Machine Design

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Machine Design

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1 DESIGN AGAINST STATIC LOAD

1.1 Introduction

1.1.1 Type of Loads

Static Load:

Magnitude, direction & point of Application of load does not change with time or remains constant with time (t)
E.g., Load acting on the beam and columns of the room.

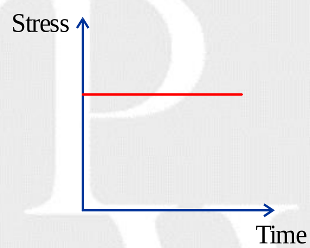


Fig.1.1: Static stress

Fluctuating Load:

Magnitude, Direction, 'or' Point of application of load changes with time.
E.g., Load on any bridge; load on the piston of an IC engine

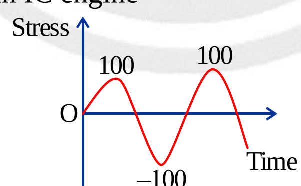


Fig.1.2: Fluctuating stress

1.1.2 Strength of Material

The magnitude of stress corresponding to which if the value of induced stress exceeds; the material fails. That particular value of stress is known as strength of material.
It is a property of material & depends on the nature of loading i.e., load is fixed.

$$\text{Induced Stress} > \text{Strength} \leftarrow \text{fail}$$

$$\text{Induced Stress} < \text{Strength} \leftarrow \text{safe}$$

For ductile material: Failure criteria is Yield strength.
For brittle material: Failure criteria is ultimate strength.

1.1.3 Design under 1-D Tensile Stress Condition

Fig.1.3: Axial tensile stress

Tensile Strength: (Strength of material under 1D tensile stress condition):

$$\sigma_{ft}\begin{cases} = S_{yt}\,(\text{for Ductile}) \\ = S_{ut}\,(\text{for Brittile}) \end{cases}$$

For safe design: $\sigma \le \sigma_{ft}$

For more safer design (we use factor of safety)

Factor of safety = $\dfrac{\text{Failure stress}}{\text{Allowable stress}} = \dfrac{\text{Strength of material}}{\text{Working stress}}\{\text{FOS} > 1\}$

$$\sigma \le \frac{\sigma_{ft}}{\text{Fos}}$$

1.1.4 Design under 1-D Compressive Stress Condition

Fig.1.4: Axial compressive stress

Compressive Strength

Compressive Strength (Strength of material under 1D tensile stress condition):

$$\sigma_{fc}\begin{cases} = S_{yc}\,(\text{for Ductile}) \\ = S_{uc}\,(\text{for Brittile}) \end{cases}$$

For safe design;

$$\sigma \le \sigma_{fc}$$

Design must be safer:

$$\sigma \le \frac{\sigma_{fc}}{FOS}$$

Note:

1. Margin of safety = Factor of safety –1

1.1.5 Even Material & Uneven Material

Even Material

Material whose strength in compression & strength in tension are equal to each other; then the material is an even Material. i.e., For even Material;

$$\sigma_{ft} = \sigma_{fc}$$

Generally ductile Material are even material.

Uneven Material

Material whose strength in compression & strength in tension are not same; then the material is known as uneven material.
i.e., for Uneven Material;

$$\sigma_{fc} \neq \sigma_{fc}$$

Generally, brittle material is uneven material.

$$S_{uc} > S_{ut}$$

1.2 Theory of Failure (TOF)

There is no need of theory of failure if there is uniaxial 1-D loading condition since we can estimate the strength by tensile test. But in complex stress condition as shown in figure 1.5 testing is not practically possible and for such cases, we require theory of failure.

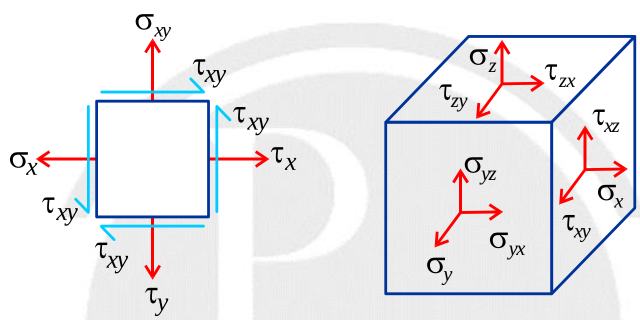
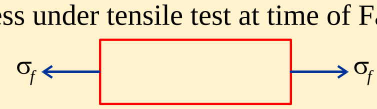


Fig.1.5: Complex Stress Condition

Theory of failure compares the actual complex stress condition of critical point with the tensile test by using some criteria of failure. Various theory of failures (TOF) is:

(1) Max. Principal stress or Max-Normal Stress TOF or Rankine TOF
(2) Max Shear stress TOF or Tresca and J.J. Guest TOF
(3) Strain Energy TOF or Haigh's TOF
(4) Distortion Energy TOF or VON Misses TOF
(5) Principal Strain TOF or St. Venant TOF.

Note:
Principal Stress under tensile test at time of Failure



{$\sigma_f \rightarrow$ failure stress}

$$\sigma_f = \begin{cases} S_{yt} \text{ (For ductile material)} \\ S_{ut} \text{ (For brittle material)} \end{cases}$$

$$\text{Principle stress} = \begin{bmatrix} \sigma_1 = \sigma_f \\ \sigma_2 = 0 \\ \sigma_3 = 0 \end{bmatrix}$$

1.2.1 Rankine 'or' Maximum Principal Stress 'or' Maximum Normal Stress TOF

Criteria of failure: Maximum principal stress or Maximum normal stress

If Maximum normal stress of the machine element crosses the maximum normal stress under tensile test at the time of failure, then; machine element will fail according to this TOF.

or

According to Rankine TOF, Machine element will be safe if the maximum normal stress induced in the Machine element is less than the maximum normal stress under tensile test at the time of failure.

For safe design:

$$(\sigma_{max})_{actual} \le (\sigma_{max})_f$$

$$(\sigma_{max})_{actual} \le \sigma_f$$

For design

$$(\sigma_{max})_{actual} \le \frac{\sigma_f}{fos}$$

$$(\sigma_{max})_{actual} \le \sigma_{perm.}$$

> **Note:**
> This theory of failure is suitable for brittle materials as brittle materials are weak in tension, hence they fail due to maximum principal stress.

1.2.2 Tresca & J.J. Guest TOF or Maximum Shear stress TOF

Criteria of failure:

Maximum Shear Stress

Maximum shear stress:

$$\tau_{max} = \max\left[\left|\frac{\sigma_1 - \sigma_2}{2}\right|, \left|\frac{\sigma_2 - \sigma_3}{2}\right|, \left|\frac{\sigma_3 - \sigma_1}{2}\right|\right]$$

Maximum shear stress under tensile test at the time of failure:

Under tensile test:

$$\sigma_1 = \sigma_f, \sigma_2 = 0, \sigma_3 = 0$$

$$(\tau_{max})_f = \max\left[\left|\frac{\sigma_f - 0}{2}\right|, \left|\frac{0 - 0}{2}\right|, \left|\frac{0 - \sigma_f}{2}\right|\right] = 0.5\sigma_f$$

Statement:

If Maximum shear stress of the machine element crosses the maximum shear stress under tensile test at the time of failure, then; machine element will fail according to this TOF.

or

Machine element will be safe if the maximum shear stress induced in the Machine element is less than the maximum shear stress under tensile test at the time of failure.

For safe design

$$(\tau_{max})_{actual} \leq (\tau_{max})_f$$

At time of design, for safer design

$$(\tau_{max})_{actual} \leq \frac{0.5\sigma_f}{FOS}$$

1.2.3 Haigh's 'or' Strain Energy TOF

Criteria of Failure:

Strain energy per unit volume.

Strain Energy per unit Volume at any point:

$$u = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] \text{ J/m}^3$$

Strain Energy per unit Volume at any point under tensile test at the time of failure:

Under tensile test: $\sigma_1 = \sigma_f, \sigma_2 = 0, \sigma_3 = 0$

$$u_f = \frac{1}{2E}\left[\sigma_f^2\right] \text{ J/m}^3$$

Statement:

According to Haigh's TOF. if Maximum strain energy per unit volume at the critical point of the machine element crosses the strain energy per unit volume under tensile test at the time of failure, then machine element will fail according to this TOF.

or

Machine element will be safe if the strain energy per unit volume at the critical point induced in the machine element is less than the strain energy per unit volume under tensile test at the time of failure.

For safe design:

$$u \leq u_f$$

$$\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] \leq \frac{1}{2E}\left[\sigma_f^2\right]$$

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) \leq \sigma_f^2$$

$$\sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)} \leq \sigma_f$$

For safer design:

$$\sqrt{\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)}\le\left(\frac{\sigma_f}{\text{Fos}}\right)$$

For 2D stress condition: $\sigma_3 = 0$

$$\sqrt{\sigma_1^2+\sigma_2^2-2\mu\sigma_1\sigma_2}\le\left(\frac{\sigma_f}{\text{Fos}}\right)$$

1.2.4 Distortion Energy 'or' Von-mises TOF

Criteria of Failure:

Distortion energy per unit volume.

Distortion Energy per unit Volume at any point:

$$u_d=\frac{1}{6G}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\right]\text{ J/m}^3$$

Distortion Energy per unit Volume at any point under tensile test at the time of failure:

Under tensile test: $\sigma_1=\sigma_f, \sigma_2=0, \sigma_3=0$

$$u_{df}=\frac{1}{2E}\left[\sigma_f^2\right]\text{ J/m}^3$$

Statement:

According to Von-mises TOF. if Maximum distortion energy per unit volume at the critical point of the machine element crosses the distortion energy per unit volume under tensile test at the time of failure, then machine element will fail according to this TOF.

or

Machine element will be safe if the distortion energy per unit volume at the critical point induced in the Machine element is less than the distortion energy per unit volume under tensile test at the time of failure.

For safe design:

$$u_d\le u_{df}$$

$$\frac{1}{6G}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\right]\le\frac{1}{6G}\sigma_f^2$$

$$\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\right]\le\sigma_f^2$$

$$\sqrt{\sigma_1^2+\sigma_2^2+\sigma_3^2-(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)}\le\sigma_f$$

For safer design:

$$\sqrt{\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\right]}\le\left(\frac{\sigma_f}{\text{Fos}}\right)$$

For 2D stress condition: $\sigma_3 = 0$

$$\sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2} \le \left(\frac{\sigma_f}{\text{Fos}}\right)$$

1.2.5 Maximum Principal Strain 'or 'Maximum normal Strain or St. Venant TOF

Criteria of Failure:

Max. principal strain or Max normal strain

Maximum normal strain at any point:

$$\varepsilon_{max} = \max\begin{cases} \frac{1}{E}(\sigma_1 - \mu\sigma_2 - \mu\sigma_3) \\ \frac{1}{E}(\sigma_2 - \mu\sigma_1 - \mu\sigma_3) \\ \frac{1}{E}(\sigma_3 - \mu\sigma_1 - \mu\sigma_2) \end{cases}$$

Maximum normal strain at any point under tensile test at the time of failure:

Under tensile test: $\sigma_1 = \sigma_f, \sigma_2 = 0, \sigma_3 = 0$

$$(\varepsilon_{max})_f = \max\begin{cases} \frac{1}{E}(\sigma_f) \\ \frac{1}{E}(\mu\sigma_f) = \frac{\sigma_f}{E} \\ \frac{1}{E}(\mu\sigma_f) \end{cases}$$

Statement:

According to St. Venant's TOF. if maximum normal strain at the critical point of the machine element crosses the maximum normal strain under tensile test at the time of failure, then machine element will fail according to this TOF.

or

Machine element will be safe if the maximum normal strain at the critical point induced in the Machine element is less than the maximum normal strain under tensile test at the time of failure.

For safe design:

$$(\varepsilon_{max})_a \le (\varepsilon_{max})_f$$

$$(\varepsilon_{max})_a \le \frac{\sigma_f}{E}$$

For safer design:

$$(E_{max})_a \le \frac{\sigma_f}{E \times Fos}$$

> **Note:**
> **Steps of Design**
> (a) Identify critical point.
> (b) Find principal stress at critical point.
> (c) Select and use suitable theory of failure.

1.3 Combined Bending & Torsion

Solid Circular shaft	Hollow Circular Shaft
$\sigma_{max} = \sigma_1 = \frac{16}{\pi D^3}[M + \sqrt{M^2 + T^2}]$	$\sigma_{max} = \sigma_1 = \frac{16}{\pi D_0^3(1-k^4)}\left[M + \sqrt{M^2 + T^2}\right]$
$\sigma_2 = \frac{16}{\pi D^3}\left[M - \sqrt{M^2 + T^2}\right]$	$\sigma_2 = \frac{16}{\pi D_0^3(1-k^4)}\left[M - \sqrt{M^2 + T^2}\right]$
$\sigma_3 = 0$	$\sigma_3 = 0$
$\tau_{max} = \frac{16}{\pi D^3}\left[\sqrt{M^2 + T^2}\right] = \left[\frac{\sigma_1 - \sigma_2}{2}\right]$	$\tau_{max} = \left\lvert\frac{\sigma_1 - \sigma_2}{2}\right\rvert = \frac{16}{\pi D_0^3(1-k^4)}\left[\sqrt{M^2 + T^2}\right]$

1.3.1 Equivalent Bending Moment (M_e)

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]$$

1.3.2 Equivalent Torsional Moment (T_e)

$$T_e = \sqrt{M^2 + T^2}$$

1.4 Region of Safety for 2D stress condition

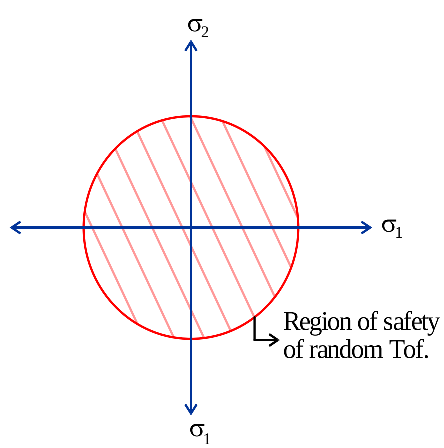
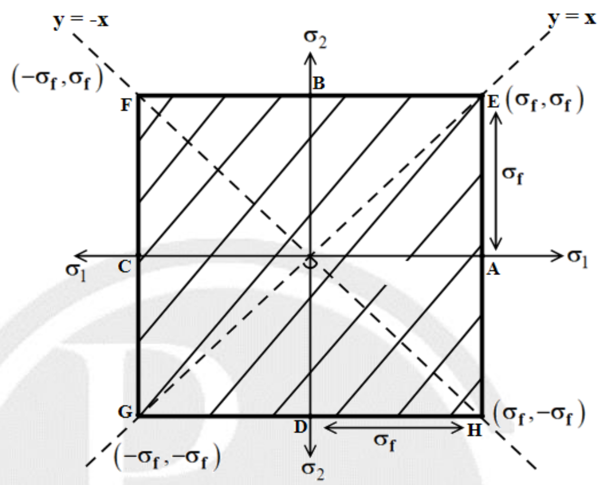
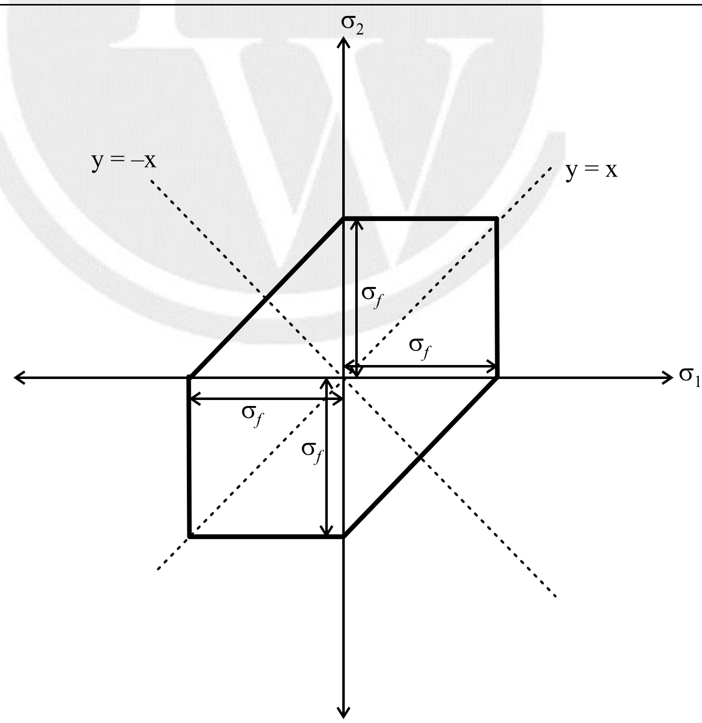
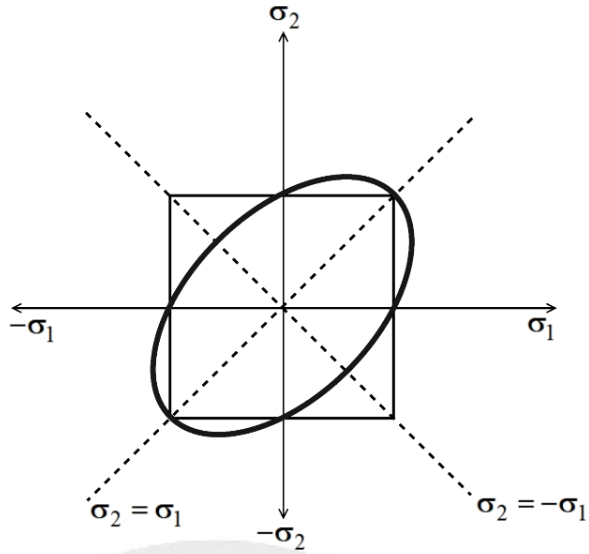
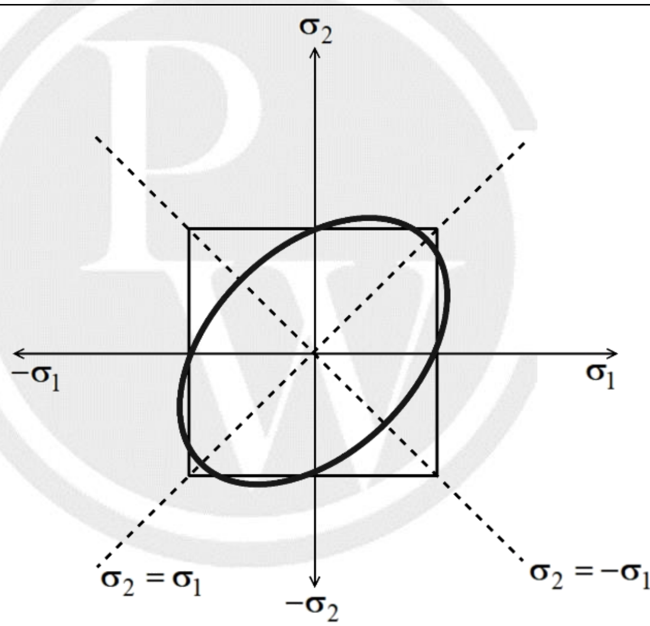


Fig.1.6: Region of safety

- If the actual loading lies under region of safety; Material is safe with FOS greater than 1.
- If the actual loading lies just on the boundary of region of safety; Material is just safe with FOS equal to 1.
- If actual loading lies outside region of safety; Material is unsafe.

1.4.1 Region of safety of Various TOF under 2 D Stress condition

TOF	Region of safety
Rankine TOF	 Fig.1.7: Safety region for Rankine TOF
Tresca TOF	 Fig.1.8: Safety region for Tresca TOF

Vonmises TOF	 **Fig.1.8: Safety region for Vonmises TOF**
Strain energy TOF	 **Fig.1.9: Safety region for Strain energy TOF**

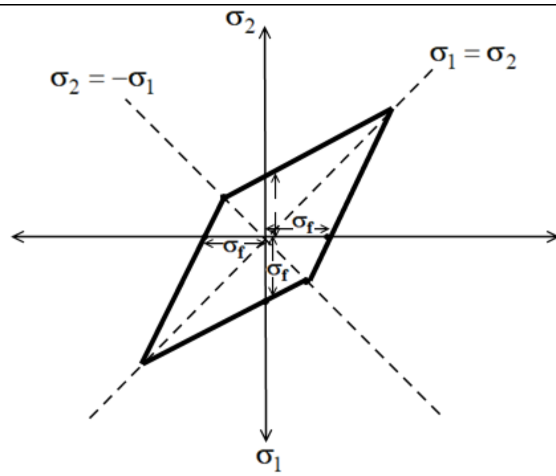
<table>
<tr>
<td>Maximum normal strain TOF</td>
<td>

Fig.1.10: Safety region for Maximum normal strain TOF</td>
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1.4.2 Comparison of Tresca's, Rankine & Von-Mises TOF for 2-D Stress Condition

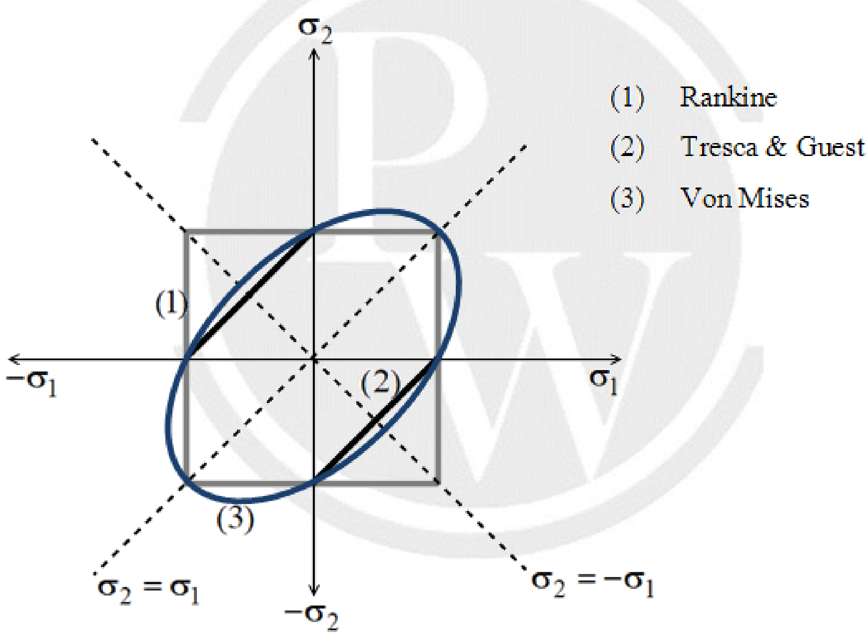


Fig.1.11: Safety region for Tresca, Rankine & Von Mises TOF

1. The theory of failure whose region of safety is smaller it will be safer.

   (a) Safer TOF in II & IV Quadrant: $\xrightarrow[\text{Safety increases}]{\text{Rankine} < \text{Von Mises} < \text{Tresca \& Guest}}$

   (b) Safer TOF in I & III Quadrant (Except $\sigma_1 = \sigma_2$) $\xrightarrow[\text{Safety increases}]{\text{Von Mises} < \text{Rankine} = \text{Tresca \& Guest}}$

   (c) Safer TOF in I & III Quadrant (when $\sigma_1 = \sigma_2$): $\xrightarrow[\text{Safety increases}]{\text{Von Mises} = \text{Rankine} = \text{Tresca \& Guest}}$

2. For any given stress condition FOS will be less for safer theory of failure.

   (a) II & IV Quadrant:

   $$\xrightarrow[\text{FOS}]{\text{Rankine} > \text{Von Mises} > \text{Tresca \& Guest}}$$

   (b) I & III Quadrant (Except $\sigma_1 = \sigma_2$):

   $$\xrightarrow[\text{FOS}]{\text{Von Mises} > \text{Rankine} = \text{Tresca \& Guest}}$$

   (c) I & III Quadrant ($\sigma_1 = \sigma_2$):

   $$\xrightarrow[\text{FOS}]{\text{Von Mises} = \text{Rankine} = \text{Tresca \& Guest}}$$

6. For any given FOS, design will be strong for safer theory of failure.

7. Safer theory of failure is less Economical.

1.5 Shear Strength under pure Shear for Various TOF

Theory of failure	Shear strength
Rankine TOF	$\tau_f = \sigma_f$
Tresca & Guest TOF	$\tau_f = 0.5\sigma_f$
Von-Mises TOF	$\tau_f = 0.577\sigma_f$
Haigh TOF	$\tau_f = \sigma_f / \sqrt{2(1+\mu)}$
St. Venant TOF	$\tau_f = \sigma_f / \sqrt{(1+\mu)}$

1.6 Equivalent Normal Stress using Various theory of Failure

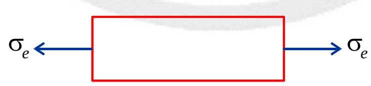


Fig.1.12: Equivalent Normal Stress

(1) Rankine TOF

$$\sigma_e = (\sigma_{max})_{actual}$$

(2) Tresca& J.J. Guest TOF

$$\frac{\sigma_e}{2} = (\tau_{max})_{actual} \Rightarrow \sigma_e = 2(\tau_{max})_{actual}$$

(3) Von-Mises TOF

$$(u_d)_{eq.} = (u_d)_{actual} \Rightarrow \frac{1}{6G} \times \sigma_e^2 = (u_d)_{actual}$$

$$\Rightarrow \frac{1}{6G}[\sigma_e^2] = \frac{1}{6G}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

$$\Rightarrow \sigma_e = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}$$

(4) Haigh TOF

$$u_{eq} = u_{actual}$$

$$\Rightarrow \frac{1}{2E}\sigma_e^2 = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

$$\Rightarrow \sigma_e = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}$$

(5) St. Venant TOF

$$(\varepsilon_{\max})_e = (\varepsilon_{\max})_{actual}$$

$$\frac{\sigma_e}{E} = (\varepsilon_{\max})_{actual}$$

1.7 Suitable TOF for various material

(1) For Brittle Material, Rankine TOF is more suitable.

(2) For ductile material, Tresca and Von mises TOF are more suitable. Out of which Tresca TOF is safer and Von-mises. TOF is more economical.

(3) Under Hydrostatic stress condition ($\sigma_1 = \sigma_2 = \sigma_3 = \sigma$), we cannot use Tresca and Von-mises TOF for any type of material, because under hydrostatic stress condition value of maximum shear stress and distortion energy per unit volume is zero.

❑❑❑

2 DESIGN AGAINST FLUCTUATING LOAD

2.1 Introduction

Fluctuating Stress: It Stress at any point continuously fluctuates with respect to time.

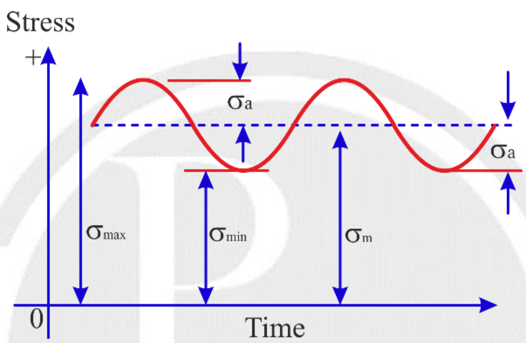


Fig.2.1: Fluctuating stress

2.1.1 Various terminology of fluctuating stress condition

1. **Average or Mean Stress** (σ_m)**:** $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$

2. **Amplitude Stress (σ_a):** $\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$

3. **Stress Ratio *(R)***: $R = \frac{\sigma_{min}}{\sigma_{max}}$ $\{0 \le R \le 1\}$

4. **Amplitude Ratio *(A)*:** $A = \frac{\sigma_a}{\sigma_m} = \frac{\sigma_{max} - \sigma_{min}}{\sigma_{max} + \sigma_{min}} = \frac{1-R}{1+R}$

Note:

Put σ_{min} & σ_{max} values with appropriate sign as per the nature of stress. These are positive for tensile stress and negative for compressive stress.

2.1.2 Special types of fluctuating stress condition

Repeating Stress/Repeated Stress condition

Special type of fluctuating stress where stress at the critical point varies from 0 to σ. (i.e., $\sigma_{min} = 0$ & $\sigma_{max} = \sigma$)

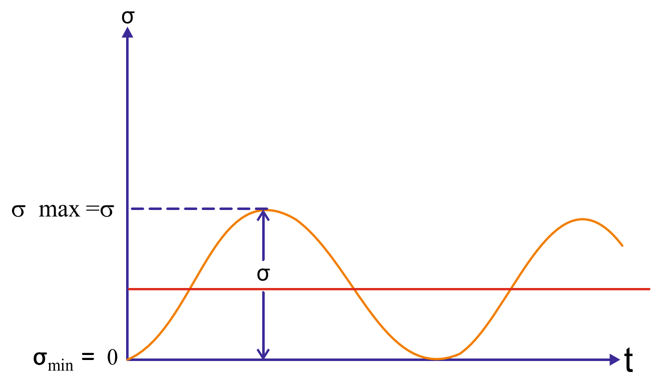


Fig.2.2: Repeating stress

For repeating stress condition: $\sigma_m = \frac{\sigma_{max}}{2}, \sigma_a = \frac{\sigma_{max}}{2}$, R = 0, A = 1

Reverse Stress Condition

Special type of fluctuating Stress condition where stress at critical point varies from ($-\sigma_1$) to ($+\sigma_2$) i.e., out of σ_{min} & σ_{max} one is compressive & other is tensile ($\sigma_{min} = -\sigma_1; \sigma_{max} = \sigma_2$).

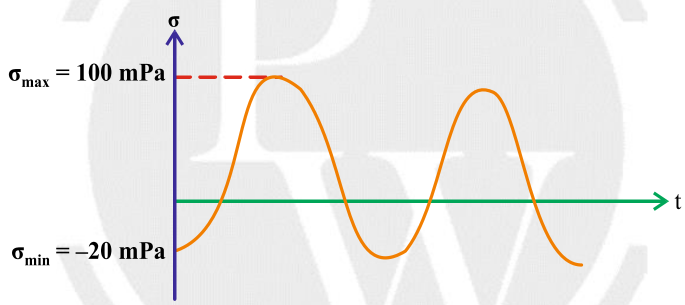


Fig.2.3: Reverse stress

Completely Reverse Stress condition:

If stress at critical points varies from $-\sigma$ to σ i.e. both σ_{min} & σ_{max} are of equal magnitude & opposite in nature, then that stress condition is known as completely reverse stress condition.

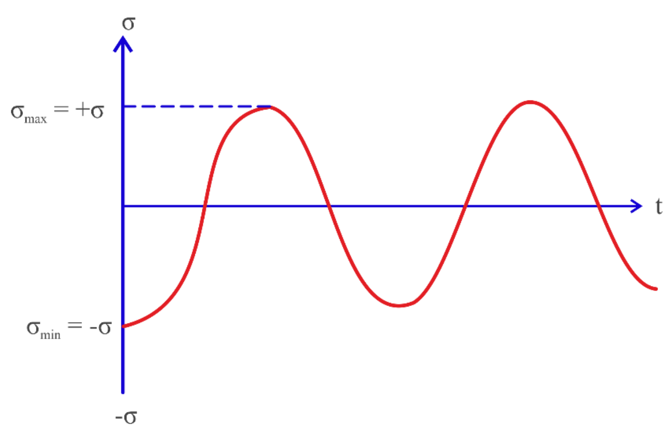


Fig.2.4: Completely Reverse stress

For repeating stress condition: $\sigma_m = 0, \sigma_a = \sigma$, R = –1, A = ∞

Note:
The bending stress at any point on a rotating shaft is state of completely reversed stress condition.

2.1.3 Fatigue Failure

- It is the failure of a machine element due to fluctuating stress at a stress level lower than the yield or ultimate strength of the material.
- Microcracks are developed in the material due to stresses. As the load is repeated, the cracks increase in size and during the final load cycle, when the material is not able to support the load, failure occurs.
- Microcracks are developed near the discontinuity where the stress is maximum due to stress concentration.

2.2 Stress concentration

Stress concentration is defined as the localization of high stresses due to the irregularities present in the component and abrupt changes of the cross section. Due to effect of stress concentration stress near irregularity will be more than nominal stress σ_0 (stress calculated by using elementary equation studied in subject strength of material with assumptions no irregularity is present is known as nominal stress)

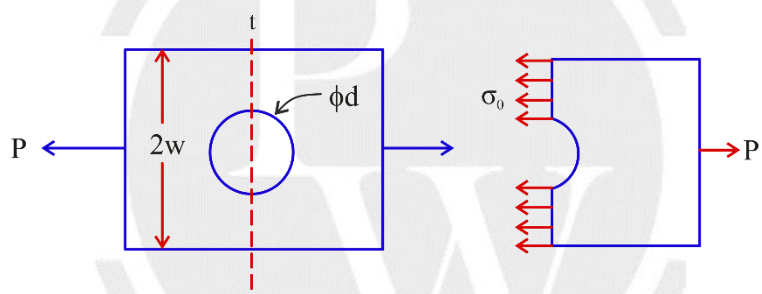


Fig.2.5: Nominal stress

Nominal stress near irregularity: $\sigma_0 = \dfrac{P}{(2w - d)t}$

Since irregularity is present in the above system therefore stress near irregularity will be more than nominal stress due to stress concentration.

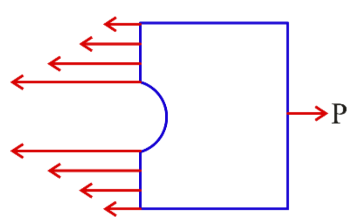


Fig.2.6: Effect of stress concentration with irregularities

Stress concentration occurs near irregularity due to disturbance in the path of stress flow due to which stress flow lines comes near and stress increases. (It is just like, when in flowing fluid stream lines comes nearer due to disturbance and velocity of flow increases.)

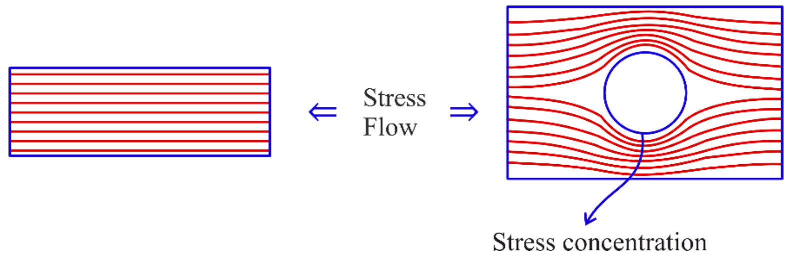
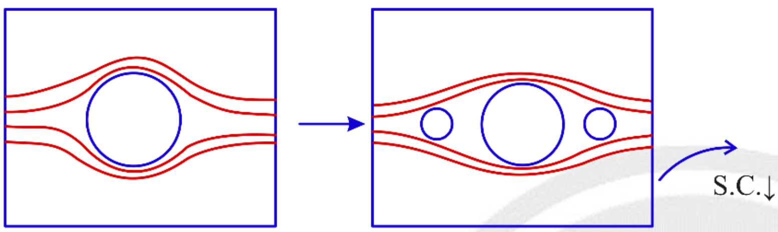


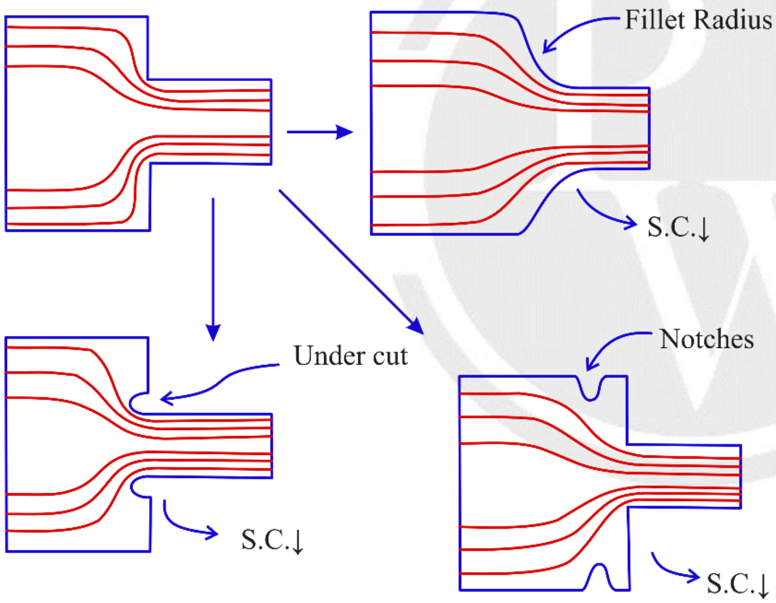
Fig.2.7: Stress flow

Methods to Reduce Stress Concentration (S.C.) when irregularity present (try to smoothen the stress flow):

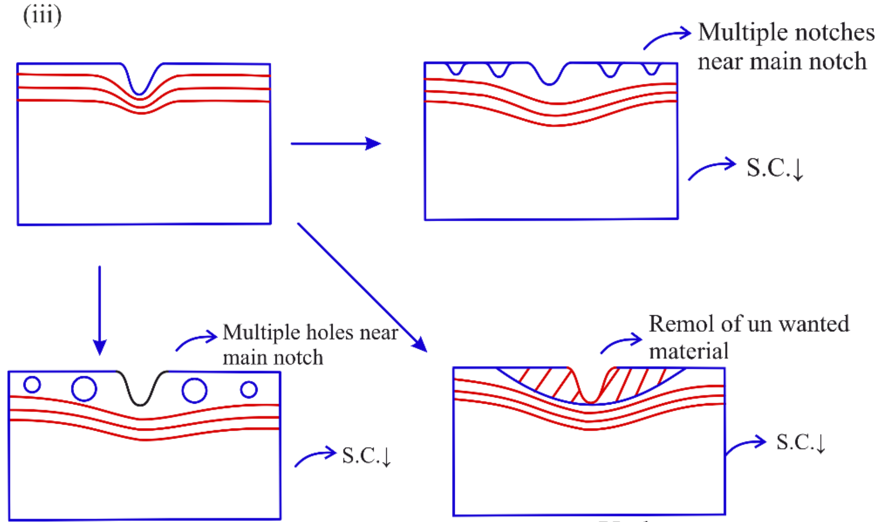
(i)



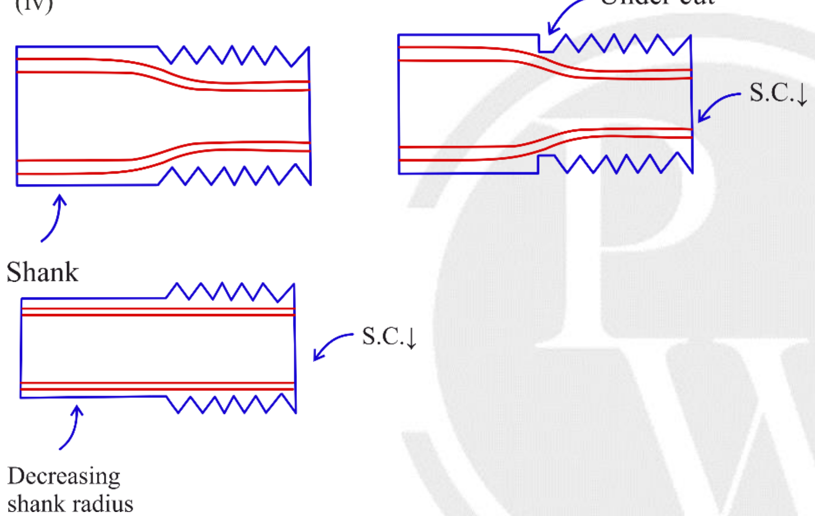
(ii)



(iii)



(iv)



2.2.1 Stress Concentration factor

Stress Concentration factor:

(a) Theoretical stress concentration factor (K_t)

(b) Actual stress concentration factor (K_f)

Theoretical Stress Concentration Factor (K_t):

- To consider the effect of stress concentration and find out localized stresses, a factor called theoretical stress concentration factor is used. It is denoted by K_t. It is also called Geometry Stress Concentration Factor.
- It depends on the geometry of irregularity corresponding to type of loading and it will not depend on the material.
- The theoretical stress concentration factor and charts are based on photo-elastic analysis of the epoxy model.
- Since theoretical stress concentration factor is calculated for epoxy material (which is assumed 100% sensitive for stress concentration), hence it will relate maximum possible stress near irregularity due to stress concentration with nominal stress, because actual stress near irregularity will also depend upon material.
- Theoretical stress concentration factor is defined as:

$$K_t = \frac{\text{Maximum possible value of stress near irregularity}}{\text{Nominal stress obtained by elementary equations}} = \frac{\sigma_{max}}{\sigma_0} \Rightarrow \sigma_{max} = K_t \sigma_0 .$$

Where, $\sigma_0 \rightarrow$ nominal stress

$\sigma_{max} \rightarrow$ Maximum possible stress near irregularity due to stress concentration

- **Empirical relation:** Theoretical stress concentration factor for elliptical hole under axial load by assuming width of plate >>> size of hole:

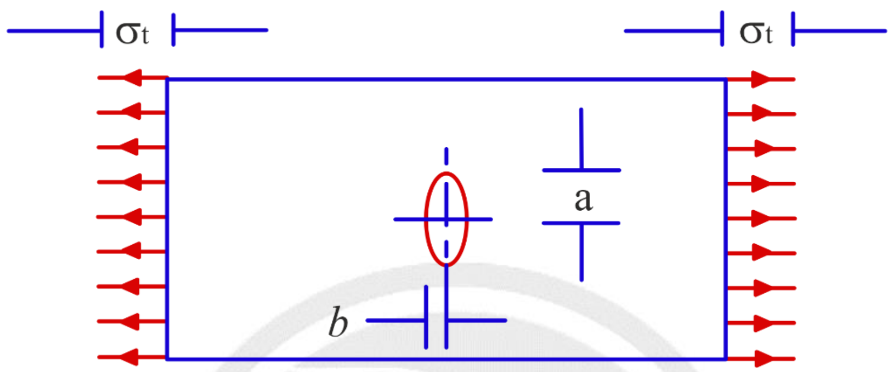


Fig.2.8: Stress concentration with elliptical hole

$$K_t = 1 + 2\left(\frac{a}{b}\right)$$

b → length of semi axis along the applied load.

a → length of semi axis perpendicular to applied load.

For circular hole: a = b: $K_t = 3$

Fatigue/Actual Stress Concentration Factor

- Experiments have shown that the actual stress concentration factor (K_f) is less than (K_t).
- It depends on the size of the stress concentration and the material.
- Fatigue stress concentration factor is defined as:

$$K_f = \frac{\text{Actual stress near irregularity due to stress concentration}}{\text{Nominal stress obtained by elementary equations}} = \frac{\sigma_a}{\sigma_0} \Rightarrow \sigma_a = K_f \sigma_0$$

Notch Sensitivity Factor

- The notch sensitivity of a material is a measure of how sensitive a material is towords notches or geometric discontinuities.
- It is defined as the ratio of increase of actual stress over nominal stress and increase of theoretical stress (or maximum possible stress due to stress concentration) over nominal stress.

$$q = \frac{K_f \sigma_0 - \sigma_0}{K_t \sigma_0 - \sigma_0} \Rightarrow q = \frac{K_f - 1}{K_t - 1} \qquad (0 < q < 1)$$

$$\Rightarrow \qquad K_f = 1 + q(K_t - 1)$$

For 100% sensitivity: $K_f = K_t$	For 0% Sensitivity: $K_f = 1$
$1 \le K_f \le K_t$	

Effect of Stress concentration on Ductile and Brittle material

(a) Effect of Stress Concentration on Ductile Material under Static Load.

Stress constriction effect can be neglected in ductile material under static load due to redistribution of stress near irregularity which occurs due to plastic deformation.

(b) Effect of Stress Concentration on brittle Material under Static Load.

Stress constriction effect cannot be neglected in brittle material under static load, because brittle material does not fail due to plastic deformation. In brittle material if we will neglect the stress concentration, then crack may develop near irregularity which propagates further and material can fail, hence we cannot neglect the effect of stress concentration in brittle material under static load.

(c) Effect of Stress Concentration on ductile and brittle material under fluctuating Load

We cannot neglect the effect of stress concentration in both ductile & brittle material under fluctuating load, because material under fluctuating Stress condition fails due to fatigue.

2.3 Design under completely reverse stress condition

2.3.1 Fatigue strength, life and endurance limit

Fatigue Strength (S_f):

The maximum possible magnitude of amplitude of completely reverse stress condition for life N number of stress cycles without fatigue failure is known as fatigue strength corresponding to N-cycles life.

Endurance Strength or Endurance Limit (S_e)

- Endurance strength of a material defined as the maximum amplitude of completely reversed stress that the material can sustain for an unlimited number of cycles without fatigue failure.
- It is determined either by rotating beam experiment or by using approximate relation between endurance strength and ultimate strength.

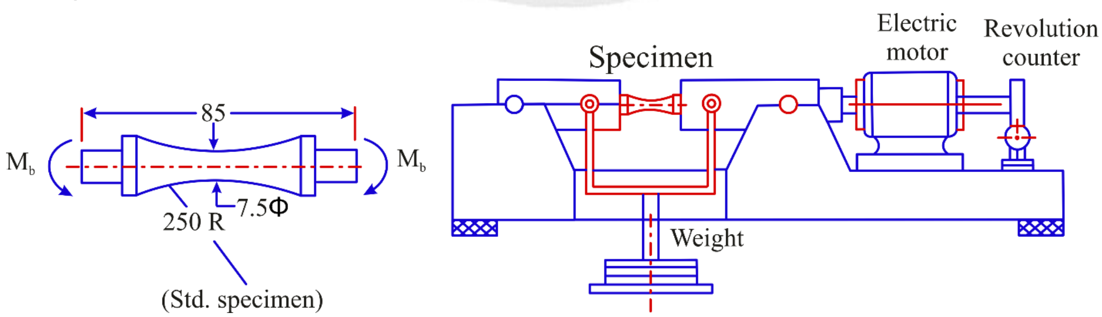


Fig.2.9: Rotating beam experiment

S-N Curve

- Through rotating beam experiment, S-N curve is drawn on log paper of base 10. S-N curve for ferrous Material is shown in figure:

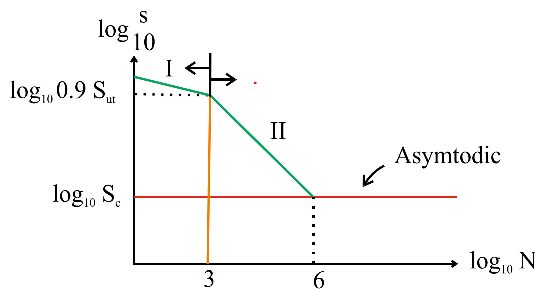


Fig.2.10: S-N Curve for ferrous material

I ⇒ Low Cycle Fatigue

II⇒ High Cycle Fatigue

I ⇒ Low Cycle Fatigue: Any fatigue failure when the number of stress cycles are less than 1000. Components are designed on the basis of ultimate tensile strength or yield strength with a suitable factor of safety.

II ⇒ High Cycle Fatigue: Any fatigue failure when the number of stress cycles are more than 1000. Components designed on the basis of fatigue strength for finite life and endurance strength for infinite life.

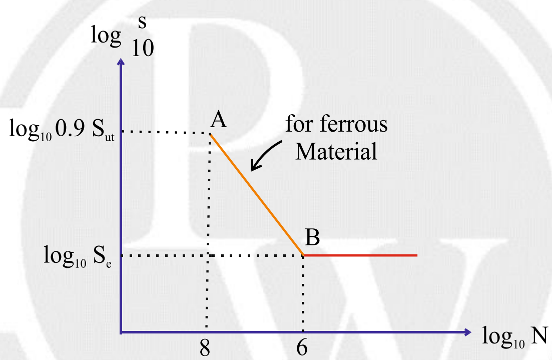


Fig.2.11: High cycle fatigue region of S-N curve for ferrous material

For ferrous material S-N curve becomes asymptotic corresponds to life 10^6 cycle (point B in figure 2.11), hence fatigue strength corresponds life 10^6 will be endurance strength for ferrous material.

S-N curve for Al:

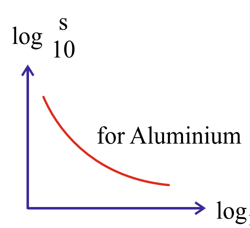


Fig.2.12: S-N curve for Al

Corrected Endurance Strength

- The endurance limit of actual component is different from the endurance limit of a rotating beam specimen due to a number of factors.
- The difference arises due to the fact that there are standard specifications and working conditions for the rotating beam specimen, while the actual components have different specifications and work under different conditions.
- Different modifying factors known as derating factor are used in practice to account this difference.

  Corrected endurance strength, $S_e = k_a\, k_b\, k_c\, k_d\, k_e\, (S_e)_{std}$

k_a = Surface finish factor

k_b = Size factor

k_c = Reliability factor

k_d = Factor to consider stress concentration

k_e = Temp. Factor

$K_L \Rightarrow$ Load factor

(i) Surface finish factor (k_a): This factor depends upon the surface finish of the machine element. For mirror polished surface like specimen (k_a =1) and if the surface finish is poor ($k_a < 1$)

(ii) Size factor (k_b): This factor depends upon the size of the body if the size increases chances of irregularities increase and endurance strength decrease.

If $\phi \leq 7.5$mm, $k_b = 1$ and if $\phi \geq 7.5$mm, $k_b < 1$

(iii) Reliability factor (k_c)

- The endurance strength determined by rotating beam experiment or the approximated relation is only 50% reliable, means 50% components will survive with this endurance strength.
- To ensure that more than 50% components survive, the component must be designed with lower endurance strength.
- As the reliability (% of components that survive) increases, reliability factor (k_c) decreases.

(iv) Factor to consider stress concentration. (k_d)

$$k_d = \frac{1}{k_f}$$

$k_f \Rightarrow$ actual stress concentration factor

$k_f = 1 + q(k_t - 1)$

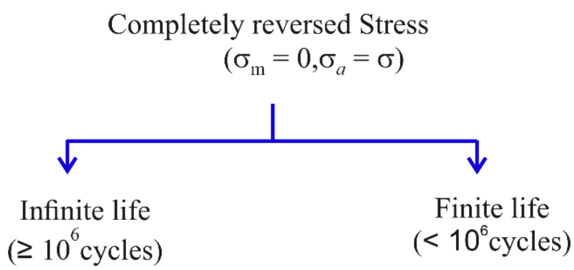
If q not given; then take $q = 1$

(v) Load factor (K_L)

From testing endurance strength is calculated for completely reversed bending stress condition

For completely reversed axial stressed condition we consider load factor which is generally 0.8. In exam load factor will be given and if not given then take load factor =1.

Design Process:



2.3.2 Design under completely Reversed stress for infinite life

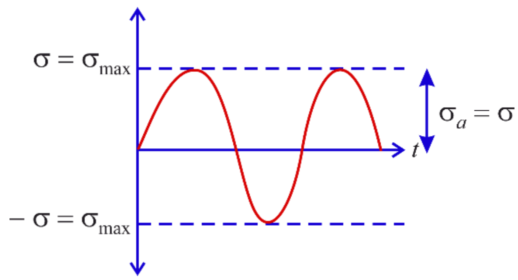


Fig.2.13: Completely Reversed stress

For infinite life: Criteria of failure is corrected endurance strength (*Se*)

Safe condition:

Amplitude stress of critical point under completely reverse stress condition should be less than corrected endurance strength (S_e).

$$\sigma_a \le S_e$$

For design:

$$\sigma_a \le \frac{S_e}{FOS}$$

2.3.3 Design under completely Reversed stress for finite life (say N)

For finite life: Criteria of failure is fatigue strength corresponds to required life (S_f).

Safe condition:

Amplitude stress of critical point under completely reverse stress condition should be less than fatigue strength corresponds to required life (S_f).

$$\sigma_a \le S_f$$

For design:

$$\sigma_a = \frac{S_f}{FOS}$$

2.3.4 Miner's Equation (Cumulative Damage in Fatigue)

Stress	Fraction	Individual Life	Actual Life for that ± σ acting
$\pm \sigma_1$	α_1	N_1	$n_1 = \alpha_1 N$
$\pm \sigma_2$	α_2	N_2	$n_2 = \alpha_2 N$
$\pm \sigma_3$	α_3	N_3	$n_3 = \alpha_3 N$

$$\alpha_1 + \alpha_2 + \alpha_3 = 1 \Rightarrow \sum \alpha_i = 1$$

$$\frac{\alpha_1}{N_1} + \frac{\alpha_2}{N_2} + \frac{\alpha_3}{N_3} = \frac{1}{N} \Rightarrow \sum \frac{\alpha_i}{N_i} = \frac{1}{N}$$

2.4 Design under fluctuating stress condition other than completely reverse stress condition

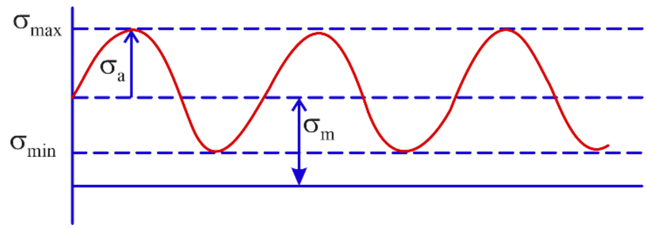


Fig.2.14: Fluctuating stress condition other than completely reverse stress condition

Mean and amplitude stress at critical point

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}, \ \sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

For fluctuating stress condition other than completely reverse stress condition, there are infinite combinations are possible and for all combinations testing is not a good idea. So, scientists have given their criteria to design under fluctuating load condition like Soderberg criteria, Goodmen criteria, Gerber criteria etc.

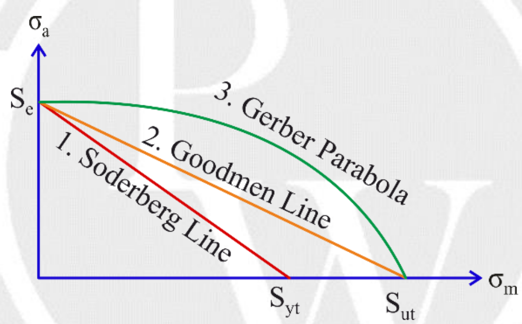


Fig.2.15: Soderberg, Goodmen and Gerber parabola

2.4.1 Soderberg criteria/Line

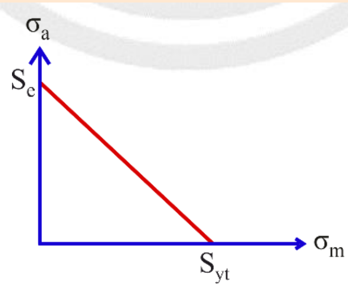


Fig.2.16: Soderberg line

Equation of Soderberg line:

$$\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_e} = 1$$

For design: use FOS = n and replace S_{yt} by $\frac{S_{yt}}{n}$ and S_e by $\frac{S_e}{n}$.

$$\frac{\sigma_m}{\frac{S_{yt}}{n}} + \frac{\sigma_a}{\frac{Se}{n}} = 1$$

$\Rightarrow$ $$\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_e} = \frac{1}{n}$$ **(Soderberg criteria for design)**

2.4.2 Goodmen criteria/Line

Goodmen Criteria

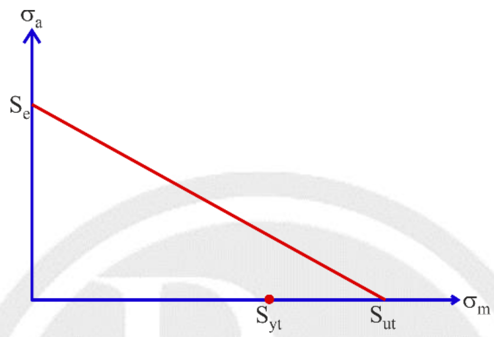


Fig.2.17: Goodmen line

Equation of Goodmen Line:

$$\frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e} = 1$$

For design: use FOS = n and replace S_{ut} by $\frac{S_{ut}}{n}$ and S_e by $\frac{S_e}{n}$.

$$\frac{\sigma_m}{\frac{S_{ut}}{n}} + \frac{\sigma_a}{\frac{Se}{n}} = 1$$

$\Rightarrow$ $$\frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e} = \frac{1}{n}$$ **(Goodman criteria of design)**

2.4.3 Gerber criteria/Parabola

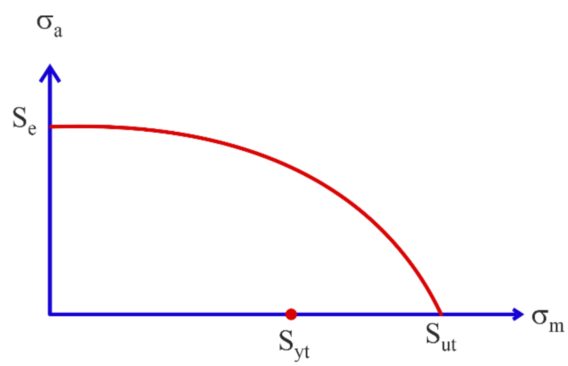


Fig.2.18: Gerber Parabola

Equation of Gerber Parabola

$$\left(\frac{\sigma_m}{S_{ut}}\right)^2 + \frac{\sigma_a}{S_e} = 1$$

For design: use FOS = n and replace S_{ut} by $\frac{S_{ut}}{n}$ and S_e by $\frac{S_e}{n}$.

$$\left(\frac{n.\sigma_m}{S_{ut}}\right)^2 + \frac{n.\sigma_a}{S_e} = 1$$ **(Gerber criteria of design)**

2.4.4 Comparison of Soderberg, Goodmen & Gerber Criteria

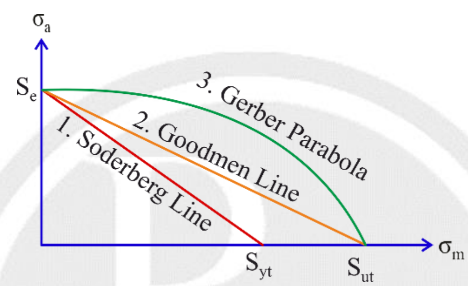


Fig.2.19: Comparison of Soderberg, Goodmen, Gerber Parabola

Region of Safety

Soderberg < Goodmen < Gerber

Key points:

- Criteria whose reason of safety is smaller is safer $\xrightarrow[\text{Safety}\downarrow]{\text{Soderberg > Goodmen > Gerber}}$.
- For any given stress condition FOS will be less for safer criteria $\xrightarrow[\text{FOS}]{\text{Gerber > Goodmen > Soderberg}}$.
- For any given FOS, design will be strong for more safer criteria.
- More safer criteria are less Economical

2.4.5 Langer criteria/Line

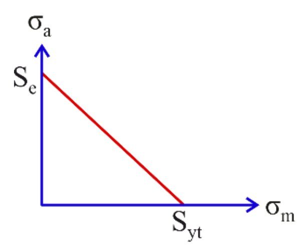


Fig.2.20: Langer criteria

Assumption:

Material fail only due to yielding. If σ_m is more then it gives good results but if σ_a is more than it does not give accurate result.

Equation of Langer Line.

$$\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_{yt}} = 1$$

For design: use FOS = n and replace S_{yt} by $\frac{S_{yt}}{n}$.

$$\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_{yt}} = \frac{1}{n} \qquad \textbf{(Langer's criteria for design)}$$

2.4.6 Modified Goodmen Criteria

Safer criteria out of Goodmen and Langer's criteria is modified goodmen criteria.

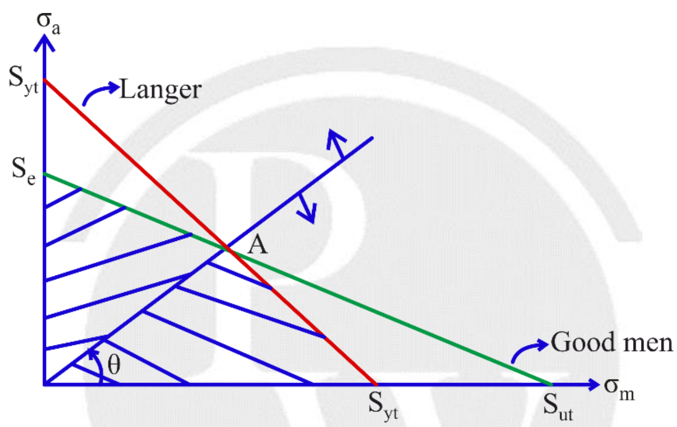


Fig.2.21: Modified Goodmen Criteria

To find FOS: Lesser FOS out of Langer criteria and Goodmen criteria will be FOS by Modified Goodmen criteria.

To design: Stronger design out of Langer criteria and Goodmen criteria will be design by Modified Goodmen criteria.

2.4.7 ASME Criteria/Ellipse

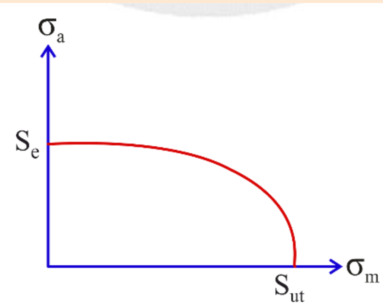


Fig.2.22: ASME criteria

Equation of ASME Ellipse;

$$\left(\frac{\sigma_m}{S_{ut}}\right)^2 + \left(\frac{\sigma_a}{S_e}\right)^2 = 1$$

For design: use FOS = n and replace S_{ut} by $\frac{S_{ut}}{n}$ and S_e by $\frac{S_e}{n}$.

$$\left(\frac{\sigma_m}{S_{ut}}\right)^2 + \left(\frac{\sigma_a}{S_{ut}}\right)^2 = \frac{1}{n^2}$$ ⇒ **ASME criteria for design.**

2.4.8 Design Against Fluctuating Stress condition under combined loading condition

- We have studied Soderberg; Goodman etc. criteria for the 1–D fluctuating loading condition, but here 2-D & 3-D stress are present.
- In order to use the above criteria, we need to convert 2-D & 3-D Stress condition into equivalent 1-D stress condition using theory of failure for equivalent stress studied in static loading one by one for both amplitude stress condition & Mean Stress conditions.
- After that we can easily design using the suitable criteria for fluctuating stress condition.

2.5 Significance of mean compressive stress under fluctuating stress condition

σ_a ⇒ always +ve

σ_m ⇒ can be +ve; 0, –ve

If σ_m is +ve; it will lead to propagation of crack & will result in fracture.

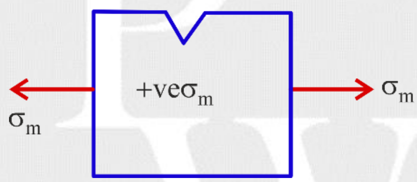


Fig.2.23: Propagation of crack under tensile stress

And if σ_m is –ve; it will not try to propagate the crack, hence results in less chance of failure.

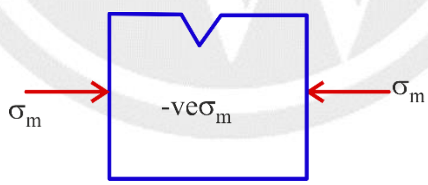


Fig.2.24: Under compressive stress

In order to increase the fatigue strength under fluctuating load we can induce the residual compressive stress by process like cold working; burnishing; shot pinning; case hardening; Cloning etc.

❑❑❑

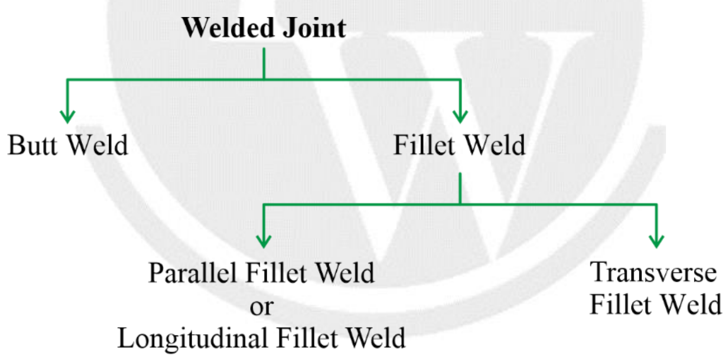
3 Design of Welded Joint

3.1 Introduction of Welded Joints

A welded joint is a permanent joint which is obtained by the fusion of the edges of the two parts to be joined together, with or without the application of pressure and a filler material.

- It is stronger than riveted joint.
- Weight addition is less compared to riveted joints
- Cheaper than riveted joints
- Due to application of heat during welding, material properties changes
- Stress concentration occurs due to uneven heating.

3.2 Classification of Welded Joints



3.2.1 Butt weld

When two plates which are in same plane are welded together, then weld is known as Butt weld.

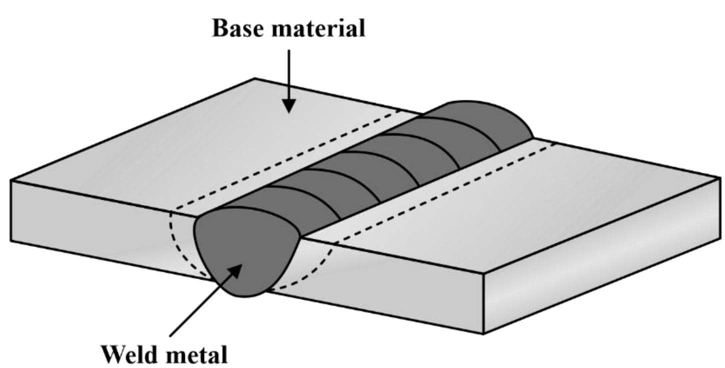


Fig.3.1: Butt weld

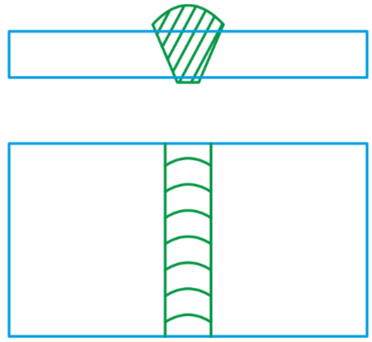


Fig. 3.2: Front and top view

3.2.2 Fillet Weld

Two plates kept in overlapping planes.

Filled weld is further classified in two types:

1. **Parallel fillet weld**

   When direction of the weld length is parallel to the direction of the force acting on the joint, then it is called a parallel joint.

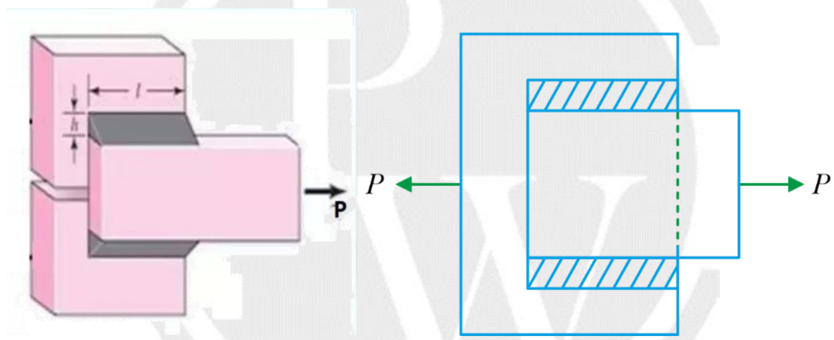


Fig. 3.3: Parallel fillet weld

2. **Transverse fillet weld**

   When direction of the weld length is perpendicular to the direction of the force acting on the joint, then it is called a transverse joint.

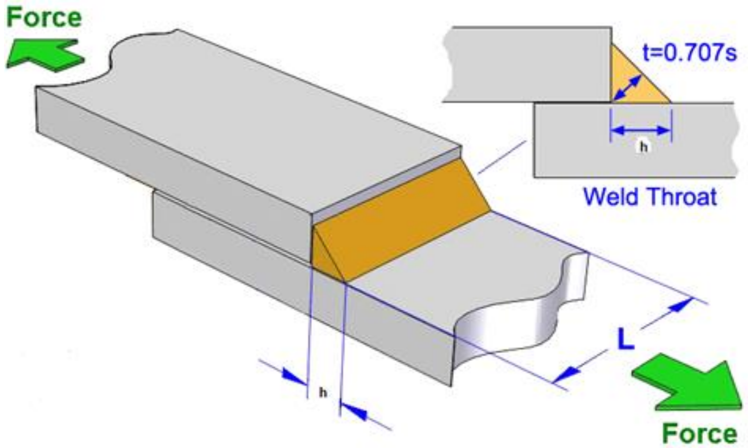


Fig. 3.4: Transverse fillet weld

Terminologies of Fillet weld:

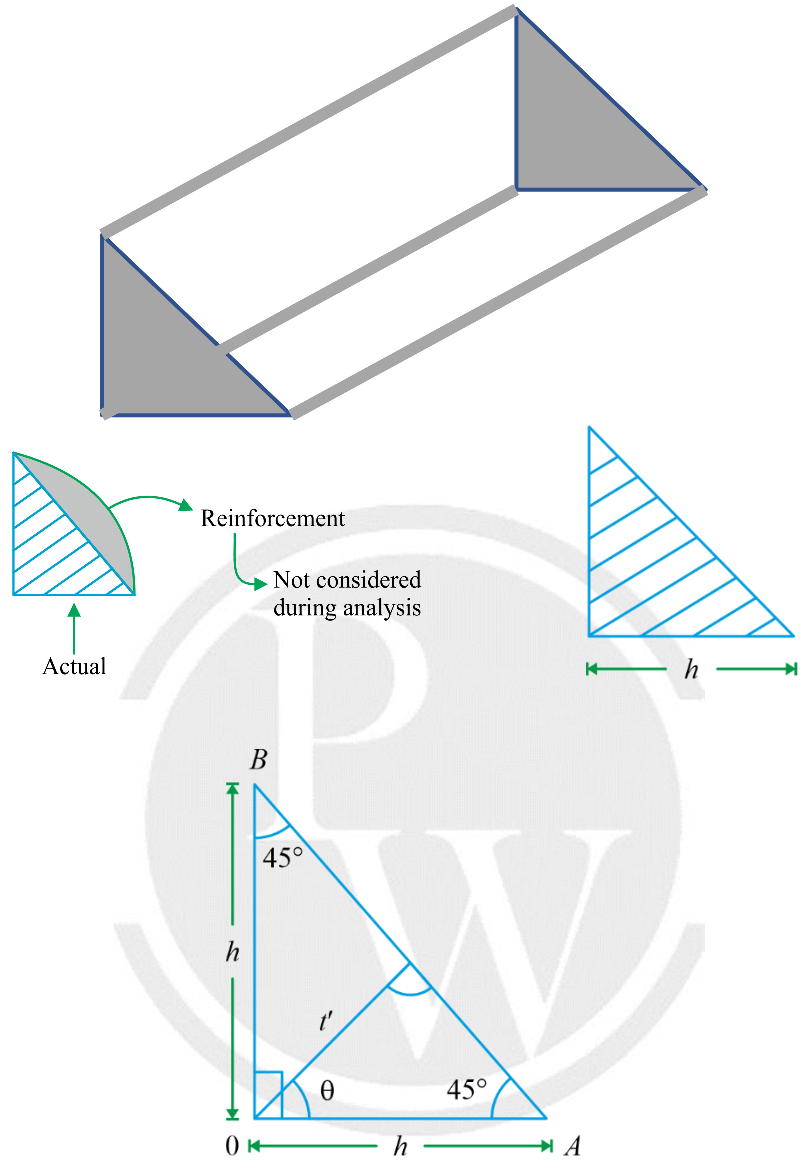


Fig. 3.5: Terminology of fillet weld

L = Length of weld

h = size or leg of fillet weld

$t' = \dfrac{h}{\sin\theta + \cos\theta}$ = thickness at θ angle from leg

$t = \dfrac{h}{\sqrt{2}}$ = throat of the fillet weld (or minimum thickness of fillet weld) = thickness at $\theta = 45^0$

A_t = Area of throat = tL

3.3 Analysis of Butt weld

Fig.3.6: Butt weld under force P

Minimum thickness of the weld is called throat. Which is thickness *t* of the weld as shown in figure. In throat reinforcement part is not consider because it is provided to compensate the flaws in the weld.

∴ Tensile stress in the weld is,

$$\sigma_t = \frac{P}{tL} \Rightarrow \boxed{P = \sigma_t \, t \, L}$$

Where,

σ_t = tensile stress in the weld

L = length of the weld

t = throat thickness

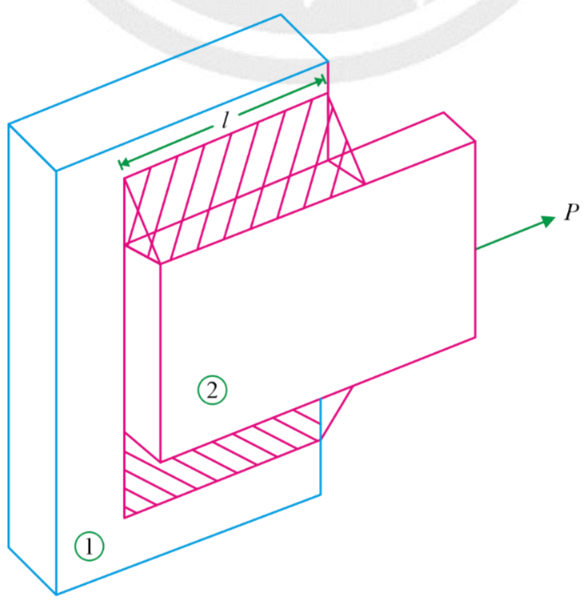
P = axial force in weld

If efficiency (η) of the joint is given, then,

$$\boxed{P = \sigma_t \, t \, L \eta}$$

3.4 Analysis of Fillet weld

3.4.1 Symmetrically loaded parallel fillet weld



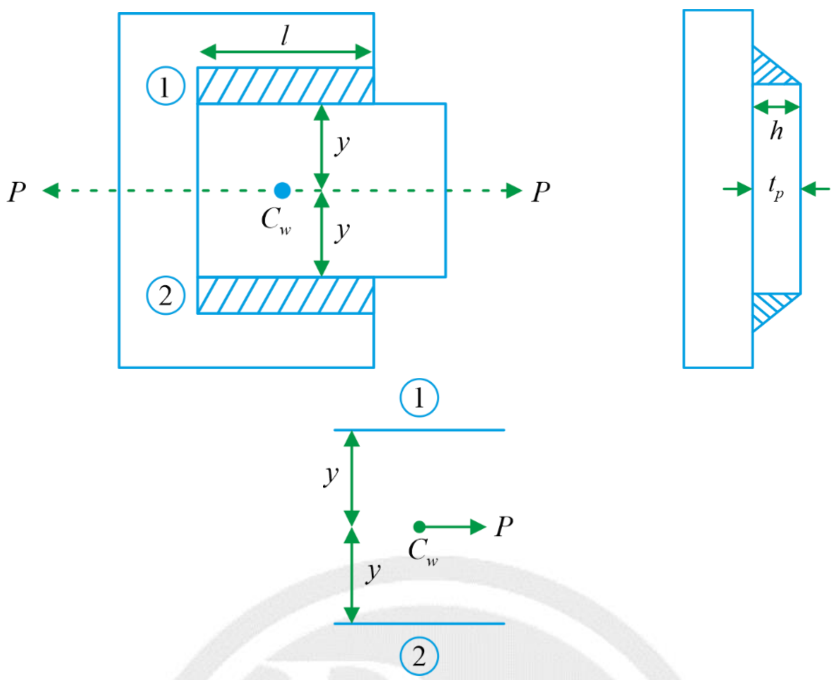


Fig.3.7: Symmetrically loaded Parallel fillet weld

l = Length of weld

h = size or leg of fillet weld = t_p

t $= \frac{h}{\sqrt{2}}$ = throat of the fillet weld (or minimum thickness or fillet weld) = thickness at $\theta = 45^0$

A_t = Total area of throat = 2 tl

- Since line of action of force P is passing through combine centroid of the weld, hence only primary stress (or direct stress) will induce in the weld.
- Whenever force P is parallel to the weld, shear stress will be induced in the weld which will be maximum at throat and expression to calculate shear stress at throat is.

$$\tau_{max} = \frac{P}{\text{total area of throat}}$$

$$\Rightarrow \quad \tau_{max} = \frac{P}{2tl}$$

For design; $\tau_{max} \le \tau_P$ (Permissible shear stress)

$$\Rightarrow \quad \frac{P}{2tl} \le \tau_P$$

$$\Rightarrow \quad \boxed{P \le \tau_P (2tl)}$$

Note:
In welded joint:
- Stress due to force = **Primary stress**
- Stress due to moment = **Secondary stress**

3.4.2 Unsymmetrical loaded parallel fillet weld

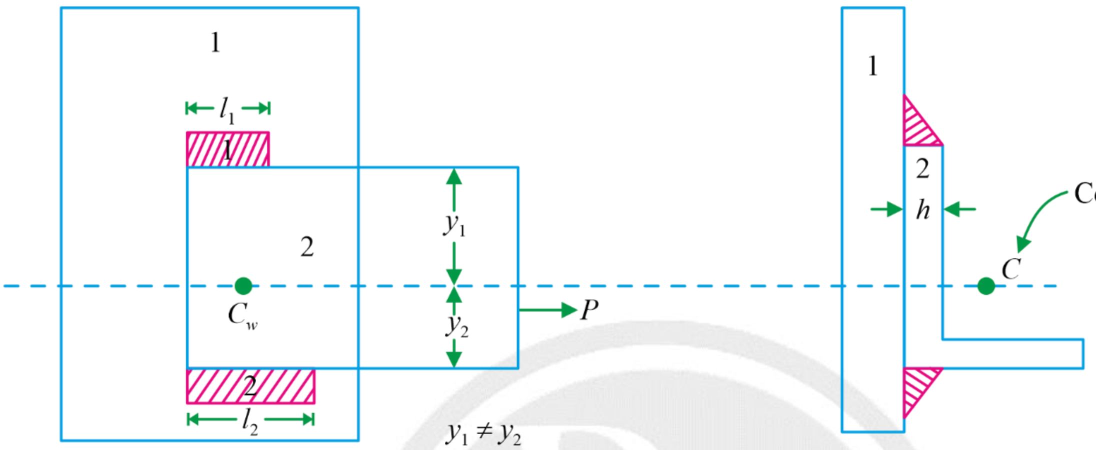


Fig.3.8: Unsymmetrical loaded parallel fillet weld.

- To ensure that no secondary stress (stress due to moment) should induce in the weld, line of action of force P pass through the combine centroid of the weld and for that following condition must be satisfied:

$$\boxed{l_1 y_1 = l_2 y_2}$$

- Since force P is passing through combine centroid of weld, only primary shear stress will induce in the weld, whose expression is

$\Rightarrow$ $$\tau_{max} = \frac{P}{\text{total area of throat}}$$

$\Rightarrow$ $$\tau_{max} = \frac{P}{(l_1 + l_2)\,t}$$

$\Rightarrow$ $$\boxed{P = \tau_{max} \times (l_1 + l_2)\,t}$$

3.4.3 Transverse fillet weld

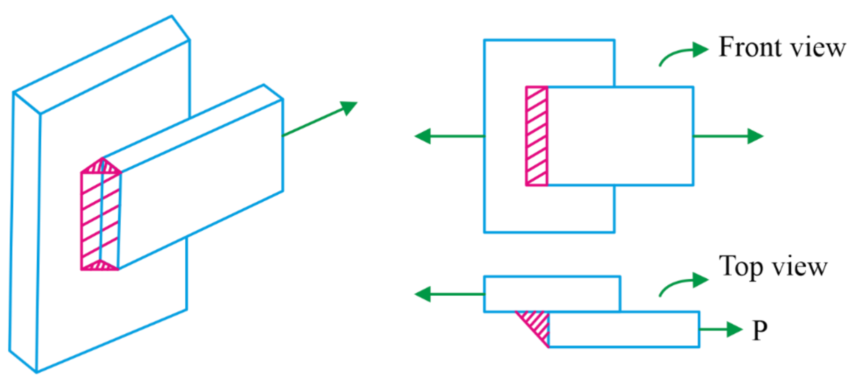
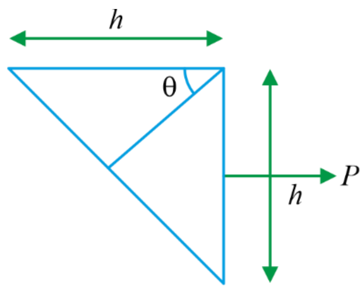


Fig.3.9: Analysis of Transverse fillet weld



- Both normal and shear stress induces in the weld.
- Actual maximum shear stress in the weld: $\tau_{max} = \frac{1.207\,P}{hL}$ [at θ = 22.5°]
- Actual maximum tensile stress in the weld: $\sigma_{t\,max} = \frac{1.207\,P}{hL}$ [at θ = 67.5 °]
- To be on safer side, instead of calculating actual maximum shear and tensile stress due to force P in transverse fillet weld, we calculate primary shear stress and tensile stress in the weld by dividing throat area in the force P as given below during analysis.

Maximum shear stress, $$\tau_{max} = \frac{P}{A_t}$$

⇒ $$\tau_{max} = \frac{P}{tL} = \frac{\sqrt{2}P}{hL}$$

⇒ $$\boxed{P = \tau_{max} \times \left(\frac{h}{\sqrt{2}} L\right)}$$

Maximum tensile stress,

$$\sigma_{t\,max} = \frac{P}{A_t} = \frac{P}{tL}$$

$$P = \sigma_{t\,max}\, tL$$

⇒ $$\boxed{P = \sigma_{t\,max} \frac{h}{\sqrt{2}} L}$$

3.5 Eccentric loaded weld joint

3.5.1 Eccentric loaded weld joint type I

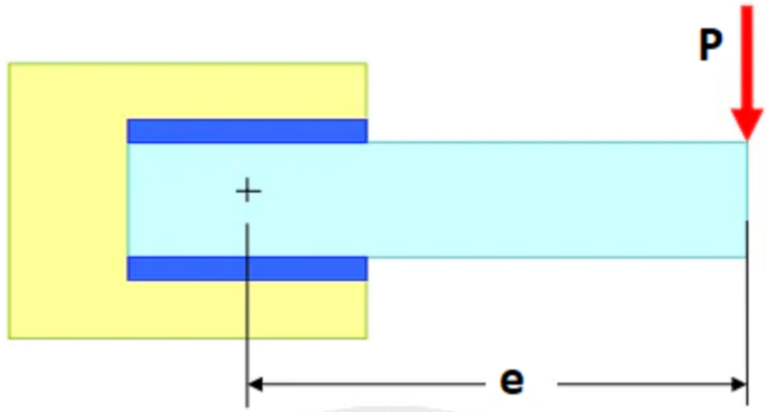


Fig. 3.10: Eccentric loaded weld joint – Type I

Any eccentric force can be replaced by an equal and same direction force (P) acting through the Centre of Gravity (COG) + a couple (M = P · e) lying in the same plane.

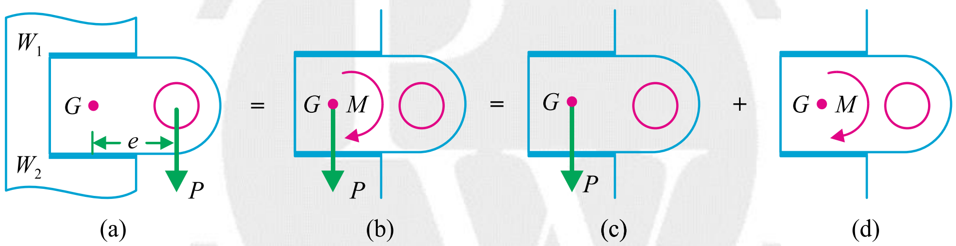


Fig.3.11: Shifting of eccentric load at center of gravity

The total stress felt by this weldment will be:

(1) Due to force P

(2) Due to moment M = P·e

Effect of Force P

- Primary shear stress (τ′ will induce)
- **Magnitude**: Primary shear stress will be same at every point as given below,

$$\tau' = \frac{P}{\text{total area of throat}} = \frac{P}{A_t}$$

- **Direction:** Direction of primary shear will be same at each point and will be opposite to force *P*.

  In figure 3.12 direction of primary shear stress of point *A* is drawn.

Effect of Moment M = P × e

- Due to moment $M = P \times e$, secondary shear stress (τ'') will induce in the weld.
- **Magnitude:**

$$\tau'' = \frac{M}{J} r$$

Where, J = Polar moment of inertia of the weld about COG.

τ'' = Secondary shear stress at a distance r from COG.

- **Direction:**

Step I: Draw a line which is perpendicular to the line joining the point whose secondary shear stress we are drawing and COG.

Step II: In this line take the sense of secondary shear stress such that it should try to rotate the weldment about COG opposite to the sense in which moment $M = P \times e$ it trying to rotate the weldment.

In figure 3.12 direction of secondary shear stress of point A is drawn.

Resultant Shear Stress

Find the vector resultant of primary and secondary shear stress.

Resultant shear stress of point A,

$$\tau_A = \sqrt{(\tau'_A)^2 + (\tau''_A)^2 + 2\tau'_A \tau''_A \cos\theta_A}$$

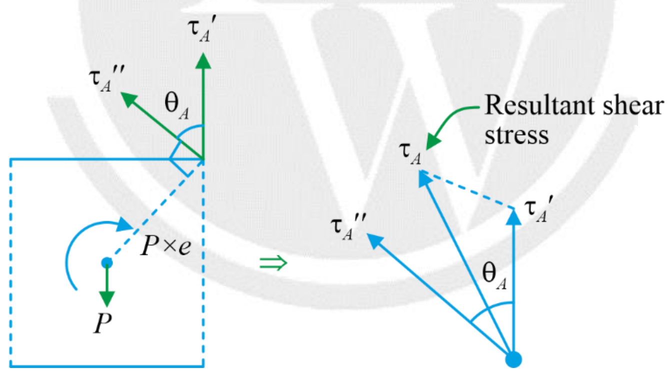


Fig.3.12: Shear stress at point A

Critical Point

During design we need to find resultant shear stress on critical point. Critical point is the point, where resultant shear stress is maximum.

Steps to find critical point:

Step I: Consider all the farthest points from COG.

Step II: Draw the direction of primary and secondary shear stress in these points.

Step III: In the selected points mark the points whose distance from COG (i.e. r) is maximum.

Step IV: In the selected points mark the points whose vector angle between primary and secondary shear stress (i.e. θ) is minimum.

Step V: Points where r is maximum and θ is minimum (i.e., common points of step III and IV) are critical points.

Note:

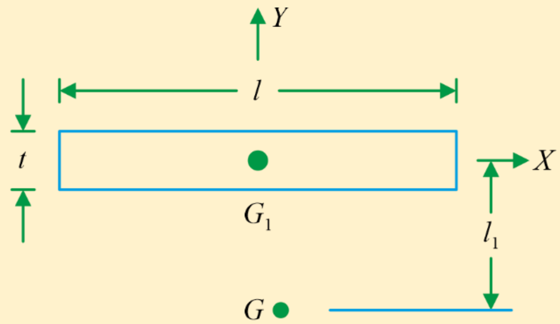
Method of find polar moment of inertia

Here, l = length of weld

t = throat of weld

G_1 = centre of gravity of weld

G = centre of gravity of group of welds



$\therefore \quad I_{xx} = \dfrac{lt^3}{12}$

$I_{yy} = \dfrac{tl^3}{12}$

$J_{G1} = I_{xx} + I_{yy} = I_{yy}$ [I_{xx} is very small]

$\Rightarrow \quad J_{G1} = \dfrac{tl^3}{12}$

$\therefore$ By Parallel axis theorem, $J_G = J_{G1} + Ar^2 = \dfrac{tl^3}{12} + ltr_1^2 = lt\left[\dfrac{l^2}{12} + r_1^2\right]$

If there are no of weld with polar moment of inertia, J_1, J_2, J_3 . . . . . . J_n, then

$$J_{resultant} = J_1 + J_2 + J_3 + \ldots\ldots + J_n$$

3.5.2 Eccentric loaded weld joint type II

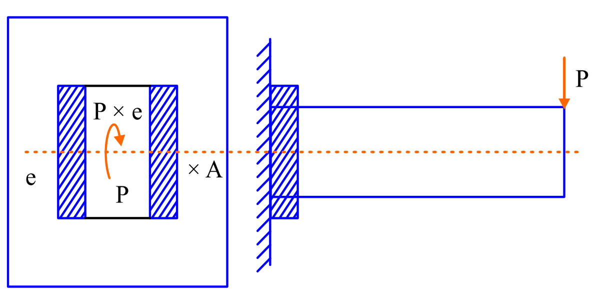


Fig.3.13: Eccentric loaded weld joint – Type II

Effect of Force P

Due to force P primary shear stress (τ') will induce in the weldment which is calculated by,

$$\tau' = \frac{P}{\text{Area of throat}} = \frac{P}{A_t}$$

Effect of bending moment ($M_b = P \times e$):

Due to moment ($M_b = P \times e$) bending stress or secondary normal stress will induce in the weldment which is calculated by

$$\sigma_b = \frac{M_b}{I} y$$

where,

I=MOI of combine weld about neutral axis (which is centroidal x axis in this case)

y=Perpendicular distance from neutral axis.

For design:

- Calculate primary shear stress (τ') and maximum bending stress ($\sigma_{b,m}$) i.e. bending stress at the point where y is maximum.

$$\tau' = \frac{P}{A_t} \ \& \ \ \sigma_{b,m} = \frac{M_b}{I} y_{max}$$

- Find principal stresses and maximum shear stresses at the critical point and use appropriate Theory of failure for design.

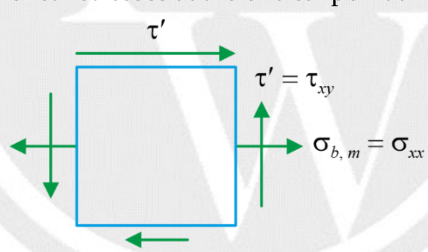


Fig.3.14 Stress condition at critical point

$$\sigma_{xx} = \sigma_{b,m}$$

$$\sigma_{yy} = 0$$

$$\tau_{xy} = \tau'$$

Principle Stresses

$$\sigma_{1,2} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$\Rightarrow$

$$\boxed{\sigma_{1,2} = \frac{\sigma_{b,m}}{2} \pm \sqrt{\left(\frac{\sigma_{b,m}}{2}\right)^2 + (\tau')^2}}$$

and $\sigma_3 = 0$

Maximum shear stress,

$$\tau_{max} = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$$\Rightarrow \quad \tau_{max} = \sqrt{\left(\frac{\sigma_{b,m}}{2}\right)^2 + (\tau')^2}$$

3.5.3 Important relation for circumferential weld

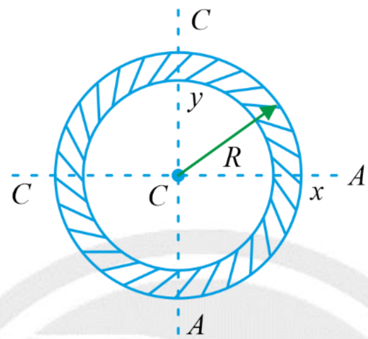


Fig.3.15: Circumferential weld

t = Throat of weld

A_t = Area of throat = $2\pi Rt$

I_{CXA} = $I_{CYA} = \pi + \pi t R^3$

J = Polar MOI about COG

= $2\pi\, t\, R^3$

Circumferential fillet weld under torsion:

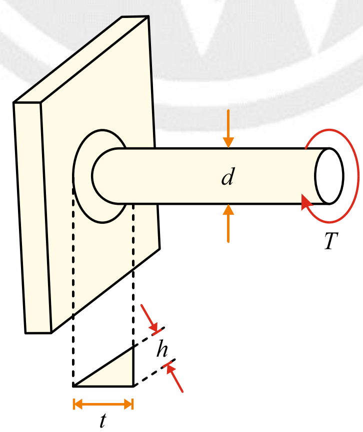


Fig.3.16: Circumferential weld under torsion

Maximum shear stress induces in the weld,

$$\tau_{max} = \frac{T}{J} r_{max}$$

$$\Rightarrow \quad \tau_{max} = \frac{T}{2\pi t R^3} \times R = \frac{T}{2\pi t R^2}$$

$$\Rightarrow \quad \tau_{max} = \frac{T}{2\pi\left(\frac{h}{\sqrt{2}}\right)\left(\frac{d}{2}\right)^2}$$

$$\Rightarrow \quad \boxed{\tau_{max} = \frac{2\sqrt{2}\,T}{\pi h d^2}}$$

Circumferential fillet weld under bending:

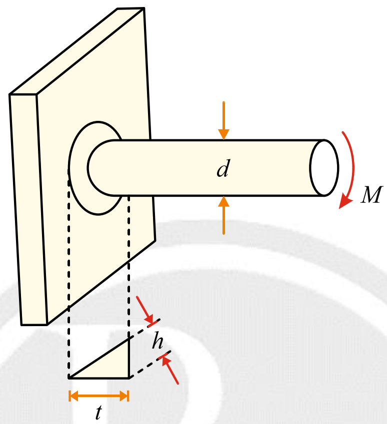


Fig.3.17: Circumferential weld under bending

Maximum bending stress induce in the weld,

$$\sigma_{b,max} = \frac{M}{I}\,y_{max}$$

$$\Rightarrow \quad \sigma_{b,max} = \frac{M}{\pi t R^3} \times R = \frac{M}{\pi t R^2}$$

$$\Rightarrow \quad \sigma_{b,max} = \frac{M}{\pi\left(\frac{h}{\sqrt{2}}\right)\left(\frac{d}{2}\right)^2}$$

$$\Rightarrow \quad \boxed{\sigma_{b,max} = \frac{4\sqrt{2}\,M}{\pi h d^2}}$$

❑❑❑

4 DESIGN OF BOLTED & RIVETED JOINT

4.1 Introduction of bolted joint

Threaded Joints has numerous applications; eg lead screw in Lathe; In Screw Jack; Bolted Joints etc.
Here we would restrict our study to Bolted Joints Only.
Bolted / Threaded Joints are TEMPORARY JOINT because both plates and joint are reusable.

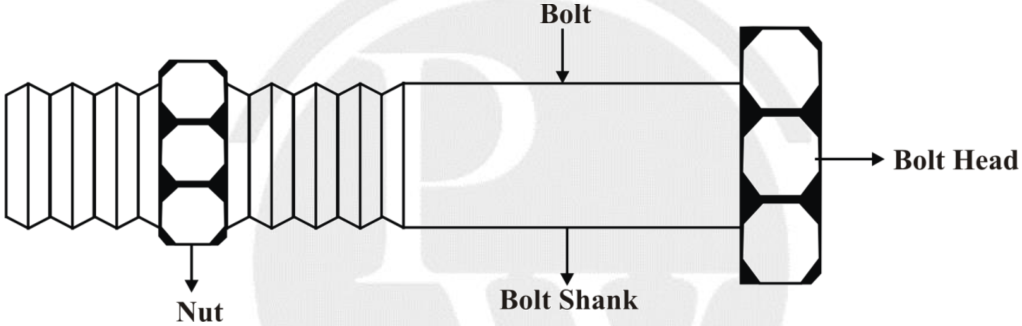


Fig.4.1: Bolted joint

4.1.1 Various terminology of bolted joint

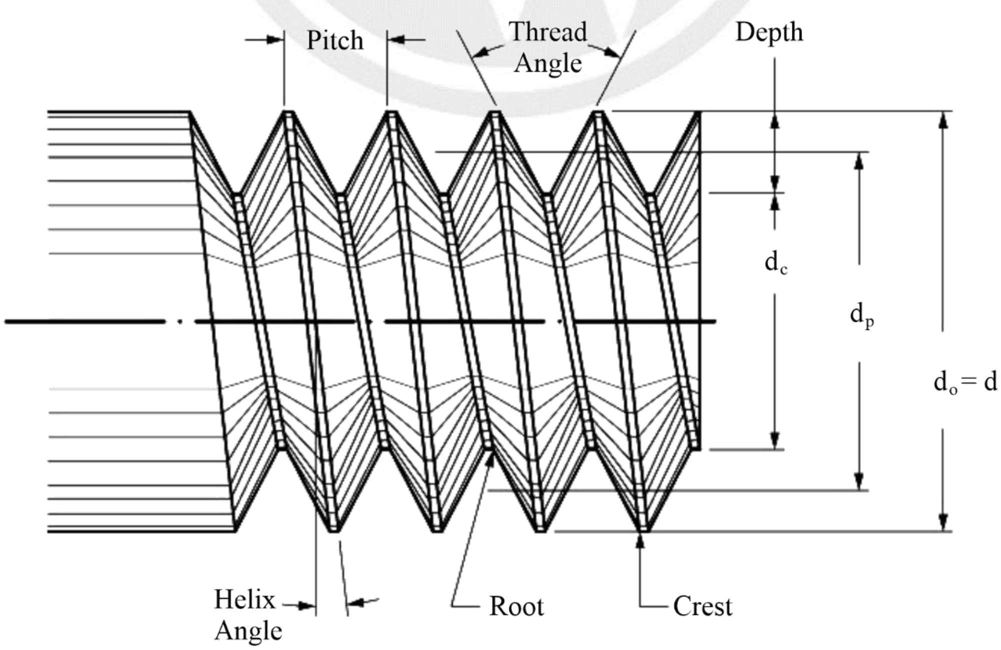


Fig.4.2: Terminologies of bolted joint

Terminologies of Screw Threads:

Crest: Peak point of thread profile

Root: Bottom-most point of thread profile

Major Diameter $\left(d_0 = d\right)$**:** Major diameter is the diameter of an imaginary cylinder that bounds the crest of an external thread. It is also called as nominal diameter

Minor/Core Diameter $\left(d_i = d_c\right)$**:** Minor diameter is the diameter of an imaginary cylinder that bounds the roots of an external thread. It is also called as core diameter.

Pitch $\left(p\right)$**:** Distance between two similar points on adjacent threads measured parallel to the axis of the thread.

Pitch diameter $\left(d_p\right)$**:** It is diameter of an imaginary cylinder the surface of which would pass through the threads at such points as to make the width of the threads (AB) equal to the width of the spaces cut by the surface of cylinder between two threads (BC). i.e. AB = BC.

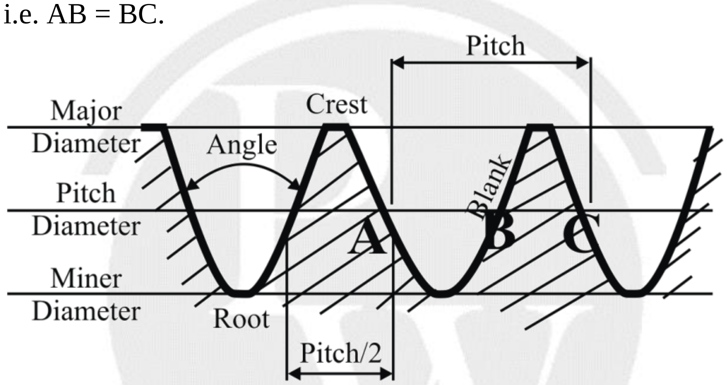


Fig.4.3: Threads of bolt

4.1.2 Various types of threads

(i) Square thread:

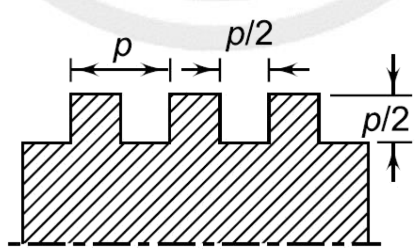


Fig.4.4: Square thread

- Used for transmission of power.
- Used in machine tool spindle, screw jack etc.
- Less wear
- Difficult to manufacture.
- Low strength.

(ii) Trapezoidal thread:

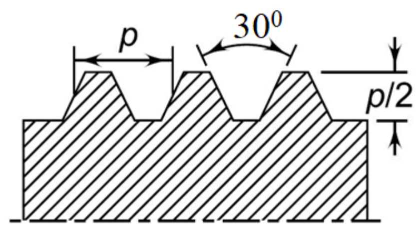


Fig.4.5: Trapezoidal thread

- Modification of square thread.
- Stronger than square thread due to more thickness at core.
- Easier to manufacture.
- More wear

(iii) Acme thread:

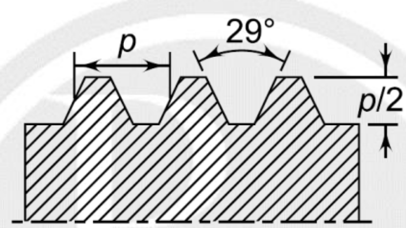


Fig.4.6: Acme thread

- Special type of trapezoidal thread called Acme Thread
- In an acme thread, the thread angle is 29° instead of 30°

(iv) Buttress thread:

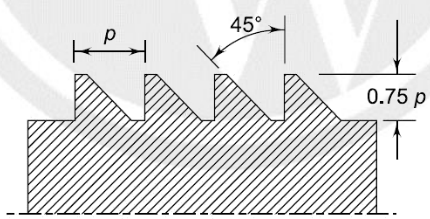


Fig.4.7: Buttress thread

- Combined advantages of square and ACME threads.

4.1.3 Lead (L)

Lead is the distance that the nut moves parallel to the axis of the screw, when the nut is given one turn. Or lead is axial movement due to 1 complete rotation of nut.

Lead = n * pitch

Where, n = number of starts in thread

n = 1 for single start thread

n = 2 for double start thread

n = 3 for triple start thread and so on

Axial movement (x) due to θ_C angular rotation of nut or k turns of nut:

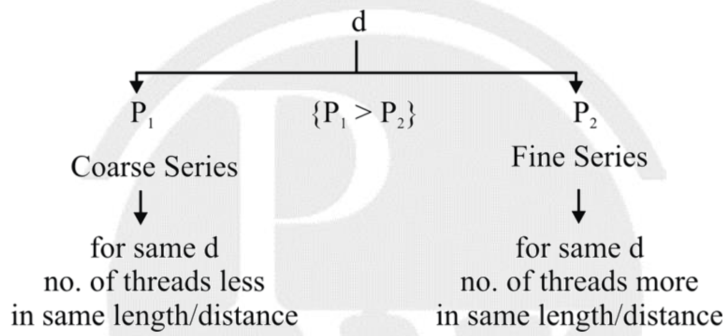
Angular rotation	Number of turns	Axial movement
2π	1	L
1 rad.	1	$\frac{L}{2\pi}$
θ_C	kL	$\frac{L}{2\pi}\theta_C$

$$x = \frac{L}{2\pi}\theta_c$$

4.1.4 Coarse and fine series of thread

There are two types of threads on the basis of their application:

1. **Coarse series-**recommended for general industrial applications
2. **Fine series-**recommended for sophisticated and delicate parts



Both these thread types have different designations:

1. **Coarse series-** Designated by the letter 'M' followed by the value of nominal diameter in mm. (Do not forget units)
   So, M12 means the nominal diameter of the thread is 12 mm.
2. **Fines Series-** Designated by the letter 'M' followed by the value of nominal diameter and then pitch in mm followed by 'X' symbol.
   So, M12 × 1.25 means nominal diameter = 12 mm and pitch = 1.25 mm.

Note:
The letter 'M' stands for ISO-Metric thread.

4.2 Preload in Nut & Bolt Assembly due to tightening

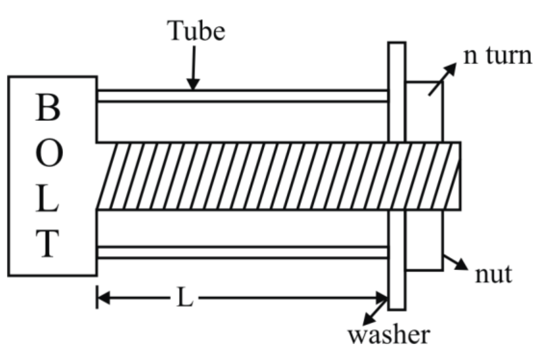


Fig.4.8: Preload in Nut and bolt

L_b = $L_t = L$

A_t = Cross sectional area of connecting member under compression due to tightening

A_b = Cross sectional Area of bolt or core area of bolt $\frac{\pi}{4}d_c^2$

E_{cm} = Young's Modulus of Elasticity of connecting member

E_b = Young's Modulus of Elasticity of bolt

Axial stiffness of bolt; $\boxed{k_b = \frac{A_b E_b}{L_b}}$

Axial stiffness of connecting member under compression due to tightening; $\boxed{k_{cm} = \frac{A_{cm} E_{cm}}{L_{cm}}}$

Axial movement of nut during tightening $= x$

- Initially the tube is at its free length i.e., washer is just touching the tube; No force is applied by nut & washer against bolt head to the tube.
- Due to tightening connecting member will be in compression and bolt will be in tension. Magnitude of force induced on the bolt is equal but opposite in nature to that of force induced in the connecting member due to tightening of the nut.

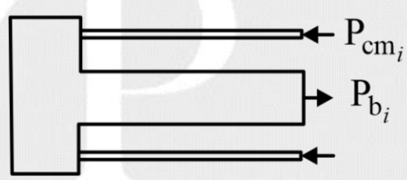


Fig.4.9: Axial force induced due to tightening

$$|P_{t_i}| = |P_{b_i}| = P_i$$

Compression Nature   Tensile Nature

Axial movement of nut during tightening:

$x = |\text{Compression in conneting member}| + |\text{Elongation of bolt}|$

$$\boxed{x = \left|\frac{P_{t_i}}{K_t}\right| + \left|\frac{P_{b_i}}{K_b}\right|}$$

$$\boxed{x = \frac{P_i}{K_t} + \frac{P_i}{K_b}}$$

Tensile stress on bolt due to preload = $(\sigma_{b_i})_{preload} = \frac{P_i}{A_b} = \frac{P_i}{\frac{\pi}{4}d_c^2}$

Compressive stress on connecting member due to preload = $(\sigma_{cm})_{preload} = \frac{P_i}{A_{cm}}$

Stresses in Bolt due to Bolt Preload:

1. Tensile stress in the cross-section: $\sigma_t = \dfrac{P_i}{\frac{\pi}{4} d_c^2}$

2. Torsional shear stress developed in the cross-section: $\tau = \dfrac{16T}{\pi d_c^3}$

3. Direct shear stress across the threads: $\tau = \dfrac{P_i}{\pi d_c t n}$

   Where, t = thickness of threads, n= no. of threads in contact with threads

4.3 Bolted Joint under External Load without Preload

4.3.1 Bolted Joint under External Tensile Load without Preload

Connecting members are connected by one bolted joint:

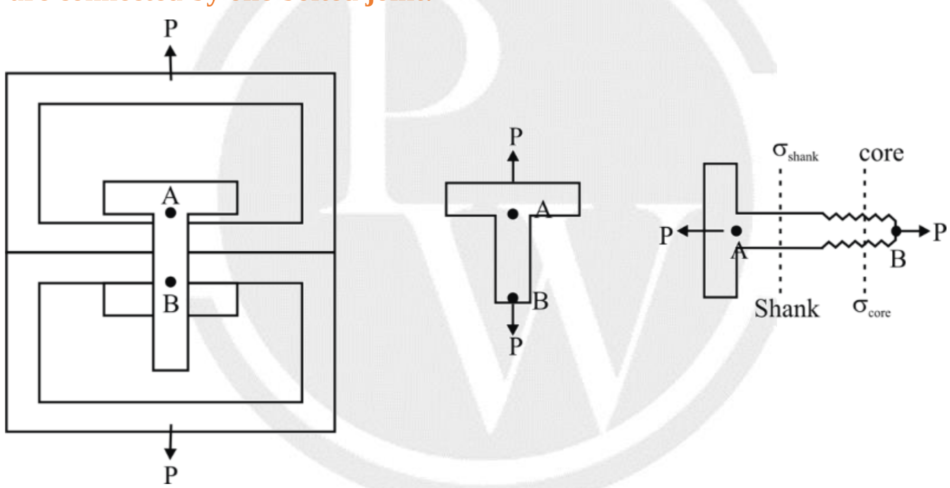


Fig.4.10: Bolted joint under tensile load without preload

- Without preload all the external tensile force will transfer in the bolts.
- Tensile force induced on bolt = P
- Maximum tensile stress induced in the bolt = $\dfrac{P}{A_c = A_b}$

  Where, $A_c = A_b$ = Core area of bolt

Connecting members are connected by n identical bolts:

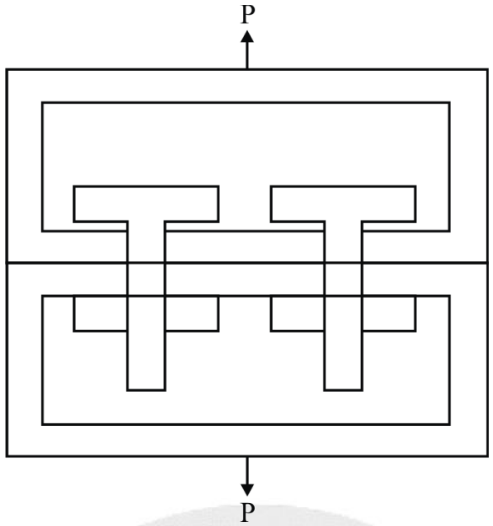


Fig.4.11: Connecting member with *n* identical bolts under external tensile load without preload

- Tensile force induced on each bolt = $\frac{P}{n}$
- Maximum tensile stress induced in the bolt = $\frac{P/n}{A_c = A_b}$

Where, $A_c = A_b$ = Core area of bolt

4.3.2 Bolted Joint under External Shear Load without Preload

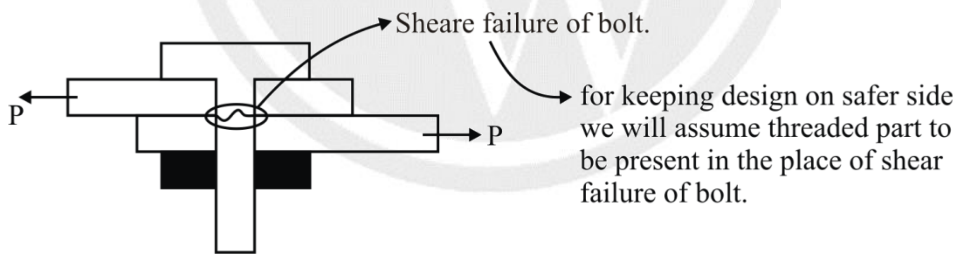


Fig.4.12: Bolted joint with external shear load without preload

$$\tau = \frac{P}{A_C}$$

For design $\tau \le \tau_P$

If more than one bolt is present;

$$\tau = \frac{P}{nA_C} \{n \Rightarrow \text{no. of bolt}\}$$

$$\frac{P}{nA_C} \le \tau_P$$

4.4 Bolted Joint under External Tensile Load with Preload

4.4.1 Case I: Connecting members are connected by one bolted joint

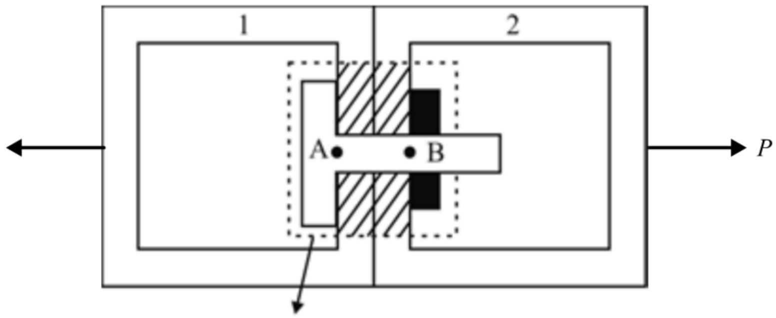


Fig.4.13: Bolted joint under external tensile load with preload

P_i = Preload on bolt

Effect of Preload on bolt = P_i (tensile)

Effect of preload on connecting member = – P_i (compressive)

K_b = Stiffness of bolt {only that part which elongates)

K_{CM} = Equivalent stiffness of that part of connecting members which is under contraction due to preload.

$C = \frac{K_b}{K_{CM} + K_b}$ = Stiffness correction factor

P_b = Effect of external load on bolt

P_{cm} = Effect of external load on part of connecting members which is under contraction due to preload

To find the effect of external tensile load with preload, part of connecting members which is under contraction due to preload and bolt will be treated as composite bar in parallel.

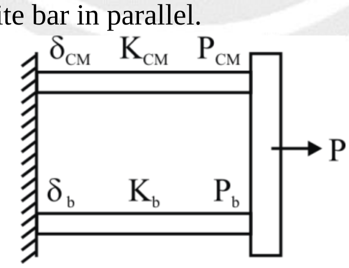


Fig.4.14: Effect of external tensile load with preload

Due to external load P,

$$\delta_{CM} = \delta_b = \delta$$

$$\frac{P_{CM}}{K_{CM}} = \frac{P_b}{K_b} = \frac{P}{K_{eq}} \quad \{K_{eq} = K_b + K_{CM})$$

$$\boxed{P_b = \left(\frac{K_b}{K_{CM} + K_b}\right) P = CP}$$

And,

$$P_{CM} = \left(\frac{K_{CM}}{K_{CM} + K_b}\right)P$$

$$P_{CM} = (1 - C)P$$

Total force on bolt: $(P_b)_{total} = P_i + P_b = P_i + CP$

Total load on connecting member: $(P_{cm})_{total} = -P_i + P_{cm} = -P_i + (1 - C)P$

Stress in bolt = $\sigma_b = \frac{(P_b)_{total}}{A_b} = \frac{P_i + CP}{A_b}$

Stress in connecting member = $\sigma_{CM} = \frac{(P_{cm})_{total}}{A_{CM}} = \frac{-P_i + (1 - C)P}{A_{CM}}$

For leak proof joint: $\sigma_{CM} < 0 \Rightarrow -P_i + (1 - C)P < 0$

4.4.2 Case II: Connecting members are connected by n identical bolts

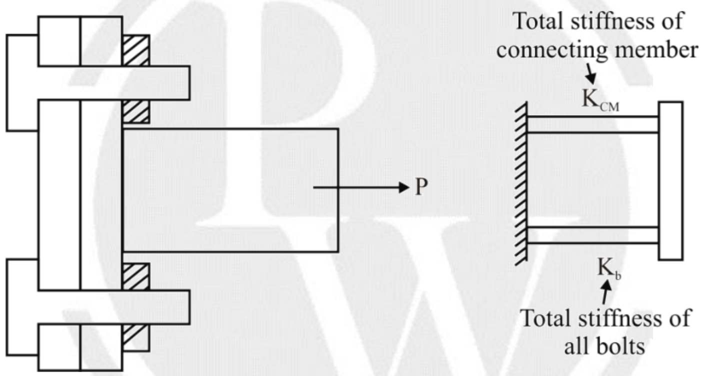


Fig.4.15: Connecting members with n identical bolts and under external tensile load with preload

Consider

n = Number of bolts

$K_{b'}$ = Stiffness of each bolt

$K_b = nK_{b'}$ = Equivalent stiffness of all bolts= Combine stiffness of all the bolts

$A_b = A_C = \frac{\pi}{4}d_C^2$ = Core area of each bolt.

P_i = Preload on each bolt

Effect of Preload on bolt = + P_i (tensile)

Effect of preload on connecting member = $-nP_i$ (compressive)

Due to external force P ($\because$ preload present) combine stiffness of all the bolt and the connecting member will be treated as composite bar in parallel.

Effect of external force on each bolt, $P_b = \frac{CP}{n}$

Effect of external force on connecting member, $P_{CM} = (1 - C)P$

Total force on each bolt: $(P_b)_{total} = P_i + P_b = P_i + \frac{CP}{n}$

Total load on connecting member: $(P_{cm})_{total} = -nP_i + P_{cm} = -nP_i + (1-C)P$

Stress in each bolt = $\sigma_b = \frac{(P_b)_{total}}{A_b} = \frac{P_i + \frac{CP}{n}}{A_b}$

Stress in connecting member = $\sigma_{CM} = \frac{(P_{cm})_{total}}{A_{CM}} = \frac{-nP_i + (1-C)P}{A_{CM}}$

For leak proof joint: $\sigma_{CM} < 0 \Rightarrow -nP_i + (1-C)P < 0$

4.5 Effect of preload under fluctuating external load condition

Preload on each bolt $= P_i$

Consider, External load fluctuating from 0 to P.

Then, in bolt, external load will fluctuate from 0 to CP (Assuming connecting members are connected with one bolted joint)

Total load fluctuation in bolt $= P_i$ to $(P_i + CP)$

$\sigma_{min} = \frac{P_i}{A_b}$ and $\sigma_{max} = \frac{P_i + CP}{A_b}$

Amplitude stress, $\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{CP}{2A_b}$

Mean stress, $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{P_i}{A_b} + \frac{CP}{2A_b}$

If $P_i >> CP$, then $\sigma_a << \sigma_m$ and if $\sigma_a << \sigma_m$ then fluctuation of stress will be less with respect to mean stress. Hence preload reduces the effect of fluctuation.

Note:

Advantage of Preload

- Leak Proof Joint { $F_{CM} < 0 \Rightarrow [-nP_i + (1-c)P] < 0$ }
- Reduces fluctuation in Load
- Increases fatigue Life
- Reduces effect of External Load = $[(P_b)_{total} = P_i + CP]$

4.6 Bolt of uniform strength

Bolts of uniform strength:

If you look at any regular bolt, if the diameter of shank will be more than the effective diameter of the threaded region. The reason for this is the diameter of threaded part has somewhat reduced due to thread making. Hence, when a load P acts through the bolt, threaded part is stressed more, so the strain energy stored in threaded part will be more than the strain energy stored in unthreaded part. To increase the amount of strain energy that this bolt can store, we have to increase the stress in unthreaded (shank) part as well.

So, we make some changes in the bolt so that the value of stress is same at every cross section, be it threaded part or unthreaded part. Such an ideal bolt is known as **bolt of uniform strength.**

Option 1: Either by reducing the unthreaded cross-sectional area (to increase stress) by turning operation as you can see in image(A) of figure 4.16.

Option 2: Either by reducing the unthreaded cross-sectional area by drilling a hole through it as you can see in image (B) of figure 4.16.

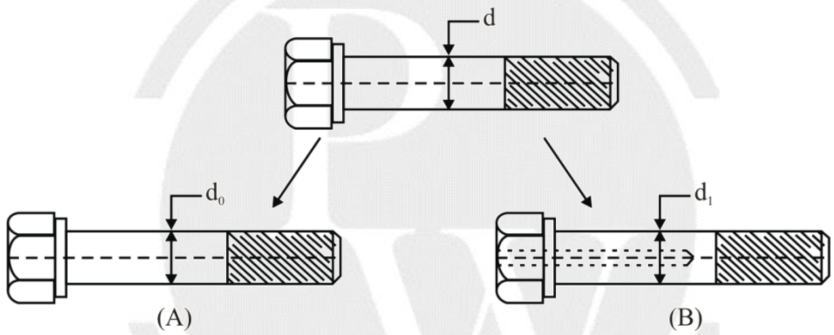


Fig.4.16: Bolts of uniform strength

Method in image (A) is preferred because:

Reason 1: Drills come in standard sizes so we cannot drill any hole of desired diameter directly. We might have to go for reaming after that. No such problem in turning.

Reason 2: Drilling a hole will cause unwanted stress concentration.

4.7 Introduction of riveted joint

A rivet has a cylindrical shank with a head at one end. It is used to produce permanent joints between two plates.

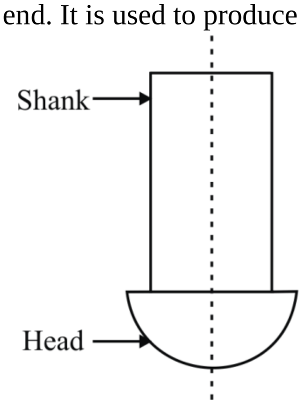


Fig.4.17: Rivet

Rivet is inserted into the holes of plates (which are to be joined) and then its protruding part is upset by a hammer.

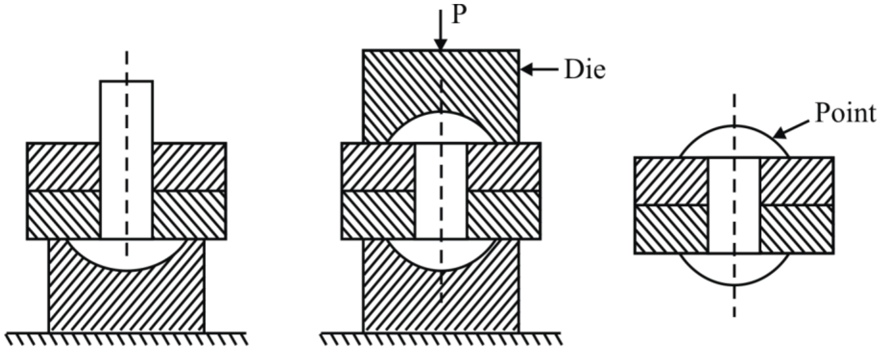


Fig.4.18: Riveting process

Joints made by rivets are called permanent because the parts can be only the dismantled by damaging rivets.

Note:
(i) Diameter of shank < Diameter of rivet hole
(ii) A rivet is specified by shank diameter of the rivet. A 20 mm rivet means a rivet having 20 mm as the shank diameter

4.7.1 Classification of riveted joint

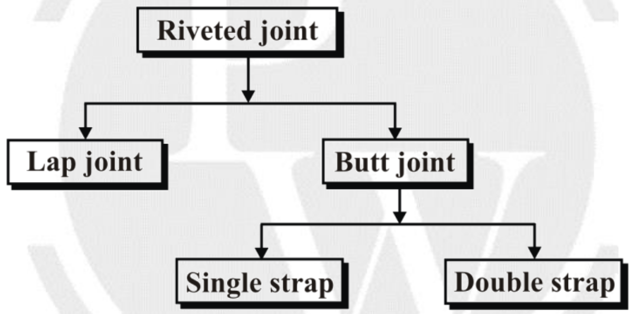


Fig.4.19: Classification of riveted joint

I. Lap Joint: Riveted joint between two overlapping plates.

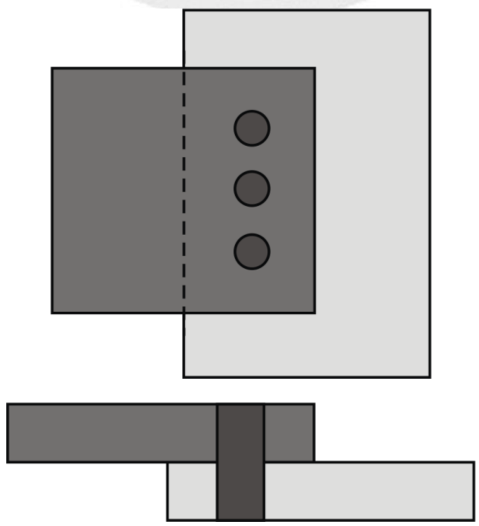


Fig.4.20: Lap Joint

II. **Butt Joint:** In this type of riveting, the plates to be joined are kept in the same plane, without forming an overlap. Another plate known as **cover or strap plate** is placed over either one side or on both side of the main plates, then it is riveted with main plates.

If only one cover plate is placed on the main plate, then that butt joint is known as **Single strap butt joint**.

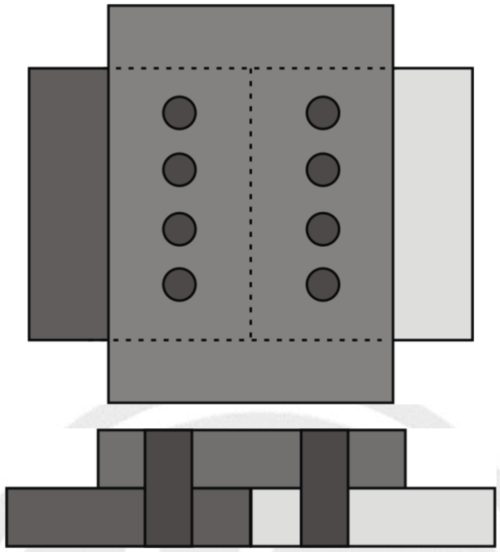


Fig.4.21: Single strap Butt joint

If cover or strap plate is placed on the both sides of main plate, then that butt joint is known as **Double strap butt joint**.

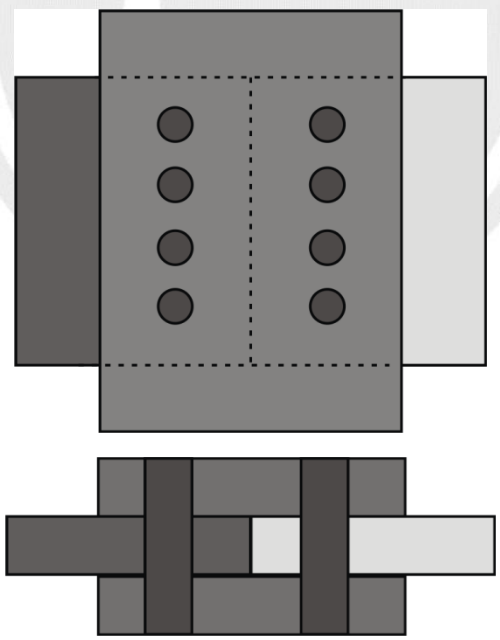


Fig.4.22: Double strap butt joint

Classification of riveted joint Based on how many rows of rivets in joint:

Single riveted joint: This riveted joint has one row of the rivet in a lap joint or only one row of the rivet on each plate of butt joint. **(Refer figure 4.23)**

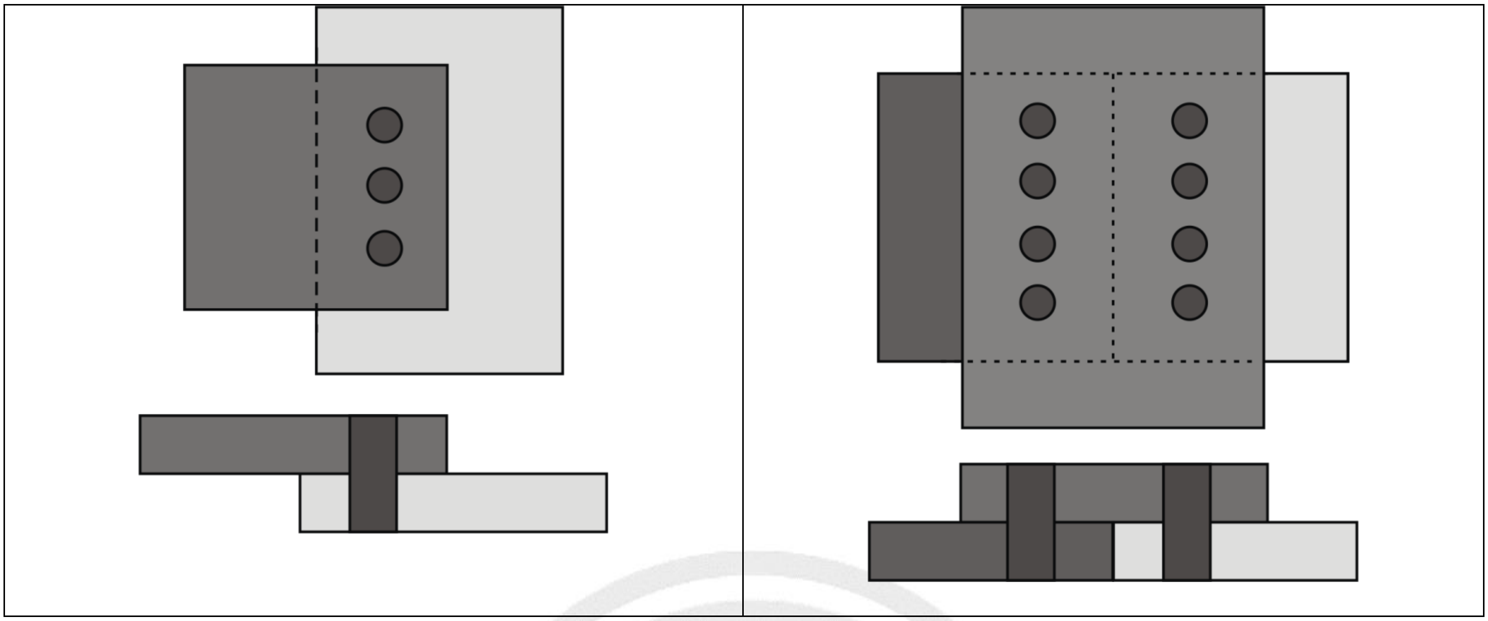


Fig.4.23: Single riveted joint

Double riveted joint: Two rows of rivets are used in a lap joint or two rows of rivet are used in each main plate of butt joint. Similarly, there are triple riveted, quadruple riveted joints are there.

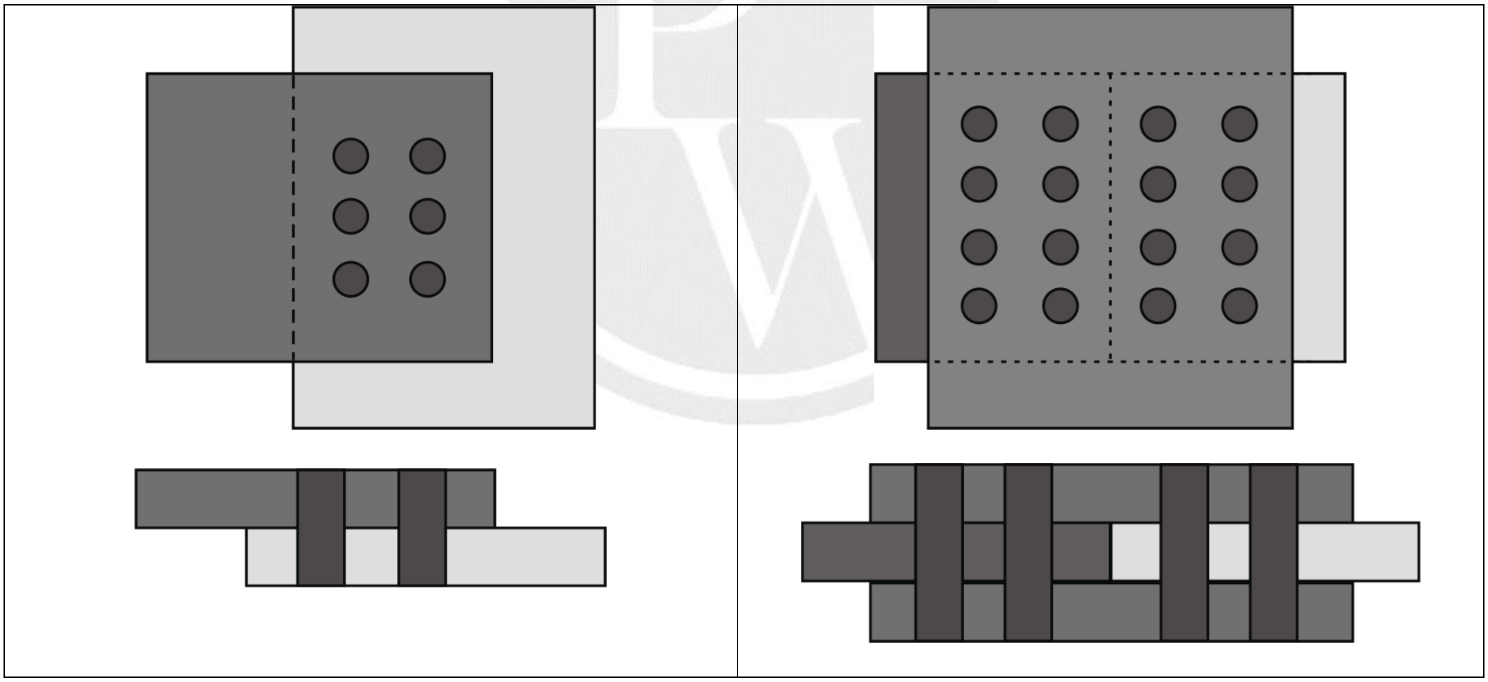
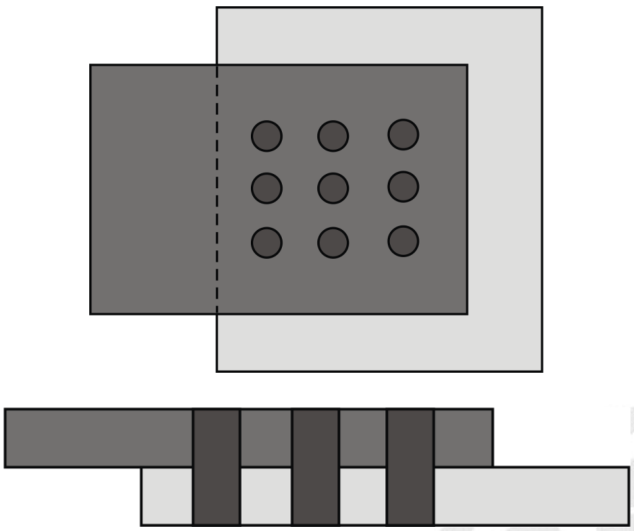
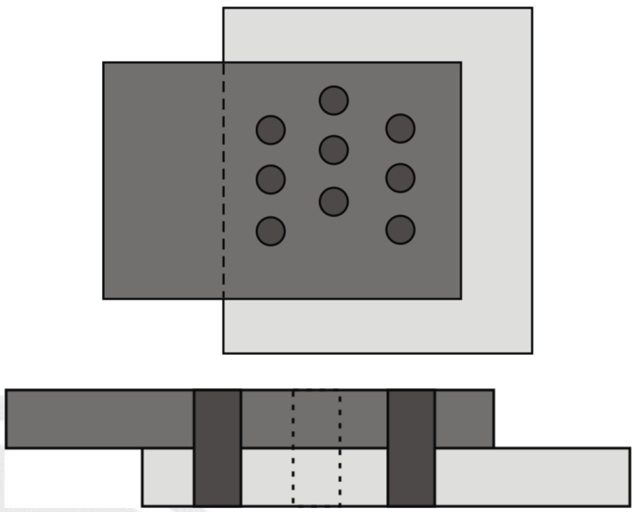


Fig.4.24: Double riveted joint

Classification of riveted joint based on the arrangement of rivets in adjacent rows of rivet

Chain riveted joints: The rivets in the adjacent rows are opposite to each other (in same transverse line).	**Zigzag riveted joints:** When the rivets in adjacent rows are not in chain arrangement.
 Fig.4.25: Chain riveted join	 Fig.4.26: Zigzag riveted joint

4.7.2 Terminology of riveted joint

Pitch(*p*): The pitch of the rivet is defined as the distance between the center of one rivet to the center of adjacent rivet in the same row.

Margin / Edge distance (*m*):

The margin is the distance between the edge of the plate to the center line of the rivets in the nearest row.

Transverse pitch (p_b):

Transverse pitch also called back pitch or row pitch. It is the distance between two consecutive rows of the rivet on the same plate.

Diagonal pitch (p_d):

Diagonal pitch is the distance between the center of one rivet to the center of adjacent rivet located in the adjacent row.

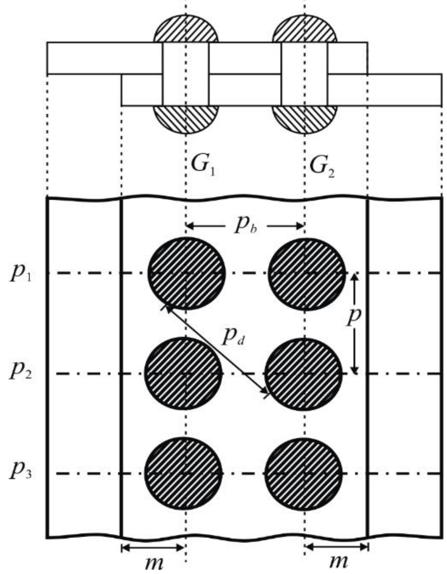


Fig.4.27: Terminologies of riveted joint

4.8 Load Carrying capacity of riveted joint

4.8.1 When number of rivets in each row is same

Consider,

w = width of each plate

t = thickness of each plate

d = diameter of rivet

d_h = diameter of rivet hole

m = Number of rows of rivet

n = Number of rivets in each row

N = mn = total number of rivets

> **Note:**
> (i) For Butt riveted joint, find value of M, n and N only for one plate
> (ii) For figure 4.28 m is 2, n is 3 and N is 6.

σ_{tp} = Permissible tensile stress of plate

τ_p = Permissible shear stress of rivet

σ_{cp} = Permissible crushing stress of rivet.

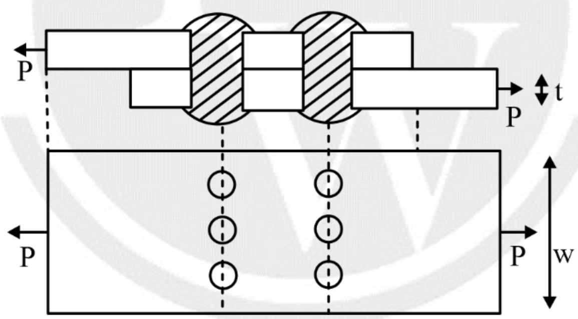


Fig.4.28: Riveted joint with tensile load

1. **Tearing load carrying capacity (or tearing strength) of the plate (P_t):**
   When number of rivets in each row is same then, tearing failures occurs from the row which is nearest to the load as shown in figure.

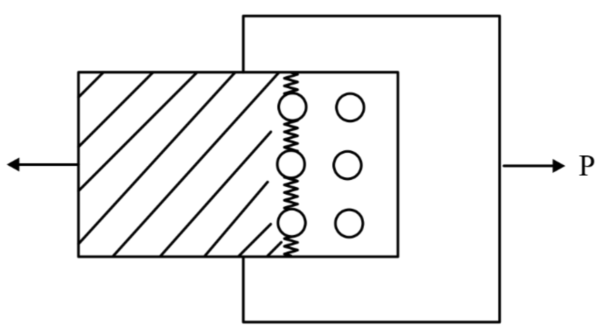


Fig.4.29: Tearing failure

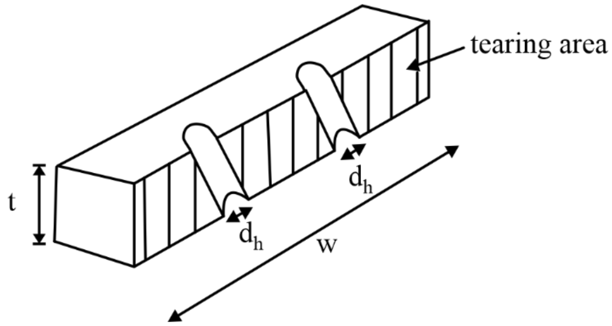


Fig.4.30: Tearing area view

Tensile stress in the plate of nearest row

$$\sigma_t = \frac{P}{\text{tearing area}} = \frac{P}{(w - nd_h)t} \le \sigma_{tp}$$

$$\Rightarrow P \le \sigma_{tp}(w - nd_h)t$$

Hence load carrying capacity to avoid tearing failure,

$$P_t = \sigma_{tp}(w - nd_h)t$$

2. **Shearing load carrying capacity or shearing strength (P_s):**

Shearing failures occur when shearing of all rivets occurs.

Shear stress in rivet,

$$\tau = \frac{P}{\text{total shear area}}$$

Note:

Total shear area of rivets

$$= k \times \frac{\pi}{4} d^2 \times N$$

Where,

k = Number of shear area in one rivet.

For single shear, k = 1(refer figure 4.31)

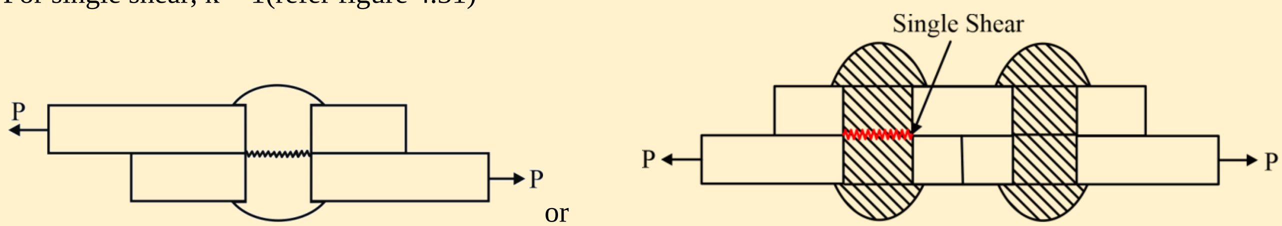


Fig.4.31: Single Shearing Failure

For double shear, k = 2

(Refer figure 4.32)

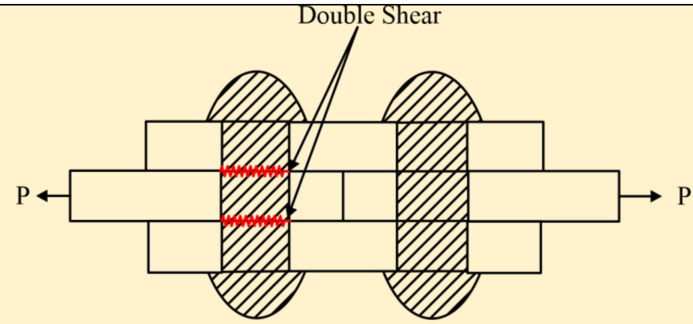


Fig.4.32: Double Shear Failure

Hence, shear stress in rivet,

$$\tau = \frac{P}{k \times \frac{\pi}{3} d^2 \times N} \le \tau_p$$

$$P \le \tau_p \times \left(k \times \frac{\pi}{4} d^2\right) \times N$$

Hence load carrying capacity to avoid shearing failure is

$$P_S = \tau_p \left(k \times \frac{\pi}{4} d^2\right) \times N$$

3. Crushing load carrying capacity or crushing strength (P_C):

In crushing failure rivets fails due to compressive stress and crushing failure occur when crushing of all rivets occurs.

Crushing stress in rivet:

$$\sigma_C = \frac{P}{\text{Total crushing area}}$$

Note:

Crushing area of one rivet = dt

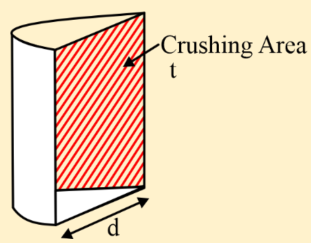


Fig.4.33: Crushing area

Total crushing area = dt × N

Hence,

$$\sigma_c = \frac{P}{dtN} \le \sigma_{cp}$$

$$\Rightarrow P \le \sigma_{cp} dtN$$

Hence load carrying capacity to avoid crushing failure is

$$\boxed{P_C = \sigma_{cp} dtN}$$

4. Actual load carrying capacity (P_{LCC})

$P_{LCC} =$ Minimum of load carrying capacity by considering all type of failure.

$$\boxed{P_{LCC} = \text{Min}(P_t, P_s, P_c)}$$

5. Efficiency (η)

It is the ratio of actual load carrying capacity (P_{LCC}) to the load carrying capacity of solid plate without rivet and hole $(P_{\text{Solid Plate}})$

$$\eta = \frac{P_{LCC}}{P_{\text{Solid Plate}}}$$

$P_{\text{Solid Plate}}$:

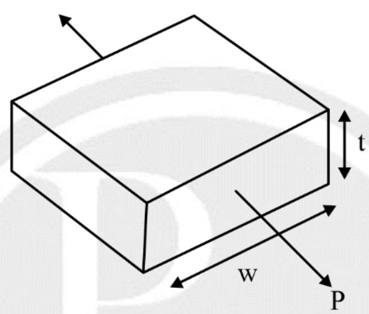


Fig.4.34: Solid plate without rivet and hole

Tensile stress, $\sigma_t = \frac{P}{wt} \le \sigma_{tP}$

$\Rightarrow P \le \sigma_{tP} wt$

Hence, $P_{\text{Solid Plate}} = \sigma_{tP} wt$

Efficiency of joint:

$$\boxed{\eta = \frac{P_{LCC}}{\sigma_{tP} wt}}$$

Note:

If width of the plate is not given, then to find efficiency of the joint calculate load carrying capacity per pitch length. To calculate load carrying capacity per pitch length replace width w by pitch p and number of rivets in each row i.e. n by 1.

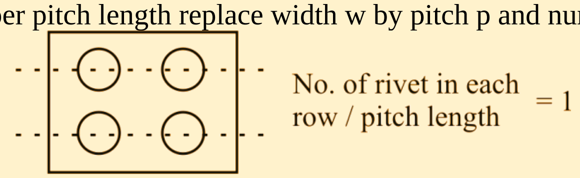


Fig.4.35

Tearing strength / Pitch length: $P_{t/PL} = \sigma_{tp}(p - d_h)t$

Shearing strength / Pitch length: $P_{S/PL} = \tau_p \times k \times \frac{\pi}{4} d^2 \times m$

Crushing strength / Pitch length: $P_{C/PL} = \sigma_{cp} \times dt \times m$

Actual load carrying capacity: $P_{LCC/PL} = \text{Min}(P_{t/PL}, P_{S/PL}, P_{C/P_L})$

Efficiency: $\eta = \frac{P_{LCC/PL}}{\sigma_{tP} \times pt}$

4.8.2 When number of rivets in each rows are different (Diamond Pattern)

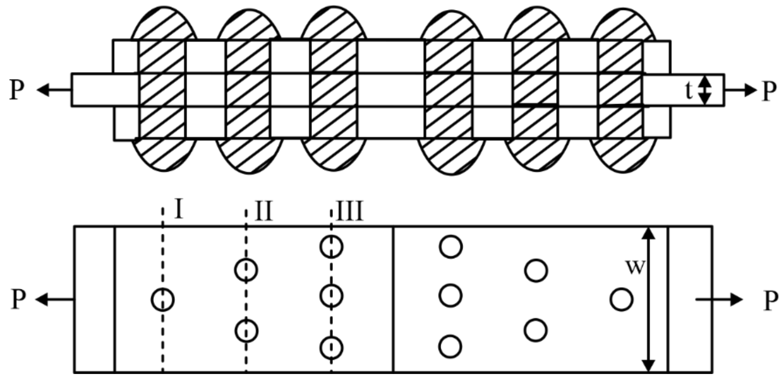


Fig.4.36: Diamond pattern rivet joint

1. **Shearing load carrying capacity (P_s)**

   $P_s = \tau_p \times \text{Total shear area}$

   [In figure 4.36 k = 2 (double shear), total number of rivets = 6]

   $$P_S = \tau_p \times \left(2 \times \frac{\pi}{4} d^2\right) \times 6$$

2. **Crushing load carrying capacity** (P_C)

   $P_C = \sigma_{cp} \times (\text{Total crushing area})$

   [For Figure 4.36 total number of rivets = 6]

   $$\boxed{P_C = \sigma_{cp} \times dt \times 6}$$

3. **Load carry capacity to avoid tearing from first row** $\left(P_{t_I}\right)$:

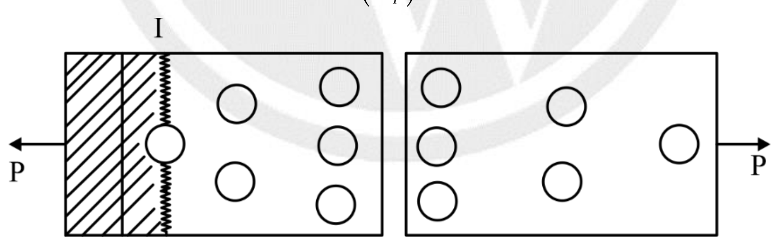


Fig.4.37: Tearing of the first row

   $P_{t_I} = \sigma_{tp} \times (w - d_h) t$

4. **Load carrying capacity to avoid tearing of second row** $(P_{t_{II}})$

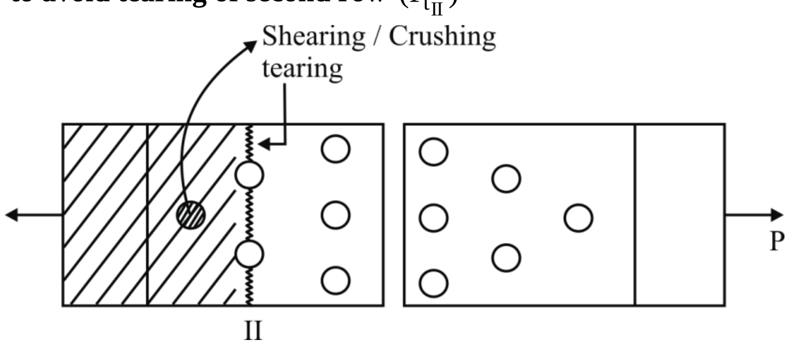


Fig.4.38: Tearing of the second row

$$P_{t_{II}} = \sigma_{tp}(w - 2d_h)t + \min(\tau_P \times k \times \frac{\pi}{4}d^2, \sigma_{cp} \times dt)$$

5. Load carrying capacity to avoid tearing of 3rd row ($P_{t_{III}}$)

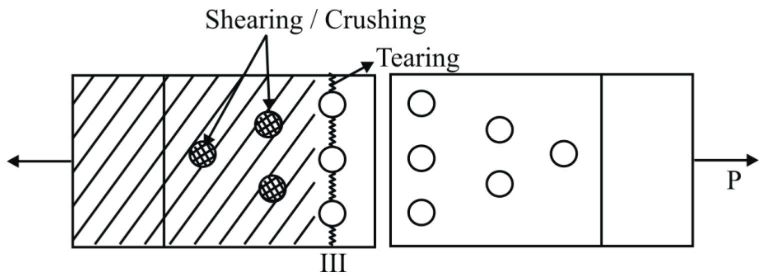


Fig.4.39: Tearing of the 3rd row

$$P_{t_{III}} = \sigma_{tp}(w - 3d_h)t + \min(\tau_P \times k \times \frac{\pi}{4}d^2 \times 3, \sigma_{cp} \times dt \times 3)$$

6. Actual load carrying capacity (P_{LCC}):

$$P_{LCC} = \min(P_S, P_C, P_{t_I}, P_{t_{II}}, P_{t_{III}})$$

7. Efficiency (η): $\eta = \frac{P_{LCC}}{\sigma_{tP} wt}$

4.9 Eccentric loading in bolted and riveted joint

4.9.1 Type I: Eccentricity in the plane of cross-section of bolt or rivet

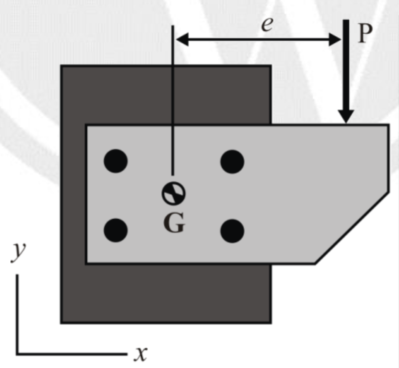


Fig.4.40: Eccentric load on the rivet

Shift the force P at the COG:

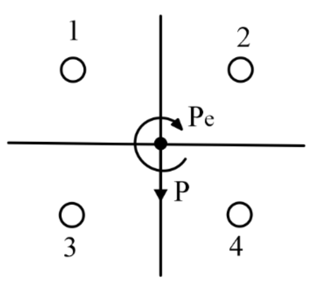


Fig.4.41: Shifting of load

(i) Direct force P will act at the COG which causes primary shear in each rivet/bolt.
(ii) Moment M = Pe will act at COG which causes secondary shear in each rivet/bolt.

Effect of force P:

- Primary shear will induce.
- **Magnitude of primary shear force (P_S):** Primary shear force in each rivet/bolt will be same & is calculated by:

$$P_S' = \frac{P}{\text{No. of rivet / bolt}}$$

- **Magnitude of primary shear stress (τ′):** Primary shear stress will be same in each bolt/rivet & is calculated by

$$\tau'_1 = \tau'_2 = \tau'_3 = \tau'_4 = \frac{\text{Primary shear force on each bolt/rivet}}{A}$$

- **Direction:** Direction of primary shear stress will be same at each point of bolt/rivet & will be opposite to force P. As shown in figure 4.42.

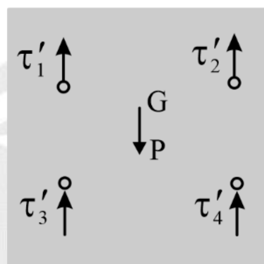


Fig.4.42: Direction of primary shear stress

Effect of moment $M = P \times e$:

- Due to moment $M = P \times e$, secondary shear will induce in each rivet/bolt.
- Magnitude of secondary shear force (P''_{s_i}) and secondary shear stress (τ''_i)

$$\mathbf{P''_{s_i} = Cr_i}$$

Where P''_{s_i} = Secondary shear force on i^{th} bolt/rivet.

r_i = distance of center of i^{th} rivet/ bolt from center of gravity,

C is proportional constant & is calculated by, $C = \frac{Pe}{(r_1^2 + r_2^2 + r_3^2 + ...r_n^2)}$ N/m

Bolt/Rivet	Secondary shear force	Secondary shear Stress
1.	$P_{s_1}'' = Cr_1$	$\tau_1'' = \frac{P_{s_1}''}{A}$
2.	$P_{s_2}'' = Cr_2$	$\tau_2'' = \frac{P_{s_2}''}{A}$
3.	$P_{s_3}'' = Cr_3$	$\tau_3'' = \frac{P_{s_3}''}{A}$
4.	$P_{s_4}'' = Cr_4$	$\tau_4'' = \frac{P_{s_4}''}{A}$

Direction

Step I: Draw a line which is perpendicular to the line joining the center of bolt/rivet whose secondary shear stress we are drawing & COG.

Step II: In this line take the sense of secondary shear stress such that it should try to rotate the rivet/bolt opposite to the sense in which moment $M = P \times e$ if trying to rotate the rive/bolt.

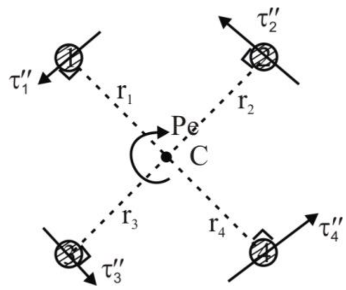


Fig.4.43: Direction of secondary shear stress

Resultant Shear Stress:

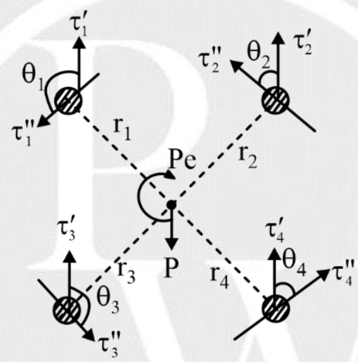


Fig.4.44: Combined primary and secondary shear stress

Find the vector resultant of primary & secondary shear stress.

Resultant shear stress of bolt/rivet 1,

$$\tau_1 = \sqrt{(\tau_1')^2 + (\tau_2'')^2 + 2\tau_1'\tau_2''\cos\theta_1}$$

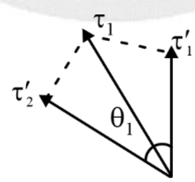


Fig.4.45: Direction and magnitude of resultant shear stress

Critical Rivet/bolt

During design we need to find resultant shear stress on critical Rivet/bolt. Critical Rivet/bolt is the Rivet/bolt where resultant shear stress is maximum.

Steps to find critical Rivet/bolt:

Step I: Draw the direction of primary and secondary shear stress of each rivet/bolt.

Step II: Select the bolt/rivet whose distance from COG (i.e. r) is maximum.

Step III: Select the bolt/rivet whose vector angle between primary & secondary shear stress (i.e. θ) is minimum.

Step IV: Bolt/rivet where r is maximum and θ is minimum (i.e. common rivets/bolts of step III & IV) will be critical bolt/rivet).

4.9.2 Type II: Eccentric load perpendicular to the axis of bolts

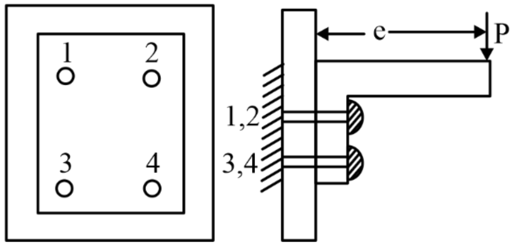


Fig.4.46: Eccentric load perpendicular to axis of the bolts

In this case, moment due to force P will try to till the bracket about tilting point C as shown in figure 4.47.

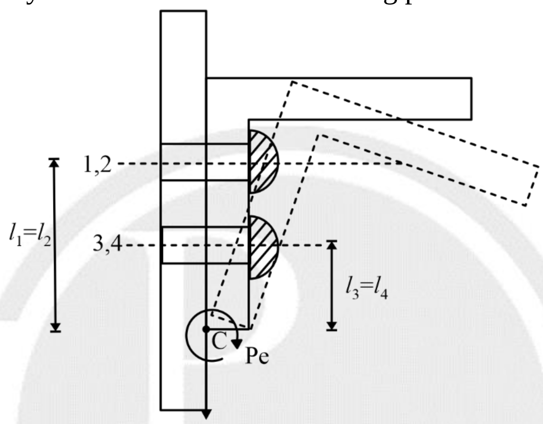


Fig.4.47: Tilting of rivet

Hence, in this case we will transfer the force P at tilting point C.

After transferring at tilting point C, point C will be subject to

(i) Direct fore P which will cause primary shear in each bolt/rivet

(ii) Moment $M_t = P \times e$ which causes secondary tensile in each bolt/rivet.

Effect of force P:

- Primary shear will induce.
- Magnitude of primary shear force (P_S'): Primary shear force in each rivet/bolt will be same & is calculated by:

$$P_S' = \frac{P}{\text{No.of rivet / bolt}}$$

- **Magnitude of primary shear stress (τ′):** Primary shear stress will be same in each bolt/rivet & is calculated by:

$$\tau'_1 = \tau'_2 = \tau'_3 = \tau'_4 = \frac{\text{Primany shear force on each bolt/rivet}}{A}$$

Effect of moment $M_t = P \times e$:

- Due to moment $M_t = P \times e$, secondary tensile fore (P_t'') will induce in each bolt/rivet which is calculated by:

$$P_{t_i}'' = k \times \ell_i$$

Where, P_{t_i}'' = secondary tensile force of i^{th} bolt/rivet

ℓ_i = Distance of i^{th} bolt/rivet from tilting point C.

k is proportional constant & is calculated by $k = \dfrac{P \times e}{\ell_1^2 + \ell_2^2 + ... + \ell_n^2}$

Bolt/Rivet	Secondary tensile force	Secondary tensile stress
1	$P_{t_1}'' = k\ell_1$	$\sigma_{t_1}'' = \frac{P_{t_1}''}{A}$
2	$P_{t_2}'' = k\ell_2$	$\sigma_{t_2}'' = \frac{P_{t_2}''}{A}$
3	$P_{t_3}'' = k\ell_3$	$\sigma_{t_3}'' = \frac{P_{t_3}''}{A}$
4	$P_{t_4}'' = k\ell_4$	$\sigma_{t_4}'' = \frac{P_{t_4}''}{A}$

For design:

- Calculate stresses on critical bolt/rivet. Bolt/rivet whose secondary tensile stress is maximum (say $\sigma_{t,m}$) will be critical bolt/rivet because primary shear stress is same for all bolt/rivet (say τ_s'). For secondary tensile stress to be maximum, ℓ should be maximum. Hence Bolt/rivet whose ℓ is maximum will be critical bolt/rivet.
- Find principal stress & maximum shear stress at the critical point & use appropriate theory of failure.

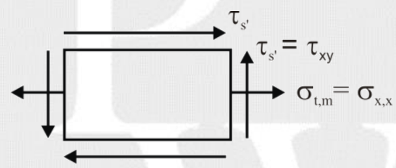


Fig.4.48: Combined tensile and shear stress

$$\sigma_{xx} = \sigma_{t,m},\ \sigma_{yy} = 0,\ \tau_{xy} = \tau_s'$$

Principle stresses:

$$\sigma_{1,2} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{\sigma_{t,m}}{2} \pm \sqrt{\left(\frac{\sigma_{t,m}}{2}\right)^2 + (\tau_s')^2}$$

and

$$\sigma_3 = 0$$

Maximum shear stress:

$$\tau_{max} = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_{t,m}}{2}\right)^2 + (\tau_s')^2}$$

Note:

Steps for designing bolt or rivet for Type II Eccentric Loading

(a) Find ℓ for each bolt & decide critical bolt or rivet

(b) Find primary and secondary shear stress for critical bolt or rivet.

(c) Find principal stresses and maximum shear stress for critical bolt or rivet.

(d) Design the bolt or rivet according to given TOF

4.9.3 Type III: Eccentric load parallel to the axis of bolts

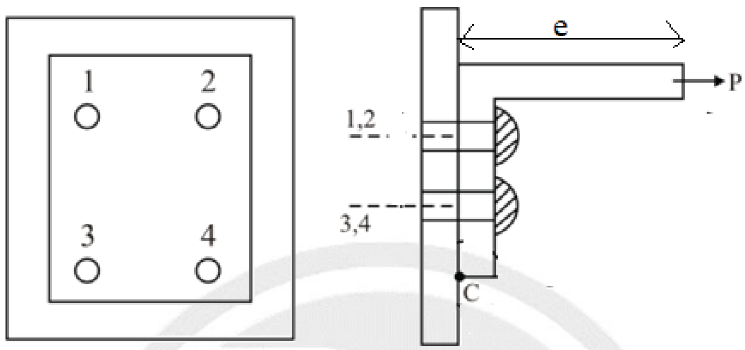


Fig.4.49: Eccentric load parallel to the axis of the bolt

Transfer the force P at tilting point C:

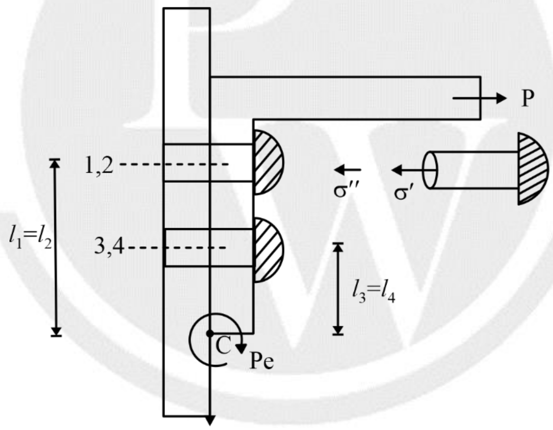


Fig.4.50: Transfer of load

After transferring, the point C will be subjected to

(i) Direct force P which causes primary tensile in each bolt/rivet.

(ii) Moment $M_t = P \times e$ which causes secondary tensile in each bolt/rivet

Effect of force P:

- Due to force P each bolt/rivet will be subjected to primary tensile.
- Magnitude of primary tensile force:
  It will be same in each bolt/rivet & is calculated by

$$P_{t_1}' = P_{t_2}' = P_{t_3}' = P_{t_4}' = P_t' = \frac{P}{\text{no.of bolt / rivet}}$$

Primary tensile stress in each bolt/rivet

$$\sigma_{t_1}' = \sigma_{t_2}' = \sigma_{t_3}' = \sigma_{t_4}' = \sigma_t' = \frac{P_t'}{A}$$

Effect of moment $M_t = P \times e$:

- Due to moment $M_t = P \times e$, secondary tensile fore (P_t'') will induce in each bolt/rivet which is calculated by: $P_{t_i}'' = k \times \ell_i$

  Where, P_{t_i}'' = secondary tensile force of i^{th} bolt/rivet

  ℓ_i = Distance of i^{th} bolt/rivet from tilting point C.

  k is proportional constant & is calculated by $k = \dfrac{P \times e}{\ell_1^2 + \ell_2^2 + ... + \ell_n^2}$

Bolt/Rivet	Secondary tensile force	Secondary tensile stress
1	$P_{t_1}'' = k\ell_1$	$\sigma_{t_1}'' = \dfrac{P_{t_1}''}{A}$
2	$P_{t_2}'' = k\ell_2$	$\sigma_{t_2}'' = \dfrac{P_{t_2}''}{A}$
3	$P_{t_3}'' = k\ell_3$	$\sigma_{t_3}'' = \dfrac{P_{t_3}''}{A}$
4	$P_{t_4}'' = k\ell_4$	$\sigma_{t_4}'' = \dfrac{P_{t_4}''}{A}$

Resultant tensile stress of any i^{th} bolt/rivet (σ_{t_i}):

$$\sigma_{t_i} = \sigma_{t_i}' + \sigma_{t_i}''$$

For design: Find the resultant tensile stress in the critical bolt/rivet. Bolt/rivet whose distance from tilting point i.e. ℓ is maximum will be critical bolt/rivet.

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5 DESIGN OF CLUTCH

5.1 Introduction

It is used to **connect or disconnect** the source of power from the remaining parts of the power transmission system at the will of the operator.

It should be capable to transmit required torque or power from input shaft to output shaft in engaged position.

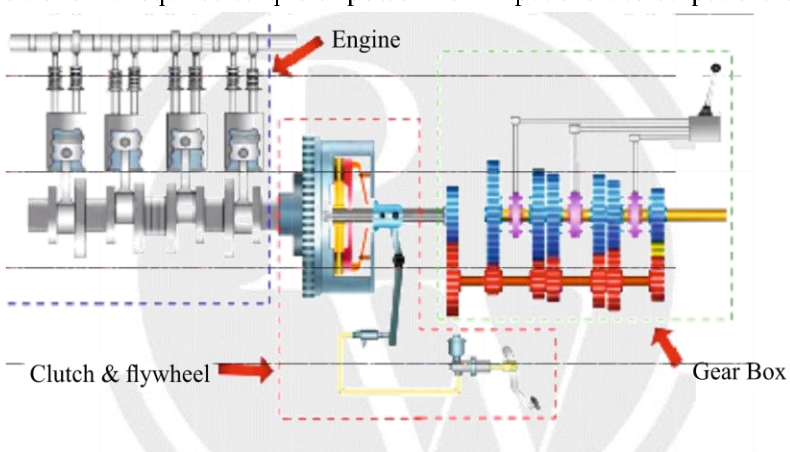

Fig.5.1: Power transmission system

5.1.1 Types of Clutch

Types of Clutch
- Positive Clutch
- Friction Clutch

(I) Positive Clutch:

- Positive engagement (no slip) due to presence of teeth. (Refer figure 5.2 and 5.3)
- High torque transmitting capacity.
- Sudden engagement.
- Cannot be engaged at high speed to avoid jerk.
- Used in power press, rolling mills etc.
- It is further classified as:

(a) Square jaw clutch: It can transmit torque in both directions.

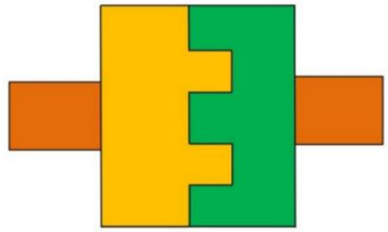

Fig.5.2: Square jaw clutch

(b) Spiral jaw clutch: It can transmit torque in one direction only.

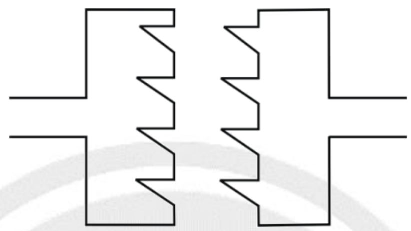

Fig.5.3: Spiral jaw clutch

(II) Friction Clutch:

- The power is transmitted by the friction between surfaces attached to the driving and driven shafts.
- Slip during engagement
- Smooth engagement
- Torque transmitting capacity depends upon axial force applied in engages position.
- Can engaged at high speed without jerk.
- Used in automobiles.

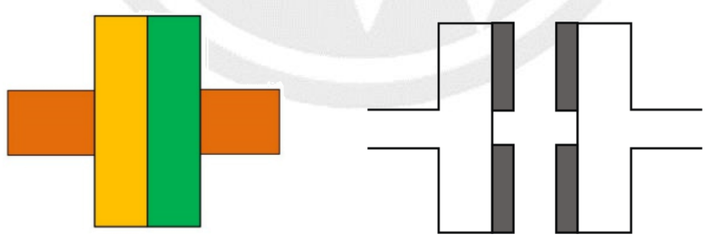

Fig.5.4: Friction clutch

Types of Friction Clutches

- Plate Clutch
  - Single Plate
  - Multi Plate Clutch
- Cone Clutch
- Centrifugal Clutch

5. 2 Plate Clutch

5. 2. 1 Introduction of Plate Clutch

Plate Clutch:

- The shape of the clutch is like a circular flat plate.
- It is an axial clutch.
- Depending upon the no. of plates on the shaft plate clutches are of two types:

  (a) Single plate clutch: Number of plates on driven shaft is 1.

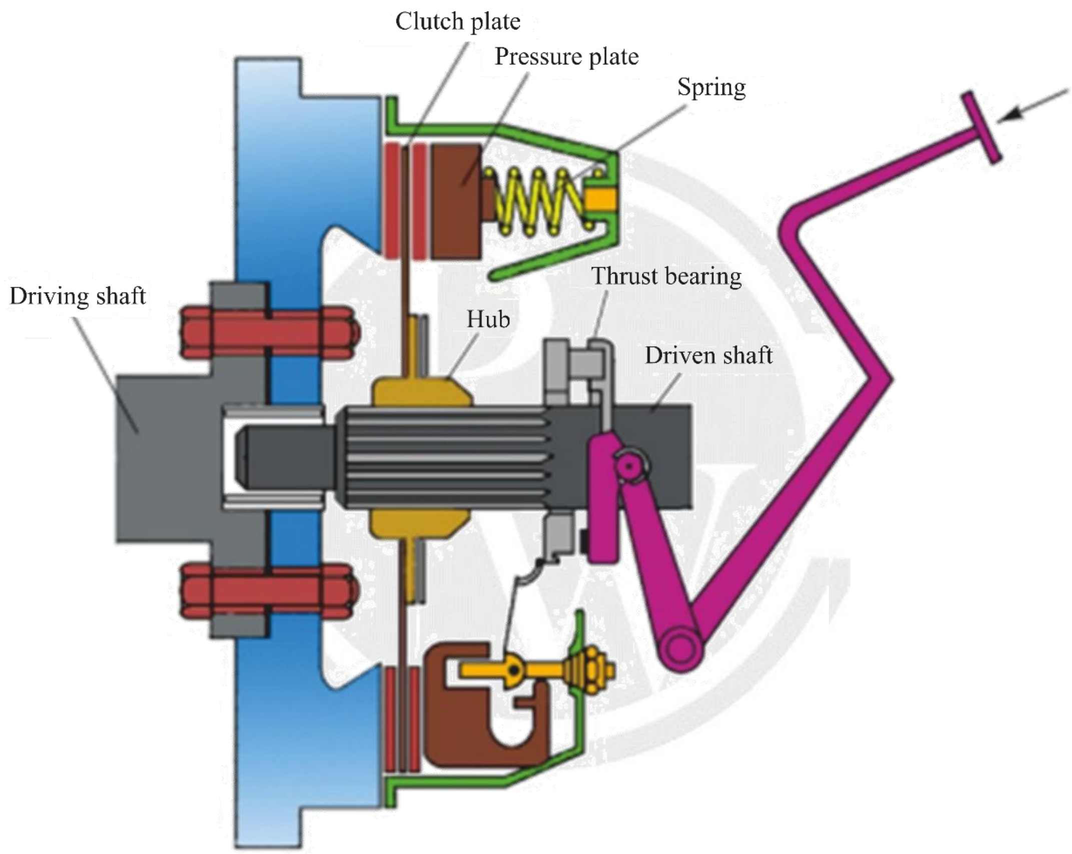

Fig.5.5: Single plate clutch

  (b) Multiplate Clutch: Multiple plates on the driving & driven shaft.

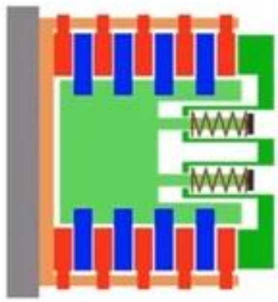

Fig.5.6: Multiple clutch

5. 2. 2 Analysis of Plate Clutch

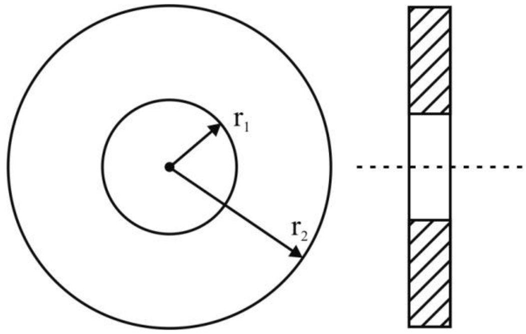

Fig.5.7: Cross-section view of the clutch

r_i = Inner radius of friction lining.

r_0 = Outer radius of friction lining.

W = Axial force on clutch at engaged position

T = Maximum torque transferred without slip.

p = Pressure on clutch plate at a radial distance r

p_{max} = Maximum pressure

μ = Coefficient of friction.

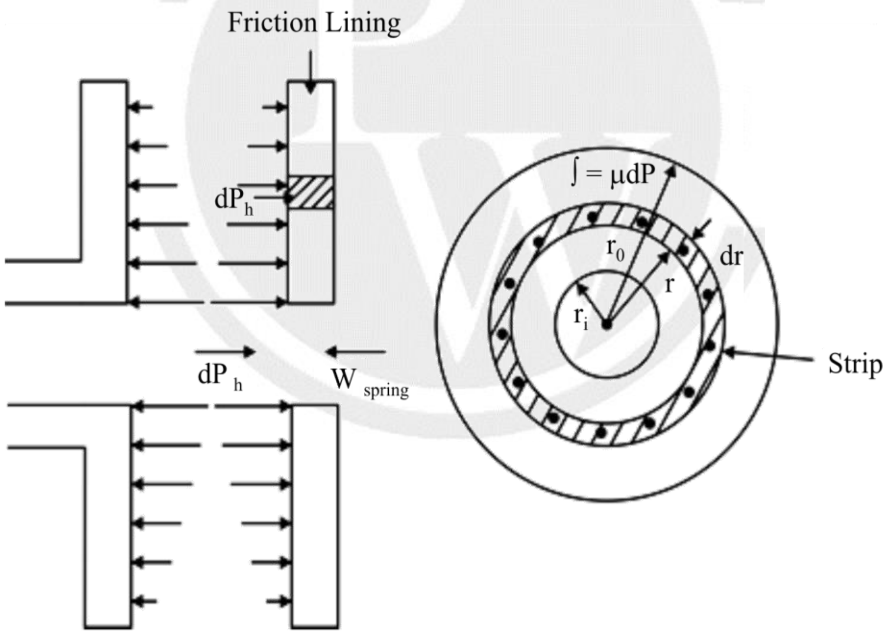

Fig.5.8: Analysis of plate clutch

(i) Total pressure (or normal) force on lining (P_n) and spring force (W)

$$W = P_n = \int_{r_i}^{r_o} p \times 2\pi r\,dr$$

(ii) Torque transmitted by clutch (T)

$$T = n\int_{r_i}^{r_o} \mu p \times 2\pi r^2 dr$$

In order to find reaction between *p* and r we have two theories:

1. Uniform Pressure theory
2. Uniform wear theory

Uniform Pressure Theory (UPT):

According to this theory if the clutch is new and there is no significant wear on the surface of the clutch, we can assume that the pressure is uniform over the area.

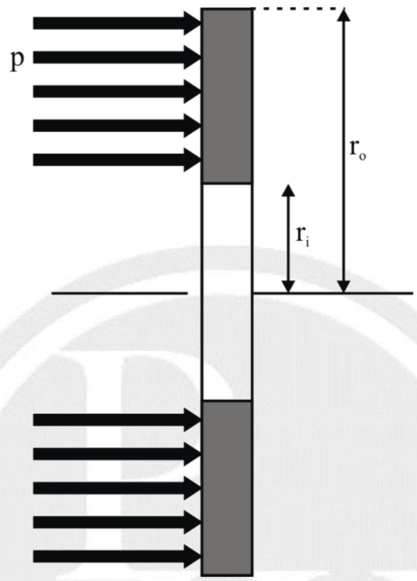

Fig.5.9: Uniform pressure

For UPT:

(i) $W = P_n = p\pi(r_o^2 - r_i^2)$

(ii) $T = n\mu W\left(\frac{2}{3}\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}\right)$

$\Rightarrow$ $T = n\mu W R_m$

Where, $R_m = \frac{2}{3}\frac{r_o^3 - r_i^3}{r_o^2 - r_o^2}$ = mean radius according to **UPT**.

Uniform Wear Theory (UWT):

According to this theory when the clutch becomes old and there is wear on the surface, then rate of wear can be assumed to be uniform.

Rate of wear = constant

Rate of wear $\propto pr$

Hence in UWT:

$pr = \text{constant} = C = p_{max} r_i$

$\Rightarrow p = \frac{C}{r}$

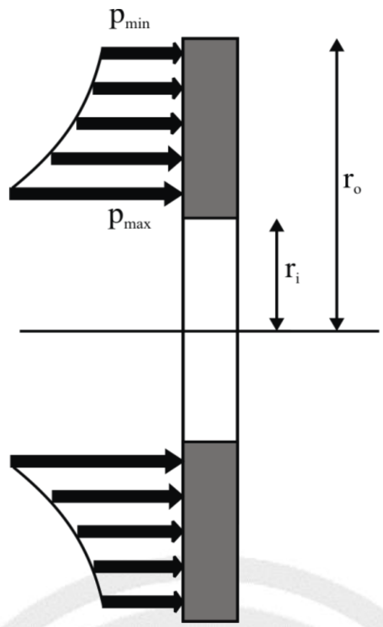

Fig.5.10: Uniform wear

(i) $W = P_n = 2\pi C(r_o - r_i)$

where, $C = pr = p_{mar} r_i$

(ii) $T = n \times \mu W\left(\frac{r_o + r_i}{2}\right) = n \times \mu W R_m$

Where, $R_m = \frac{r_o + r_i}{2}$ = mean radius according to UWT.

Note:

(i) **Number of pairs of contacting surfaces:**

Plate Clutch	n
Single plate clutch with one side effective	1
Single plate clutch with both side effective	2
Multiple Clutch	$n_1 + n_2 - 1$ where, n_1 = Number of plates on driving shaft n_2 = Number of plates on driven shaft

(ii) In plate clutch, if n (number of pairs of contacting surfaces), coefficient of friction (μ), permissible pressure and outer radius (r_o) due to space limitation are fixed, then to have maximum torque transmitting capacity inner radius (r_i) of the friction lining should be: $r_i = \frac{r_o}{\sqrt{3}}$

(iii) The only important thing to understand in friction clutches is the knowledge when to apply which theory. Generally, question will mention which theory to use but if it does not, use this to determine which theory to use:

a. If question mentions that clutch is **new**, use uniform pressure theory.

b. If question mentions that clutch is **old**, use uniform wear theory.

c. If question mentions that clutch is **worn-out**, use uniform pressure theory.

d. In questions when it is mentioned that clutch is being used for **power transmission**, use uniform wear theory.

e. If question mentions **nothing** then also use uniform wear theory.

5.2.3 Friction radius (r_f)

Hypothetical or Imaginary radius where total frictional force is assumed to be acting such that torque corresponding to this condition is same as actual torque.

Friction Radius	
UPT	**UWT**
$r_f = \frac{2}{3}\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}$	$r_f = \frac{r_o + r_i}{2}$

5.2.4 Single Plate vs Multi Plate Clutch

S. No.	Single Plate Clutch	Multi Plate Clutch	
1	Less torque transmitting capacity for same size	1.	More torque transmitting capacity for same size
2.	Less heat generation, hence no coolant is required. **(Dry clutch)**	2.	High heat generation, hence we need coolant. **(Wet clutch)**
3.	Used in big automobiles like buses, cares etc.	3.	Used in small automobiles like bikes due to limitation of size

5. 3 Cone Clutches

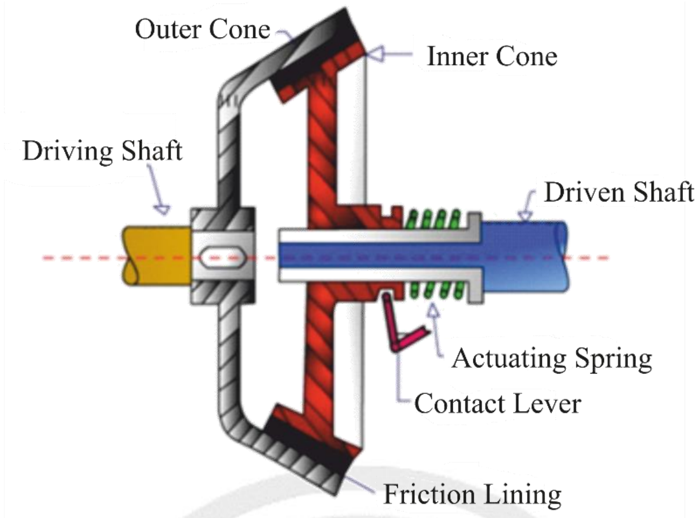

Fig.5.11: Cone clutch

5.3.1 Analysis of Cone Clutch

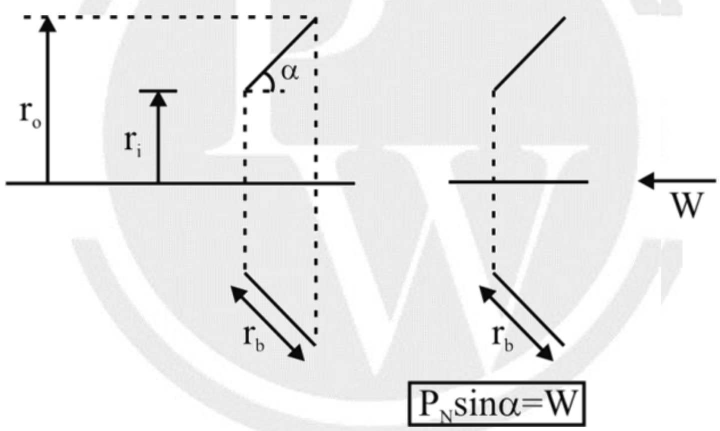

Fig.5.12: Analysis of Cone clutch

(i) Face width, $b = \dfrac{r_o - r_i}{\sin\alpha}$

(ii) Total pressure or normal force on friction lining (P_n)

$$P_n = \int_{r_i}^{r_o} p_x 2\pi r \frac{dr}{\sin\alpha} = \frac{W}{\sin\alpha}$$

(iii) Spring or axial force at engaged position (W)

$$W = \int_{r_i}^{r_o} p \times 2\pi r\, dr$$

(iv) Torque transmitted by clutch (T),

$$T = \int_{r_i}^{r_o} \mu p \times 2\pi r^2 \frac{dr}{\sin\alpha}$$

For UPT:

(i) $W = p \times \pi(r_o^2 - r_i^2)$

(ii) $P_n = \dfrac{W}{\sin\alpha}$

(iii) $T = \mu W \times \left(\dfrac{2}{3}\dfrac{r_o^3 - r_i^3}{r_o^2 - r_i^2}\right) \times \dfrac{1}{\sin\alpha}$

For UWT:

(i) $W = 2\pi C(r_o - r_i)$

where, $C = pr = P_{mar}\ r_i$

(ii) $P_n = \dfrac{W}{\sin\alpha}$

(iii) $T = \mu W\left(\dfrac{r_o + r_i}{2}\right) \times \dfrac{1}{\sin\alpha}$

5.3.2 Advantage and disadvantage of cone clutch

Advantage of Cone Clutch:

For same size (r_O & r_i), friction lining (μ); permissible pressure and effort (W_{spring}) torque transmitting capacity increases for cone clutch by factor $\left(\dfrac{1}{\sin\alpha}\right)$ in comparison to single plate clutch.

Disadvantages of Cone Clutch:

- Multi plate cone clutch is not possible
- Engaging & disengaging is not so smooth & chances of self-locking.
  $\alpha \downarrow$ changes of self-locking $\uparrow$

Note:

For smooth engagement and disengagement: $\alpha > \tan^{-1}(\mu)$

5.4 Centrifugal clutch

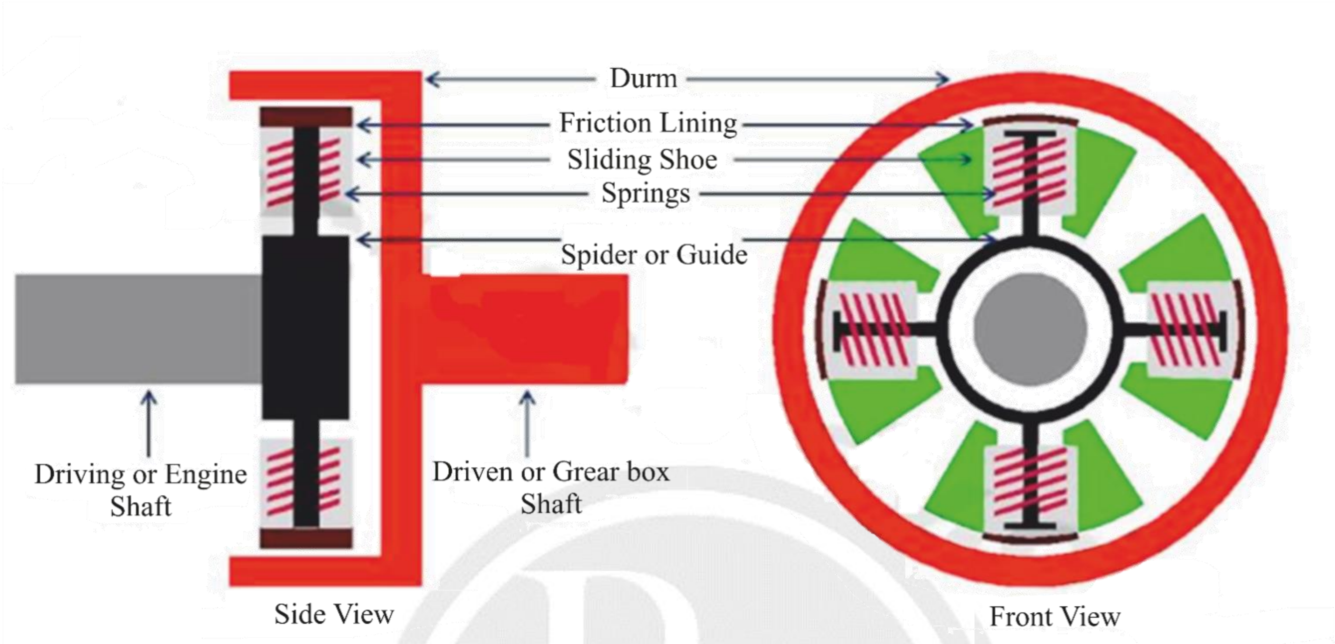

Fig.5.13: Centrifugal clutch

5.4.1 Analysis of centrifugal clutch

Key points:

- Centrifugal clutch is an automatic clutch where the clutch is engaged or disengaged automatically based on the speed of the driving shaft.
- At two speeds centrifugal force & spring force are equal, net force between drum and shoe will be zero and clutch will be in disengaged position.
- As the speed increases both centrifugal force & spring force F_c & F_s increases, at a certain speed ω_1, spring force reaches its maximum value (F_S). If the speed is increased beyond ω_1 only centrifugal force increases (F_C) & spring force remains constant at its max value (F_S). Hence there is a net force $F_C - F_S$ on each shoe which engages the clutch.
- It is a radial clutch.

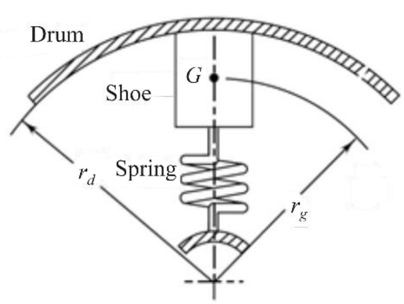

Fig.5.14: Analysis of centrifugal clutch

m = mass of each shoe (kg)

r_g = radius of the centre of gravity of the shoe in engaged position

r_d = inner radius of drum.

z = number of shoes

ω_1 = Speed at which just engagement starts

ω = Actual speed

F_S = Maximum spring force or spring force at all engaged position = centrifugal force at just engaged position $(F_{c_1} = m r_g \omega_1^2)$

$\Rightarrow F_S = F_{c_1} = m r_g \omega_1^2$

If $\omega < \omega_1 \leftarrow$ Disengaged position, and if $\omega > \omega_1 \leftarrow$ engaged position

At engaged position $(\omega > \omega_1)$:

Centrifugal force at each shoe, $F_C = m r_g \omega^2$

Spring force at each shoe, $F_S = F_{C_1} = m r_g w_1^2$

Net normal force between shoe and drum,

$F_C - F_S = m r_g (\omega^2 - \omega_1^2)$

Friction force = $\mu(F_C - F_S) = \mu m r_g (\omega^2 - \omega_1^2)$

Torque transmitted by each shoe,

$= \mu(F_C - F_S) r_d = \mu m r_g (\omega^2 - \omega_1^2) r_d$

Total torque transmitted (T)

$$\boxed{T = \mu(F_C - F_S) r_d \times z}$$

$\Rightarrow T = \mu m r_g (\omega^2 - \omega_1^2) r_d \times z$

5. 4. 2 Application of centrifugal clutch

Application:

1. Centrifugal clutches are used in light weight vehicles such as Elalf carts, move pads, lawn movers etc.
2. In centrifugal clutch the engagement is very smooth.
3. Centrifugal clutches are also used in heavy duties applications, cranes, cement mills etc. where the engine has to be started at no load condition.

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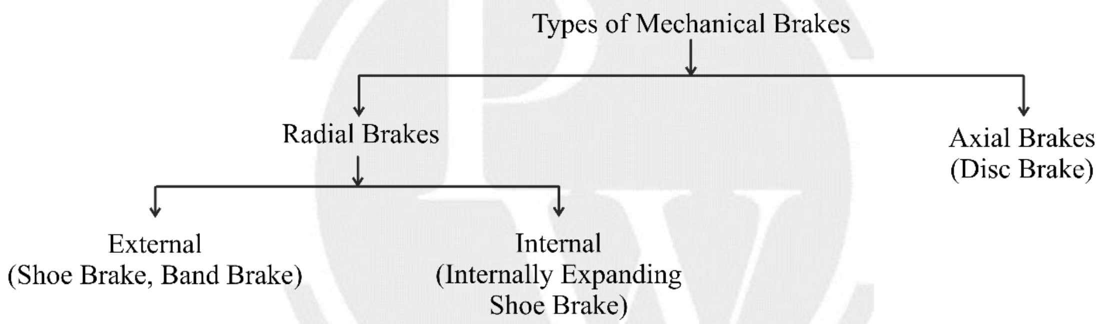
6 DESIGN OF BRAKE

6.1 Introduction of Brake

- A brake is defined as a device, which is used to absorb the energy possessed by a moving system or mechanism by means of friction to slow down or completely stop the motion of a moving system, such as a rotating drum, machine or vehicle
- Types of Brakes – Mechanical Brakes, Hydraulic Brakes, Electrical Brakes etc.
- Our study is limited to only mechanical brakes.



6.2 Shoe Brake

6.2.1 Introduction of Shoe Brake

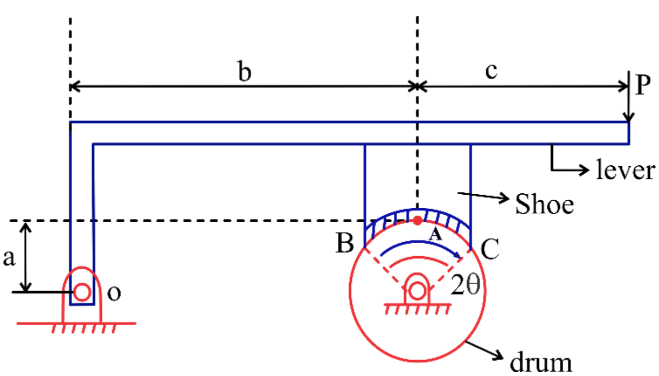


Fig. 6.1: Shoe Brake

- **Fulcrum (O):** Point at which lever is hinged.
- **Braking effort (P):** Effort required in lever to get braking action in drum.
- FBD of lever and drum.

  Shoe of the lever and drum in region BAC are contacting each other. For simplicity we can consider only point A (midpoint of region BAC) is contacting each other to make the FBD of lever and drum.
- If FBD is drawn by considering single contact point A, then use equivalent coefficient of friction (μ_e) to calculate friction force which is calculated as discussed below:

For short shoe Brake $\left(2\theta < \frac{\pi}{4}\right)$	For long shoe brake $\left(2\theta > \frac{\pi}{4}\right)$
$\mu_e = \mu$	$\mu_e = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta}$

Note:

(i) 2θ: Block angle in rad.

(ii) If block angle (2θ) is not given, then consider brake as short shoe brake.

6.2.2 Analysis of Shoe Brake

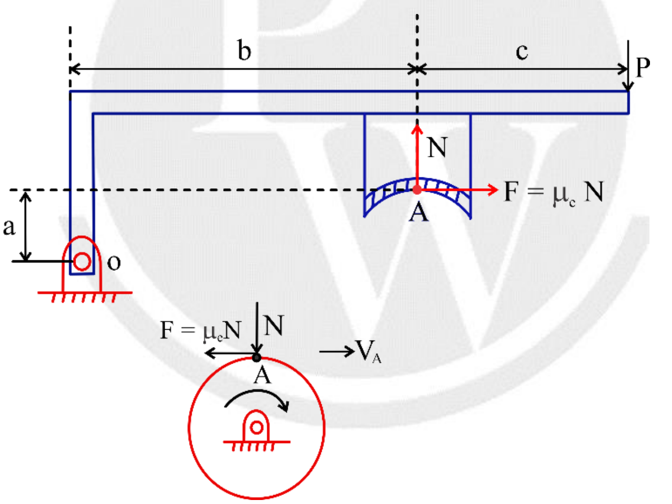


Fig. 6.2: FBD of shoe brake

Step I: First draw the FBD of drum:

Normal force (N): Normal force N in the drum will be towards centre of drum

Friction force (F = μ_eN): In the FBD of drum friction force (F = μ_eN) will be opposite to direction of velocity of point A.

Step II: Draw the FBD of lever:

In the FBD of lever contact force will be equal and opposite to the contact forces drawn on the FBD of drum.

- Braking torque (T_B)

  $$T_B = F \times r = \mu_e N \times r$$

- Relation between P & N:

  In the FBD of lever,

  $$\sum M_{fulcrum,O} = 0$$

  $$\Rightarrow P(b+c) - N(b) + \mu_e Na = 0$$

  $$\Rightarrow P = \frac{Nb - \mu_e Na}{b+c}$$

Note:

Always take the $\sum M_{fulcrum,O} = 0$ to find relation between P & N. Don't try to memorise above relation

Other important relation:

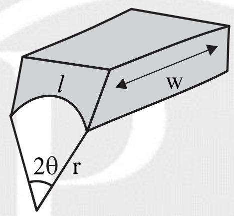


Fig. 6.3

Maximum pressure,

$$p_{max} = \frac{N}{w \times r \times 2\theta} \text{ (For long shoe brake)}$$

$$p_{max} = \frac{N}{wl} \text{ (For short shoe brake)}$$

6.2.3 Self-energising brake and Self-locking Condition

Self-energising brake:

When friction reduces the braking effort, then brake is known as self-energising brake.

Self-locking Condition:

If braking effort P become zero or negative, then brake automatically get locked with the drum and this condition is called self-locking condition.

To avoid self-locking condition, P > 0

Now we will discuss various cases of Shoe brake.

6.2.4 Line of action of friction force passes through fulcrum and drum is rotating CW

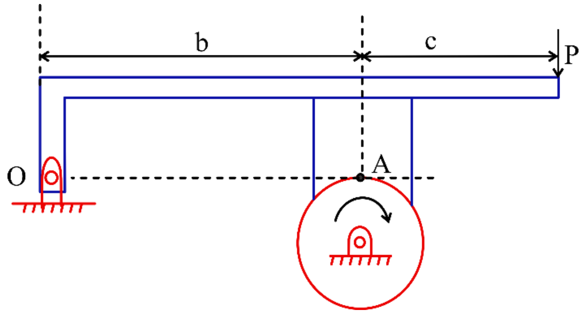


Fig. 6.4: Line of action of friction force passes through fulcrum O

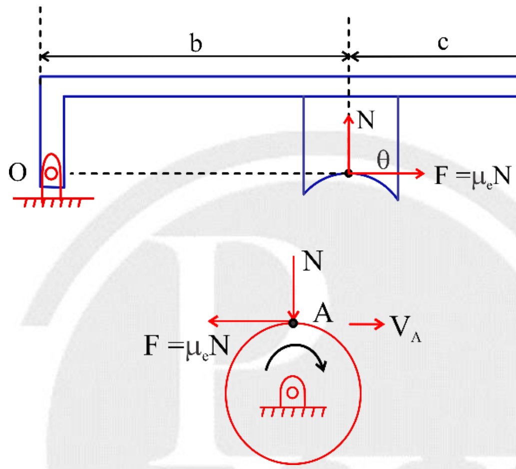


Fig. 6.5: FBD of lever and drum

In FBD of lever,

$$\sum M_O = 0 \Rightarrow P(b+c) - Nb = 0$$

$$\Rightarrow P = \frac{Nb}{b+c}$$

6.2.5 Line of action of friction is above fulcrum O and drum is rotating CW

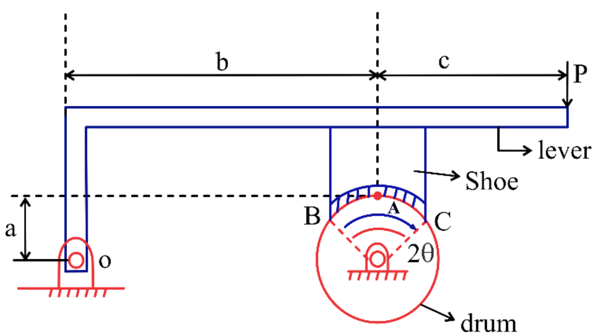


Fig. 6.6: Line of action of friction force is above fulcrum and CW drum rotation

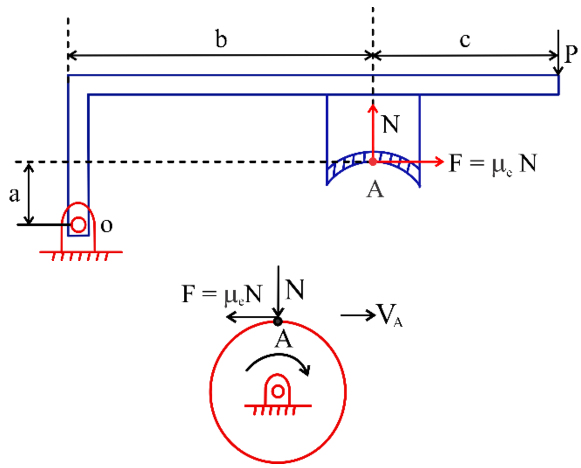


Fig. 6.7: FBD lever and drum

In the FBD of lever,

$$\sum M_{fulcrum,O} = 0$$

$$\Rightarrow P(b+c) - N(b) + \mu_e Na = 0$$

$$\Rightarrow P = \frac{Nb - \mu_e Na}{b+c}$$

Important Points:

- Since friction force is reducing braking effort, hence above brake is self-energising brake.
- If P becomes 0 or negative, then brake will be in self-locking condition (SIC),

To avoid SLC:

$$P > 0 \Rightarrow b - \mu_e a > 0$$

$$\Rightarrow b > \mu_e a$$

6.2.6 Line of action of friction is above fulcrum O and drum is rotating ACW

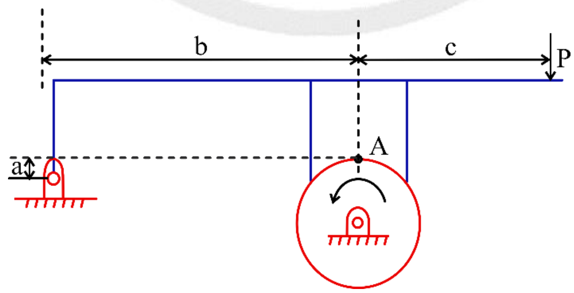


Fig. 6.8: Line of action of friction force is above fulcrum O and ACW drum rotation

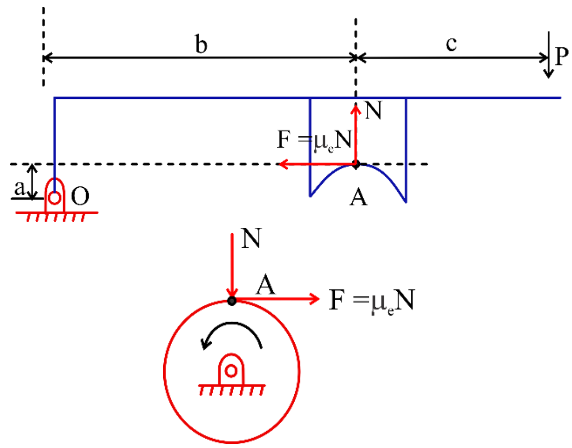


Fig. 6.9: FBD of lever and drum

In FBD of lever

$$\sum M_o = 0 \Rightarrow P(b+c) - Nb - \mu_e Na = 0$$

$$\Rightarrow P = \frac{N(b+\mu_e a)}{b+c}$$

Since friction is not reducing braking effort, hence above brake is not a self-energizing brake.

6.2.7 Line of action of friction is below fulcrum O and drum is rotating CW

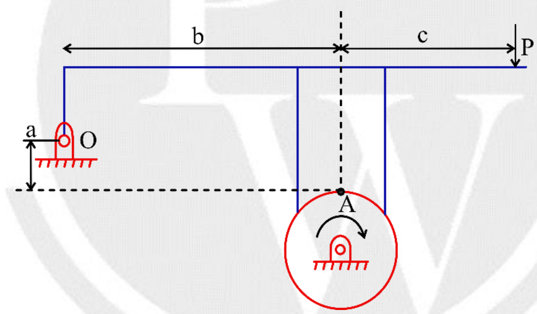


Fig. 6.10: Line of action of friction force below fulcrum and CW drum rotation

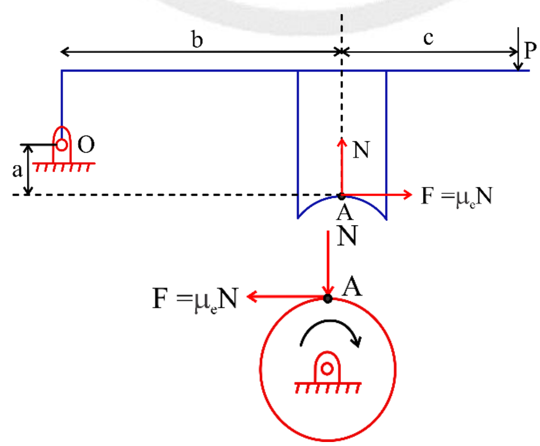


Fig. 6.11: FBD of lever and drum

$$\sum M_o = 0 \Rightarrow P(b+c) - Nb - \mu_e Na = 0$$

$$\Rightarrow P = \frac{Nb + \mu_e Na}{b+c}$$

Since friction is not reducing braking effort, hence above brake is not a self-energizing brake.

6.2.8 Line of action of friction is below fulcrum O and drum is rotating ACW

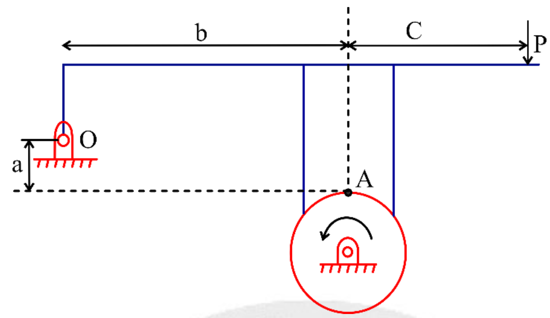


Fig. 6.12: Line of action of friction force is below fulcrum and ACW drum rotation

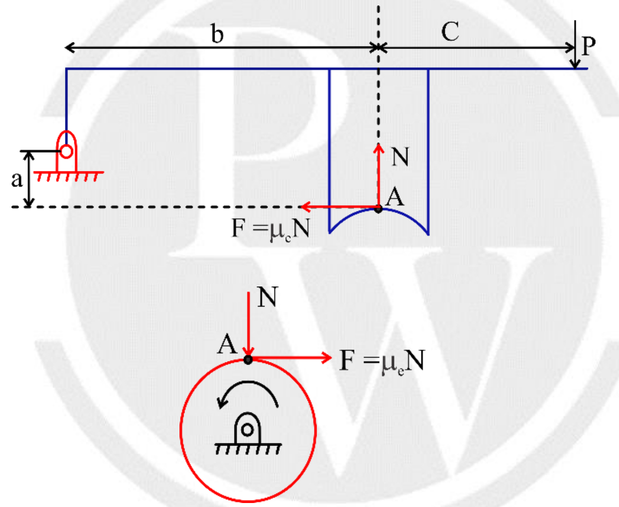


Fig. 6.13: FBD of below offset ACW drum rotation

In FBD of lever,

$$\sum M_o = 0$$

$$\Rightarrow P(b+c) - Nb + \mu_e Na = 0$$

$$\Rightarrow P = \frac{Nb - \mu_e Na}{b+c}$$

Important Points:

- Since friction force is reducing braking effort, hence above brake is self-energising brake.
- If P becomes 0 or negative, then brake will be in self-locking condition, (SIC)

To avoid SLC:

$P > 0 \Rightarrow b - \mu_e a > 0$

$\Rightarrow b > \mu_e a$

Note:

(i) If direction of moment of friction force & braking effort (P) about fulcrum O in the FBD of lever is same, then brake will act as a self-energizing brake.

(ii) If brake is not a self-energizing brake, then brake can never be in self-locking condition.

(iii) If brake is self-energizing brake then, brake may or may not be in self-locking condition.

- If $P \leq 0$ (Brake will be in self locking condition)
- If $P > 0$ (Brake will not be in self locking condition)

6.3 Band Brake

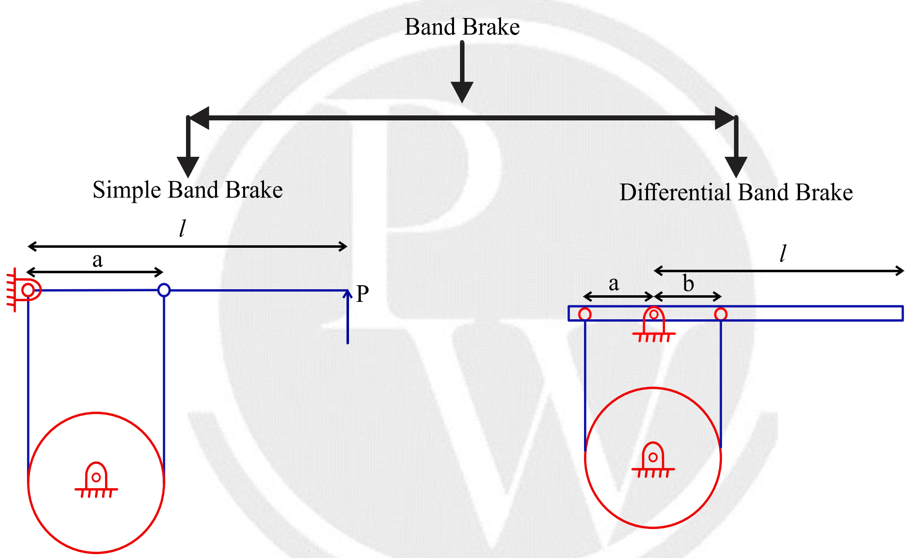


Fig. 6.14: Band brake

Identifying tight side and slack side of the band: Side opposite to the direction of drum rotation will be tight side and other side will be slack side in the FBD of drum and band **(Refer figure 6.15)**

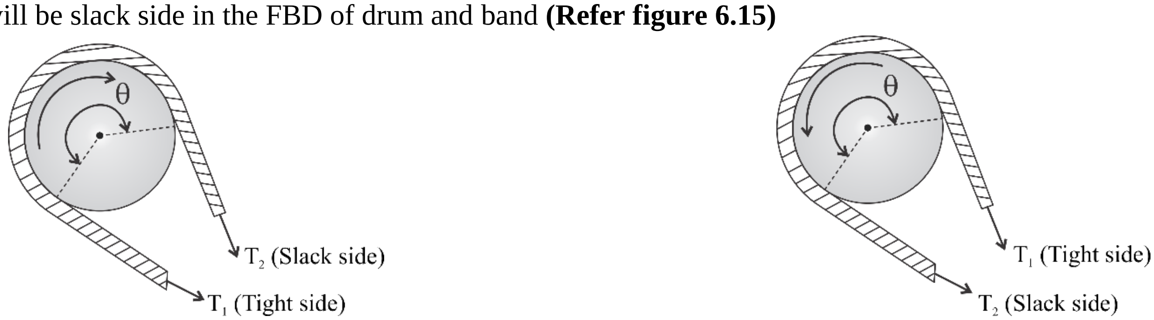


Fig. 6.15: Identifying Tight and slack side

Important Relations:

T_1 = Tension in tight side of band

T_2 = Tension in slack side of band

r = radius of drum

t = thickness of band

$r_e = r + \frac{t}{2} \approx r$ (If t is not given)

b = width of band

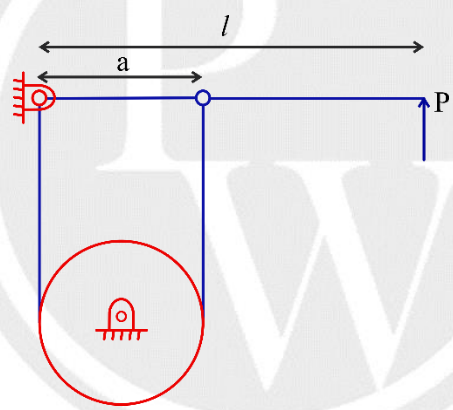
θ = angle of wrap

Braking torque (T_B): $T_B = (T_1 - T_2) r_e$

Relation between T_1 and T_2: $\frac{T_1}{T_2} = e^{\mu\theta}$

Maximum tensile stress in band: $\sigma_{t,max} = \frac{T_1}{bt}$

6.3.1 Analysis of simple band brake



For CW rotation of drum

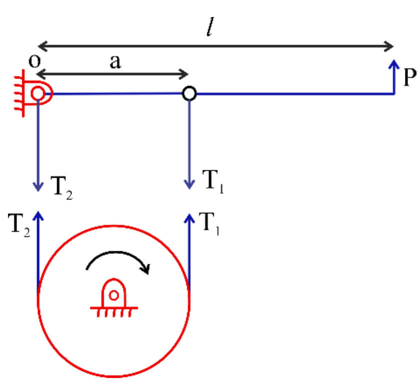


Fig. 6.16: Analysis of simple Band brake for CW rotation of drum

In FBD of lever,

$$\sum M_O = 0 \Rightarrow Pl - T_1 a = 0$$

$$\Rightarrow P = \frac{T_1 a}{l}$$

For ACW rotation of drum

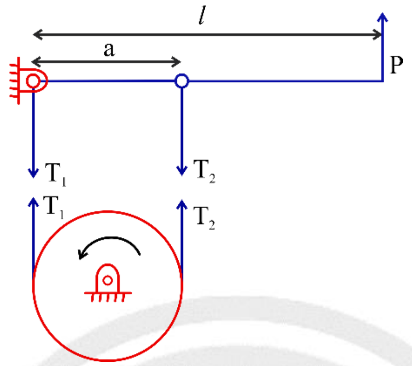


Fig. 6.17: Analysis of Simple Band brake for ACW rotation

In FBD of lever,

$$\sum M_O = 0 \Rightarrow Pl - T_2 a = 0$$

$$\Rightarrow P = \frac{T_2 a}{l}$$

6.3.2 Analysis of differential band brake

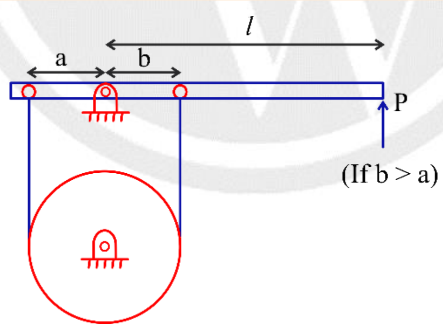


Fig. 6.18: Differential band brake (b > a)

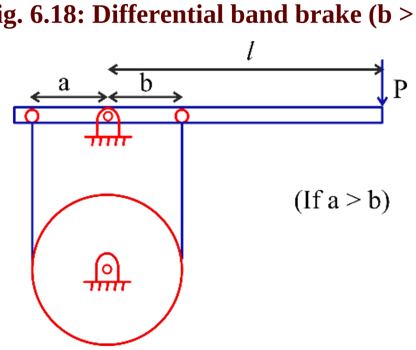


Fig. 6.19: Differential band brake (a > b)

Analysis for b > a (figure 6.18) and for CW rotation of drum:

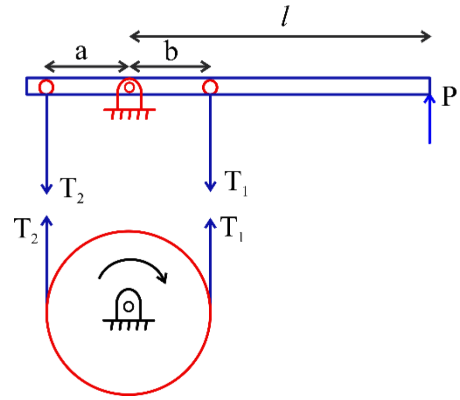


Fig. 6.20: FBD of differential band brake for CW rotation

In FBD of lever,

$$\sum M_O = 0 \Rightarrow Pl - T_1 b + T_2 a = 0$$

$$\Rightarrow P = \frac{T_1 b - T_2 a}{l}$$

Key points:

- Since $T_2 b$ part is reducing braking effort, hence above brake is self-energizing brake.
- To avoid SLC

$P > 0 \Rightarrow T_1 b > T_2 a$

$$\Rightarrow \frac{T_1}{T_2} > \frac{a}{b}$$

$$\Rightarrow e^{\mu\theta} > \frac{a}{b}$$

LHS = $e^{\mu\theta} > 1$ and since b > a therefore $RHS = \frac{a}{b} < 1$, therefore LHS (i.e. $e^{\mu\theta}$) will always be greater than RHS (i.e. $\frac{a}{b}$).

Hence, above condition will always satisfy & brake will never be in self-locking condition.

Analysis for b > a (figure 6.18) and for ACW rotation of drum

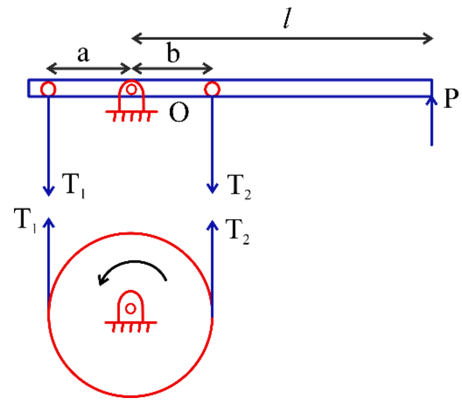


Fig. 6.21: FBD of differential band brake for ACW rotation

In FBD of lever,

$$\sum M_O = 0 \Rightarrow Pl - T_2 b + T_1 a = 0$$

$$\Rightarrow P = \frac{T_2 b - T_1 a}{l}$$

Key Points

- Since T_1a part is reducing braking effort therefore above brake is self-energizing brake.
- To avoid SLC,

  $P > 0 \Rightarrow T_2b - T_1a > 0$

  $$\Rightarrow \frac{T_2}{T_1} > \frac{a}{b} \Rightarrow \frac{T_1}{T_2} < \frac{b}{a}$$

❑❑❑

7 DESIGN OF ROLLING & SLIDING CONTACT BEARING

7.1 Introduction of Bearing

Bearings are machine elements that support a moving element (shaft or axle) with minimum friction.

(a) Function of bearings-

- Support the load actiing on shaft.
- Permit rotation of shaft with minimum friction.

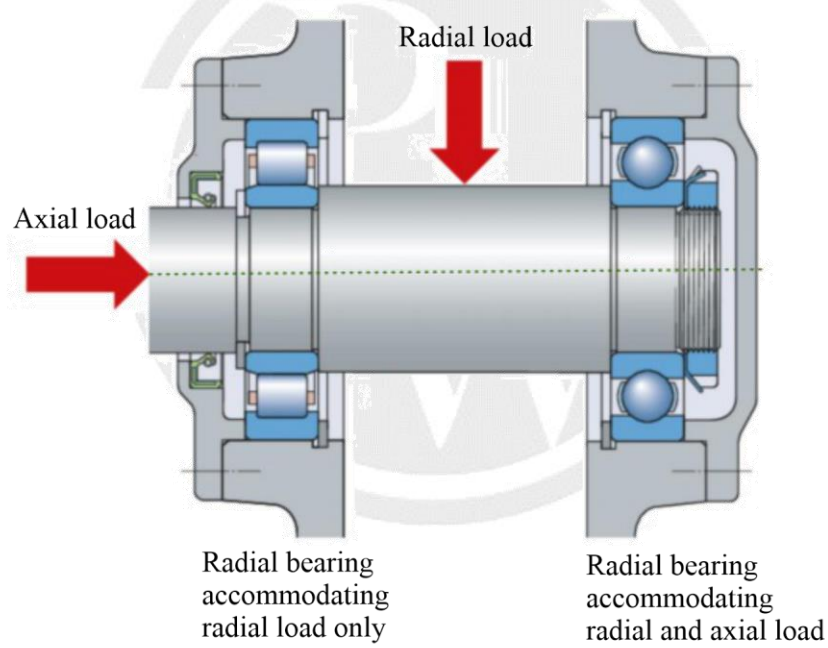


Fig. 7.1

Types of Bearings

- **Rolling Contact Bearings**
- **Sliding Contact Bearings**

7.2 Rolling Contact Bearings

- Rolling motion between the fixed and moving surfaces.
- Low starting and running friction, hence they are also known as Antifriction Bearings.
- More noise at very high speeds.
- More initial cost.
- Used in machine tool spindles, automobile front, and rear axles, gearbox, and small-size electric motors.
- In rolling contact bearings, the shafts are supported on the bearing surface through rolling elements such as balls and rollers.
- The relative motion between the shaft and bearing surface is rolling motion hence the friction is very low
- At low speeds the friction is negligible hence these bearings are also called as anti-friction bearings.

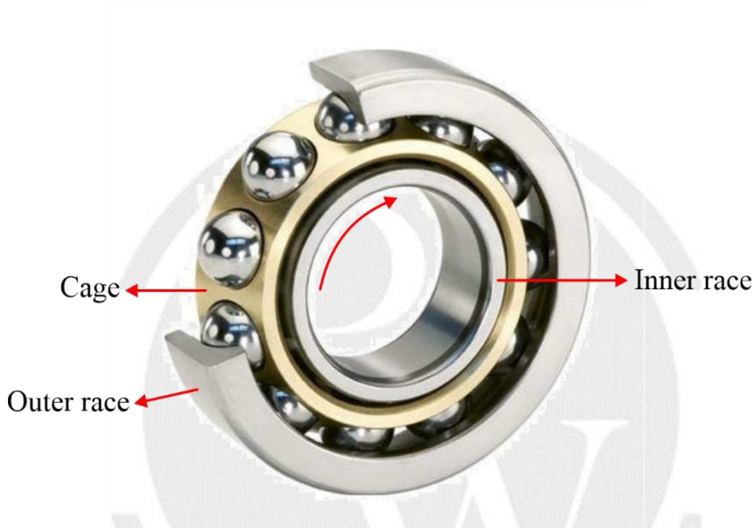


Fig. 7.2: Rolling contact bearing

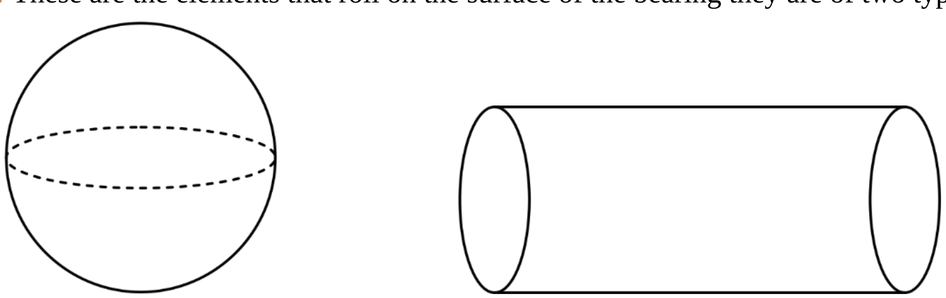
7.2.1 Construction of rolling contact bearing

(1) Inner Race: It is a circular ring connected to the shaft through interference fit it rotates with the shaft.

(2) Outer Race: It is a circular ring connected to the fix support through interference if it remains stationary.

(3) Cage: It is used to hold the rolling elements together so that the distance between them is maintained.

(4) Rolling Elements: These are the elements that roll on the surface of the bearing they are of two types:



(a) Ball **(b) Rollers**

Fig. 7.3: Rolling elements of rolling contact bearing bearing

- In case of balls less contact area, less friction & less load and it is point contact and bearing is known as ball bearing.
- In case of rollers more contact area, more load carrying capacity and more friction and it is line contact and bearings is known as roller bearing.

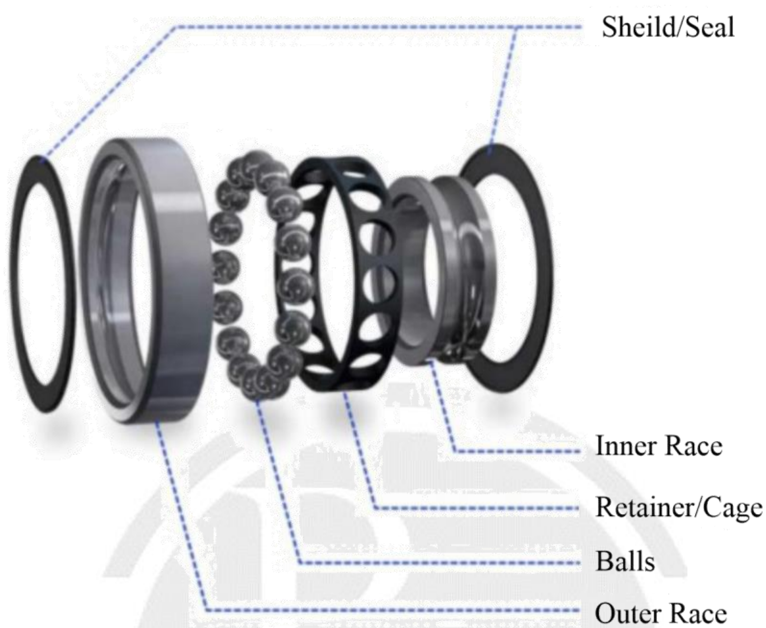


Fig. 7.4: Parts of rolling contact bearing

7.2.2 Types of Rolling Contact Bearings:

(1) Ball Bearing:

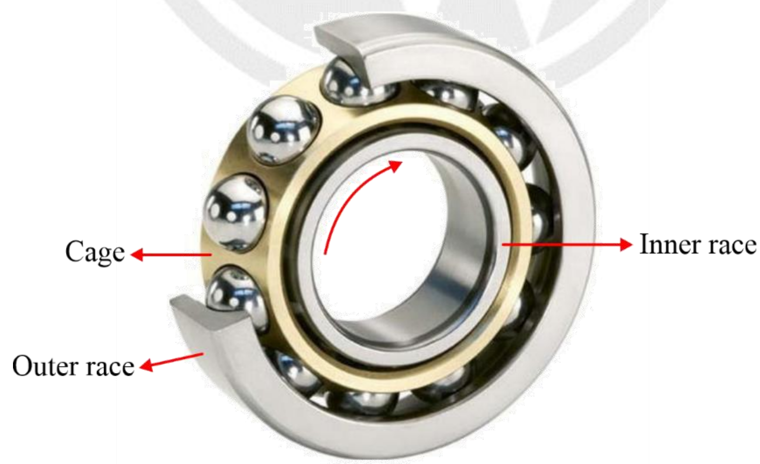


Fig. 7.5: Ball bearing

- Point contact
- Less load carrying capacity
- Less friction

(2) Roller Bearing:

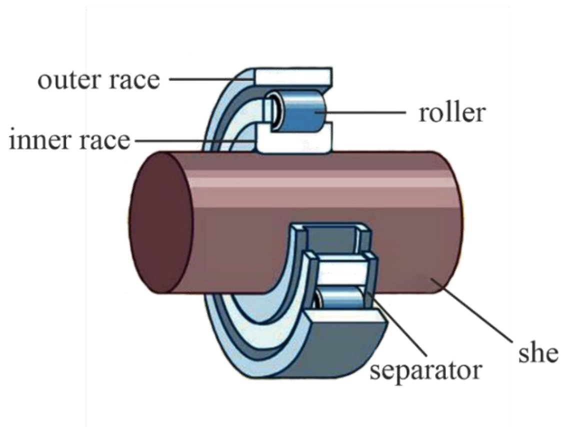


Fig. 7.6: Roller bearing

- Line contact
- High load carrying capacity
- High friction

7.2.3 Types of Ball Bearings:

(1) Deep Groove Ball Bearing:

- These are the most frequently used ball bearings

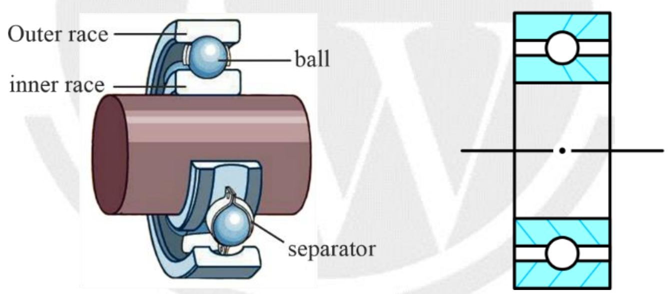


Fig. 7.7: Deep groove ball bearing

- Due to the depth of grooves balls are constrained in axial direction hence they can support radial as well as axial loads.
- Compared to axial loads they can support large radial loads.
- They have poor rigidity compared to roller bearing.

(2) Angular contact Ball Bearing:

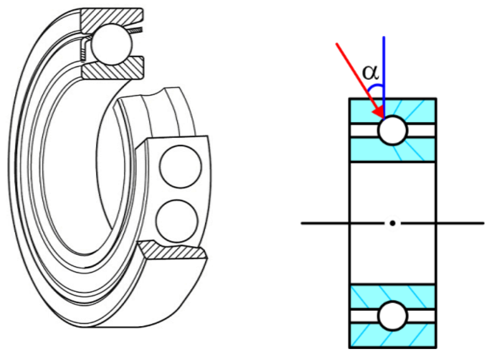


Fig. 7.8: Angular contact Ball Bearing

- In angular contact bearings the grooves in inner and outer race are so shaped that the line of reaction at the contact between the balls and races makes an angle with axis of bearing.
- This reaction has a radial and axial component therefore these bearings can support radial as well as axial roads.
- These bearings can only support axial load in one direction. Hence, they are often used in pair so that they can support axial load in both directions.

(3) Self-Aligning Ball Bearing:

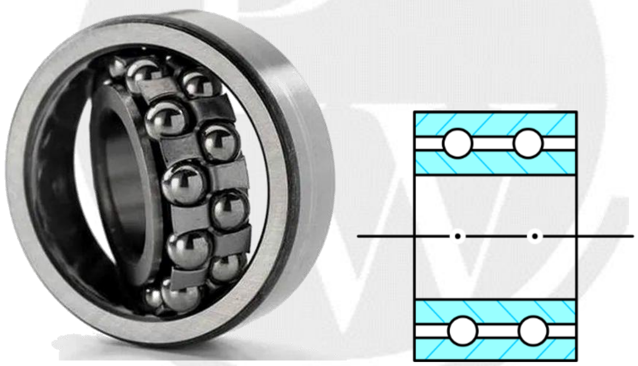


Fig. 7.9: Self Aligning Ball Bearing

- Self-aligning bearings are used where the axis of shaft and bearing can be misaligned due to tolerance or excessive deformation.

(4) Thrust Ball Bearing:

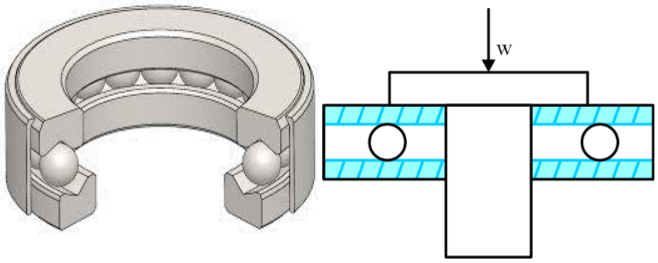


Fig. 7.10: Thrust Ball Bearing

7.2.4 Types of Roller Bearings

(1) Cylindrical Roller Bearings:

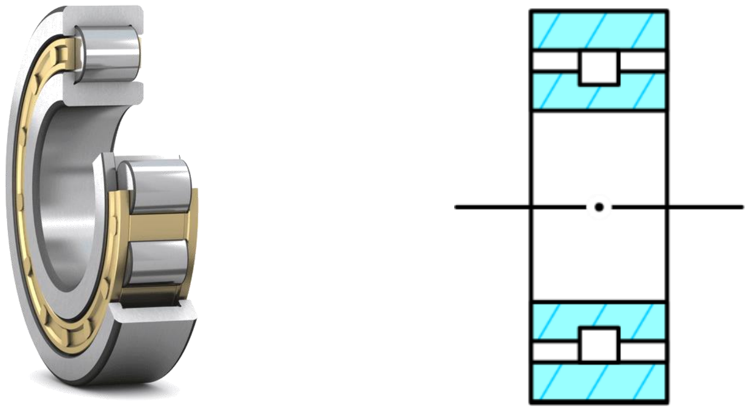


Fig. 7.11: Cylindrical roller bearing

- Cylindrical roller bearings can only support radial loads
- Due to the line contact between the rollers and races they have high radial load carrying capacity.
- These bearings are more rigid than ball bearings

(2) Needle Bearing:

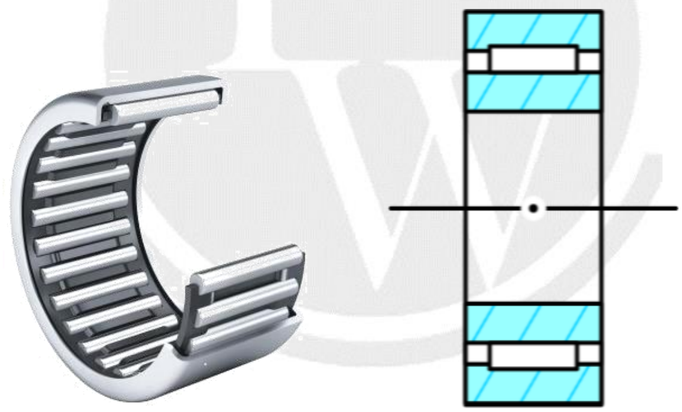


Fig. 7.12: Needle bearing

- If length of rolling element (l) is >>> than diameter of roller (d) $\left[\text{Generally if } \frac{l}{d} > 4\right]$, then that rolling contact bearing is known as Needle Bearing.
- Used where radial space is less
- Can support oscillating loads.

(3) Tapered roller bearing:

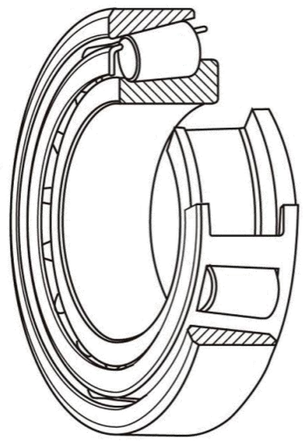


Fig. 7.13: Tapered roller bearing

- The line of reaction between the rollers and the races make an angle with the axes of bearings hence these bearings can support radial as well as axial loads.

(4) Self aligning roller bearing:

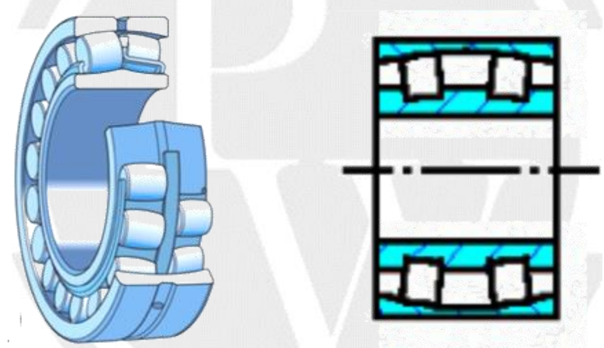


Fig. 7.14: Self aligning roller bearing

- Self- aligning roller bearings are used where the axis of shaft and bearing can be misaligned due to tolerance or excessive deformation.

7. 3. Design of rolling contact bearing

7. 3.1 Static Load Carrying Capacity

- Static load is the load acting on the bearing when the shaft is stationary.
- It produces permanent deformation in balls and races, which increases with increasing load.
- It is defined as the static load which corresponds to a total permanent deformation of balls and races, at the most heavily stressed point of contact, equal to 0.0001 of the ball diameters.

$$C_0 \frac{k.d^2 n}{5}$$

k ⟶ constant which depends on material
d ⟶ diameter of ball
n ⟶ number of balls
C_0 ⟶ static load carrying capacity

7.3.2 Life of Bearing

- Life of a bearing is defined as the number of revolutions that the bearing can make before it fails. It is denoted in million revolutions.

Life at reliability R(L_R):

Life at reliability R(L_R) at any load P means at least R percent of bearing will sustain stated life under load P.

(a) Rated life ($L_{90\%}$ or L): Life for 90% Reliability

(b) Average life ($L_{50\%}$ or L_{av}): Life for 50% reliability.

Relationship between life at different reliablity under any load

$$\frac{L_{R_2}}{L_{R_1}} = \left[\frac{\ln\left(\frac{1}{R_2}\right)}{\ln\left(\frac{1}{R_1}\right)}\right]^{1/1.17}$$

Relation between average life ($L_{50\%}$) and rated life ($L_{90\%}$): $L_{50\%} = 5L_{90\%}$

Relation between life in hours and life in mR:

$$\text{Life in } mR = \frac{\text{Life in } hr \times 60 \times N}{10^6}$$

where N = speed of the shaft in rpm.

7.3.3 Equivalent Load (P)

- The equivalent dynamic load is defined as the constant radial load in radial bearings (or thrust load in thrust bearings), which if applied to the bearing would give same life as that which the bearing will attain under actual condition of forces.

$$P = XVF_r + YF_a$$

Where,

F_r = radial load

F_a = axial load

X = radial load factor

Y = Axial load factor

V = Race rotation factor

= 1 (when inner race rotates)

= 1.2 (when outer race rotates)

7.3.4 Relation between equivalent load and rated life

$$\frac{L_2}{L_1} = \left(\frac{P_1}{P_2}\right)^k$$

Where, L_2 = rated life under load P_2

L_1 = rated life under load P_1

k = 3 for ball bearing and k = $\frac{10}{3}$ for roller bearing.

7.3.5 Dynamic Load Carrying Capacity:

- It is the equivalent load at which the rated life of bearing is one million revolutions.

Relation between life (L) corresponds to load P and dynamic load carrying capacity (C):

$$L = \left(\frac{C}{P}\right)^k$$

Where, L = rated life corresponds to equivalent load P

C = dynamic load carrying capacity

7.3.6 Cyclic loading in rolling contact bearing

Loads	Rpm	%time	Revolutions in 1 min.
P_1	N_1	α_1	$n_1 = \alpha_1 N_1$
P_2	N_2	α_2	$n_2 = \alpha_2 N_2$
P_3	N_3	α_3	$n_3 = \alpha_3 N_3$
			$N_e = \sum n$

Equivalent speed in rpm = $N_e = \sum n$.

Equivalent load $P = \left[\frac{P_1^k n_1 + P_2^k n_2 + ...}{n_1 + n_2 + ...}\right]^{1/k}$

Where k = 3 for ball bearing and $\frac{10}{3}$ for roller bearing

7.4 Designation of Bearings:

P – Q – R – S

Here,

- P = Type of bearing:
  0, 1, 6, 8 – ball bearing
  2, 3, 4, 5, 7, 9 – roller bearing

- Q = Indicate series:
  1 – Extra light 2 – Light
  3 – Medium 4 – Heavy
- RS = when multiply by 5, it gives shaft diameter in mm.

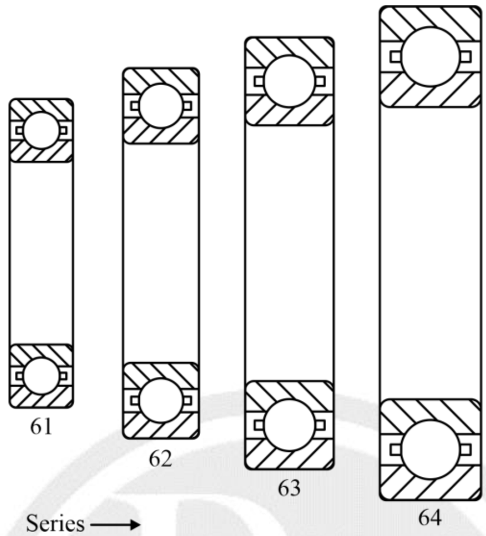


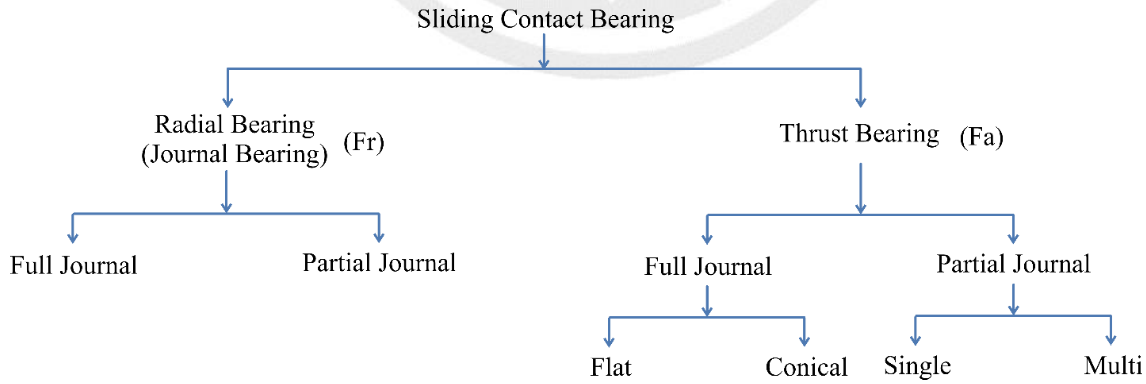
Fig. 7.15: Designation of bearing

- Due to large size balls, load carrying capacity is high.

7. 5 Introduction of sliding contact bearings

- Relative motion between the shaft and the support is sliding motion.
- Due to sliding motion friction, wear and heat generation is very high, hence lubrication is required.

Applications: Crankshaft bearings in petrol and diesel engines, centrifugal pumps, large size electric motors steam and gas turbines.



7.6 Types of Sliding contact bearing:

7.6.1 Journal Bearings

(1) Full Journal Bearing

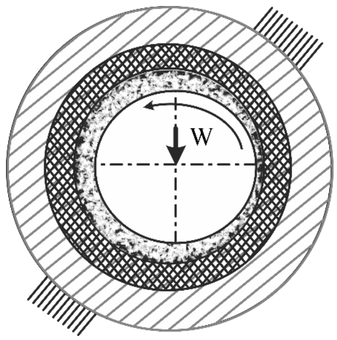


Fig. 7.16: Full journal bearing

- These bearings can support load in any direction
- This journal bearing is fully covered.

(2) Partial Journal Bearing

- These bearings can support radial load in one direction.
  Ex: Axles of Railway wagons

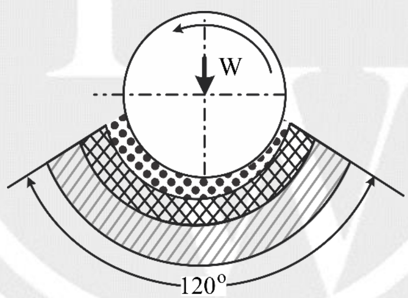


Fig. 7.17: Partial journal bearing

7.6.2 Thrust Bearings

(1) Pivot Bearings

The end of the shaft is in contact with bearing surface.

(a) Flat pivot Bearing:

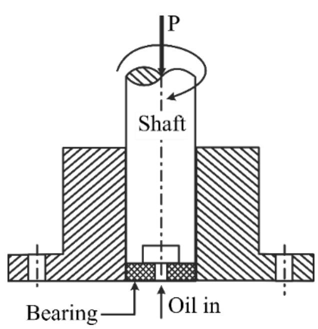


Fig. 7.18: Flat pivot bearing

(b) Conical pivot Bearing: It has more contact area so more friction and has less pressure.

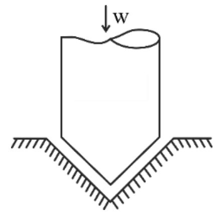


Fig. 7.19: Conical pivot Bearing

(2) Collar Bearings

The shaft is supported by the collars which are in contact with bearings

(a) **Single Collar:**

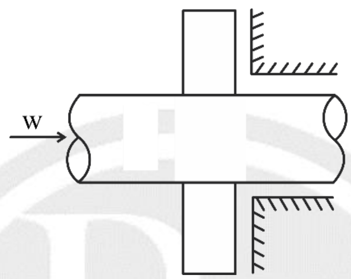


Fig. 7.20: Collar bearing

(b) **Multi Collar:**

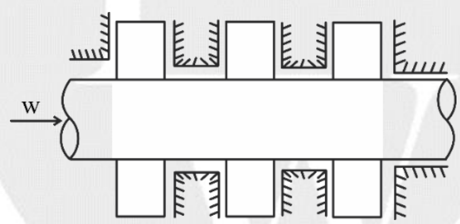
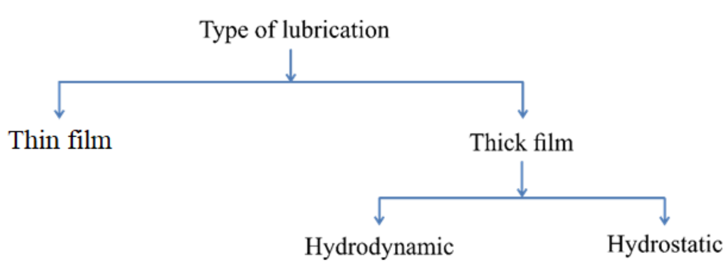


Fig. 7.21: Multi collar bearing

7.7 Lubrication in sliding contact bearing

7.7.1 Types of Lubrication



(1) Thin Film Lubrication:

- There is partial contact between journal bearing surface as the thickness of oil is very less
- It is used where the load & the speed of the journal is low.

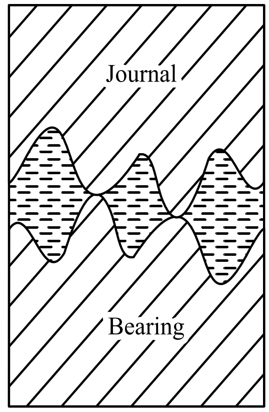


Fig. 7.22: Thin film lubrication

(2) Thick Film lubrication

- Due to thick film of lubricating oil there is no contact between journal & bearing surface
- It is used where the load & the speed of the journal is high

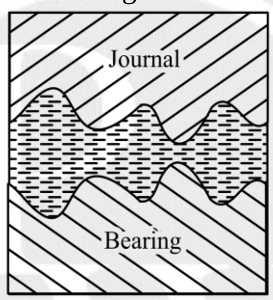


Fig. 7.23: Thick Film lubrication

(a) Hydrodynamic Lubrication:

In Hydro dynamic lubrication, the lubrication is done by the speed of journal. Initially at low speeds of journal there is a partial contact between the surface as the journal starts to rotate a concurring film is developed in the direction of motion where the pressure starts to build and at certain speed the journal is completely separated from the surface.

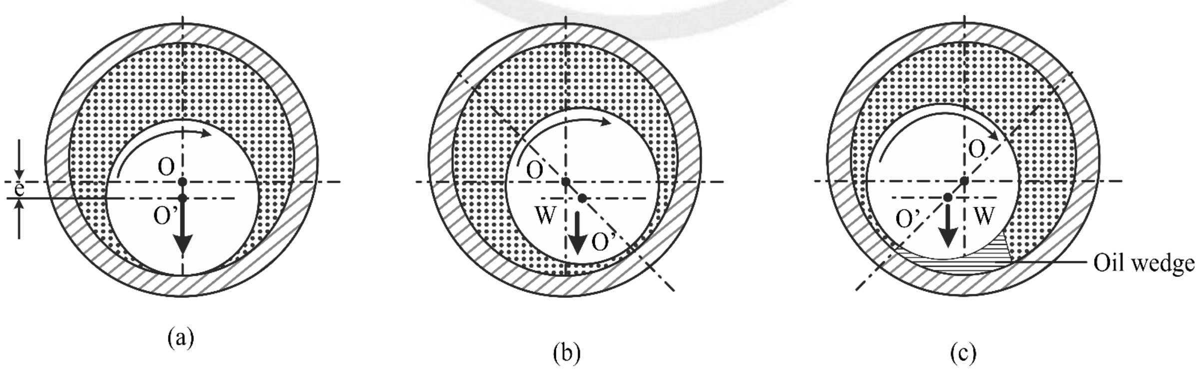


Fig. 7.24: Hydrodynamic Lubrication

Fig. (a): Journal is at rest: Full contact between journal and bearing.
Fig. (b): Journal starts to rotate: Partial contact between journal and bearing.
Fig. (c): Journal rotating at high speed: No contact between journal and bearing.

(b) Hydrostatic Lubrication

In hydrostatic lubrication, the lubricating oil is pressurised by external source as the speed of the journal is not enough.

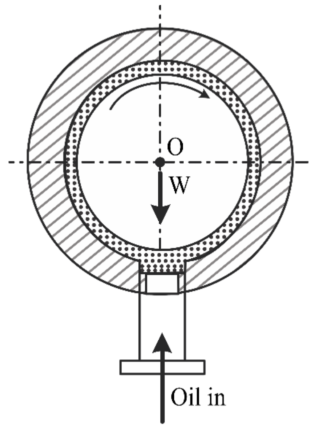


Fig. 7.25: Hydrostatic Lubrication

7.8 Analysis of Hydrodynamic Journal Bearing

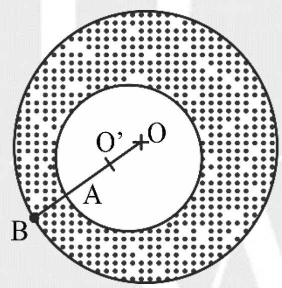


Fig. 7.26: Hydrodynamic journal bearing

R = Radius of Bearing = OB

D = Diameter of Bearing = 2R

r = radius of shaft / journal = O'A

d = diameter of shaft / journal = 2r

L = Length of Bearing

ω = speed of shaft in rad/s $= \dfrac{2\pi N}{60}$

N = speed of shaft in rpm

n_s = speed of shaft in rps $= \dfrac{N}{60}$

W = Radial load in bearing

p = Bearing pressure

z = dynamic viscosity of lubricant in Pa – s

μ_e = Equivalent coefficient of friction

e = eccentricity = OO'

- **Radial Clearance (C_R):** $C_R = R - r$
- **Diametral Clearance:** $C_D = D - d = 2C_R$
- **Minimum Oil thickness (h_0):**

  OB = OO' + O'A + AB

  $\Rightarrow R = e + r + h_0$

  $\Rightarrow h_0 = R - r - e$

  $\Rightarrow h_0 = C_R - e$
- **Eccentricity ratio (ε)**

  $$\varepsilon = \frac{e}{C_R} = \frac{C_R - h_0}{C_R} = 1 - \frac{h_0}{C_R}$$
- **Bearing pressure (p):** $p = \frac{W}{Ld}$
- **Equivalent coefficient of friction (μ_e) as per Petroff's equation:**

  $$\mu_e = 2\pi^2 \left(\frac{zn_s}{p}\right)\left(\frac{r}{C_R}\right)$$

  Where, μ_e = equivalent coefficient of friction
- **Bearing characteristic number (BCN):** $BCN = \frac{zn_s}{p}$
- **Frictional torque loss due to friction:** $T_f = \mu_e w_r$
- **Power Loss:** $\omega T_f = 2\pi n_s T_f = \frac{2\pi N}{60} T_f$
- **Heat Generation rate (Hg):** H_g = Power loss
- **Some field number (S):** $S = \left(\frac{zn_s}{p}\right)\left(\frac{r}{C_R}\right)^2$

7.9 McKee's Investigation Curve

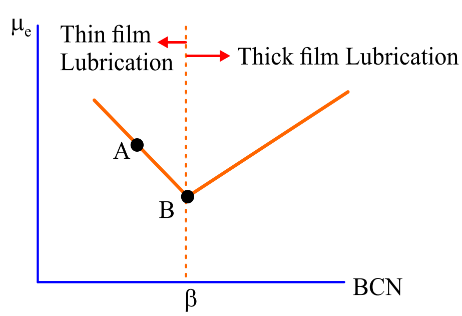


Fig. 7.27: McKee's Curve

- Bearing modulus **(β):** BCN for least coefficient of friction is known as bearing modulus.
- If we keep BCN at B, any reduction in velocity will shift the point toward A. This will increase μ_e, which will increase Temperature. Increase in temperature will reduce viscosity z, which will reduce BCN. Reduction in BCN will shift point more towards left side of A. This will keep happening until shaft will come in contact with the bearing. So BCN is not kept near B.
- Generally, BCN is kept between 3 β to 15 β.

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8 DESIGN OF GEAR

8.1 Introduction of spur gear

Spur Gear

- Used to connect parallel shafts.
- Teeth are parallel to axis.
- Due to sudden engagement, noise and vibration at high speed.
- Only radial load, no axial load.

Fig.8.1: Spur gear

8.2 Force analysis in spur gear

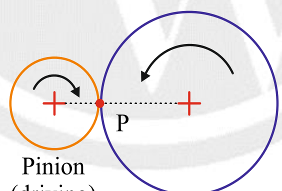

Fig.8.2: Spur gear line diagram

Consider,

m = module of gear

T_p = Number of teeth on pinion

T_w = Number of teeth of wheel

R_p = pitch circle radius of pinion $= \frac{mT_p}{2}$

R_w = pitch circle radius of wheel $= \frac{mT_w}{2}$

Φ = pressure angle

G = Gear ratio $= \frac{T_w}{T_p}$

N_p = Speed of pinion in rpm

ω_p = Speed of pinion in rad/s $= \frac{2\pi N_p}{60}$

N_w = Speed of wheel in rpm

ω_w = Speed of wheel in rad/s $= \frac{2\pi N_w}{60}$

F_t = tangential component of force

F_r = radial component of force

F_n = Normal force

M_{tp} = Torque in pinion

M_{tw} = Torque transmitted to wheel

- Since the driving gear rotates due to some external force its tangential component of force is opposite in direction of rotation
- In driven gear tangential component is in the same direction of rotation.
- F_r is drawn from pitch point towards centre of gear

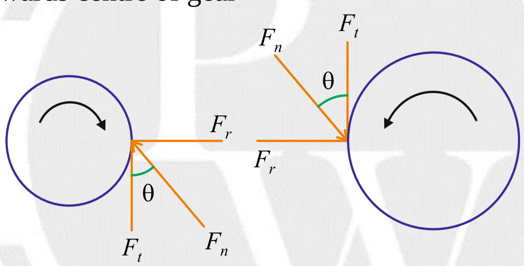

Fig.8.3: Force components on spur gear

- $F_t = F_n \cos\phi$
- $F_r = F_n \sin\phi = F_t \tan\phi$
- $M_{tp} = F_t R_p$
- $M_{tw} = F_t R_w$
- $\frac{N_p}{N_w} = \frac{T_w}{T_p}$
- $(\text{Power})_p = (\text{Power})_w$

$$\Rightarrow \omega_p M_{tp} = \omega_w M_{tw}$$

$$\Rightarrow \frac{2\pi N_p}{60} M_{tp} = \frac{2\pi N_w}{60} M_{tw}$$

$$\Rightarrow N_p M_{tp} = N_w M_{tw}$$

8.3 Design of spur gear

8. 3. 1 Beam strength of gear

Beam Strength:

Beam strength of a Spur gear is maximum value of tangential force that the teeth of Gear can sustain without failure.

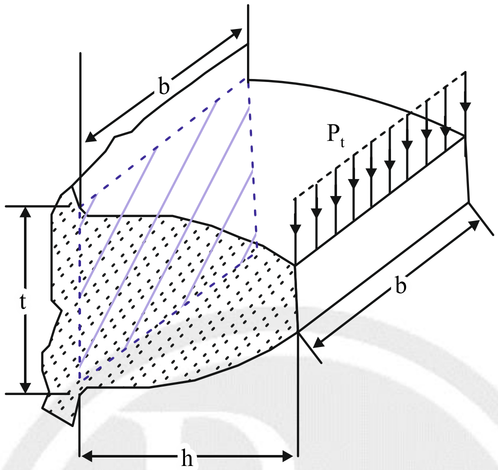

Fig.8.3: Spur gear tooth

Assumptions:

(a) Axial compressive stress due to F_N which is negligible

(b) F_T is uniformly distributed over the width

(c) Only one pair of teeth are in contact at a time

(d) F_T is static

Lewis Formula:

$$P_t = mb\sigma_b Y \frac{K_1}{K_2}$$

Where,

m = module
b = face width of gear
σ_b = bending stress in gear tooth
Y = Lewis form factor
K_1 = product of factors which are less than 1
K_2 = product of factor which are greater than 1
If no factors are given then take K_1 & K_2 = 1

For Design

- Out of pinion and gear, design by using the gear which is weaker.
- If material of both wheel and pinion is same, then pinion will always be weaker.

- If material of wheel and pinion is different, calculate product of permissible bending stress and Lewis form factor for pinion $\left[\left(\sigma_{b,p}Y\right)_p\right]$ and wheel $\left[\left(\sigma_{b,p}Y\right)_w\right]$

  If $\left(\sigma_{b,p}Y\right)_p < \left(\sigma_{b,p}Y\right)_w$, then pinion will be weaker.

  If $\left(\sigma_{b,p}Y\right)_p > \left(\sigma_{b,p}Y\right)_w$, then wheel will be weaker.

- For weaker gear during design, put σ_b = permissible bending stress of that gear $\left(\sigma_{b,p}\right)$, then we will get the expression of beam strength:

  $$P_t = mb\sigma_{b,p}Y\frac{K_1}{K_2}$$

8. 3. 2 Wear strength of spur gear (S_w)

Consider,

D_p = pitch circle diameter of pinion,
D_w = pitch circle diameter of wheel,
b = face width
E_p = Young Modulus of pinion,
E_w = Young Modulus of wheel,
σ_{es} = surface endurance limit
BHN = Brinell hardness Number

Wear strength of spur gear (S_w)

$$S_w = D_p bQK$$

Where,
D_p = pitch circle diameter of pinion
b = face width

Q = ratio factor $= \frac{2G}{G \pm 1} = \frac{2D_w}{D_w \pm D_p}$ [+ sign is used for external gearing and – sign is used for internal gearing]

$$K = \text{Load Stress Factor} = \frac{\sigma_{es}^2 \sin\phi\left[\frac{1}{E_p} + \frac{1}{E_w}\right]}{1.4} = 0.16\left(\frac{BHN}{100}\right)^2$$

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9 DESIGN OF SPRING

9.1 Introduction

A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed.

The important functions and applications of springs are:

- to absorb shocks and vibrations e.g. vehicle suspension to store energy, e.g. clocks, toys
- to control motion by maintaining contact between two elements e.g. cams and followers
- to measure force, weighing balances and scales.

9. 2 Helical Spring

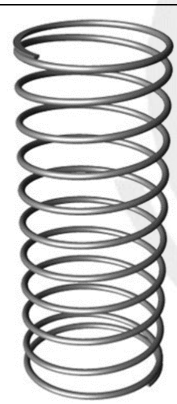

Fig.9.1: Open coil Helical spring

Fig.9.2: Close Coil Helical spring

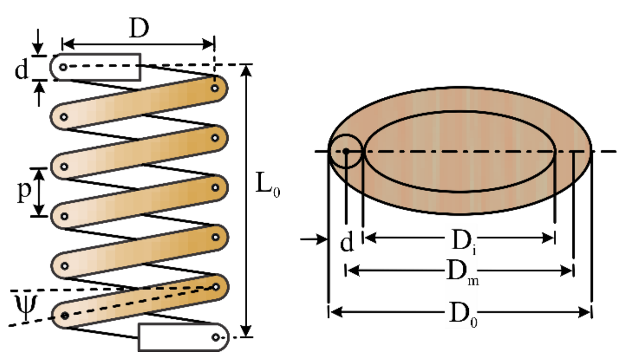

Fig.9.3: Helical spring

In the helical spring shown in figure 9.1.

$p =$ Pitch,

$L_o =$ Free length of spring,

$\Psi =$ Coil angle or helix angle,

$n =$ Number of actives coils,

$d =$ Wire diameter,

$D_0 =$ Outer diameter of coil,

$D_i =$ Inner diameter of coil and

$D_m = D =$ Mean diameter of coil $= 2R$,

$C =$ Spring index $= \dfrac{D}{d}$

Note:

- If $\Psi < 10^\circ \rightarrow$ Close coil helical spring
- If $\Psi > 10^\circ \rightarrow$ Open coil helical spring

9. 2. 1 Analysis of close coil Helical spring

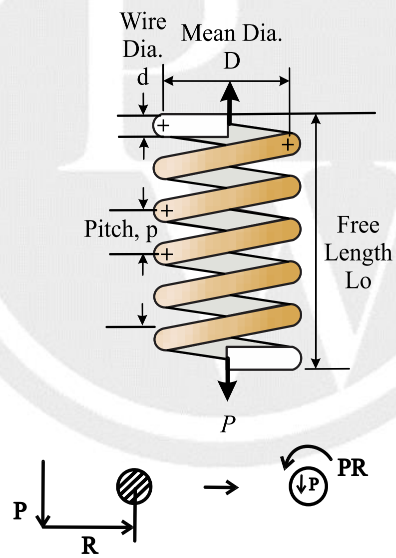

Fig.9.4: Analysis of close coil Helical spring

(i) Maximum Shear Stress (τ_{max})

- $\tau_{max} = \dfrac{16PR}{\pi d^3} + \dfrac{4P}{\pi d^2}$

  [Neglecting Curvature effect]

- $\tau_{max} = \dfrac{16PR}{\pi d^3}$

  [Neglecting curvature effect and effect of direct shear]

- $\tau_{max} = k_w \frac{16PR}{\pi d^3}$

  [Considering Curvature effect and effect of primary shear] where, k_w is wharl's correction factor.

  $$k_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

(ii) Stiffness of spring (k)

$$k = \frac{Gd^4}{64R^3n}$$

(iii) Deflection of spring (δ)

$$\delta = \frac{P}{k} = \frac{64PR^3n}{Gd^4}$$

(iv) Strain energy (U)

$$U = \frac{1}{2}P\delta = \frac{1}{2}k\delta^2 = \frac{32P^2R^3n}{Gd^4}$$

9.3 Spring in series and parallel

Case 1: Spring are in series:

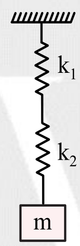

Fig.9.5: Spring in series

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$$

Case 2: Springs are in parallel:

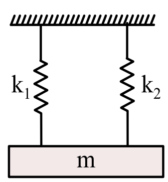

Fig.9.6: Spring in parallel

$$k_e = k_1 + k_2$$

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