RD Sharma Solutions Class 10 Maths Chapter 5 Exercise 5.1: Chapter 5 of RD Sharma's Class 10 Maths focuses on Trigonometric Ratios, introducing students to fundamental trigonometric concepts based on right-angled triangles. Exercise 5.1 specifically covers the basic trigonometric ratios - sine, cosine, tangent, cotangent, secant, and cosecant—defined as ratios of various sides of a right triangle (opposite, adjacent, and hypotenuse).

This exercise teaches students how to use these ratios to relate angles to side lengths, laying the groundwork for solving more complex trigonometric problems. It also involves understanding reciprocal relationships between ratios and solving questions to build confidence in applying these concepts effectively.

RD Sharma Solutions Class 10 Maths Chapter 5 Exercise 5.1 Overview

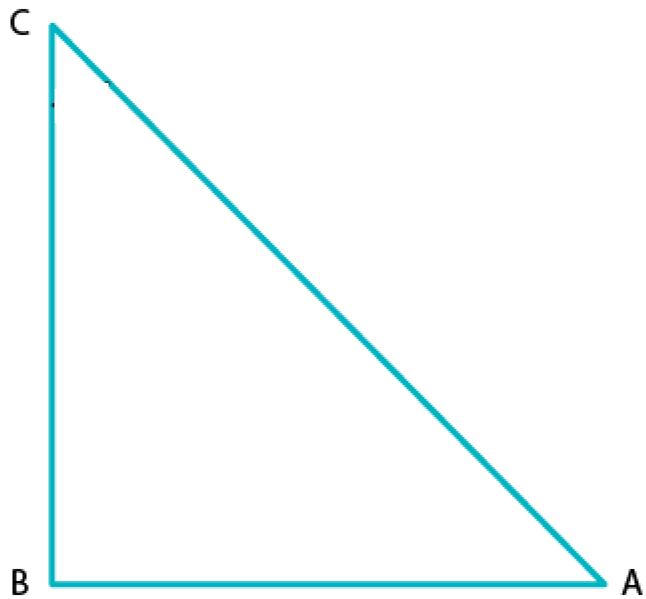
Chapter 5, Exercise 5.1 in RD Sharma's Class 10 Maths covers the basics of trigonometric ratios, focusing on fundamental concepts like sine, cosine, and tangent functions based on right-angled triangles. This exercise is crucial because understanding trigonometric ratios is foundational for higher-level mathematics, including calculus, geometry, and physics.

By practicing these problems, students learn how to relate angles to side lengths, allowing them to solve real-life applications involving heights, distances, and angles. Mastery of these concepts strengthens analytical thinking and problem-solving skills, setting a solid base for advanced trigonometry and various applications in science and engineering.

RD Sharma Solutions Class 10 Maths Chapter 5 Exercise 5.1 Trigonometric Ratios

Below is the RD Sharma Solutions Class 10 Maths Chapter 5 Exercise 5.1 Trigonometric Ratios -

1. In each of the following, one of the six trigonometric ratios s given. Find the values of the other trigonometric ratios.



(i) sin A = 2/3

Solution:

We have,

sin A = 2/3 ………... (1)

As we know, by sin definition,

sin A = Perpendicular/ Hypotenuse = 2/3 ….(2)

By comparing eq. (1) and (2), we have

Opposite side = 2 and Hypotenuse = 3

Now, on using Pythagoras theorem in Δ ABC

$AC^2 = AB^2 + BC^2$

Putting the values of perpendicular side (BC) and hypotenuse (AC) and for the base side as (AB), we get

$\Rightarrow 3^2 = AB^2 + 2^2$

$AB^2 = 3^2 - 2^2$

$AB^2 = 9 - 4$

$AB^2 = 5$

$AB = \sqrt{5}$

Hence, Base = $\sqrt{5}$

By definition,

cos A = Base/Hypotenuse

$\Rightarrow \cos A = \sqrt{5}/3$

Since, cosec A = 1/sin A = Hypotenuse/Perpendicular

$\Rightarrow \operatorname{cosec} A = 3/2$

And, sec A = Hypotenuse/Base

$\Rightarrow \sec A = 3/\sqrt{5}$

And, tan A = Perpendicular/Base

$\Rightarrow \tan A = 2/\sqrt{5}$

And, cot A = 1/ tan A = Base/Perpendicular

$\Rightarrow \cot A = \sqrt{5}/2$

(ii) cos A = 4/5

Solution:

We have,

cos A = 4/5 ……….. (1)

As we know, by cos definition,

cos A = Base/Hypotenuse …. (2)

By comparing eq. (1) and (2), we get

Base = 4 and Hypotenuse = 5

Now, using Pythagoras' theorem in Δ ABC,

$AC^2 = AB^2 + BC^2$

Putting the value of base (AB) and hypotenuse (AC) and for the perpendicular (BC), we get

$5^2 = 4^2 + BC^2$

$BC^2 = 5^2 – 4^2$

$BC^2 = 25 – 16$

$BC^2 = 9$

BC= 3

Hence, Perpendicular = 3

By definition,

sin A = Perpendicular/Hypotenuse

⇒ sin A = 3/5

Then, cosec A = 1/sin A

⇒ cosec A= 1/ (3/5) = 5/3 = Hypotenuse/Perpendicular

And, sec A = 1/cos A

⇒ sec A =Hypotenuse/Base

sec A = 5/4

And, tan A = Perpendicular/Base

⇒ tan A = 3/4

Next, cot A = 1/tan A = Base/Perpendicular

∴ cot A = 4/3

(iii) tan θ = 11/1

Solution:

We have, tan θ = 11……… (1)

By definition,

tan θ = Perpendicular/ Base…. (2)

On Comparing eq. (1) and (2), we get;

Base = 1 and Perpendicular = 5

Now, using Pythagoras' theorem in Δ ABC,

$AC^2 = AB^2 + BC^2$

Putting the value of base (AB) and perpendicular (BC) to get hypotenuse(AC), we get

$AC^2 = 1^2 + 11^2$

$AC^2 = 1 + 121$

$AC^2 = 122$

AC= √122

Hence, hypotenuse = √122

By definition,

sin = Perpendicular/Hypotenuse

⇒ sin θ = 11/√122

And, cosec θ = 1/sin θ

⇒ cosec θ = √122/11

Next, cos θ = Base/ Hypotenuse

⇒ cos θ = 1/√122

And, sec θ = 1/cos θ

⇒ sec θ = √122/1 = √122

And, cot θ = 1/tan θ

∴ cot θ = 1/11

(iv) sin θ = 11/15

Solution:

We have, sin θ = 11/15 ………. (1)

By definition,

$\sin \theta$ = Perpendicular/ Hypotenuse …. (2)

On Comparing eq. (1) and (2), we get,

Perpendicular = 11 and Hypotenuse= 15

Now, using Pythagoras' theorem in Δ ABC,

$AC^2 = AB^2 + BC^2$

Putting the value of perpendicular (BC) and hypotenuse (AC) to get the base (AB), we have

$15^2 = AB^2 + 11^2$

$AB^2 = 15^2 - 11^2$

$AB^2 = 225 - 121$

$AB^2 = 104$

$AB = \sqrt{104}$

$AB = \sqrt{(2\times2\times2\times13)}$

$AB = 2\sqrt{(2\times13)}$

$AB = 2\sqrt{26}$

Hence, Base = $2\sqrt{26}$

By definition,

$\cos \theta$ = Base/Hypotenuse

$\therefore \cos\theta = 2\sqrt{26}/ 15$

And, $\operatorname{cosec} \theta = 1/\sin \theta$

$\therefore \operatorname{cosec} \theta = 15/11$

And, $\sec\theta$ = Hypotenuse/Base

$\therefore \sec\theta = 15/ 2\sqrt{26}$

And, $\tan \theta$ = Perpendicular/Base

$\therefore \tan\theta = 11/ 2\sqrt{26}$

And, cot θ = Base/Perpendicular

∴ cotθ =2√26/ 11

(v) tan α = 5/12

Solution:

We have, tan α = 5/12 …. (1)

By definition,

tan α = Perpendicular/Base…. (2)

On Comparing eq. (1) and (2), we get

Base = 12 and Perpendicular side = 5

Now, using Pythagoras' theorem in Δ ABC,

$AC^2 = AB^2 + BC^2$

Putting the value of base (AB) and the perpendicular (BC) to get hypotenuse (AC), we have

$AC^2 = 12^2 + 5^2$

$AC^2 = 144 + 25$

$AC^2= 169$

AC = 13 [After taking sq root on both sides]

Hence, Hypotenuse = 13

By definition,

sin α = Perpendicular/Hypotenuse

∴ sin α = 5/13

And, cosec α = Hypotenuse/Perpendicular

∴ cosec α = 13/5

And, cos α = Base/Hypotenuse

∴ cos α = 12/13

And, sec α =1/cos α

∴ sec α = 13/12

And, tan α = sin α/cos α

∴ tan α=5/12

Since, cot α = 1/tan α

∴ cot α =12/5

(vi) sin θ = √3/2

Solution:

We have, sin θ = √3/2 …………. (1)

By definition,

sin θ = Perpendicular/ Hypotenuse….(2)

On Comparing eq. (1) and (2), we get

Perpendicular = √3 and Hypotenuse = 2

Now, using Pythagoras' theorem in Δ ABC,

$AC^2 = AB^2 + BC^2$

Putting the value of perpendicular (BC) and hypotenuse (AC) and get the base (AB), we get

$2^2 = AB^2 + (\sqrt{3})^2$

$AB^2 = 2^2 – (\sqrt{3})^2$

$AB^2 = 4 – 3$

$AB^2 = 1$

AB = 1

Thus, Base = 1

By definition,

cos θ = Base/Hypotenuse

∴ cos θ = 1/2

And, cosec θ = 1/sin θ

Or cosec θ= Hypotenuse/Perpendicualar

∴ cosec θ =2/√3

And, sec θ = Hypotenuse/Base

∴ sec θ = 2/1

And, tan θ = Perpendicula/Base

∴ tan θ = √3/1

And, cot θ = Base/Perpendicular

∴ cot θ = 1/√3

(vii) cos θ = 7/25

Solution:

We have, cos θ = 7/25 ……….. (1)

By definition,

cos θ = Base/Hypotenuse

On Comparing eq. (1) and (2), we get;

Base = 7 and Hypotenuse = 25

Now, using Pythagoras' theorem in Δ ABC,

$AC^2 = AB^2 + BC^2$

Putting the value of base (AB) and hypotenuse (AC) to get the perpendicular (BC),

$25^2 = 7^2 + BC^2$

$BC^2 = 25^2 – 7^2$

$BC^2 = 625 – 49$

$BC^2 = 576$

BC= $\sqrt{576}$

BC= 24

Hence, Perpendicular side = 24

By definition,

sin θ = perpendicular/Hypotenuse

∴ sin θ = 24/25

Since, cosec θ = 1/sin θ

Also, cosec θ= Hypotenuse/Perpendicualar

∴ cosec θ = 25/24

Since, sec θ = 1/cosec θ

Also, sec θ = Hypotenuse/Base

∴ sec θ = 25/7

Since, tan θ = Perpendicular/Base

∴ tan θ = 24/7

Now, cot = 1/tan θ

So, cot θ = Base/Perpendicular

∴ cot θ = 7/24

(viii) tan θ = 8/15

Solution:

We have, tan θ = 8/15 …………. (1)

By definition,

tan θ = Perpendicular/Base …. (2)

On Comparing eq. (1) and (2), we get

Base = 15 and Perpendicular = 8

Now, using Pythagoras' theorem in Δ ABC,

$AC^2 = 15^2 + 8^2$

$AC^2 = 225 + 64$

$AC^2 = 289$

$AC = \sqrt{289}$

AC = 17

Hence, Hypotenuse = 17

By definition,

Since, sin θ = perpendicular/Hypotenuse

∴ sin θ = 8/17

Since, cosec θ = 1/sin θ

Also, cosec θ = Hypotenuse/Perpendicular

∴ cosec θ = 17/8

Since, cos θ = Base/Hypotenuse

∴ cos θ = 15/17

Since, sec θ = 1/cos θ

Also, sec θ = Hypotenuse/Base

∴ sec θ = 17/15

Since, cot θ = 1/tan θ

Also, cot θ = Base/Perpendicular

∴ cot θ = 15/8

(ix) cot θ = 12/5

Solution:

We have, cot θ = 12/5 …………. (1)

By definition,

$\cot \theta = 1/\tan \theta$

$\cot \theta$ = Base/Perpendicular ……. (2)

On Comparing eq. (1) and (2), we have

Base = 12 and Perpendicular side = 5

Now, using Pythagoras' theorem in Δ ABC,

$AC^2 = AB^2 + BC^2$

Putting the value of base (AB) and perpendicular (BC) to get the hypotenuse (AC),

$AC^2 = 12^2 + 5^2$

$AC^2 = 144 + 25$

$AC^2 = 169$

$AC = \sqrt{169}$

AC = 13

Hence, Hypotenuse = 13

By definition,

Since, $\sin \theta$ = perpendicular/Hypotenuse

$\therefore \sin \theta = 5/13$

Since, $\operatorname{cosec} \theta = 1/\sin \theta$

Also, $\operatorname{cosec} \theta$ = Hypotenuse/Perpendicualar

$\therefore \operatorname{cosec} \theta = 13/5$

Since, $\cos \theta$ = Base/Hypotenuse

$\therefore \cos \theta = 12/13$

Since, $\sec \theta = 1/\cos\theta$

Also, $\sec \theta$ = Hypotenuse/Base

∴ sec θ = 13/12

Since, tanθ = 1/cot θ

Also, tan θ = Perpendicular/Base

∴ tan θ = 5/12

(x) sec θ = 13/5

Solution:

We have, sec θ = 13/5………. (1)

By definition,

sec θ = Hypotenuse/Base…………. (2)

On Comparing eq. (1) and (2), we get

Base = 5 and Hypotenuse = 13

Now, using Pythagoras' theorem in Δ ABC,

$AC^2 = AB^2 + BC^2$

And putting the value of base side (AB) and hypotenuse (AC) to get the perpendicular side (BC),

$13^2 = 5^2 + BC^2$

$BC^2 = 13^2 – 5^2$

$BC^2 = 169 – 25$

$BC^2 = 144$

BC= √144

BC = 12

Hence, Perpendicular = 12

By definition,

Since, sin θ = perpendicular/Hypotenuse

∴ sin θ= 12/13

Since, cosec θ= 1/ sin θ

Also, cosec θ= Hypotenuse/Perpendicular

∴ cosec θ = 13/12

Since, cos θ= 1/sec θ

Also, cos θ = Base/Hypotenuse

∴ cos θ = 5/13

Since, tan θ = Perpendicular/Base

∴ tan θ = 12/5

Since, cot θ = 1/tan θ

Also, cot θ = Base/Perpendicular

∴ cot θ = 5/12

(xi) cosec θ = $\sqrt{10}$

Solution:

We have, cosec θ = $\sqrt{10}/1$ ……..… (1)

By definition,

cosec θ = Hypotenuse/ Perpendicualar ……….(2)

And, cosecθ = 1/sin θ

On comparing eq.(1) and(2), we get

Perpendicular side = 1 and Hypotenuse = $\sqrt{10}$

Now, using Pythagoras' theorem in Δ ABC,

$AC^2 = AB^2 + BC^2$

Putting the value of perpendicular (BC) and hypotenuse (AC) to get the base side (AB),

$(\sqrt{10})^2 = AB^2 + 1^2$

$AB^2 = (\sqrt{10})^2 - 1^2$

$AB^2 = 10 – 1$

$AB = \sqrt{9}$

AB = 3

So, Base side = 3

By definition,

Since, sin θ = Perpendicular/Hypotenuse

∴ $\sin \theta = 1/\sqrt{10}$

Since, cos θ = Base/Hypotenuse

∴ $\cos \theta = 3/\sqrt{10}$

Since, sec θ = 1/cos θ

Also, sec θ = Hypotenuse/Base

∴ $\sec \theta = \sqrt{10}/3$

Since, tan θ = Perpendicular/Base

∴ tan θ = 1/3

Since, cot θ = 1/tan θ

∴ cot θ = 3/1

(xii) cos θ =12/15

Solution:

We have; cos θ = 12/15 ………. (1)

By definition,

cos θ = Base/Hypotenuse……… (2)

By comparing eq. (1) and (2), we get;

Base =12 and Hypotenuse = 15

Now, using Pythagoras' theorem in Δ ABC, we get

$AC^2 = AB^2 + BC^2$

Putting the value of base (AB) and hypotenuse (AC) to get the perpendicular (BC),

$15^2 = 12^2 + BC^2$

$BC^2 = 15^2 – 12^2$

$BC^2 = 225 – 144$

$BC^2 = 81$

$BC = \sqrt{81}$

BC = 9

So, Perpendicular = 9

By definition,

Since, sin θ = perpendicular/Hypotenuse

∴ sin θ = 9/15 = 3/5

Since, cosec θ = 1/sin θ

Also, cosec θ = Hypotenuse/Perpendicular

∴ cosec θ= 15/9 = 5/3

Since, sec θ = 1/cos θ

Also, sec θ = Hypotenuse/Base

∴ sec θ = 15/12 = 5/4

Since, tan θ = Perpendicular/Base

∴ tan θ = 9/12 = 3/4

Since, cot θ = 1/tan θ

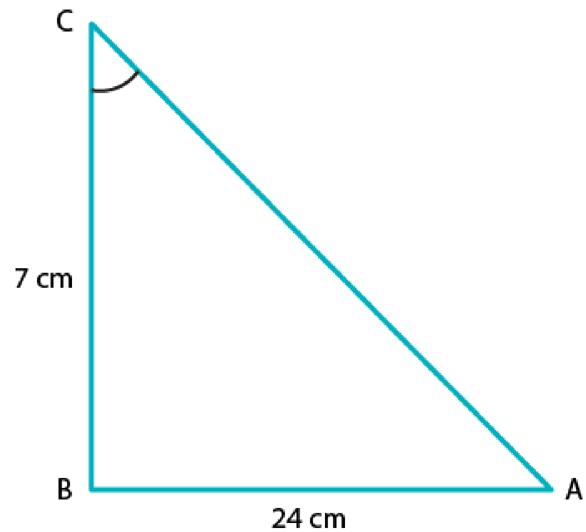
Also, cot θ = Base/Perpendicular

∴ cot θ = 12/9 = 4/3

2. In a △ ABC, right-angled at B, AB = 24 cm , BC = 7 cm. Determine

(i) sin A , cos A (ii) sin C, cos C

Solution:



(i) Given: In △ABC, AB = 24 cm, BC = 7cm and ∠ABC = 90°

To find: sin A, cos A

By using Pythagoras' theorem in △ABC, we have

$AC^2 = AB^2 + BC^2$

$AC^2 = 24^2 + 7^2$

$AC^2 = 576 + 49$

$AC^2 = 625$

AC = √625

AC= 25

Hence, Hypotenuse = 25

By definition,

sin A = Perpendicular side opposite to angle A/ Hypotenuse

sin A = BC/ AC

sin A = 7/ 25

And,

cos A = Base side adjacent to angle A/Hypotenuse

cos A = AB/ AC

cos A = 24/ 25

(ii) Given: In △ABC , AB = 24 cm and BC = 7cm and ∠ABC = 90^{o}

To find: sin C, cos C

By using Pythagoras' theorem in △ABC, we have

$AC^2 = AB^2 + BC^2$

$AC^2 = 24^2 + 7^2$

$AC^2 = 576 + 49$

$AC^2 = 625$

AC = √625

AC= 25

Hence, Hypotenuse = 25

By definition,

sin C = Perpendicular side opposite to angle C/Hypotenuse

sin C = AB/ AC

sin C = 24/ 25

And,

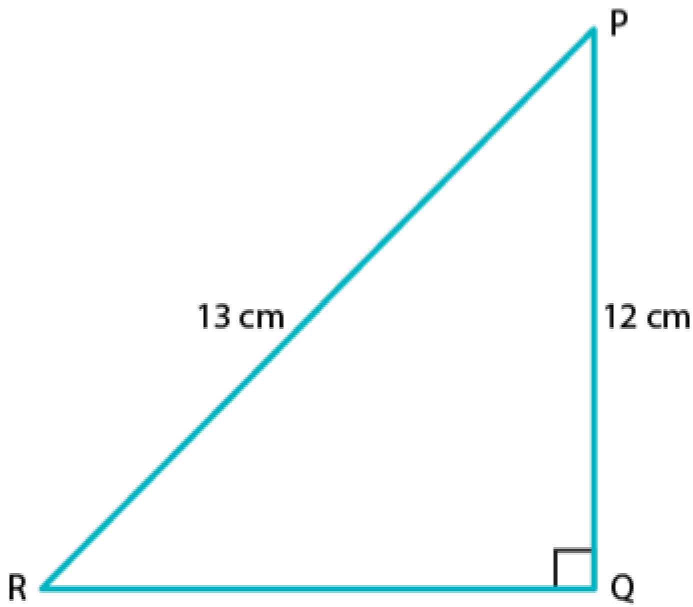
cos C = Base side adjacent to angle C/Hypotenuse

cos A = BC/AC

cos A = 7/25

3. In fig. 5.37, find tan P and cot R. Is tan P = cot R?

Solution:



By using Pythagoras theorem in △PQR, we have

$PR^2 = PQ^2 + QR^2$

Putting the length of given side PR and PQ in the above equation,

$13^2 = 12^2 + QR^2$

$QR^2 = 13^2 – 12^2$

$QR^2 = 169 – 144$

$QR^2 = 25$

$QR = \sqrt{25} = 5$

By definition,

tan P = Perpendicular side opposite to P/ Base side adjacent to angle P

tan P = QR/PQ

tan P = 5/12 ………. (1)

And,

cot R= Base/Perpendicular

cot R= QR/PQ

cot R= 5/12 …. (2)

When comparing equation (1) and (2), we can see that R.H.S of both the equation is equal.

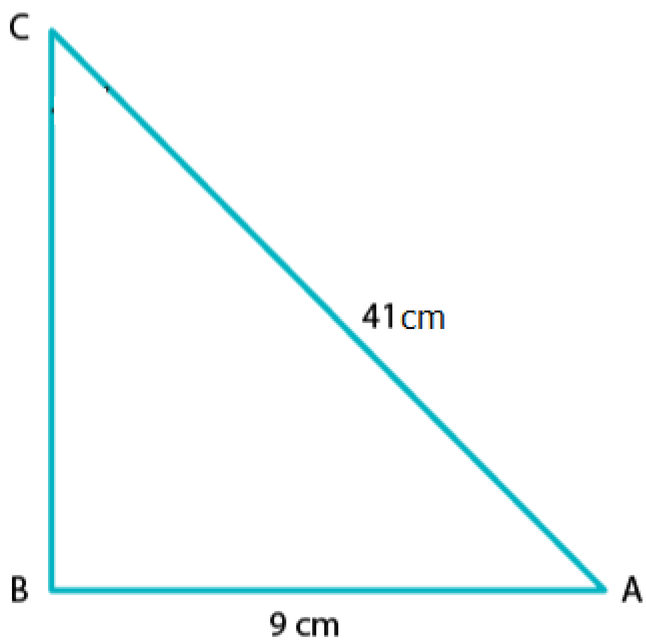
Therefore, L.H.S of both equations should also be equal.

∴ tan P = cot R

Yes, tan P = cot R = 5/12

4. If sin A = 9/41, compute cos A and tan A.

Solution:



Given that, sin A = 9/41 …………. (1)

Required to find: cos A, tan A

By definition, we know that

sin A = Perpendicular/ Hypotenuse…………..(2)

On Comparing eq. (1) and (2), we get

Perpendicular side = 9 and Hypotenuse = 41

Let's construct △ABC as shown below,

And, here the length of base AB is unknown.

Thus, by using Pythagoras theorem in △ABC, we get

$AC^2 = AB^2 + BC^2$

$41^2 = AB^2 + 9^2$

$AB^2 = 41^2 – 9^2$

$AB^2 = 168 – 81$

AB= 1600

AB = √1600

AB = 40

⇒ Base of triangle ABC, AB = 40

We know that,

cos A = Base/ Hypotenuse

cos A =AB/AC

cos A =40/41

And,

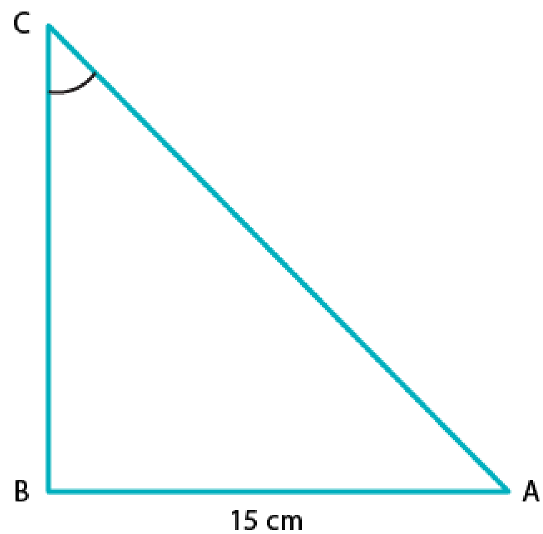
tan A = Perpendicular/ Base

tan A = BC/AB

tan A = 9/40

5. Given 15cot A= 8, find sin A and sec A.

Solution



We have, 15cot A = 8

Required to find: sin A and sec A

As, 15 cot A = 8

⇒ cot A = 8/15 …….(1)

And we know,

cot A = 1/tan A

Also by definition,

Cot A = Base side adjacent to ∠A/ Perpendicular side opposite to ∠A …. (2)

On comparing equation (1) and (2), we get

Base side adjacent to ∠A = 8

Perpendicular side opposite to ∠A = 15

So, by using Pythagoras theorem to △ABC, we have

$AC^2 = AB^2 + BC^2$

Substituting values for sides from the figure

$AC^2 = 8^2 + 15^2$

$AC^2 = 64 + 225$

$AC^2 = 289$

$AC = \sqrt{289}$

$AC = 17$

Therefore, hypotenuse =17

By definition,

sin A = Perpendicular/Hypotenuse

⇒ sin A= BC/AC

sin A= 15/17 (using values from the above)

Also,

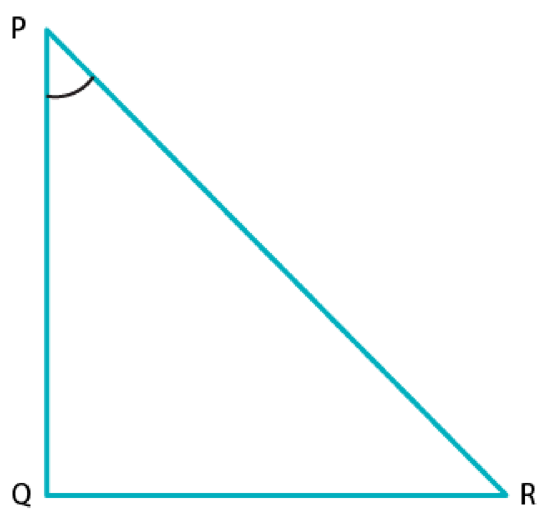
sec A= 1/ cos A

⇒ secA = Hypotenuse/ Base side adjacent to ∠A

∴ sec A= 17/8

6. In △PQR, right-angled at Q, PQ = 4cm and RQ = 3 cm. Find the value of sin P, sin R, sec P and sec R.

Solution:



Given:

△PQR is right-angled at Q.

PQ = 4cm

RQ = 3cm

Required to find: sin P, sin R, sec P, sec R

Given △PQR,

By using Pythagoras theorem to △PQR, we get

$PR^2 = PQ^2 + RQ^2$

Substituting the respective values,

$PR^2 = 4^2 + 3^2$

$PR^2 = 16 + 9$

$PR^2 = 25$

PR = √25

PR = 5

⇒ Hypotenuse =5

By definition,

sin P = Perpendicular side opposite to angle P/ Hypotenuse

sin P = RQ/ PR

⇒ sin P = 3/5

And,

sin R = Perpendicular side opposite to angle R/ Hypotenuse

sin R = PQ/ PR

⇒ sin R = 4/5

And,

sec P=1/cos P

secP = Hypotenuse/ Base side adjacent to ∠P

sec P = PR/ PQ

⇒ sec P = 5/4

Now,

sec R = 1/cos R

secR = Hypotenuse/ Base side adjacent to ∠R

sec R = PR/ RQ

⇒ sec R = 5/3

7. If cot θ = 7/8, evaluate

(i) (1+sin θ)(1–sin θ)/ (1+cos θ)(1–cos θ)

(ii) $\cot^2\theta$

Solution:

(i) Required to evaluate:

$$\frac{(1+sin\theta)(1-sin\theta)}{(1+cos\theta)(1-cos\theta)}$$, given = cot θ = 7/8

Taking the numerator, we have

(1+sin θ)(1–sin θ) = 1 – $\sin^2\theta$ [Since, (a+b)(a-b) = $a^2 - b^2$]

Similarly,

(1+cos θ)(1–cos θ) = 1 – $\cos^2\theta$

We know that,

$\sin^2\theta + \cos^2\theta = 1$

⇒ $1 - \cos^2\theta = \sin^2\theta$

And,

$1 - \sin^2\theta = \cos^2\theta$

Thus,

$(1+\sin\theta)(1-\sin\theta) = 1 - \sin^2\theta = \cos^2\theta$

$(1+\cos\theta)(1-\cos\theta) = 1 - \cos^2\theta = \sin^2\theta$

⇒

$$\frac{(1+sin\theta)(1-sin\theta)}{(1+cos\theta)(1-cos\theta)}$$

$= \cos^2\theta / \sin^2\theta$

$= (\cos\theta/\sin\theta)^2$

And, we know that $(\cos\theta/\sin\theta) = \cot\theta$

⇒

$$\frac{(1+sin\theta)(1-sin\theta)}{(1+cos\theta)(1-cos\theta)}$$

= $(\cot\theta)^2$

$= (7/8)^2$

$= 49/64$

(ii) Given,

$\cot\theta = 7/8$

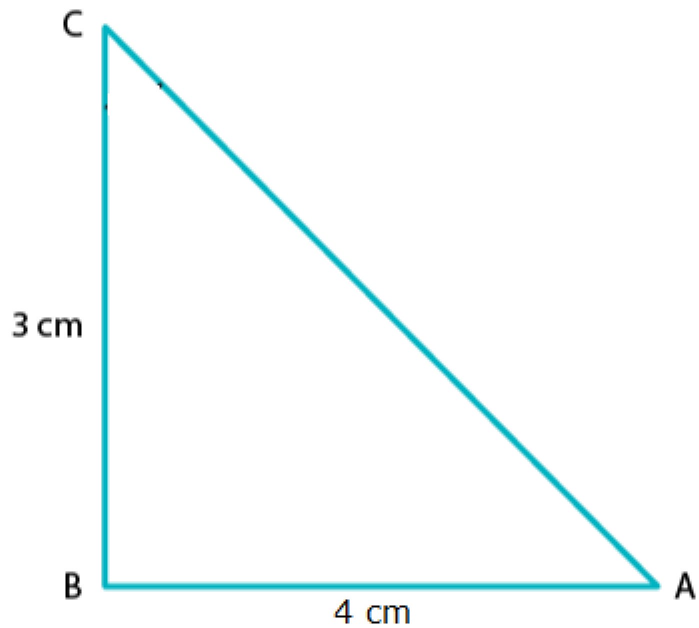
So, by squaring on both sides we get

$(\cot\theta)^2 = (7/8)^2$

$\therefore \cot\theta^2 = 49/64$

8. If 3cot A = 4, check whether $(1-\tan^2 A)/(1+\tan^2 A) = (\cos^2 A - \sin^2 A)$ or not.

Solution:



Given,

3cot A = 4

⇒ cot A = 4/3

By definition,

tan A = 1/ Cot A = 1/ (4/3)

⇒ tan A = 3/4

Thus,

Base side adjacent to ∠A = 4

Perpendicular side opposite to ∠A = 3

In ΔABC, Hypotenuse is unknown.

Thus, by applying Pythagoras theorem in ΔABC,

We get

$AC^2 = AB^2 + BC^2$

$AC^2 = 4^2 + 3^2$

$AC^2 = 16 + 9$

$AC^2 = 25$

$AC = \sqrt{25}$

$AC = 5$

Hence, hypotenuse = 5

Now, we can find that

sin A = opposite side to ∠A/ Hypotenuse = 3/5

And,

cos A = adjacent side to ∠A/ Hypotenuse = 4/5

Taking the LHS,

$$\text{L.H.S} = \frac{1-\tan^2 A}{1+\tan^2 A}$$

Putting value of tan A

We get,

$$\text{L.H.S} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}$$

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}$$

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}}$$

Take L.C.M of both numerator and denominator;

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}}$$

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{7}{25}$$

Thus, LHS = 7/25

Now, taking RHS,

R.H.S = $\cos^2 A - \sin^2 A$

Putting value of sin A and cos A

R.H.S= $(\frac{4}{5})^2 - (\frac{3}{5}^2)$

$\cos^2 A - \sin^2 A = (\frac{4}{5})^2 - (\frac{3}{5}^2)$

$\cos^2 A - \sin^2 A = \frac{16}{25} - \frac{9}{25}$

$\cos^2 A - \sin^2 A = \frac{16-9}{25}$

$\cos^2 A - \sin^2 A = \frac{7}{25}$

Therefore,

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$$

Hence Proved

9. If tan θ = a/b, find the value of (cos θ + sin θ)/ (cos θ – sin θ)

Solution:

Given,

tan θ = a/b

And we know by definition that

tan θ = opposite side/ adjacent side

Thus, by comparison,

Opposite side = a and adjacent side = b

To find the hypotenuse, we know that by Pythagoras theorem that

$\text{Hypotenuse}^2 = \text{opposite side}^2 + \text{adjacent side}^2$

⇒ Hypotenuse = $\sqrt{(a^2 + b^2)}$

So, by definition

sin θ = opposite side/ Hypotenuse

sin θ = a/ $\sqrt{(a^2 + b^2)}$

And,

cos θ = adjacent side/ Hypotenuse

cos θ = b/ $\sqrt{(a^2 + b^2)}$

Now,

After substituting for cos θ and sin θ, we have

$$\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{(a+b)/\sqrt{(a^2+b^2)}}{(a-b)/\sqrt{(a^2+b^2)}}$$

$$\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{(a+b)}{(a-b)}$$

∴

Hence, proved.

$$\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$$

10. If 3 tan θ = 4, find the value of

Solution:

Given, 3 tan θ = 4

⇒ tan θ = 4/3

$$\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$$

From, let's divide the numerator and denominator by cos θ.

We get,

(4 – tan θ) / (2 + tan θ)

⇒ (4 – (4/3)) / (2 + (4/3)) [using the value of tan θ]

⇒ (12 – 4) / (6 + 4) [After taking LCM and cancelling it]

⇒ 8/10 = 4/5

$$\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$$

∴ = 4/5

$$\frac{4\sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta}$$

11. If 3 cot θ = 2, find the value of

Solution:

Given, 3 cot θ = 2

⇒ cot θ = 2/3

$$\frac{4\sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta}$$

From, let's divide the numerator and denominator by sin θ.

We get,

(4 –3 cot θ) / (2 + 6 cot θ)

⇒ (4 – 3(2/3)) / (2 + 6(2/3)) [using the value of tan θ]

⇒ (4 – 2) / (2 + 4) [After taking LCM and simplifying it]

⇒ 2/6 = 1/3

$$\frac{4\sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta}$$

∴ = 1/3

$$\frac{a \sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2 - b^2}{a^2 + b^2}$$

12. If tan θ = a/b, prove that

Solution:

Given, tan θ = a/b

$$\frac{a \sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$$

From LHS, let’s divide the numerator and denominator by cos θ.

And we get,

(a tan θ – b) / (a tan θ + b)

⇒ (a(a/b) – b) / (a(a/b) + b) [using the value of tan θ]

⇒ $(a^2 - b^2)/b^2$ / $(a^2 + b^2)/b^2$ [After taking LCM and simplifying it]

⇒ $(a^2 - b^2)/ (a^2 + b^2)$

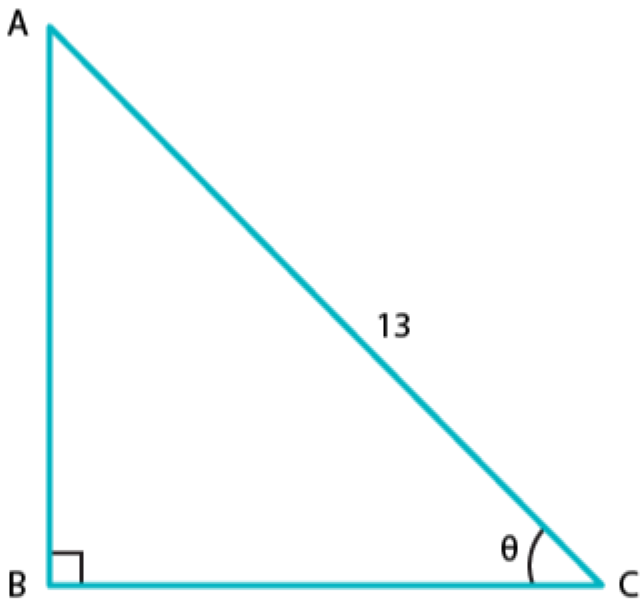
= RHS

Hence, proved

$$\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = 3$$

13. If sec θ = 13/5, show that

Solution:



Given,

sec θ = 13/5

We know that,

sec θ = 1/ cos θ

⇒ cos θ = 1/ sec θ = 1/ (13/5)

∴ cos θ = 5/13 ……. (1)

By definition,

cos θ = adjacent side/ hypotenuse ….. (2)

Comparing (1) and (2), we have

Adjacent side = 5 and hypotenuse = 13

By Pythagoras theorem,

Opposite side = $\sqrt{(\text{hypotenuse})^2 - (\text{adjacent side})^2}$

= $\sqrt{13^2 - 5^2}$

= $\sqrt{169 - 25}$

= $\sqrt{144}$

= 12

Thus, the opposite side = 12

By definition,

tan θ = opposite side/ adjacent side

∴ tan θ = 12/ 5

$$\frac{2 \sin \theta - 3 \cos \theta}{4 \sin \theta - 9 \cos \theta}$$

From, let’s divide the numerator and denominator by cos θ.

We get,

(2 tan θ – 3) / (4 tan θ – 9)

⇒ (2(12/5) – 3) / (4(12/5) – 9) [using the value of tan θ]

⇒ (24 – 15) / (48 – 45) [After taking LCM and cancelling it]

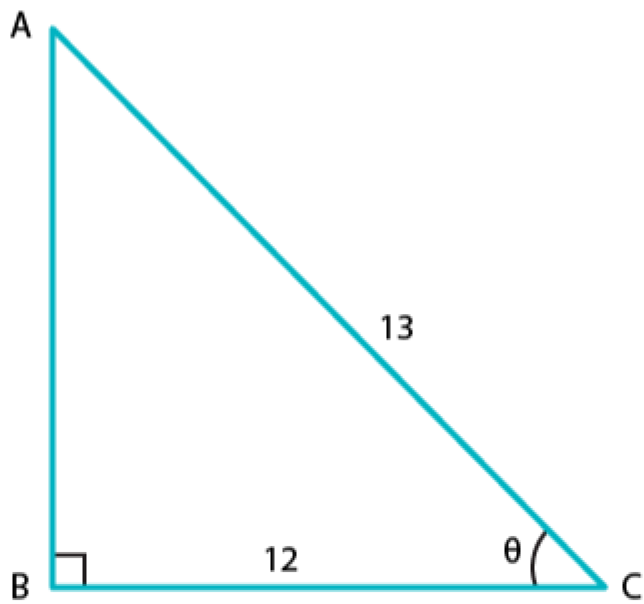
⇒ 9/3 = 3

$$\frac{2 \sin \theta - 3 \cos \theta}{4 \sin \theta - 9 \cos \theta}$$

∴ = 3

14. If cos θ = 12/13, show that sin θ(1 – tan θ) = 35/156

Solution:



Given, $\cos \theta = 12/13$…… (1)

By definition, we know that

$\cos \theta$ = Base side adjacent to $\angle\theta$ / Hypotenuse……. (2)

When comparing equation (1) and (2), we get

Base side adjacent to $\angle\theta = 12$ and Hypotenuse = 13

From the figure,

Base side BC = 12

Hypotenuse AC = 13

Side AB is unknown here, and it can be found by using Pythagoras theorem.

Thus, by applying Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$13^2 = AB^2 + 12^2$

Therefore,

$AB^2 = 13^2 – 12^2$

$AB^2 = 169 – 144$

$AB^2 = 25$

AB = √25

AB = 5 …. (3)

Now, we know that

sin θ = Perpendicular side opposite to ∠θ / Hypotenuse

Thus, sin θ = AB/AC [from figure]

⇒ sin θ = 5/13… (4)

And, tan θ = sin θ / cos θ = (5/13) / (12/13)

⇒ tan θ = 12/13… (5)

Taking L.H.S we have

L.H.S = sin θ (1 – tan θ)

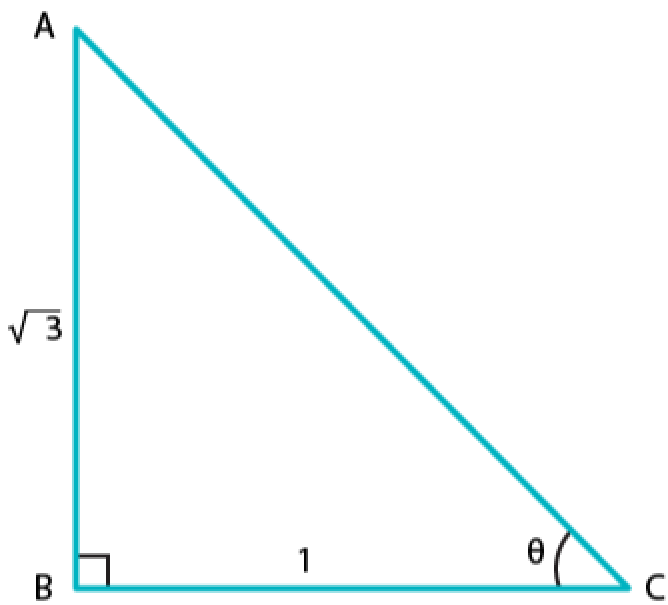
Substituting the value of sin θ and tan θ from equation (4) and (5),

We get,

$$\text{If } \cot\theta = \frac{1}{\sqrt{3}}, \text{ show that } \frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}$$

15.

Solution:



Given, cot θ = 1/√3……. (1)

By definition, we know that,

cot θ = 1/ tan θ

And, since tan θ = perpendicular side opposite to ∠θ / Base side adjacent to ∠θ

⇒ cot θ = Base side adjacent to ∠θ / perpendicular side opposite to ∠θ …… (2)

[Since they are reciprocal to each other]

On comparing equation (1) and (2), we get

Base side adjacent to ∠θ = 1 and Perpendicular side opposite to ∠θ = √3

Therefore, the triangle formed is

On substituting the values of known sides as AB = √3 and BC = 1,

$AC^2 = (\sqrt{3}) + 1$

$AC^2 = 3 + 1$

$AC^2 = 4$

$AC = \sqrt{4}$

Therefore, AC = 2 …   (3)

Now, by definition,

sin θ = Perpendicular side opposite to ∠θ / Hypotenuse = AB / AC

⇒ sin θ = √3/ 2 ……(4)

And, cos θ = Base side adjacent to ∠θ / Hypotenuse = BC / AC

⇒ cos θ = 1/ 2 ….. (5)

Now, taking L.H.S, we have

$$\mathrm{L.H.S} = \frac{1-\cos^2\theta}{2-\sin^2\theta}$$

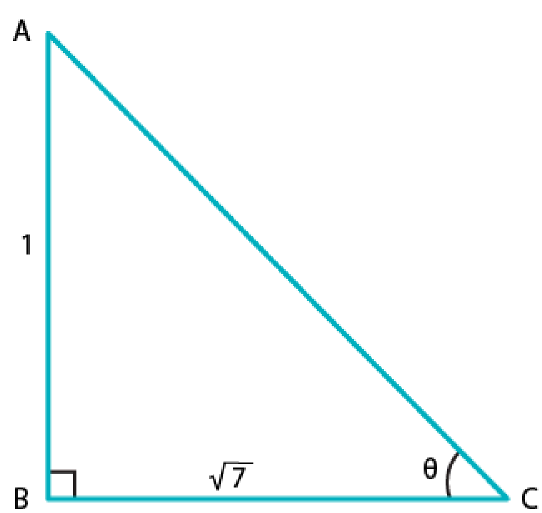
Substituting the values from equation (4) and (5), we have

$$\text{If } \tan\theta = \frac{1}{\sqrt{7}}, \text{then show that } \frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{3}{4}$$

16.

Solution:

Given, tan θ = 1/ √7 …..(1)



By definition, we know that

tan θ = Perpendicular side opposite to ∠θ / Base side adjacent to ∠θ ……(2)

On comparing equations (1) and (2), we have

The perpendicular side opposite to ∠θ = 1

Base side adjacent to ∠θ = √7

Thus, the triangle representing ∠ θ is,

Hypotenuse AC is unknown, and it can be found by using Pythagoras theorem.

By applying Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$AC^2 = 1^2 + (\sqrt{7})^2$

$AC^2 = 1 + 7$

$AC^2 = 8$

AC = √8

⇒ AC = 2√2

By definition,

sin θ = Perpendicular side opposite to ∠θ / Hypotenuse = AB / AC

⇒ sin θ = 1/ 2√2

And, since cosec θ = 1/sin θ

⇒ cosec θ = 2√2 …….. (3)

Now,

cos θ = Base side adjacent to ∠θ / Hypotenuse = BC / AC

⇒ cos θ = √7/ 2√2

And, since sec θ = 1/ sin θ

⇒ sec θ = 2√2/ √7 ……. (4)

Taking the L.H.S of the equation,

$$\text{L.H.S} = \frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$$

Substituting the value of cosec θ and sec θ from equation (3) and (4), we get

$$\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$$

17. If sec θ = 5/4, find the value of

Solution:

Given,

sec θ = 5/4

We know that,

sec θ = 1/ cos θ

⇒ cos θ = 1/ (5/4) = 4/5 …… (1)

By definition,

cos θ = Base side adjacent to ∠θ / Hypotenuse …. (2)

On comparing equation (1) and (2), we have

Hypotenuse = 5

Base side adjacent to ∠θ = 4

Thus, the triangle representing ∠ θ is ABC.

Perpendicular side opposite to ∠θ, AB is unknown, and it can be found by using Pythagoras theorem.

By applying Pythagoras' theorem, we have

$AC^2 = AB^2 + BC^2$

$AB^2 = AC^2 + BC^2$

$AB^2 = 5^2 – 4^2$

$AB^2 = 25 – 16$

AB = √9

⇒ AB = 3

By definition,

sin θ = Perpendicular side opposite to ∠θ / Hypotenuse = AB / AC

⇒ sin θ = 3/ 5 …..(3)

Now, tan θ = Perpendicular side opposite to ∠θ / Base side adjacent to ∠θ

⇒ tan θ = 3/ 4 ……(4)

And, since cot θ = 1/ tan θ

⇒ cot θ = 4/ 3 ……(5)

Now,

Substituting the value of sin θ, cos θ, cot θ and tan θ from the equations (1), (3), (4) and (5), we have

$$\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta} = \frac{\frac{3}{5} - 2\left(\frac{4}{5}\right)}{\frac{3}{4} - \frac{4}{3}}$$

= 12/7

Therefore,

$$\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta} = \frac{12}{7}$$

$$\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$$

18. If tan θ = 12/13, find the value of

Solution:

Given,

tan θ = 12/13 …….. (1)

We know that by definition,

tan θ = Perpendicular side opposite to ∠θ / Base side adjacent to ∠θ …… (2)

On comparing equation (1) and (2), we have

The perpendicular side opposite to ∠θ = 12

Base side adjacent to ∠θ = 13

Thus, in the triangle representing ∠ θ, we have,

Hypotenuse AC is the unknown, and it can be found by using Pythagoras theorem.

So, by applying Pythagoras' theorem, we have

$AC^2 = 12^2 + 13^2$

$AC^2 = 144 + 169$

$AC^2 = 313$

⇒ $AC = \sqrt{313}$

By definition,

sin θ = Perpendicular side opposite to ∠θ / Hypotenuse = AB / AC

⇒ $\sin \theta = 12/ \sqrt{313}$…..(3)

And, cos θ = Base side adjacent to ∠θ / Hypotenuse = BC / AC

⇒ $\cos \theta = 13/ \sqrt{313}$ …..(4)

Now, substituting the value of sin θ and cos θ from equation (3) and (4), respectively, in the equation below,

$$\frac{2\sin\theta\cos\theta}{\cos^2\Theta\ -\sin^2\theta} = \frac{2\times\frac{13}{\sqrt{313}}\times\frac{12}{\sqrt{313}}}{\left(\frac{13}{\sqrt{313}}\right)^2-\left(\frac{12}{\sqrt{313}}\right)^2}$$

$$= \frac{\frac{312}{313}}{\frac{25}{313}}$$

$$= \frac{312}{25}$$

Therefore,

$$\frac{2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta} = \frac{312}{25}$$

Benefits of Solving RD Sharma Solutions Class 10 Maths Chapter 5 Exercise 5.1

Solving RD Sharma Solutions for Class 10 Maths Chapter 5, Exercise 5.1 on Trigonometric Ratios offers several benefits:

Strong Foundation in Trigonometry: Exercise 5.1 introduces students to basic trigonometric ratios like sine, cosine, tangent, and their reciprocals. Practicing this helps build a solid foundation for more advanced concepts in trigonometry.

Enhanced Problem-Solving Skills: Working through RD Sharma's exercises encourages a step-by-step approach to solving problems, which improves analytical thinking and problem-solving skills.

Understanding Real-Life Applications: Trigonometry is widely used in fields like physics, engineering, architecture, and astronomy. By mastering the basics, students can better understand these applications.

Preparation for Competitive Exams: The exercises in RD Sharma's book are similar to those in competitive exams, providing students with a strong base for exams that require trigonometric knowledge.

Concept Clarity: Detailed solutions and explanations aid in clearing any doubts regarding trigonometric concepts, making it easier to retain and apply them in higher classes.