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Mathematics

DPP: 8

Binomial Theorem

Q1 If $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots, + {}^nC_nx^n$ then the value of $\sum_{r=0}^{s}\sum_{s=1}^{n} {}^nC_s.\,{}^sC_r$ is equal to (where $r \leq s$, is)

(A) $3^n - 1$ (B) $3^n + 1$

(C) 3^n (D) None of these

Q2 Let $(5+2\sqrt{6})^n = p + f$ where $n, p \in N$ and $0 < f < 1$ then the value of $f^2 - f + pf - p$ is

(A) a natural number

(B) a negative integer

(C) a prime number

(D) are irrational number

Q3 In the expansion of $\left(1+3x+2x^2\right)^6$ the coefficient of x^{11} is

(A) 144 (B) 288

(C) 216 (D) 576

Q4 If the maximum value of the term independent of t in the expansion of $\left(t^2x^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$, $x \geqslant 0$, is K, then $8K$ is equal to

(A) 3003 (B) 6006

(C) 4004 (D) none of these

Q5 If $k \in R^+$ and the middle term of $\left(\frac{k}{2}+2\right)^8$ is 1120 , then value of k is:

(A) 3 (B) 2

(C) 1 (D) 4

Q6 For $r = 0, 1, \ldots, 10$, let A_r, B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1+x)^{10}, (1+x)^{20}$ and $(1+x)^{30}$. Then $\sum_{r=1}^{10} A_r\left(B_{10}B_r - C_{10}A_r\right)$ is equal to -

(A) $B_{10} - C_{10}$

(B) $A_{10}\left(B_{10}^2 - C_{10}\,A_{10}\right)$

(C) 0

(D) $C_{10} - B_{10}$

Q7 The coefficient of x^7 in the expression $(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + \ldots$ is $+ x^{10}$

(A) 120 (B) 330

(C) 210 (D) 420

Q8 The remainder when $(11)^{1011} + (1011)^{11}$ is divided by 9 is

(A) 1 (B) 4

(C) 6 (D) 8

Q9 If $\left(1+x+x^2\right)^{25} = a_0 + a_1x + a_2x^2 + \ldots.. + a_{50}\cdot x^{50}$ then $a_0 + a_2 + a_4 + \ldots.. + a_{50}$ is

(A) even

(B) odd and of the form $3n$

(C) odd and of the form $(3n-1)$

(D) odd and of the form $(3n+1)$

Q10 The term independent of a in the expansion of $\left(1+\sqrt{a}+\frac{1}{\sqrt{a}-1}\right)^{-30}$ is

(A) ${}^{30}C_{20}$

(B) 0

(C) ${}^{30}C_{10}$

(D) None of these

Q11 If $\sum_{r=0}^{n}\left(\frac{r^3+2r^2+3r+2}{(r+1)^2}\right)^{n}C_r=\frac{2^4+2^3+2^2-2}{3}$ then the value of n is :

(A) 2
(B) 2^2
(C) 2^3
(D) 2^4

Q12 The number $N={}^{20}C_7-{}^{20}C_8+{}^{20}C_9-{}^{20}C_{10}+\ldots\ldots-{}^{20}C_{20}$ is not divisible by :

(A) 3 (B) 7
(C) 11 (D) 19

Q13 The term independent of y in the binomial expansion of $\left(\frac{1}{2}\cdot y^{1/3}+y^{-1/5}\right)^8$, is

(A) sixth (B) seventh
(C) fifth (D) None of these

Q14 The ratio of the coefficient x^{10} in $\left(1-x^2\right)^{10}$ and the term independent of x in $\left(x-\frac{2}{x}\right)^{10}$, is

(A) $1:16$
(B) $1:32$
(C) $1:64$
(D) None of these

Q15 The value of $99^{50}-99.98^{50}+\frac{99.98}{1.2}(97)^{50}-\ldots.+99$ is

(A) 0 (B) -1
(C) -2 (D) -3

Q16 $\frac{C_0}{1}+\frac{C_2}{3}+\frac{C_4}{5}+\frac{C_6}{7}+\ldots=$

(A) $\frac{2^{n+1}}{n+1}$
(B) $\frac{2^{n-1}-1}{n+1}$
(C) $\frac{2^n}{n+1}$
(D) None of these

Q17 If $(1+x)^{10}=a_0+a_1x+a_2x^2+\ldots\ldots,+a_{10}x^{10}$ then value of $\left(a_0-a_2+a_4-a_6+a_8-a_{10}\right)^2$ is $+\left(a_1-a_3+a_5-a_7+a_9\right)^2$

(A) 2^{10} (B) 2
(C) 2^{20} (D) None of these

Q18 Which of the following is NOT CORRECT?

(A) The greatest integer less than or equal to $(\sqrt{2}+1)^6$ is 197
(B) The integer next above $(\sqrt{3}+1)^{2m}$ contains 2^{m+1} as factor
(C) The greatest integer less than or equal to the number $(7+4\sqrt{3})^m$ is a multiple of 2
(D) If R $R=(6\sqrt{6}+14)^{2n+1}$ and $f=R-[R]$ where $[R]$ is integer and $0\le f<1$ then $Rf=20^{2n+1}$.

Q19 Let m and n be the coefficients of seventh and thirteenth terms respectively in the expansion of $\left(\frac{1}{3}x^{\frac{1}{3}}+\frac{1}{2x^{\frac{2}{3}}}\right)^{18}$. Then $\left(\frac{n}{m}\right)^{\frac{1}{3}}$ is :

(A) $\frac{1}{4}$
(B) $\frac{1}{9}$
(C) $\frac{9}{4}$
(D) $\frac{4}{9}$

Q20 The number of terms which are free from radical signs in the expansion of $\left(y^{1/5}+x^{1/10}\right)^{55}$ is

(A) 5 (B) 6
(C) 7 (D) None

Q21 If $(1+x)^n={}^nC_0+{}^nC_1x+{}^nC_2x^2+\ldots.+{}^nC_nx^n$ then $\frac{{}^nC_1}{{}^nC_0}+2\cdot\frac{{}^nC_2}{{}^nC_1}+3\cdot\frac{{}^nC_3}{{}^nC_2}+\ldots..+n\cdot\frac{{}^nC_n}{{}^nC_{n-1}}$ is

(A) $\frac{n(n-1)}{2}$
(B) $\frac{n(n+2)}{2}$

(C) $\frac{n(n+1)}{2}$

(D) $\frac{(n-1)(n-2)}{2}$

Q22 If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n, x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is :-

(A) 729 (B) 64

(C) 2187 (D) 243

Q23 The expression $\left[x+\left(x^3-1\right)^{1/2}\right]^5+\left[x-\left(x^3-1\right)^{1/2}\right]^5$ is a polynomial of degree

(A) 5 (B) 6

(C) 7 (D) 8

Q24 If $(1+x)^n={}^nC_0+{}^nC_1x+{}^nC_2x^2+\ldots, +{}^nC_nx^n$ then the value of $\sum_{r=0}^{n}\sum_{s=0}^{n}{}^nC_r\cdot{}^nC_s$ is equal to

(A) 2^n (B) 2^{n+1}

(C) 2^{2n} (D) 2^{3n}

Q25 The middle term in the expansion of $\left(x^2+\frac{1}{x^2}+2\right)^n$, is

(A) $\frac{n!}{((n/2)!)^2}$

(B) $\frac{2n!}{((n/2)!)^2}$

(C) $\frac{1.3.5\ldots(2n-1)}{n!}\cdot 2^n$

(D) $\frac{1.3.5\ldots(2n+1)}{n!}\cdot 2^n$

Answer Key

Q1	(A)
Q2	(B)
Q3	(D)
Q4	(B)
Q5	(B)
Q6	(D)
Q7	(B)
Q8	(D)
Q9	(A)
Q10	(B)
Q11	(A)
Q12	(C)
Q13	(A)
Q14	(B)
Q15	(A)
Q16	(C)
Q17	(A)
Q18	(C)
Q19	(C)
Q20	(B)
Q21	(C)
Q22	(A)
Q23	(C)
Q24	(C)
Q25	(C)