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Maths

Matrices

DPP: 1

Q1 If A and B are symmetric matrices of the same order and $X = AB + BA$ and $Y = AB - BA$, then $(XY)^T$ is equal to
(A) XY (B) YX
(C) $-YX$ (D) none of these

Q2 How many 3×3 matrices M with entries from {0,1,2} are there, for which the sum of the diagonal entries of $M^T M$ is 5 ?
(A) 198 (B) 162
(C) 126 (D) 135

Q3 Let A and B be two non-singular matrices such that $\mathrm{A} \neq \mathrm{I}, \mathrm{B}^3 = \mathrm{I}$ and $\mathrm{AB} = \mathrm{BA}^2$, where I is the identity matrix, the least value of k such that $\mathrm{A}^k = \mathrm{I}$ is
(A) 3 (B) 5
(C) 7 (D) 9

Q4 A square matrix P satisfies $P^2 = I - P$, where I is identity matrix. If $P^n = 5I - 8P$, then n is :
(A) 4 (B) 5
(C) 6 (D) 7

Q5 Let $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ is an orthogonal matrix and $abc = \lambda(< 0)$.
The equation whose roots are a, b, c, is
(A) $x^3 - 2x^2 + \lambda = 0$
(B) $x^3 - \lambda x^2 + \lambda x + \lambda = 0$
(C) $x^3 - 2x^2 + 2\lambda x + \lambda = 0$
(D) $x^3 \pm x^2 - \lambda = 0$

Q6 If $A_1, A_3, \ldots, A_{2n-1}$ are n skew-symmetric matrices of same order, then $B = \sum_{r=1}^{n} \left(2r - 1\right) \left(A_{2r-1}\right)^{2r-1}$ will be
(A) symmetric
(B) skew-symmetric
(C) neither symmetric nor skew-symmetric
(D) data not adequate

Q7 Let A, B be two 2×2 matrices. Let $\alpha = \det(A) + \det(B) - \det(A + B)$ and $\beta = \mathrm{tr}(AB) - (\mathrm{tr}\, A)(\mathrm{tr}\, B)$, then
(A) $\alpha = \beta$
(B) $\alpha + \beta = 0$
(C) $\alpha^2 = \beta$
(D) $\alpha = \beta^2$

Q8 Let α be a root of equation $x^2 + x + 1 = 0$ and the matrix $A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$, then the matrix A^{31} is equal to :
(A) A^3 (B) A^2
(C) I_3 (D) A

Q9 If A is a square matrix of order 5 and $2A^{-1} = A^T$, then the remainder when $|adj.\,(adj.\,(adj.\,A))|$ is divided by 7 is
(A) 2 (B) 3
(C) 4 (D) 5

Q10 If $A = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}$, then $\left|A^2 + 3A + 2I\right| + \mathrm{tr}(B + C)$ where B and C are matrices of order 2 with integer elements and $A = B^3 + C^3$, is
(A) -3 (B) 3
(C) 0 (D) 2

Q11 If A and B are two non-singular matrices of order 3 such that $AA^T = 2I$ and $A^{-1} = A^T - A \cdot$ adj. $\left(2B^{-1}\right)$, then det. (B) is equal to
(A) 4 (B) $4\sqrt{2}$
(C) 16 (D) $16\sqrt{2}$

Q12 Let $A=\begin{bmatrix}0 & \alpha\\ 0 & 0\end{bmatrix}$ and $\left(A+I\right)^{50}-50A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$. Then the value of $a+b+c+d$ is

(A) 2 (B) 1

(C) 4 (D) none of these

Q13 If $\begin{bmatrix}4a^2 & 4a & 1\\ 4b^2 & 4b & 1\\ 4c^2 & 4c & 1\end{bmatrix}\begin{bmatrix}f(-1)\\ f(1)\\ f(2)\end{bmatrix}=\begin{bmatrix}3a^2+3a\\ 3b^2+3b\\ 3c^2+3c\end{bmatrix}$, $f(x)$ is a quadratic function and its maximum value occurs at a point V. A is a point of intersection of $y=f(x)$ with the x-axis and point B is such that chord AB subtends a right angle at V. The value of $f(1)$ is:

(A) $3/4$ (B) $-3/4$

(C) 1 (D) None of these

Q14 If $A^{-1}=\begin{bmatrix}\sin^2\alpha & 0 & 0\\ 0 & \sin^2\beta & 0\\ 0 & 0 & \sin^2\gamma\end{bmatrix}$ and $B^{-1}=\begin{bmatrix}\cos^2\alpha & 0 & 0\\ 0 & \cos^2\beta & 0\\ 0 & 0 & \cos^2\gamma\end{bmatrix}$ where α,β,γ are any real numbers and $C=\left(A^{-5}+B^{-5}\right)+5A^{-1}B^{-1}\left(A^{-3}+B^{-3}\right)+10A^{-2}B^{-2}\left(A^{-1}+B^{-1}\right)$ then find |C|.

(A) 0 (B) 1

(C) 2 (D) 3

Q15 Let $P=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$, $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ and $Q=PAP^T$. If $P^TQ^{2007}P=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, then $2a+b-3c-4d$ equal to

(A) 2004 (B) 2005

(C) 2007 (D) 2006

Q16 Let M and N be two 3×3 nonsingular skew-symmetric matrices such that $MN=NM$. If P^T denotes the transpose of P, then $M^2N^2\left(M^{\mathrm{T}}N\right)^{-1}\left(MN^{-1}\right)^T$ is equal to

(A) M^2

(B) $-N^2$

(C) $-M^2$

(D) MN

Q17 A is a 2×2 matrix such that $A\begin{bmatrix}1\\ -1\end{bmatrix}=\begin{bmatrix}-1\\ 2\end{bmatrix}$ and $A^2\begin{bmatrix}1\\ -1\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}$.

The sum of the elements of A is

(A) -1 (B) 0

(C) 2 (D) 5

Q18 Let $P=\begin{bmatrix}1 & 0 & 0\\ 4 & 1 & 0\\ 16 & 4 & 1\end{bmatrix}$ and I be the identity matrix of order 3. If $Q=[q_{ij}]$ is a matrix such that $P^{50}-Q=I$, then $\frac{q_{31}+q_{32}}{q_{21}}$ equals

(A) 52 (B) 103

(C) 201 (D) 205

Q19 Let A and B be two 3×3 matrices such that AB = I and $|A|=\frac{1}{8}$, then $|adj\,(Badj\,(2A))|$ is equal to

(A) 16 (B) 32

(C) 64 (D) 128

Q20 Let P = $\begin{bmatrix}\cos\frac{\pi}{9} & \sin\frac{\pi}{9}\\ -\sin\frac{\pi}{9} & \cos\frac{\pi}{9}\end{bmatrix}$ and $\alpha,\ \beta,\ \gamma$ be non-zero real numbers such that $\alpha P^6+\beta P^3+\gamma I$ is the zero matrix. Then the value of $\left(\alpha^2+\beta^2+\gamma^2\right)^{(\alpha-\beta)\ (\beta-\gamma)\ (\gamma-\alpha)}$

(A) 1 (B) 2

(C) 3 (D) 4

Q21

$A = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$ and $\left(A^8 + A^6 + A^4 + A^2 + I\right) V = \begin{bmatrix} 31 \\ 62 \end{bmatrix}$

(where I is the (2×2) identity matrix),then the product of all elements of matrix V is

(A) 2 (B) 1

(C) 3 (D) 6

Q22 If α and β are the roots of the equation

$$\begin{bmatrix} 1 & 5 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -4 & 7 \end{bmatrix}^2 \begin{bmatrix} \frac{7}{19} & -\frac{3}{19} \\ \frac{4}{19} & \frac{1}{19} \end{bmatrix}^4 \begin{bmatrix} 1 & 3 \\ -4 & 7 \end{bmatrix}^2 \begin{bmatrix} x^2 - 5x + 5 \\ -3 \end{bmatrix} = [-4]$$

then the value of $(2 - \alpha)(2 - \beta)$ is

(A) 51 (B) –12

(C) 12 (D) –7

Q23 If A & B are two matrices of order 3 such that $2A + 3BB^T = I$ and $B^{-1} = A^T$. Let $\alpha = det\left(A^{-1} - 3B^3 + BA\right)$ and $\beta = Tr\left(A^{-1} - 3B^3 - BA\right)$ then value of $\frac{\alpha}{\beta}$ is

(A) 9 (B) 18

(C) 27 (D) 45

Q24 If A is a nilpotent matrix of order 3, such that $|A - 3I| = 27$, $A^2 = O$ and $\frac{|A+3I|^{50}}{3^{147}} = K\left[\frac{1}{10}\sum_{n=1}^{10}\left((-1)^n\left(\sum_{r=1}^{2n-1}\left((n-1)^2 + r\right)\right)\right)\right]$

Then value of K, is equal to

(A) 0.27

(B) 0.36

(C) 0.45

(D) 0.89

Q25 Let A be a 3×3 matrix such that $a_{11} = a_{33} = 2$ and all other $a_{ij} = 1$. Let $A^{-1} = xA^2 + yA + zI$, then the value of $(x + y + z)$ where I is a unit matrix of order 3.

(A) 1 (B) 2

(C) 3 (D) 4

Answer Key

Q1	(C)
Q2	(A)
Q3	(C)
Q4	(C)
Q5	(D)
Q6	(B)
Q7	(A)
Q8	(A)
Q9	(A)
Q10	(A)
Q11	(D)
Q12	(A)
Q13	(A)
Q14	(B)
Q15	(B)
Q16	(C)
Q17	(D)
Q18	(B)
Q19	(C)
Q20	(A)
Q21	(A)
Q22	(B)
Q23	(A)
Q24	(A)
Q25	(A)