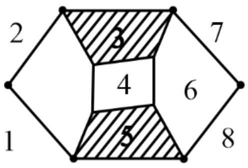
f = 3 (8–1) – 2 × 10 – 0
$\boxed{F = 1}$

(3)



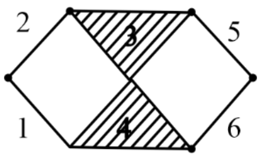
l = 4
j = 4
h = 0
f = 3 (l–1) – 2j – h
= 3 (4–1) – 8 – 0
$\boxed{F = 1}$

(4)



l = 8
j = 10
h = 0
f = 3(8–1) – 2 × 10 – 0
$\boxed{F = 1}$

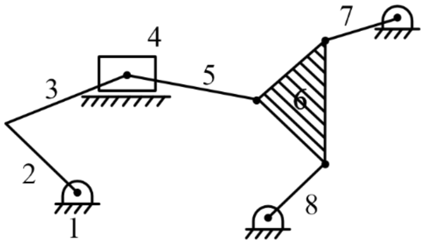
(5)



l = 6

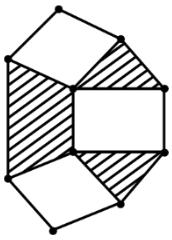
j = 7
h = 0
f = 3(6–1) – 2 × 7 – 0
$\boxed{F=1}$

(6)

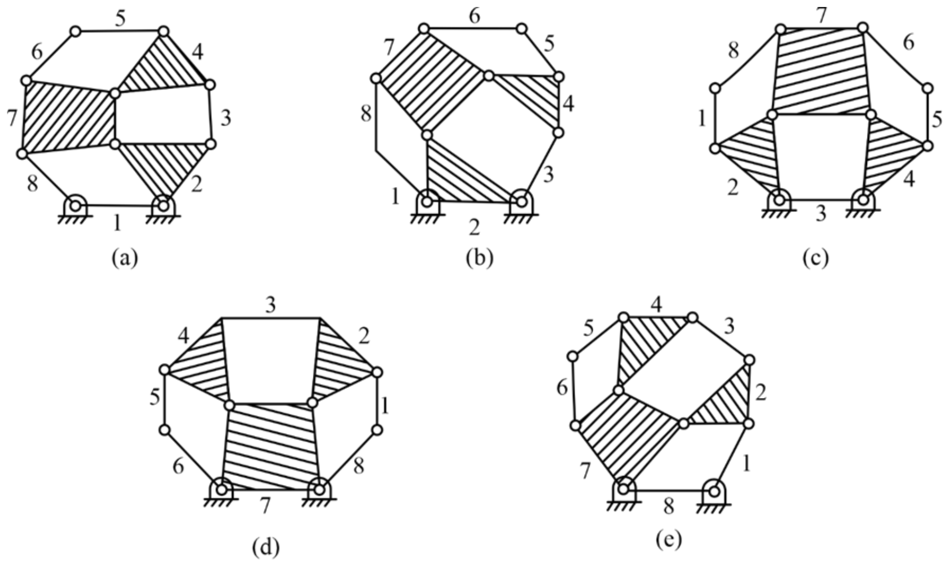


l = 8
j = 10
h = 0
f = 3(8–1) – 2 × 10 – 0
$\boxed{F=1}$

(c) (ii) Determine and show the structurally distinct (unique) inversions of the following 8-link kinematic chain:



Sol. **(c) (ii)** The kinematic chain consists of 8 links. By fixing one link to the ground each time and excluding symmetric mechanisms, a unique mechanism is created. The chain is symmetric about links 3 and 7, resulting in identical mechanisms when links 2, 1, 8, or 4, 5, 6 are fixed. Additionally, two more unique mechanisms can be derived from the 8-link kinematic chain by fixing links 3 and 7 in turn, as illustrated in the figure below. Hence, a total of 5 unique inversions may be obtained.



(d) The turbine rotor of a ship has a mass of 2.5 tonnes and rotates at 1750 r.p.m. clockwise when viewed from the aft. The radius of gyration of the rotor is 320 mm. Determine the gyroscopic couple and its effect when (i) the ship turns right at a radius of 250 m with a speed of 30 km/hr, (ii) the ship pitches with the bow rising at an angular velocity of 0.7 rad/s and (iii) the ship rolls at an angular velocity of 0.2 rad/s.

Sol. (d)

m = 2.5 tonnes = 2500 kg

$$\omega = 2\pi \times \frac{1750}{60} = 183.25 \text{ rad/s (cw)}$$

k = 0.32 m

(i) When ship turns right:

R = 250 m

$$V = 30 \text{ km/hr} = 30 \times \frac{5}{18} \text{ m/s} = 8.33 \text{ m/s}$$

$$T = I\omega\omega_p = (mk^2)\omega \times \frac{V}{R}$$

$$\Rightarrow T = (2500 \times 0.32^2) \times 183.25 \times \left(\frac{8.33}{250}\right)$$

$$\Rightarrow \boxed{T = 1563.12 \text{ N.m}}$$

Effect ⇒ Bow of ship will go down.

(ii) When ship pitches with bow rising:

$T = I\omega\omega_p$

$\Rightarrow T = 2500 \times 0.32^2 \times 183.25 \times 0.7 = 32838.4$ N-m

Effect ⇒ Bow will be thrown towards right.

(iii) When ship rolls ⇒ No gyroscopic effect

(e) What is the relationship between tensile and shear yield stresses as per (i) von Mises' criterion and (ii) Tresca's criterion?

The above relationships are to be derived by considering yielding under uniaxial tensile loading and under pure torsion.

Sol. (e) Tensile strength (σ_y)

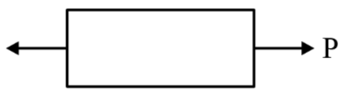
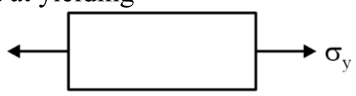


Figure: Uniaxial tensile load

Stress in critical point at yielding



Principal stresses under uniaxial tensile load at yielding.

$\sigma_1 = \sigma_y$, $\sigma_2 = 0$, $\sigma_3 = 0$

Shear strength (τ_y)

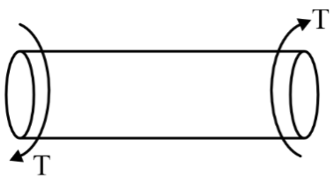
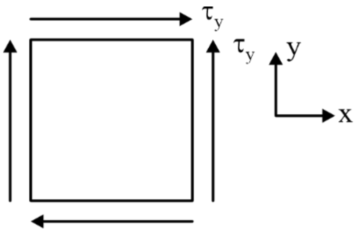


Figure: pure torsion

Stress critical point at yielding



$\sigma_{xx} = 0$

$\sigma_{yy} = 0$

$\tau_{xy} = \tau_y$

Principal stresses under pure torsion at yielding

$$\sigma_{1,2} = \frac{\sigma_{xx} + \sigma_{xy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{xy}}{2}\right)^2 + \tau_{xy}^2} \text{ and } \sigma_3 = 0$$

$$\sigma_{1,2} = \frac{0+0}{0} \pm \sqrt{\left(\frac{0-0}{2}\right)^2 + \tau_y^2} \text{ and } \sigma_3 = 0$$

$\sigma_{1,2} = 0 \pm \tau_y$ and $\sigma_3 = 0$

$\sigma_1 = \tau_y, \sigma_2 = -\tau_y, \sigma_3 = 0$

(i) Von-Mises criteria

Criteria of failure: Distortion energy/volume (u_d)

Distortion energy/volume under uniaxial load [u_d) $_{uniaxial}$]

$$u_{d_{uniaxial}} = \frac{1}{6G}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)\right]$$

$$= \frac{1}{6G}\left[\sigma_y^2 + 0 + 0 - (0+0+0)\right]$$

$$u_{d_{uniaxial}} = \frac{1}{6G}\left[\sigma_y^2\right]$$

Distortion energy/volume under pure torsion [u_d) $_{torsion}$]

$$u_{d_{torsion}} = \frac{1}{6G}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)\right]$$

$$u_{d_{torsion}} = \frac{1}{6G}\left[\tau_y^2 + (-\tau_y)^2 + 0 - ((\tau_y)(-\tau_y) + 0 + 0)\right]$$

$$u_{d\,pure\ torsion} = \frac{1}{6G}\left[3\tau_y^2\right]$$

As per von Mises criteria

$$u_{d_{pare\ torsion}} = u_{d_{uniaxial}}$$

$$\Rightarrow \frac{1}{6G}(3\tau_y^2) = \frac{1}{6G}(\sigma_y^2)$$

$$\Rightarrow \tau_y^2 = \frac{\sigma^2{}_y}{3}$$

$$\Rightarrow \boxed{\tau_y = \frac{\sigma_y}{\sqrt{3}}}$$

(ii) **Tresa's criteria**

Criteria of failure : maximum shear stress τ_{max}

Maximum shear stress under uni-axial load $(\tau_{max})_{uniaxial}$

$$\tau_{max_{uniaxial}} = \max\left[\left|\frac{\sigma_1-\sigma_2}{2}\right|, \left|\frac{\sigma_2-\sigma_3}{2}\right|, \left|\frac{\sigma_1-\sigma_3}{2}\right|\right]$$

$$= \max\left[\left|\frac{\sigma_y-0}{2}\right|, \left|\frac{0-0}{2}\right|, \left|\frac{\sigma_y-0}{2}\right|\right]$$

$$\tau_{max_{uniaxial}} = \frac{\sigma_y}{2}$$

Maximum shear stress under pure torsion

$$\left[(\tau_{max})_{pure\ torsion}\right]$$

$$\left[(\tau_{max})_{pure\ torsion}\right] = \max\left[\left|\frac{\sigma_1-\sigma_2}{2}\right|, \left|\frac{\sigma_2-\sigma_3}{2}\right|, \left|\frac{\sigma_1-\sigma_3}{2}\right|\right]$$

$$= \max\left[\left|\frac{\tau_y-(-\tau_y)}{2}\right|, \left|\frac{-\tau_y-0}{2}\right|, \left|\frac{\tau_y-0}{2}\right|\right]$$

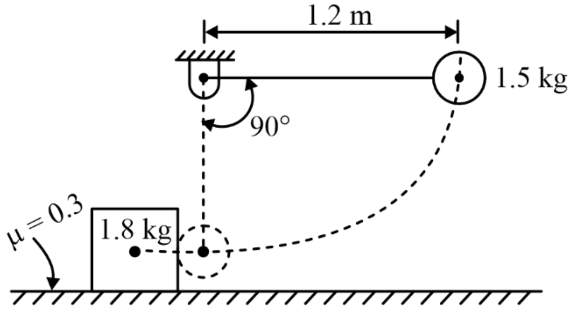
$$(\tau_{max})_{pure\ torsion} = \tau_y$$

As per Tresa's criteria

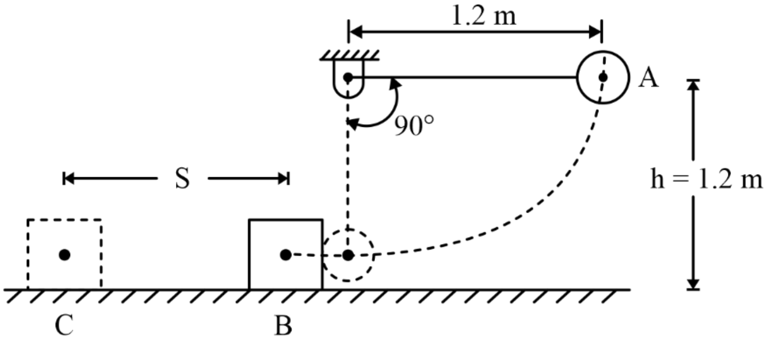
$$(\tau_{max})_{pure\ torsion} = (\tau_{max})_{uniaxial}$$

$$\Rightarrow \boxed{\tau_y = \frac{\sigma_y}{2}}$$

Q.2. **(a)** A smooth sphere of mass 1.5 kg is released from rest in the position when the flexible string attached to it is horizontal. It hits centrally a stationary block of mass 1.8 kg kept on a surface, with the coefficient of friction between the block and the surface being 0.3. If the coefficient of restitution is 0.8, how far would the block move after impact?



Sol. **(a)** m = 1.5 kg, M = 1.8 kg, μ = 0.3, e = 0.8



speed of sphere at position A (v_A) = 0

By conservation of energy

$$\frac{1}{2}mv_A^2 + mgh = \frac{1}{2}mv_B^2$$

$$\Rightarrow 0 + 1.5 \times 9.81 \times 1.2 = \frac{1}{2} \times 1.5 \times v_B^2$$

$\Rightarrow$ v_B = 4.852 m/s

Velocity of block before impact (V_B) = 0

let v_B and V_B' are the velocities of sphere and block after the impact.

By conservation of linear momentum

$$\Rightarrow m v_B + M V_B = m v_B' + M V_B'$$

$$\Rightarrow 1.5 \times 4.852 + 0 = 1.5 v_B' + 1.8 V_B'$$

$$\Rightarrow 1.5 v_B' + 1.8 V_B' = 7.278 \quad \text{--- (i)}$$

Coefficient of restitution is given as:

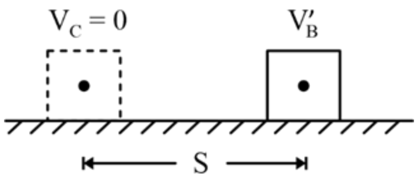
$$e = \frac{V_B' - v_B'}{v_B - V_B}$$

$$\Rightarrow 0.8 = \frac{V_B' - v_B'}{4.852 - 0}$$

$$\Rightarrow V_B' - v_B' = 3.882 \quad \text{--- (ii)}$$

from equation (i)and (ii)

$v_B' = 0.088$ m/s $\qquad V_B' = 3.97$ m/s



Velocity of block at position C (v_C) = 0

By work energy theorem;

Work done by friction = Change in kinetic energy of block

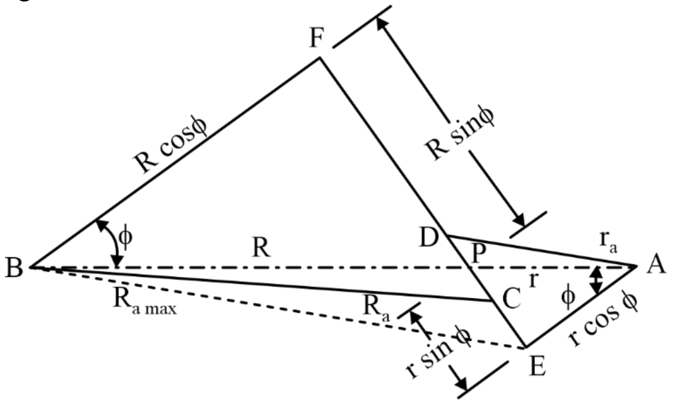
$$-f \times S = \frac{1}{2}M V_C^2 - \frac{1}{2}M V_B'^2$$

$$\Rightarrow -\mu mg \times S = \frac{1}{2}M\left(V_C^2 - V_B'^2\right)$$

$$\Rightarrow -0.3\times1.8\times9.81\times S = \frac{1}{2}\times1.8\times\left(0^2 - 3.97^2\right)$$

$$\Rightarrow S = 2.68 \text{ m}$$

(b) (i) Derive the formulation for path of contact of two gears A and B in contact as shown in the figure:



Sol. **(b)** **(i)** Assuming pinion is driver in the given figure. The contact between the two teeth occurs where the addendum circle of the wheel intersects the line of action EF, at point C. The contact is broken where the addendum circle of the pinion intersects the line of action EF, at point D. Thus, CD represents the path of contact.

CD = CP + PD

Where, CP is the path of approach and PD is the path of recess.

Path of approach (CP)

In ΔBCF;

$BC^2 = BF^2 + CF^2$

$$\Rightarrow R_a^2 = (CP + PF)^2 + (R\cos\phi)^2$$

$$\Rightarrow (CP + R\sin\phi)^2 = R_a^2 - (R\cos\phi)^2$$

$$\Rightarrow \boxed{CP = \sqrt{R_a^2 - (R\cos\phi)^2} - R\sin\phi}$$

Similarly

Path of recess (PD)

In ΔADE;

$AD^2 = DE^2 + AE^2$

$$\Rightarrow r_a^2 = (DP + PE)^2 + (r\cos\phi)^2$$

$$\Rightarrow (DP + r\sin\phi)^2 = r_a^2 - (r\cos\phi)^2$$

$$\Rightarrow \boxed{DP = \sqrt{r_a^2 - (r\cos\phi)^2} - r\sin\phi}$$

Hence, Path of contact is given as;

$$\boxed{DC = \sqrt{r_a^2 - (r\cos\phi)^2} - r\sin\phi + \sqrt{R_a^2 - (R\cos\phi)^2} - R\sin\phi}$$

(b) (ii) Each of two gears in a mesh has 48 teeth and a module of 8 mm. The teeth are of 20° involute profile. The arc of contact is 2.25 times the circular pitch. Determine the addendum and contact ratio.

Sol. **(b)** **(i)** t = 48, T = 48

m = 8 mm, $\phi = 20°$

AOC = 2.25 P_C

$$r = R = \frac{mt}{2} = \frac{8 \times 48}{2} = 192 \text{ mm}$$

$$\text{Contact ratio} = \frac{AOC}{P_C} = 2.5$$

Path of contact;

POC = KP + PL = 2KP (∵ KP=PL in this case)

$$\Rightarrow (AOC)\cos\phi = 2\sqrt{r_A^2 - (r\cos\phi)^2} - r\sin\phi$$

$$\Rightarrow (2.25 \times \pi \times 8)\cos 20° = 2\sqrt{r_A^2 - (192\cos 20°)^2} - (192\sin 20°)$$

$$\Rightarrow \boxed{r_A = 202.63 \text{ mm}}$$

Addendum = $r_A - r$ = 202.63 – 192

$$\Rightarrow \boxed{\text{Addendum} = 10.63\text{mm}}$$

(c) A cam having a lift of 1.2 cm operates the suction valve of a four-stroke petrol engine. The least radius of the cam is 2 cm and nose radius is 0.3 cm. The crank angle of the engine when suction valve opens is 4° after t.d.c. and it is 50° after b.d.c. when the suction valve closes. The camshaft has a speed of 960 r.p.m. The cam is of circular type with circular nose and flanks. It is integral with camshaft and operates a flat-faced follower. Calculate (i) the maximum velocity of the valve, (ii) the maximum acceleration and retardation of the valve and (iii) the minimum force to be exerted by the spring to overcome inertia of the valve parts which weigh 250 g.

Sol. **(c)** **(i)** Lift (x) = 1.2 cm = 12 mm

least radius r_1 = 20 mm

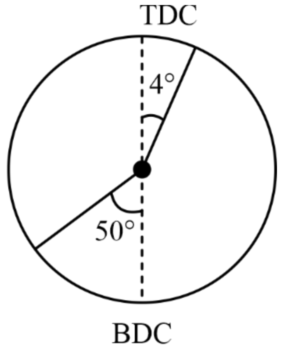
nose radius r_2 = 3 mm

camshaft speed, N = 960 rpm

suction value opens 4° after TDC

suction value opens 50° after BDC

valve part inertia = 0.25 kg



Crank angle diagram

Angular displacement of crank when suction valve is open

= 180° – 4° + 50° = 226°

In four-stroke engines, cam shaft speed is half of the crankshaft speed.

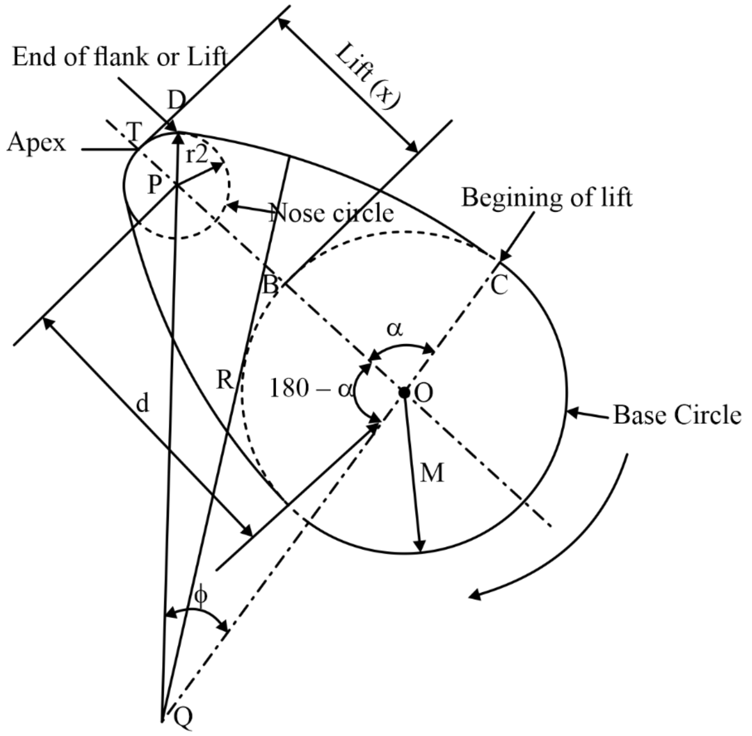
∴ angular displacement of cam shaft during opening of the suction valve

$= \frac{226}{2} = 113° = 2\alpha$

Since the cam is a symmetrical;

angle of ascent = angle of descent

Angle of ascent $\alpha = \frac{113°}{2} = 56.5°$



$OP + r_2 = \text{lift} + r_1$

$\Rightarrow OP = 12 + 20 - 3 = 29 \text{ mm} = d$

flank radius;

$$R = \frac{r_1^2 - r_2^2 + d^2 - 2r_1 d\cos\alpha}{2(r_1 - r_2 - d\cos\alpha)}$$

$$\Rightarrow R = \frac{20^2 - 3^2 + 29^2 - 2\times 20\times 29\cos 56.5°}{2(20 - 3 - 29\cos 56.5)}$$

$$\Rightarrow R = \frac{591.753}{1.98765}$$

$\Rightarrow R = 297.714$ mm

flank angle from triangle OQP

$$\frac{PO}{\sin\phi} = \frac{PQ}{\sin(180-\alpha)}$$

$$\Rightarrow \sin\phi = \frac{\sin(180 - 56.5°)\times 29}{(297.714 - 3)}$$

$\Rightarrow \sin\ \phi = 0.082054$

$\Rightarrow \phi = 4.7066°$

(i) Maximum velocity when follower leaves the flank ($\alpha = \phi$)

$v_{max} = \omega(R - r_1)\sin\phi$

$= \frac{2\pi N}{60} \times (297.7149 - 20)\sin 4.7066$

$= \frac{2\pi \times 960}{60} \times (297.7149 - 20)\sin 4.7066$

= 2.29 m/s

(ii) Maximum acceleration of value when $\phi = 0$

acceleration $a = \omega^2 (R - r_1)\cos\phi$

$= \left(\frac{2\pi \times 960}{60}\right)^2 \times (297.714 - 20)\cos 0^\circ$

= 2806.718 m/s^2

Retardation is maximum when $(\alpha - \phi) = 0$

$a = -\omega^2 d$

$= \left(\frac{2\pi \times 960}{60}\right)^2 \times 29 \times 10^{-3}$

= –293.087 m/s^2

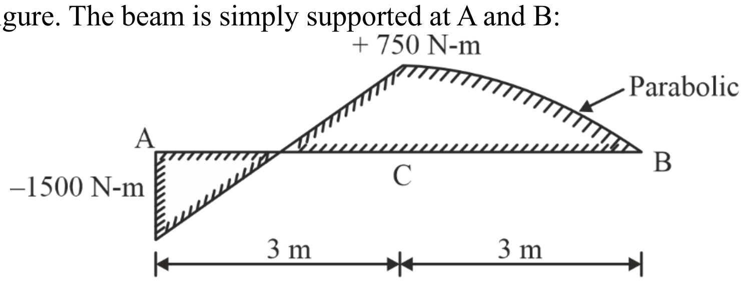
(iii) Minimum force to be exerted by the spring to overcome inertia of the valve parts

Minimum force = Mass × Retardation

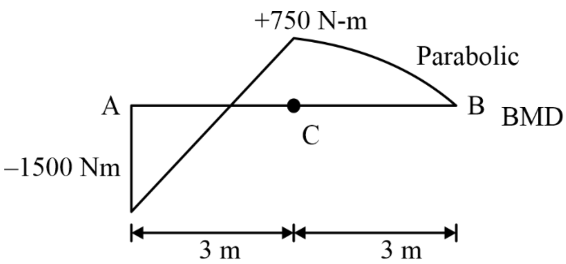
= 0.25 × (293.087)

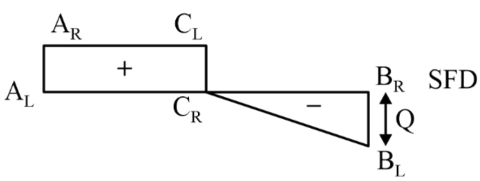
minimum force = 73.271 N

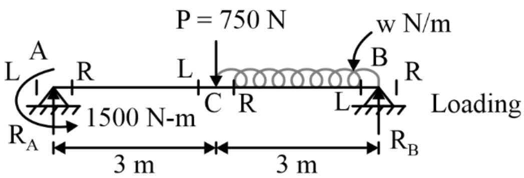
Q.3. (a) Show the loading on the beam corresponding to the bending moment diagram shown in the figure. The beam is simply supported at A and B:



Sol. 3. (a)

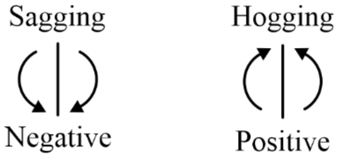




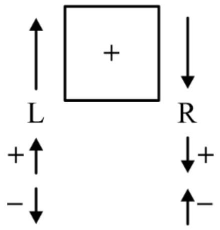


Sign convection

Bending moment



Shear Force



At A, bending moment is dropping suddenly by 1500 Nm, therefore at A, a concentrated couple of 1500 Nm acts in ACW direction.

Between AC, BMD is inclined line, which means the slope of BMD is constant, therefore in AC region shear force is constant (Say F_{AC}).

$$F_{AC} = \text{Slope of BMD is AC region}$$

$$\Rightarrow \quad F_{AC} = \frac{B.M.)_C - B.M.)_{A,R}}{3}$$

$$\Rightarrow \quad F_{AC} = \frac{750 - (-1500)}{3}$$

$$\Rightarrow \quad F_{AC} = 750 \text{ N}$$

Reaction calculation:

$$S_{A,R} - S_{A,L} = +R_A$$

$$720 - 0 = +R_A$$

$$R_A = 750 \text{ N}$$

At just right of C, slope of BMD is 0, therefore shear force at just right of C should be 0.

Now at C, there is sudden of shear force of height 750 N, therefore concentrated force of P = 750 N should act in downward direction AC.

From just right of C(C, R) to just left of B(B, L) i.e., in CB region sign of slope of BMD is negative, therefore shear force value in CB region is negative.

And from C, R to B, L magnitude of slope of BMD is increasing therefore magnitude of shear force will increase as shown in figure.

Since in CB region SFD in inclined line with negative slope, therefore in CB region UDL of intensity w N/m will act in downward direction.

$$\Sigma M_B = 0$$

$$\Rightarrow \quad R_A \times 6 - 1500 - 750 \times 3 - w \times 3 \times \frac{3}{2} = 0$$

$$\Rightarrow \quad 750 \times 6 - 1500 - 2250 - 4.5\,w = 0$$

$$\Rightarrow \quad 750 = 4.5\,w$$

$$\Rightarrow \quad w = \frac{750}{4.5}$$

$$\Rightarrow \quad w = 166.667 \text{ N/m}$$

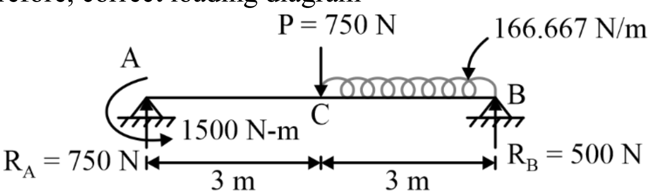
Now, $\Sigma F_y = 0$

$\Rightarrow$ $R_A - 750 - w \times 3 + R_B = 0$

$\Rightarrow$ $750 - 750 - 166.667 \times 3 + R_B = 0$

$\Rightarrow$ $R_B = 500$ N

Therefore, correct loading diagram



(b) A shaft 1.7 cm diameter and 1.2 m long is held in long bearings. The weight of a disc at the centre of the shaft is 20 kg. The eccentricity of the centre of gravity of the disc from centre of rotor is 0.03 cm. The Young's modulus of material of the shaft is 2×10^6 kg/cm². The permissible stress in the shaft material is 750 kg/cm². Calculate (i) the critical speed of the shaft and (ii) the range of speed over which it is unsafe to run the shaft. Neglect weight of the shaft.

Sol. (3) (b)

$m = 20$ kg, $L = 1.2$ m

$d = 1.7$ cm $= 0.017$ m

$e = 0.03$ cm $= 0.0003$ m

Assuming $g = 9.81$ m/s²

$\sigma_{per} = 750$ kg/cm² $= 73.575$ MPa

$E = 2 \times 10^6$ kg/cm² $= 196200$ MPa

(i) Since the shaft is supported by long bearings, it can be considered fixed at both ends.

Static deflection of the shaft due to the weight of the disc is given as;

$$\Delta = \frac{mgL^3}{192EI}$$

$$\Rightarrow \Delta = \frac{20 \times 9.81 \times 1.2^3}{192 \times 196200 \times 10^6 \times \frac{\pi}{64} \times (0.017)^4} = 0.002195\,\text{m}$$

Critical Speed = Natural frequency

$$\Rightarrow f_c = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta}} = \frac{1}{2\pi}\sqrt{\frac{9.81}{0.002195}} = 10.64\,\text{Hz}$$

Critical speed in rpm;

$$N_c = 60 f_c = 60 \times 10.64 = 638.4 \text{ rpm}$$

(ii) When the shaft rotates, the additional dynamic load can be determined using the following relationship:

$$\frac{M}{I} = \frac{\sigma_{per}}{y}$$

$$\Rightarrow \frac{\frac{W_1 L}{8}}{\frac{\pi}{64} \times d^4} = \frac{\sigma_{per}}{d/2}$$

$$\Rightarrow \frac{W_1 \times 1.2}{8 \times \frac{\pi}{64} \times (0.017)^4} = \frac{73.575 \times 10^6}{\left(\frac{0.017}{2}\right)}$$

$\Rightarrow W_1 = 236.584$ N

Additional deflection due to this load;

$$y = \frac{W_1}{mg} \times \Delta = \frac{236.584}{20 \times 9.81} \times 0.002195$$

$\Rightarrow y = 0.002647$ m

Additional deflection is given by relation;

$$y = \frac{\pm e}{\left(\frac{\omega_n}{\omega}\right)^2 - 1}$$

$$\Rightarrow 0.002647 = \frac{\pm 0.0003}{\left(\frac{N_C}{N}\right)^2 - 1}$$

$$\Rightarrow \left(\frac{638.4}{N}\right)^2 = 1 \pm 0.11334$$

(On considering + Sign)

$$\Rightarrow \left(\frac{638.4}{N_1}\right)^2 = 1.11334$$

$\Rightarrow N_1 = 605.033$ rpm

(On considering – Sign)

$$\left(\frac{638.4}{N_2}\right)^2 = 1 - 0.11334 = 0.88666$$

$\Rightarrow N_2 = 677.98$ rpm

Thus, the range of unsafe speed is from 605.033 rpm to 677.98 rpm.

(c) (i) A bolted joint is used to connect two components. The combined stiffness of the two components is twice the stiffness of the bolt. The initial tightening of the nut results in a preload of 10 kN in the bolt. The external force of 7.5 kN creates further tension in the bolt. The bolt is made of plain carbon steel 30C8, having tensile yield strength of 400 N/mm². There are coarse threads on the bolt. Calculate the tensile stress area of the bolt. The factor of safety specified is 3.

Sol. (3) (c)

(i) Stiffness of connecting member = 2 (stiffness of bolt)

$\Rightarrow K_{cm} = 2 K_b$

Initial tightening load, $P_i = 10$ kN = 10000 N

External tensile load, P = 7.5 kN = 7500 N

Tensile yield strength of bolt, $S_{yt} = 400$ N/mm²

FOS = 3

Tensile stress area of bolt (A_b) = ?

Stiffness correction factor(C),

$$C = \frac{K_b}{K_b + K_{cm}}$$

$$\Rightarrow C = \frac{K_b}{K_b + 2K_b}$$

$$\Rightarrow C = \frac{1}{3}$$

Total tensile force on bolt (P_b)

$P_b = P_i + CP$

$$\Rightarrow P_i = 10000 + \frac{1}{3} \times 7500$$

$\Rightarrow P_C = 12500$ N

Tensile stress on bolt (σ_t)

$$\sigma_t = \frac{P_i}{A_b}$$

$$\Rightarrow \sigma_t = \frac{12500}{A_b}$$

Now,

$$\sigma_t = \frac{S_{yt}}{FOS}$$

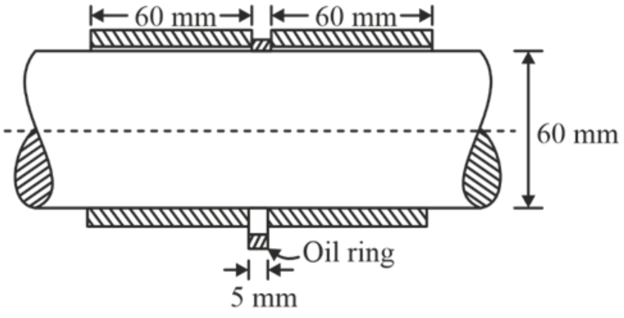
$$\Rightarrow \frac{12500}{A_b} = \frac{400}{3}$$

$$\boxed{\Rightarrow A_b = 93.75 \text{ mm}^2}$$

(ii) An oil ring of a shaft transmitting power is shown in the figure. There is no hydrodynamic action over 5 mm width of the oil ring. The total radial load on the journal is 21 kN and the journal rotates at 1440 r.p.m.

$$\frac{c}{r} = 0.8 \times 10^{-3};\ \frac{h_0}{c} = 0.2$$

where c = radial clearance, r = radius and h_o = minimum oil thickness.
For the instant case, Sommerfeld number (S) = 0.0446.
For this case, calculate the viscosity of lubricant:



Sol. **(3)** **(c)**

(ii)

Radial load, W = 21 kN = 21000 N

Speed, N = 1440 rpm

$$\frac{c}{r} = 0.8 \times 10^{-3}$$

$$\frac{h_o}{c} = 0.2$$

S = 0.0446

Viscosity of lubricant, z = ?

From figure,

Length of bearing in which hydrodynamic action is occurring,

L = 60 + 60 = 120 mm = 0.12 m

Diameter of shaft/journal, d = 60 mm = 0.06 m

Now,

$$S = \left(\frac{zn_s}{p}\right)\left(\frac{r}{c}\right)^2 \text{ ....(i)}$$

here,

n_s = speed of shaft in rps

$$\Rightarrow n_s = \frac{N}{60} = \frac{1440}{60}$$

$$\Rightarrow n_s = 24 \text{ rps}$$

p = bearing pressure

$$\Rightarrow p = \frac{W}{Ld}$$

$$\Rightarrow p = \frac{21000}{0.12 \times 0.06}$$

$$\Rightarrow P = 2916666.667 \text{ P}_a$$

$$\frac{r}{c} = \frac{1}{c/r}$$

$$\Rightarrow \frac{r}{c} = \frac{1}{0.8 \times 10^{-3}}$$

$$\Rightarrow \frac{r}{c} = 1250$$

From equation (i)

$$0.0446 = \frac{z \times 24}{2916666.667} \times (1250)^2$$

$$\Rightarrow z = \frac{0.0446 \times 2916666.667}{24 \times (1250)^2}$$

$$\Rightarrow \boxed{z = 3.469 \times 10^{-3} \text{ Pa} - \text{s}}$$

Q.4. **(a)**

A thick cylinder of 225 mm internal diameter has to be designed for a safe internal pressure of 50 MPa. Calculate the thickness of the cylinder wall using maximum shear stress theory. The axial stress may be neglected in the calculation. The yield stress of the cylinder material is 260 MPa and the factor of safety is 2.

Sol. **(4)** **(a)** Internal diameter, $D_i = 225$ mm

Internal radius, $R_i = \frac{D_i}{2} = \frac{225}{2} = 112.5$ mm

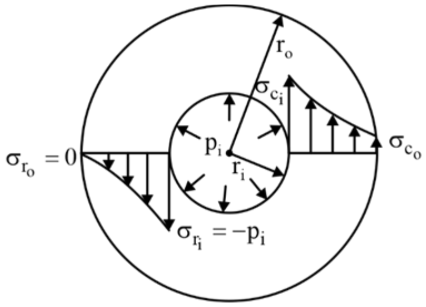
Internal pressure $p_i = 50$ MPa

Axial stress, $\sigma a \approx 0$ (neglecting)

yield stress, $\sigma_y = 260$ MPa

FOS = 2

Thickness, t =?



As per Lami's theorem

$$\sigma_c = \frac{B}{r^2} + A..............(i)$$

$$\sigma_r = -\frac{B}{r^2} + A..............(ii)$$

at $r = r_0 \Rightarrow p = p_0 = 0$, from equation (ii)

$$0 = -\frac{B}{r_0^2} + A$$

$$\Rightarrow B = Ar_0^2 \qquad ....(iii)$$

at $r = r_i \Rightarrow \sigma_r = -p_i$, from equation (ii)

$$-p_i = -\frac{B}{r_i^2} + A$$

From equation (iii)

$$-p_i = \frac{-Ar_0^2}{r_i^2} + A$$

$$\Rightarrow -p_i = -A\left(\frac{r_0^2 - r_i^2}{r_i^2}\right) \;.......... \text{(iv)}$$

Put value of A in equation (ii),

$$B = Ar_0^2$$

$$B = \frac{p_i r_i^2 r_0^2}{r_0^2 - r_i^2} \quad ......(v)$$

Put value of A and B in equation (i) and (ii)

from equation (i)

$$\sigma_c = \frac{p_i r_i^2 r_0^2}{\left(r_0^2 - r_i^2\right) r^2} + \frac{p_i r_i^2}{r_0^2 - r_i^2}$$

$$\sigma_c = \frac{p_i r_i^2}{r_0^2 - r_i^2}\left[\left(\frac{r_0}{r}\right)^2 + 1\right] \quad .... (vi)$$

from equation (vi) σ_c is positive and as r increases σ_c decreases as shown in figure

From equation (ii)

$$\sigma_r = \frac{-p_i r_i^2 r_0^2}{\left(r_0^2 - r_i^2\right) r^2} + \frac{p_i r_i^2}{r_0^2 - r_i^2}$$

$$\Rightarrow \sigma_r = \frac{-p_i r_i^2}{r_0^2 - r_i^2}\left[\left(\frac{r_o}{r}\right)^2 - 1\right] \quad .... (vii)$$

from equation (vii) σ_r is negative and as r increase, $|\sigma_r|$ decrease as shown in figure

At $r = r_i$, $\sigma_c = \sigma_{ci}$ is maximum in tensile nature and $\sigma_r = \sigma_{ri}$ is maximum in compression therefore shear stress will be maximum at $r = r_i$.

From equation (vi) at $r = r_i$

$$\sigma_{ci} = \frac{p_i r_i^2}{r_0^2 - r_i^2}\left[\left(\frac{r_o}{r_i}\right)^2 + 1\right]$$

$$\sigma_{ci} = \frac{p_i r_i^2}{r_0^2 - r_i^2}\left[\frac{r_0^2 + r_i^2}{r_i^2}\right]$$

$$\sigma_{ci} = p_i\left(\frac{r_0^2 + r_i^2}{r_0^2 - r_i^2}\right)$$

at $r = r_i$, $\sigma_{ri} = -p_i$ (already known)

∴ Maximum shear stress (τ_{max})

$$\tau_{max} = \left|\frac{\sigma_{ci} - \sigma_{ri}}{2}\right|$$

$$\tau_{max} = \left|\frac{p_i\left(\frac{r_0^2 + r_i^2}{r_0^2 - r_i^2}\right) - (-p_i)}{2}\right|$$

$$\tau_{max} = \frac{p_i r_0^2}{r_0^2 - r_i^2}$$

As per maximum shear stress theory

$$\tau_{max} = \frac{0.5\,\sigma_y}{FOS}$$

$$\Rightarrow \frac{p_i r_0^2}{r_0^2 - r_i^2} = \frac{0.5 \times 260}{2}$$

$$\Rightarrow \frac{50 \times r_0^2}{r_0^2 - 112.5^2} = 65$$

$$\Rightarrow 50 r_0^2 = 65 r_0^2 - 822656.25$$

$$\Rightarrow 15 r_0^2 = 822656.25$$

$$\Rightarrow r_0^2 = 54843.75$$

$$\Rightarrow r_0 = 234.187 \text{mm}$$

$\therefore$ thickness of cylinder (t)

$t = r_0 - r_i$

$\Rightarrow t = 234.187 - 112.5$

$\Rightarrow \boxed{t = 121.687 \text{ mm}}$

(b) A riveting machine is driven by a motor of 4 kW. The actual time to complete one riveting operation is 1.5 seconds and it absorbs 12 kN-m of energy. The moving parts including the flywheel are equivalent to 220 kg at 0.5 m radius. Determine the speed of the flywheel immediately after riveting, if it is 380 r.p.m. before riveting. Also determine the number of rivets closed per minute.

Sol. 4. (b)

P = 4000 W

$E_{required/riveting}$ = 12000 J

Exact punching time = 1.5 sec.

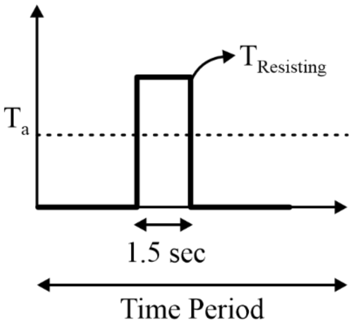
m = 220 kg

r = 0.5 m

$I = mr^2 = 220 \times 0.5^2 = 55 \text{ kg.m}^2$

ω_{max} = 380 rpm, ω_{min} = ?

No. of cycle/minute =?



Motor supplies the total energy during entire cycle uniformly;

Power × time period = $E_{required/cycle}$

$\Rightarrow$ 4000 × t = 12000

$\Rightarrow \boxed{t = 3 \text{sec}}$

No. of cycle/minute $\Rightarrow \frac{60}{3} = \boxed{20 \text{ cycle/minute}}$

Therefore, 20 rivets will be closed per minute.

$E_{available}$ by motor in 1.5 sec $\Rightarrow 4000 \times 1.5 = 6000$ J

$\Delta E_{max} = E_{required} - E_{available}$ in 1.5 sec by motor

$\Rightarrow \Delta E_{max} = 12000 - 6000 = 6000$ J

Maximum fluctuation of energy is given as;

$$\Delta E_{max} = \frac{1}{2} I\left(\omega_{max}^2 - \omega_{min}^2\right)$$

$$\Rightarrow 6000 = \frac{1}{2} \times 55 \left(\frac{2\pi}{60}\right)^2 \left(380^2 - N_{min}^2\right)$$

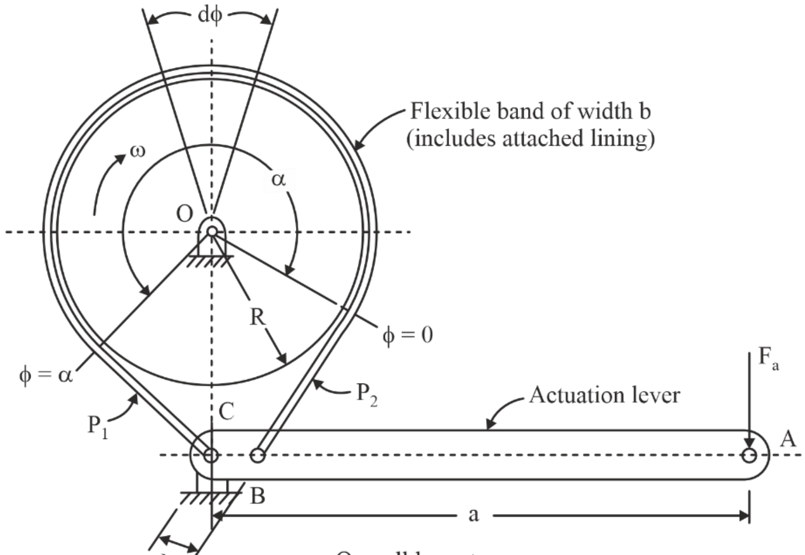
$$\Rightarrow \boxed{N_{min} = 352.85 \text{ rpm}}$$

(c) Refer to the following figure of the drum brake.

Prove that the braking torque (T_f) can be expressed as

$T_f = P_2R(e^{\mu\alpha}-1)$ and $T_f = bR^2 p_{max}(1-e^{-\mu\alpha})$

where R drum radius, b width, p = pressure at any point in the arc of contact and P = tensile force in the band at the same point:



Overall layout

Sol. 4. (c) FBD of drum for strip

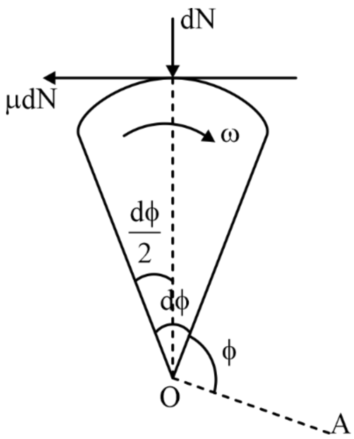


Figure (a)

FBD of band for strip

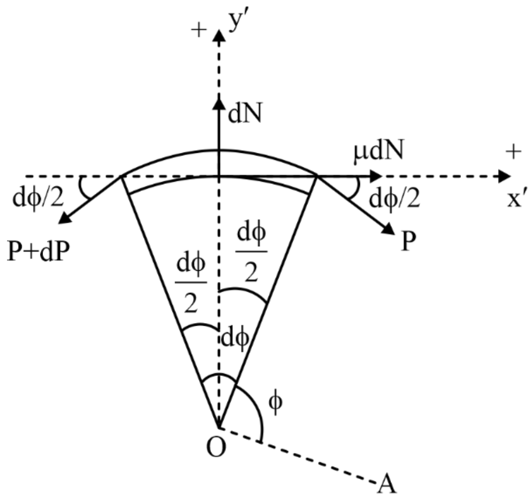


Figure (b)

P → tension in band at ϕ

P + dP → tension in band at ϕ + dϕ

Normal force on strip (dN)

dN = P × area of strip

$\Rightarrow$ dN = p × dA

$\Rightarrow$ dN = p × RdØ × b ....(i)

In FBD of band

$$\sum F_{x'} = o$$

$$P\cos\frac{d\phi}{2} + \mu dN - (p + dP)\cos\left(\frac{d\phi}{2}\right) = 0$$

$$\left[\cos\left(\frac{d\phi}{2}\right) \approx 1\right]$$

$\Rightarrow$ P + μdN – (P + dP) = 0

$\Rightarrow$ P + μdN – (P – dP) = 0

$\Rightarrow$ dP = μdN .... (ii)

and

$$\sum F_{y'_i} = 0$$

$$dN - P\sin\frac{d\phi}{2} - (P + dP)\sin\frac{d\phi}{2} = 0$$

$$\left[\sin\frac{d\phi}{2} \Rightarrow \frac{d\phi}{2}\right]$$

$$\Rightarrow dN - \frac{pd\phi}{2} - \frac{pd\phi}{2} - \frac{dpd\phi}{2} = 0\left[\frac{dpd\phi}{2} \approx 0\right]$$

$\Rightarrow$ dN – Pdϕ = 0

$\Rightarrow dN - Pd\phi$ .... (iii)

Put $dN = Pd\phi$ from equation (iii) to (ii)

$dP = \mu \times Pd\phi$

$$\frac{dP}{P} = \mu d\phi$$

Integrating for contact region between band and drum

$$\int_{P_2}^{P_1} \frac{dP}{P} = \int_0^{\alpha} \mu d\phi$$

$$\Rightarrow [\ln P]_{P_2}^{P_1} - \mu[\phi]_0^{\alpha}$$

$$\Rightarrow \ln \frac{P_1}{P_2} = \mu\alpha$$

$$\Rightarrow \frac{P_1}{P_2} = e^{\mu\alpha} \text{ ....(iv)}$$

From equation (iv)

$e^{\mu\alpha} > \alpha \Rightarrow P_1 > P_2$

Therefore $P_1 = P_{max}$ = tight side tension and, $P_2 = P_{min}$ = stack side tension

Put $dN = Pd\phi$ from equation (iii) to (i)

$Pd\phi = p \times Rd\phi \times b$

$$\Rightarrow p = \frac{P}{Rb} \text{ ....(v)}$$

$\therefore$ p α P, therefore $p = p_{max}$ = Maximum pressure

when $P = P_{max} = P_1$ = maximum tension from equation (v)

$$p_{max} = \frac{P_1}{rb} \text{ .... (vi)}$$

From FBD of drum (figure a) braking torque on strip (dT_f)

$dT_f = \mu dN \times R$ .... (vii)

from equation (ii) put $\mu dN = dP$ in equation (vii)

$dT_f = dP \times R$

Total braking torque (T_f)

$$T_f = \int_{P_2}^{P_1} dP \times R$$

$$\Rightarrow T_f = [P]_{P_2}^{P_1} \times R$$

$$\Rightarrow T_f = (P_1 - P_2) \times R \text{ ..... (viii)}$$

$$\left[\frac{P_1}{P_2} = e^{\mu\alpha}\right]$$

$$\Rightarrow \left[T_F = P_2 R\left(e^{\mu\alpha} - 1\right)\right]$$ First relation proved

From equation (viii)

$T_F = (P_1 - P_2)\,R$

$$\Rightarrow T_f = P_1\left(1 - \frac{P_2}{P_1}\right)R$$

$$\left[\because \frac{P_1}{P_2} = e^{\mu\alpha} \Rightarrow \frac{P_2}{P_1} = e^{-\mu\alpha}\right]$$

$$\left[p_{max} = \frac{P_1}{Rb} \Rightarrow P_1 = p_{max}Rb\right]$$

$$T_f = p_{max} \times Rb\left[1 - e^{-\mu\alpha}\right]R$$

$$\Rightarrow \boxed{T_f = bR^2 p_{max}\left(1 - e^{-\mu\alpha}\right)}$$

Hence second relation proved

SECTION-B

Q.5. **(a)** Zirconium has an HCP crystal structure and a density of 6.51 g/cm³. The atomic weight of zirconium is 91.22 g/mol. Answer the following:

(i) What is the volume of its unit cell in cubic metres?

(ii) If the c/a ratio is 1.593, compute the values of c and a.

Sol. **(a)** (i) The volume of the Zr unit cell may be computed using Equation

$$V_C = \frac{nA_{Zr}}{\rho N_A}$$

Now, for HCP, $n = 6$ atoms/unit cell, and for Zr, $A_{Zr} = 91.22$ g/mol. Thus,

$$V_C = \frac{(6\ \text{atoms/unit cell})\ (91.22\ \text{g/mol})}{(6.51\ \text{g/cm}^3)\ (6.022 \times 10^{23}\ \text{atoms/mol})}$$

$$= 1.396 \times 10^{-22}\ \text{cm}^3/\text{unit cell} = 1.396 \times 10^{-28}\ \text{m}^3/\text{unit cell}$$

(ii) The solution for HCP

$$V_C = 6R^2 c\sqrt{3}$$

But, since a = 2R, (i.e., R = a/2) then

$$V_C = 6\left(\frac{a}{2}\right)^2 c\sqrt{3} = \frac{3\sqrt{3}a^2 c}{2}$$

But, since c = 1.593a

$$V_C = \frac{3\sqrt{3}(1.593)a^3}{2} = 1.396 \times 10^{-22}\ \text{cm}^3/\text{unit cell}$$

Now, solving for a

$$a = \left[\frac{(2)\ (1.396 \times 10^{-22}\ \text{cm}^3)}{(3)\ (\sqrt{3})\ (1.593)}\right]^{1/3}$$

$$= 3.23 \times 10^{-8}\ \text{cm} = 0.323\ \text{nm}$$

And finally

$$c = 1.593a = (1.593)\ (0.323\ \text{nm}) = 0.515\ \text{nm}$$

(b) Give at least four comparisons between honing and lapping. Also list at least three functions performed by electrolyte in electrochemical machining (ECM) process.

Sol. (b) 1. Purpose and Application:

- **Honing:** Used to improve the geometric form of a surface and to provide a precise surface finish. This process is often used for internal cylindrical surfaces such as automobile engine cylinders and hydraulic cylinders.
- **Lapping:** Primarily aimed at achieving a highly finished surface, with extremely tight tolerances for flatness or roundness. It is typically employed for finishing the surfaces of, metal seals, and fine mechanical bearings.

2. Tooling and Abrasives:

- **Honing:** Utilizes a set of abrasive stones or sticks mounted on a tool that rotates and moves back and forth over the workpiece. The abrasives are often made of aluminum oxide or silicon carbide.
- **Lapping:** Employs a lapping plate and loose abrasive grains that are rolled or rubbed against the workpiece, along with a lapping compound. The abrasives may include aluminum oxide, silicon carbide, diamond dust, or other specialized materials.

3. Surface Contact:

- **Honing:** The abrasive stones are in continuous contact with the workpiece, applying a controlled amount of pressure to create the desired surface.
- **Lapping:** The abrasives are loosely held and the contact is less direct, with the paste or slurry allowing the grains to move freely for a gentler and more diffuse finishing process.

4. Process Characteristics:

- **Honing:** Produces a cross-hatched surface pattern that helps in retaining lubrication, which is ideal for engine cylinders or components where lubrication retention is crucial.
- **Lapping:** Yields a mirror-like finish with very high precision, which is essential for applications demanding minimal surface irregularities and high dimensional accuracy.

Functions of Electrolyte in Electrochemical Machining (ECM) Process:

1. Conducting Electricity:

- The electrolyte acts as a conductor for the electric current between the tool (cathode) and the workpiece (anode). This is crucial for initiating and sustaining the electrochemical reaction necessary for material removal.

2. Removing Material:

- As the electric current passes through the electrolyte, it facilitates the removal of metal from the workpiece through an electrochemical process. The metal atoms at the workpiece surface lose electrons (oxidize) and dissolve into the electrolyte.

3. **Cooling the Tool and the Workpiece:**

- During the ECM process, heat is generated due to the electrical resistance and chemical reactions. The electrolyte helps in dissipating this heat, thereby cooling both the tool and the workpiece. This prevents thermal damage and maintains the integrity of the machined surface.

(c) List five causes of service failure giving example of at least one mechanical component in which it occurs. Also list at least five causes of vibration in mechanical system.
An automobile has four tyres. The constant failure rates of tyres 1, 2, 3 and 4 are 0.00001 failure/hour, 0.00002 failure/hour, 0.00003 failure/hour and 0.00003 failure/hour respectively. The automobile cannot be driven when any one of the tyres punctures. Find the mean time to failure of the automobile with respect to tyres and reliability for operating the automobile for 500 hours without failure of tyres.

Sol. (c) Fatigue:

Example: Crankshaft in an engine

Description: Repeated loading and unloading cycles cause small cracks to develop and propagate over time, leading to eventual failure.

Wear:

Example: Bearings in a rotary machine

Description: Continuous friction between surfaces causes material loss, leading to increased clearance and eventual malfunction.

Corrosion:

Example: Pipeline in a chemical plant

Description: Chemical reactions with the environment cause the metal to deteriorate, weakening the structure and causing leaks or bursts.

Overloading:

Example: Gear in a transmission system

Description: Operating under excessive load beyond the designed capacity leads to deformation or breakage of the gear teeth.

Material Defects:

Example: Turbine blade in a jet engine

Description: Inherent flaws in the material, such as inclusions or voids, can lead to unexpected failure under operational stresses. Causes of Vibration in Mechanical Systems:

Unbalanced Rotating Components:

Description: When the mass distribution is not even around the axis of rotation, it causes periodic forces leading to vibration.

Misalignment:

Description: Shafts or other components that are not aligned properly can induce lateral or axial vibrations.

Loose or Worn Components:

Description: Parts that are not securely fastened or are worn out can move irregularly, causing vibrations.

Resonance:

Description: When the natural frequency of a system coincides with the operating frequency, it can lead to amplified vibrations.

Gear Defects:

Description: Issues such as worn teeth, incorrect gear meshing, or damaged gears can cause uneven torque transmission and vibrations.

Bearing Defects:

Description: Faulty bearings can lead to irregular motion and cause vibrations due to the uneven movement of the rotating parts

Calculation of Mean time to failure (MTTF)

For a series system, the overall failure rate λ is the sum of the individual failure rates:

$$\lambda = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$$

$$= 0.00001 + 0.00002 + 0.00003 + 0.0003 = 0.00009 \text{ failures/hr}$$

The mean time to failure is the reciprocal of the failure rate:

$$\text{MTTF} = \frac{1}{\lambda} = \frac{1}{0.00009} = 11111.11 \text{ hrs}$$

Calculation of reliability for 500 hours.

$$R(t) = e^{-\lambda t}$$

For $t = 500$ hours

$$R(500) = e^{-0.00009 \times 500} = 95.56\%$$

(d) You are a consultant for operations of a firm that deals with just one item that costs ₹45. The firm buys the item wholesale from a supplier and sells retail. You have compiled the following details for the item:

Parameters	**Values**
Annual demand	4380
Workdays/year	365
Opportunity cost of investment in inventory	12.5%
Fixed cost of order generation per order	₹22
Cost of inspecting items received	₹3
Cost due to breakage or spoilage	9.5%
Warehouse rental	6.5%
Insurance costs	1.5%

The following two options are available to you:

Option 1: The supplier can supply all items at once

Option 2: The supplier can supply 15 items per day

Which of the options would you recommend to the firm and why?

Sol. **(d)** Find the holding cost (C_H)

C_H = Unit cost × (opportunity cost + warehouse rental + insurance)

C_H = 45 (12.5% + 9.5% + 1.5%) = ₹10.575

Option 1 (Single order)

Order cost = Fixed order generation + inspecting (O.C) = 22 + 22 = ₹44

Holding cost = 2190 × 10.575 = ₹23159.25 (H.C)

Breakage/spoilage cost 4380 × 3 = 13140

Total cost = O.C + H.C + Breakage

= 44 + 23159.25 + 13140 = ₹36343.25

Option 2 (Daily supply of 15 items)

Daily demand = $\frac{4380}{365} \cong 12$ unit / day

Order frequency → Daily so no holding cost

Order cost = 44 × 365 = 16060

Breakage = 4380 × 3 = 13140

Total cost = O.C + Breakage = 16060 + 13140 = ₹29200/

Conclusion: Option 2 is recommended due to significantly lower total cost.

(e) Compare between hydraulic and electrical actuators' characteristics in the following points in brief:

(i) Stiffness of the actuators

(ii) Need of reduction gear

(iii) Need of braking device

(iv) Working in low and high temperature

(v) Working of the actuators

(vi) Maintenance need of the actuators

Sol. **(e)** **(i)** **Stiffness of the Actuators**

- **Hydraulic Actuators:** Tend to have high stiffness due to the incompressibility of the hydraulic fluid. The system's inherent rigidity makes it well-suited for applications requiring precise control under high load conditions.
- Electrical Actuators: Generally have lower stiffness compared to hydraulic actuators. The elasticity of mechanical components like gears and belts can introduce some play and flexibility, which might be a drawback in applications demanding high precision.

(ii) **Need of Reduction Gear**

- **Hydraulic Actuators:** Typically do not require reduction gears because they can generate high torque at low speeds directly through hydraulic pressure.
- **Electrical Actuators:** Often need reduction gears to achieve high torque. Electric motors usually operate at high rotational speeds with lower torque, necessitating gears to reduce the speed and increase the torque to usable levels.

(iii) Need of Braking Device

- **Hydraulic Actuators:** Often require braking devices to hold positions, as hydraulic systems can drift if not properly sealed or if the valve control is not precise.
- **Electrical Actuators:** Electric motors can inherently provide braking through electromagnetic forces. Additionally, many electric actuators can be designed to hold position without power by using worm gears or other self-locking mechanisms.

(iv) Working in Low and High Temperature

- **Hydraulic Actuators:** Can be sensitive to temperature extremes. Low temperatures can thicken hydraulic oil, reducing its efficiency, while high temperatures might lead to overheating or seals failure.
- **Electrical Actuators:** Typically more robust across a broader range of temperatures, especially with recent advances in materials and components. However, at very high temperatures, insulation materials and other components may degrade.

(v) Working of the Actuators

- **Hydraulic Actuators:** Work by pressurizing hydraulic fluid in a cylinder, which creates linear or rotational motion. This process is highly effective for generating large forces and is commonly used in heavy-duty applications.
- **Electrical Actuators:** Convert electrical energy into mechanical motion, generally using rotary motors that are either converted into linear motion through screws or belts, or used as-is for rotary applications. Suitable for precise, clean, and quiet operations.

(vi) Maintenance Need of the Actuators

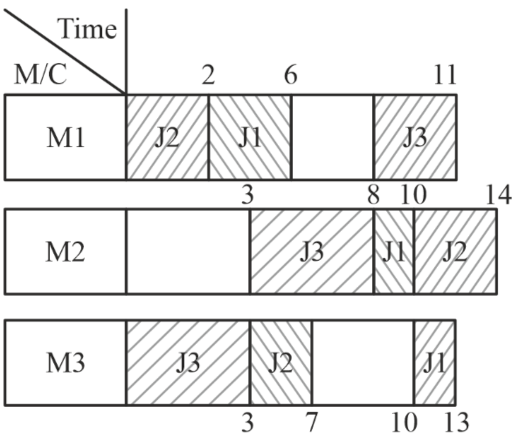
- **Hydraulic Actuators:** Require regular maintenance to check for leaks, replace hydraulic fluid, and maintain seals and filters. This maintenance is crucial for performance and longevity, and lack of it can lead to system failure.
- **Electrical Actuators:** Generally have lower maintenance needs as they have fewer moving parts and do not require fluid replacements. However, they still need periodic inspection of electrical components and lubrication of mechanical parts.

Q.6. (a) (i) Three jobs are to be processed in a job shop consisting of three machines. Each job requires three operations and they are to be carried out in 1→2→3 order. The following table indicates the machines required as well as processing time (in hours) required for each operation. Initially, all jobs and machines are available. Compute the make span by drawing Gantt chart indicating every operation of each job using shortest processing time dispatching rule and break ties with least work remaining rule:

	Machine required for operation			Processing time of operation		
	1	2	3	1	2	3
Job 1	M1	M2	M3	4	2	3
Job 2	M1	M3	M2	2	4	4
Job 3	M3	M2	M1	3	5	3

Sol. (a) (i)

	Machine required for operation			Processing time of operation		
	1	2	3	1	2	3
Job 1	M1	M2	M3	4	2	3
Job 2	M1	M3	M2	2	4	4
Job 3	M3	M2	M1	3	5	3



Make span time = 14 hours

(a) (ii) Explain the physics of arc initiation in arc welding. Why is arc initiation difficult in plasma arc welding? Why is plasma arc welding called as plasma arc welding despite the fact that plasma is present in all other arc welding processes?

Sol. (a) (ii) Physics of Arc Initiation in Arc Welding

Arc initiation in welding generally involves creating an electric arc between an electrode and the workpiece. This arc is formed when a sufficiently high voltage is applied, overcoming the dielectric breakdown of the air or shielding gas between the electrode and the workpiece. When the breakdown occurs, it ionizes the air or gas, turning it into a conductor which allows current to pass and heat the material to welding temperatures.

Challenges in Arc Initiation in Plasma Arc Welding

Plasma arc welding (PAW) uses a constricted plasma jet as the heat source, which can be initiated by either transferring an arc from a pilot arc or by direct initiation at the torch. Here's why it's particularly challenging:

- Constricted arc: The plasma in PAW is tightly constricted by a fine-bore copper nozzle, which raises the temperature and velocity of the plasma jet. This constriction makes the initial ionization of the gas more difficult compared to other forms of arc welding where the arc spreads more freely.
- Stability requirement: The stability of the pilot arc, which is crucial for initiating the main welding arc in non-contact starts, requires precise control over the torch setup and gas flow. This adds complexity to arc initiation.
- Shielding and plasma gas: The need to maintain precise flows and balances between shielding gas and plasma gas further complicates the initiation process.

Why is it called "Plasma Arc Welding"?

Plasma arc welding is named for the very distinct role plasma plays in the process. Although all arc welding processes generate some plasma, PAW uniquely manipulates the plasma itself as a more defined and controlled heat source.

(b) (i) Briefly describe the techniques that may be used for galvanic protection. Also explain why cold-worked metals are more susceptible to corrosion than non-cold-worked metals.

Sol. (b) (i) **Techniques for Galvanic Protection:** Galvanic protection involves using electrochemical principles to prevent metal corrosion. Here are the primary techniques used:

Sacrificial Anode Protection (Cathodic Protection): Principle: A more anodic (reactive) metal is connected to the metal to be protected, which acts as the cathode.

Process: The sacrificial anode corrodes instead of the protected metal.

Application: Commonly used in marine environments, underground pipelines, and water heaters.

Materials: Typical sacrificial anodes include zinc, magnesium, and aluminum.

Impressed Current Cathodic Protection (ICCP):

Principle: An external power source supplies current to counteract the corrosive currents.

Process: An inert anode material is used with a direct current power source, which provides electrons to the protected metal.

Application: Used for large structures like pipelines, ship hulls, and steel-reinforced concrete.

Materials: Inert anodes such as titanium, mixed metal oxides, or platinum-coated anodes.

Cold-Worked Metals and Corrosion Susceptibility

Cold-worked metals are more susceptible to corrosion compared to non-cold-worked (annealed) metals due to the following reasons:

Increased Dislocation Density:

Effect: Cold working introduces a high density of dislocations and defects in the metal's crystal structure.

Corrosion Mechanism: These dislocations serve as pathways for corrosion, increasing the metal's susceptibility to localized corrosion such as pitting and stress corrosion cracking.

Residual Stresses:

Effect: Cold working generates residual stresses within the metal.

Corrosion Mechanism: These stresses can create micro-cracks or weaken the metal, providing sites for corrosive agents to penetrate and initiate corrosion.

Altered Microstructure:

Effect: Cold working can lead to grain refinement and introduce inhomogeneities.

Corrosion Mechanism: The altered microstructure may have regions of different electrochemical potentials, leading to galvanic corrosion within the metal itself.

Decreased Passivity:

Effect: The passive oxide layer on metals like stainless steel can be disrupted by cold working.

Corrosion Mechanism: The disruption of the passive layer exposes the metal to the environment, increasing its vulnerability to corrosion.

(b) (ii) Write the possible oxidation and reduction half-reactions that occur when magnesium is immersed in each of the following solutions:

(1) HCI

(2) HCl solution containing dissolved oxygen

(3) HCl solution containing dissolved oxygen and in addition Fe2+ ions

In which of the above solutions would you expect the magnesium to oxidize rapidly and why?

Sol. (b) (ii) When magnesium is immersed in various solutions, different oxidation and reduction half-reactions can occur depending on the constituents of the solution. Here are the possible half-reactions for each scenario:

Possible Oxidation and Reduction Half-Reactions

(1) Magnesium in HCI (Hydrochloric Acid) Solution

Oxidation half-reaction (anode):

$$Mg\ (s) \rightarrow Mg^{2+}\ (aq) + 2e^-$$

Reduction half-reaction (cathode):

$$2H^+\ (aq) + 2e^- \rightarrow H_2(g)$$

(2) Magnesium in HCI Solution Containing Dissolved Oxygen

Oxidation half-reaction (anode):

$$Mg\ (s) \rightarrow Mg^{2+}\ (aq) + 2e^-$$

Reduction half-reactions (cathode):

- **For hydrogen ions:**

  $2H^+ (aq) + 2e^- \rightarrow H_2(g)$

- **For dissolved oxygen:**

  $O_2(g) + 4H^+ (aq) + 4e^- \rightarrow 2H_2O(1)$

(3) Magnesium in HCI Solution Containing Dissolved Oxygen and Fe^2 + Ions

Oxidation half-reaction (anode):

$Mg (s) \rightarrow Mg^{2+} (aq) + 2e^-$

Reduction half-reactions (cathode):

- **For hydrogen ions:**

  $2H^+ (aq)+ 2e^- \rightarrow H_2(g)$

- **For dissolved oxygen:**

  $O_2(g) + 4H^+ (aq) + 4e^- \; 2H_2O(1)$

- **For iron(II) ions:**

  $Fe^{3+} (aq) + e^- \rightarrow Fe^{2+}(aq)$

Magnesium would oxidize most rapidly in the HCl solution containing dissolved oxygen and Fe^{2+} ions. This is because:

(i) **Multiple Reduction Reactions:** The presence of multiple reduction reactions (hydrogen ion reduction, oxygen reduction, and Fe^{3+} reduction) provides several pathways for the consumption of electrons. This enhances the overall corrosion rate of magnesium.

(ii) **Dissolved Oxygen:** Dissolved oxygen is a strong oxidizing agent, which can significantly accelerate the corrosion process through the reduction of oxygen, thereby increasing the rate of magnesium oxidation.

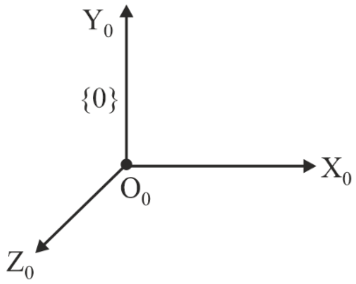
(iii) **Presence of Fe^{2+} Ions:** The presence of Fe^{2+} ions further accelerates the process because Fe^{3+} ions can be reduced to Fe^{2+}, providing an additional reduction reaction that consumes electrons, thus enhancing the oxidation of magnesium.

In summary, the HCl solution containing dissolved oxygen and Fe^{2+} ions creates an environment with multiple active reduction reactions, which collectively increase the rate of electron consumption, leading to a more rapid oxidation of magnesium.

(c) (i) The forward kinematic model of a planar 2 DOF (RR) manipulator with link lengths $a_1 = a_2 = 10$ units, is given by the matrix

$$^{o}T_2 = \begin{bmatrix} 0 & -1 & 0 & 10/\sqrt{2} \\ 1 & 0 & 0 & 10+10/\sqrt{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Draw the last frame {2}, with respect to {0} frame, by locating its position and its orientation. The initial frame, frame {0} is given as



Sol. **(c)** **(i)** $a_1 = a_2 = 10$ units

$$T_2^o = \begin{bmatrix} 0 & -1 & 0 & 10/\sqrt{2} \\ 1 & 0 & 0 & 10+10/\sqrt{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

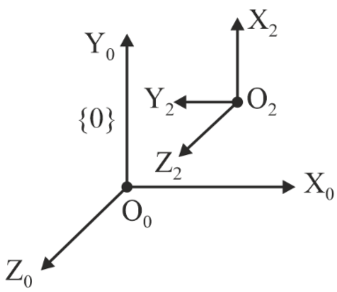
Rotational part (upper-left 3×3 block); Translational part (upper-right 3×1 column)

$\Rightarrow$ Rotation indicate a 90° counterclockwise rotation about z-axis.

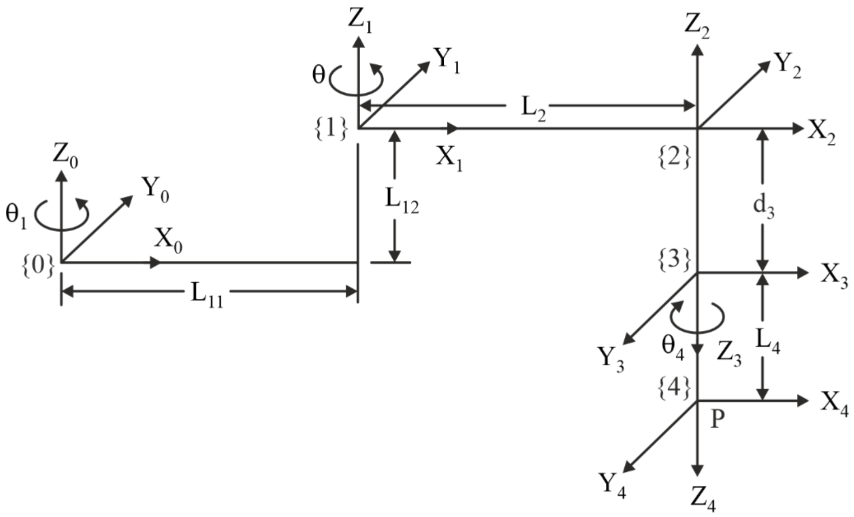
$\Rightarrow$ Translation is $\left(\frac{10}{\sqrt{2}}, 10+\frac{10}{\sqrt{2}}\right)$

Frame {2}

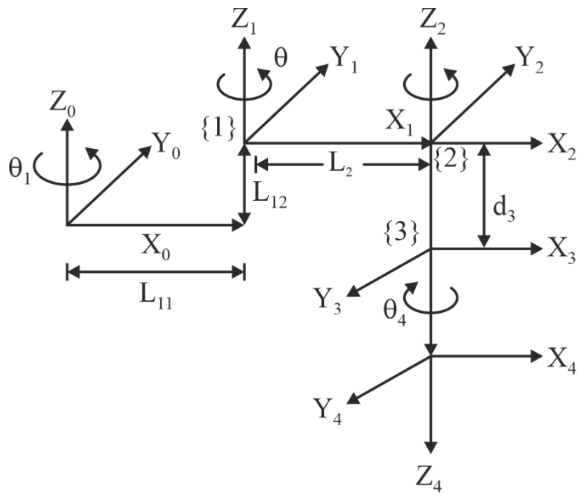
Start from origin O_0 of frame {0} moving along the original x-axis by $\frac{10}{\sqrt{2}}$ units and then parallel to the original y-axis by $10+\frac{10}{\sqrt{2}}$ units.



(c) **(ii)** For the given frames of SCARA manipulator, generate the DH parameters table:



Sol. **(c)** **(ii)**



θ_I → Angle between the previous z-axis and the current z-axis, about the previous x-axis.

d_i → The offset along the previous z-axis to the common normal.

a_i → The length of the common normal.

α_I → Angle between the previous x-axis and the current x-axis, about the common normal.

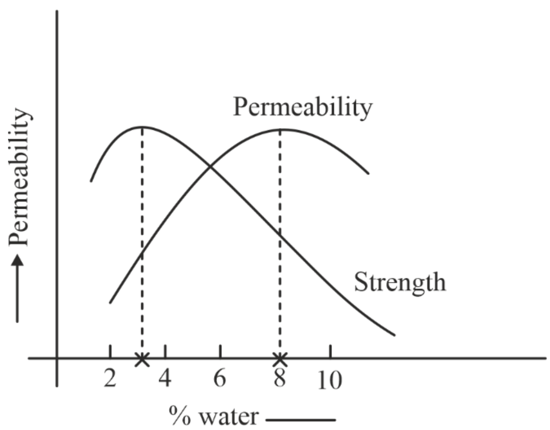
i = 1, 2, 3, 4.

DH Parameters table

Joint i	θ_i	d_i	a_i	α_i (degree)
1	θ_1	0	L_{11}	0
2	θ_2	0	L_{12}	0
3	0	d_3	0	0
4	θ_4	L_4	0	0

Q.7. **(a)** **(i)** How does permeability of molding sand vary with the moisture content? Explain with the help of neat sketches. Also explain the role of adding organic additives in the molding sand.

Sol. **(a)** **(i)** **Permeability:** Gases are formed due to mould metal reaction and also due to vapourization. The mould must be porous to permit the gases to pass off, otherwise will have gas holes defect. If the grain size of sand is large, there will be larger void size between particles. This leads to larger permeability.



As shown is figure if the water content is less, these voids will be filled with smaller sand particles which results in lower permeability. By increasing the water content, smaller sand particles are also combine together, this increase permeability. Once the percentage of water goes beyond 8%, water start accumulating in these voids and the sand becomes pasty. This decreases permeability and decrease in strength of the mould. Permeability is quantified by Permeability Number (PN) which is defined as

$$PN = \frac{VH}{PAT}$$

PN is the ratio in millimeters at which air will pass through the sand under a standard condition of pressure.

Role of Additives

- Cereal binder up to 2% increases the strength.
- Pitch if used up to 3% would improve the hot strength.
- Saw dust up to 2% may improve the collapsibility by slowly burning and increase the permeability.
- Other materials: Sea Coal, Asphalt, Fuel Oil, Graphite, Molasses, Iron oxide, etc.

(a) (ii) A dimension 57.975 mm is required to be set with the help of slip gauge blocks as accurately as possible. Two slip gauge block sets M45 (Grade 0) and M112 (Grade II) are available. The range and number of pieces in each set are given below:

Set M45 (Grade 0)			**Set M112 (Grade II)**		
Range (mm)	**Steps (mm)**	**Number of blocks**	**Range (mm)**	**Steps (mm)**	**Number of blocks**
1.001 to 1.009	0.001	9	1.0005	–	1
1.01 to 1.09	0.01	9	1.001 to 1.009	0.001	9
1.1 to 1.9	0.1	9	1.01 to 1.49	0.01	49
1.0 to 9.0	1.0	9	0.5 to 24.5	0.5	49
10.0 to 90.0	10.0	9	25.0 to 100.0	25.0	4

The permissible errors in 1/100000 mm units in the mean length of Grade 0 and Grade II are given below:

Length	0 to 20	20 to 60
Grade II	+50 –20	+80 –50
Grade 0	±10	±15

Find the slip gauge that you will prefer, with reasons.

Sol. **(a)** **(ii)** For more accuracy we will chose grade '0' as the tolerance build as will be less

Grade 0		**Grade II**	
57.975		57.975	
– 1.005	(1)	– 1.005	(1)
56.97		56.97	
– 1.07	(2)	– 1.47	(2)
55.9		55.5	
1.9	(3)	– 5.5	(3)
54		50	(4)
– 4	(4)		
50	(5)		

Grade 0:

Tolerance build up

$= 4 \times 10 + 15 = \pm 55$

(110) width of tolerance zone

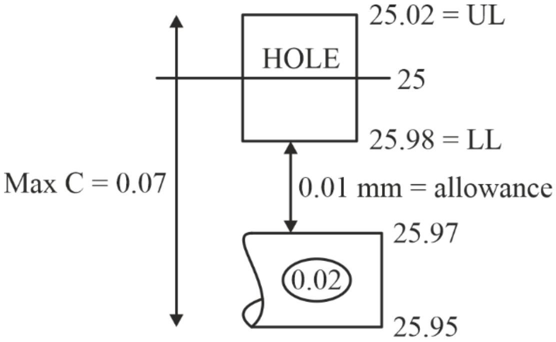
Grade II:

Tolerance build up

$$\left.\begin{aligned} +50\times 3+80 &= +230 \\ -20\times 3-50 &= -110 \end{aligned}\right\} \text{Width of tolerance}$$

(b) **(i)** It is possible to drill a 25 mm nominal hole to an accuracy of $25^{+0.02}_{-0.02}$ mm using standard drill and drilling machine available. A shaft is to be machined to obtain a clearance fit in the above hole such that minimum allowance should be 0.01 mm and maximum clearance should not be more than 0.07 mm. Find the tolerance on the shaft. Also state why hole basis system of fits is generally preferred over shaft basis system of fits.

Sol. **(b)** **(i)**



Minimum Hole Size = 25 − 0.02 = 24.98 mm

Maximum Hole Size = 25 + 0.02 = 25.02 mm

Minimum Allowance = 0.01 mm

Maximum Clearance = 0.07 mm

Maximum Shaft Size = 24.98 mm − 0.01 mm = 24.97 mm

Minimum Shaft Size = 25.02 mm − 0.07 mm = 24.95 mm

The tolerance on the shaft is = 24.97 − 24.95 = 0.02 mm

Reason for Preference of Hole Basis System:

- **Manufacturing Convenience:** Holes are generally easier to control in terms of size during manufacturing processes, especially when drilling, reaming, or boring. This is because the tools for creating holes tend to have standardized sizes.

- **Tooling Costs:** Standardized hole sizes mean that tools and gauges are readily available and less expensive compared to custom sizes required for shaft basis systems.
- **Interchangeability:** With a hole basis system, a variety of shaft sizes can be used with a single hole size, improving interchangeability and reducing inventory costs.
- **Machining Consistency:** It is often easier to maintain consistency in hole sizes because machining operations like drilling and reaming produce more uniform results compared to turning operations used for shafts.

(b) (ii) List the manufacturing situations where FMS technology can be successfully employed. Also give at least four differences between dedicated and random-order FMS.

Sol. (b) (ii) Situations for Successful Employment of FMS:

1. **Variable Product Mix:** FMS is ideal in environments where the product mix changes frequently, requiring the manufacturing process to adapt quickly without significant downtime.
2. **Moderate Volume, High Variety:** FMS systems excel in situations where the production volume is not high enough to justify dedicated assembly lines, but the variety of products demands flexibility in production capabilities.
3. **Custom Manufacturing:** Companies that offer customized or bespoke products can benefit from FMS due to its ability to switch between different product designs quickly.
4. **Just-In-Time Manufacturing:** FMS supports JIT manufacturing practices by reducing setup times and enabling efficient response to production demands without the need for large inventories.
5. **Complex Machining Tasks:** Industries requiring complex, precise machining tasks (like aerospace or medical devices) find FMS valuable because of its precision and repeatability across diverse tasks.

Differences Between Dedicated and Random-Order FMS:

1. **Flexibility in Operation:**
   - **Dedicated FMS:** Configured to produce a specific range of parts or products, with limited flexibility. It is optimized for a particular set of similar operations or products.
   - **Random-Order FMS:** Designed to handle any order of products within a mix at any time, providing higher flexibility and the ability to adapt to changing production requirements quickly.
2. **System Layout:**
   - **Dedicated FMS:** Typically has a fixed layout optimized for the efficient production of a particular set of products.
   - **Random-Order FMS:** Features a more dynamic layout that can be adjusted based on the production schedule and is not tied to a specific sequence of operations.

3. **Production Capacity and Scalability:**
   - **Dedicated FMS:** Generally has higher production capacity for specific products but is less scalable since adapting to new products may require significant reconfiguration or additional investment.
   - **Random-Order FMS:** More scalable in terms of handling various products as market demands shift, though it might operate with less operational efficiency compared to a perfectly tuned dedicated system.
4. **Capital Investment and Operating Costs:**
   - **Dedicated FMS:** Might involve lower initial capital investment for specific product lines but can incur higher costs if product specifications change significantly.
   - **Random-Order FMS:** Requires a higher initial investment due to the need for more sophisticated control systems and equipment that can accommodate a wide variety of products, but potentially lower long-term costs by avoiding the need for future significant investments.
5. **Typical Applications:**
   - **Dedicated FMS:** Best suited for stable product demands with little variation in type and design, such as automotive or electronic component manufacturing.
   - **Random-Order FMS:** Ideal for industries like consumer electronics or furniture where product designs and demands can change rapidly and unpredictably.

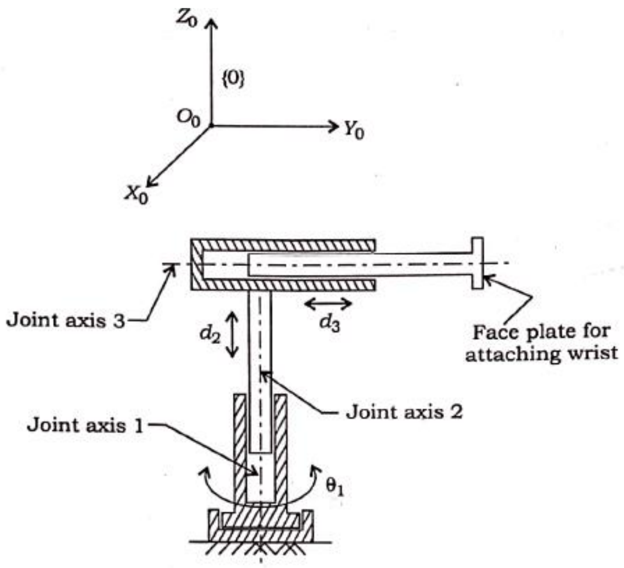
(c) Formulate the forward kinematic model of the 3 DOF (RPP) manipulator arm, shown in the figure, by

(i) generating and drawing the frames using DH rules;

(ii) generating the DH parameters table from the assigned frames;

(iii) generating the individual transformation matrices ${}^0T_1$, ${}^1T_2$, ${}^2T^3$ and the overall transformation matrix ${}^0T_3$.

Also draw the last frame {3}, if θ_1, d_2 and d_3 are given respectively as 0°, 10 units and 10 units, with reference to the given initial frame:



The homogeneous transformation matrix ${}^{i-1}T_i$ is given as

$$ {}^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & -s\theta_i s\alpha_i & \alpha_i c\theta_i \\ s\theta_i & -c\theta_i c\alpha_i & -c\theta_i s\alpha_i & \alpha_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Sol. (c) (i)

- **Joint 1 (Revolute):** Rotates around z-axis, joint 2 prismatic. Extends along the y-axis and joint 3 (Prismatic): Extends along the y-axis.

Frames:

- **Frame 0:** At the base, with z_0 up, frame 1: located at joint 1, rotates with it frame 2: located at the base of joint 2, extends in y-axis and frame 3: located at the base of joint 3, extends in y-axis.

Drewing Frames:

- **For joint 1:** x_1 along y_0, z_1 aligned with z_0.

  For joint 2: x_2 along x_1, z_2 along y_1

  For joint 3: x_3 along y_2, x_3 along y_2.

DH Parameters

Joint	θ	d	a	α
1	θ_1	0	0	90°
2	0	d_2	0	0
3	0	d_3	0	0

Individual and overall transformation matrixes.

${}^0T_1$ for joint 1:

$$ {}^0T_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

${}^1T_2$ for joint 2:

$$ {}^1T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

${}^2T_3$ for joint 3:

$$ {}^2T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Overall transformation at matrix ${}^0T_3$

$$ {}^0T_3 = {}^0T_1 \times {}^1T_2 \times {}^2T_3 $$

$$= \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & -\sin\theta_1\, d_2 \\ \sin\theta_1 & \cos\theta_1 & 0 & \cos\theta_1\,(d_1 + d_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q.8. **(a)** A 12.7 mm diameter steel wire is drawn to obtain 35.5% reduction in area by drawing through a conical die of 6º semi-cone angle. The coefficient of friction between the wire material and die material at conical portion of die is 0.1 and there is no back pull. The tensile yield strength of the original specimen is 207 MPa and is 414 MPa at a strain of 0.5. Assuming linear stress relationship for the wire material and efficiency of electrical motor as 98%, find the drawing power and maximum possible reduction.

Sol. **(a)** Given, $d_i = 12.7$ mm, $\alpha = 6°$, $A_o = \frac{\pi}{4} \times d_i^2 = \frac{\pi}{4} \times 12.7^2 = 126.67\ \text{mm}^2$

$A_f = 0.645 \times A_i$

$= 0.645 \times 126.67$

$= 81.70\ \text{mm}^2$

Stress-strain relationship is linear

$$\sigma_{yf} = 207 + \frac{414 - 207}{0.5} \times 0.438 = 388\ \text{N/mm}^2$$

$$\varepsilon = \ln\frac{A_i}{A_f} = \ln\left(\frac{126.67}{81.70}\right) = 0.438$$

$B = \mu \cot\alpha = 0.1 \cot 6° = 0.95$

$$\sigma_o = \frac{207 + 388}{2} = 297.5\ \text{MPa}$$

$$\sigma_d = \sigma_o\left(\frac{1+B}{B}\right)\left[1 - \left(\frac{A_f}{A_i}\right)^B\right]$$

$$\Rightarrow \quad 297.5 = \left(\frac{1+0.95}{0.95}\right)\left(1 - \left(\frac{81.70}{126.67}\right)^{0.95}\right)$$

$$= 297.5 \times \frac{1.95}{0.95}\left[1 - (0.64)^{0.95}\right]$$

$= 610.65\ [1 – 0.65]$

$= 211.57$ MPa

For power, Assume v = 1 m/s

$$P = \frac{\sigma_d \times v \times A_f}{0.98} \Rightarrow \frac{211.57 \times 1 \times 81.70}{0.98} = 17.63\ \text{kW}$$

Maximum possible reduction

$\sigma_d = \sigma_o$ (for maximum reduction)

$$1 = \left(\frac{0.95+1}{0.95}\right)\left[1-\left(\frac{A_f}{A_i}\right)^{0.95}\right]$$

$$0.487 = 1-\left(\frac{A_f}{A_i}\right)^{0.95}$$

$$0.512 = \left(\frac{A_f}{A_i}\right)^{0.95}$$

$$0.4942 = \frac{A_f}{A_i}$$

$$\%\text{ reduction } = \left(\frac{A_i - A_f}{A_i}\right)\times 100 = (1-0.4942)\times 100 = 50.57\%$$

(b) **(i)** The transformation of frame {i-1} to frame {i} consists of four basic transformations as following:

(1) A rotation about Z_{i-1} axis by an angle θ_i

(2) A translation along Z_{i-1} axis by distance d_i

(3) A translation along X_i axis by distance a_i

(4) A rotation about X_i axis by an angle α_i

Generate the individual transformation matrices and also the composite transformation matrix ${}^{i-1}T_i$, due to the above successive transformations. If all the above parameters (DH) are zero, what will be the composite transformation matrix?

Sol. **(b)** **(i)** Transformation matrices for moving from frame i-1 to frame i using DH convection.

Determine the composite transformation matrix when all parameter are zero.

(1) Rotation about Z_{i-1} axis by angle θ_i

$$R_\theta = \begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(2) Translation along Z_{i-1} axis by distance d_i

$$T_d = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(3) Translation along X_i axis by distance a_i

$$T_a = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4) Rotation about X_i axis by angle α_i

$$R_\alpha = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_i & \sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Composite transformation matrix

$$T_i^{i-1} = R_\theta \cdot T_d \cdot T_a \cdot R_\alpha$$

The individual matrix for zero parameters are

$$R_\theta = R_\alpha = T_d = \alpha = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_i^{i-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) (ii) Explain the following sensor characteristics in brief:

(1) Range (2) Resolution

(3) Reliability (4) Repeatability

(5) Sensitivity

Sol. (b) (ii) (i) **Range:** The range of a sensor refers to the span of input values that it can accurately measure.

Example: If a temperature sensor has a range of -50°C to 150°C, it means it can accurately measure temperatures within that interval.

(ii) **Resolution:** Resolution is the smallest change in the measured quantity that the sensor can detect.

Example: If a pressure sensor has a resolution of 0.1 psi, it means it can detect changes in pressure as small as 0.1 psi.

(iii) **Reliability:** Reliability indicates the sensor's ability to perform consistently under specified conditions over time.

Example: A reliable sensor will consistently produce accurate readings without frequent failure or need for recalibration.

(iv) **Repeatability:** Repeatability refers to the sensor's ability to produce the same output for the same input when measured multiple times under the same conditions.

Example: If a sensor repeatedly measures a weight of 100 grams as 100 grams in multiple trials, it has high repeatability.

(v) **Sensitivity:** Sensitivity is the ratio of the change in output to the change in input. It indicates how much the sensor's output will change in response to a change in the measured quantity.

Example: If a sensor's output changes by 5 units for every 1 unit change in input, the sensitivity is 5 units per unit of input.

(c) (i) Generate a forward kinematic model of the given two degrees of freedom (RP) planar manipulator.

Sol. (c) (i) By using Denovit-Hartenberg parameter method.

1^{st} Joint 1 (Revolute joint) → Rotates about z-axis

Jiont 2 (Prismatic joint) → Extends along x-axis

2^{nd} **Frame 0:** Fixed at the base of the manupulator.

The z-axis is along the axis of the rotation of the first joint.

Frame 1: Aligned with frame 0 initially but rotates about the z-axis of frame based on the joint angle θ_1.

Frame 2: Located at the end of the prismatic joint, translates along the x-axis of frame 1 by distance d2.

DH Parameter

θ_i → Angle about the previous z-axis to the current z-axis.

d_i → Offset along the previous z-axis to the common normal.

a_i → Length of the common normal.

α_i → Angle about common normal from the previous z-axis.

Joint i	θ_i	d_i	a_i	α_i
1	θ_1	0	0	0°
2	0	d_2	0	0°

Transformation matrices

(a) Transformation from frame 0 to frame 1 (T_1^0)

$$T_1^0 = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) Transformation from frame 1 to frame 2 (T_2^1)

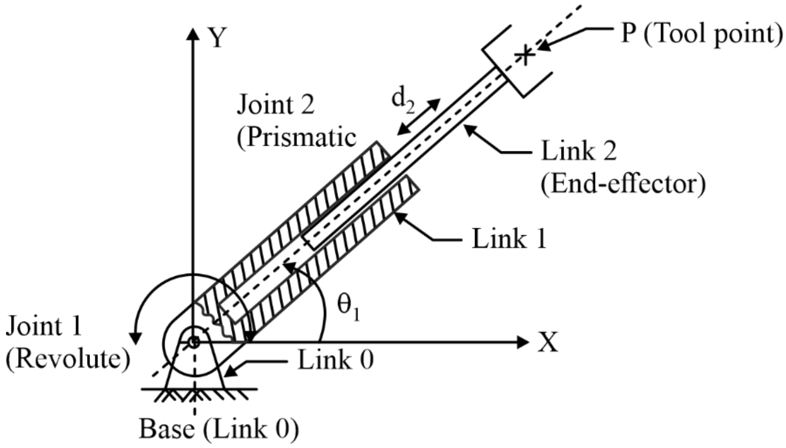
$$T_1^1 = \begin{bmatrix} 1 & 0 & 0 & d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = T_1^0 \cdot T_2^1$$

$$= \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 & d_2\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & d_2\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(c) **(ii)** Determine the joint variables (θ_1, d_2) for the above manipulator using inverse kinematic model, if the position and orientation of the end-effector are given by the following matrix:

$$T_E = \begin{bmatrix} 0.707 & 0 & 0.707 & 70.71 \\ 0.707 & 0 & -0.707 & -70.71 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Given that

$${}^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Sol. **(c)** **(ii)** $T_E = \begin{bmatrix} -0.707 & 0 & -0.707 & -70.71 \\ 0.707 & 0 & -0.707 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

From matrix the position vector = (P_x, P_y, P_z) = (–70.70, 0, 1)

$$\theta_1 = \tan^{-1}\left(\frac{0.707}{-0.707}\right)$$

θ_1 = 225° or –135°

$$d_2 = \frac{-70.70}{\cos(225)} = 100 \text{ units}$$

❑❑❑

THANK YOU!