Part-A: 1-Mark Questions

1. If the Hamiltonian of a classical particle is $H = \dfrac{p_x^2 + p_y^2}{2m} + xy$, then $\langle x^2 + xy + y^2 \rangle$ at temperature T is equal to

(A) $k_B T$.

(B) $\dfrac{1}{2} k_B T$.

(C) $2\, k_B T$.

(D) $\dfrac{3}{2} k_B T$.

2. Two equal positive charges of magnitude $+q$ separated by a distance d are surrounded by a uniformly charged thin spherical shell of radius $2d$ bearing a total charge $-2q$ and centred at the midpoint between the two positive charges. The net electric field at distance r from the midpoint ($\gg d$) is

(A) zero.

(B) proportional to d.

(C) proportional to $1/r^3$.

(D) proportional to $1/r^4$.

3. Let $\Lambda = \begin{pmatrix} 1 & 0 \\ 0 & 11 \end{pmatrix}$ and $M = \begin{pmatrix} 10 & 3i \\ -3i & 2 \end{pmatrix}$. Similarity transformation of M to Λ can be performed by

(A) $\dfrac{1}{\sqrt{10}} \begin{pmatrix} 1 & 3i \\ 3i & 1 \end{pmatrix}$.

(B) $\dfrac{1}{\sqrt{9}} \begin{pmatrix} 1 & -3i \\ 3i & 11 \end{pmatrix}$.

(C) $\dfrac{1}{\sqrt{10}} \begin{pmatrix} 1 & 3i \\ -3i & 11 \end{pmatrix}$.

(D) $\dfrac{1}{\sqrt{9}} \begin{pmatrix} 1 & 3i \\ -3i & 1 \end{pmatrix}$.

4. After the detonation of an atom bomb, the spherical ball of gas was found to be of 15 meter radius at a temperature of 3×10^5 K. Given the adiabatic expansion coefficient $\gamma = 5/3$, what will be the radius of the ball when its temperature reduces to 3×10^3 K?

(A) 156 m

(B) 50 m

(C) 150 m

(D) 100 m

5. A possible Lagrangian for a free particle is

(A) $L = \dot{q}^2 - q^2$.

(B) $L = \dot{q}^2 - q\dot{q}$.

(C) $L = \dot{q}^2 - q$.

(D) $L = \dot{q}^2 - \frac{1}{q}$.

6. If the ground state wavefunction of a particle moving in a one dimensional potential is proportional to $\exp(-x^2/2)\cosh(\sqrt{2}x)$, then the potential in suitable units such that $\hbar = 1$, is proportional to

(A) x^2.

(B) $x^2 - 2\sqrt{2}x \tanh\left(\sqrt{2}x\right)$.

(C) $x^2 - 2\sqrt{2}x \tan\left(\sqrt{2}x\right)$.

(D) $x^2 - 2\sqrt{2}x \coth\left(\sqrt{2}x\right)$.

7. $\phi_0(x)$ and $\phi_1(x)$ are respectively the orthonormal wavefunctions of the ground and first excited states of a one dimensional simple harmonic oscillator. Consider the normalised wave function $\psi(x) = c_0\phi_0(x) + c_1\phi_1(x)$, where c_0 and c_1 are real. For what values of c_0 and c_1 will $\langle\psi(x)|x|\psi(x)\rangle$ be maximized?

(A) $c_0 = c_1 = +1/\sqrt{2}$

(B) $c_0 = -c_1 = +1/\sqrt{2}$

(C) $c_0 = +\sqrt{3}/2,\ c_1 = +1/2$

(D) $c_0 = +\sqrt{3}/2,\ c_1 = -1/2$

8. What is the equation of the plane which is tangent to the surface $xyz = 4$ at the point $(1, 2, 2)$?

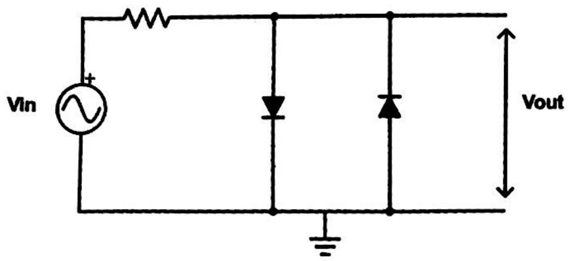
(A) $x + 2y + 4z = 12$

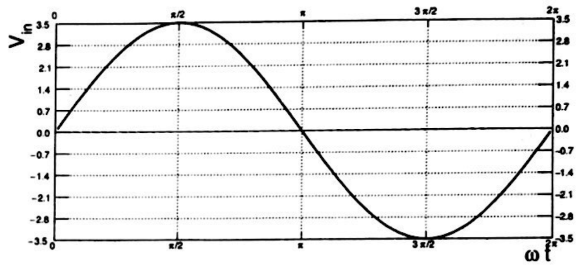
(B) $4x + 2y + z = 12$

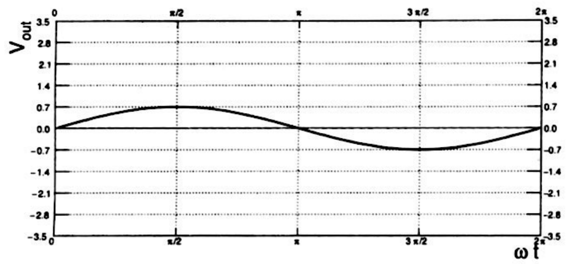
(C) $x + 4y + z = 0$

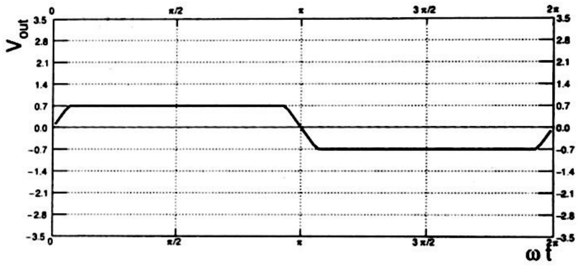
(D) $2x + y + z = 6$

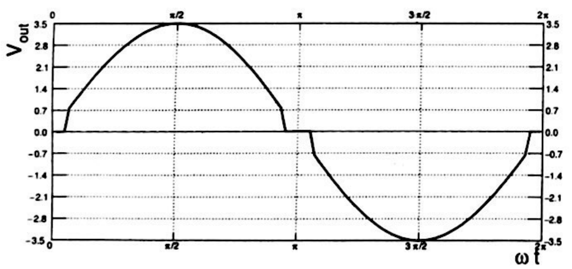
9. In the following silicon diode circuit ($V_B = 0.7$ V), determine the output voltage waveform (V_{out}) for the given input wave.

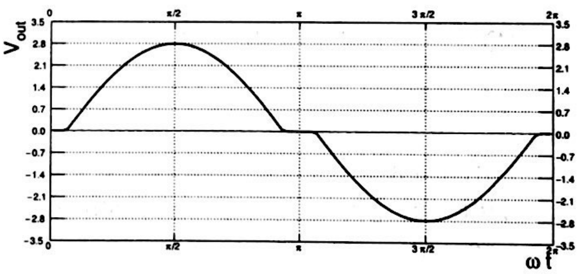




(A) 

(B) 

(C) 

(D) 

10. (Q_1, Q_2, P_1, P_2) and (q_1, q_2, p_1, p_2) are two sets of canonical coordinates, where Q_i and q_i are the coordinates and P_i and p_i are the corresponding conjugate momenta. If $P_1 = q_2$ and $P_2 = p_1$, then which of the following relations is true?

(A) $Q_1 = q_1, Q_2 = p_2$

(B) $Q_1 = p_2, Q_2 = q_1$

(C) $Q_1 = -p_2, Q_2 = q_1$

(D) $Q_1 = q_1, Q_2 = -p_2$

11. A bead of mass M slides along a parabolic wire described by $z = 2(x^2 + y^2)$. The wire rotates with angular velocity Ω about the z-axis. At what value of Ω does the bead maintain a constant nonzero height under the action of gravity along $-\hat{z}$?

(A) $\sqrt{3g}$

(B) $\sqrt{g}$

(C) $\sqrt{2g}$

(D) $\sqrt{4g}$

12. A thin air film of thickness d is formed in a glass medium. For normal incidence, the condition for constructive interference in the reflected beam is (in terms of wavelength λ and integer $m = 0, 1, 2...$)

(A) $2d = (m - 1/2)\lambda$.

(B) $2d = m\lambda$.

(C) $2d = (m - 1)\lambda$.

(D) $2\lambda = (m - 1/2)d$.

13. Consider magnetic vector potential $\vec{A}$ and scalar potential Φ which define the magnetic field $\vec{B}$ and electric field $\vec{E}$. If one adds $-\vec{\nabla}\lambda$ to $\vec{A}$ for a well-defined λ, then what should be added to Φ so that $\vec{E}$ remains unchanged up to an arbitrary function of time, $f(t)$?

(A) $\frac{\partial \lambda}{\partial t}$

(B) $-\frac{\partial \lambda}{\partial t}$

(C) $\frac{1}{2}\frac{\partial \lambda}{\partial t}$

(D) $-\frac{1}{2}\frac{\partial \lambda}{\partial t}$

14. $\int_{-\infty}^{+\infty} (x^2 + 1)\, \delta(x^2 - 3x + 2)\, dx = ?$

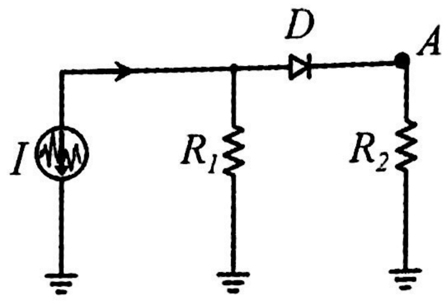
(A) 1

(B) 2

(C) 5

(D) 7

15. Consider the circuit shown in the figure where $R_1 = 2.07k\Omega$ and $R_2 = 1.93k\Omega$. Current source I delivers 10 mA current. The potential across the diode D is 0.7 V. What is the potential at A?



(A) 10.35 V

(B) 9.65 V

(C) 19.30 V

(D) 4.83 V

16. Suppose that we toss two fair coins hundred times each. The probability that the same number of *heads* occur for both coins at the end of the experiment is

(A) $\left(\frac{1}{4}\right)^{100} \sum_{n=0}^{100} \binom{100}{n}$

(B) $2\left(\frac{1}{4}\right)^{100} \sum_{n=0}^{100} \binom{100}{n}^2$

(C) $\frac{1}{2}\left(\frac{1}{4}\right)^{100} \sum_{n=0}^{100} \binom{100}{n}^2$

(D) $\left(\frac{1}{4}\right)^{100} \sum_{n=0}^{100} \binom{100}{n}^2$

17. If the mean square fluctuations in energy of a system in equilibrium at temperature T is proportional to T^α, then the energy of the system is proportional to

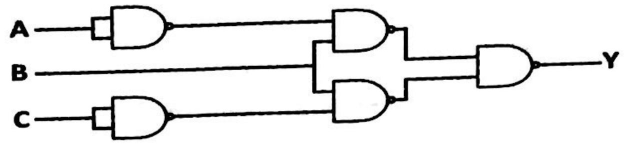
(A) $T^{\alpha-2}$.

(B) $T^{\frac{\alpha}{2}}$.

(C) $T^{\alpha-1}$.

(D) T^α.

18. What is Y for the circuit shown below?



(A) $Y = \overline{(A+\bar{B})(\bar{B}+C)}$

(B) $Y = \overline{(A+\bar{B})(B+C)}$

(C) $Y = \overline{(\bar{A}+B)(\bar{B}+C)}$

(D) $Y = \overline{(A+B)(\bar{B}+C)}$

19. What is the dimension of $\frac{\hbar}{i}\frac{\partial \psi}{\partial x}$, where ψ is a wavefunction in two dimensions?

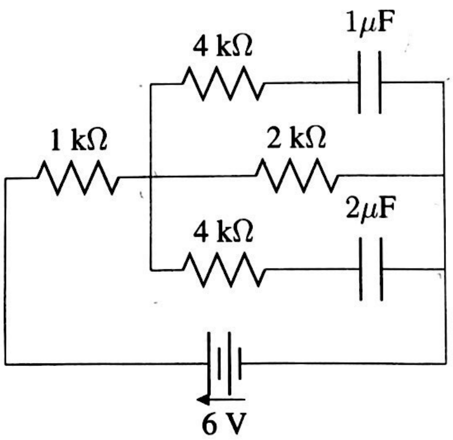
(A) kg m^{-1} s^{-2}

(B) kg s^{-2}

(C) kg m^2 s^{-2}

(D) kg s^{-1}

20. Consider the following circuit in steady state condition. Calculate the amount of charge stored in 1μF and 2μF capacitors respectively.



(A) 4 μC and 8 μC

(B) 8 μC and 4 μC

(C) 3 μC and 6 μC

(D) 6 μC and 3 μC

21. Which one is the image of the complex domain $\{z|xy \geq 1, x+y > 0\}$ under the mapping $f(z) = z^2$, if $z = x + iy$?

(A) $\{z|xy \geq 1, x+y > 0\}$

(B) $\{z|x \geq 2, x+y > 0\}$

(C) $\{z|y \geq 2 \; \forall \; x\}$

(D) $\{z|y \geq 1 \; \forall \; x\}$

22. A rod of mass m and length l is suspended from two massless vertical springs with a spring constants k_1 and k_2. What is the Lagrangian for the system, if x_1 and x_2 be the displacements from equilibrium position of the two ends of the rod?

(A) $\frac{m}{8}(\dot{x}_1^2 + 2\dot{x}_1\dot{x}_2 + \dot{x}_2^2) - \frac{1}{2}k_1x_1^2 - \frac{1}{2}k_2x_2^2$

(B) $\frac{m}{2}(\dot{x}_1^2 + \dot{x}_1\dot{x}_2 + \dot{x}_2^2) - \frac{1}{4}(k_1 + k_2)(x_1^2 + x_2^2)$

(C) $\frac{m}{6}(\dot{x}_1^2 + \dot{x}_1\dot{x}_2 + \dot{x}_2^2) - \frac{1}{2}k_1x_1^2 - \frac{1}{2}k_2x_2^2$

(D) $\frac{m}{4}(\dot{x}_1^2 - 2\dot{x}_1\dot{x}_2 + \dot{x}_2^2) - \frac{1}{4}(k_1 - k_2)(x_1^2 + x_2^2)$

23. A plane electromagnetic wave propagating in air with $\vec{E} = (8\hat{i} + 6\hat{j} + 5\hat{k})e^{i(\omega t+3x-4y)}$ is incident on a perfectly conducting slab positioned at $x = 0$. $\vec{E}$ field of the reflected wave is

(A) $(-8\hat{i} - 6\hat{j} - 5\hat{k})e^{i(\omega t+3x+4y)}$.

(B) $(-8\hat{i} + 6\hat{j} - 5\hat{k})e^{i(\omega t+3x+4y)}$.

(C) $(-8\hat{i} + 6\hat{j} - 5\hat{k})e^{i(\omega t-3x-4y)}$.

(D) $(-8\hat{i} - 6\hat{j} - 5\hat{k})e^{i(\omega t-3x-4y)}$.

24. Suppose the spin degrees of freedom of a 2-particle system can be described by a 21-dimensional Hilbert subspace. Which among the following could be the spin of one of the particles?

(A) $\frac{1}{2}$

(B) 3

(C) $\frac{3}{2}$

(D) 2

25. Water is poured at a rate of R m^3/hour from the top into a cylindrical vessel of diameter D. The vessel has a small opening of area $a(\sqrt{a} \ll D)$ at the bottom. What should be the minimum height of the vessel so that water does not overflow?

(A) ∞

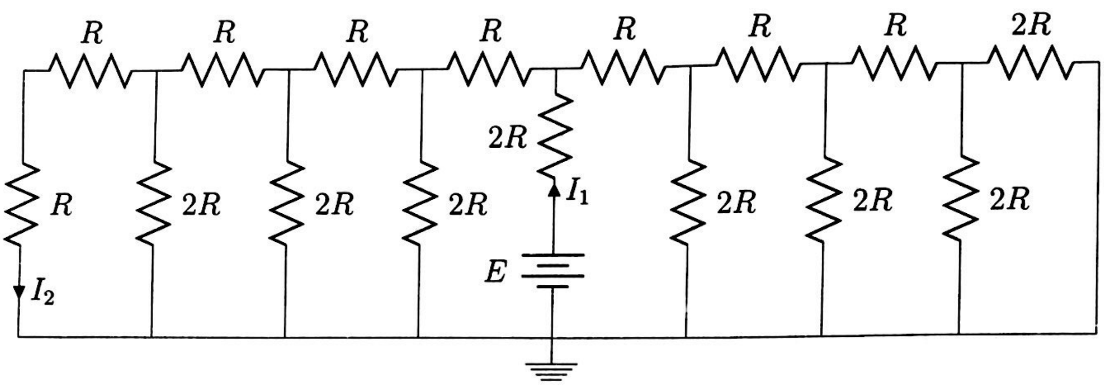
(B) $\dfrac{R^2}{2ga^2}$

(C) $\dfrac{R^2}{2gaD^2}$

(D) $\dfrac{8R^2}{\pi D^2 g^2}$

Part-B: 3-Mark Questions

1. For the circuit shown below, what is the ratio $\frac{I_1}{I_2}$?

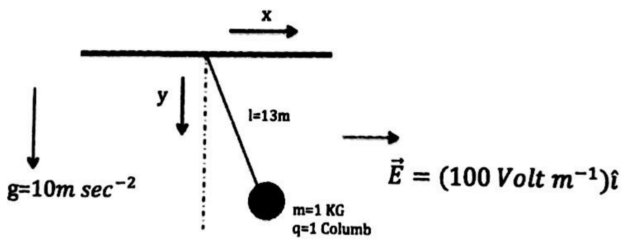


2. Suppose that the number of microstates available to a system of N particles depends on N and the combined variable UV^2, where U is the internal energy and V is the volume of the system. The system initially has volume 2m^3 and energy 200J. It undergoes an isentropic expansion to volume 4m^3. What is the final pressure of the system in SI units?

3. A sphere of inner radius 1 cm and outer radius 2 cm, centered at origin has a volume charge density $\rho_0 = \frac{K}{4\pi r}$, where K is a nonzero constant and r is the radial distance. A point charge of magnitude 10^{-3} C is placed at the origin. For what value of K in units of C/m^2, the electric field inside the shell is constant?

4. If $\hat{x}(t)$ be the position operator at a time t in the Heisenberg picture for a particle described by the Hamiltonian, $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$, what is $e^{i\omega t}\langle 0|\hat{x}(t)\hat{x}(0)|0\rangle$ in units of $\frac{\hbar}{2m\omega}$ where $|0\rangle$ is the ground state?

5. A simple pendulum has a bob of mass 1 Kg and charge 1 Coulomb. It is suspended by a massless string of length 13 m. The time period of small oscillations of this pendulam is T_0. If an electric field $\vec{E} = 100\hat{x}$ V/m is applied, the time period becomes T. What is the value of $(T_0/T)^4$?



6. Let a particle of mass 1×10^{-9} Kg, constrained to have one dimensional motion, be initially at the origin($x = 0$ m). The particle is in equilibrium with a thermal bath ($k_BT = 10^{-8}$ J). What is $\langle x^2\rangle$ of the particle after a time $t = 5$ s?

7. A ball of mass 0.1 kg and density 2000 kg/m^3 is suspended by a massless string of length 0.5 m under water having density 1000 kg/m^3. The ball experiences a drag force, $\vec{F_d} = -0.2(\vec{v_b} - \vec{v_w})$, where $\vec{v_b}$ and $\vec{v_w}$ are the velocites of the ball and water respectively. What will be the frequency of small oscillations for the motion of pendulum, if the water is at rest?

8. The temperature in a rectangular plate bounded by the lines $x = 0$, $y = 0$, $x = 3$ and $y = 5$ is $T = xy^2 - x^2y + 100$. What is the maximum temperature difference between two points on the plate?

9. A particle is described by the following Hamiltonian

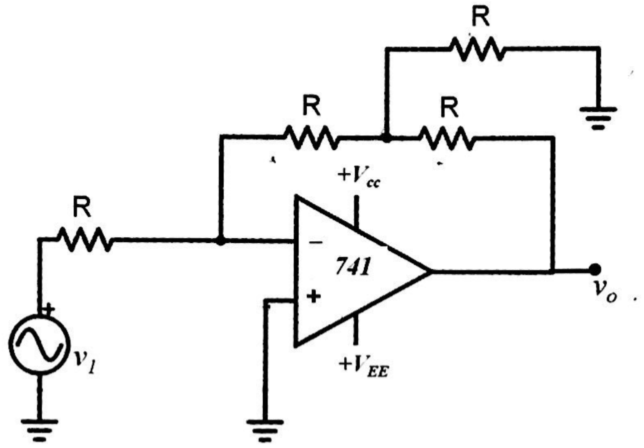
$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 + \lambda\hat{x}^4,$$

where the quartic term can be treated perturbatively. If ΔE_0 and ΔE_1 denote the energy correction of $O(\lambda)$ to the ground state and the first excited state respectively, what is the fraction $\Delta E_1/\Delta E_0$?

10. A solid, insulating sphere of radius 1 cm has charge 10^{-7} C distributed uniformly over its volume. It is surrounded concentrically by a conducting thick spherical shell of inner radius 2 cm, outer radius 2.5 cm and is charged with -2×10^{-7} C. What is the electrostatic potential in Volts on the surface of the sphere?

Part-C: 3-Mark Questions

1. Consider a 741 operational amplifier circuit as shown below, where $V_{CC} = V_{EE} = +15V$ and $R = 2.2k\Omega$. If v_I = 2 mV, what is the value of v_O with respect to the ground?



(A) - 1 mV

(B) - 2 mV

(C) - 3 mV

(D) - 4 mV

2. The integral $I = \int_1^\infty \frac{\sqrt{x-1}}{(1+x)^2}\, dx$ is

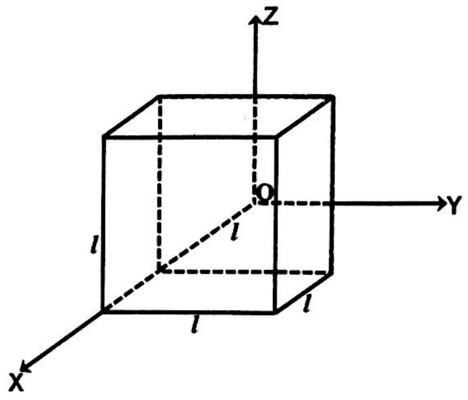
(A) $\frac{\pi}{\sqrt{2}}$.

(B) $\frac{\pi}{2\sqrt{2}}$.

(C) $\frac{\sqrt{\pi}}{2}$.

(D) $\sqrt{\frac{\pi}{2}}$.

3. For an electric field $\vec{E} = k\sqrt{x}\hat{x}$ where k is a non-zero constant, total charge enclosed by the cube as shown below is



(A) 0.

(B) $k\epsilon_0 l^{5/2}(\sqrt{3} - 1)$.

(C) $k\epsilon_0 l^{5/2}(\sqrt{5} - 1)$.

(D) $k\epsilon_0 l^{5/2}(\sqrt{2} - 1)$.

4. Consider a particle confined by a potential $V(x) = k|x|$, where k is a positive constant. The spectrum E_n of the system, within the WKB approximation, is proportional to

(A) $(n + \frac{1}{2})^{3/2}$.

(B) $(n + \frac{1}{2})^{2/3}$.

(C) $(n + \frac{1}{2})^{1/2}$.

(D) $(n + \frac{1}{2})^{4/3}$.

5. A cylinder at temperature $T = 0$ is separated into two compartments A and B by a free sliding piston. Compartments A and B are filled by Fermi gases made of spin 1/2 and 3/2 particles respectively. If particles in both the compartments have same mass, the ratio of equilibrium density of the gas in compartment A to that of gas in compartment B is

(A) 1.

(B) $\frac{1}{3^{2/5}}$.

(C) $\frac{1}{2^{2/5}}$.

(D) $\frac{1}{2^{2/3}}$.

6. Consider the Hamiltonian

$$H(t) = \alpha \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} + \beta(t) \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -2 \end{pmatrix}.$$

The time dependent function $\beta(t) = \alpha$ for $t \leq 0$ and zero for $t > 0$. Find $|\langle\Psi(t < 0)|\Psi(t > 0)\rangle|^2$, where $|\Psi(t < 0)\rangle$ is the normalized ground state of the system at a time $t < 0$ and $|\Psi(t > 0)\rangle$ is the state of the system at $t > 0$.

(A) $\frac{1}{2}(1 + cos(2\alpha t))$

(B) $\frac{1}{2}(1 + cos(\alpha t))$

(C) $\frac{1}{2}(1 + sin(2\alpha t))$

(D) $\frac{1}{2}(1 + sin(\alpha t))$

7. If $\rho = [I + \frac{1}{\sqrt{3}}(\sigma_x + \sigma_y + \sigma_z)]/2$, where σ's are the Pauli matrices and I is the identity matrix, then the trace of ρ^{2017} is

(A) 2^{2017}.

(B) 2^{-2017}.

(C) 1.

(D) $\frac{1}{2}$.

8. Consider a grounded conducting plane which is infinitely extended perpendicular to the y-axis at $y = 0$. If an infinite line of charge per unit length λ runs parallel to x-axis at $y = d$, then surface charge density on the conducting plane is

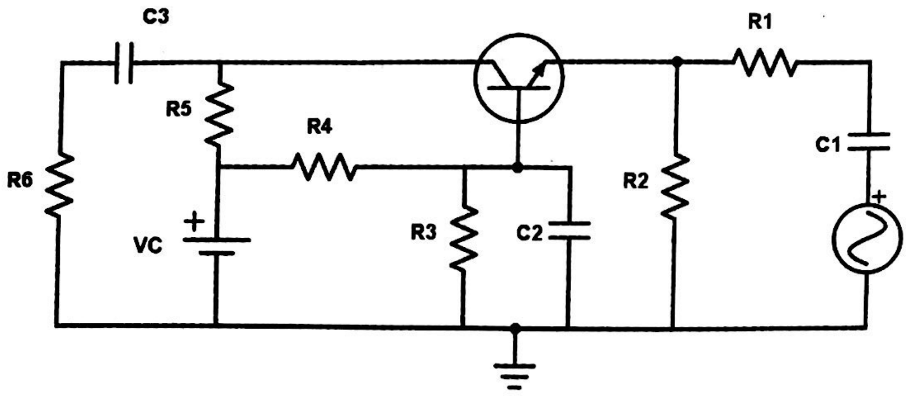
(A) $\frac{-\lambda d}{(x^2 + d^2 + z^2)}$.

(B) $\frac{-\lambda d}{(x^2 + d^2 + z^2)}$.

(C) $\frac{-\lambda d}{\pi\,(x^2 + d^2 + z^2)}$.

(D) $\frac{-\lambda d}{2\pi\,(x^2 + d^2 + z^2)}$.

9. What is the DC base current (approximated to nearest integer value in μA) for the following *n-p-n* silicon transtisor circuit, given $R_1 = 75\Omega$, $R_2 = 4.0k\Omega$, $R_3 = 2.1k\Omega$, $R_4 = 2.6k\Omega$, $R_5 = 6.0k\Omega$, $R_6 = 6.8k\Omega$, $C_1 = 1\mu F$, $C_2 = 2\mu F$, $V_C = +15V$ and $\beta_{dc} = 75$?



(A) 20

(B) 24

(C) 16

(D) 32

10. The Fourier transform of the function $\dfrac{1}{x^4 + 3x^2 + 2}$ up to a proportionality constant is

(A) $\sqrt{2}\exp(-k^2) - \exp(-2k^2)$.

(B) $\sqrt{2}\exp(-|k|) - \exp\left(-\sqrt{2}|k|\right)$.

(C) $\sqrt{2}\exp\left(-\sqrt{|k|}\right) - \exp\left(-\sqrt{2|k|}\right)$.

(D) $\sqrt{2}\exp\left(-\sqrt{2}k^2\right) - \exp(-2k^2)$.

11. Consider a point particle A of mass m_A colliding elastically with another point particle B of mass m_B at rest, where $m_B/m_A = \gamma$. After collision, the ratio of the kinetic energy of particle B to the initial kinetic energy of particle A is given by

(A) $\dfrac{4}{\gamma + 2 + 1/\gamma}$.

(B) $\dfrac{2}{\gamma + 1/\gamma}$.

(C) $\dfrac{2}{\gamma + 2 - 1/\gamma}$.

(D) $\dfrac{1}{\gamma}$.

12. Two classical particles are distributed among N (> 2) sites on a ring. Each site can accommodate only one particle. If two particles occupy two nearest neighbour sites, then the energy of the system is increased by ϵ. The average energy of the system at temperature T is

(A) $\dfrac{2\epsilon e^{-\beta\epsilon}}{(N-3)+2e^{-\beta\epsilon}}$.

(B) $\dfrac{2N\epsilon e^{-\beta\epsilon}}{(N-3)+2e^{-\beta\epsilon}}$.

(C) $\dfrac{\epsilon}{N}$.

(D) $\dfrac{2\epsilon e^{-\beta\epsilon}}{(N-2)+2e^{-\beta\epsilon}}$.

13. The function $f(x) = \cosh x$ which exists in the range $-\pi \leq x \leq \pi$ is periodically repeated between $x = (2m-1)\pi$ and $(2m+1)\pi$, where $m = -\infty$ to $+\infty$. Using Fourier series, indicate the correct relation at $x = 0$.

(A) $\displaystyle\sum_{n=-\infty}^{\infty} \frac{(-1)^n}{1-n^2} = \frac{1}{2}\left(\frac{\pi}{\cosh\pi} - 1\right)$

(B) $\displaystyle\sum_{n=-\infty}^{\infty} \frac{(-1)^n}{1-n^2} = 2\frac{\pi}{\cosh\pi}$

(C) $\displaystyle\sum_{n=-\infty}^{\infty} \frac{(-1)^{-n}}{1+n^2} = 2\frac{\pi}{\sinh\pi}$

(D) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{1+n^2} = \frac{1}{2}\left(\frac{\pi}{\sinh\pi} - 1\right)$

14. A system of particles on N lattice sites is in equilibrium at temperature T and chemical potential μ. Multiple occupancy of the sites is forbidden. The binding energy of a particle at each site is $-\epsilon$. The probability of no site being occupied is

(A) $\dfrac{1-e^{\beta(\mu+\epsilon)}}{1-e^{(N+1)\beta(\mu+\epsilon)}}$.

(B) $\dfrac{1}{[1+e^{\beta(\mu+\epsilon)}]^N}$.

(C) $\dfrac{1}{[1+e^{-\beta(\mu+\epsilon)}]^N}$.

(D) $\dfrac{1-e^{-\beta(\mu+\epsilon)}}{1-e^{-(N+1)\beta(\mu+\epsilon)}}$.

15. A toy car is made from a rectangular block of mass M and four disk wheels of mass m and radius r. The car is attached to a vertical wall by a massless horizontal spring with spring constant k and constrained to move perpendicular to the wall. The coefficient of static friction between the wheels of the car and the floor is μ. The maximum amplitude of oscillations of the car above which the wheels start slipping is

(A) $\frac{\mu g(M+2m)(M+4m)}{mk}$.

(B) $\frac{\mu g(M^2-m^2)}{Mk}$.

(C) $\frac{\mu g(M+m)^2}{2mk}$.

(D) $\frac{\mu g(M+4m)(M+6m)}{2mk}$.