

**CA FOUNDATION**



**MARATHON**

**JUNE 2024**

**Quantitative Aptitude**

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**By Anurag Chauhan**





# *Chapters* to be covered



- ① Statistical Description of Data & Sampling
- ② Central Tendency and Dispersion
- ③ Correlation & Regression
- ④ Index Number

2022-J → 9 marks  
2022-D → 4 marks  
2023-J → 5 marks  
2023-D ⇒ 7 marks



# Statistical Description of Data





# Statistics Originated From

Status  $\Rightarrow$  Latin

Statista  $\Rightarrow$  Italian

Statistik  $\Rightarrow$  German

Statistique  $\Rightarrow$  French

one n  
models  
facts

Parabola  
Latius Rectum  
Status

Swastika  
卐





# Definition of Statistics



Singular  
Sense

Plural  
Sense



# Definition of Statistics (Singular sense)

## Scientific Method

- Collection Of data ✓✓
- Classification ✓✓
- Presentation ✓✓
- Analysis ✓✓
- Interpretation ✓✓







# Definition of Statistics (Plural sense)

**Systematically collected numerical facts**







# Limitation of Statistics



- Deals in aggregates only
- Does not deal in qualitative data
- Not always true
- Expert consultation required
- They can be misused





# DATA



Information of  
Some character

Qualitative  
Character

Attributes

g

Color

g

Nationality

g

Caste

Quantitative  
Character

Variable

Discrete

g

No. of Accidents

g

Goals in a match

g

Salary

Continuous

g

Weight

g

Height

g

Age



# COLLECTION OF DATA



Primary

Collected For  
First Time

Secondary

Public (Existing)  
Information used by  
others

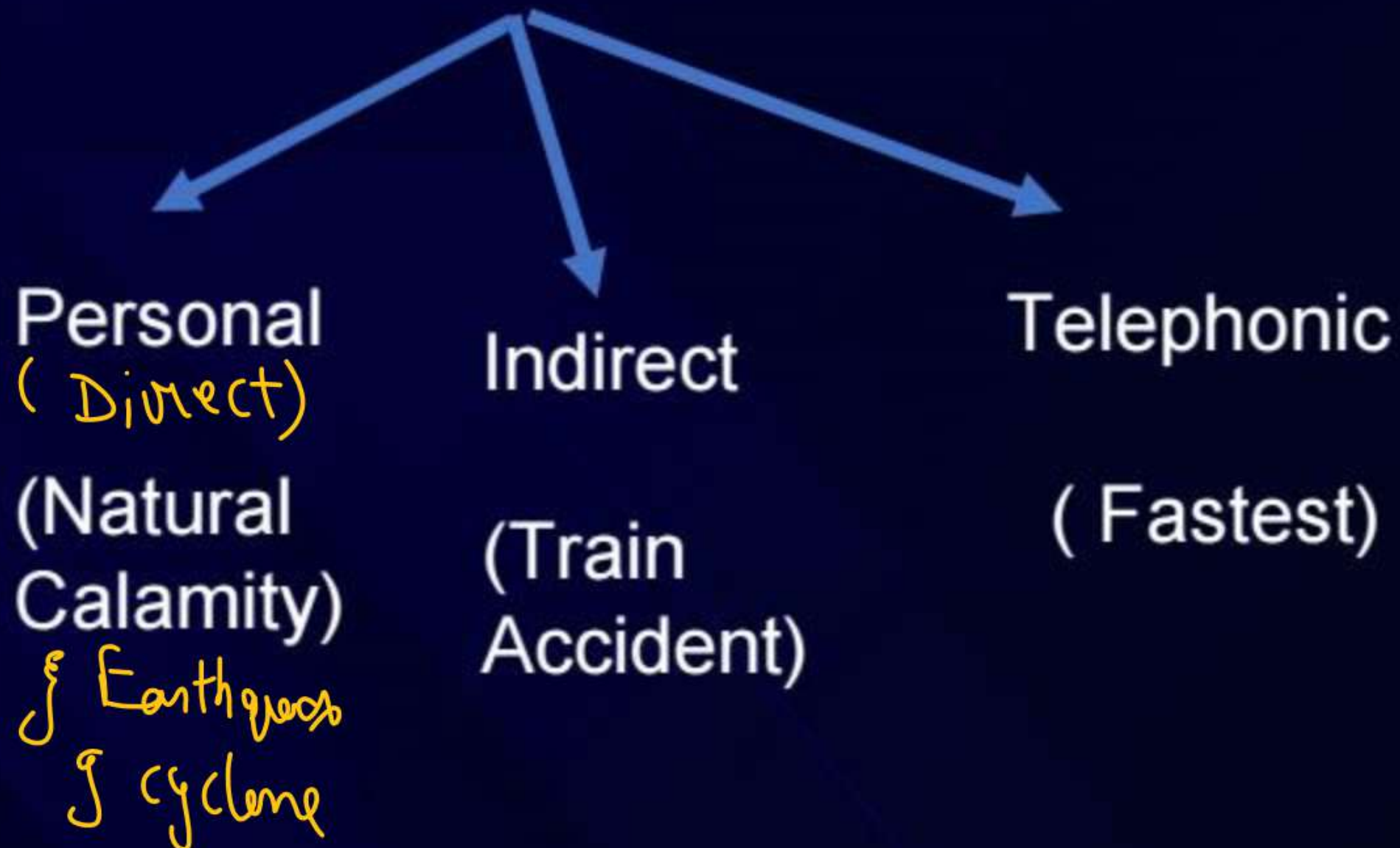




# COLLECTION OF PRIMARY DATA



## 1) Interview Method





## 2) Questionnaire Method

### i) Mailed Questionnaire Method

Questionnaire is mailed to all target respondents

### ii) Enumerator Method

Questionnaire filled by enumerator  
On behalf of respondents



#### Customer Satisfaction Survey Questionnaire

##### I. Questions

Directions: Please indicate your level of agreement or disagreement with each of these statements regarding QRZ Family Restaurant. Place an "X" mark in the box of your answer.

Q1: How many times per year do you visit QRZ Family Restaurant?

Q2: Do you visit QRZ Family Restaurant with family or friends?

☐ Yes

☐ No

1. The store is accessibly located.

2. Store hours are convenient for my dining needs.

3. Advertised dish was in stock.

Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree



### 3) Observation Method

Investigator's own direct observation  
without asking respondents

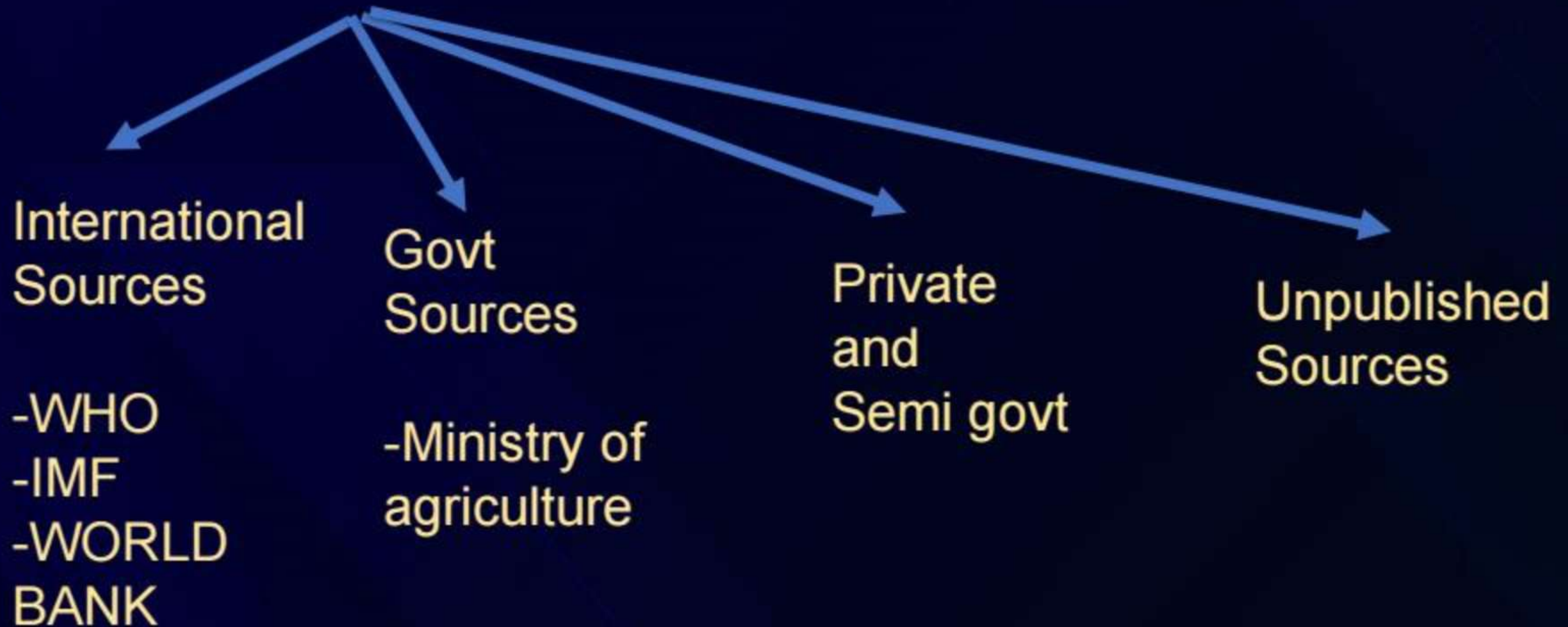
Used for research together data  
about people, objects, events &  
behaviors etc.

Accurate Data  
Time Consuming





## COLLECTION OF SECONDARY DATA







जाँच

# Scrutiny Of Data



Verification of

✓ - Accuracy

✓ - Consistency Of Data



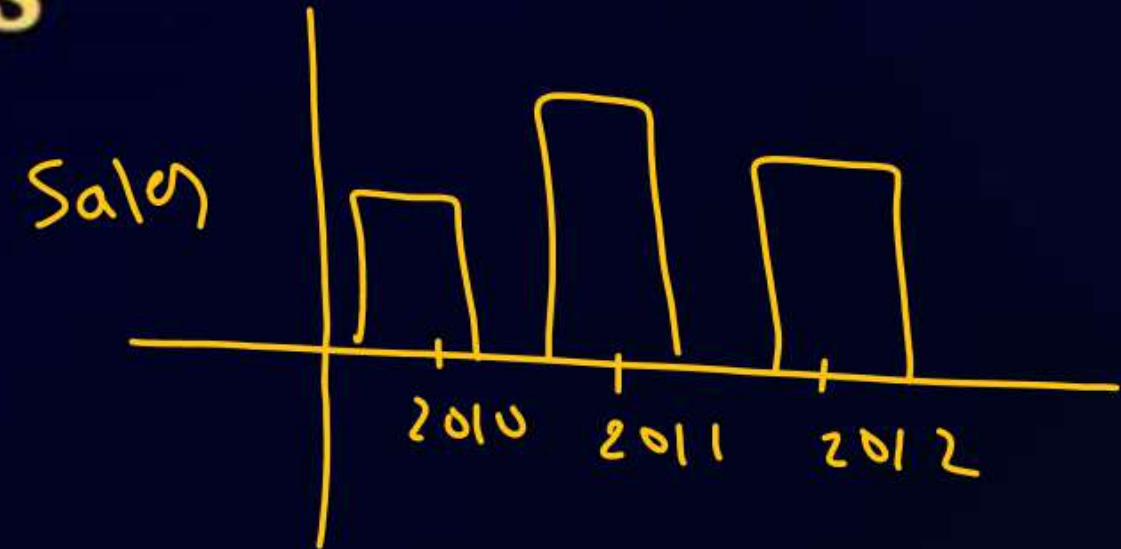


# Classification Of Data

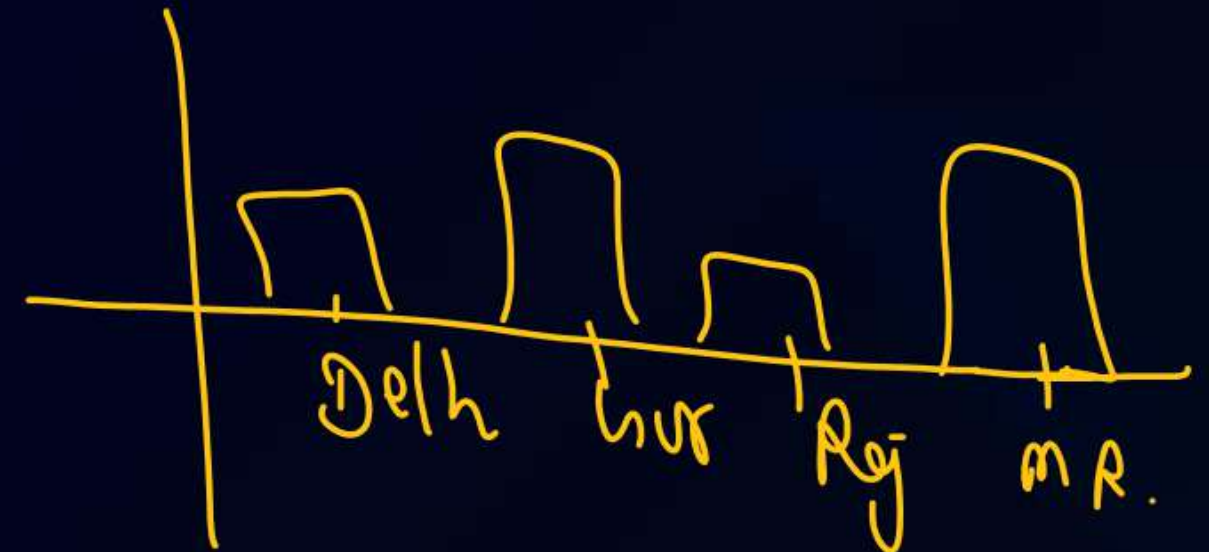


# Data may be classified as

i) Temporal → on the Basis of time  
(Chronological)



ii) Geographical → on the Basis of Place  
(Spatial)

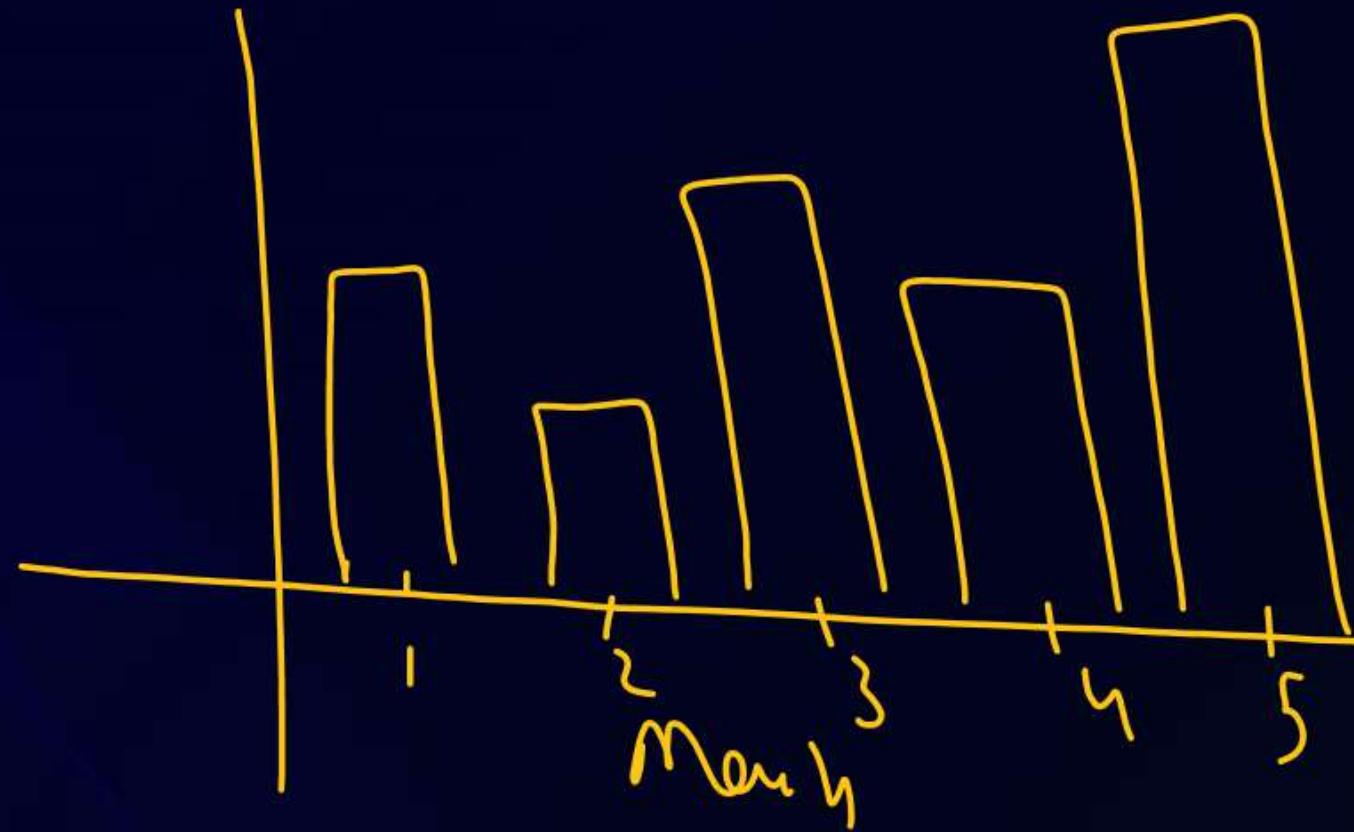




### iii) Qualitative ( Ordinal )



### iv) Quantitative ( Cardinal )





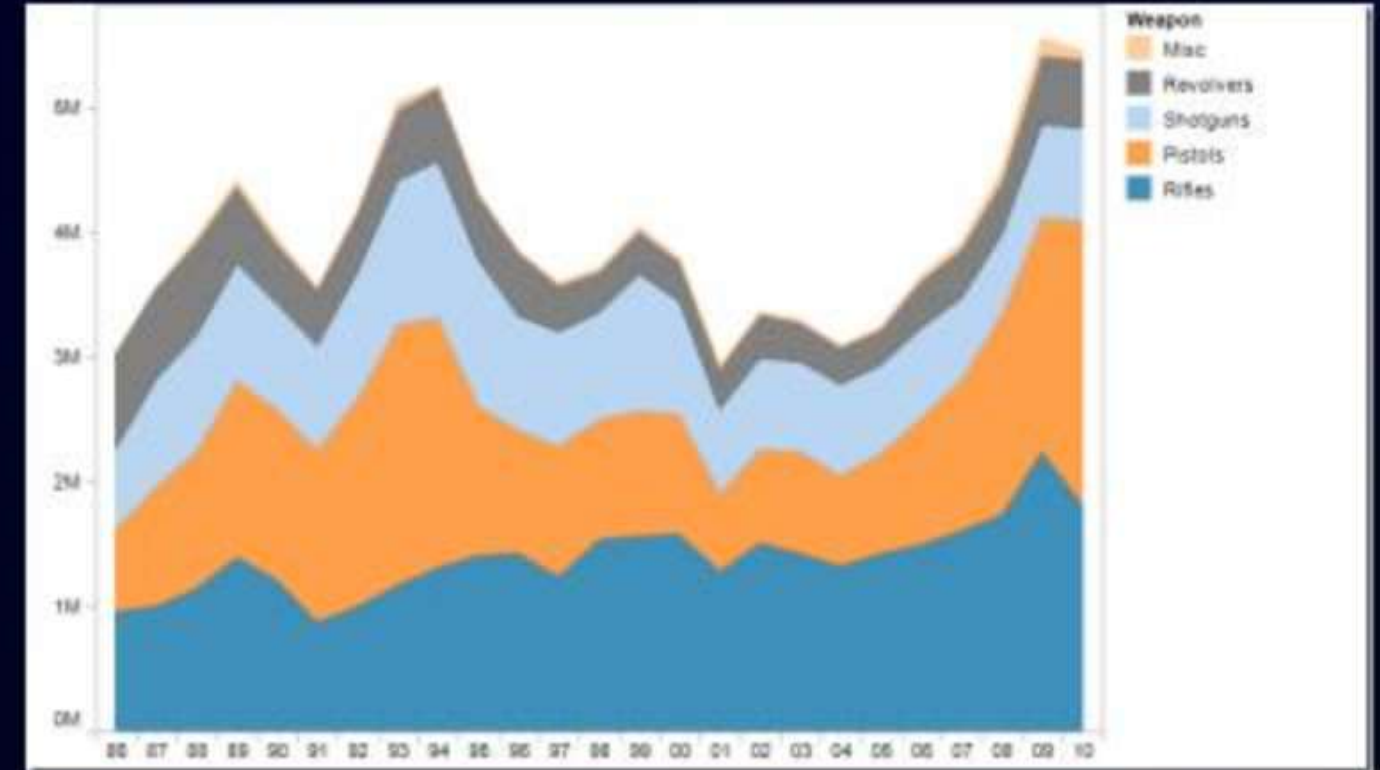
# Presentation of Data



1> Textual Presentation : E.g. Paragraph

2> Tabular Presentation : E.g. Table  
(This Is Best Method)

3> Diagrammatic Presentation : Eg Bar diagram , Pie Chart  
(This is most attractive) ( Hidden Trend Can Be Noticed)





# #Textual Presentation

Paragraph

In 2009, out of a total of five thousand workers of Roy Enamel Factory, four thousand and two hundred were members of a Trade Union. The number of female workers was twenty per cent of the total workers out of which thirty per cent were members of the Trade Union. In 2010, the number of workers belonging to the trade union was increased by twenty per cent as compared to 2009 of which four thousand and two hundred were male. The number of workers not belonging to trade union was nine hundred and fifty of which four hundred and fifty were females.



# #Table



Best method



- 1) Table Number
- 2) Title – Explains content of table
- 3) Head Note – Given in brackets provides information of units
- 4) Column Heading-(Caption)- Describes columns and subcolumns
- 5) Row Heading (Stubs) – Describes Rows
- 6) Body Of Table - Numeric Information
- 7) Source
- 8) Footnote – Specific feature which is not explained



**Table 1.1** ← Table Number

**Literacy in Bihar by Sex and Locations** ← Title  
(Percent) ← Head Note

Column Heading (Caption) →

Row Heading (Stub) →

Row Entries (Stub Entries) →

Column Total →

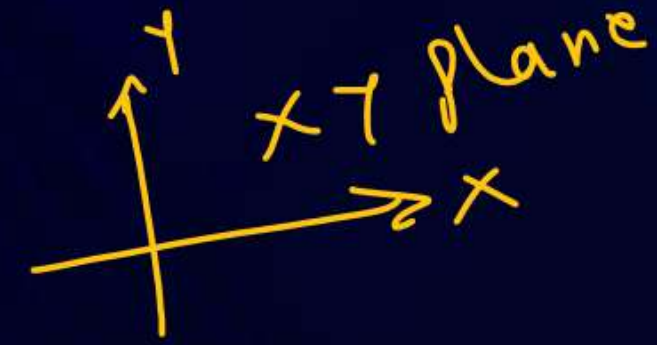
Row Total ←

Sex	LOCATION		Total
	Rural	Urban	
Males	69.67	82.56	71.20
Females	44.30	61.95	51.50
Total	59.78	76.86	61.80

Source Note → Source Note: Census of India 2011, Provisional Population Totals

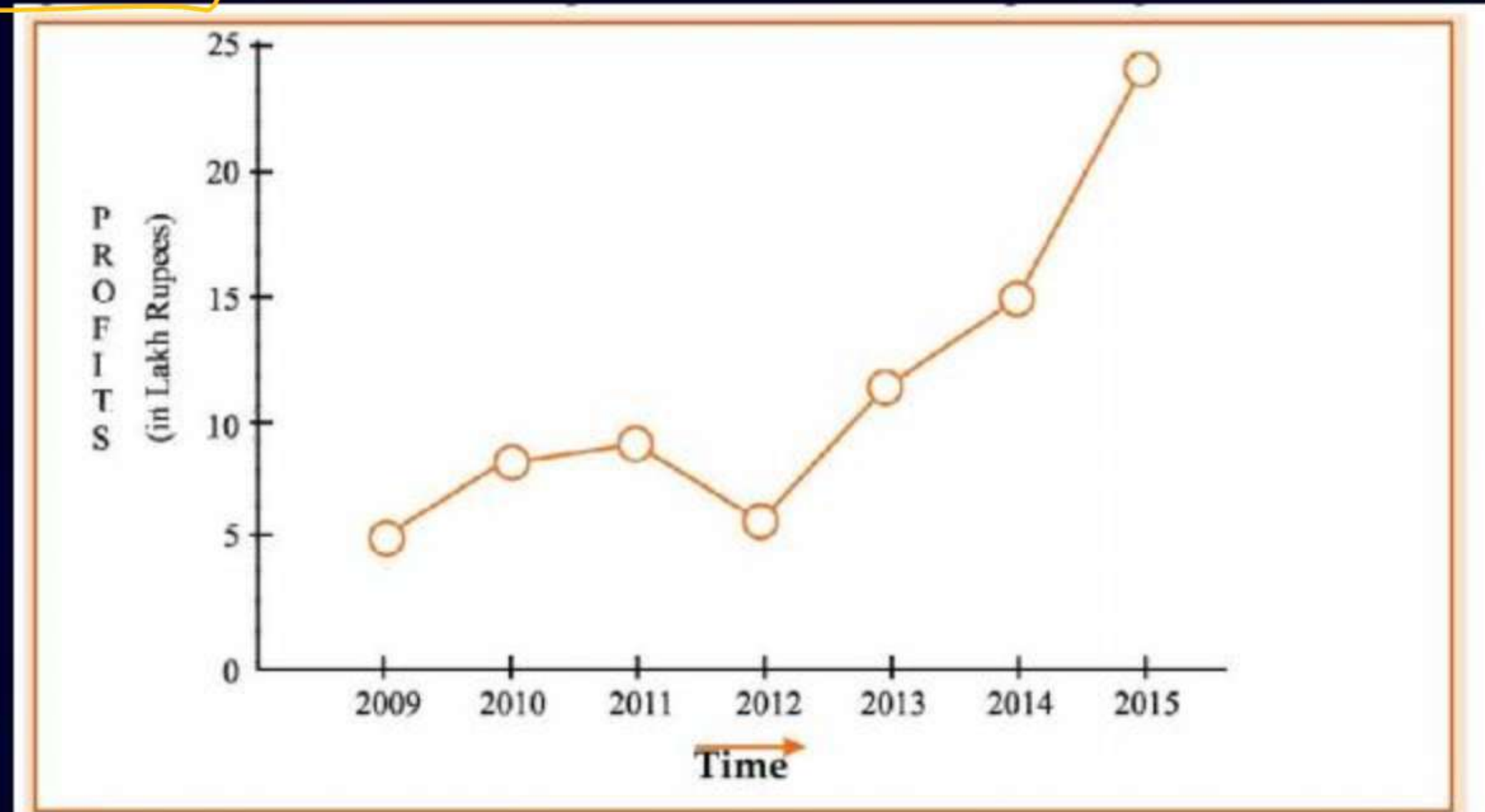
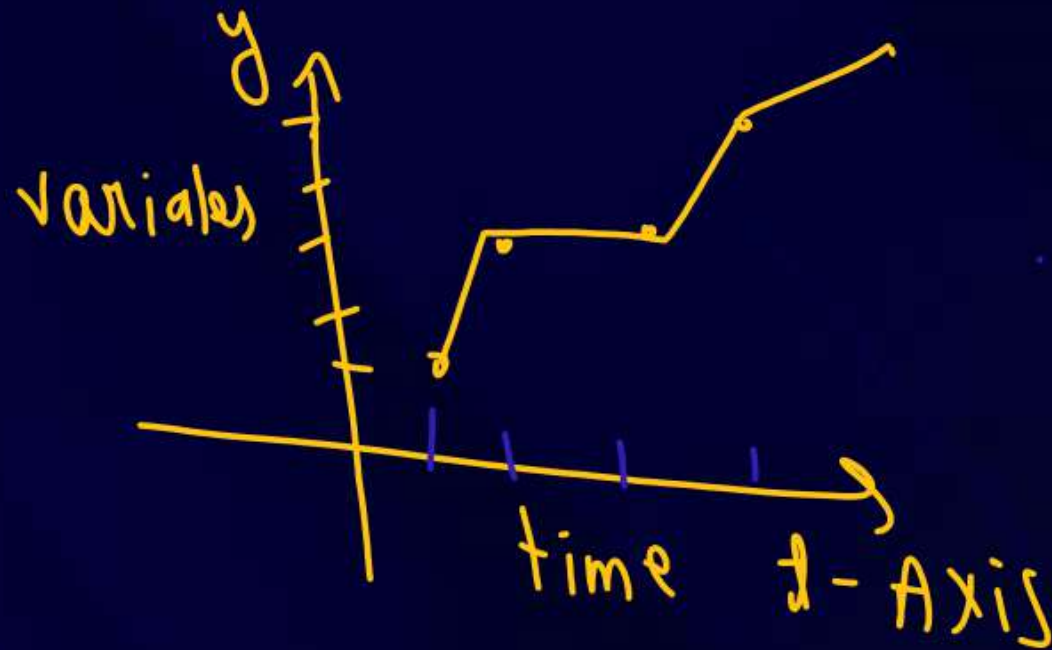
Footnote → Note: Figures are rounded to two digits after decimal.

# #Diagrammatic Presentation



## 1) Line Diagram (Historiagram)

- One Dimensional
- 1-D Plane
- Time Series

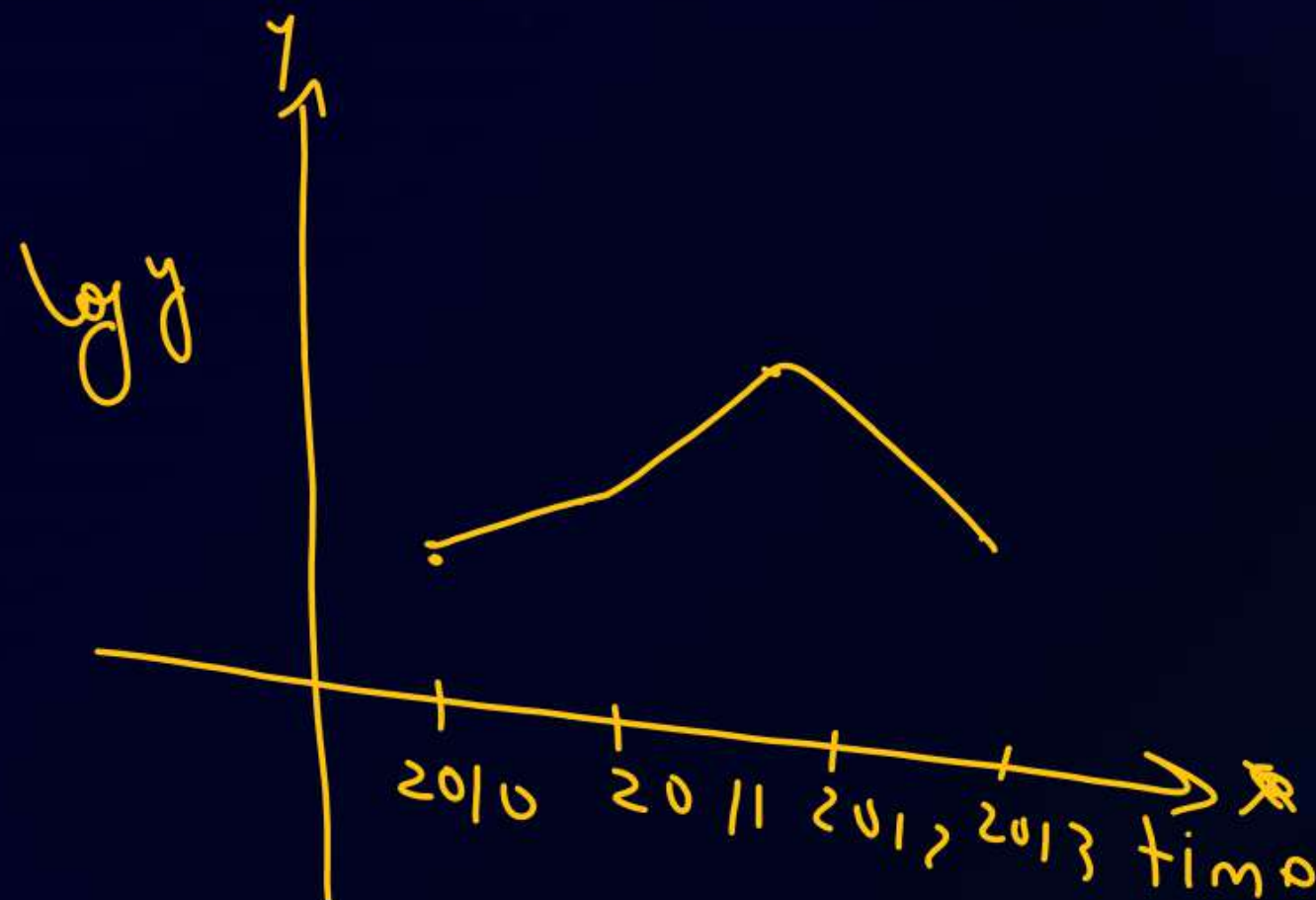




## 2) Ratio Chart

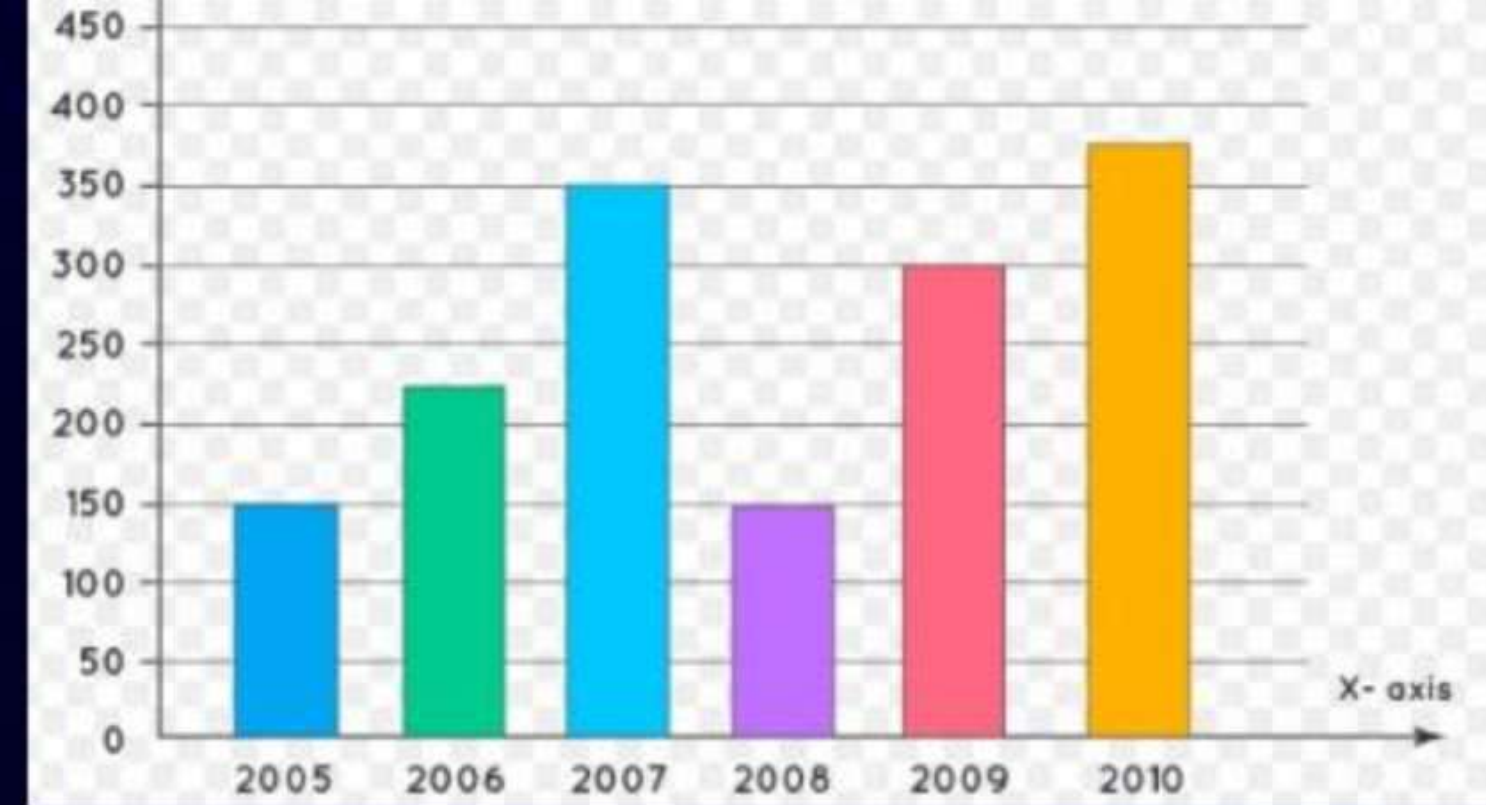
When time series have fluctuations we use ratio chart

(t) time	(y) Salary	$\log y_i$
2010	20	1.3010
11	30	1.4771
12	50	.
13	60	-



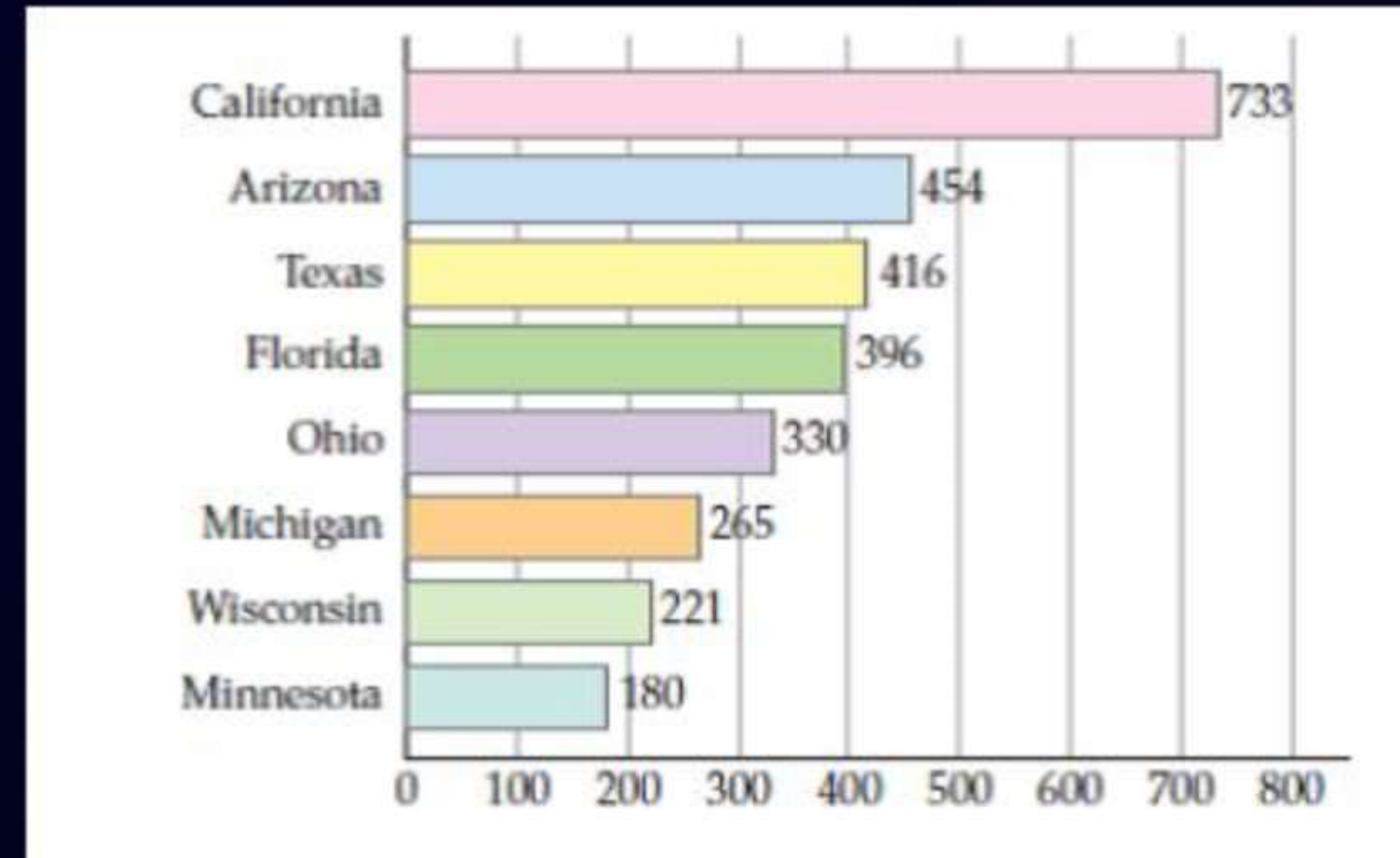
### 3) Bar Diagram

- ✓ - Rectangular bars( Only Length matters)
- ✓ - One Dimensional



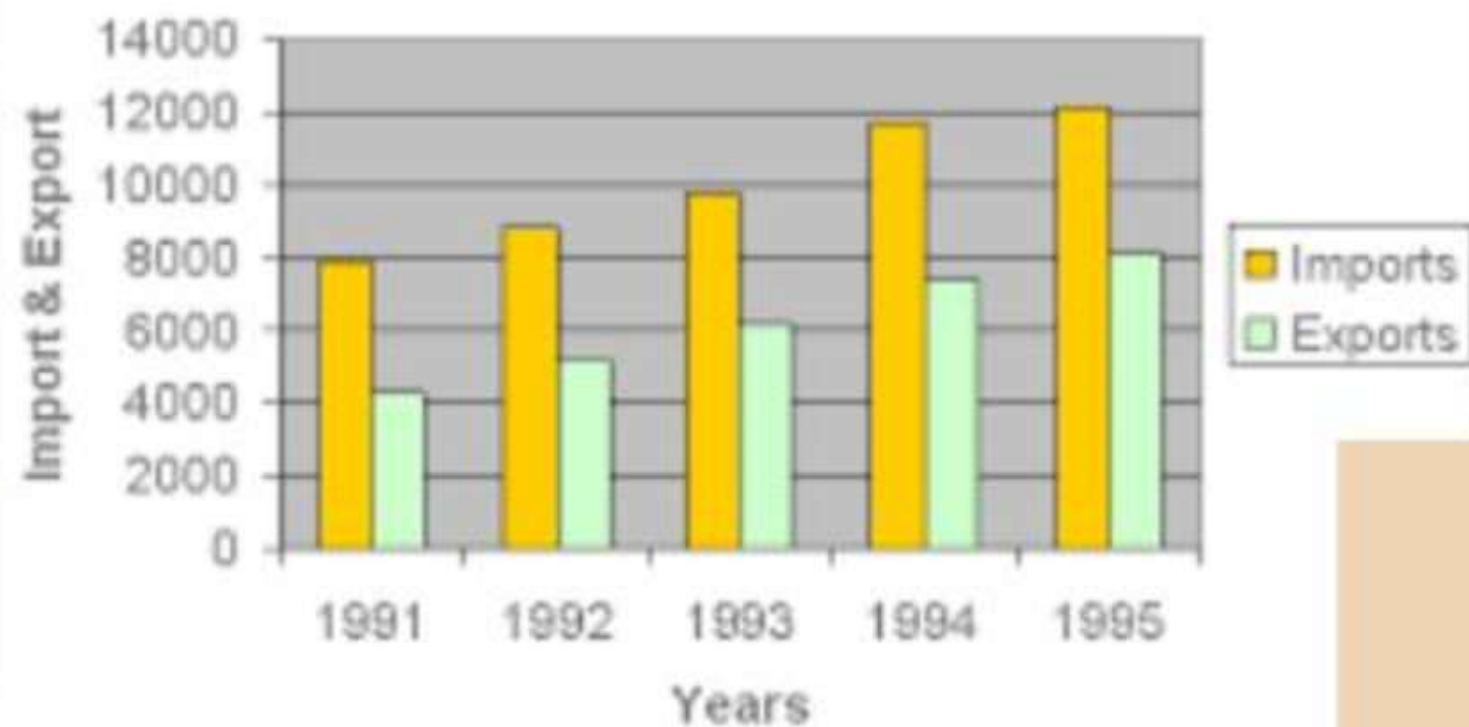
Vertical Bar Diagram – Quantitative data  
(When data vary over time)  
(Temporal)

Horizontal Bar Diagram – Qualitative Data  
(When data vary over space)  
Spatial





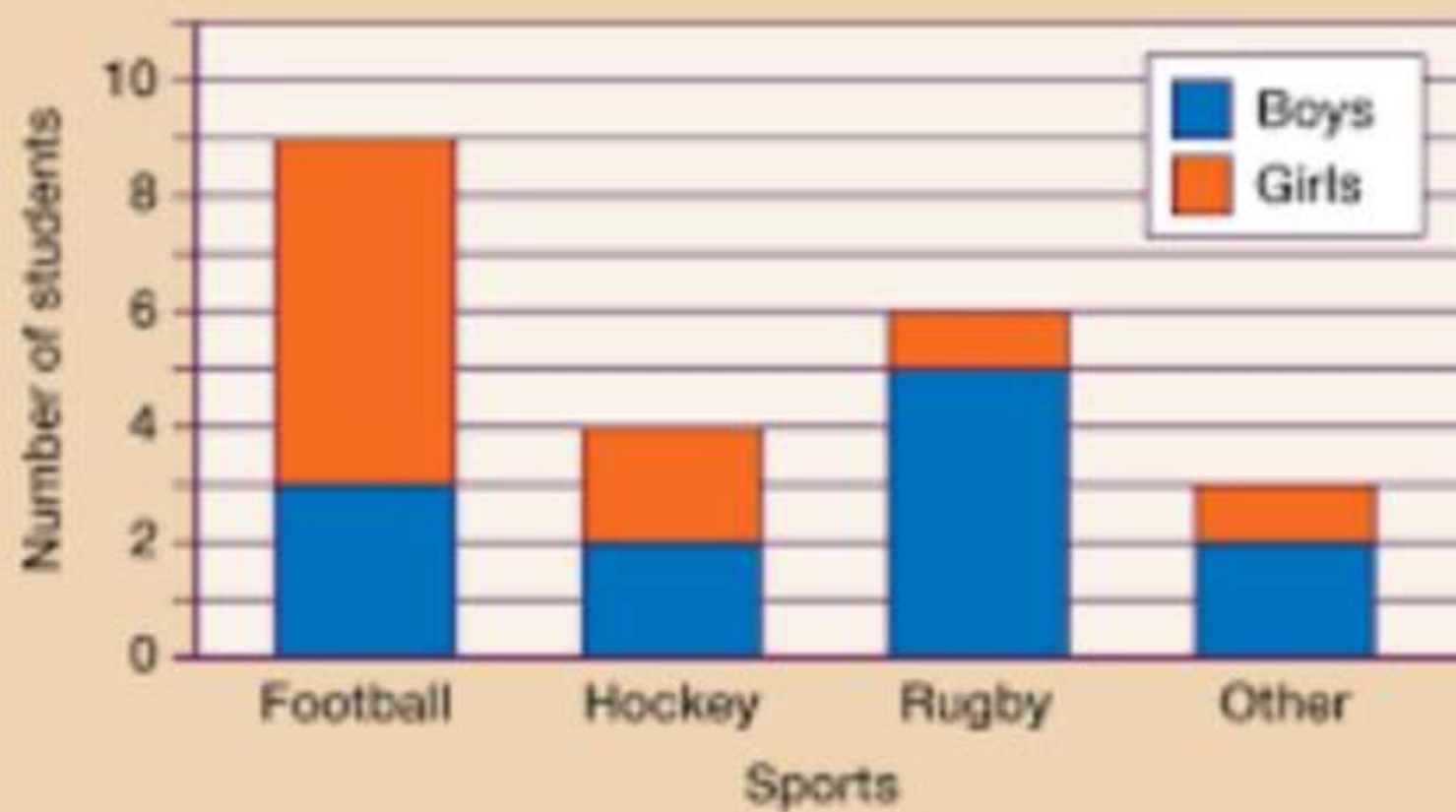
### Multiple Bar Chart



Component Bar Diagram

Multiple Bar Diagram

### Favourite sports



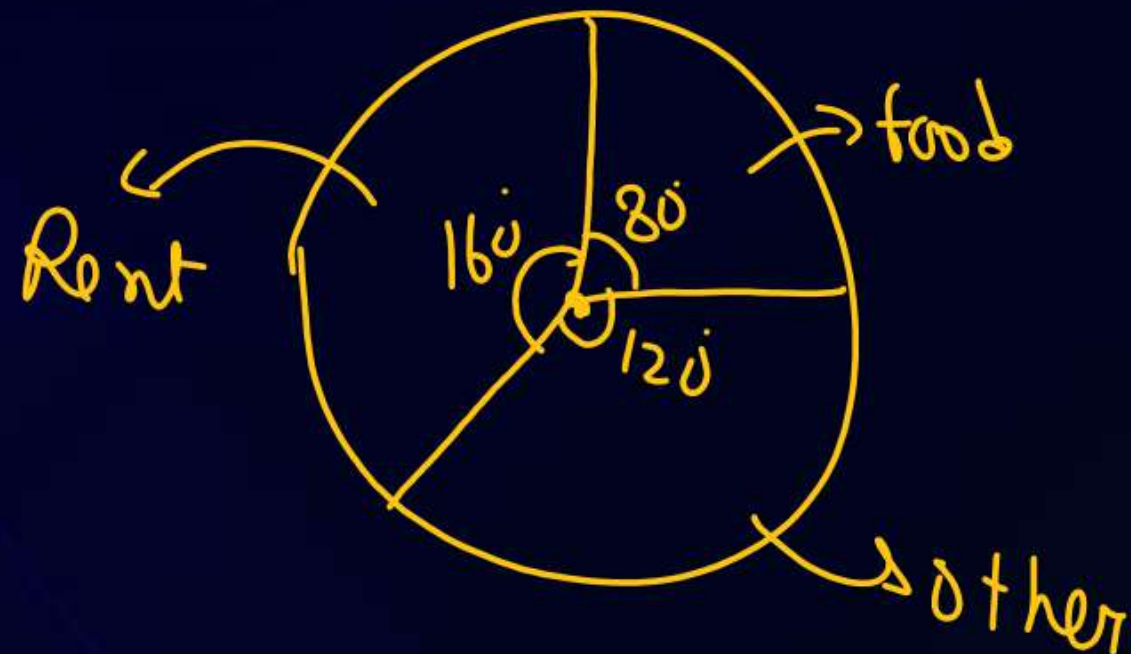
## 4) Pie Chart

Displays data in a circular-shaped graph

- 2 Dimensional

	<u>Expenses</u>	<u>Central angle</u>
Food	2000	$\frac{2000}{9000} \times 360 = 80$
Rent	4000	$\frac{4000}{9000} \times 360 = 160$
Other	3000	$\frac{3000}{9000} \times 360 = 120$
	<u>9000</u>	

360







# Frequency Distribution



Tabular presentation  
of Statistical Data

**Number of  
accidents in  
last 15 days**  
**1,2,1,1,1,2,3,3,2**  
**1,1,1,3,2,2**

<u>No. of Accidents (<math>x_i</math>)</u>	<u>Tally Bar</u>	<u>frequency</u>
1		7
2		5
3		3



# Types of Series

Individual  
series

||

marks: 1, 2, 3, 1, 4  
5, 2, 4, 5, 1, 3  
6, 1, 2, 5, 2, 2

Describe  
series

||

<u>marks (<math>x_i</math>)</u>	<u>frequency (<math>f_i</math>)</u>
1	2
2	3
3	4
4	2
5	1
$N = 12$	

Continuous  
series

||

Intervals



# Continuous series

Exclusive

⇓

<u>C.I</u>	<u>f<sub>i</sub></u>
0-2	3
2-4	2
4-6	5
6-8	1

Inclusive

<u>C.I</u>	<u>f<sub>i</sub></u>
1-6	3
7-12	5
13-18	4
19-24	1

Open ended series

<u>C.I</u>	<u>f<sub>i</sub></u>
less than 10	2
10-12	3
12-14	1
14-16	4
more than 16	5

<u>CI</u>	<u>fi</u>
3 - 8	6
9 - 14	2
15 - 20	5

<u>CI</u>	<u>fi</u>
2.5 - 8.5	6
8.5 - 14.5	2
14.5 - 20.5	5

3 - 8  
 ↙ ↘  
 lower class limit (LCL)      upper class limit (UCL)

2.5 - 8.5  
 ↙ ↘  
 L.C.L & lower class Boundary      upper class Boundary




$$\# \text{ frequency density} = \frac{\text{frequency of class interval}}{\text{class length}}$$

$$\# \text{ Relative frequency} = \frac{\text{frequency class interval}}{\text{Total frequency}}$$

Q

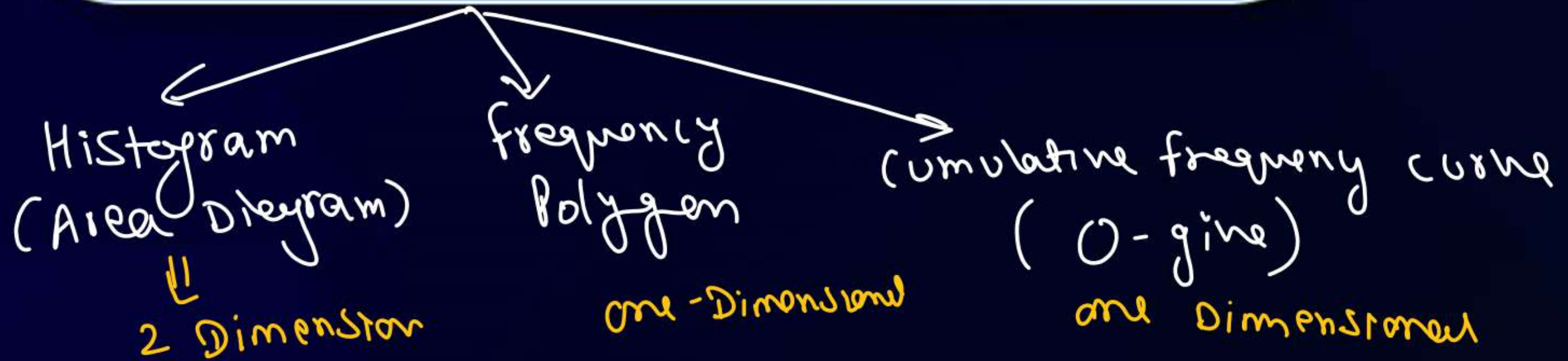
<u>C.I</u>	<u><math>f_i</math></u>	<u>Frequency Density</u>	<u>Relative frequency</u>
0-5	3	$\frac{3}{5} = 0.6$	$\frac{3}{20} = 0.15$
5-10	6	$\frac{6}{5} = 1.2$	$\frac{6}{20} = 0.30$
10-15	2	$\frac{2}{5} = 0.4$	$\frac{2}{20} = 0.10$
15-20	9	$\frac{9}{5} = 1.8$	$\frac{9}{20} = 0.45$
	20		

1



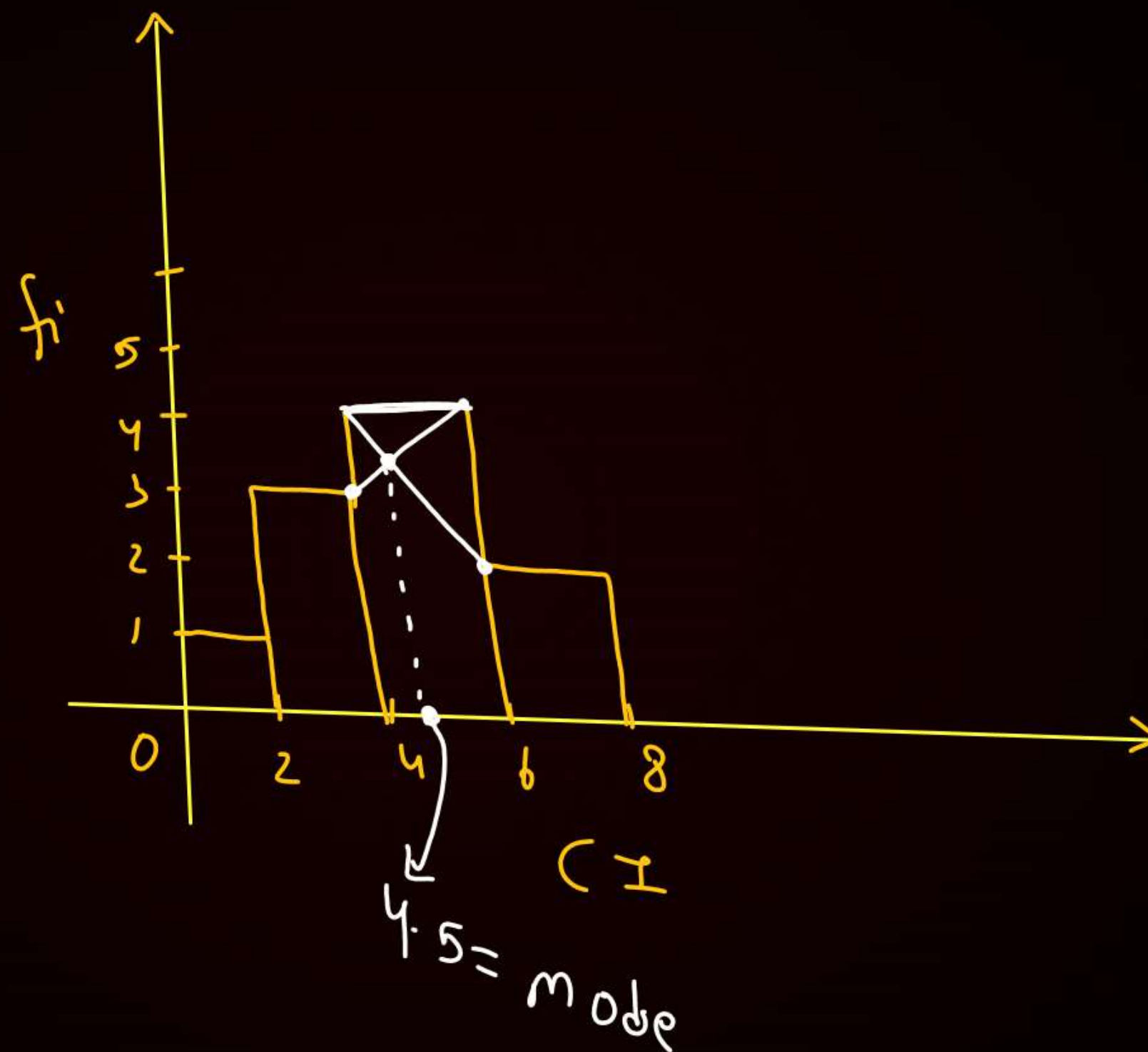


# Graphical Representation of a Frequency Distribution



Ex

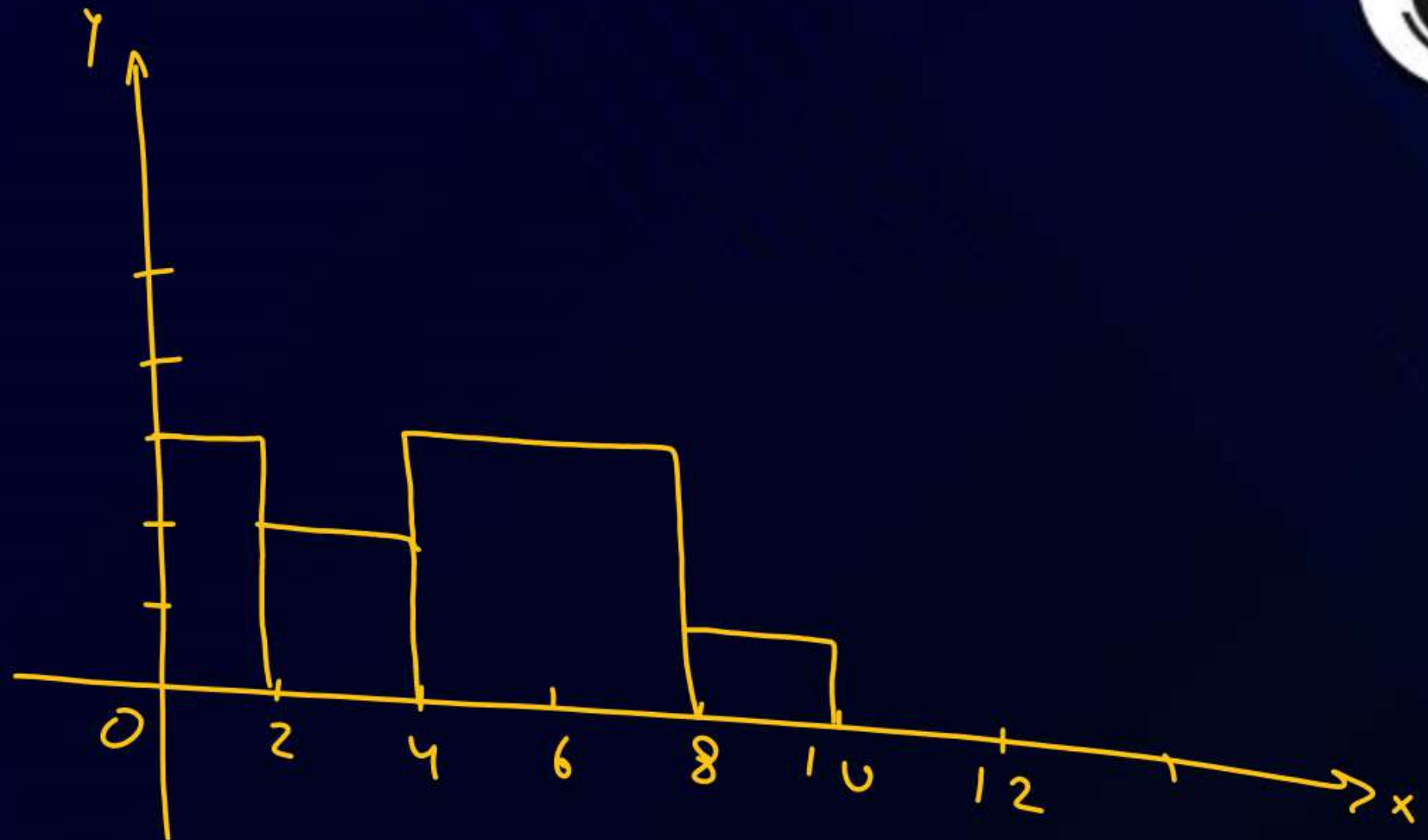
CI	$f_i$
0-2	1
2-4	3
4-6	4
6-8	2





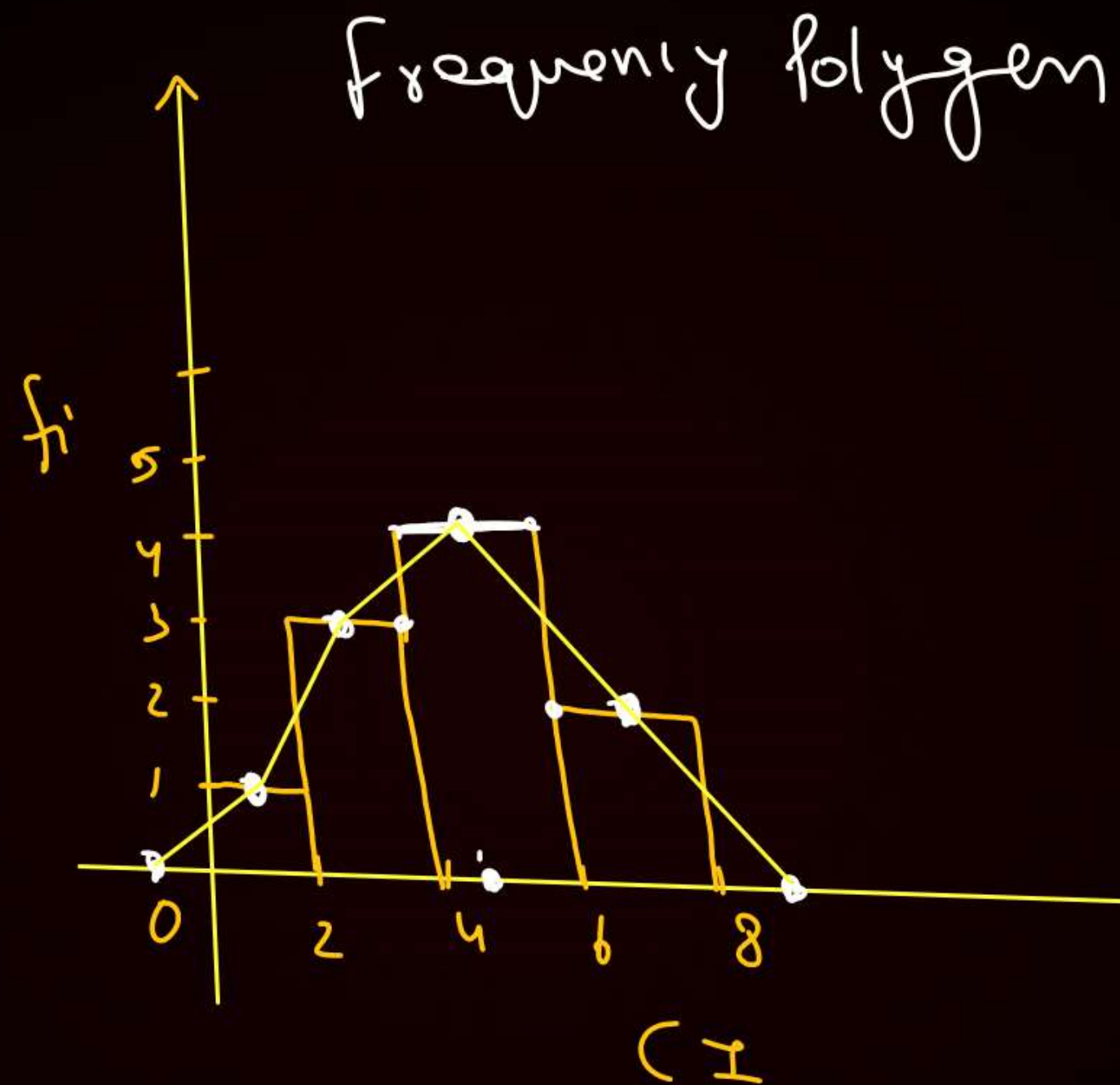
Ex

<u>CI</u>	<u><math>f_i</math></u>
0-2	3
2-4	2
4-8	6
8-10	1



$$\xi$$

$C \pm$	$f_i$
0-2	1
2-4	3
4-6	4
6-8	2



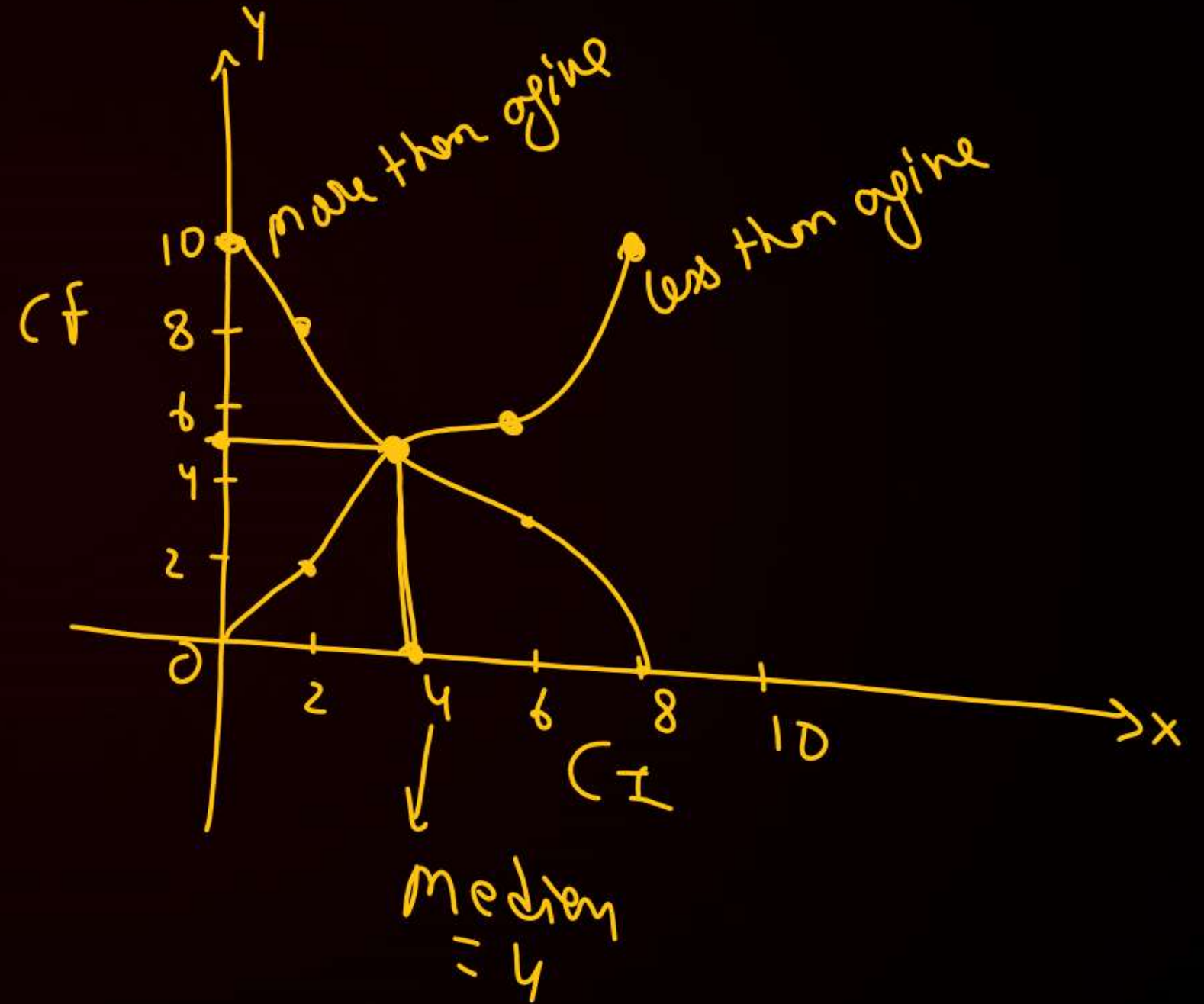


# Cumulative frequency curve (ogive)

C.I	$f_i$	less than	Cf	more than	Cf
0-2	2	2	2	0	10
2-4	3	4	5	2	8
4-6	1	6	6	4	5
6-8	4	8	10	6	4

$$N = 10$$

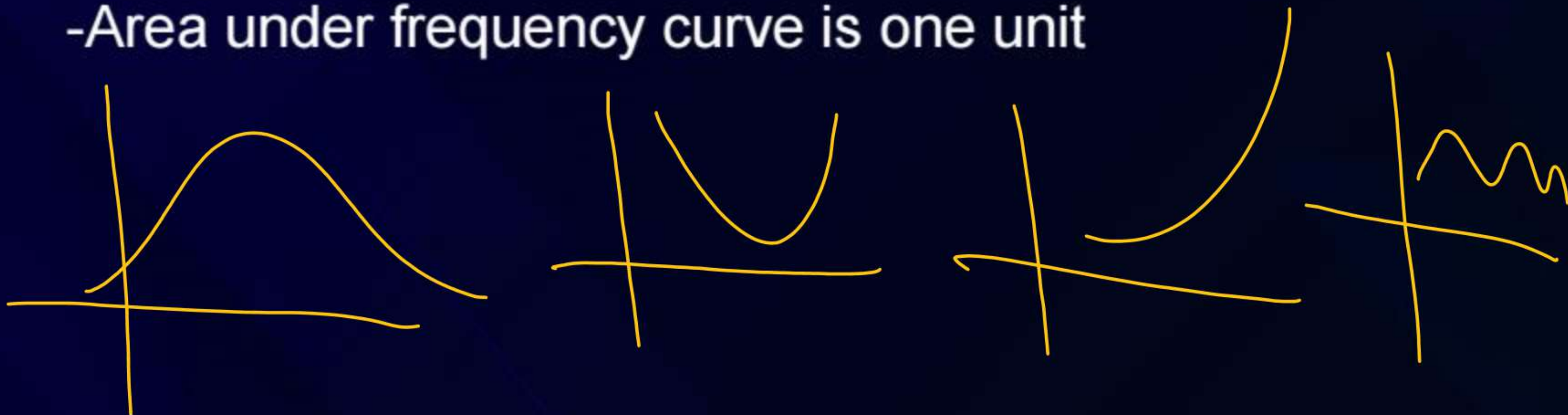
$$\frac{N}{2} = 5$$





# Frequency Curve

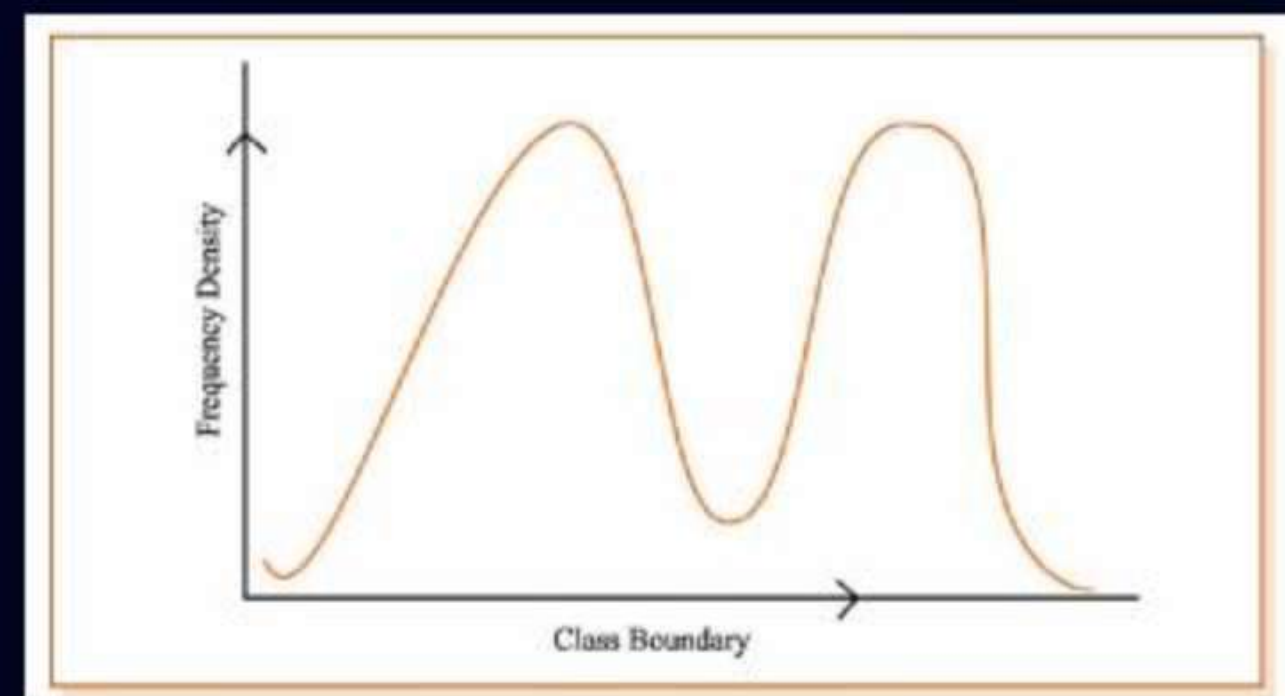
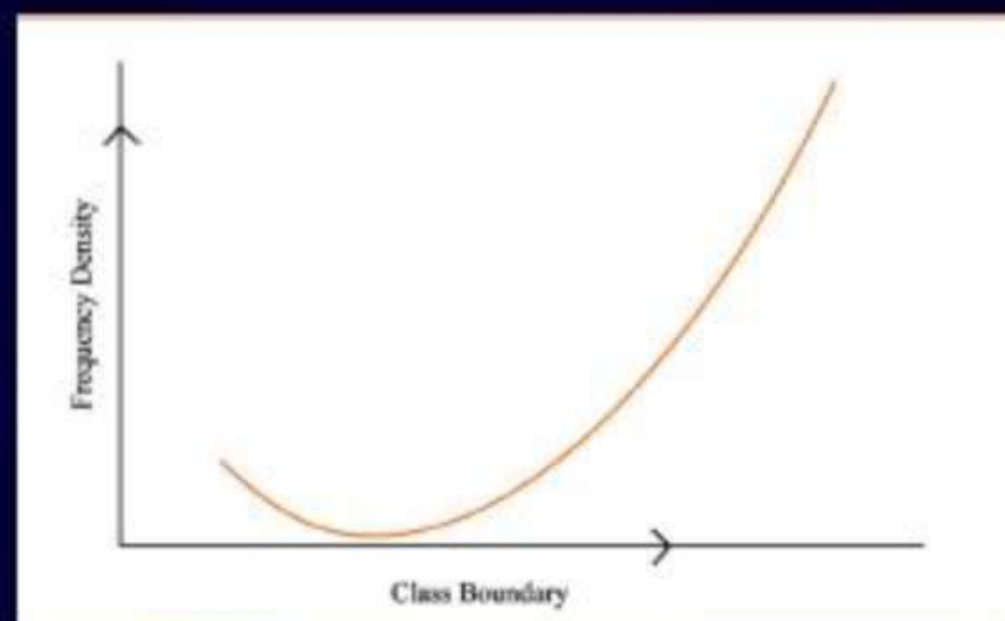
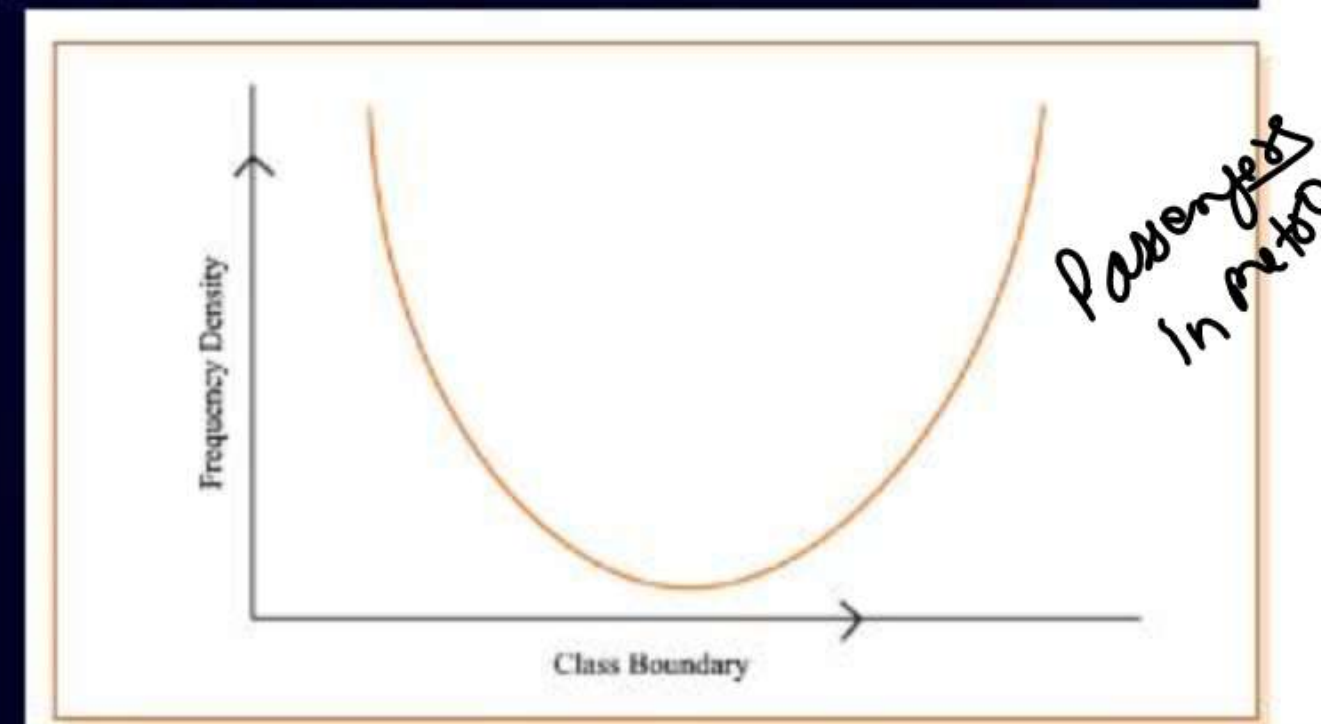
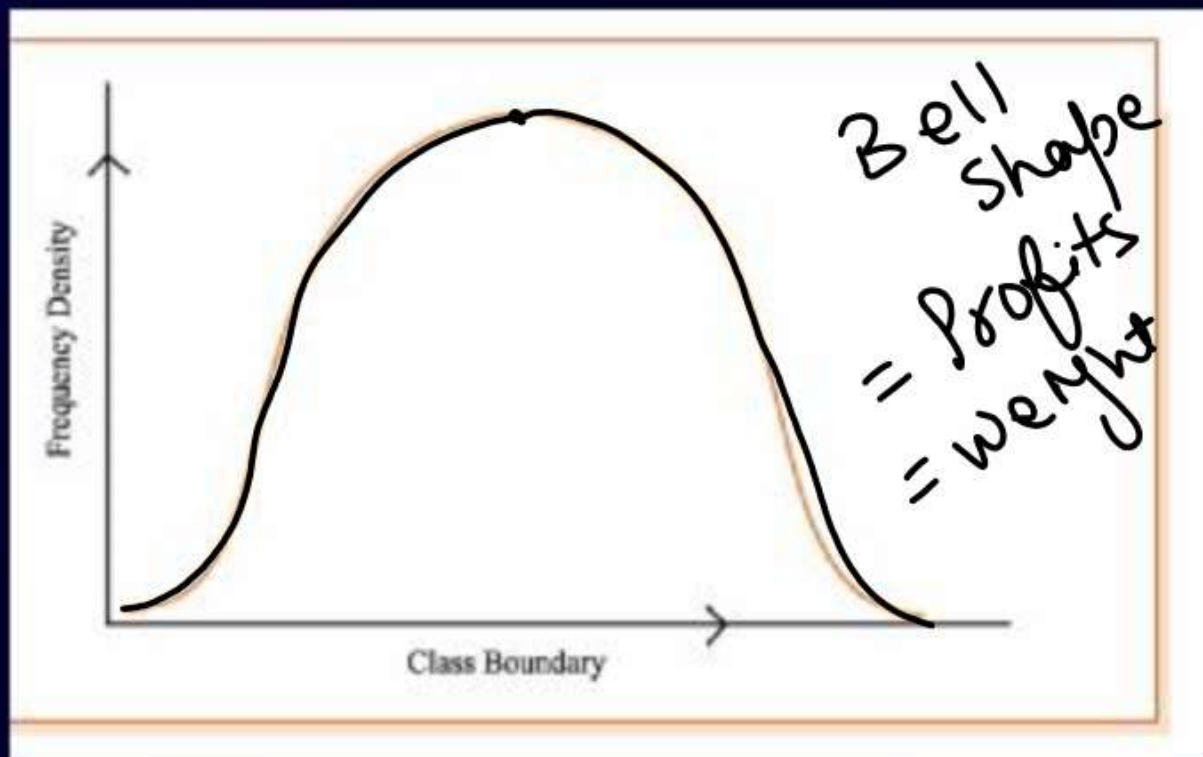
- Smooth curve obtained by joining midpoint of upper side of rectangles in histogram
- Area under frequency curve is one unit





There exist four types of frequency curves namely

- (a) Bell-shaped curve;
- (b) U-shaped curve;
- (c) J-shaped curve;
- (d) Mixed curve.





## QUESTION



The quickest method to collect primary data is

- (a) Personal Interview
- (b) Indirect Interview
- (c) Mailed Questionnaire Method
- (d) Telephonic Interview ✓✓

## QUESTION



Which of the following statement is true?

- (a) Statistics is derived from the French word 'Statistik'
- (b) Statistics is derived from the Italian word 'Statista'. ✓✓
- (c) Statistics is derived from the Latin word 'Statistique'.
- (d) None of these



## QUESTION



Relative frequency for a particular class lies between :

- (a) 0 and 1 ✓✓
- (b) 0 and 1, both inclusive
- (c) - 1 and 0 ✓
- (d) - 1 and 1 ✗

## QUESTION



The graphical representation of a cumulative frequency distribution is called :

- (a) Histogram
- (c) Both

- (b) Ogive ✓✓
- (d) None



## QUESTION



Cost of sugar in a month under the heads raw materials, labour, direct production and others were 12, 20, 35 and 23 units respectively. What is the difference between the central angles for the largest and smallest components of the cost of sugar ?

- (a)  $72^\circ$
- (c)  $56^\circ$

(b)  $48^\circ$

(d)  $92^\circ$

Handwritten calculations:

$$\begin{array}{r} 12 \\ 20 \\ 35 \\ 23 \\ \hline 90 \end{array}$$
$$\frac{12}{90} \times 360 = 48$$
$$\frac{35}{90} \times 360 = 140$$
$$140 - 48 = 92$$

## QUESTION



Frequency density corresponding to a class interval is the ratio of :

- (a) Class Frequency to the Total Frequency
- (b) Class Frequency to the Class Length ✓
- (c) Class Length to the Class Frequency
- (d) Class Frequency to the Cumulative Frequency



An area diagram is :

- |                       |                   |
|-----------------------|-------------------|
| (a) Histogram //      | (b) Ogive         |
| (c) Frequency Polygon | (d) None of these |

## QUESTION



The distribution of profits of a company follows

- (a) J - shaped frequency curve
- (b) U - shaped frequency curve
- (c) Bell - shaped frequency curve ✓✓
- (d) Any of these



Median of a distribution can be obtained from ;

- |                              |                       |
|------------------------------|-----------------------|
| (a) Histogram                | (b) Frequency Polygon |
| (c) Less than type Ogives // | (d) None of these     |

## QUESTION



The column headings of a table are known as

(a) Body

(b) Stub

(c) Box - head

(d) Caption ✓✓



## QUESTION



From the following data find the number class intervals if class length is given as 5.

73, 72, 65, 41, 54, 80, 50, 46, 49, 53.

(a) 6

(b) 5

(c) 7

(d) 8

(1 mark)

$$\begin{array}{r} 80 - 41 \\ \hline = 39 \\ = 7.8 \approx 8 \end{array}$$

41-46  
46-51  
51-56  
56-61  
61-66  
66-71  
71-76  
76-81

## QUESTION



The most appropriate diagram to represent the data relating to the monthly expenditure on different items by a family is

- |                       |                    |          |
|-----------------------|--------------------|----------|
| (a) Histogram         | (b) Pie-diagram. ✓ |          |
| (c) Frequency polygon | (d) Line graph.    | (1 mark) |



## QUESTION



Which of the following is a statistical data ?

- (a) Ram is 50 years old. ✗
- (b) Height of Ram is 5'6" and of Shyam and Hari is 5'3" and 5'4" respectively. ✓
- (c) Height of Ram is 5'6" and weight is 90kg ✗
- (d) Sale of A was more than B and C. ✗

(1 mark)

## QUESTION



Which of the following is not a two-dimensional figure ?

- (a) Line Diagram  $\rightarrow 1$  (b) Pie Diagram  $\rightarrow 2$   
(c) Square Diagram  $\rightarrow 2$  (d) Rectangle Diagram  $\rightarrow 2$



## QUESTION



Arrange the dimensions of Bar diagram, Cube diagram, Pie diagram in sequence.

(a) 1, 3, 2 ✓✓

(c) 2, 3, 1

(b) 2, 1, 3

(d) 3, 2, 1

(1 mark)

## QUESTION



With the help of histogram one can find.

- |             |                    |
|-------------|--------------------|
| (a) Mean    | (b) Median         |
| (c) Mode ✓✓ | (d) First Quartile |



Nationality of a person is :

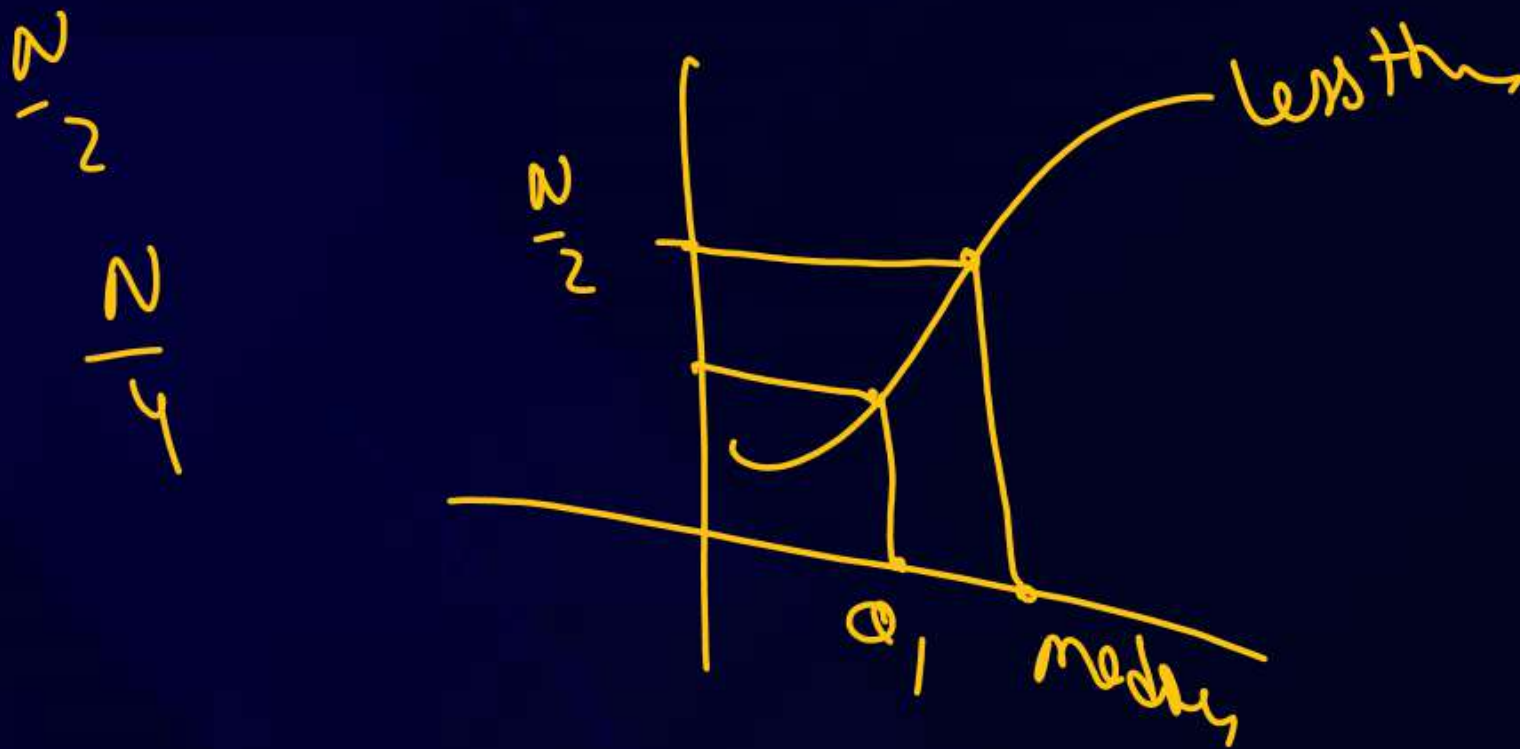
- |                         |              |                               |
|-------------------------|--------------|-------------------------------|
| (a) Discrete variable   | <del>/</del> | <del>/</del> (b) An attribute |
| (c) Continuous variable | <del>/</del> | (d) None                      |

## QUESTION



Using Ogive Curve, we can determine

- (a) Median
- (b) Quartile
- (c) Both (a) and (b) ✓✓
- (d) None.





## QUESTION



Histogram is used for the presentation of the following type of series

- (a) Time series
- ☒ (b) Continuous frequency distribution
- (c) Discrete frequency distribution
- (d) Individual observation

(1 mark)

## QUESTION



Classification is of \_\_\_\_\_ kinds.

(a) Two

(b) Three

(c) One

(d) Four



## QUESTION



The chart that uses logarithm of variable is known as:

- ☒ (a) Ratio chart
- ☐ (b) Line chart
- ☐ (c) Multiple line chart
- ☐ (d) Component line chart

'Stub' of a table is the

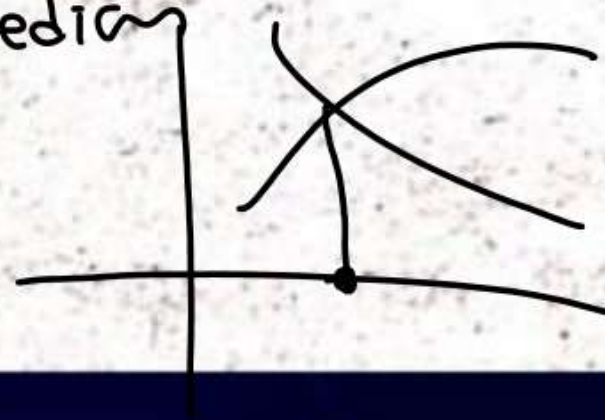
- (a) Left part of the table describing the columns
- (b) Right part of the table describing the columns
- (c) Right part of the table describing the rows
- ☒ (d) Left part of the table describing the rows.

## QUESTION



A perpendicular drawn from the point of intersection of two Ogive on the horizontal axis gives the value of:

- ☒ (a) 2<sup>nd</sup> Quartile = median
- (b) 3<sup>rd</sup> Quartile
- (c) Mode
- (d) 1<sup>st</sup> Quartile





## QUESTION



The frequency of visitor in an office is given below:

Time	9 AM-11 AM	11 AM – 1 PM	1 PM- 3 PM	3 PM – 5 PM
Frequency	5	18	7	12

Find the cumulative frequency of visitors for the time 11 AM – 1 PM?

- (a) 5
- ☒ (b) 23
- (c) 18
- (d) 30

Handwritten calculations and arrows:  
Arrows point from the question to the frequency values 5, 18, 7, and 12.  
Below 5 is a handwritten '5'.  
Below 18 is a handwritten '23'.  
Below 7 is a handwritten '30'.  
Below 12 is a handwritten '42'.



# Central Tendency & Dispersion



# Central Tendency

A central value which represent all the observations  
Of a series is called central tendency

→ AM

→ median

→ mode

→ G.M.

→ H.M.





# Central Tendency

**A central value which represent all the observations  
Of a series is called central tendency**



## Criteria for An ideal Central Tendency

- 1) Easy calculation ✓✓
- 2) Easy to understand ✓✓
- 3) Based on all the observations ✓✓
- 4) Rigidly defined ✓✓
- 5) Have mathematical properties ✓✓
- 6) Least affected by extreme values



$$\frac{1 \times 2}{1 + 1 + 2}$$

$$\frac{0 \times 2}{0 + 1 + 2}$$





# Arithmetic Mean



Direct method

$$\bar{X} = \frac{\sum x_i}{N}$$

$$\bar{X} = \frac{\sum f_i x_i}{N}$$

Shortcut method

Assume mean

$$\Rightarrow d_i = x_i - A$$

then

$$\bar{X} = A + \frac{\sum f_i d_i}{N}$$

Step Deviation method

$$\Rightarrow u_i = \frac{x_i - A}{h}$$

then

$$\bar{X} = A + \frac{\sum f_i u_i}{N} \times h$$



eg

$x_i$
1
2
3
4
5

$$\begin{aligned}\bar{x} &= \frac{\sum x_i}{n} \\ &= \frac{15}{5} \\ &= 3\end{aligned}$$

eg

$x_i$	$f_i$	$fix_i$
1	3	3
2	2	4
3	4	12
4	1	4
10		23

$$\begin{aligned}\bar{x} &= \frac{\sum fix_i}{n} \\ &= \frac{23}{10} \\ &= 2.3\end{aligned}$$

eg

CI	$f_i$	$x_i$	$fix_i$
0-2	3	1	3
2-4	8	3	24
4-6	4	5	20
6-8	5	7	35
N=20			82

$$\begin{aligned}\bar{x} &= \frac{\sum fix_i}{n} = \frac{82}{20} \\ &= 4.1\end{aligned}$$

g

$x_i$	$f_i$	$fix_i$
6.7	5	33.5
7.7	4	30.8
8.7	3	26.1
9.7	8	77.6
10.7	5	53.5
$N = 25$		221.5

$$\bar{X} = \frac{\sum fix_i}{N} = \frac{221.5}{25} = 8.86$$

$x_i$	$f_i$	$d_i = x_i - 8.7$	$f_i d_i$
6.7	5	-2	-10
7.7	4	-1	-4
8.7	3	0	0
9.7	8	1	8
10.7	5	2	10
$N = 25$			4

$$\begin{aligned} \bar{X} &= A + \frac{\sum f_i d_i}{N} \\ &= 8.7 + \frac{(4)}{25} \\ &= 8.86 \end{aligned}$$

Assume mean method



g

<u>C.I</u>	<u>f<sub>i</sub></u>	<u>x<sub>i</sub><sup>o</sup></u>	<u>f<sub>i</sub>x<sub>i</sub></u>
2-5	3	3.5	10.5
5-8	2	6.5	13
8-11	6	9.5	57
11-14	4	12.5	50
			<u>130.5</u>

N = 15

$$\begin{aligned}\bar{X} &= \frac{\sum f_i x_i^o}{N} \\ &= \frac{130.5}{15} \\ &= 8.7\end{aligned}$$

g

<u>C.I</u>	<u>f<sub>i</sub></u>	<u>x<sub>i</sub></u>	<u>U<sub>i</sub></u>	<u>f<sub>i</sub>U<sub>i</sub></u>
2-5	3	3.5	-1	-3
5-8	2	6.5 = A	0	0
8-11	6	9.5	1	6
11-14	4	12.5	2	8
	<u>15</u>			<u>11</u>

$$\begin{aligned}\bar{X} &= A + \frac{\sum f_i U_i}{N} \times h \\ &= 6.5 + \frac{(11)}{15} \times 3 \\ &= 8.7\end{aligned}$$



Q

<u>C.I</u>	<u>f<sub>i</sub></u>
2-5	3
5-8	7
8-11	6
11-14	4

mean = 8.7

find missing frequency.

Sol:

<u>x<sub>i</sub></u>	<u>f<sub>i</sub></u>	<u>f<sub>i</sub>x<sub>i</sub></u>
3.5	3	10.5
6.5	f	6.5f
9.5	6	57
12.5	4	50

$N = 13 + f$	$117.5 + 6.5f$
--------------	----------------

$$\bar{x} = \frac{\sum f_i x_i}{n}$$

$$8.7 = \frac{117.5 + 6.5f}{13 + f}$$

$$8.7(13 + f) = 117.5 + 6.5f$$

$$113.1 + 8.7f = 117.5 + 6.5f$$

$$2.2f = 4.4$$

$$f = 4.4 / 2.2$$

$f = 2$
---------

## Combined mean

Group-A

$N_1$

$\bar{X}_1$

Group-B

$N_2$

$\bar{X}_2$

combined  
mean

$$\bar{X}_{12} = \frac{N_1 \bar{X}_1 + N_2 \bar{X}_2}{N_1 + N_2}$$

Ex

Girls

$N_1 = 5$

$\bar{X}_1 = 25$

Boys

$N_2 = 15$

$\bar{X}_2 = 30$

find combined mean

Sol.

$$\begin{aligned}\bar{X}_{12} &= \frac{5(25) + 15(30)}{5 + 15} \\ &= \frac{575}{20} \\ &= 28.75\end{aligned}$$

Change of origin



If  $k$  is added  
to each item then  
new mean will be  
 $= \text{old mean} + k$

Change of scale



If each item of  
series is multiplied by  $k$

New mean  
 $= \text{old mean} \times k$

$$\text{If } y_i = a + bx_i;$$

$$\text{then } \bar{y} = a + b\bar{x}$$



change of origin

change of scale

mean

✓

✓

median

✓

✓

mode

✓

✓

Q

$$\bar{x} = 25$$

$$y_i = 2x_i + 10$$

find  $\bar{y} = ?$

Sol.

$$\bar{y} = 2\bar{x} + 10$$

$$= 2(25) + 10$$

$$= 50 + 10$$

$$= 60$$

# If each item is 'k'  
then A.M. is also 'k'.

---

$$\# \sum_{\text{over}} (x_i - \bar{x}) = 0$$

Sum of deviations from  
arithmetic mean is always zero

---

$$\# \sum (x_i - A)^2 \text{ will be minimum when } A = \bar{x}$$

Sum of Squares of Deviation is  
minimum when deviations are taken from A.M.

$$\sum (x_i - m)^2$$

$$\sum (x_i - \text{modu})^2$$

$$\sum (x_i - \bar{x})^2$$

Minimum



# weighted mean

$$\text{weighted mean} = \frac{\sum w_i x_i}{\sum w_i}$$

g

$x_i$	$w_i$	$w_i x_i$
1	3	3
2	5	10
3	2	6
	<u>10</u>	<u>19</u>

$$\text{weighted mean} = \frac{19}{10} = 1.9$$

## QUESTION



The mean of 'n' observation is 'x'. If k is added to each observation, then the new mean is.

(a) k

(b) xk

(c) x-k

(d) x+k

$$\text{mean}(\bar{x}) = x$$

$$\text{new mean} = x + k$$

(1 mark)

## QUESTION



If there are 3 observations 15, 20, 25 then the sum of deviation of the observations from their AM is

(a) 0

(b) 5

(c) -5

(d) 10

$$\sum (x_i - \bar{x}) = 0$$

(1 mark)



# QUESTION

CA

Find the mean of the following data

Class interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	9	13	6	4	6	2	3

(a) 23.7

(c) 39.7

(b) 35.7

(d) 43.7

(1 mark)

$x_i$	$f_i$	$U_i$	$f_i u_i$
15	9	-3	-27
25	13	-2	-26
35	6	-1	-6
45	4	0	0
55	6	1	6
65	2	2	4
75	3	3	9
	<u>43</u>		<u>-40</u>

$$\begin{aligned}\bar{X} &= A + \frac{\sum f_i u_i}{n} \times h \\ &= 45 + \frac{(-40)}{43} \times 10 \\ &= 35.69\end{aligned}$$

## QUESTION

CA

If mean of 5 observations  $x + 1$ ,  $x + 3$ ,  $x + 5$ ,  $x + 7$  and  $x + 9$  is given 15, then the value of  $x$  will be:

- (a) 10
- (b) 12
- (c) 8
- (d) 11

$x_i$

(1 mark)

$$x+1$$

$$x+3$$

$$x+5$$

$$x+7$$

$$x+9$$

$$5x + 25$$

$$\bar{x} = \frac{\sum x_i}{n}$$

$$15 = \frac{5x + 25}{5}$$

$$75 = 5x + 25$$

$$50 = 5x$$

$$x = 10$$



## QUESTION

CA

The mean of the first three terms is 17 and mean of next four terms is 21. Calculate the mean of seven terms.

- (a) 18.28
- (b) 19.78
- (c) 19.58
- (d) 19.28

(1 mark)

$$\begin{array}{l} \underbrace{1 \quad 2 \quad 3}_3 \\ \bar{x}_1 = 17 \\ N_1 = 3 \end{array} \quad \begin{array}{l} \underbrace{4 \quad 5 \quad 6 \quad 7}_4 \\ \bar{x}_2 = 21 \\ N_2 = 4 \end{array}$$

$$\bar{x}_{12} = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2} = \frac{17(3) + 21(4)}{3+4} = 19.28$$



## QUESTION

CA

The mean of a set of 20 observations is 18.3. The mean is reduced by 0.6 when a new observation is added to the set. The new observation is:

(a) 17.6

(b) 18.9

(c) 5.7

(d) 24.6

$$N = 20$$
$$\bar{x} = 18.3$$

$$N = 21$$
$$\text{new mean} = 18.3 - 0.6$$
$$= 17.7$$

(1 mark)

$$\sum_{i=1}^{20} x_i = N \bar{x}$$
$$= 20 \times 18.3$$
$$= 366$$

$$\sum_{i=1}^{21} x_i = 21 \times 17.7$$
$$= 371.7$$

$$371.7 - 366 = 5.7$$

## QUESTION

CA

A Professor has given assignment to students in a statistics class. A student computer the arithmetic mean and standard deviation for 100 students as 50 and 5 respectively. Later on She points out the student that he has made mistake in taking one observation as 100 instead of 50. What would be the consent mean if the wrong observation is correct?

(a) 50.5

(b) 49.9

(c) 49.5

(d) 50.1

(1 mark)

$$\bar{x} = 50$$

$$\sigma = 5$$

$$N = 100$$

$$\bar{x} = \frac{\sum x_i}{N}$$

$$50 = \frac{\sum x_i}{100}$$

wrong  $\sum x_i = 5000$

correct  $\sum x_i = 5000 - 100 + 50$

$$\sum x_i = 4950$$

Correct mean

$$= \frac{4950}{100}$$

$$= 49.50$$



## QUESTION

CA

There are  $n$  numbers. When 50 is subtracted from each of these number the sum of the numbers so obtained is -10. When 46 is subtracted from each of the original  $n$  numbers, then the sum of numbers so obtained is 70. What is the mean of the original  $n$  numbers?

- (a) 56.8
- (b) 25.7
- (c) 49.5
- (d) 53.8

(1 mark)

$$\sum (x_i - 50) = -10 \quad \& \quad \sum (x_i - 46) = 70$$

$$\sum x_i - 50n = -10 \quad \& \quad \sum x_i - 46n = 70$$

$$\sum x_i = -10 + 50n \quad \& \quad \sum x_i = 46n + 70$$

$$\sum (x_i - k)$$

$$\sum x_i - kn$$



$$50n - 10 = 46n + 70$$

$$4n = 80$$

$$n = 20$$

nn

$$\sum x_i = -10 + 50n$$

$$= -10 + 50(20)$$

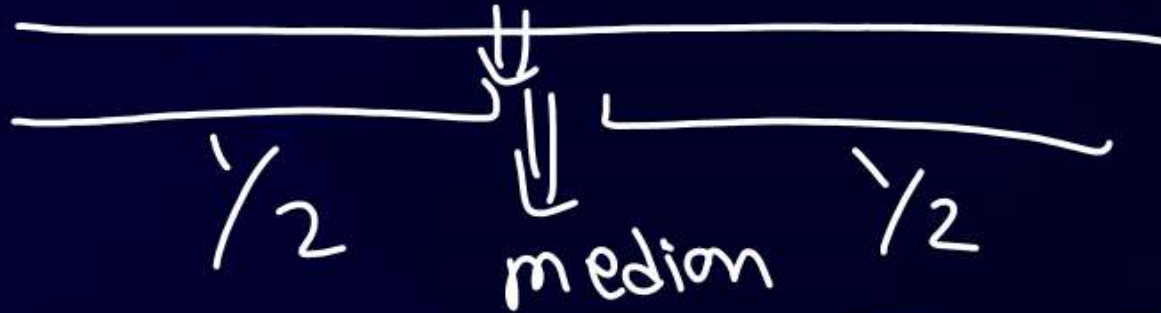
$$= -10 + 1000$$

$$= 990$$

$$\begin{aligned}\bar{x} &= \frac{\sum x_i}{n} \\ &= \frac{990}{20} \\ &= 49.5\end{aligned}$$



# Median $\Rightarrow 50\%$



Individual series or Described series

$n = \text{odd}$        $n = \text{even}$

$$\text{Median} = \left( \frac{n+1}{2} \right)^{\text{th}} \text{ term}$$

$$\text{Median} = \frac{\left( \frac{n}{2} \right)^{\text{th}} + \left( \frac{n}{2} + 1 \right)^{\text{th}}}{2}$$

## Continuous Series

→ find  $\frac{N}{2}$

→ locate  $\frac{N}{2}$  in cf

→ select median class

$$\rightarrow \text{median} = l + \left\{ \frac{\frac{N}{2} - cf}{f} \right\} \times h$$



## QUESTION



For the given data set: 5, 10, 3, 6, 4, 8, 9, 3, 15, 2, 9, 4, 19, 11, 4, what is the median

(a) 8

(b) 6

(c) 4

(d) 9

(1 mark)

$x_i: 2, 3, 3, 4, 4, 4, 5, 6, 8, 9, 9, 10, 11, 15, 19$

$$N = 15 (\text{odd})$$

$$\text{median} = \left( \frac{15+1}{2} \right)^{\text{th}} = 8^{\text{th}} = 6$$

eg

$x_i$	$f_i$	$cf$
1	3	3 (1-3)
2	2	5 (4-5)
3	5	10 (6-10)
4	4	14 (11-14)
5	2	16 (15-16)
$N=16$		

find median

Sol.  $N=16$  (even)

1, 1, 1, 2, 2, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5

$$\text{median} = \frac{\left(\frac{16}{2}\right)^{\text{th}} + \left(\frac{16}{2} + 1\right)^{\text{th}}}{2}$$

$$= \frac{8^{\text{th}} + 9^{\text{th}}}{2}$$

$$= \frac{3 + 3}{2} = \frac{6}{2} = 3$$



# QUESTION

CA

The Median of the following frequency distribution is:

x	0-10	10-20	20-30	30-40	40-50
f(x)	3	5	20	12	7

(a) 27.75

(b) 9.35

(c) 8.25

(d) 10.01

median

$$= l + \left\{ \frac{\frac{N}{2} - cf}{f} \right\} \times h$$

$$= 20 + \left\{ \frac{23.5 - 8}{20} \right\} \times 10$$

$$= 27.75$$

$$\frac{N}{2} = \frac{47}{2} = 23.5$$

Ci	fi	cf
0-10	3	3
10-20	5	8
20-30	20	28
30-40	12	40
40-50	7	47
	<u>47</u>	

median class



#  $\sum |x_i - A|$  will be minimum if  $A = \text{median}$

Sum of absolute deviations  
taken from median is always minimum

$$\sum |x_i - \bar{x}| \quad \text{or} \quad \sum |x_i - \text{mode}| \quad \text{or} \quad \sum |x_i - \text{median}|$$

$\times$                        $\times$                        $\Downarrow$   
minimum

## QUESTION



Mean deviation is minimum when deviations are taken from:

- (a) Mean
- (b) Median
- (c) Mode
- (d) Range

## QUESTION



If two variable 'x' and 'y' are related as  $2x - y = 3$ , if the median of 'x' is 10, what is median of 'y'?

(a) 4

(b) 17

(c) 5

(d) 6

$$2x - y = 13$$

$$\text{med. of } x = 10$$

now

$$2x - y = 3$$

$$2x - 3 = y$$

$$y = 2x - 3$$

$$\begin{aligned} \text{Med of } y &= 2(10) - 3 \\ &= 17 \end{aligned}$$

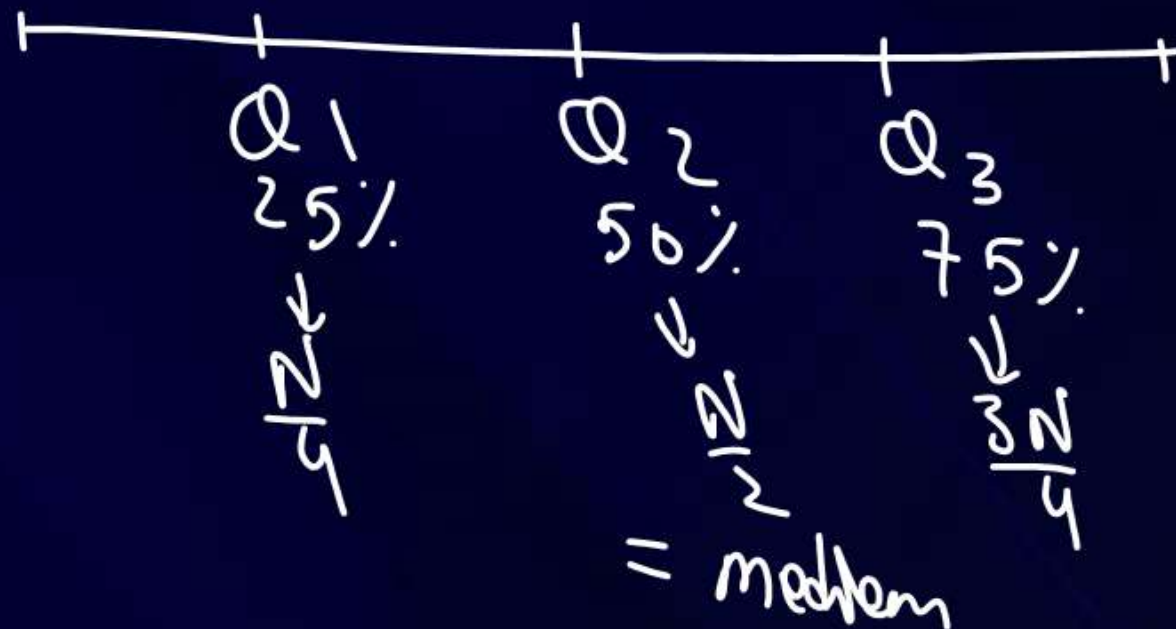




# Fractiles

## Quantiles

Divide the series  
in 4 parts



## Individual Series / Discrete

$$Q_1 = \left( \frac{N+1}{4} \right)^{th} \text{ \& } Q_3 = \left[ 3 \left( \frac{N+1}{4} \right) \right]^{th}$$

## Continuous series

→ locate  $\frac{N}{4}$  &  $\frac{3N}{4}$  in cf

→ select  $Q_1$  &  $Q_3$  class

$$\rightarrow Q_1 = l + \left\{ \frac{\frac{N}{4} - cf}{f} \right\} \times h$$

$$\& \ Q_3 = l + \left\{ \frac{\frac{3N}{4} - cf}{f} \right\} \times h$$

g marks

$$\begin{array}{r}
 2 \\
 [ 8 \\
 12 \\
 25 \\
 30 \\
 [ 36 \\
 39 \\
 50 \\
 \hline
 N=8
 \end{array}$$

$$\begin{aligned}
 Q_1 &= \left( \frac{N+1}{4} \right)^{th} \\
 &= \left( \frac{8+1}{4} \right)^{th} \\
 &= (2.25)^{th} \\
 &= 2^{nd} + 0.25(3^{rd} - 2^{nd}) \\
 &= 8 + 0.25(12 - 8) \\
 &= 8 + 1 \\
 &= 9
 \end{aligned}$$

$$\begin{aligned}
 Q_3 &= \left[ 3 \left( \frac{N+1}{4} \right) \right]^{th} \\
 &= \left[ 3 \left( \frac{8+1}{4} \right) \right]^{th} \\
 &= (6.75)^{th} \\
 &= 6^{th} + 0.75(7^{th} - 6^{th}) \\
 &= 36 + 0.75(39 - 36) \\
 &= 36 + 2.25 \\
 &= 38.25
 \end{aligned}$$



1  
1  
1  
2  
2  
2  
2  
2  
2  
3  
3  
3  
3  
3  
4  
4  
4  
4  
4  
4

$$Q_1 = \left( \frac{23+1}{4} \right)^{th} = 6^{th} = 2$$

1



<u>CI</u>	<u><math>f_i</math></u>	<u>cf</u>
0-8	2	2
8-16	4	6
16-24	6	12
24-32	2	14
32-40	6	20
<u><math>N=20</math></u>		

$$\frac{N}{4} = \frac{20}{4} = 5$$

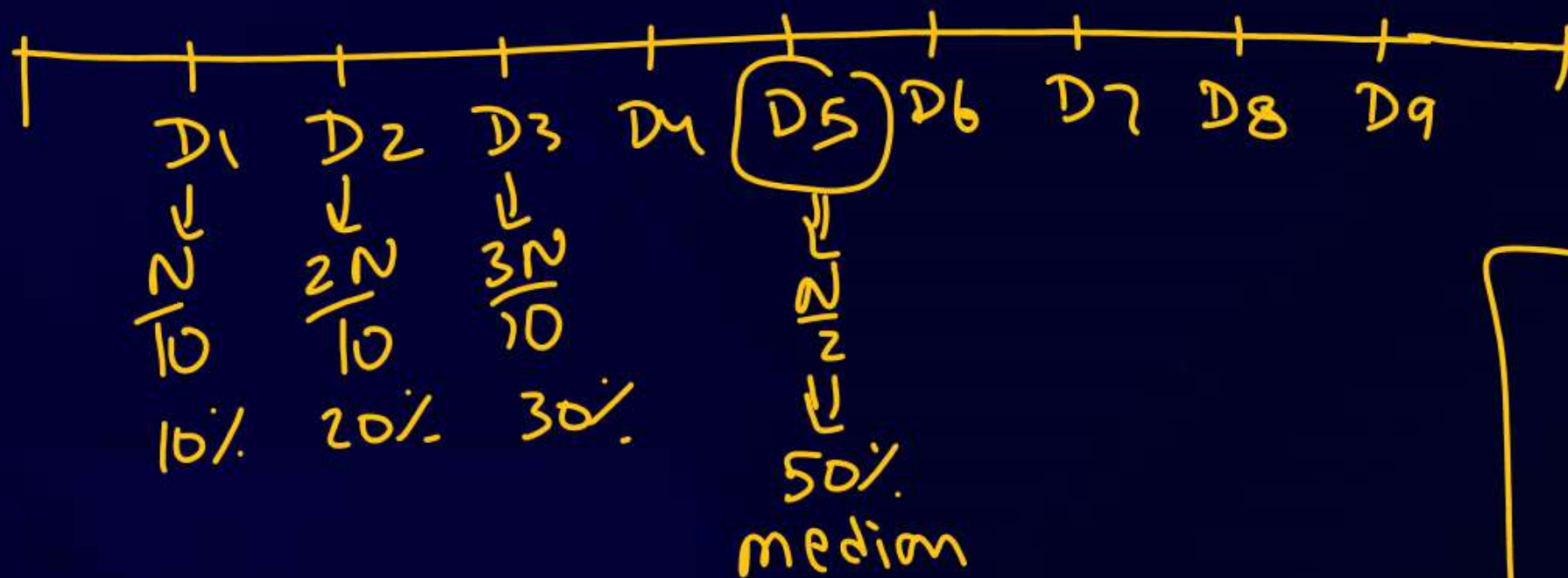
$$\frac{3N}{4} = \frac{3 \times 20}{4} = 15$$

$$\begin{aligned}
 Q_1 &= l + \left\{ \frac{\frac{N}{4} - cf}{f} \right\} \times h \\
 &= 8 + \left\{ \frac{5 - 2}{4} \right\} \times 8 \\
 &= 8 + 6 \\
 &= 14
 \end{aligned}$$

$$\begin{aligned}
 Q_3 &= l + \left\{ \frac{\frac{3N}{4} - cf}{f} \right\} \times h \\
 &= 32 + \left\{ \frac{15 - 14}{6} \right\} \times 8 \\
 &= 33.33
 \end{aligned}$$



# Deciles



## Continuous

for  $D_1$

→ locate  $\frac{N}{10}$  in cf

→ Select  $D_1$  class

$$\rightarrow D_1 = l + \left\{ \frac{\frac{N}{10} - cf}{f} \right\} \times h$$

for  $D_7$

→ locate  $\frac{7N}{10}$  in cf

$$\rightarrow D_7 = l + \left\{ \frac{\frac{7N}{10} - cf}{f} \right\} \times h$$

## # Individual Series or Discrete series

$$D_1 = \left( \frac{N+1}{10} \right)^{th}$$

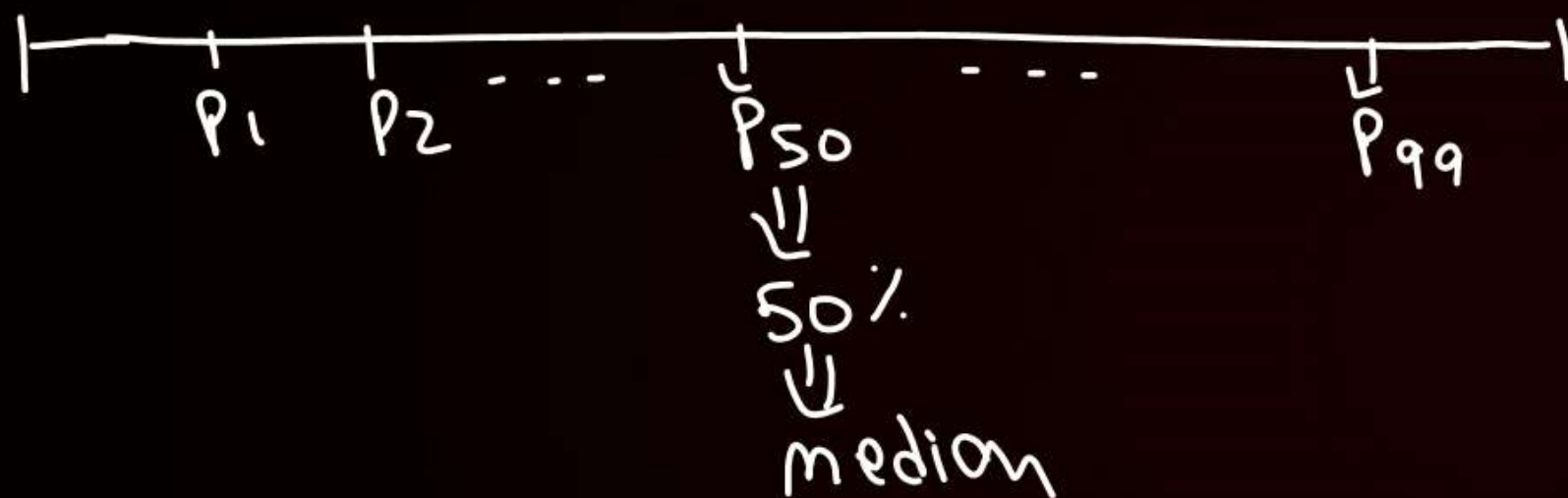
$$D_6 = \left[ \left( \frac{N+1}{10} \right) \right]^{th}$$

$$D_3 = 3 \left( \frac{N+1}{10} \right)$$

$$D_9 = \left[ 9 \left( \frac{N+1}{10} \right) \right]^{th}$$

# # Percentiles

Divides series  
in 100 parts



Individual / Discrete series

$$P_1 = \left( \frac{N+1}{100} \right)^{th}, \quad P_{62} = 62 \left( \frac{N+1}{100} \right)^{th}$$

Continuous series

for  $P_3$

→ locate  $\frac{3N}{100}$  in cf

$$\rightarrow P_3 = l + \left\{ \frac{\frac{3N}{100} - cf}{f} \right\} \times h$$



## QUESTION

CA

The 3<sup>rd</sup> decile for the numbers  
15, 10, 20, 25, 18, 11, 9, 12 is

- (a) 13
- (b) 10.70
- (c) 11.00
- (d) 11.50

9, 10, 11, 12, 15, 18, 20, 25  
 $N = 8$

$$D_3 = \left[ 3 \left( \frac{N+1}{10} \right) \right]^{th}$$

$$D_3 = 3 \left( \frac{8+1}{10} \right)^{th}$$

$$= (2.7)^{th}$$

$$= 2^{nd} + 0.7 (3^{rd} - 2^{nd})$$

$$= 10 + 0.7 (11 - 10)$$

$$= 10 + 0.7$$

$$= 10.7$$



# Mode



Observation with  
Highest frequency

g marks

2, 3, 1, 5, 2, 1, 3, 2, 5, 2, 1

1, 2, 1, 2, 4, 2, 5, 3, 2

mode = 2

g	marks $x_i$	No of students ( $f_i$ )
	1	4
	2	6
	3	2
	4	5
	5	1

mode = 2



<u>CI</u>	<u>fi</u>
0-4	3
4-8	4 $\rightarrow f_0$
<u>8-12</u>	7 $\rightarrow f_1$
12-16	5 $\rightarrow f_2$
16-20	1

modal class  $\rightarrow$  8-12

$$\text{mode} = l + \left\{ \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right\} \times h$$

$$= 8 + \left\{ \frac{7 - 4}{2(7) - 4 - 5} \right\} \times 4$$

$$= 8 + \left\{ \frac{3}{5} \right\} \times 4$$

$$= 10.4$$



$$\# \left\{ 3 \text{ median} = \text{mode} + 2 \text{ mean} \right\}$$

## QUESTION



Which of the following measures of central tendency cannot be calculated by graphical method?

☒ (a) Mean

(b) Mode

(c) Median

(d) Quartile

(1 mark)

ogive  
→ median  
→ Quartile  
→ Decile  
→ Percentile

Histogram  
↓  
mode



## QUESTION



Relation between mean, median and mode is

- ~~(a)~~ mean-mode = 2 (mean-median)
- ~~(b)~~ mean-median = 3(mean-mode)
- ~~(c)~~ mean-median = 2 (mean-mode)
- ~~(d)~~ mean-mode = 3(mean-median)



$$3 \text{ median} = \text{mode} + 2 \text{ mean}$$

## QUESTION



One hundred participants expressed their opinion on recommending a new product to their friends using the attributes : most unlikely, not sure, likely, most likely. The appropriate measure of central tendency that can be used here is

- (a) Mean
- (b) Mode
- (c) Geometric mean
- (d) Harmonic mean

ML  
LS  
NS  
MU

(1 mark)



## QUESTION



If the mean and median of a moderately asymmetrical series are 26.8 and 27.9 respectively, then the most probable mode is:

- (a) 35.4
- (b) 30.1
- (c) 34.3
- (d) 70.8

$$\begin{aligned}\bar{X} &= 26.8 \\ \text{med} &= 27.9 \\ \text{mode} &= ?\end{aligned}$$

$$\begin{aligned}3 \text{ median} &= \text{mode} + 2 \text{ mean} \\ 3(27.9) &= \text{mode} + 2(26.8) \\ 83.7 - 53.6 &= \text{mode} \\ \text{mode} &= 30.1\end{aligned}$$





# Geometric Mean



$$G.M. = \left[ x_1 \times x_2 \times x_3 \times \dots \times x_n \right]^{\frac{1}{n}}$$

eg G.M of 2 & 8

$$= (2 \times 8)^{\frac{1}{2}}$$
$$= (16)^{\frac{1}{2}}$$
$$= 4$$

eg G.M of 2, 10 & 25

$$= (2 \times 10 \times 25)^{\frac{1}{3}}$$
$$= (500)^{\frac{1}{3}}$$
$$= 7.945$$

$$(x)^{\frac{1}{n}} = ?$$

→  $\sqrt{\quad}$  12 times

→  $-$

→  $\div n$

→  $+$

→  $\boxed{x =}$  12 times

g

$x_i$	$f_i$
2	4
3	2
4	3
	9

$$h_m = \left( 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 4 \times 4 \times 4 \right)^{1/9}$$

g

CI	$f_i$	$x_i$
0-4	2	2
4-8	1	6
8-12	3	10

$$h_m = \left( 2 \times 2 \times 6 \times 10 \times 10 \times 10 \right)^{1/6}$$





#> If all observation are same (let  $k$ )  
then  $sm$  is also  $k$

e.g.  $sm$  of  $2, 2, 2, 2, 2$  is also  $2$

$$\# \log(sm) = \frac{\sum \log x_i}{N}$$

$$sm = AL \left( \frac{\sum \log x_i}{N} \right)$$

$$\# sm(x \cdot y) = sm \text{ of } x + sm \text{ of } y$$

$$\# sm\left(\frac{x}{y}\right) = \frac{sm \text{ of } x}{sm \text{ of } y}$$

## QUESTION



The geometric mean of 3, 7, 11, 15, 24, 28, 30, 0 is:

- (a) 6
- (b) 0
- (c) 9
- (d) 12

$$\begin{aligned} & \left( 3 \times 7 \times 11 \times 15 \times 24 \times 28 \times 30 \times 0 \right)^{1/8} \\ & = (0)^{1/8} = 0 \end{aligned}$$



# Harmonic Mean



“Reciprocal of Average of the reciprocal of all observations”

$$HM = \frac{1}{\left( \frac{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}}{n} \right)}$$



$$Hm = \frac{N}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}}$$

or

$$Hm = \frac{N}{\sum \left( \frac{1}{x_i} \right)}$$

$$Hm = \frac{N}{\sum \left( \frac{f_i}{x_i} \right)}$$

eg Hm of 2 & 8

$$= \frac{2}{\frac{1}{2} + \frac{1}{8}}$$

$$= \frac{2}{\frac{4+1}{8}}$$

$$= \frac{16}{5}$$

$$= 3.2$$

g Hm of 2, 5 & 10

$$= \frac{3}{\frac{1}{2} + \frac{1}{5} + \frac{1}{10}}$$

g

$x_i$	$f_i$	$\frac{f_i}{x_i}$
1	4	4
2	2	1
3	6	2
	12	7

$$Hm = \frac{N}{\sum \left( \frac{f_i}{x_i} \right)}$$

$$= \frac{12}{7}$$

$$= 1.7142$$

---

# Weighted Harmonic mean

$$= \frac{\sum w}{\sum \left( \frac{w_i}{x_i} \right)}$$

#

Combined Hm

$$= \frac{N_1 + N_2}{\frac{N_1}{H_1} + \frac{N_2}{H_2}}$$

#

$$Am \geq Gm \geq Hm$$

#

If all the items are same  
then  $Am = Gm = Hm$

#

If all the items are different  
then  $Am > Gm > Hm$

#

for any two numbers  
 $Hm \times Am = (Gm)^2$





## QUESTION



A man travels at a speed of 20 km/hr and then returns at a speed of 30 km/hr. His average speed of the whole journey is :

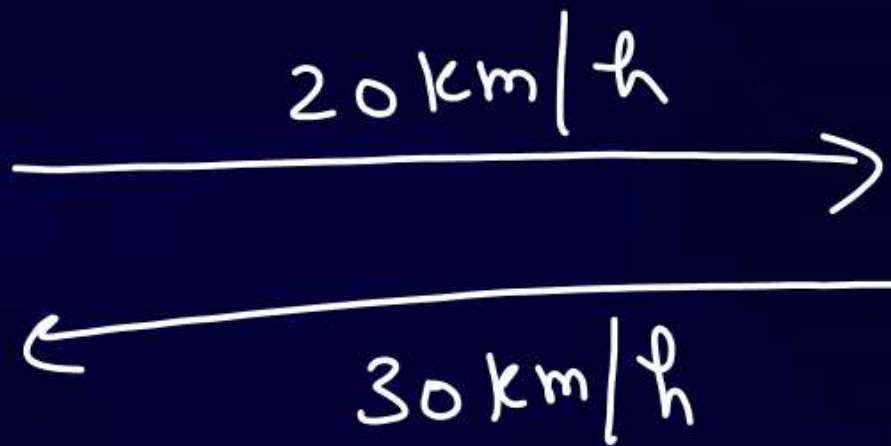
(a) 25 km/hr

(b) 24.5 km/hr

☒ (c) 24 km/hr

(d) None

(1 mark)



$$\begin{aligned} \text{Avg speed} &= \text{HM of } 20 \text{ \& } 30 \\ &= \frac{2}{\frac{1}{20} + \frac{1}{30}} \\ &= \frac{2}{\frac{30+20}{600}} = \frac{1200}{50} = 24 \end{aligned}$$

## QUESTION



Given the weights for the numbers 1,2,3.....n are respectively  $1^2, 2^2, 3^2, \dots, n^2$  then weighted HM is \_\_\_\_\_.

(a)  $\frac{2n+1}{4} = \frac{7}{4}$

(b)  $\frac{2n+1}{6} = \frac{7}{6}$

~~(c)  $\frac{2n+1}{3} = \frac{7}{3}$~~

(d)  $\frac{2n+1}{2} = \frac{7}{2}$

$x_i$	$w_i$	$\frac{w_i}{x_i}$
1	1	1
2	4	2
3	9	3
<hr/>		<hr/>
14		6

weighted HM  

$$= \frac{\sum w}{\sum \left( \frac{w_i}{x_i} \right)}$$

$$= \frac{14}{6} = \frac{7}{3}$$

(1 mark)



## QUESTION

CA

The harmonic mean A and B is  $\frac{1}{3}$  and harmonic mean of C and D is  $\frac{1}{5}$ . The harmonic mean of ABCD is

(a)  $\frac{8}{15}$

(b)  $\frac{1}{4}$

(c)  $\frac{1}{15}$

(d)  $\frac{5}{3}$

(1 mark)

A & B	C & D	Combined HM
$H_1 = \frac{1}{3}$	$H_2 = \frac{1}{5}$	$= \frac{N_1 + N_2}{\frac{N_1}{H_1} + \frac{N_2}{H_2}}$
$N_1 = 2$	$N_2 = 2$	$= \frac{2 + 2}{\left(\frac{2}{\frac{1}{3}}\right) + \left(\frac{2}{\frac{1}{5}}\right)} = \frac{4}{6 + 10} = \frac{4}{16} = \frac{1}{4}$

## QUESTION



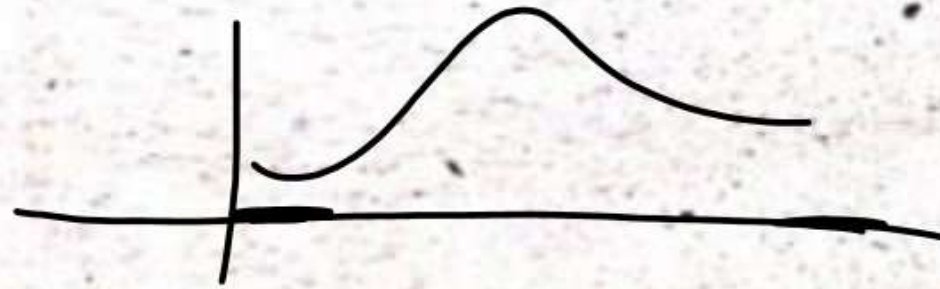
Which one of these is least affected by extreme values?

(a) Mean

(b) Median

(c) Mode

(d) None



(1 mark)



## QUESTION



From the record on sizes of shoes sold in a shop, one can compute the following to determine the most preferred shoe size.

- (a) Mean
- (b) Median
- (c) Mode
- (d) Range

(1 mark)



## QUESTION



If A.M. and G.M. of two positive numbers (a) and (b) are 12 and 12, respectively, find the numbers.

(a) 18 and 6

(b) 15 and 9

(c) 16 and 8

(d) 12 and 12

$$Am = 12$$

$$Gm = 12$$

$$a = b = 12$$

(1 mark)

## QUESTION

CA

Which measure is suitable for open-end classification?

- (a) Median
- (b) Mean
- (c) Mode
- (d) GM

<u>C</u>	<u>f<sub>i</sub></u>
less than 10	1
10-12	6
12-14	7
14-16	8
more than 16	2

## QUESTION



The AM and HM of two numbers are 5 and 3.2 respectively, then GM will be:

- (a) 4.4
- (b) 4.2
- (c) 4.0
- (d) 3.8

(1 mark)

$$AM = 5$$

$$HM = 3.2$$

$$GM = ?$$

$$GM^2 = AM \times HM$$
$$= 5 \times 3.2$$

$$GM^2 = 16$$

$$GM = 4$$



## QUESTION



Which of the following measure does not possess mathematical properties?

- (a) Arithmetic mean
- (b) Geometric mean
- (c) Harmonic mean
- (d) Median

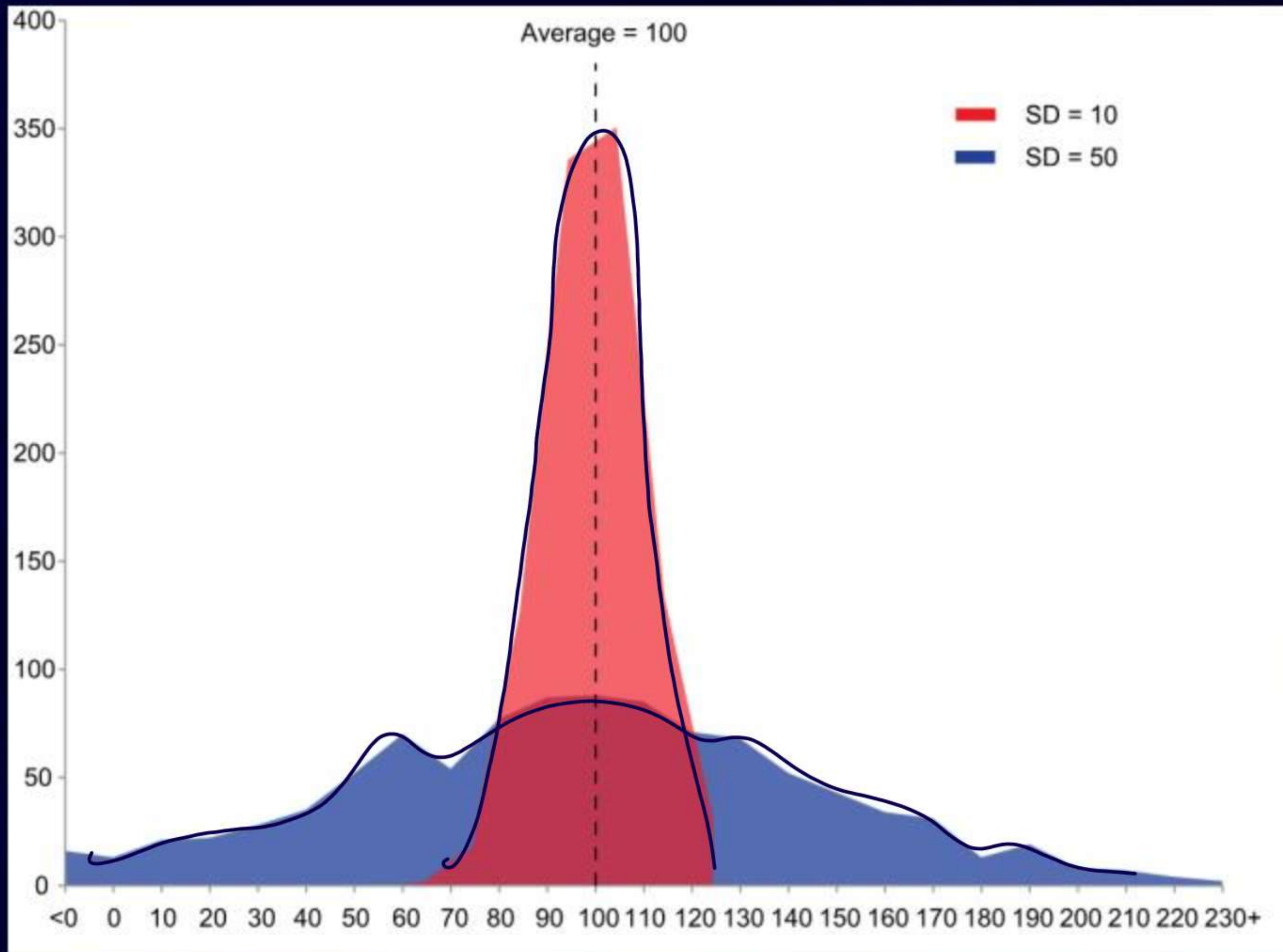
(1 mark)



# Dispersion



“The Degree Of the **scatterness** or **spread** or **variation** of the variable about a **central value** is called **dispersion**”





# Measures of Dispersion



Absolute measurement  
↓

→ Range

→ Q.D.

→ M.D.

→ S.D.

Relative measurement  
↓

→ coefficient of Range

→ coeff of Q.D.

→ coeff of M.D.

→ coeff of variance



# RANGE



Range = Largest - Smallest

$$\text{Range} = L - S$$

$$\text{Coeff. of Range} = \left( \frac{L - S}{L + S} \right) \times 100$$



g marks: 2, 5, 7, 8, 9

$$\text{Range} = 9 - 2 = 7$$

$$\begin{aligned} \text{Coeff of Range} &= \frac{9-2}{9+2} \times 100 \\ &= 63.63\% \end{aligned}$$

$\Sigma$	$\frac{x_i}{S}$	$\frac{f_i}{L}$
	1	6
	2	3
	3	5
	4	2
	5	8

$$\text{Range} = 5 - 1 = 4$$

$$\begin{aligned} \text{Coeff of Range} &= \frac{5-1}{5+1} \times 100 \\ &= 66.67\% \end{aligned}$$

$\Sigma$	$\frac{C_i}{S}$	$\frac{f_i}{L}$
	2-4	3
	4-6	5
	6-8	1
	8-10	3

$$\text{Range} = 10 - 2 = 8$$

$$\text{Coeff of Range} = \frac{10-2}{10+2} \times 100$$



Range, Q.D, M.D & S.D

→ Change of origin  $\Rightarrow$  No impact

Ex Range of  $x_i = 5$   
 $y_i = x_i + 10$   
 Range of  $y_i = ?$

Sol.: Range of  $y_i =$  Range of  $x_i$

Ex	$x_i$	$y_i = x_i + 10$
	1	11
	2	12
	3	13
	6	16
	Range = $6 - 1$ $= 5$	Range = $16 - 11$ $= 5$

Range, QD, MD, SD



CA WALLAH



## Change of scale

if all the items are multiplied by any number 'k'

then new Range =  $|k| \times \text{old Range}$

new SD =  $|k| \times \text{old SD}$

new MD =  $|k| \times \text{old MD}$

new QD =  $|k| \times \text{old QD}$

g Range of  $x_i = 5$

$$y = 2x_i$$

Range of  $y_i$

$$= 2 \times \text{Range of } x_i$$

$$= 2 \times 5$$

$$= 10$$

g Range of  $x_i = 5$

$$y_i = -2x_i$$

Range of  $y_i$

$$= |-2| \times \text{Range of } x_i$$

$$= 2 \times 5$$

$$= 10$$



## QUESTION



If the range of a data is 20 and its smallest value is 5, then what is the largest value of data is?

(a) 20

(b) 25

(c) 5

(d) 30

(1 mark)

$$\text{Range} = 20$$

$$L - S = 20$$

$$L - 5 = 20$$

$$L = 25$$

## QUESTION

CA

If the relationship between  $x$  and  $y$  is given by  $2x + 3y = 10$  and the range of  $y$  is 10, then what is the range of  $x$ ?

(a) 10

(b) 18

(c) 8

(d) 15

(1 mark)

$$2x + 3y = 10$$

find value of  $x$  in terms of  $y$

$$2x = -3y + 10$$

$$x = -\frac{3y}{2} + \frac{10}{2}$$

$$\text{Range of } y = 10$$

$$\text{Range of } x = \left| -\frac{3}{2} \right| \times \text{Range of } y$$

$$= \frac{3}{2} \times 10$$

$$= 15$$





# Quartile Deviation



$$\text{Quartile Range} = Q_3 - Q_1$$

$$\text{Quartile Deviation} = \frac{Q_3 - Q_1}{2}$$

(Semi-Quartile Range)

$$\text{Coeff of Q.D} = \left( \frac{Q_3 - Q_1}{Q_3 + Q_1} \right) \times 100$$

If Distribution is symmetric

$$\frac{Q_1 + Q_3}{2} = Q_2 = \text{median}$$



## QUESTION



If the quartile deviation is 12 and the first quartile is 25, then the value of the third quartile is:

(a) 37

(b) 49

(c) 61

(d) 60

(1 mark)

$$QD = 12 \text{ \& } Q_1 = 25, Q_3 = ?$$

$$\frac{Q_3 - Q_1}{2} = 12$$

$$\frac{Q_3 - 25}{2} = 12 \Rightarrow Q_3 = 49$$

## QUESTION

CA

If 'x' and 'y' are related as  $3x - 4y = 20$  and the quartile deviation of 'x' is 12, then the quartile deviation of 'y' is:

- (a) 9
- (b) 8
- (c) 7
- (d) 6

Sol.

$$3x - 4y = 20$$

$$QD \text{ of } x = 12$$

$$QD \text{ of } y = ?$$

$$3x - 4y = 20$$

$$-4y = -3x + 20$$

$$y = \frac{-3x}{-4} + \frac{20}{-4}$$

$$y = \frac{3}{4}x - 5$$

Now

$$QD \text{ of } y = \frac{3}{4} \times QD \text{ of } x$$

$$= \frac{3}{4} \times 12$$

$$= 9$$





# Mean Deviation



MD about  
mean

$$MD_{\bar{x}} = \frac{\sum f_i |x_i - \bar{x}|}{N}$$

$$\text{Coff of MD} = \frac{MD_{\bar{x}}}{\bar{x}} \times 100$$

MD about  
median

$$MD_m = \frac{\sum f_i |x_i - m|}{N}$$

$$\text{Coff of MD} = \frac{MD_m}{m} \times 100$$



## QUESTION

CA

Find mean deviation about mean for the data

12, 16, 21, 30, 35, 39, 40

(a) 9.14

(b) 9.63

(c) 8.91

(d) 9.81

$x_i$	$ x_i - \bar{x} $
12	15.57
16	11.57
21	6.57
30	2.43
35	7.43
39	11.43
40	12.43
<u>193</u>	<u>67.43</u>

$$\bar{x} = \frac{\sum x_i}{n} = \frac{193}{7} = 27.57$$

$$MD = \frac{\sum |x_i - \bar{x}|}{n}$$

$$= \frac{67.43}{7}$$

$$= 9.63$$

Q.  $x_i: 2, 6, 8, 12, 15, 17$   
find MD about median

Sol.

$x_i$	$ x_i - 10 $
2	8
6	4
8	2
12	2
15	5
17	7
	<hr/> 28

$N = 6$  (even)

$$\begin{aligned} \text{Median} &= \frac{\left(\frac{6}{2}\right)^{\text{th}} + \left(\frac{6}{2} + 1\right)^{\text{th}}}{2} \\ &= \frac{3^{\text{rd}} + 4^{\text{th}}}{2} \\ &= \frac{8 + 12}{2} = 10 \end{aligned}$$

$$\begin{aligned} MD_m &= \frac{\sum |x_i - m|}{N} \\ &= \frac{28}{6} \\ &= 4.6666 \end{aligned}$$





# Standard Deviation

( $\sigma$ )



$$SD(\sigma) = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{N}}$$

$$SD(\sigma) = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2}$$

or

$$\sqrt{\frac{\sum f_i x_i^2}{N} - (\bar{x})^2}$$

$$d_i = x_i - A$$

$$SD = \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2}$$

$$U_i = \frac{x_i - A}{h}$$

$$SD = \sqrt{\frac{\sum f_i U_i^2}{N} - \left(\frac{\sum f_i U_i}{N}\right)^2} \times h$$



$$\begin{array}{c|c} \sum & \begin{array}{c} x_i \\ 2 \\ 4 \\ 6 \end{array} \end{array} \quad \begin{array}{c} x_i^2 \\ 4 \\ 16 \\ 36 \end{array}$$


---


$$\begin{array}{c|c} 12 & 56 \end{array}$$

$$\begin{aligned} \text{mean} &= \frac{\sum x_i}{N} \\ &= \frac{12}{3} \\ &= 4 \end{aligned}$$

$$\begin{aligned} SD &= \sqrt{\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2} \\ &= \sqrt{\frac{56}{3} - \left(\frac{12}{3}\right)^2} \\ &= 1.63299 \end{aligned}$$

$$\begin{aligned} &= \sqrt{\frac{n^2-1}{3}} \\ &= \sqrt{\frac{3^2-1}{3}} \\ &= 1.63299 \end{aligned}$$

g

$x_i$	$d_i^0 = x_i - 0.23$	$d_i^2$
1.23	1	1
2.23	2	4
3.23	3	9
	<u>6</u>	<u>14</u>

$$SD = \sqrt{\frac{\sum d_i^2}{N} - \left(\frac{\sum d_i^0}{N}\right)^2}$$

$$= \sqrt{\frac{14}{3} - \left(\frac{6}{3}\right)^2} = 0.8164$$

$$\begin{aligned} SD &= \sqrt{\frac{n^2 - 1}{12}} \\ &= \sqrt{\frac{3^2 - 1}{12}} \\ &= 0.8164 \end{aligned}$$

g

<u>Ci</u>	<u>fi</u>	<u>xi</u>	<u>fixi</u>	<u>fixi<sup>2</sup></u>
0-4	3	2	6	12
4-8	2	6	12	72
8-12	1	10	10	100
12-16	4	14	56	784
	10		84	968

$$\begin{aligned}
 SD &= \sqrt{\frac{\sum fixi^2}{N} - \left(\frac{\sum fixi}{N}\right)^2} \\
 &= \sqrt{\frac{968}{10} - \left(\frac{84}{10}\right)^2} = 5.1224
 \end{aligned}$$



<u>C1</u>	<u>f<sub>i</sub></u>	<u>x<sub>i</sub></u>	<u>u</u>	<u>f<sub>i</sub>u<sub>i</sub></u>	<u>f<sub>i</sub>u<sub>i</sub><sup>2</sup></u>
0-4	3	2	-1	-3	3
4-8	2	6=A	0	0	0
8-12	1	10	1	1	1
12-16	4	14	2	8	16
	10			6	20

$$\begin{aligned}
 SD &= \sqrt{\frac{\sum f_i u_i^2}{N} - \left(\frac{\sum f_i u_i}{N}\right)^2} \times h \\
 &= \sqrt{\frac{20}{10} - \left(\frac{6}{10}\right)^2} \times 4 \\
 &= 1.28 \times 4 \\
 &= 5.1224
 \end{aligned}$$

$$\text{Variance} = (\text{S.D.})^2$$

$$\text{S.D.} = \sqrt{\text{Variance}}$$

# Coefficient of variance

$$= \frac{\text{S.D.}}{\text{mean}} \times 100$$

more C.V.  
more variability  
& less consistency

less C.V.  
→ less variability  
→ more consistency

$$QD : MD : SD = 10 : 12 : 15$$

$$\frac{QD}{MD} = \frac{10}{12} \quad / \quad \frac{MD}{SD} = \frac{12}{15} \quad / \quad \frac{QD}{SD} = \frac{10}{15}$$



$$\text{S.D. of two numbers } a \text{ \& } b = \frac{|a-b|}{2}$$

$$\text{eg SD of } 2 \text{ \& } 7 = \left| \frac{2-7}{2} \right| = 2.5$$

$$\text{eg}$$

$\frac{x_i}{2}$	$\frac{x_i^2}{4}$
2	4
7	49
<u>9</u>	<u>53</u>

$$\text{SD} = \sqrt{\frac{53}{2} - \left(\frac{9}{2}\right)^2} = 2.5$$

# S.D. of first 'n' natural no =  $\sqrt{\frac{n^2 - 1}{12}}$

# S.D. of first 'n' even  
natural no  
or  
odd natural no =  $\sqrt{\frac{n^2 - 1}{3}}$

$x_i$	$x_i^2$
1	1
3	9
5	25
<u>9</u>	<u>35</u>

$$SD = \sqrt{\frac{35}{3} - \left(\frac{9}{3}\right)^2}$$
$$= 1.63299$$

# If all the observations are same

then Range = 0

$$MD = 0$$

$$QD = 0$$

$$SD = 0$$



Combined S.D.

$$= \sqrt{\frac{N_1(\sigma_1^2 + d_1^2) + N_2(\sigma_2^2 + d_2^2)}{N_1 + N_2}}$$

where

$$d_1 = \bar{X}_{12} - \bar{X}_1$$

$$d_2 = \bar{X}_{12} - \bar{X}_2$$

Ex

$N_1 = 5$	$N_2 = 5$
$\sigma_1 = 2$	$\sigma_2 = 1$
$\bar{X}_1 = 25$	$\bar{X}_2 = 35$

combined SD = ?

Now

$$\sigma_{12} = \sqrt{\frac{N_1(\sigma_1^2 + d_1^2) + N_2(\sigma_2^2 + d_2^2)}{N_1 + N_2}}$$

$$= \sqrt{\frac{5(4 + 25) + 5(1 + 25)}{5 + 5}}$$

$$= \sqrt{\frac{275}{10}}$$

$$= 5.24$$

Sol.

$$\bar{X}_{12} = \frac{N_1 \bar{X}_1 + N_2 \bar{X}_2}{N_1 + N_2}$$

$$= \frac{5(25) + 5(35)}{5 + 5}$$

$$\bar{X}_{12} = \frac{300}{10} = 30$$

Now

$d_1 = \bar{X}_{12} - \bar{X}_1$	$d_2 = \bar{X}_{12} - \bar{X}_2$
$= 30 - 25$	$= 30 - 35 = -5$
$= 5$	



## QUESTION

CA

If the sum of square of the values equals to 3390, Number of observations are 30 and Standard deviation is 7, what is the mean value of the above observations?

- (a) 14
- (b) 11
- (c) 8
- (d) 5

(1 mark)

$$\begin{aligned}\sum x_i^2 &= 3390 \\ N &= 30 \\ \sigma &= 7 \\ \bar{x} &= ?\end{aligned}$$

$$\begin{aligned}\sigma &= \sqrt{\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2} \\ 7 &= \sqrt{\frac{3390}{30} - (\bar{x})^2}\end{aligned}$$

$$\begin{aligned}49 &= 113 - (\bar{x})^2 \\ (\bar{x})^2 &= 64 \\ \bar{x} &= 8\end{aligned}$$



## QUESTION

CA

If the Standard Deviation of data 2, 4, 5, 6, 8, 17 is 4.47, then Standard Deviation of the data 4, 8, 10, 12, 16, 34 is

(a) 4.47

(b) 8.94

(c) 13.41

(d) 2.24

(1 mark)

$x_i$	$2x_i$
2	4
4	8
5	10
6	12
8	16
17	34

$SD = 4.47$        $SD = 2 \times 4.47 = 8.94$

## QUESTION

CA

For the given set of normally distributed data, the following statistical data are known: Mean = 6; Standard Deviation = 2.6; Median = 5 and Q deviation = 1.5, then the coefficient of quartile deviation equals to

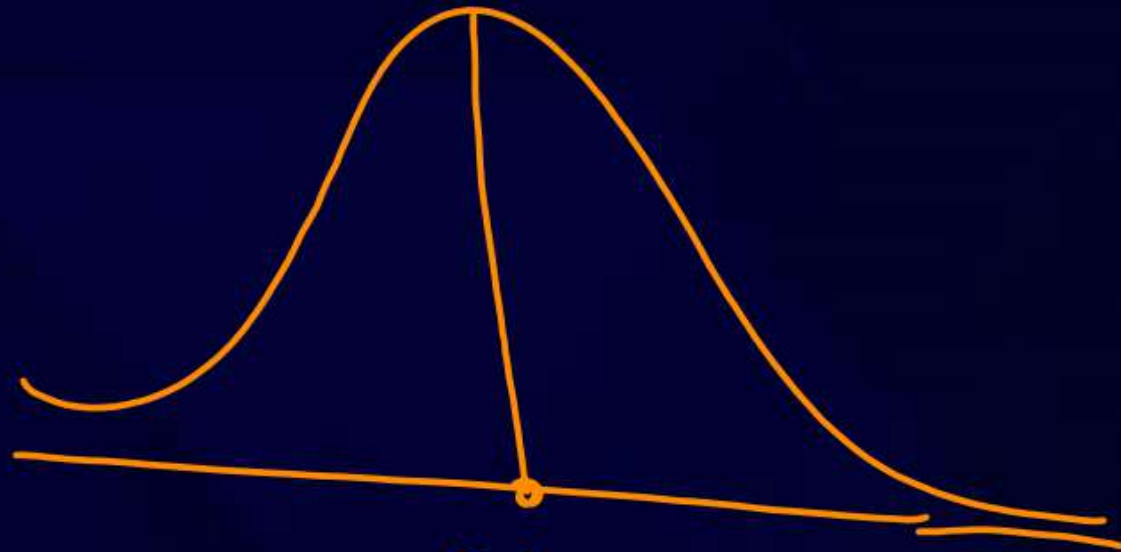
(a) 30

(b) 32

(c) 25

(d) 39

(1 mark)



mean  
= median  
= mode

Skewness  
= zero

$$\bar{X} = 6$$

$$\sigma = 2.6$$

$$\text{median} = 5$$

$$QD = 1.5$$

$$\text{Coeff of } QD = ?$$

Coeff of QD

$$= \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

$$= \frac{\frac{Q_3 - Q_1}{2}}{\frac{Q_3 + Q_1}{2}} \times 100$$

$$= \frac{QD}{\text{median}} \times 100$$

$$= \frac{1.5}{5} \times 100$$

$$= 30$$



## QUESTION



If two samples of sizes 30 and 20 have means as 55 and 60 and variances as 16 and 25 respectively, then what would be the S. D. of the combined sample size 50 ?

(a) 5.33

(b) 5.17

(c) 5.06

(d) 5

(1 mark)

$$N_1 = 30$$

$$\bar{X}_1 = 55$$

$$\sigma_1^2 = 16$$

$$\sigma_1 = 4$$

$$N_2 = 20$$

$$\bar{X}_2 = 60$$

$$\sigma_2^2 = 25$$

$$\sigma_2 = 5$$

combined SD = ?





# Coefficient of Variance



$$CV = \frac{SD}{mean} \times 100$$

## QUESTION



If the coefficient of variation and standard deviation are 30 and 12 respectively, then the arithmetic mean of the distribution is:

(a) 40

(b) 36

(c) 25

(d) 19

(1 mark)

$$CV = 30$$

$$\sigma = 12$$

$$\bar{X} = ?$$

$$CV = \frac{\sigma}{\bar{X}} \times 100$$

$$30 = \frac{12}{\bar{X}} \times 100$$

$$\bar{X} = 40$$

## QUESTION



The probable value of mean deviation when  $Q_3 = 40$  and  $Q_1 = 15$  is:

- (a) 15
- (b) 18.75
- (c) 17.50
- (d) 0

(1 mark)

$$mD = ?$$

$$Q_3 = 40, Q_1 = 15$$

$$\begin{aligned} QD &= \frac{Q_3 - Q_1}{2} \\ &= \frac{40 - 15}{2} \\ &= 12.5 \end{aligned}$$

$$QD : mD : SD = 10 : 12 : 15$$

$$\begin{aligned} \frac{QD}{mD} &= \frac{10}{12} \\ \frac{12.5}{mD} &= \frac{10}{12} \end{aligned}$$

$$mD = 15$$



## QUESTION



Which one of the following is not a method of measures of dispersion?

- (a) Standard deviation
- (b) Mean deviation .
- (c) Range
- (d) Concurrent deviation method

## QUESTION



The standard deviation of 1 to 9 natural number is:

- (a) 6.65
- (b) 2.58
- (c) 6.75
- (d) 5.62

$$\begin{aligned} & \sqrt{\frac{n^2 - 1}{12}} \\ &= \sqrt{\frac{9^2 - 1}{12}} \\ &= 2.58 \end{aligned}$$



## QUESTION



It is given that the mean ( $\bar{X}$ ) is 10 and standard deviation (s.d.) is 3.2. If the observations are increased by 4, then the new mean and standard deviations are:

(a)  $\bar{X} = 10$ , s.d. = 7.2

(b)  $\bar{X} = 10$ , s.d. = 3.2

(c)  $\bar{X} = 14$ , s.d. = 3.2

(d)  $\bar{X} = 14$ , s.d. = 7.2

(1 mark)

$$\bar{X} = 10$$

$$\sigma = 3.2$$

$$y_i = x_i + 4$$

origin

$$\text{New mean} = 10 + 4 = 14$$

$$\text{New SD} = 3.2$$



## QUESTION



The best statistical measure used for comparing two series is

- (a) Mean absolute deviation X
- (b) Range X
- (c) Coefficient of variation ✓✓✓
- (d) Standard deviation

(1 mark)

## QUESTION



The sum of the squares of deviations of a set of observations has the smallest value, when the deviations are taken from their :

(a) A . M.

(b) H. M.

(c) G. M.

(d) None

(1 mark)

$$\sum (x_i - A)^2 \text{ is small (minimum)}$$

$\downarrow$   
 $A = A_m$

## QUESTION



Which of the following result hold for a set of distinct positive observations?

(a)  ~~$A . M. \geq G . M. \geq H . M.$~~

(b)  ~~$G . M. > A . M. > H . M.$~~

(c)  ~~$G . M. \geq A . M. \geq H . M.$~~

☒ (d)  $A . M. > G . M. > H . M.$  (1 mark)



## QUESTION



If variance of  $x$  is 5, then find the variance of  $(2 - 3x)$

(a) 10

(b) 45

(c) 5

(d) -13

(1 mark)

$$\sigma_x^2 = 5$$

$$\sigma_x = \sqrt{5}$$

$$\text{SD of } (2 - 3x)$$

$$= |-3| \times \text{SD of } x$$

$$\text{SD} = 3 \times \sqrt{5}$$

$$\begin{aligned} \text{Variance} &= (3\sqrt{5})^2 \\ &= 9 \times 5 = 45 \end{aligned}$$



**THANK YOU**





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# Sampling



# Branches Of Statistics



**Descriptive Stats**-Gathering , Classifying & summarizing the data from samples

अनुसंधान

**Inferential Stats** - It involves drawing conclusions, generalization or making prediction from the gathered data





# Population

- Group of people
- Group of objects
- Group of events & observations

Ex. Height of male students

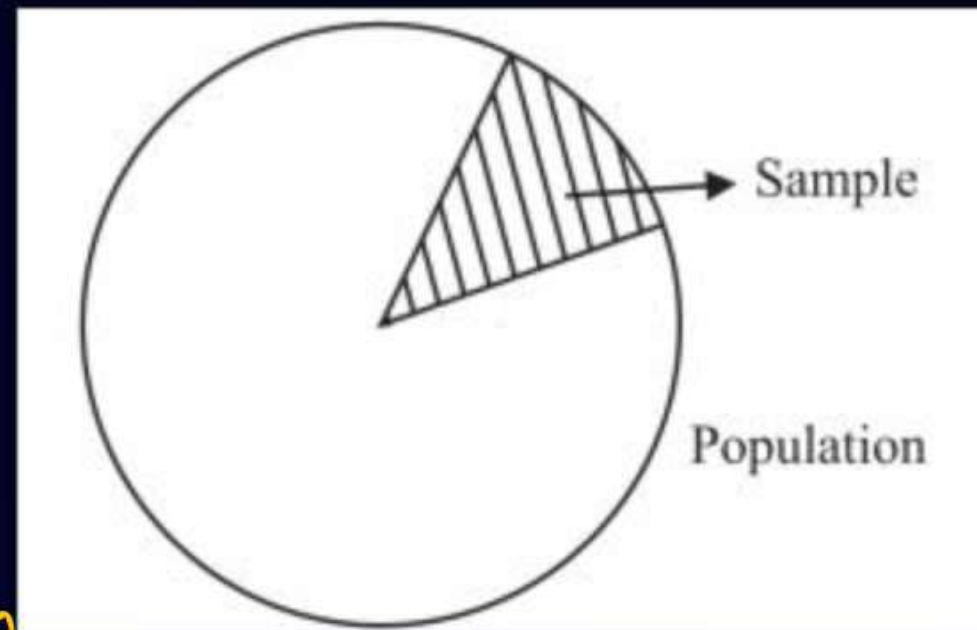
Ex. Blood pressure of females b/w age 40 & 60 years

$N = 50 \text{ lakh}$   $\leftarrow$  50,00,000  $\rightarrow$   $n = 1 \text{ lakh}$

# Sample

- It is a subset of Population

“Small group of elements selected from population”



Number of elements in a sample is called the sample size.



# Parameters:

Measurable characters of populations are called parameters

Eg Population Mean =  $\mu = \mu$

Eg Population Variance =  $\sigma^2 = \sigma^2$

S.D. =  $\sigma$   
Proportion =  $p$

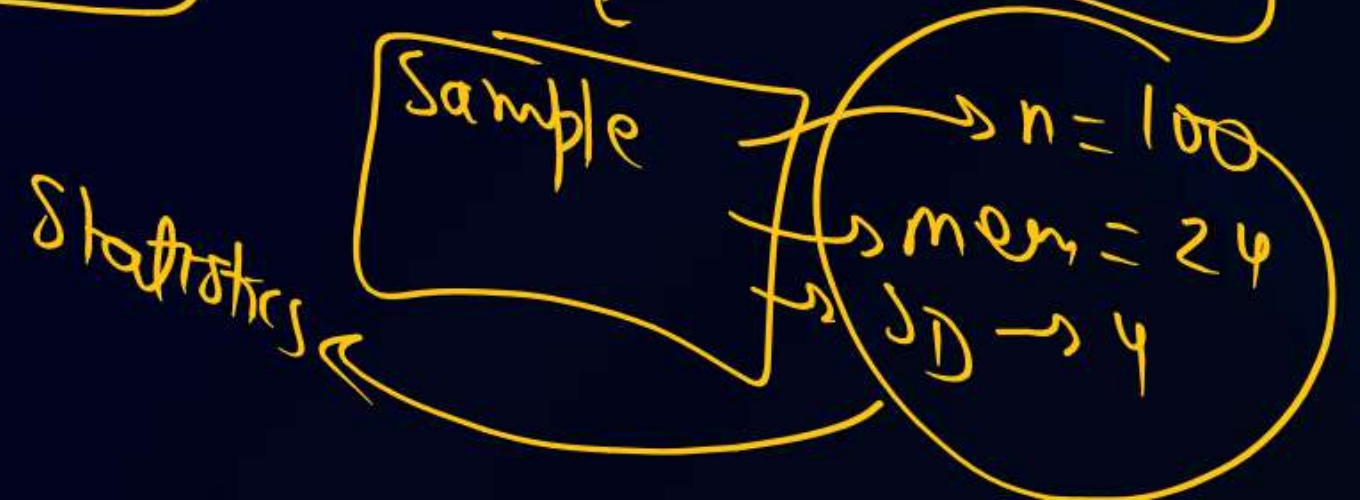
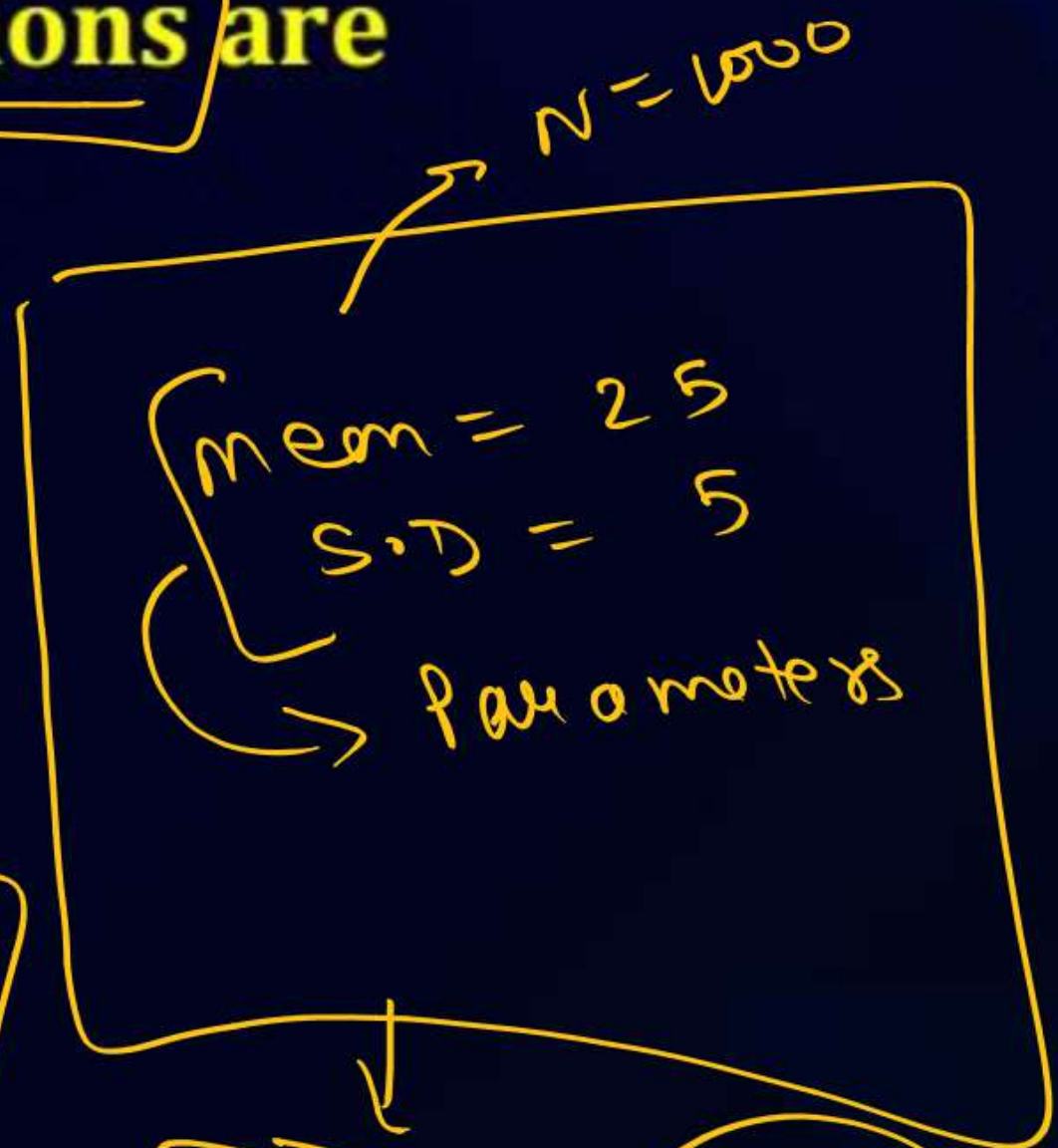
# Statistics:

Measurable characters of Sample

Eg Sample Mean =  $\bar{x}$

Eg Sample Variance =  $s^2$

S.D. =  $s$





Parameter is a characteristic of:

- (a) Population ✓
- (b) Sample
- (c) Probability distribution
- (d) Both (a) and (b).

Sampling is preferred than population in the following case:

- (a) Testing of items is destructive in nature
- (b) Testing of items needs equipment of high costs
- (c) Population is very large
- ☒ (d) All of the above.



# Principles Of Sampling



Sampling is the procedure of selecting elements for a sample from population so that inferences can be ~~drawn~~ drawn about population from sample



## Some Basis principles of sampling

### 1. Law of statistical Regularity :

This law suggests that if a large sample is taken randomly from population, it will possess almost same characters of population.

### 2. Law of Inertia of Large Numbers:

This law is the corollary of “Law of statistical Regularity”

This law says that “Larger the size of sample, more accurate the results one”



### 3. Principle of optimization:

Maximum efficiency at minimum cost can be achieved only when appropriate "sampling design" is selected

→ method of selection of sample

max profit  
minimum cost  
Time minimum

### 4. Principle of validity:

According to this law sampling Design is valid only if it is possible to obtain accurate estimates about population

!! Probabilistic method





# Sampling v/s Census:



1. **Speed** : Sampling takes less time

2. **Cost** : Sampling is least expensive

3. **Accuracy & Reliability** : Census gives more Accurate & Reliable results unless there is no bias or error in collecting information





# Types Of Sampling



(Random)

**Probability  
Sampling**

when every element  
of population has equal  
chances of being selected  
in sample

**Non Probability  
Sampling**



# Probability (Random Sampling)



## i) Simple Random sampling (SRS):

In this sampling each member of population has an equal chance of being selected in sample.

Ex. There are 1000 students in a college. You assign a number to every student & Then Randomly select a sample of 100 students.



# Simple Random Sampling without Replacement

S R S W O R

||

Element selected once can not be selected again.

g m, R, S, C  
selection of 2 students

$$= mR, mS, mC \\ RS, RC, SC = {}^4C_2 = \frac{4!}{2!2!} = 6$$

g Selection of 3 elements  
from 5 elements

$$= {}^5C_3 \\ = \frac{5!}{3!2!} \\ = 10$$

Total ways of selecting 'n' elements  
from a population of 'N' elements  $= {}^NC_n$



# Simple Random Sampling with Replacement

S R S W R

element selected once can be selected again.

g A, B, C, D  
Selection of 2 elements

A A	B A	C A	D A
A B	B B	C B	D B
A C	B C	C C	D C
A D	B D	C D	D D

Total ways of selecting 'n' elements from a population of 'N' elements  $= (N)^n = 4^2 = 16$

g Selection of a sample of 3 elements with replacement from a population of 5 elements

Sol:  $5^3 = 125$

Simple random sampling is

- (a) A probabilistic sampling  
(c) A mixed sampling

- (b) A non- probabilistic sampling  
(d) Both (b) and (c).



## (ii) Systematic Sampling(Quasi):

In this sampling every member of the population is assigned a number. The first member is selected randomly & then instead of choosing other Randomly we chose them in regular intervals.

Ex. There are 1000 employees in a company, they are assigned numbers from 1 to 1000 we randomly select number '6' from first 10 numbers & after that every 10th person is selected that is 6, 16, 26, 36, .....

$$\frac{1000}{100} = 10$$

6, 16, 26, 36, 46, - - - / 7, 17, 27, 37, 47, - - -



50 people



10 sample

1, 2, 3, 4, 5

$$\frac{50}{10} = 5$$

2, 7, 12, 17, 22, 27, 32, 37, 42, 47



 $\frac{2}{T}$  $\frac{12}{T}$  $\frac{22}{T}$  $\frac{32}{T}$  $\frac{42}{T}$ 

Which sampling is affected most if the sampling frame contains an undetected periodicity?

(a) Simple random sampling

(b) Stratified sampling

(c) Multistage sampling

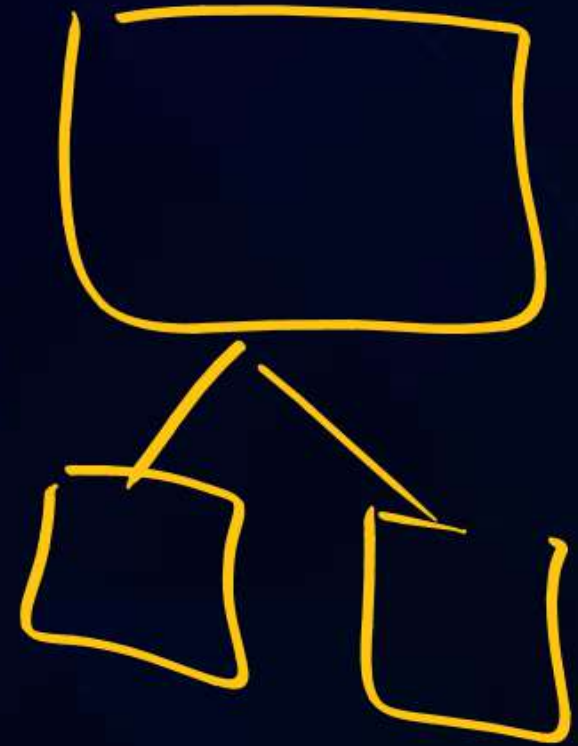
(d) Systematic sampling



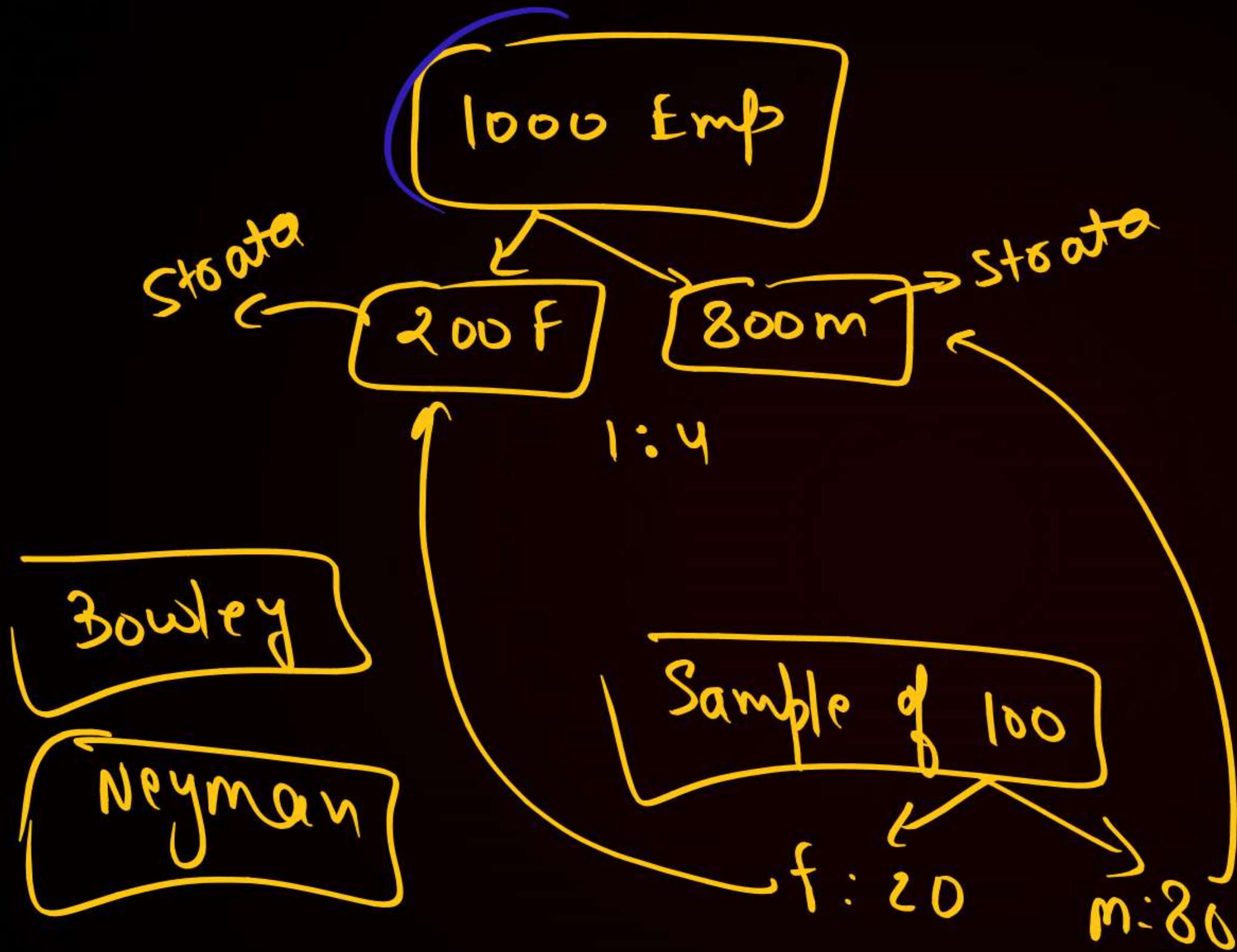
### (iii) Stratified Sampling :

In this sampling members are divided into sub group called strata based on gender, age & income etc. After that members are selected using Random or systematic sampling from each sub group.

Ex. He say there are 1000 employees out of which 800 are males & 200 females. A sample of 100 employees reflecting gender balance of the company is made by Dividing the employees in males & females then selecting 80 male & 20 female employer.







Which sampling provides separate estimates for population means for different segments and also an over all estimate?

- (a) Multistage sampling
- (b) Stratified sampling
- (c) Simple random sampling
- (d) Systematic sampling

According to Neyman's allocation, in stratified sampling

- ☒ (a) Sample size is proportional to the population size
- (b) Sample size is proportional to the sample SD
- (c) Sample size is proportional to the sample variance
- (d) Population size is proportional to the sample variance.

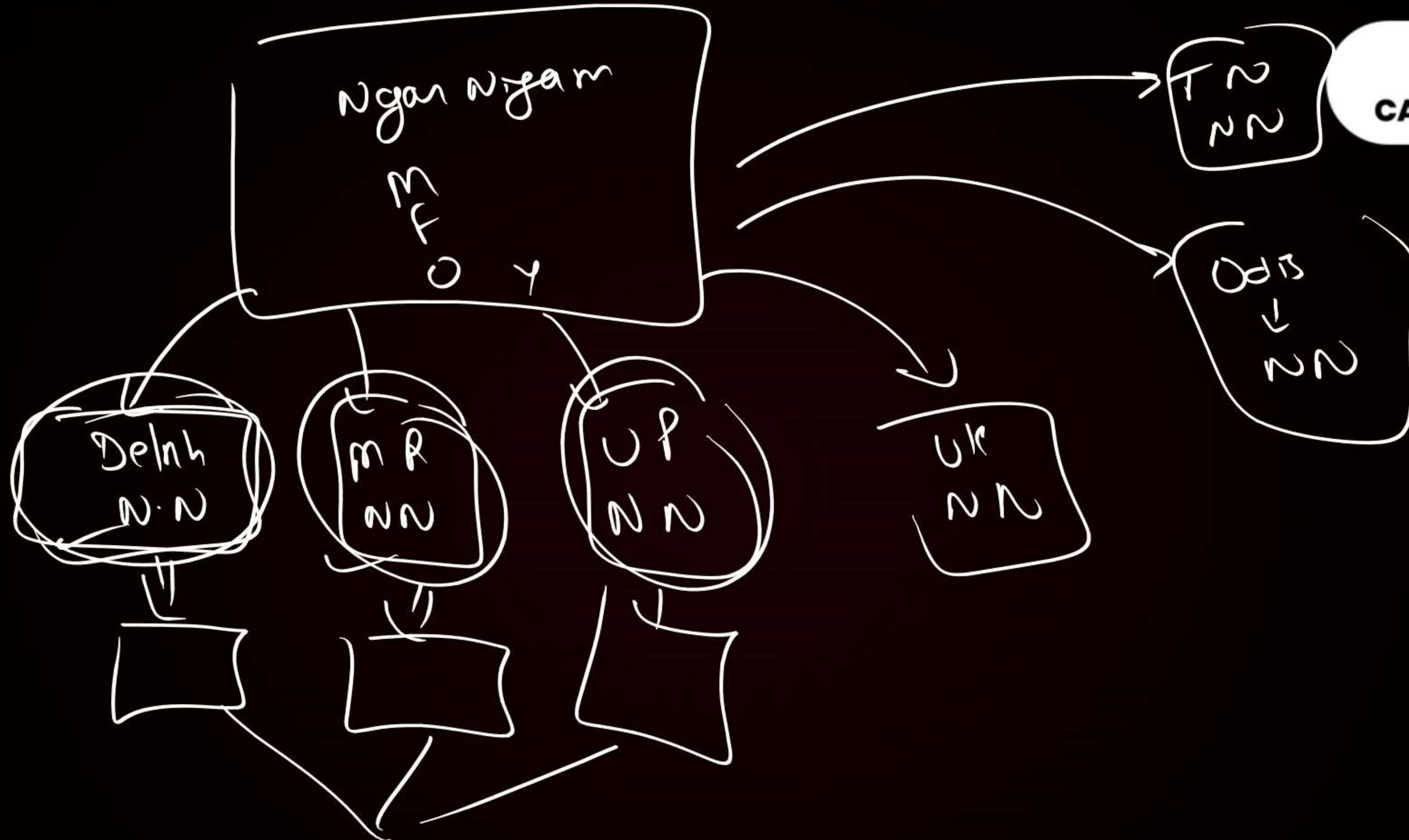


#### (iv) Cluster Sampling (Multi Stage Sampling)

When population size is large, divide the population in sub groups (each sub group has similar characteristic of the whole population). Then some groups are selected randomly & then members are selected from them for sample.

Ex. You want to study the behavior of Govt. employees of Nagar Nigam. Nagar Nigam Dept. of each state is a cluster (Sample). Then you select any 3 Department of Nagar Nigam & then select 20 members from each Department. Thus sample of 60 item is formed.







Which sampling adds flexibility to the sampling process?

- (a) Simple random sampling
- (b) Multistage sampling
- (c) Stratified sampling
- (d) Systematic sampling

# Non Probability Sampling



1) Purposive Judgement Sampling: Based on the opinion of expert  
Ex. Indian idol

Which sampling is subjected to the discretion of the sampler?

~~(a)~~ Systematic sampling

~~(b)~~ Simple random sampling

☒ (c) Purposive sampling

~~(d)~~ Quota sampling.



**2) Convenience Sampling:** Those elements are selected which are easily accessible to researcher. Ex. Asking your students to complete survey regarding services provided by universities.



### (3) Volunteer response sampling:

People who are themselves ready to conduct the survey collect the sample Data

(4) **Snow ball sampling:** First select some members, then with the help of them select some more & process continues.





# Sampling Errors:

Difference b/w Sample statistics & population parameter because sample was not the true representative of population

- ✘ ➤ Faulty sampling method
- ✘ ➤ Faulty Demarcation of sampling units
- ✘ ➤ Replacing sampling units with unsuitable unit
- ✘ ➤ Wrong choice of statistic

$N = 100000$   
Population  
mean  
 $= 25$

$n = 100$   
Sample  
mean  
 $= 22$



## **Non Sampling errors:**

These are human errors  
census & sampling both can have these errors.

- Lapse of memory
- Preference for certain units
- Wrong measurements
- Untrained interviewer
- Biased opinion





# # Population

- Aggregate of all units under consideration
- Population size is denoted by "N"

**# Finite Population**

**# Infinite Population**



# # Population

(imaginary)  
अन्योन्य



## # Existent Population



- No of students in a schools
- Male population of Delhi

## # Hypothetical Population



- g A coin is tossed 100 times
- g A dice is thrown 200 times



➤ Sample size is denoted by 'n'

➤ Detailed & Complete list of all potential sampling units is known as Sampling Frame

Sampling frame is a term used for

- ☐ (a) a list of random numbers
- ☐ (b) a list of voters
- ☒ (c) a list of sampling units of population
- ☐ (d) None of the above.

Population  
= 10,000  
Audition

1000 → Sampling frame

## Parmeter:

Characteristic of Population

$$\Rightarrow \underline{\text{Population mean (u)}} = \frac{\sum_{i=1}^N X_i}{N}$$

$$\Rightarrow \text{Population Proportion (P)} = \frac{X}{N}$$

$$\Rightarrow \text{Population S.D } (\sigma) = \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$$



$$\mu = \frac{\sum x_i}{n}$$

$$P = \frac{x}{n}$$

$$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{n}}$$



## Statistics:



### Measurable Characters of Sample

$$\hat{\mu} = \text{Sample mean} = \bar{x} = \frac{\sum x_i}{n}$$

$$\bar{x} = \text{Sample S.D} = S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

$$\hat{p} = \text{Sample Proportion} = P = \frac{x_i}{n}$$

# Total no of samples with replacement =  $(N)^n$

# Total no of Sample without Replacement =  ${}^N C_n$

$$= \binom{N}{n}$$

**Sampling Fluctuation:** Value of Sample Statistics may be different in different samples, this variation is called sampling fluctuation

g 1, 3, 5

2 numbers are selected without Replacement.  
Write down all possible samples & their A.M.

$${}^3C_2 = \frac{3!}{2!1!} = 3$$

Sol:

		mean
$A_1$	1 & 3	$\frac{4}{2} = 2$
$A_2$	1 & 5	$\frac{6}{2} = 3$
$A_3$	3 & 5	$\frac{8}{2} = 4$



➤ A population comprises of following units : a, b, c & d

Draw all possible samples of

(i) Size two without replacement

(ii) Size two with replacement

Populations = a, b, c, d

i> w/o Replacement

a, b

a, c

a, d

b, c

b, d

c, d

$${}^N C_n$$

$$= 4$$

$${}^N C_2$$

$$= \frac{4!}{2!2!} = 6$$

ii> with Replacement  
 $= (N)^n = (4)^2 = 16$

aa	ba	ca	da
ab	bb	cb	db
ac	bc	cc	dc
ad	bd	cd	dd



# Sampling Distribution



- We can make many samples of same size with a given population.
- Sample statistic will be different in each sample.
- If this sample statistic is considered as a Random variable we can make probability distribution. .
- This probability Distribution is known as sampling Distribution



Q Population: 1, 3 & 5  
2 elements selected for sample.  
with Replacement

$X_i$ : Random variable  
(sample mean) = 1, 2, 3, 4, 5

Sol.

S.no.	Sample	( $\bar{X}$ ) mean
1	(1, 1)	1
2	(1, 3)	2
3	(1, 5)	3
4	(3, 1)	2
5	(3, 3)	3
6	(3, 5)	4
7	(5, 1)	3
8	(5, 3)	4
9	(5, 5)	5

Sampling Distribution

$X_i$	$P_i$	$P_i X_i$	$P_i X_i^2$
1	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
2	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{8}{9}$
3	$\frac{3}{9}$	$\frac{9}{9}$	$\frac{27}{9}$
4	$\frac{2}{9}$	$\frac{8}{9}$	$\frac{32}{9}$
5	$\frac{1}{9}$	$\frac{5}{9}$	$\frac{25}{9}$
		$\frac{27}{9} = 3$	$\frac{93}{9} = 10.33$

$$\begin{aligned}\text{Mean} &= E(X) \\ &= \sum P_i X_i \\ &= 3\end{aligned}$$

$$\begin{aligned}\text{S.D (Standard Error)} &= \sqrt{\sum P_i X_i^2 - (\sum P_i X_i)^2} \\ &= \sqrt{\frac{93}{9} - (3)^2} \\ &= 1.15\end{aligned}$$



➤ The mean of Sampling Distribution is known as Expectation denoted by  $E(x) = \sum p_i x_i$

➤ Standard deviation of this sampling Distribution is known as "Standard Error" =  $\sqrt{\sum p_i x_i^2 - (\sum p_i x_i)^2}$

Standard Error of mean ( $\bar{x}$ ) =  $\frac{\sigma}{\sqrt{n}}$  for SRSWR

Standard Error of mean =  $\frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$  for SRSWOR

SRSWR → Simple Random Sampling with Replacement

SRSWOR → Simple Random Sampling without Replacement



eg

1, 3, 5

if samples of size 2 are made with replacement

find standard error of mean.

Sol.

$x_i$	$x_i^2$
1	1
3	9
5	25
<u>9</u>	<u>35</u>

$$SD(\sigma) = \sqrt{\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2}$$

$$\begin{aligned}\sigma &= \sqrt{\frac{35}{3} - \left(\frac{9}{3}\right)^2} \\ &= \sqrt{2.6666} \\ \sigma &= 1.63299\end{aligned}$$

S.E of mean

$$= \frac{\sigma}{\sqrt{n}}$$

$$\begin{aligned}&= \frac{1.63299}{\sqrt{2}} \\ &= 1.1546\end{aligned}$$

## Standard Error for Proportion:

$$SE(P) = \sqrt{\frac{Pq}{n}}$$

SRSWR

$$SE(P) = \sqrt{\frac{Pq}{n}} \sqrt{\frac{N-n}{N-1}}$$

SRSWOR



A measure of precision obtained by sampling is given by

- ☒ (a) Standard error
- ☒ (b) Sampling fluctuation
- ☒ (c) Sampling distribution
- ☒ (d) Expectation.

As the sample size increases, standard error

- ☐ (a) Increases
- ☒ (b) Decreases
- ☐ (c) Remains constant
- ☐ (d) Decreases proportionately.

Standard error can be described as

- (a) The error committed in sampling
- (b) The error committed in sample survey
- (c) The error committed in estimating a parameter
- ☒ (d) Standard deviation of a statistic.

A Population has 3 elements 1, 5 & 3

Draw all possible sample of size two

(i) With Replacement (ii) Without Replacement

$$\begin{aligned} & {}^N P_n \\ &= 3^2 \\ &= 9 \end{aligned}$$

$$\begin{aligned} & {}^N C_n \\ &= {}^3 C_2 \\ &= 3 \end{aligned}$$



A population comprises 5 members. The number of all possible samples of size 2 that can be drawn from it with replacement is

- (a) 100 (b) 15  
(c) 125 (d) 25

$$N^n = 5^2 = 25$$

If from a population with 25 members, a random sample without replacement of 2 members is taken, the number of all such samples is

- (a) 300 (b) 625  
(c) 50 (d) 600

$${}^N C_n = {}^{25} C_2 = \frac{25!}{2! \cdot 23!} = \frac{25 \times 24}{2} = 300$$

Sol:  $N = 3$

$$SD(\sigma) = \sqrt{\frac{8}{3}}$$

$$n = 2$$

i)  $\frac{WR}{S.F.} =$

$$S.F. = \frac{\sigma}{\sqrt{n}}$$

$$= \frac{\sqrt{\frac{8}{3}}}{\sqrt{2}} = \frac{1.632}{1.41} = 1.1539$$

ii)  $WOR$

$$S.F. = \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$$

$$= \frac{\sqrt{\frac{8}{3}}}{\sqrt{2}} \sqrt{\frac{3-2}{3-1}}$$

$$= \frac{\sqrt{\frac{8}{3}}}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 0.8164$$



## QUESTION



Sampling fluctuations may be described as:

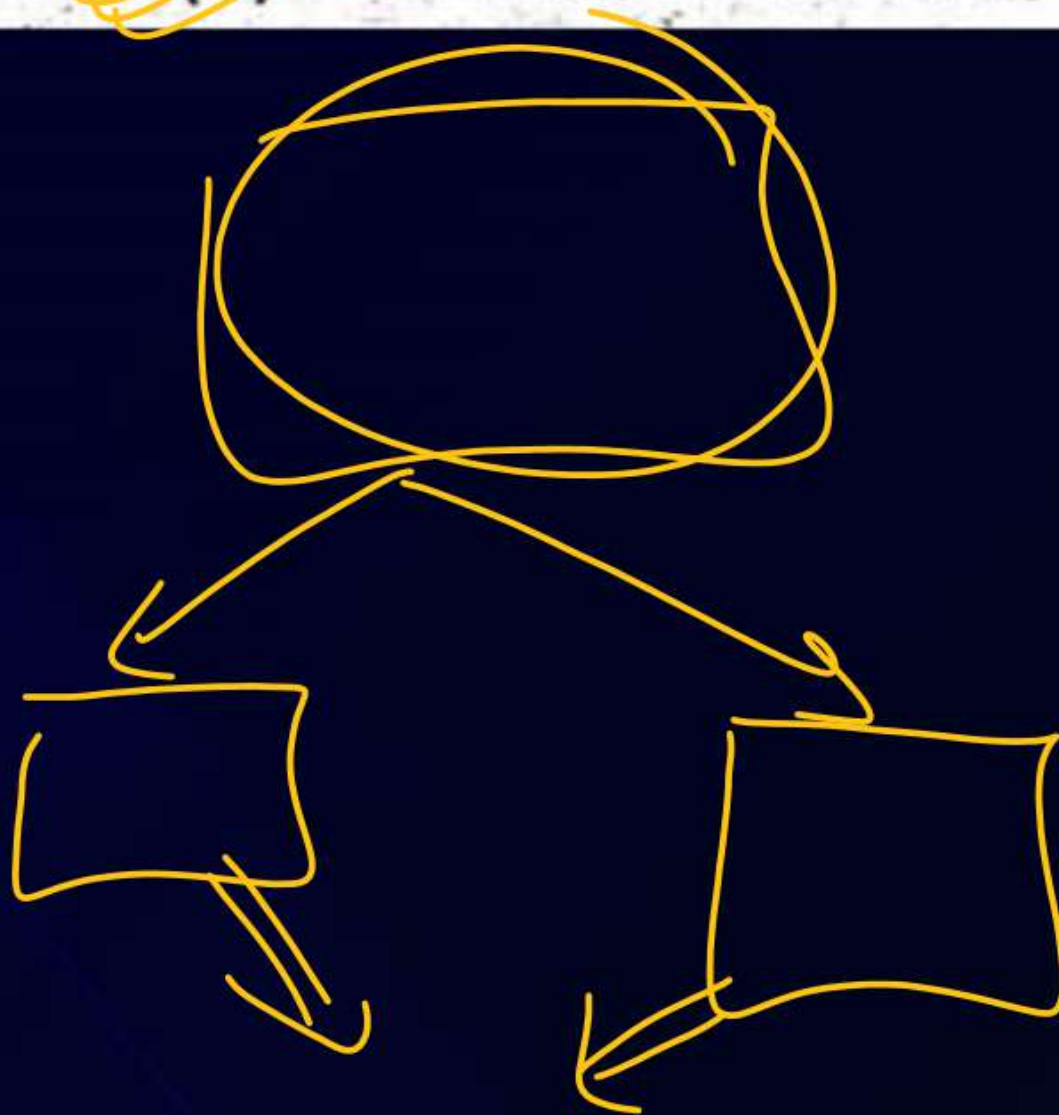
- (a) ~~The variation in the values of a sample~~
- (b) ~~The differences in the values of a parameter~~
- (c) The variation in the values of a statistic
- (d) ~~The variation in the values of observations.~~

## QUESTION



Which sampling provides separate estimates for population means for different purposes and also an over all estimate ?

- (a) Multistage sampling
- (b) Simple random sampling
- (c) Systematic sampling
- (d) Stratified sampling





## QUESTION



If the population S.D. is known to be 5 for a population containing 80 units, then the standard error of sample mean for a sample of size 25 without replacement is :

(a) 0.83

(b) 0.80

(c) 0.93

(d) 0.74

$$\begin{aligned}\sigma &= 5 \\ N &= 80 \\ n &= 25\end{aligned}$$

$$\begin{aligned}SE &= \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}} \\ &= \frac{5}{\sqrt{25}} \sqrt{\frac{80-25}{80-1}} \\ &= \frac{5}{5} \sqrt{\frac{55}{79}} = 0.83\end{aligned}$$

## QUESTION



A Population comprises of 20 members. The number of all possible samples of size 2 that can be drawn from it without replacement.

(a) 210

(b) 380

(c) 190

(d) 400

$${}^{20}C_2$$





**CA WALLAH**

## QUESTION



Distribution formed of all possible value of statistics is called\_\_\_\_\_.

- (a) Sampling Distribution
- (b) Classification
- (c) Tabulation
- (d) None



## QUESTION



In sampling, standard error is:

- (a) Standard deviation
- (b) Quartile deviation
- (c) Mean deviation
- (d) Coefficient of variation

## QUESTION



If every 9<sup>th</sup> unit is selected from universal set then this type of sampling is known as:

- (a) Quota Sampling
- (b) Systematic Sampling
- (c) Stratified Sampling
- (d) None of these



## QUESTION



The method of sampling in which each unit of the population has an equal chance of being selected in the sample is

- (a) Random sampling ✓✓
- (b) Stratified sampling ✗
- (c) Systematic sampling ✗
- (d) None of the above.

## QUESTION



The standard deviation of sampling distribution is known as

- ☒ (a) Standard error
- (b) Mean
- (c) Variance
- (d) Mode



## QUESTION

CA

If a random sample of size 2 with replacement is taken from the population containing the units 3, 6 and 1, then the samples would be

- ☒ (a) (3, 6), (3, 1), (6, 1)
- ☒ (b) (3, 3), (6, 6), (1, 1)
- ☒ (c) (1, 1), (1, 3), (1, 6), (6, 1), (6, 2), (6, 3), (6, 6), (6, 1)
- ☒ (d) (1, 1), (1, 3), (1, 6), (3, 1), (3, 3), (3, 6), (6, 1), (6, 3), (6, 6)

3, 6, 1

$$N^n = 3^2 = 9$$

## QUESTION



Method used to test the human blood is called in Statistical terminology \_\_\_\_

- (a) Census Investigation
- (b) Blood Investigation
- (c) Sample Investigation
- (d) None of these