



JEE Mains (Dropper)

Sample Paper - V

DURATION : 180 Minutes

M. MARKS : 300

ANSWER KEY

PHYSICS

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83. (8)
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86. (8)
87. (52)
88. (300)
89. (256)
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1. (3)
Work done by the gas = Area of ΔABC

$$= \frac{1}{2} \times (AB)(BC)$$

$$= \frac{1}{2} \times \frac{(450-200)}{10^6} \times (200-120) \times 1000$$

$$= \frac{(250)(80) \times 1000}{2 \times 10^6} = 10 \text{ J}$$
 Work done by the cycle is taken to be negative if the cycle is anticlockwise. $\therefore W = -10 \text{ J}$
2. (4)
The electric field at a distance r from the line charge of linear density λ is given by

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$
 Hence, the field at the negative charge,

$$E_1 = \frac{(4.0 \times 10^{-4})(2 \times 9 \times 10^9)}{0.02} = 3.6 \times 10^8 \text{ N C}^{-1}$$
 The force on the negative charge,

$$F_1 = (3.6 \times 10^8)(2.0 \times 10^{-8})$$

$$= 7.2 \text{ N towards the line charge}$$
 Similarly, the field at the positive charge, i.e., at $r = 0.022 \text{ m}$ is

$$E_2 = 3.3 \times 10^8 \text{ N C}^{-1}$$
 The force on the positive charge,

$$F_2 = (3.3 \times 10^8) \times (2.0 \times 10^{-8})$$

$$= 6.6 \text{ N away from the line charge.}$$
 Hence, the net force on the dipole

$$= 7.2 \text{ N} - 6.6 \text{ N}$$

$$= 0.6 \text{ N towards the line charge}$$
3. (3)
Displacement = area of bigger triangle – area of smaller triangle + area of rectangle

$$= \left[\frac{1}{2}(3 \times 2) - \frac{1}{2}(1 \times 2) + (1 \times 2) \right] = 4 \text{ m}$$
4. (2)
Suppose the current is I in the indicated direction. Applying Kirchhoff's loop law,

$$\epsilon_1 - Ir_1 + \epsilon_2 - Ir_2 + \epsilon_3 - Ir_3 + \dots + \epsilon_n - Ir_n = 0$$
 or,
$$I = \frac{\epsilon_1 + \epsilon_2 + \epsilon_3 + \dots + \epsilon_n}{r_1 + r_2 + r_3 + \dots + r_n}$$

$$= \frac{\epsilon_1 + \epsilon_2 + \epsilon_3 + \dots + \epsilon_n}{k(\epsilon_1 + \epsilon_2 + \epsilon_3 + \dots + \epsilon_n)} = \frac{1}{k}$$
 The potential difference between the terminals of the i^{th} battery is $\epsilon_i - Ir_i = \epsilon_i - \left(\frac{1}{k}\right)(k\epsilon_i) = 0$
5. (2)
Given : $v = ax^{3/2}$ where, $a = 5 \text{ m}^{-1/2} \text{ s}^{-1}$
 Acceleration = $\frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v \frac{dv}{dx} \left(\because v = \frac{dx}{dt} \right)$

$$\text{As } v^2 = a^2 x^3$$

Differentiating both sides with respect to x , we get

$$2v \frac{dv}{dx} = 3a^2 x^2 \text{ or, Acceleration} = \frac{3}{2} a^2 x^2$$

$$\text{Force, } F = \text{Mass} \times \text{Acceleration} = \frac{3}{2} ma^2 x^2$$

$$\text{Work done, } W = \int F dx = \int_0^2 \frac{3}{2} ma^2 x^2 dx$$

$$W = \frac{3}{2} ma^2 \left[\frac{x^3}{3} \right]_0^2 = \frac{3}{2} \times 0.5 \times 5^2 \times \frac{8}{3} = 50 \text{ J}$$

6. (4)

$$\text{Here, } m_A = \frac{m}{2}, m_B = m$$

$$\mu_A = 0.2, \mu_B = 0.1$$

Let both the blocks are moving with common acceleration a . Then,

$$a = \frac{\mu_A m_A g}{m_A} = \mu_A g = 0.2g$$

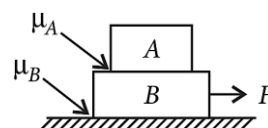
$$\text{and } F - \mu_B (m_B + m_A)g = (m_B + m_A)a$$

$$F = (m_B + m_A)a + \mu_B (m_B + m_A)g$$

$$= \left(m + \frac{m}{2} \right) (0.2g) + (0.1) \left(m + \frac{m}{2} \right) g$$

$$= \left(\frac{3}{2} m \right) (0.2g) + \left(\frac{3}{2} m \right) (0.1g)$$

$$= \frac{0.9}{2} mg = 0.45mg$$



7. (2)

Given: $r = 0.1 \text{ m}$, $N = 10$, $B_H = 0.314 \times 10^{-4} \text{ T}$
 Magnetic field at the centre of current carrying circular coil

$$B = \frac{\mu_0 NI}{2r}$$

Since at neutral point, the magnetic field due to circular coil is completely cancelled by the horizontal component of earth's magnetic field. Therefore,

$$B = B_H \text{ or } \frac{\mu_0 NI}{2r} = B_H$$

$$\therefore I = \frac{2rB_H}{\mu_0 N} = \frac{2 \times 0.1 \times 0.314 \times 10^{-4}}{(4\pi \times 10^{-7}) \times 10} = 0.5 \text{ A}$$

8. (3)

$$\text{Here, } l = 50 \text{ cm} = 50 \times 10^{-2} \text{ m}$$

$$m = 50 \text{ g} = 50 \times 10^{-3} \text{ kg}; I = 5.0 \text{ A}$$

Tension in the wires is zero if the force on the rod due to magnetic field is equal and opposite to the weight of the rod.

$$\text{i.e., } mg = BIl \Rightarrow B = \frac{mg}{Il}$$

Substituting the given values, we get

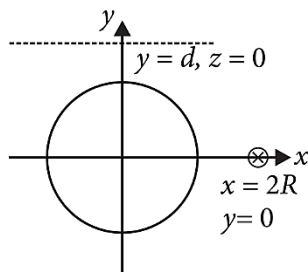
$$B = \frac{50 \times 10^{-3} \times 10}{5 \times 50 \times 10^{-2}} = 0.2 \text{ T}$$

9. (2)

An axis passing through $x = 2R, y = 0$ is in \otimes direction as shown in figure. Moment of inertia about this axis will be

$$I_1 = \frac{1}{2}mR^2 + m(2R)^2 = \frac{9}{2}mR^2 \quad \dots (i)$$

Axis passing through $y = d, z = 0$ is shown by dotted line in figure. Moment of inertia about this axis will be



$$I_2 = \frac{1}{4}mR^2 + md^2 \quad \dots (ii)$$

By equations (i) and (ii), we get

$$\frac{1}{4}mR^2 + md^2 = \frac{9}{2}mR^2 \text{ or } d = \frac{\sqrt{17}}{2}R$$

10. (1)

Here, total length $l = 40 \text{ cm} = 40 \times 10^{-2} \text{ m}$,

Resistivity $= 1.7 \times 10^{-8} \Omega \text{ m}$

The area A of the loop

$$= \left(\frac{40 \text{ cm}}{4} \right) \left(\frac{40 \text{ cm}}{4} \right) = 0.01 \text{ m}^2$$

If the magnetic field at an instant is B , the flux through the frame at that instant will be $\phi = BA$. As the area remains constant, the magnitude of the emf induced will be

$$\varepsilon = \frac{d\phi}{dt} = A \frac{dB}{dt}$$

$$= (0.01 \text{ m}^2) (0.02 \text{ T s}^{-1}) = 2 \times 10^{-4} \text{ V}$$

Resistance of the loop,

$$R = \frac{(1.7 \times 10^{-8} \Omega \text{ m})(40 \times 10^{-2} \text{ m})}{3.14 \times 1 \times 10^{-6} \text{ m}^2}$$

$$= 2.16 \times 10^{-3} \Omega$$

Hence, the current induced in the loop will be

$$I = \frac{2 \times 10^{-4} \text{ V}}{2.16 \times 10^{-3} \Omega} = 9.3 \times 10^{-2} \text{ A} \approx 0.1 \text{ A}$$

11. (1)

Given, $\tau = a \times L + b \times I/\omega$

$$\therefore [a] = \frac{[\tau]}{[L]} = \frac{[I\alpha]}{[I\omega]} = \frac{[\text{T}^{-2}]}{[\text{T}^{-1}]} = [\text{T}^{-1}]$$

$$[b] = \frac{[\tau]}{[I/\omega]} = \frac{[I\alpha]}{[I/\omega]} = [\alpha\omega]$$

$$= [\text{T}^{-2}][\text{T}^{-1}] = [\text{T}^{-3}]$$

$$\therefore [a \times b] = [\text{T}^{-1}][\text{T}^{-3}] = [\text{M}^0 \text{L}^0 \text{T}^{-4}]$$

12. (1)

The electric field at the surface of the sphere is Aa and being radial it is along the outward normal. The flux of the electric field is, therefore,

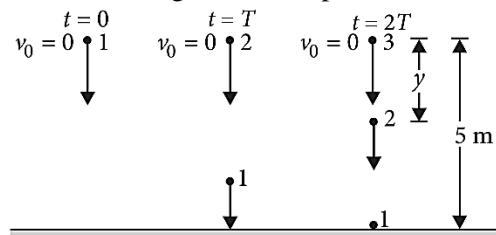
$$\Phi = \oint E dS \cos 0^\circ = Aa(4\pi a^2)$$

From Gauss's law, the charge contained in the sphere is

$$\begin{aligned} Q_{\text{inside}} &= \varepsilon_0 \Phi = 4\pi \varepsilon_0 Aa^3 \\ &= \frac{1}{9 \times 10^9} \times 100 \text{ V m}^{-2} \times (0.20)^3 \\ &= 8.89 \times 10^{-11} \text{ C} \end{aligned}$$

13. (3)

Let T be the time interval between the drops (1, 2, 3) falling from the tap as shown in the figure.



Since distance covered by the first drop in time $2T$ is 5 m,

$$5 = \frac{1}{2}g(2T)^2 = 2gT^2 \quad \dots (i)$$

Further, distance covered by the second drop in time T (from $t = T$ to $t = 2T$),

$$y = \frac{1}{2}gT^2 \quad \dots (ii)$$

From eqns. (i) and (ii), $y = 1.25 \text{ m}$

Distance of the second drop from the ground

$$= 5 - y = 5 - 1.25 = 3.75 \text{ m}$$

14. (3)

The 30 cm length of the scale reads upto 60 kg

$$\therefore \text{Maximum force, } F = 60 \text{ kg wt} = 60 \times 9.8 \text{ N} = 588 \text{ N}$$

and maximum extension, $x = 30 - 0 = 30 \text{ cm}$

$$= 30 \times 10^{-2} \text{ m}$$

Spring constant of the spring balance is

$$k = \frac{F}{x} = \frac{588 \text{ N}}{30 \times 10^{-2} \text{ m}} = 1960 \text{ N/m}$$

Let a body of mass m is suspended from this balance.

Then, time period of oscillation,

$$T = 2\pi\sqrt{\frac{m}{k}} \text{ or } T^2 = \frac{4\pi^2 m}{k}$$

$$\therefore m = \frac{T^2 k}{4\pi^2} = \frac{(0.8)^2 \times (1960)}{4 \times (3.14)^2} = 31.8 \text{ kg}$$

$$\begin{aligned} \text{Weight of the body} &= mg = (31.8 \text{ kg}) (9.8 \text{ m/s}^2) \\ &= 311.64 \text{ N} \end{aligned}$$

15. (4)

$$R_{BC} \text{ (right hand side)} = \frac{8 \times 8}{8+8} \Omega$$

$$= 4 \Omega \text{ (as } 2 \Omega + 4 \Omega + 2 \Omega = 8 \Omega)$$

R_{AD} (right hand side) is again 4Ω .

Equivalent resistance of the circuit,

$$R = 3 \Omega + 4 \Omega + 2 \Omega = 9 \Omega$$

$$\text{Current drawn from battery, } I = \frac{V}{R} = \frac{9}{9} = 1 \text{ A}$$

At A, I is equally divided ($I/2$) between 8Ω resistance and the remaining circuit of 8Ω . At B, ($I/2$) is equally divided ($I/4$) between the 8Ω resistor and the remaining circuit of resistance 8Ω .

Thus, current through 4Ω resistor is $I/4$, i.e., 0.25 A .

16. (1)

Here, $m = 1 \text{ kg}$, $v_i = 2 \text{ ms}^{-1}$, $k = 0.5 \text{ J}$

Initial kinetic energy,

$$K_i = \frac{1}{2} m v_i^2 = \frac{1}{2} \times (1 \text{ kg}) (2 \text{ ms}^{-1})^2 = 2 \text{ J}$$

Work done by retarding force

$$W = \int F_r dx = \int_{0.1}^{2.01} -\frac{k}{x} dx = -k [\ln x]_{0.1}^{2.01}$$

$$= -k \ln \left(\frac{2.01}{0.1} \right) = -0.5 \ln (20.1) = -15 \text{ J}$$

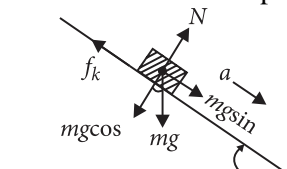
According to work-energy theorem

$$W = K_f - K_i$$

$$\text{or } K_f = W + K_i = -15 \text{ J} + 2 \text{ J} = -13 \text{ J}$$

17. (4)

Let μ_s and μ_k be the coefficient of static and kinetic friction between the box and the plank respectively.



When the angle of inclination θ reaches 30° , the block just slides,

$$\therefore \mu_s = \tan \theta = \tan 30^\circ = \frac{1}{\sqrt{3}} = 0.6$$

If a is the acceleration produced in the block, then

$$ma = mg \sin \theta - f_k$$

$$= mg \sin \theta - \mu_k N$$

(where f_k is force of kinetic friction as $f_k = \mu_k N$)

$$a = g(\sin \theta - \mu_k \cos \theta) \quad (\text{as } N = mg \cos \theta)$$

As $g = 10 \text{ ms}^{-2}$ and $\theta = 30^\circ$

$$\therefore a = (10 \text{ ms}^{-2})(\sin 30^\circ - \mu_k \cos 30^\circ) \quad \dots(i)$$

If s is the distance travelled by the block in time t , then

$$s = \frac{1}{2} a t^2 \text{ (as } u = 0) \text{ or } a = \frac{2s}{t^2}$$

But $s = 4.0 \text{ m}$ and $t = 4.0 \text{ s}$ (given)

$$\therefore a = \frac{2(4.0 \text{ m})}{(4.0 \text{ s})^2} = \frac{1}{2} \text{ ms}^{-2}$$

Substituting this value of a in equation (i), we get

$$\frac{1}{2} \text{ ms}^{-2} = (10 \text{ ms}^{-2}) \left(\frac{1}{2} - \mu_k \frac{\sqrt{3}}{2} \right)$$

$$\frac{1}{10} = 1 - \sqrt{3} \mu_k \text{ or } \sqrt{3} \mu_k = 1 - \frac{1}{10} = \frac{9}{10} = 0.9$$

$$\mu_k = \frac{0.9}{\sqrt{3}} = 0.5$$

18. (4)

The time period T of oscillation of a magnet is given by

$$T = 2\pi \sqrt{\frac{I}{MB}}$$

As I and M remain the same,

$$\therefore T \propto \frac{1}{\sqrt{B}} \text{ or } \frac{T_2}{T_1} = \sqrt{\frac{B_1}{B_2}}$$

According to given problem,

$$B_1 = 24 \mu\text{T}, B_2 = 24 \mu\text{T} - 18 \mu\text{T} = 6 \mu\text{T}, T_1 = 2 \text{ s}$$

$$\therefore T_2 = (2 \text{ s}) \sqrt{\frac{24 \mu\text{T}}{6 \mu\text{T}}} = 4 \text{ s}$$

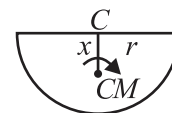
19. (4)

We know, $I_C = mr^2/2$

Using parallel axes theorem,

$$I_C = I_{CM} + mx^2$$

$$\therefore I_{CM} = I_C - mx^2 = mr^2/2 - mx^2$$



20. (4)

If I_1 is the current through R_1 and I_2 is the current through L and R_2 , then $I_1 = \frac{\varepsilon}{R_1}$ and

$$I_2 = I_0(1 - e^{-t/\tau}),$$

$$\text{Where } \tau = \frac{L}{R_2} = \frac{400 \times 10^{-3}}{2} = 0.2 \text{ s}$$

$$\text{and } I_0 = \frac{\varepsilon}{R_2} = \frac{12}{2} = 6 \text{ A}$$

Thus, $I_2 = 6(1 - e^{-t/0.2})$

Potential drop across L , i.e.,

$$e - R_2 I_2 = 12 \text{ V} - 2 \times 6(1 - e^{-t/0.2}) \text{ V}$$

$$= (12e^{-5t}) \text{ V}$$

21. (5)

22. (2)

Case I : The bus acts as the source

$$u_s = 5 \text{ m s}^{-1}, u = 335 \text{ m s}^{-1}, v = 165 \text{ Hz}$$

$$v' = \frac{u \times v}{u - u_s} = \frac{335 \times 165}{335 - 5} = \frac{335}{2} \text{ Hz}$$

Case II: For reflected sound from the wall, bus acts as listener $u_L = 5 \text{ m s}^{-1}$

$$v'' = \frac{(u + u_L)v'}{u} = \frac{(335 + 5) \times 335}{335 \times 2} = 170 \text{ Hz}$$

$$\therefore \text{Number of beats heard per second} = v'' - v$$

$$= 170 - 168 = 2$$

23. (4)

Maximum kinetic energy

$$= \frac{1}{2}mv_{\max}^2 = 9 \text{ eV}$$

Energy of photon emitted during transition of electron in an atom is

$$h\nu = E_i - E_f = \frac{13.6}{n_i^2} - \left(-\frac{13.6}{n_f^2} \right)$$

$$= \frac{-13.6}{3^2} + \frac{13.6}{1^2} = -1.51 + 13.6$$

$$\text{or } h\nu = 12.09 \text{ eV}$$

By Einstein's photoelectric equation

$$= \frac{1}{2}mv_{\max}^2 = h\nu - W_0$$

$$\text{or } W_0 = h\nu - \frac{1}{2}mv_{\max}^2 = 12.09 - 9$$

$$\text{or } \frac{hc}{\lambda_0} = 3.09 \text{ eV} = 3.09 \times 1.6 \times 10^{-19} \text{ J}$$

$$\text{or } \lambda_0 = \frac{hc}{3.09 \times 1.6 \times 10^{-19}}$$

$$\text{or } \lambda_0 = 4 \times 10^{-7} \text{ m} = 4000 \text{ \AA}$$

24. (198)

$$\text{As } E = \frac{1}{2}mv^2, v = \sqrt{\frac{2E}{m}} \approx 2500 \text{ m s}^{-1}$$

Time taken to cover a distance of 5 m i.e.,

$$dt = \frac{5 \text{ m}}{2500 \text{ m s}^{-1}} = 2 \times 10^{-3} \text{ s},$$

$$\text{As } -\frac{dN}{dt} = \lambda N,$$

$$-\frac{dN}{N} = \lambda dt = \frac{0.693}{T_{1/2}} (2 \times 10^{-3} \text{ s})$$

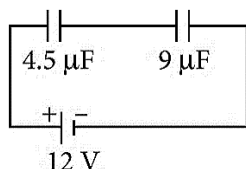
$$= \frac{0.693(2 \times 10^{-3} \text{ s})}{700 \text{ s}} = 198 \times 10^{-8}$$

25. (3)

26. (8)

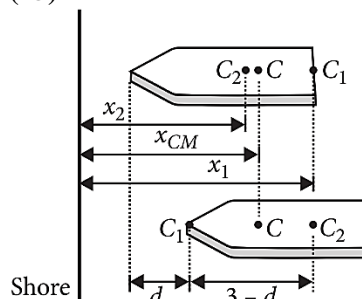
The given arrangement can be redrawn as

\therefore Potential difference across $4.5 \mu\text{F}$ capacitor



$$= \frac{9 \mu\text{F}}{(9 \mu\text{F} + 4.5 \mu\text{F})} \times 12 \text{ V} = 8 \text{ V}$$

27. (75)



As shown in figure, let C_1 , C_2 and C be the centres of mass of the boy, boat and the system (boy and boat) respectively. Let x_1 and x_2 be the distances of C_1 and C_2 from the shore. Then the centre of mass will be at a distance,

$$x_{CM} = \frac{30x_1 + 90x_2}{30 + 90}$$

As the boy moves from the stern to the bow, the boat moves backward through a distance d so that position of the centre of mass of the system remains unchanged.

$$x_{CM} = \frac{30[x_1 - (3 - d)] + 90(x_2 + d)}{30 + 90}$$

$$\text{As } x_{CM} = x_{CM}$$

$$\frac{30(x_1 - 3 + d) + 90(x_2 + d)}{120} = \frac{30x_1 + 90x_2}{120}$$

$$\text{or } -90 + 30d + 90d = 0$$

$$\text{or } d = 0.75 \text{ m}$$

28. (6)

Pressure for soap bubble A

$$P_A = P_0 + \frac{4S}{r_A}$$

$$P_A = 8 \text{ N m}^{-2} + \frac{4 \times 0.04 \text{ N m}^{-1}}{2 \times 10^{-2} \text{ m}}$$

$$P_A = 16 \text{ N m}^{-1}$$

$$P_B = 8 \text{ N m}^{-2} + \frac{4 \times 0.04 \text{ N m}^{-1}}{4 \times 10^{-2} \text{ m}}$$

$$P_B = 12 \text{ N m}^{-2}$$

$$V_A = \frac{4}{3}\pi r_A^3 \text{ and } V_B = \frac{4}{3}\pi r_B^3$$

$$\frac{V_B}{V_A} = \frac{r_B^3}{r_A^3} = \left(\frac{2}{1} \right)^3 \quad \dots (i)$$

Using ideal gas equation

$$PV = nRT$$

$$\text{For bubble A, } P_A V_A = n_A RT \quad \dots (ii)$$

$$\text{For soap bubble B, } P_B V_B = n_B RT \quad \dots (iii)$$

From equation (i), (ii) and (iii)

$$\frac{n_B}{n_A} = \frac{P_B}{P_A} \left(\frac{V_B}{V_A} \right)$$

$$= \frac{12}{16} \times \left(\frac{2}{1} \right)^3 \Rightarrow \frac{n_B}{n_A} = 6$$

29. (45)

Power of the drill,

$$P = 0.2 \text{ hp} = (0.2) (750 \text{ W}) = 150 \text{ W}$$

Work done (W) by the drill in 20 second

$$= P \times 20 \text{ s (as } P = \text{work/time)}$$

$$\text{or } W = (150 \text{ W}) (20 \text{ s}) = 3000 \text{ J} \quad \dots (i)$$

Mass of iron, $m = 100 \text{ g} = 0.1 \text{ kg}$

Specific heat of iron, $c = 450 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$

If ΔT is the rise in temperature of iron,

$$Q = mc \Delta T$$

$$= 0.1 \times 450 \times \Delta T$$

$$= (45 \Delta T) \text{ J} \quad \dots (ii)$$

From equations (i) and (ii),

$$\text{or } 45 \Delta T = 3000$$

$$\text{or } \Delta T = \frac{3000}{45} \text{ }^\circ\text{C}$$

30. (15)

$$\frac{1}{2}mv_{\max}^2 = eV_0 = \frac{hc}{\lambda} - W_0$$

$$\text{or } W_0 = \frac{hc}{\lambda} - eV_0$$

$$\therefore \frac{hc}{\lambda_1} - eV_1 = \frac{hc}{\lambda_2} - eV_2$$

$$\text{or } V_2 = \frac{hc}{e} \left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right) + V_1$$

$$\text{or } V_2 =$$

$$\frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 10^{-9}} \left[\frac{1}{427.2} - \frac{1}{640.2} \right] + 0.54$$

$$\text{or } V_2 = 12.375 \times 10^2 \times \left[\frac{640.2 - 427.2}{427.2 \times 640.2} \right] + 0.54$$

$$\text{or } V_2 = 12.375 \times 10^2 \times \frac{213}{427.2 \times 640.2} + 0.54$$

$$\text{or } V_2 = 1.5 \text{ V}$$

CHEMISTRY

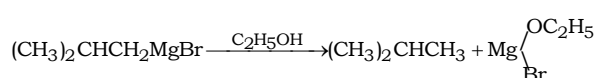
31. (2)

When the temperature is increased energy in form of heat is supplied which increases the kinetic energy of the reacting molecules. This will increase the number collisions and ultimately the rate of reaction will be enhanced.

32. (2)

In lanthanides, there is poorer shielding of 5d electrons by 4f electrons resulting in greater attraction of the nucleus over 5d electrons and contraction for the atomic radii.

33. (2)



34. (3)

Using the relation $K_p = K_c \cdot (RT)^{\Delta n}$, we get

$$\frac{K_p}{K_c} = (RT)^{\Delta n}$$

Thus $\frac{K_p}{K_c}$ will be highest for the reaction having

highest value of Δn . The Δn values for various reactions are:

$$(1) \Delta n = 1 - \left(1 + \frac{1}{2} \right) = -\frac{1}{2}$$

$$(2) \Delta n = 2 - (1 + 1) = 0$$

$$(3) \Delta n = (1 + 1) - 1 = 1$$

$$(4) \Delta n = (2 + 4) - (7 + 2) = -3$$

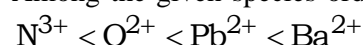
Thus, maximum value of $\Delta n = 1$

35. (4)

According to Fajan's rule: Covalent character

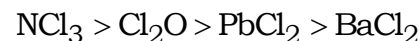
$$\propto \frac{1}{\text{size of cation}} \propto \text{size of anion}$$

Among the given species order of size of cations



Order of size of anion $\text{O}^{2-} > \text{Cl}^-$.

Hence the order of covalent character is



$\therefore \text{BaCl}_2$ is most ionic in nature.

36. (3)

Element	%
C	49.3
H	6.84
O	43.86

Relative no. of atoms

$$49.3 / 12 = 4.1$$

$$1.5 \times 2 = 3$$

$$6.84 / 1 = 6.84$$

$$43.86 / 16 = 2.74$$

Simplest ratio of atoms

$$4.1 / 2.74 = 1.5$$

$$6.84 / 2.74 = 2.5 = 2.5 \times 2 = 5$$

$$2.74 / 2.74 = 1 \quad 1 \times 2 = 2$$

$$\therefore \text{Empirical formula} = \text{C}_3\text{H}_5\text{O}_2$$

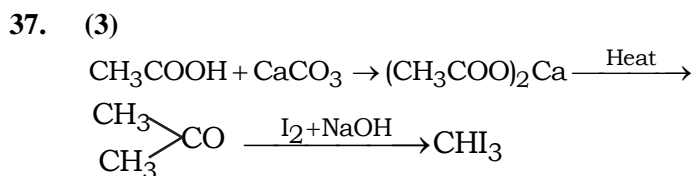
Empirical formula mass

$$= (3 \times 12) + (5 \times 1) + (2 \times 16) = 36 + 5 + 32 = 73$$

$$\text{Molecular} = 2 \times \text{Vapour density} = 2 \times 73 = 146$$

$$n = \frac{\text{molecular mass}}{\text{empirical formula mass}}$$

$$= 146 / 73 = 2 = (\text{C}_3\text{H}_5\text{O}_2) \times 2 = \text{C}_6\text{H}_{10}\text{O}_4$$



38. (2)

$$\Delta G = \Delta H - T\Delta S$$

At equilibrium, $\Delta G = 0$

$$\Rightarrow 0 = (170 \times 10^3 \text{ J}) - T(170 \text{ JK}^{-1})$$

$$\Rightarrow T = 1000 \text{ K}$$

For spontaneity, ΔG is $^{-ve}$, which is possible only if $T > 1000 \text{ K}$

39. (2)

According to gas law $PV = nRT$, $n = \frac{PV}{RT}$

$$\frac{n_A}{n_B} = \frac{\frac{P_1 V_1}{RT_1}}{\frac{P_2 V_2}{RT_2}}; \frac{n_A}{n_B} = \frac{P_1 V_1}{T_1} \times \frac{T_2}{P_2 V_2}$$

$$\frac{n_A}{n_B} = \frac{2P \times 2V}{2T} \times \frac{T}{PV}; \frac{n_A}{n_B} = \frac{2}{1}$$

40. (2)

Due to inert pair effect oxidation state decrease by 2 while going down the group in p-block.

41. (1)

Carbon atom is connected with four different groups in chiral structure.

42. (3)

Sr^{90} is harmful radiological pollutant.

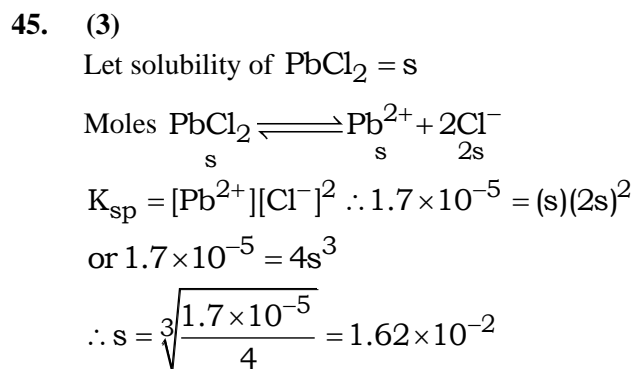
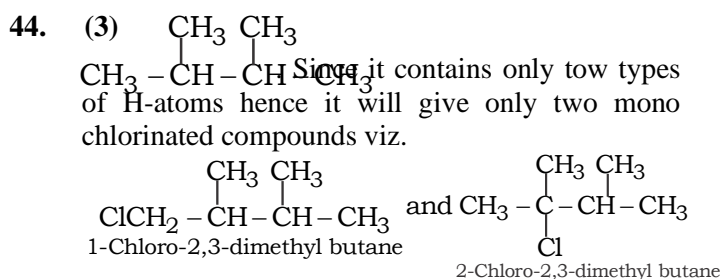
43. (4)

Here, A_2B_3 can also be written as A_4B_6

Since, hcp has six atoms, so 'B' forms hcp lattice and 'A' is present in void.

Total tetrahedral voids = 12

\therefore Fraction of tetrahedral voids occupied by A = $\frac{4}{12} = \frac{1}{3}$



46. (4)

d^4 in high spin octahedral complex

$e_g \uparrow \quad -$

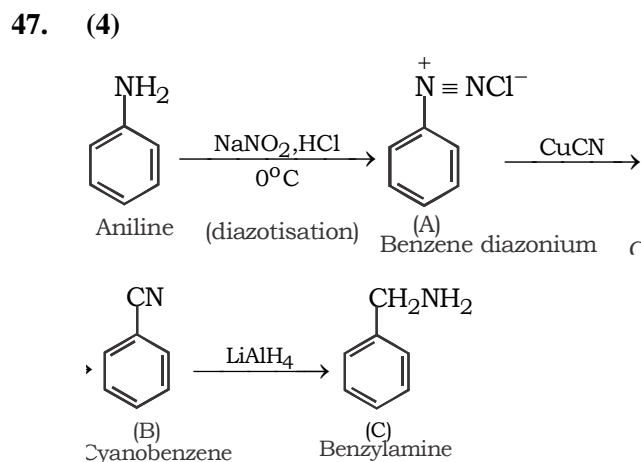
$t_{2g} \uparrow \quad \uparrow \quad \uparrow$

$$\text{CFSE} = (-0.4x + 0.6y)\Delta_0$$

Where, $x \rightarrow$ electrons in t_{2g} orbital

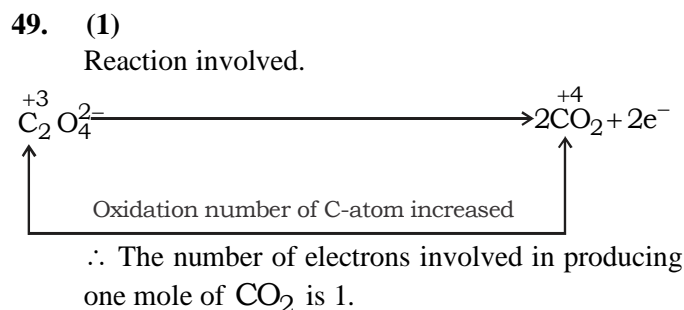
$y \rightarrow$ electrons in e_g orbital

$$\text{CFSE} = [0.6 \times 1] + [-0.4 \times 3] = -0.6 \Delta_0$$



48. (2)

Nylon is a polyamide polymer



50. (4)

$$\lambda = \frac{h}{mv}$$

$$\therefore mv = \frac{h}{\lambda} = \frac{6.625 \times 10^{-34}}{0.33 \times 10^{-9}} = 2.01 \times 10^{-24} \text{ kg m sec}^{-1}$$

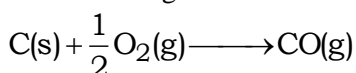
51. (300)

ΔH = Heat of formation at constant pressure

ΔE = Heat of formation at constant volume

$$T = 27^\circ\text{C} = 27 + 273 = 300\text{K}$$

$$R = 2 \text{ cal/degree/mole}$$



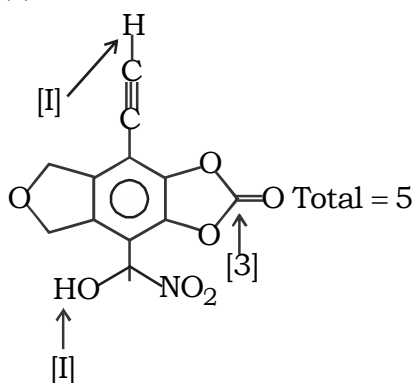
$$\Delta n = n_p - n_r = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\Delta H = \Delta E + \Delta n_g RT \text{ or } \Delta H - \Delta E = \Delta n_g RT$$

$$= \frac{1}{2} \times 2 \times 300 = 300 \text{ cal}$$

\therefore Heat of formation of CO at constant pressure and at constant volume at 27°C will differ from one another by 300 cal.

52. (5)



53. (32)

$$\text{O}_2\% = 20\%$$

$$\text{Metal}\% = 80\%$$

100 g of metal oxide containing 80g metal and 20g oxygen.

\therefore Eq. wt. of metal = mass of metal $\times 8$ / mass of

$$\text{oxygen} = \frac{80 \times 8}{20} = 32\text{g}$$

54. (4)

The total number of isomers for the complex compound

$[\text{Cu}^{\text{II}}(\text{NH}_3)_4][\text{Pt}^{\text{II}}\text{Cl}_4]$ is four.

These four isomers are

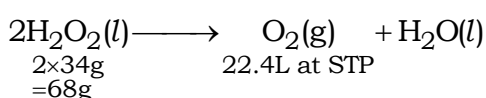
$[\text{Cu}(\text{NH}_3)_3\text{Cl}][\text{Pt}(\text{NH}_3)\text{Cl}_3]$, $[\text{Cu}(\text{NH}_3)\text{Cl}_3][\text{Pt}(\text{NH}_3)_3\text{Cl}]$,

$[\text{CuCl}_4][\text{Pt}(\text{NH}_3)_4]$ and $[\text{Cu}(\text{NH}_3)_4][\text{PtCl}_4]$.

The isomer $[\text{Cu}(\text{NH}_3)_2\text{Cl}_2][\text{Pt}(\text{NH}_3)_2\text{Cl}_2]$ does not exist due to both parts being neutral.

55. (3)

10 volume solution of H_2O_2 means that 1L of this H_2O_2 solution will give 10 L of oxygen at STP.



Thus, 22.4 L of O_2 is produced from 68g H_2O_2 at STP.

$$10\text{L of } \text{O}_2 \text{ at STP is produced from } \frac{68 \times 10}{22.4} \text{ g}$$

$$= 29.9\text{g } \text{H}_2\text{O}_2 = 30\text{g}$$

Therefore, strength of H_2O_2 in 10 volume of H_2O_2 solution, $= 30\text{g/L} = 3\% \text{H}_2\text{O}_2$ solution.

56. (100)

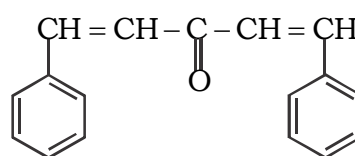
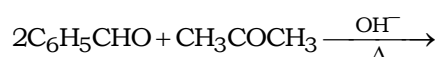
$$1 \text{ L contain} \longrightarrow 10^{-3} \text{ mole } \text{Ca}^{2+}$$

thus; moles of $\text{CaCO}_3 = 10^{-3}$ moles

mass of $\text{CaCO}_3 = 0.1\text{gm}$

$$\text{ppm of } \text{CaCO}_3 = \frac{0.1}{1000} \times 10^6 = 100\text{ppm}$$

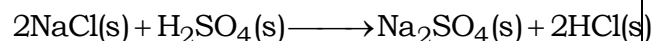
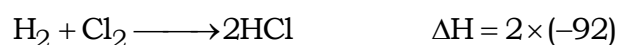
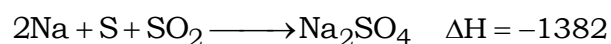
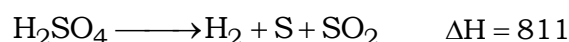
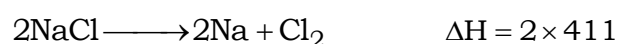
57. (87.179)



$$\text{Molecular mass} = 17 \times 10 + 14 + 16 = 234$$

$$\%C = \frac{204 \times 100}{234} = 87.179\%$$

58. (62.02)



$$\Delta H = 67$$

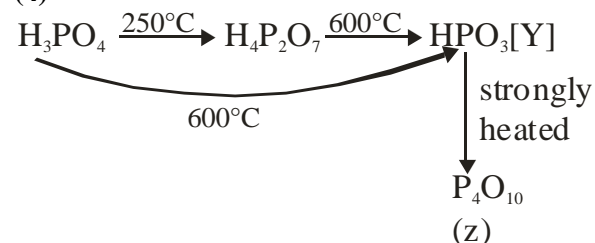
$$\Delta H = \Delta U + \frac{2 \times 8.3 \times 300}{1000}$$

$$\Delta U = 67 - 4.98 = 62.02$$

59. (3)

Only the bicarbonates of Na, K, Rb, Cs and Ammonium exist in solid state.

60. (4)



61. (2)

$$\begin{aligned} & \int_0^{\frac{\pi}{2}} (|\cos x|^3 + |\cos(\pi - x)|^3) dx \\ & \Rightarrow 2 \int_0^{\frac{\pi}{2}} |\cos x|^3 dx \\ & \Rightarrow 2 \int_0^{\frac{\pi}{2}} (\cos x)^3 dx \\ & \Rightarrow 2 \left(\frac{2}{3} \right) = \frac{4}{3} \text{ (By wallis formula)} \end{aligned}$$

62. (4)

$$\begin{aligned} & \int x \sqrt{\frac{2 \sin(x^2 - 1)(1 - \cos(x^2 - 1))}{2 \sin(x^2 - 1)(1 + \cos(x^2 - 1))}} dx \\ & = \int x \frac{\sin\left(\frac{x^2 - 1}{2}\right)}{\cos\left(\frac{x^2 - 1}{2}\right)} dx \\ & = \int x \tan\left(\frac{x^2 - 1}{2}\right) dx \\ & \text{Let } \frac{x^2 - 1}{2} = t \Rightarrow 2x dx = 2dt \\ & = \int \tan(t) dt = \ell n |\sec t| + c \\ & = \ell n \left| \sec\left(\frac{x^2 - 1}{2}\right) \right| + c \end{aligned}$$

63. (2)

Vertex is (2, 0)
 $a = 2$
 Any general point on given parabola can be taken as $(2 + 2t^2, 4t) \forall t \in \mathbb{R}$.
 (8, 6) does not lie on this.

64. (1)

$$\begin{aligned} & \frac{x^2}{\cos^2 \theta} - \frac{y^2}{\sin^2 \theta} = 1 \\ & \because e > 2 \text{ (given)} \\ & e^2 > 4 \Rightarrow 1 + \frac{\sin^2 \theta}{\cos^2 \theta} > 4 \\ & \Rightarrow 1 + \tan^2 \theta > 4 \\ & \tan^2 \theta > 3 \\ & \because \theta \in \left(\frac{\pi}{3}, \frac{\pi}{2} \right) \\ & \text{Latus rectum} = 2 \frac{\sin^2 \theta}{\cos \theta} = 2 \tan \theta \sin \theta \\ & \because \text{for } \theta \in \left(\frac{\pi}{3}, \frac{\pi}{2} \right), 2 \tan \theta \sin \theta \text{ is increasing function} \\ & \text{Hence, latus rectum} \in (3, \infty) \end{aligned}$$

65. (1)

$$\begin{aligned} & x \in \mathbb{R} - \{0, 1\} \\ & f_1(x) = \frac{1}{x}, f_2(x) = 1 - x, f_3(x) = \frac{1}{1 - x} \\ & \text{Given } f_2(J(f_1(x))) = f_3(x) \\ & 1 - J(f_1(x)) = f_3(x) \\ & J(f_1(x)) = 1 - f_3(x) = 1 - \frac{1}{1 - x} \\ & J(f_1(x)) = \frac{x}{x - 1} \\ & J\left(\frac{1}{x}\right) = \frac{x}{x - 1} = \frac{1}{1 - \frac{1}{x}} \\ & J(x) = \frac{1}{1 - x} = f_3(x) \end{aligned}$$

66. (1)

$$\begin{aligned} & \frac{dy}{dx} = \frac{y}{2y \log y + y - x} \Rightarrow \frac{dx}{dy} = \frac{2y \log y}{y} + \frac{y}{y} - \frac{x}{y} = \\ & 2 \log y + 1 - \frac{x}{y} \Rightarrow \frac{dx}{dy} + \frac{x}{y} = 2 \log y + 1 \\ & \Rightarrow y \frac{dx}{dy} + x = 2y \log y + y \\ & \text{IF} = e^{\int \frac{1}{y} dy} \\ & = e^{\log y} = y \\ & \Rightarrow xy = \int (2 \log y + 1) \cdot y dy = y^2 \log y + c \\ & \Rightarrow x = y \log y + \frac{c}{y} \end{aligned}$$

67. (4)

$$\begin{aligned} & a + ar + ar^2 = xar \\ & \text{Since } a \neq 0, \text{ so } \frac{r^2 + r + 1}{r} = x; 1 + r + \frac{1}{r} = x \\ & \because r + \frac{1}{r} \in (-\infty, -2] \cup [2, \infty) \Rightarrow x \in (-\infty, -1] \cup [3, \infty) \end{aligned}$$

68. (1)

$$\begin{aligned} & \cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2} \left(x > \frac{3}{4}\right) \\ & \cos^{-1}\left(\frac{2}{3x} \times \frac{3}{4x} - \sqrt{1 - \frac{4}{9x^2}} \sqrt{1 - \frac{9}{16x^2}}\right) = \frac{\pi}{2} \\ & \Rightarrow \frac{1}{2x^2} = \frac{\sqrt{9x^2 - 4} \sqrt{16x^2 - 9}}{12x^2} \\ & \Rightarrow 6 = \sqrt{9x^2 - 4} \sqrt{16x^2 - 9} \\ & \text{Square both side} \\ & 36 = 144x^4 - 81x^2 - 64x^2 + 36 \\ & \Rightarrow 144x^4 = 145x^2 \\ & \Rightarrow x^4 = \frac{145x^2}{144} \Rightarrow x = \pm \frac{\sqrt{145}}{12}, 0 \\ & \therefore x > \frac{3}{4} \text{ hence, } x = \frac{\sqrt{145}}{12} \end{aligned}$$

69. (2)

$$ty = x + t^2$$

$$\left| \frac{3+t^2}{\sqrt{1+t^2}} \right| = 3$$

$$\Rightarrow t = \sqrt{3}$$

$$\Rightarrow \sqrt{3}y = x + 3$$

70. (4)

By applying Cramer's Rule

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2 - 1 \end{vmatrix}$$

$$= 3(a^2 - 1) - 6 - 2(a^2 - 1) + 4$$

$$= a^2 - 1 - 2 = a^2 - 3$$

If $|a| \neq \pm\sqrt{3} \Rightarrow$ system has unique solution

$$\text{If } |a| = \sqrt{3} \quad \begin{bmatrix} x + y + z = 1 \\ 2x + 3y + 2z = 1 \\ 2x + 3y + 2z = \pm\sqrt{3} + 1 \end{bmatrix}$$

Hence, system is inconsistent for $|a| = \sqrt{3}$

71. (3)

Let the line L be $\frac{x+4}{a} = \frac{y-3}{b} = \frac{z-1}{c}$

$$L \parallel x + 2y - z - 5 = 0$$

L intersects $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$

$$\Rightarrow \begin{vmatrix} 3 & 0 & 1 \\ a & b & c \\ -3 & 2 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 2a + 0b - 6c = 0$$

$$\text{Also } a + 2b - c = 0$$

$$\therefore \frac{a}{3} = \frac{b}{-1} = \frac{c}{1}$$

$$\therefore L \text{ is } \frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$$

72. (1)

$$px + qy + r = 0$$

$$px + qy + \left(\frac{-3p - 2p}{4} \right) = 0$$

$$p \left(x - \frac{3}{4} \right) + q \left(y - \frac{2}{4} \right) = 0$$

$$x = \frac{3}{4} \text{ and } y = \frac{1}{2}.$$

73. (1)

$$(1+x)^n \cong 1 + nx \text{ (when } x \rightarrow 0)$$

$$\text{So, } \sqrt{1+y^4} = 1 + \frac{y^4}{2}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{2 + \frac{y^4}{2}} - \sqrt{2}}{y^4}$$

$$= \frac{\sqrt{2} \left(1 + \frac{y^4}{8} - 1 \right)}{y^4} = \frac{\sqrt{2}}{8} = \frac{1}{4\sqrt{2}}$$

74. (4)

Equation of required plane is

$$(x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0$$

$$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda) = 0$$

Since given plane is parallel to y -axis

$$\Rightarrow 3\lambda + 1 = 0 \Rightarrow \lambda = -\frac{1}{3}$$

Hence equation of plane is $x + 4z - 7 = 0$

75. (3)

$$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$$

By using symmetry

$$A^{-50} = \begin{bmatrix} \cos(-50\theta) & -\sin(-50\theta) \\ \sin(-50\theta) & \cos(-50\theta) \end{bmatrix}$$

$$\text{At } \theta = \frac{\pi}{12}$$

$$A^{-50} = \begin{bmatrix} \cos \frac{25\pi}{6} & \sin \frac{25\pi}{6} \\ -\sin \frac{25\pi}{6} & \cos \frac{25\pi}{6} \end{bmatrix}$$

$$= \begin{bmatrix} \cos \frac{\pi}{6} & \sin \frac{\pi}{6} \\ -\sin \frac{\pi}{6} & \cos \frac{\pi}{6} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

76. (1)

Check all option repeatedly

$$(i) (p \wedge q) \wedge (\sim p \wedge q) \equiv p \vee (q \wedge (\sim p \vee q))$$

$$\equiv p \wedge (q) \equiv p \wedge q$$

\Rightarrow (i) is correct

77. (2)

$$\begin{aligned}
 & 3(1 - \sin 2\theta)^2 + 6(1 + \sin 2\theta) + 4\sin^6 \theta \\
 &= 3(1 - 2\sin 2\theta + \sin^2 2\theta) + 6 + 6\sin 2\theta + 4\sin^6 \theta \\
 &= 9 + 3\sin^2 2\theta + 4\sin^6 \theta \\
 &= 9 + 12\sin^2 \theta \cos^2 \theta + 4(1 - \cos^2 \theta)^3 \\
 &= 9 + 12(1 - \cos^2 \theta)\cos^2 \theta + 4 \\
 &\quad (1 - 3\cos^2 \theta + 3\cos^4 \theta - \cos^6 \theta) \\
 &= 13 + 12\cos^2 \theta - 12\cos^4 \theta - 12\cos^2 \theta + \\
 &\quad 12\cos^4 \theta - 4\cos^6 \theta \\
 &= 13 - 4\cos^6 \theta
 \end{aligned}$$

78. (4)

For $x = 1$

$$\text{R.H.L} = a + b$$

$$\text{L.H.L} = 5$$

So, to be continuous at $x = 1$

$$a + b = 5 \quad \dots(1)$$

for $x = 3$

$$\text{R.H.L} = b + 15$$

$$\text{L.H.L} = a + 3b$$

$$b + 15 = a + 3b$$

$$a + 2b = 15 \quad \dots(ii)$$

for $x = 5$

$$\text{R.H.L} = 30$$

$$\text{L.H.L} = b + 25$$

$$b + 25 = 30$$

$$b = 5.$$

From equation (ii)

$$a = 10$$

But $a = 10$ and $b = 5$ does not satisfied equation (i)

So $f(x)$ is discontinuous for $a \in R$ and $b \in R$

79. (4)

$$z = \frac{3 + 2i \sin \theta}{1 - 2i \sin \theta} \times \frac{1 + 2i \sin \theta}{1 + 2i \sin \theta}$$

$$z = \frac{(3 - 4\sin^2 \theta) + 8i \sin \theta}{1 + 4\sin^2 \theta}$$

For purely imaginary real part should be zero.

$$\Rightarrow 3 - 4\sin^2 \theta = 0$$

$$\text{i.e. } \sin \theta = \pm \frac{\sqrt{3}}{2}$$

$$\theta = -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}, \text{ Sum of all values is}$$

$$-\frac{\pi}{3} + \frac{\pi}{3} + \frac{2\pi}{3} = \frac{2\pi}{3}$$

80. (1)

Length of direct common tangent for circle C_1 and C_2 is

$$AB = \sqrt{(a+b)^2 - (a-b)^2}$$

For C_2 and C_3

Length of direct common tangent is

$$BC = \sqrt{(a+c)^2 - (a-c)^2}$$

For C_1 and C_3

Length of direct common tangent is

$$AC = \sqrt{(b+c)^2 - (b-c)^2}$$

$$AB + BC = AC$$

$$\sqrt{(a+b)^2 - (a-b)^2} + \sqrt{(a+c)^2 - (a-c)^2}$$

$$= \sqrt{(b+c)^2 - (b-c)^2}$$

$$\sqrt{ab} + \sqrt{ac} = \sqrt{bc}$$

$$\frac{1}{\sqrt{c}} + \frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}}$$

81. (19)

$$a = \hat{i} - \hat{j}, b = \hat{i} + \hat{j} + \hat{k}, c = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{a} \times \vec{c} + \vec{b} = 0$$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ x & y & z \end{vmatrix} + (\hat{i} + \hat{j} + \hat{k}) = 0$$

$$\hat{i}(-z) - \hat{j}(z) + \hat{k}(y+x) + \hat{i} + \hat{j} + \hat{k} = 0$$

$$\Rightarrow 1 - z = 0 \Rightarrow z = 1$$

$$\text{Also, } x + y = -1 \text{ and } \vec{a} \cdot \vec{c} = 4$$

$$\Rightarrow x - y = 4$$

$$\Rightarrow x = \frac{3}{2}, y = \frac{5}{2}$$

$$\therefore |\vec{c}|^2 = x^2 + y^2 + z^2 = \frac{9}{4} + \frac{25}{4} + 1 = \frac{38}{4} = \frac{19}{2}$$

82. (49)

$$x \frac{dy}{dx} + 2y = x^2$$

$$\frac{dy}{dx} + \frac{2}{x}y = x$$

This is linear differential equation in $\frac{dy}{dx}$

$$\text{Integrating factor} = e^{\int \frac{2}{x} dx} = x^2$$

Solution of differential equation is $yx^2 = \int x^3 dx$

$$yx^2 = \frac{x^4}{4} + c$$

Curve passes through (1, 1)

$$\text{Then, } c = \frac{3}{4} \quad yx^2 = \frac{x^4 + 3}{4}$$

$$\text{Put } x = \frac{1}{2} \Rightarrow y\left(\frac{1}{4}\right) = \frac{\left(\frac{1}{2}\right)^4 + 3}{4}; y = \frac{49}{16}$$

83. (8)

$$\frac{2^{403}}{15} = \frac{2^3 \cdot 2^{400}}{15} = \frac{8 \cdot (1+15)^{100}}{15}$$

$$= \frac{8 \left({}^{100}C_0 + {}^{100}C_1(15) + {}^{100}C_2(15)^2 + \dots \right)}{15}$$

$$\frac{8}{15} + 8 \left({}^{100}C_1 + {}^{100}C_2(15) + \dots \right)$$

Remainder is 8.

84. (8)

$$y = x^2 + 2 \text{ and } y = 10 - x^2$$

Meet at $(\pm 2, 6)$

$$\Rightarrow m_1 = 4 \text{ and } m_2 = -4$$

$$|\tan \theta| = \frac{8}{15}$$

85. (20)

Let 5 students are x_1, x_2, x_3, x_4, x_5

$$\text{Given } \bar{x} = \frac{\sum x_i}{5} = 150 \Rightarrow \sum_{i=1}^5 x_i = 750 \dots (1)$$

$$\frac{\sum x_i^2}{5} - (\bar{x})^2 = 18 \Rightarrow \frac{\sum x_i^2}{5} - (150)^2 = 18$$

$$\Rightarrow \sum x_i^2 = (22500 + 18) \times 5$$

$$\Rightarrow \sum_{i=1}^5 x_i^2 = 112590 \dots (2)$$

Height of new student = 156 (Let x_6)

$$\text{Then } x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 750 + 156$$

$$\bar{x}_{\text{new}} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = \frac{906}{6} = 151$$

$$, \dots (3)$$

$$\text{Variance (new)} = \frac{\sum x_i^2}{6} - (\bar{x}_{\text{new}})^2$$

From equation (2) and (3)

Variance (new)

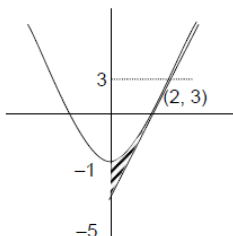
$$= \frac{112590 + (156)^2}{6} - (151)^2 = 22821 - 22801 = 20$$

86. (8)

$$\text{Area} = \int_{-5}^3 x dy - \int_{-1}^3 x dy$$

$$= \int_{-5}^3 \left(\frac{y+5}{4} \right) dy - \int_{-1}^3 \sqrt{y+1} dy$$

$$= \left| \frac{y^2}{2} + 5y \right|_{-5}^3 - \left| \frac{2}{3} (y+1)^{3/2} \right|_{-1}^3$$



$$= \left| \frac{\left(\frac{9}{2} + 15 \right) - \left(\frac{25}{2} - 25 \right)}{4} \right| - \frac{2}{3} [4]^{3/2} - 0 = \frac{8}{3}$$

87. (52)

$$S = \sum_{i=1}^{30} a_i, T = \sum_{i=1}^{15} a_{2i-1}, a_5 = 27, S - 2T = 75$$

$$\text{Let } a_i = a + (i-1)D$$

$$S = a_1 + a_2 + a_3 + \dots + a_{30}$$

$$T = a_1 + a_3 + a_5 + \dots + a_{29}$$

$$\therefore 2T = 2a_1 + 2a_3 + 2a_5 + \dots + 2a_{29}$$

$$S - 2T = (a_2 - a_1) + (a_4 - a_3) + (a_6 - a_5) + \dots + (a_{30} - a_{29}) = 75$$

$$= 15D$$

$$\text{But } S - 2T = 75 \Rightarrow 15D = 75 \Rightarrow D = 5$$

$$\text{Now, } a_5 = 27 \Rightarrow a + 4D = 27$$

$$\therefore a = 27 - 20 \Rightarrow a = 7$$

$$\text{Now, } a_{10} = a + 9D$$

$$= 7 + 45 = 52$$

88. (300)

Number of ways = Total number of ways without restriction - When two specific boys are in team without any restriction, total number of ways of forming team is ${}^7C_3 \times {}^5C_2 = 350$. If two specific boys

B_1, B_2 are in same team then total number of ways of forming team equals to ${}^5C_1 \times {}^5C_2 = 50$ ways, total ways = $350 - 50 = 300$ ways.

89. (256)

$$x^2 + 2x + 2 = 0 \Rightarrow (x+1)^2 = -1$$

$$x = -1 \pm i = \sqrt{2} e^{i\left(\pm \frac{3\pi}{4}\right)}$$

$$\therefore \alpha^{15}, \beta^{15} = (\sqrt{2})^{15} \times 2 \cos\left(15 \cdot \frac{3\pi}{4}\right)$$

$$= 2^8 \sqrt{2} \times \left(-\frac{1}{\sqrt{2}}\right) = 256$$

90. (25)

$$P(x=1) = \frac{4}{52} \times \frac{48}{52} \times 2 = \frac{24}{169}$$

$$P(x=2) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$$

$$\Rightarrow P(x=1) + P(x=2) = \frac{25}{169}$$