NUMBER SYSTEM

CLASS - IX

By Ritik Sir

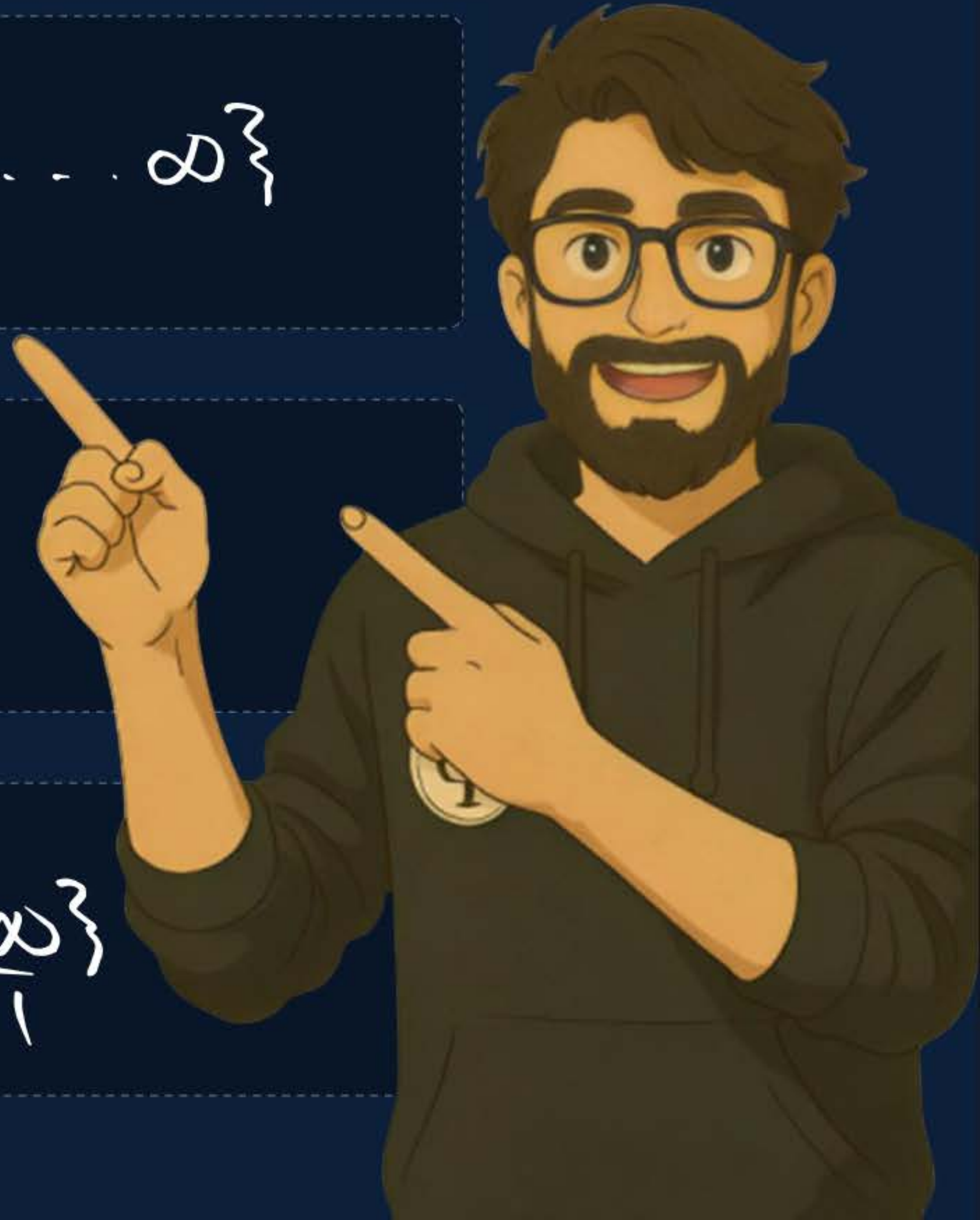
Recalling Numbers

Natural Numbers (N) → $\{1, 2, 3, 4, 5, 6 \ldots\ldots \infty\}$

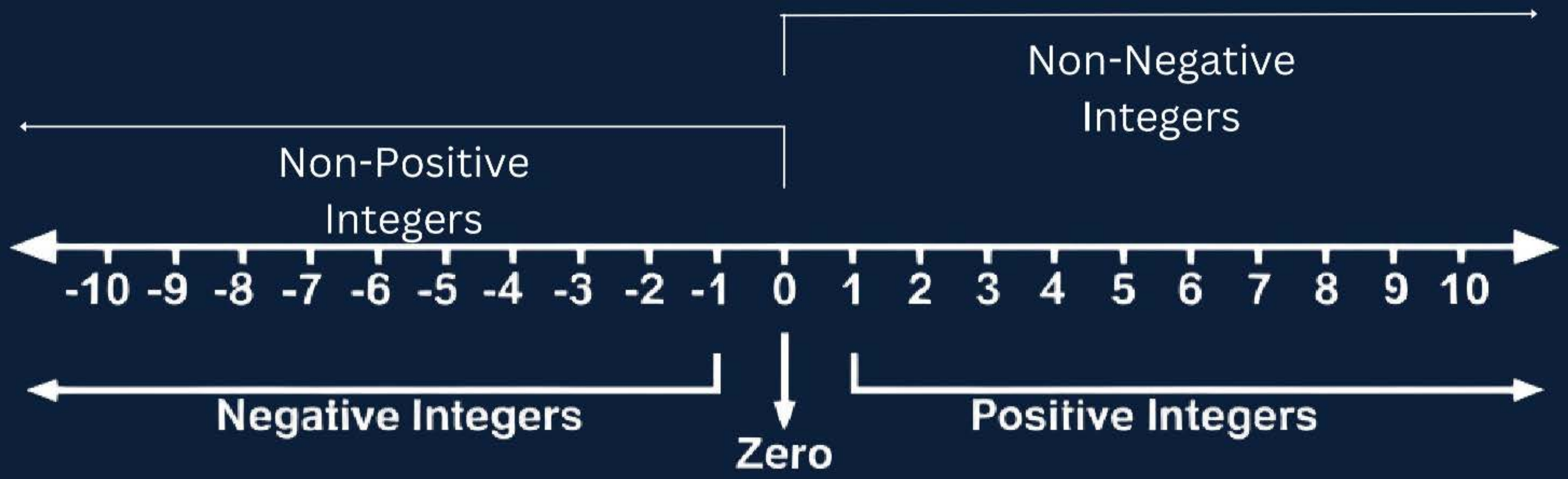
Whole Numbers (W) → $\{0, 1, 2, 3, 4 \ldots\ldots \infty\}$

I/Z

Integers → $\{-\infty \ldots \frac{-4}{1}, \frac{-3}{1}, \frac{-2}{1}, \frac{-1}{1}, \frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1} \ldots \frac{\infty}{1}\}$



Important Terms



Rational Numbers

$\frac{p}{1}$ → ① p/q ② p=0, q=1 ③ $q \neq 0$

A number that can be written in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

Example: $\frac{5}{8}, \frac{-5}{6}, \frac{7}{-5}, 3, 0, -5$

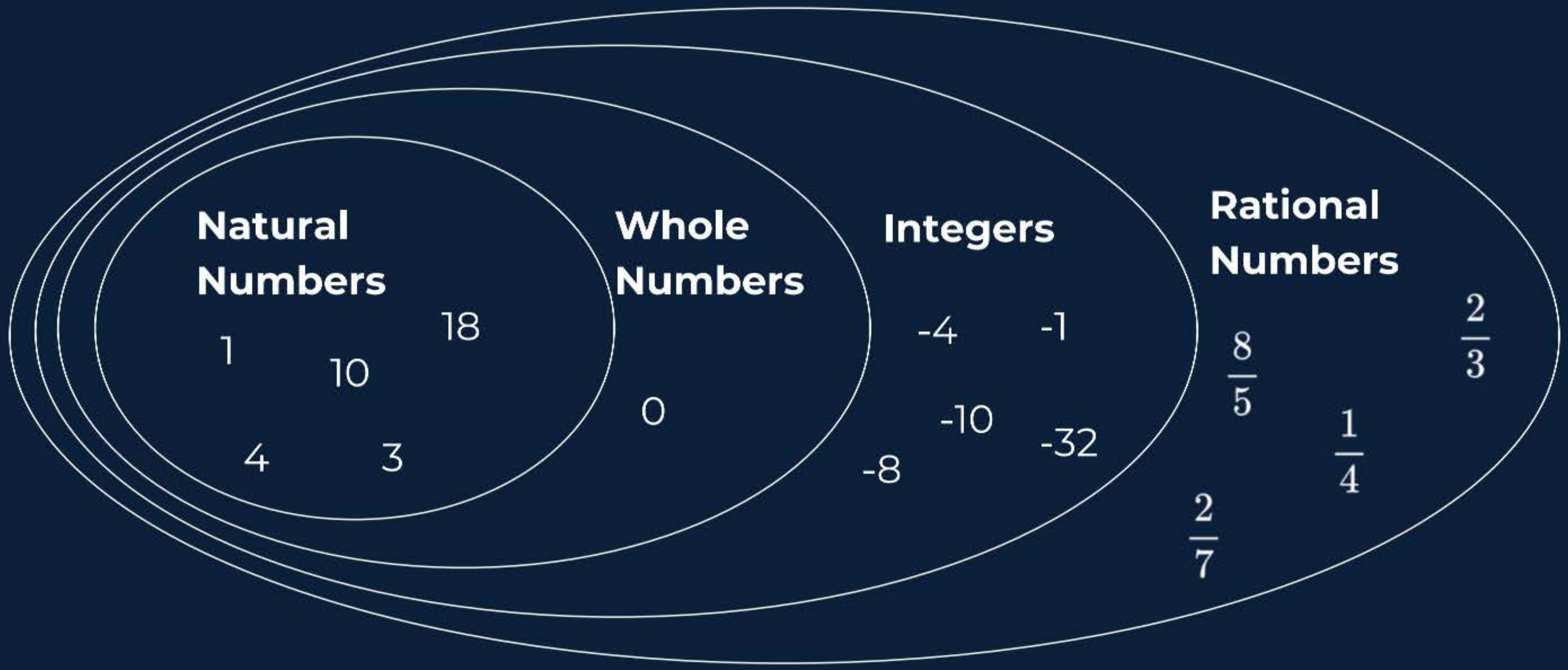
$\frac{2}{3} = \frac{p}{q}$

$= \frac{-7}{3}$

$= \frac{2}{1}$

$\frac{p}{q}$, p & q → integers, $q \neq 0$

Numbers



Natural Numbers
18
1
10
4
3
Whole Numbers
0
Integers
-4
-1
-10
-32
-8
Rational Numbers
$\frac{8}{5}$
$\frac{2}{3}$
$\frac{1}{4}$
$\frac{2}{7}$

Are the following statements true or false? Give reason for your answer.

1. Every whole number is a natural number. F
2. Every integer is a rational number. T
3. Every rational number is an integer. F
4. Every natural number is a whole number. T
5. Every integer is a whole number. F
6. Every rational number is a whole number. F

Equivalent Rational Number

$\Rightarrow \frac{3}{8} = \frac{\cancel{6}\,3}{\cancel{16}\,8}$

Q 5 E.R. No. $\frac{1}{2}$

$\frac{1}{2} = \frac{\cancel{3}\,1}{\cancel{6}\,2} = \frac{\cancel{4}\,1}{\cancel{8}\,2} = \frac{\cancel{8}\,1}{\cancel{16}\,2} = \frac{\cancel{100}\,1}{\cancel{200}\,2}$

Q $\frac{-5}{9}$ $\frac{-\cancel{30}\,5}{\cancel{54}\,9}$, $\frac{-\cancel{10}\,5}{\cancel{18}\,9}$

Inserting Rational Numbers b/w any two Rational Numbers

- **Method I :** (Average Method)

- **Method II :** (Equivalent Rational Number Method) one

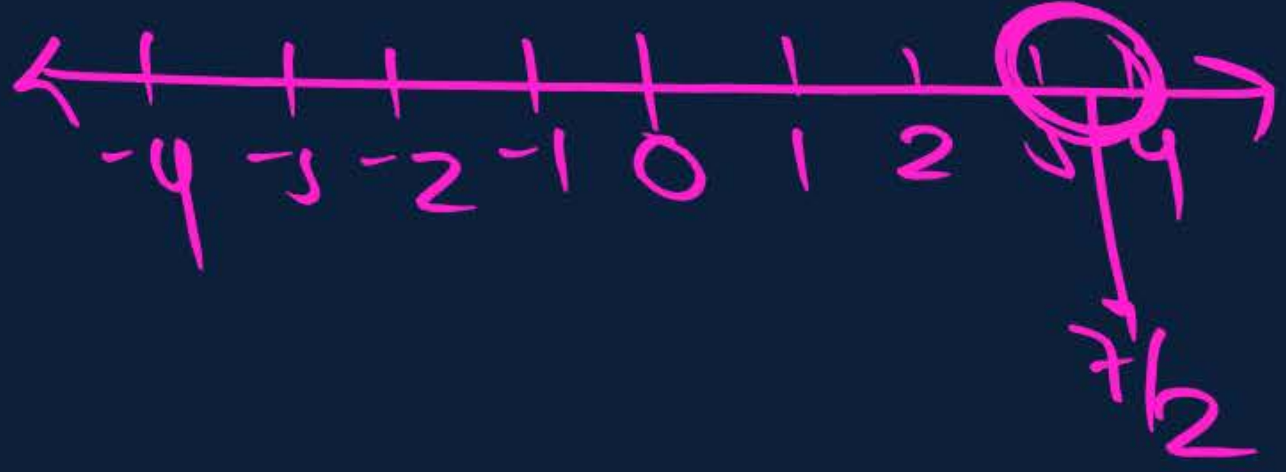
☞ **Find a rational number between 3 and 4.**

$$\frac{3+4}{2} = \frac{7}{2}$$

B

$$a, b$$

$$\frac{a+b}{2}$$



$$\frac{\frac{7}{2}+4}{2} = \frac{\frac{7+8}{2}}{2} = \frac{15}{4}$$

☞ **Find a rational number between $\frac{-2}{3}$ and $\frac{1}{4}$**

$$= \frac{\frac{-2}{3} + \frac{1}{4}}{2}$$

$$= \frac{\frac{-8+3}{12}}{\frac{2}{1}}$$

$$= \frac{\frac{-5}{12}}{\frac{2}{1}} = \frac{-5}{24}$$

Equivalent Rational Number Method

Denominators Same Banao

Numerator key beech main gap create karo

☞ **Find six rational number between 3 and 4.**

$$\frac{3}{1} \qquad \frac{4}{1}$$

$$\frac{30}{10}, \boxed{\frac{31}{10}, \frac{32}{10}, \frac{33}{10}, \frac{34}{10} \ldots} , \frac{40}{10}$$

$$\frac{3000}{1000}, \frac{3001}{1000}, \frac{3002}{1000} \ldots\ldots \frac{4000}{1000}$$

☞ **Insert 10 rational numbers between $\frac{-3}{11}$ and $\frac{8}{11}$**

$\boxed{\frac{-3}{11}}, \frac{-2}{11}, \frac{-1}{11}, \frac{0}{11}, \frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \frac{4}{11}, \frac{5}{11}, \frac{6}{11}, \frac{7}{11}, \boxed{\frac{8}{11}}$

Q 11 R.N batao, $\frac{2}{3}$, $\frac{5}{6}$

$\frac{2}{3}, \frac{5}{6}$

$\frac{4}{6}, \frac{5}{6}$

$\frac{400}{600}, \boxed{\frac{401}{600}, \frac{402}{600} \ldots\ldots \frac{499}{600}}, \frac{500}{600}$

$$\begin{array}{c|c}
3 & 6, 3 \\
\hline
2 & 2, 1 \\
 & 1, 1
\end{array}$$

L.C.M = 6

The maximum number of integers between two consecutive natural numbers is:

laqatar

A zero

B 2

C 3

D Infinite

6 7 ✗

-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5.

☞ Express $\frac{7}{8}$ in the decimal form by long division method

$$
\begin{array}{r}
.875 \\
8\,\overline{)\,70} \\
\underline{64} \\
60 \\
\underline{56} \\
40 \\
\underline{40} \\
\boxed{0}
\end{array}
$$

Remainder

$$\boxed{\frac{7}{8} = 0.875}$$

(T)

☞ Find the decimal representation of $\frac{8}{3}$

$$
\begin{array}{r}
2.6666 \\
3\,\overline{)\,8} \\
6 \\
\hline
\boxed{2}0 \\
18 \\
\hline
\boxed{2}0 \\
18 \\
\hline
\boxed{2}0 \\
18 \\
\hline
\boxed{2}0 \\
18 \\
\hline
\boxed{2}
\end{array}
$$

$\frac{8}{3} = 2.666666\ldots\ldots\ldots\ldots$

$$\boxed{\frac{8}{3} = 2.\overline{6}}$$

N.T.R

☞ **Find the decimal representation of $\frac{-16}{45}$**

$$\begin{array}{r} .355 \\ 45 \overline{)\,160\,10} \\ 135 \\ \hline 0\boxed{25}0 \\ 225 \\ \hline \boxed{25}0 \\ 225 \\ \hline \boxed{25} \end{array}$$

$$\frac{-16}{45} = -0.3555555\ldots\ldots$$

$$= \boxed{-0.3\overline{5}}$$

N.T.R

☞ Find the decimal representation of $\frac{1}{7}$

$$\frac{1}{7} = 0.142857142857$$
$$142857142857$$
$$\ldots\ldots$$

$$\boxed{\frac{1}{7} = 0.\overline{142857}}$$

N.T.R

.142857142857

$7 \,\overline{)\, 10}$

7, 30, 28, 20, 14, 60, 56, 40, 35, 50, 49, 10, 7, 30, 28, 20, 14, 60, 56, 40, 35

50, 49, 1

What have you noticed

- The remainder becomes zero.
- The remainder never becomes zero.

Rational Number

$R=0$

Terminating

$R\neq 0$

Non-terminating Repeating

(Recurring)

☞ **You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansion of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, and \frac{6}{7}, are$ without actually doing the long division? If so, how?**

$$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857}$$
$$= \boxed{0.\overline{285714}}$$

$$\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times 0.\overline{142857}$$
$$= \boxed{0.\overline{857142}}$$

Conversion of decimal numbers into Rational numbers of the form $\frac{p}{q}$

Two cases !

- **When the decimal number is of terminating nature.**
- **When the decimal representation is of non-terminating nature.**

Express each of the following numbers in the form $\frac{p}{q}$

C.I

1. 0.15

$= \frac{\cancel{15}^{3}}{\cancel{100}_{20}}$

$= \boxed{\frac{3}{20}}$

2. 0.675

$= \frac{\cancel{\cancel{\cancel{675}}}\,\cancel{135}\,27}{\cancel{1000}\,\cancel{200}\,40}$

$= \boxed{27/40}$

CASE 2

N·T·R $\rightarrow p/q$

When the decimal is non-terminating but repeating.

Pure Recurring Decimal

$0.\overline{52}$

$0.\overline{293}$

$11.\overline{29}$

$5.\overline{2463}$

Mixed Recurring Decimal

$0.5\overline{2}$

$0.9\overline{13}$

$1.2\overline{7}$

$11.29\overline{62}$

☞ Express each of the following decimals in the form $\frac{p}{q}$

1 $0.\overline{1}$

$x = 0.\overline{1}$

$x = 0.111111111\ldots$ — (1)

$10x = 1.11111111\ldots$ — (2)

(2) − (1)

$9x = 1$

$x = 1/9$

$\boxed{0.\overline{1} = 1/9}$

2 $0.\overline{6}$ → pure

$x = 0.\overline{6}$

$x = 0.666666\ldots$ — (1)

$10x = 6.666666\ldots$ — (2)

(2) − (1)

$9x = 6$

$x = 6/9$

$\boxed{0.\overline{6} = 2/3}$

Express each of the following decimals in the form $\frac{p}{q}$

1. $0.\overline{35}$

$\frac{35}{99}$

2. $0.\overline{585}$

$x = 0.585585585585\ldots \quad (1)$

$1000x = 585.\cancel{585585585}\ldots \quad (2)$

$(2) - (1)$

$999x = 585$

$x = \frac{\cancel{585}\ \cancel{195}\ 65}{\cancel{999}\ \cancel{333}\ 111}$

$\boxed{0.\overline{585} = \frac{65}{111}}$

Q $0.\overline{1234} = \frac{1234}{9999}$

Q $0.\overline{8} = 8/9$

Q $0.\overline{92} = 92/99$

☞ **Show that $1.272727 = 1.\overline{27}$ can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.**

A 127/99

B 14/11

C 16/11

D None of the above

$x = 1.\overline{27}$ — pure

$x = 1.27272727\ldots \quad (1)$

$100x = 127.272727\ldots \quad (2)$

$(2) - (1)$

$99x = 126$

$x = 126/99$

$x = 42/33 = 14/11$

$x = 1.\overline{27}$

$x = 1 + 0.\overline{27}$

$x = 1 + \frac{27}{99}$

$x = \frac{99+27}{99}$

$\boxed{x = 126/99}$

☞ **Express each of the following decimals in the form $\frac{p}{q}$**

$0.12\overline{3}$ Mixed

A 31/90

B 24/90

C 29/90

D None of the above

$x = 0.12\overline{3}$

$x = 0.123333333....$

$100x = 12.3333333...$ (1)

$1000x = 123.333333...$ (2)

(2) − (1)

$900x = 111$

$x = 111/900$

$\boxed{x = 37/300}$

$x = 0.12\overline{3}$

$100x = 12.\overline{3}$

$100x = 12 + 0.\overline{3}$

$100x = 12 + 3/9$

$100x = \frac{12}{1} + 1/3$

$100x = \frac{36+1}{3}$

$\boxed{x = \frac{37}{300}}$

☞ **Express each of the following decimals in the form $\boxed{\frac{p}{q}}$**

$\boxed{15.7\overline{12}}$ → Mixed.

Ⓐ 1037/66

Ⓑ 137/66

Ⓒ 1037/666

Ⓓ None of the above

$x = 15.7\overline{12}$

$10x = 157.\overline{12}$

$10x = 157 + 0.\overline{12}$

$10x = \frac{157}{1} + \frac{12}{99}$

$10x = \frac{15543 + 12}{99}$

$\boxed{x = 15555/990}$

Express $0.6+0.\overline{7}+0.4\overline{7}$ in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0.

$\frac{6}{10}+\frac{7}{9}+0.4\bar{7}$ — p/q

#ans in Comments

A 167/90

B 167/60

C 167/900

D None of the above

From all the discussion that we how had, we can conclude that the decimal representation of a number can be

- Terminating 1.5
- Non-terminating but repeating $1.\overline{512}$
- Non-terminating and non-repeating

2.19345601292943....... N.T.N.R

Pehchaan of Irrational no.

① N.T.N.R

② p/q ✗

③ $\sqrt{\text{not a perfect square}}$

Ex: $\sqrt{5}, \sqrt{3}, \sqrt{2}, \sqrt{11}, \sqrt{99} \ldots$

$\boxed{1, 4, 9, 16, 25, 36, 49}$ ...

$1^2 \quad 2^2 \quad 3^2 \quad 4^2 \quad 5^2 \quad 6^2 \quad 7^2$

Square root

$\sqrt{2} = 2^{1/2}$

$\sqrt{4} = 4^{1/2}$

Q $\sqrt{9} \rightarrow$ Rational

$= 9^{1/2} = (3^2)^{1/2}$

$= 3^{2 \times \frac{1}{2}} = 3^1 = 3$

Important Points

$\underline{Q}$ $2\sqrt{2}$ $R \times Irr = Irr$

$\underline{Q}$ $\frac{3}{\sqrt{2}} = Irr$

non-zero

$R + - Irr = Irr$

$R \times \div Irr = Irr$

- Sum of a rational and irrational is irrational.
- Difference of a rational and an irrational is irrational.
- Product of a non-zero rational and an irrational is irrational.
- Quotient of a non-zero rational and an irrational is irrational.
- Negative of a irrational number is irrational.

$Q \times \sqrt{2}$ $R \times I = Q$ R

Important Points

Irr $+$ $-$ Irr $=$ R/Irr
$\times$ $\div$

- Sum of two irrationals need not be an irrational
- Difference of two irrationals need not be an irrational.
- Product of two irrationals need no be an irrational.
- Quotient of two irrational need not be an irrational.

① $3+\sqrt{2}$ $\quad 5-\sqrt{2}$ $= 3+\cancel{\sqrt{2}}+5-\cancel{\sqrt{2}} = 8 \rightarrow R$

Irr. $\quad$ Irr.

② $3+\sqrt{2}$ $\quad 5+\sqrt{3}$ $= 8+\sqrt{2}+\sqrt{3}$

Irr. $\quad$ Irr. $\quad$ R $\quad$ Irr $\quad$ Irr

N.T.N.R $\quad$ N.T.N.R.

$$\begin{array}{r} 2.14923\ldots\ldots \\ 5.92653429\ldots\ldots \\ \hline \end{array}$$

N.T.N.R

(3) $5\sqrt{2}$ $6\sqrt{2}$ $= 5\sqrt{2} \times 6\sqrt{2} = 30 \cdot 2 = 60$

Irr. Irr. R

$\sqrt{2} \cdot \sqrt{2} = (\sqrt{2})^2$

$= (2^{1/2})^2$

$= 2$

(4) $5\sqrt{3} \times 6\sqrt{5} = 30\sqrt{15} =$ Irr

Irr Irr R I.

5 $\frac{5\cancel{\sqrt{2}}}{3\cancel{\sqrt{2}}} = \frac{5}{3}$

$\frac{Irr}{Irr.} = R$

6 $\frac{\sqrt{2}}{5\sqrt{3}} = Irr.$

$\sqrt{2}$ — Irr

$5\sqrt{3}$ — Irr.

Examine, whether the following numbers are rational or irrational:

1. $\sqrt{7}$ I
2. $\sqrt{4}$ R
3. $2+\sqrt{3}$ I
4. $\sqrt{3}+\sqrt{2}$ I
5. $\sqrt{3}+\sqrt{5}$ I
6. $\left(\sqrt{2}-2\right)^{2}$ I

   $=(\sqrt{2})^2+(2)^2-2(\sqrt{2})(2)$

   $=2+4-4\sqrt{2}$

   $=6-4\sqrt{2}$ Irr

   R

   $=R-\text{Irr}$

   $=\text{Irr}$
7. $\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)$ R

   $(2)^2-(\sqrt{2})^2=4-2=2$
8. $\left(\sqrt{2}+\sqrt{3}\right)^{2}$

   $=(\sqrt{2})^2+(\sqrt{3})^2+2\cdot\sqrt{2}\cdot\sqrt{3}$

   $=2+3+2\sqrt{6}$

   $=5+2\sqrt{6}$ Irr.
9. $\sqrt{5}-2$ I
10. $\sqrt{23}$ I
11. $\sqrt{225}$ R
12. 0.3796 R
13. $7.478478\ldots$ R
14. $1.101001000100001\ldots$ I

☞ Give two rational numbers lying between 0.232332333233332 ... and 0.212112111211112.

I

I

$0.213213213213\ldots\ldots = \boxed{0.\overline{213}}$

$\boxed{0.\overline{221}}$

☞ **Find one irrational number between 0.2101 and 0.2222 ...... = $0.\overline{2}$**

R R

$$\boxed{0.219456902952345\ldots\ldots I}$$

Find a rational number and also an irrational number lying between the numbers 0.30300300003 ... and 0.3010010001...

$\boxed{0.3012}$ → R.

$\boxed{0.30134956201932 65 \ldots\ldots}$ I

3010 3030

☞ **Find three different irrational numbers between the rational numbers 5/7 and 9/11**

$\frac{5}{7}$

$0.\overline{714285}$

$\frac{9}{11}$

$0.\overline{81}$

$\boxed{\begin{array}{l} 0.726923402532\ldots\ldots \\ 0.750934625\ldots\ldots \end{array}}$

State whether the given statement is true or false.

π is irrational and 22/7 is rational

A True

B False

$\pi = 3.14\ldots\ldots$

(N.T.N.R)

$\pi \approx 3.14, 22/7$

$\frac{22}{7} \to R$

$3.14 \to R$

$\pi \to I$

Real Numbers

R / Irr.

$\sqrt{5} = (\sqrt{5})^2 = 5$

+ve, 0

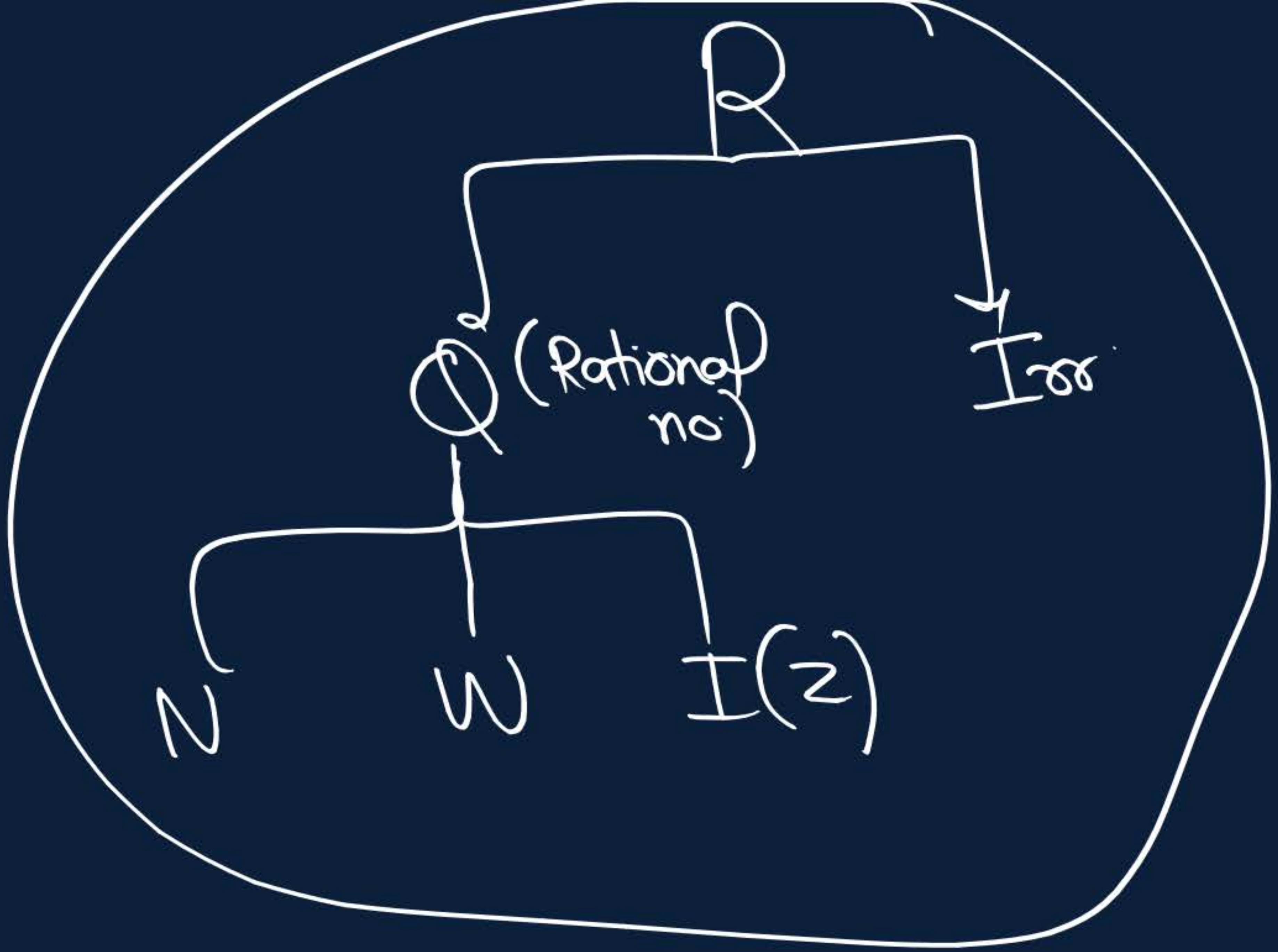
$(\sqrt{-5})^2 = -5$

Real numbers: A number whose square is non-negative, is called a real number. In fact, all rational and all irrational numbers form the collection of all real numbers. Every real number is either rational or irrational.

Consider a real number:

- If it is an integer or it has a terminating or repeating decimal representation then it is rational.
- If it has a nonterminating and nonrepeating decimal representation then it is irrational.

The totality of rationals and irrationals forms the collection of all real numbers.


R
Q (Rational no.)
Irr
N
W
I(z)

Real Numbers

Completeness property:
On the number line, each point corresponds to a unique real number. And, every real number can be represented by a unique point on the real line.

Density property:
Between any two real numbers, there exist infinitely many real numbers

Representation of irrational number on the number line

Example:

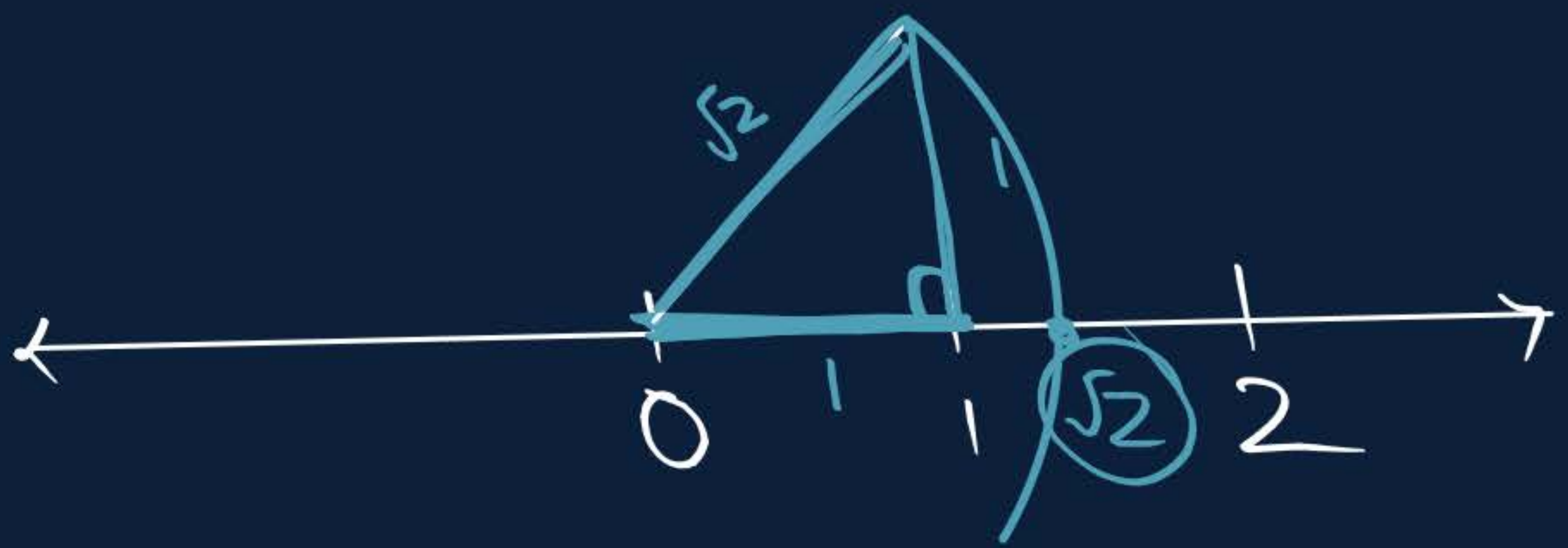
H

Represent $\boxed{\sqrt{2} \text{ on the number line.}}$

$H = \sqrt{2}$

$B = \sqrt{1} = 1$

$P = 1$



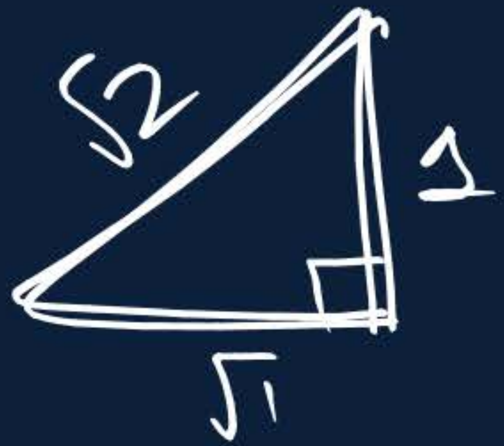
(P.T)

$$H^2 = P^2 + B^2$$

$$P = 1$$

$$B =$$

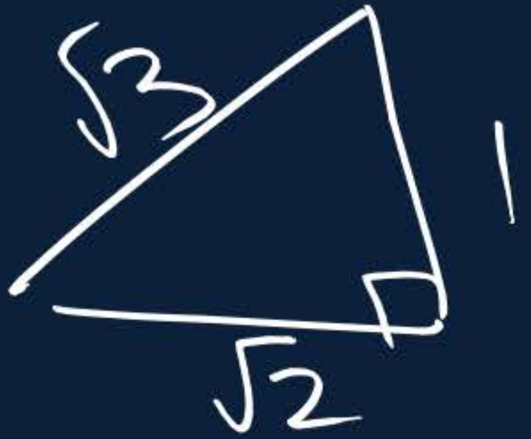
Ex:



$$1^2 + (\sqrt{1})^2 = (\sqrt{2})^2$$

$$1 + 1 = 2$$

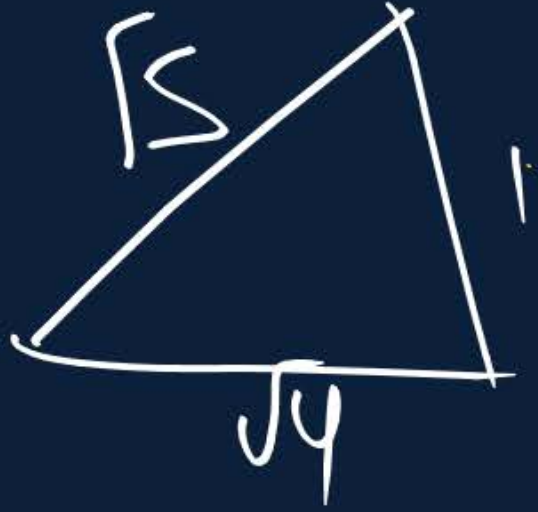
$$2 = 2$$



$$1^2 + (\sqrt{2})^2 = (\sqrt{3})^2$$

$$1 + 2 = 3$$

$$3 = 3$$

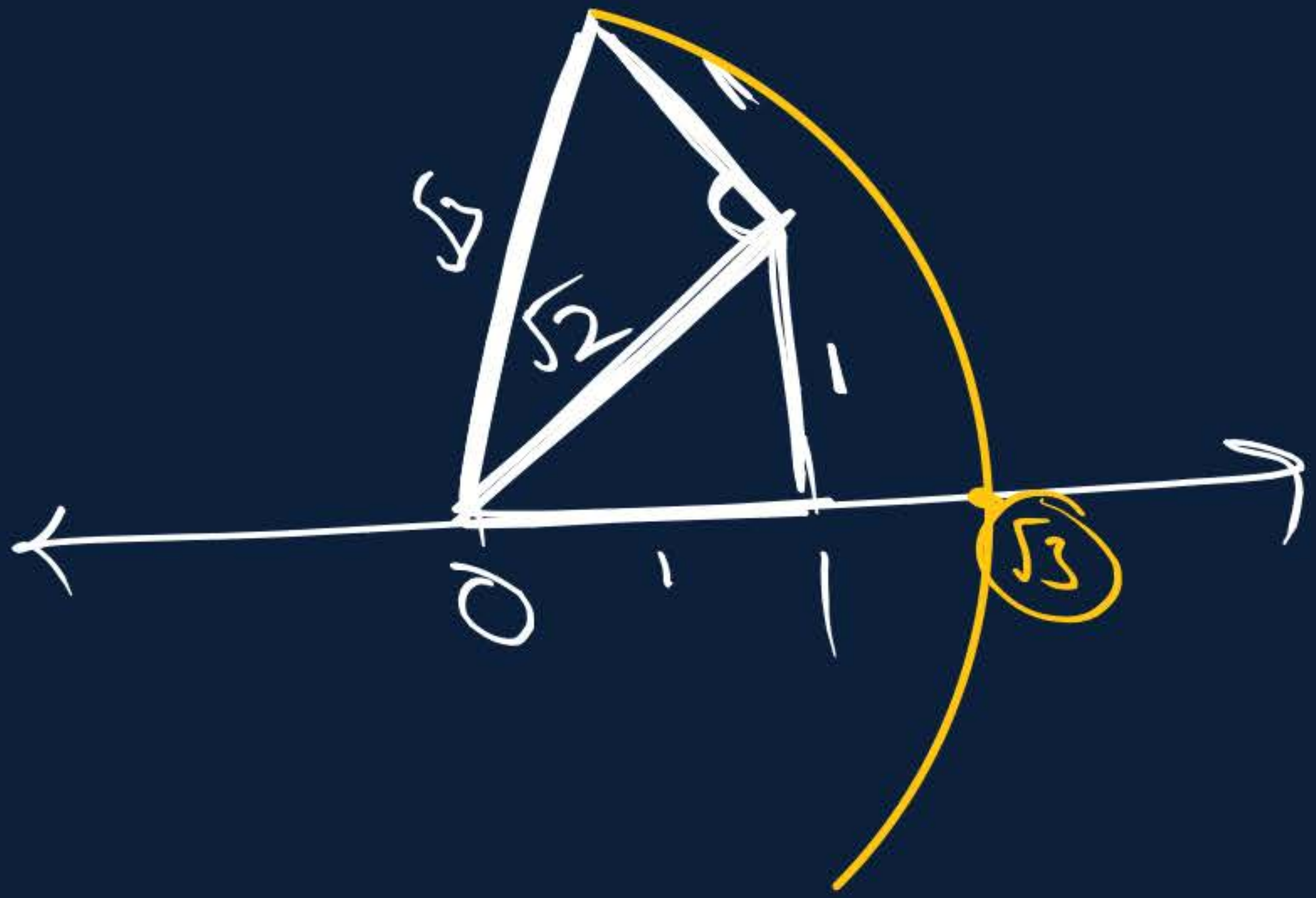


Q $\sqrt{3}$ plot karo.

$H = \sqrt{3}$

$P = 1$

$B = \sqrt{2}$

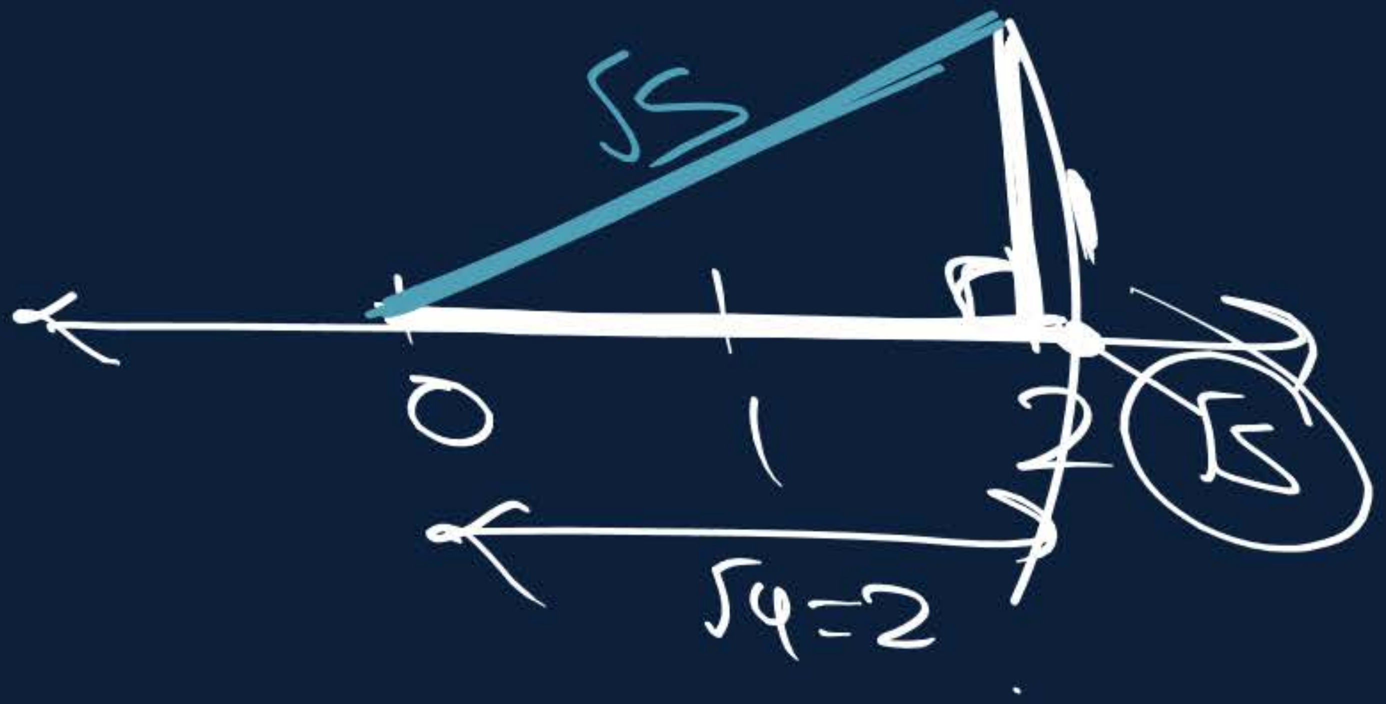


$\underline{\underline{\theta}}$ $\sqrt{5}$

$H = \sqrt{5}$

$P = 1$

$B = \sqrt{4} = 2.$

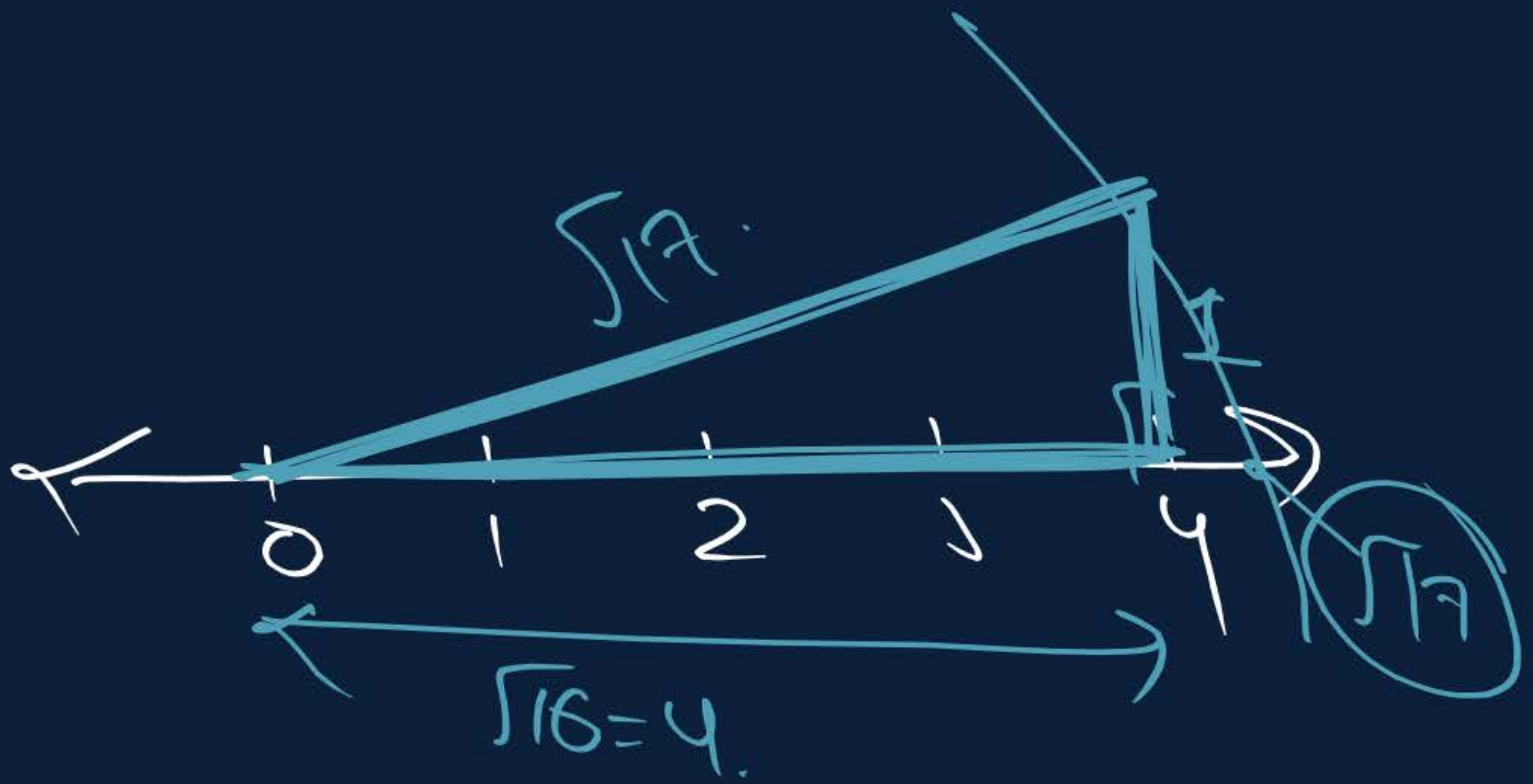


Q $\sqrt{17}$.

$H = \sqrt{17}$

$B = \sqrt{16} = 4$

$P = 1$



Q $\sqrt{6}$.

$B = \sqrt{5}$

$H = \sqrt{6}$

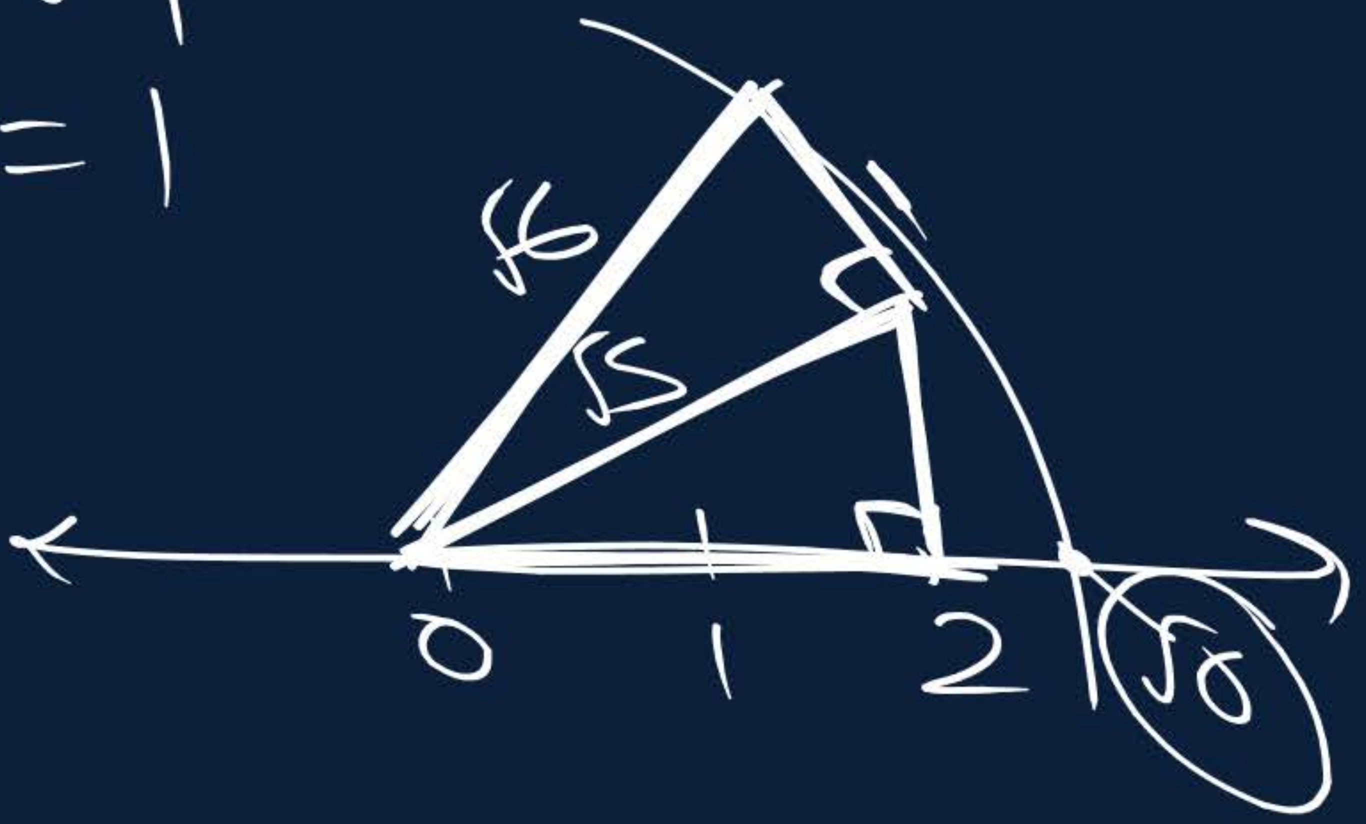
$P = 1$

$\sqrt{5}$.

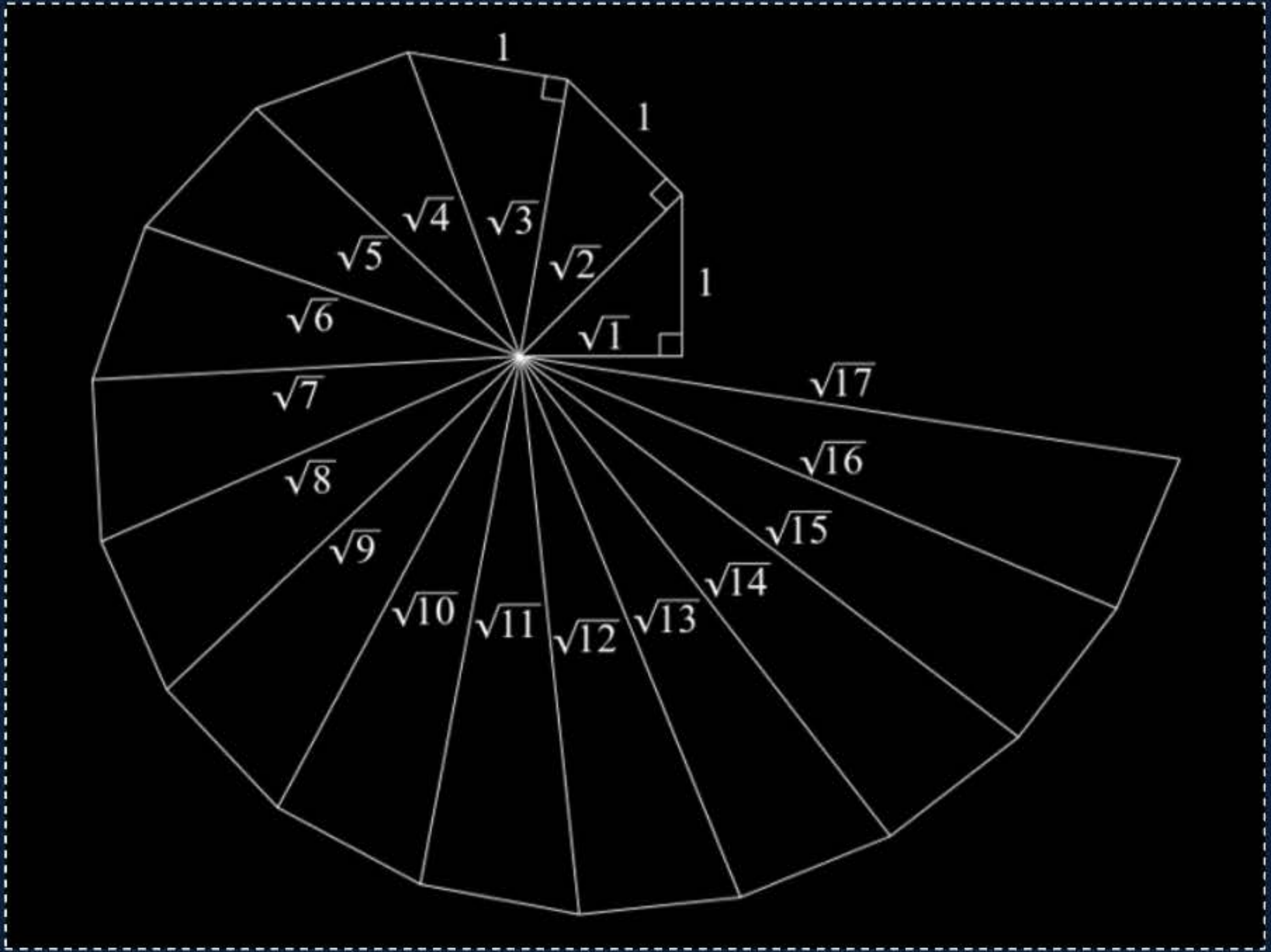
$H = \sqrt{5}$

$B = \sqrt{4} = 2$.

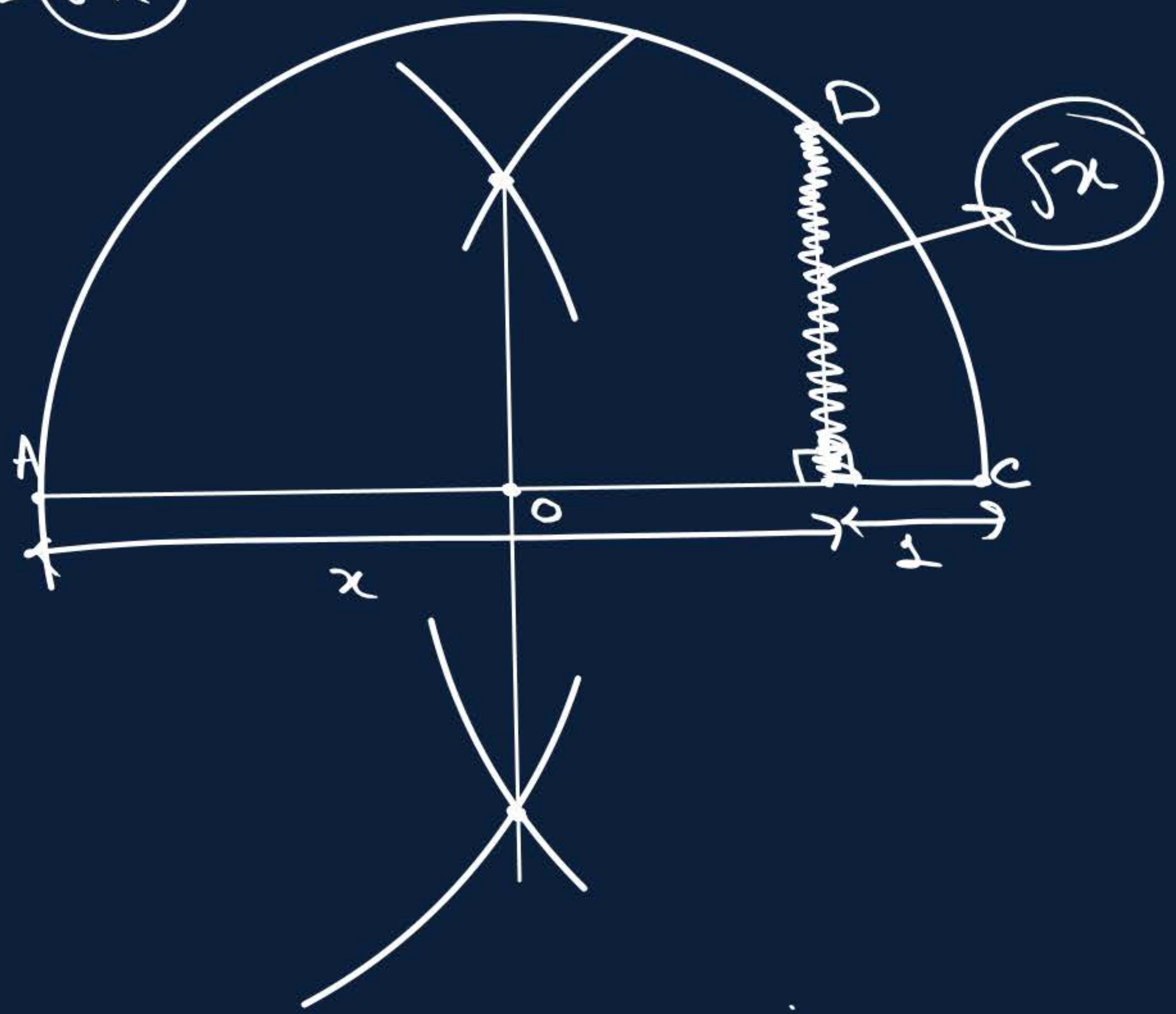
$P = 1$



Square root spiral



Q $\sqrt{9.3}$ → no line par plot karein. $= \sqrt{x}$



Example:

Represent $\sqrt{9.3}$ on the number line.

For any two real number x, we have

$$\sqrt{\left(\frac{x+1}{2}\right)^2-\left(\frac{x-1}{2}\right)^2}=\sqrt{\frac{x^2+2x+1}{4}-\frac{x^2-2x+1}{4}}=\sqrt{\frac{4x}{4}}=\sqrt{x}$$

To find the +ve square root of a positive real number, we follow the following algorithm

Algorithm

Step 1: Obtain the positive real number x

Step 2: Draw a line and mark a point A on it

Step 3: Mark a point B on the line such that AB = x units

Step 4: From a point B mark a distance of 1 unit and mark the new point as C

Step 5: Find the mid-point of AC and mark the point as O

Step 6: Draw a circle with centre O and radius OC

Step 7: Draw a line perpendicular to AC passing through B and intersecting the semi-circle at D. Length BD is equal to $\sqrt{x}$

Rationalisation

Suppose we are given a number whose denominator is irrational. The, the process of converting it into an equivalent expression whose denominator is a rational number by multiplying its numerator and denominator by a suitable number, is called rationalisation.

☞ Rationalise the denominator in each of the following:

1. $\frac{2}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{7}}{7}$

2. $\frac{2}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3 \cdot 3} = \frac{2\sqrt{3}}{9}$

☞ **Rationalize the denominator of** $\frac{5}{\sqrt{3}-\sqrt{5}} \times \frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}+\sqrt{5}}$

$$= \frac{5(\sqrt{3}+\sqrt{5})}{(\sqrt{3})^2-(\sqrt{5})^2}$$

$$= \frac{5(\sqrt{3}+\sqrt{5})}{3-5}$$

$$= \boxed{\frac{5(\sqrt{3}+\sqrt{5})}{-2}}$$

☞ **Rationalize the denominator of** $\frac{1}{7+3\sqrt{2}} \times \frac{7-3\sqrt{2}}{7-3\sqrt{2}}$

$$= \frac{7-3\sqrt{2}}{(7)^2-(3\sqrt{2})^2}$$

$$= \frac{7-3\sqrt{2}}{49-18}$$

$$= \boxed{\frac{7-3\sqrt{2}}{31}}$$

Simplify each of the following by rationalising the denominator:

1. $\frac{7+3\sqrt{5}}{7-3\sqrt{5}} \times \frac{7+3\sqrt{5}}{7+3\sqrt{5}}$

$$= \frac{(7+3\sqrt{5})^2}{(7)^2-(3\sqrt{5})^2}$$

$$= \frac{(7)^2+(3\sqrt{5})^2+2(7)(3\sqrt{5})}{49-45}$$

$$= \frac{49+45+42\sqrt{5}}{4}$$

$$= \frac{94+42\sqrt{5}}{4}$$

$$= \frac{2(47+21\sqrt{5})}{4}$$

$$= \boxed{\frac{47+21\sqrt{5}}{2}}$$

☞ Simplify each of the following by rationalising the denominator:

2 $\dfrac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}} \times \dfrac{2\sqrt{2}-3\sqrt{3}}{2\sqrt{2}-3\sqrt{3}}$

#GPK

If both a and b are rational numbers, find the values of a and b in each of the following equalities:

1. $\dfrac{\sqrt{3}-1}{\sqrt{3}+1} = a + b\sqrt{3}$

L.H.S

$= \dfrac{\sqrt{3}-1}{\sqrt{3}+1} \times \dfrac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \dfrac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2}$

$= \dfrac{(\sqrt{3})^2+(1)^2-2(\sqrt{3})(1)}{3-1}$

$= \dfrac{3+1-2\sqrt{3}}{2}$

$= \dfrac{4-2\sqrt{3}}{2}$

$= \dfrac{\cancel{2}(2-\sqrt{3})}{\cancel{2}}$

$= \boxed{2-\sqrt{3}}$

$\dfrac{\sqrt{3}-1}{\sqrt{3}+1} = a + b\sqrt{3}$

$\boxed{2-\sqrt{3} = a + b\sqrt{3}}$

On comp.

$2 + (-1)\sqrt{3} = a + b\sqrt{3}$

$a = 2,\ b = -1$

Laws of Exponents of Real Numbers

$a, b \rightarrow$ Real no.

$m, n \rightarrow$ Rational no.

1. $a^m \cdot a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $a^m \cdot b^m = (ab)^m$
4. $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$
5. $a^0 = 1$. $(a \neq 0)$
6. $a^{-m} = \frac{1}{a^m}$
7. $(a^m)^n = a^{mn}$

Evaluate each of the following:

1. $5^2 \times 5^4 = 5^{2+4} = 5^6$

2. $5^8 \div 5^3 = \frac{5^8}{5^3} = 5^{8-3} = 5^5$

3. $(3^2)^3 = 3^{2\times 3} = 3^6$

4. $\left(\frac{11}{12}\right)^3 = \frac{11^3}{12^3} = \frac{1331}{1728}$

5. $\left(\frac{3}{4}\right)^{-3} = \frac{3^{-3}}{4^{-3}} = \frac{4^3}{3^3} = \frac{64}{27}$

☞ Find the values of

1. $\left(\frac{8}{27}\right)^{\frac{1}{3}} = \left(\frac{2^3}{3^3}\right)^{1/3} = \left[\left(\frac{2}{3}\right)^3\right]^{1/3} = \left(\frac{2}{3}\right)^{\cancel{3}\times\frac{1}{\cancel{3}}} = \left(\frac{2}{3}\right)^1 = \frac{2}{3}$

2. $\left(\frac{32}{243}\right)^{\frac{1}{5}} = \left(\frac{2^5}{3^5}\right)^{1/5} = \frac{\left(2^{\cancel{5}}\right)^{1/\cancel{5}}}{\left(3^{\cancel{5}}\right)^{1/\cancel{5}}} = \frac{2}{3}$ //

3. $\left(\frac{1}{64}\right)^{6}$

3. $\left(\frac{1}{2^6}\right)^6 = \frac{1^6}{(2^6)^6} = \boxed{\frac{1}{2^{36}}}$

☞ Simplify the following:

1. $(625)^{-\frac{1}{4}} = (5^4)^{-1/4} = 5^{\cancel{4} \times -\frac{1}{\cancel{4}}} = 5^{-1} = \boxed{\frac{1}{5}}$

2. $\left(\frac{256}{81}\right)^{\frac{5}{4}} = \left(\frac{2^8}{3^4}\right)^{5/4} = \frac{(2^{\cancel{8}\,2})^{5/\cancel{4}}}{(3^{\cancel{4}})^{5/\cancel{4}}} = \frac{2^{10}}{3^5} = \frac{1024}{243}$

3. $\left(\frac{243}{32}\right)^{-\frac{4}{5}}$

$= \left(\frac{3^5}{2^5}\right)^{-4/5}$

$= \frac{(3^{\cancel{5}})^{-4/\cancel{5}}}{(2^{\cancel{5}})^{-4/\cancel{5}}} = \frac{3^{-4}}{2^{-4}} = \frac{2^4}{3^4} = \frac{16}{81}$

4. $\sqrt[5]{(32)^{-3}}$

$= \left[(32)^{-3}\right]^{1/5}$

$= 32^{-3/5}$

$= (2^{\cancel{5}})^{-3/\cancel{5}} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

$\longrightarrow$ $\sqrt{2} = 2^{1/2}$

$\longrightarrow$ $\sqrt[3]{2}$ = cube root '2' = $2^{1/3}$

$\longrightarrow$ $\sqrt[4]{2}$ = fourth root of '2' = $2^{1/4}$

$\longrightarrow$ $\sqrt[5]{3}$ = Fifth root of 3 = $3^{1/5}$

$\longrightarrow$ $\sqrt[n]{3}$ = n^{th} root of 3 = $3^{1/n}$

Q $\sqrt[10]{7} = 7^{1/10}$

Q $\sqrt[9]{5} = 5^{1/9}$

☞ **Simplify:**

$$= \left(\frac{81}{16}\right)^{-\frac{3}{4}} \times \left[\left(\frac{25}{9}\right)^{-\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$= \left(\frac{16}{81}\right)^{3/4} \times \left[\left(\frac{9}{25}\right)^{3/2} \div \left(\frac{2}{5}\right)^{3}\right]$$

$$= \left(\frac{2^4}{3^4}\right)^{3/4} \times \left[\left(\frac{3^2}{5^2}\right)^{3/2} \div \frac{2^3}{5^3}\right]$$

$$= \frac{2^3}{3^3}\left[\frac{3^3}{5^3} \times \frac{5^3}{2^3}\right] = \frac{\cancel{2^3}}{\cancel{3^3}} \times \frac{\cancel{3^3}}{\cancel{5^3}} \times \frac{\cancel{5^3}}{\cancel{2^3}} = 1$$

Q $2^3 = 2^x$

$x = ?$

on comp.

$x = 3$

Q $2^5 = 32^x$

$2^5 = (2^5)^x$

$2^5 = 2^{5x}$

on comp.

$5 = 5x$

$1 = x$

☞ **Solve the following equations:**

1. $2^{x-5} = 256$

$2^{x-5} = 2^{8}$

$x-5=8$

$x=13$ //

2. $2^{x+3} = 4^{x+1}$

$2^{x+3} = (2^2)^{x+1}$

$2^{x+3} = 2^{2x+2}$

$x+3=2x+2$

$3-2=2x-x$

$1=x$

☞ **Show that:** $\left(x^{a-b}\right)^{a+b} \cdot \left(x^{b-c}\right)^{b+c} \cdot \left(x^{c-a}\right)^{c+a} = 1$

$$= x^{(a-b)(a+b)} \cdot x^{(b-c)(b+c)} \cdot x^{(c-a)(c+a)}$$

$$= x^{a^2-b^2} \cdot x^{b^2-c^2} \cdot x^{c^2-a^2}$$

$$= x^{\cancel{a^2}-\cancel{b^2}+\cancel{b^2}-\cancel{c^2}+\cancel{c^2}-\cancel{a^2}}$$

$$= x^0$$

$$= 1$$

ONE SHOT

Thank You