

**Basic Mathematics (241)****Marking Scheme**

2023-24

**Section A**1)  $xy^2$  1

2) 20 1

3)  $\frac{1}{2}$  1

4) No Solution 1

5) 0,8 1

6) 5 Unit 1

7)  $\Delta PQR \sim \Delta CAB$  1

8) RHS 1

9)  $70^\circ$  110)  $\frac{3}{4}$  111)  $45^\circ$  112)  $\sin^2 A$  113)  $\pi : 2$  1

14) 7 cm 1

15)  $\frac{1}{6}$  1

16) 15 1

17) 3.5 CM 1

18) 12-18 1

19) Both assertion and reason are true and reason is the correct explanation of assertion. 1

20) Assertion (A) is false but reason(R) is true. 1

## SECTION B

21)  $3x+2y = 8$

$6x - 4y = 9$

$a_1=3, a_2=6, C_1=8$

$b_1=2, b_2=-4, C_2=9$

1

$$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2} \quad \frac{b_1}{b_2} = \frac{2}{-4} = \frac{-1}{2} \quad \frac{c_1}{c_2} = \frac{8}{9}$$

1/2

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

The given pair of lines is consistent.

1/2

22) Given:-  $AB \parallel CD \parallel EF$

To prove:-  $\frac{AB}{ED} = \frac{BF}{FC}$

Constant:- Join BD which intersect EF at G.

Proof:- in  $\Delta ABD$

$EG \parallel AB$  (  $EF \parallel AB$  )

$$\frac{AE}{ED} = \frac{BG}{GD} \quad (\text{by BPT}) \quad (1)$$

In  $\Delta DBC$

$GF \parallel CD$  (  $EF \parallel CD$  )

$$\frac{BF}{FC} = \frac{BG}{GD} \quad (\text{by BPT}) \quad (2)$$

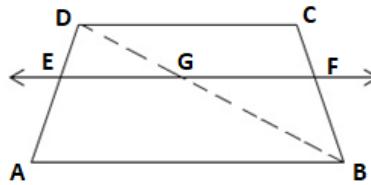
from (1) & (2)

$$\frac{AE}{ED} = \frac{BF}{FC}$$

1/2

1/2

1/2



OR

Given  $AD=6\text{cm}$ ,  $DB=9\text{cm}$

$AE=8\text{cm}$ ,  $EC=12\text{cm}$ ,  $\angle ADE=48^\circ$

To find:-  $\angle ABC=?$

Proof:

In  $\Delta ABC$

Consider,  $\frac{AD}{DB} = \frac{AE}{EC}$

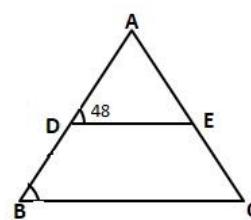
$$\frac{6}{9} = \frac{8}{12}$$

$$\frac{2}{3} = \frac{2}{3}$$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

DEIIBC (Converse of BPT)

$\angle ADE=\angle ABC$  (Corresponding angles)  $\angle ABC=48^\circ$



1

1

23) In  $\Delta OTA$ ,  $\angle OTA = 90^\circ$

By Pythagoras theorem

$$OA^2 = OT^2 + AT^2$$

$$(5)^2 = OT^2 + (4)^2$$

$$25 - 16 = OT^2$$

$$9 = OT^2$$

$$OT = 3\text{cm}$$

$$\text{radius of circle} = 3\text{cm.}$$

24)  $\sin^2 60^\circ + 2 \tan 45^\circ - \cos^2 30^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + 2(1) - \left(\frac{\sqrt{3}}{2}\right)^2$$

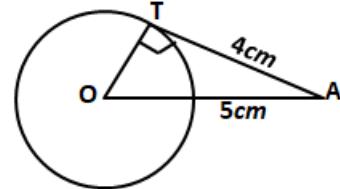
$$= \frac{3}{4} + 2 - \frac{3}{4}$$

$$= 2$$

1/2

1/2

1



25) Area of the circle = sum of areas of 2 circles

$$\pi R^2 = \pi(40)^2 + \pi(9)^2$$

$$\pi R^2 = \pi \times (40^2 + 9)^2$$

$$R^2 = 1600 + 81$$

$$R^2 = 1681$$

$$R = 41\text{ cm.}$$

$$\text{Diameter of given circle} = 41 \times 2 = 82\text{cm}$$

1/2

1/2

1/2

1/2

OR

$$r \text{ of circle} = 10\text{cm} \quad \theta = 90^\circ$$

$$A \text{ of minor segment} = \frac{\theta}{360^\circ} \pi r^2 - A \text{ of } \Delta$$

$$= \frac{\theta}{360^\circ} \times \pi r^2 - \frac{1}{2} \times b \times h$$

$$= \frac{90^\circ}{360^\circ} \times 3.14 \times 10 \times 10 - \frac{1}{2} \times 10 \times 10$$

$$= \frac{314}{4} - 50$$

$$= 78.5 - 50 = 28.5 \text{ cm}^2$$

1/2

1/2

1/2

$$A \text{ of segment} = 28.5 \text{ cm}^2$$

## Section C

26) Let  $\sqrt{3}$  be a rational number

$$\sqrt{3} = \frac{a}{b} \quad \text{where } a \text{ and } b \text{ are co-prime.}$$

1

squaring on both the sides

$$(\sqrt{3})^2 = \left(\frac{a}{b}\right)^2$$

1/2

$$3 = \frac{a^2}{b^2} = a^2 = 3b^2$$

$a^2$  is divisible by 3 so a is also divisible by 3 \_\_\_\_\_ (1)

let  $a=3c$  for any integer c.

$$(3c)^2 = 3b^2$$

1/2

$$ac^2 = 3b^2$$

$$b^2 = 3c^2$$

since  $b^2$  is divisible by 3 so, b is also divisible by 3 \_\_\_\_\_ (2)

From (1) & (2) we can say that 3 is a factor of a and b

1/2

which is contradicting the fact that a and b are co-primes.

Thus, our assumption that  $\sqrt{3}$  is a rational number is wrong.

Hence,  $\sqrt{3}$  is an irrational number.

1/2

27)  $P(S) = 4S^2 - 4S + 1$

$$4S^2 - 4S - 2S + 1 = 0$$

$$2S(2S-1) - 1(2S-1) = 0$$

$$(2S-1)(2S-1) = 0$$

$$S = \frac{1}{2} \quad S = \frac{1}{2}$$

1

$$a = 4 \quad b = -4 \quad c = 1 \quad \alpha = \frac{1}{2} \quad \beta = \frac{1}{2}$$

$$\alpha + \beta = \frac{-b}{a} \quad \alpha \beta = \frac{c}{a}$$

$$\frac{1}{2} + \frac{1}{2} = \frac{-(-4)}{4} \quad \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$$

1

$$\frac{1+1}{2} = \frac{+4}{4} \quad \frac{1}{4} = \frac{1}{4}$$

$$\frac{2}{2} = 1$$

1

$$1 = 1$$

28) Let cost of one bat be Rs x

Let cost of one ball be Rs y

1/2

ATQ

$$4x + 1y = 2050 \quad (1)$$

$$3x + 2y = 1600 \quad (2)$$

1/2

from (1)  $4x + 1y = 2050$

$$y = 2050 - 4x$$

1/2

Substitute value of  $y$  in (2)

$$[3x + 2(2050 - 4x) = 1600]$$

$$3x + 4100 - 8x = 1600$$

$$-5x = -2500$$

$$x = 500$$

Substitute value of  $x$  in (1)

$$4x + 1y = 2050$$

$$4(500) + y = 2050$$

$$2000 + y = 2050$$

$$y = 50$$

1/2

Hence

Cost of one bat=Rs 500

Cost of one ball = Rs 50

**OR**

Let the fixed charge for first 3 days= Rs  $x$

And additional charge after 3 days= RS  $y$

ATQ

$$x + 4y = 27 \quad \dots \dots \dots (1)$$

$$x + 2y = 21 \quad \dots \dots \dots (2)$$

Subtract eq<sup>n</sup> (2) from (1)

$$x + 4y = 27$$

$$x + 2y = 21$$

$$2y = 6$$

$$y = 3$$

Substitute value of  $y$  in (2)

$$x + 2y = 21$$

$$x + 2(3) = 21$$

$$x = 21 - 6$$

$$x = 15$$

Fixed charge= RS 15

Additional charge = Rs 3

1/2

1/2

1/2

1

1

1

1

1

- 29) Given circle touching sides of ABCD at P,Q,R and S

To prove-  $AB+CD=AD+BC$

Proof-

$$AP=AS \quad \dots \dots \dots (1) \quad \text{tangents from same point}$$

$$PB=BQ \quad \dots \dots \dots (2) \quad \text{to a circle are equal in length}$$

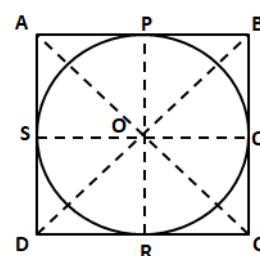
$$DR=DS \quad \dots \dots \dots (3)$$

$$CR=CQ \quad \dots \dots \dots (4)$$

Adding eq<sup>n</sup> (1),(2),(3) & (4)

$$AP+BP+DR+CR=AS+DS+BQ+CQ$$

$$AB+DC=AD+BC$$



30)  $(\cosec \theta - \cot \theta) = \frac{1-\cos\theta}{1+\cos\theta}$

$$\begin{aligned}
 \text{LHS} &= (\cosec \theta - \cot \theta)^2 && \\
 &= \left( \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2 && 1/2 \\
 &= \left( \frac{1-\cos \theta}{\sin \theta} \right)^2 && 1/2 \\
 &= \frac{(1-\cos \theta)^2}{\sin^2 \theta} \\
 &= \frac{(1-\cos \theta)^2}{1-\cos^2 \theta} && 1 \\
 &= \frac{(1-\cos \theta)^2}{(1-\cos \theta)(1+\cos \theta)} \\
 &= \frac{1-\cos \theta}{1+\cos \theta} && 1
 \end{aligned}$$

LHS = RHS

**OR**

$\sec A (1 - \sin A)(\sec A + \tan A) = 1$

$$\begin{aligned}
 \text{LHS} &= \frac{1}{\cos A} (1 - \sin A) \left( \frac{1}{\cos A} - \frac{\sin A}{\cos A} \right) && 1 \\
 &= \frac{(1-\sin A)}{\cos A} \frac{(1+\sin A)}{\cos A} \\
 &= \frac{(1-\sin A)(1+\sin A)}{\cos^2 A} \\
 &= \frac{1-\sin^2 A}{\cos^2 A} \quad (1-\sin^2 A = \cos^2 A) && 1 \\
 &= \frac{\cos^2 A}{\cos^2 A} \\
 &= 1 && 1
 \end{aligned}$$

LHS=RHS

31) Red color balls= 6

Black color balls= 4

Total ball=10 1/2

$$P(S) = \frac{\text{favourable out comes}}{\text{total no of out comes}} \quad 1/2$$

$$P(\text{Red}) = \frac{6}{10} = \frac{3}{5} \quad 1$$

$$P(\text{Not Red}) = 1 - \frac{3}{5} = \frac{5-3}{5} = \frac{2}{5} \quad 1$$

## Section D

32) Let the speed of train be  $x \text{ km/hr}$

1/2

distance= 360 km

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{Time} = \frac{360}{x}$$

1/2

New speed =  $(x + 5) \text{ km/hr}$

$$\text{Time} = \frac{D}{5}$$

$$x + 5 = \frac{360}{\left(\frac{360}{x} - 1\right)}$$

1

$$(x + 5) \left( \frac{360}{x} - 1 \right) = 360$$

$$(x + 5)(360 - x) = 360x$$

$$-x^2 - 5x + 1800 = 0$$

$$x^2 + 5x - 1800 = 0$$

1

$$x^2 + 45x - 40x - 1800 = 0$$

$$x(x + 45) - 40(x + 45) = 0$$

$$(x + 45)(x - 40) = 0$$

1

$$x + 45 = 0 \quad x - 40 = 0$$

$$x = -45 \quad x = 40$$

Speed cannot be negative

Speed of train = 40 km/hr

1

OR

Let the speed of the stream =  $x \text{ km/hr}$

1/2

Speed of boat = 18 km/hr

Upstream speed =  $(18 - x) \text{ km/hr}$

Downstream speed =  $(18 + x) \text{ km/hr}$

1/2

$$\text{Time taken (upstream)} = \frac{24}{(18-x)}$$

$$\text{Time taken (downstream)} = \frac{24}{(18+x)}$$

ATQ

$$\frac{24}{(18-x)} = \frac{24}{(18+x)} + 1$$

1

$$\frac{24}{(18-x)} - \frac{24}{(18+x)} = 1$$

$$24(18 + x) - 24(18 - x) = (18 - x)(18 + x)$$

$$24(18 + x - 18 + x) = (18)^2 - x^2$$

$$24(2x) = 324 - x^2$$

$$48x - 324 + x^2 = 0$$

$$x^2 + 48x - 324 = 0$$

1

$$x^2 - 6x + 54x - 324 = 0$$

$$x(x - 6) + 54(x - 6) = 0$$

$$(x - 6)(x + 54) = 0$$

1

$$x - 6 = 0 \quad x + 54 = 0$$

$$x = 6$$

$$x = -54$$

Speed cannot be negative

Speed of stream=6km/hr

1

33) Given  $\Delta ABC \sim \Delta DEC$

$$\text{To prove } \frac{AD}{DB} = \frac{AE}{EC}$$

Construction: join BE and CD

1/2

Draw DM  $\perp$  AC and EN  $\perp$  CD

$$\text{Proof: or } \Delta ABC = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times AD \times EN \quad \dots \dots \dots (1)$$

$$\text{Or } \Delta ABC = \frac{1}{2} \times DB \times EN \quad \dots \dots \dots (2)$$

Divide eq<sup>n</sup> (1) by (2)

$$\frac{\text{Or } \Delta ABC}{\text{Or } \Delta BDE} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \quad \dots \dots \dots (A)$$

$$\text{Or } \Delta ABC = \frac{1}{2} \times AE \times DM \quad \dots \dots \dots (3)$$

$$\text{Or } \Delta DEC = \frac{1}{2} \times EC \times DM \quad \dots \dots \dots (4)$$

Divide eq<sup>n</sup> (3) by (4)

$$\frac{\text{Or } \Delta ADE}{\text{Or } \Delta DEC} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad \dots \dots \dots (A)$$

$\Delta BDE$  and  $\Delta DEC$  are on the same as DE and between name parallel lines BC and DE

- or ( $BDE$ ) = or ( $DEC$ )

hence

$$\frac{\text{ar } \Delta ADE}{\text{ar } \Delta BDE} = \frac{\text{ar } \Delta ADE}{\text{ar } \Delta DEC}$$

$$\frac{AD}{DB} = \frac{AE}{EC} \quad (\text{from (A) and (B)})$$

Given

$$\frac{PS}{PQ} = \frac{PT}{TR}$$

$$\angle PST = \angle PRQ$$

To prove :- PQR is an isosceles  $\Delta$

$$\text{Proof : } \frac{PS}{PQ} = \frac{PT}{TR}$$

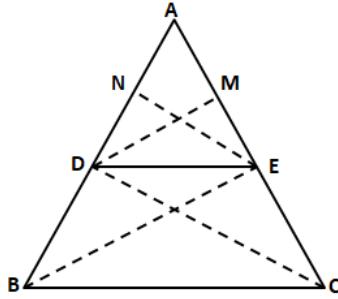
$$\angle PST = \angle PRQ \quad (\text{Corresponding angles})$$

$$\text{But } \angle PST = \angle PRQ$$

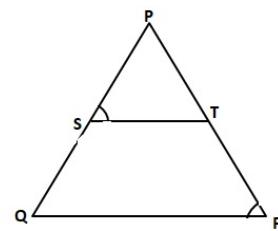
$$\angle PQR = \angle PRQ$$

PR = PQ ( sides opposite to equal angles are equal

-  $\Delta PQR$  is isosceles  $\Delta$ .



1



1

34) Diameter of cylinder and hemisphere = 5mm radius ( $r$ ) =  $\frac{5}{2}$

Total weight = 14mm

Height of cylinder =  $14 - 5 = 9\text{mm}$

CSA of cylinder =  $2\pi rh$

$$= 2 \times \frac{22}{7} \times \frac{5}{2} \times 9$$

$$= \frac{990}{7} \text{ mm}^2$$

CSA of hemispheres =  $2\pi r^2$

$$= 2 \times \frac{22}{7} \times \left(\frac{5}{2}\right)^2$$

$$= \frac{275}{7} \text{ mm}^2$$

CSA of 2 hemispheres =  $2 \times \frac{275}{7}$

$$= \frac{550}{7} \text{ mm}^2$$

Total area of capsule =  $\frac{990}{7} + \frac{550}{7}$

$$= \frac{1540}{7}$$

$$= 220 \text{ mm}^2$$

**OR**

Diameter of cylinder = 2.8 cm

$$r \text{ of cylinder} = \frac{2.8}{2} = 1.4 \text{ cm}$$

$r$  of cylinder =  $r$  of hemisphere = 1.4 cm

Height of cylinder =  $5 - 2.8$

$$= 2.2 \text{ cm}$$

Volume of 1 gulab jamun = vol. of cylinder +  $2 \times$  vol. of hemisphere

$$= \pi r^2 h + 2 \times \frac{2}{3} \pi r^2$$

$$\frac{22}{7} \times (1.4)^2 \times 2.2 + 2 \times \frac{2}{3} \times \frac{22}{7} \times (1.4)^3$$

$$= 13.55 + 11.50$$

$$= 25.05 \text{ cm}^3$$

volume of us gulab jamun =  $45 \times 25.05$

syrup jin 45 jamun =  $30\% \times 45 \times 25.05$

$$= \frac{30}{100} \times 45 \times 25.05$$

$$= 338.185 \text{ cm}^3$$

$$= 338 \text{ cm}^3$$

35)

Life time (in hours)	Number of lamps	Mid x	d	fd
1500-2000	14	1750	-1500	-21000
2000-2500	56	2250	-1000	-56000
2500-3000	60	2750	-500	-30000
3000-3500	86	3250	0	0
3500-4000	74	3750	500	37000
4000-4500	62	4250	1000	62000
4500-5000	48	4750	1500	72000
	400			64000

2

$$\text{Mean} = a + \frac{\sum fd}{\sum f}$$

1/2

$$a = 3250$$

1/2

$$\text{Mean} = 3250 + \frac{64000}{400}$$

1

$$= 3250 + 160$$

$$= 3410$$

Average life of lamp is 3410 hr

1

## Section E

36)  $a_6 = 16000$     $a_9 = 22600$

$$a+5d=16000 \quad \dots \quad (1)$$

$$a=16000-5d$$

$$a+8d=22600 \quad \dots \quad (2)$$

substitute in (2)

$$16000-5d + 8d = 22600$$

$$3d = 22600 - 16000$$

$$3d=6600$$

$$d = \frac{6600}{3} = 2200$$

$$a = 16000 - 5(2200)$$

$$a = 16000 - 11000$$

$$a = 5000$$

$$(i) a_n = 29200 \quad a = 5000 \quad d = 2200$$

$$a_n = a + (n-1)d$$

$$29200 = 5000 + (n - 1)2200$$

$$29200 - 5000 = 2200n - 2200$$

$$24200 + 2200 = 2200n$$

$$26400 = 2200n$$

$$n = \frac{264}{22}$$

$$n = 12$$

in 12<sup>th</sup> year the production was Rs 29200

$$(ii) n=8, \quad a=5000, \quad b=2200$$

$$a_n = a + (n-1)d$$

$$= 5000 + (8-1)2200$$

$$= 5000 + 7 \times 2200$$

$$= 5000 + 15400$$

$$= 20400$$

The production during 8<sup>th</sup> year is = 20400

OR

$$n = 3, \quad a = 5000, \quad b = 2200$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{3}{2} [2(5000) + (3-1) 2200]$$

$$S_3 = \frac{3}{2} (10000 + 2 \times 2200)$$

$$= \frac{3}{2} (10000 + 4400)$$

$$= 3 \times 7200$$

$$= 21600$$

The production during first 3 year is 21600

$$(iii) a_4 = a+3d$$

$$= 5000 + 3 (2200)$$

$$= 5000 + 6600$$

$$= 11600$$

$$a_7 = a+6d$$

$$= 5000 + 6 \times 2200$$

$$= 5000 + 13200$$

$$= 18200$$

$$a_7 - a_4 = 18200 - 11600 = 7400$$

37) coordinates of A (2,3)- Alia is house

coordinates of B (2,1)- Shagun is house

coordinates of C (4,1)- library

$$(i) AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(2 - 2)^2 + (1 - 3)^2}$$

$$= \sqrt{(0^2 + (-2))^2}$$

$$AB = \sqrt{0 + 4} = \sqrt{4} \text{ unit} = 2 \text{ units}$$

Alia's house from shagun's house is 2 unit

$$(ii) C(4,1), B(2,1)$$

$$CB = \sqrt{(2 - 4)^2 + (1 - 1)^2}$$

$$= \sqrt{(-2)^2 + 0^2}$$

$$= \sqrt{4 + 0} = \sqrt{4} = 2 \text{ unit}$$

$$(iii) O(0,0), B(2,1)$$

$$OB = \sqrt{(2 - 0)^2 + (1 - 0)^2}$$

$$= \sqrt{2^2 + 1^2} = \sqrt{4 + 1} = \sqrt{5} \text{ units}$$

Distance between Alia's house and Shagun's house AB = 2 units

Distance between Library and Shagun's house CB = 2 units

OB is greater than AB and CB,

For shagun, school [O] is farther than Alia's house [A] and Library [C]

**OR**

$$C(4,1) A(2,3)$$

$$CA = \sqrt{(2 - 4)^2 + (3 - 1)^2}$$

$$= \sqrt{(-2)^2 + 2^2} = \sqrt{4 + 4} = \sqrt{8}$$

$$= 2\sqrt{2} \text{ units} \quad AC^2 = 8$$

Distance between Alia's house and Shagun's house AB = 2 units

Distance between Library and Shagun's house CB = 2 units

$$AC^2 + BC^2 = 2^2 + 2^2 = 4 + 4 = 8$$

Therefore A,B and C form a right triangle.

38) (i) XY || CD and AC is transversal.

$$\angle ACD = \angle CAX (\text{alt.int } \angle S)$$

$$\angle ACD = 30^\circ$$

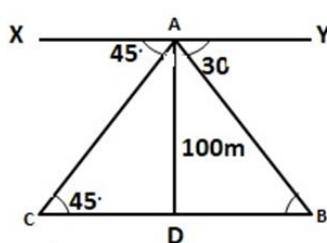
$$(ii) \angle YAB = 30^\circ$$

$$\angle ABD = 30^\circ$$

Because XY || CD and AB is a transversal

so alternate interior angles are equal

$$\angle YAB = \angle ABD$$



(iii) CD=?

$$\ln \Delta ADC \theta = 45^\circ$$

$$\tan \theta = \frac{P}{B}$$

$$\tan 45^\circ = \frac{100}{B}$$

$$1 = \frac{100}{B}$$

$$B=100\text{m}$$

$$CD = 100\text{m}$$

1/2

1/2

1

OR

BD=?

$$\ln \Delta ABD \theta = 30^\circ$$

$$\tan \theta = \frac{P}{B}$$

$$\tan 30 = \frac{100}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{BD}$$

$$BD = 100\sqrt{3} \text{ m}$$

1/2

1/2

1