# JEE MAIN 2024

ATTEMPT – 01 , 31TH JAN 2024 , SHIFT - 01

LIVE

# PAPER DISCUSSION

# Mathematics

(Easy) (LB)

If $f(x) = \begin{vmatrix} x^3+0 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$ for all $x \in R$, then $2f(0) + f'(0)$ is equal to

(Diff)

(A) 42

(B) 48

(C) 52

(D) 50

$$f(0) = \begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix} = -2(-2-4) = 12$$

$$f(x) = \begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2 & 2x & x^3+6 \\ \underset{\substack{\downarrow \\ x^2}}{x^3} & 4 & x^2-2 \end{vmatrix} + \underbrace{\begin{vmatrix} 0 & 2x^2+1 & 1+3x \\ 2 & 2x & x^3+6 \\ -x & 4 & x^2-2 \end{vmatrix}}_{\Delta}$$

$$= x^2 g(x) + \Delta$$

$$f(x)=x^2g(x)+\begin{vmatrix} 0 & 2x^2+1 & 1+3x \\ 2 & 2x & x^3+6 \\ -x & 4 & x^2-2 \end{vmatrix}$$

$$f'(x)=2xg(x)+x^2g'(x)+\begin{vmatrix} 0 & 2x^2+1 & 1+3x \\ 0 & 2x & x^3+6 \\ -1 & 4 & x^2-2 \end{vmatrix}$$

$$+\begin{vmatrix} 0 & 4x & 1+3x \\ 2 & 2 & x^3+6 \\ -x & 0 & x^2-2 \end{vmatrix}+\begin{vmatrix} 0 & 2x^2+1 & 3 \\ -2 & 2x & 3x^2 \\ x & 4 & 2x \end{vmatrix}$$

$$f'(0)=0+0+\begin{vmatrix} 0 & 1 & 1 \\ 0 & 0 & 6 \\ -1 & 4 & -2 \end{vmatrix}+\begin{vmatrix} 0 & 0 & 1 \\ 2 & 2 & 6 \\ 0 & 0 & -2 \end{vmatrix}+\begin{vmatrix} 0 & 1 & 3 \\ -2 & 0 & 0 \\ 0 & 4 & 0 \end{vmatrix}=-6+2(12)=18.$$

$$2f(0)+f'(0)=24+18=42.$$

(LB)

Find the maximum integral values of $a$ for which $\dfrac{ax^2+2(a+1)x+9a+4}{x^2+8x+32} < 0, \forall\, x \in R.$

Easy

(Quad)

(A) 0

(B) ~~-1~~

(C) 1

(D) -2

$\dfrac{ax^2+2(a+1)x+9a+4}{x^2+8x+32} < 0 \quad \forall\, x \in R$

$x^2+8x+32$: $A = 1 > 0$, $D = 64 - 128 < 0$

$\downarrow$

always +ve.

$ax^2+2(a+1)x+9a+4 < 0.$

$a < 0$ (1) & $D < 0 \Rightarrow 4(a+1)^2 - 4 \cdot a \cdot (9a+4) < 0$

$a^2+2a+1-9a^2-4a < 0$

$-8a^2-2a+1 < 0$

$8a^2+2a-1 > 0$

$$8a^2+4a-2a-1>0$$

$$(4a-1)(2a+1)>0$$

$$\begin{array}{ccccc} + & & - & & + \\ \hline & -\frac{1}{2} & & \frac{1}{4} & \end{array}$$

$$a \in \left(-\infty, -\tfrac{1}{2}\right) \cup \left(\tfrac{1}{4}, \infty\right) \quad \text{(II)}$$

(I) ∩ (II)

$$a \in \left(-\infty, -\tfrac{1}{2}\right)$$

$$\begin{array}{cccc} -3 & -2 & -1 & -\frac{1}{2} \end{array}$$

(FUNCT) Easy (LB)

If $f(x)=\frac{4x+3}{6x-4}, x\neq\frac{2}{3}$ and $(f\circ f)(x)=g(x)$, where $g: R-\left\{\frac{2}{3}\right\}\to R-\left\{\frac{2}{3}\right\}$, then $(g\circ g\circ g)(4)$ is equal to

A 2

B 4

C 6

D 8

$g(x)=(f\circ f)(x)=f(f(x))=f\left(\frac{4x+3}{6x-4}\right)=\dfrac{\frac{4(4x+3)}{6x-4}+3}{\frac{6(4x+3)}{6x-4}-4}$

$g(x)=\dfrac{16x+12+18x-12}{24x+18-24x+16}=\dfrac{34x}{34}=x$

$g(x)=x$

$g(4)=4$

$(g\circ g\circ g)(4)=g(g(\underbrace{g(4)}))=4$

(Moderate)
(Sequence)

Sum of the series $\frac{1}{1-3\cdot1^2+1^4}+\frac{2}{1-3\cdot2^2+2^4}+\frac{3}{1-3\cdot3^2+3^4}$...... upto 10 terms is ____

(A) $-\frac{55}{109}$

(B) $\frac{55}{109}$

(C) $\frac{45}{109}$

(D) $-\frac{45}{109}$

$$T_r=\frac{r}{1-3\cdot r^2+r^4}=\frac{r}{1-2r^2+r^4-r^2}=\frac{r}{(r^2-1)^2-r^2}$$

$$T_r=\frac{r}{(r^2-r-1)(r^2+r-1)}=\frac{r^2+r-1-(r^2-r-1)}{2(r^2-r-1)(r^2+r-1)}$$

$$T_r=\frac{1}{2}\left[\frac{1}{r^2-r-1}-\frac{1}{r^2+r-1}\right]$$

$$T_1=\frac{1}{2}\left[\frac{1}{-1}-\frac{1}{1}\right]$$

$$T_2=\frac{1}{2}\left[\frac{1}{1}-\frac{1}{5}\right]$$

$$T_3=1/2\left[\frac{1}{5}-\frac{1}{11}\right]$$

$$T_{10}=\frac{1}{2}\left[\frac{1}{89}-\frac{1}{109}\right]$$

$$S_{10}=\frac{1}{2}\left[-1-\frac{1}{109}\right]=\frac{-110}{2\times109}=-\frac{55}{109}$$

# JEE MAIN 2024 LIVE PAPER DISCUSSION

(ITF)

If $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \pi$ and $(\alpha+\beta+\gamma)(\alpha+\beta-\gamma) = 3\alpha\beta$, then find $\gamma$.

$$\underbrace{\sin^{-1}\alpha}_{A} + \underbrace{\sin^{-1}\beta}_{B} = \pi - \underbrace{\sin^{-1}\gamma}_{C}$$

$$\sin(A+B) = \sin C$$

$$\sin A\cos B + \cos A\sin B = \sin C$$

$$\alpha\sqrt{1-\beta^2} + \beta\sqrt{1-\alpha^2} = \gamma.$$

$$\alpha^2(1-\beta^2) + \beta^2(1-\alpha^2) + 2\alpha\beta\sqrt{1-\alpha^2}\sqrt{1-\beta^2} = \gamma^2$$

$$\cancel{\alpha^2} + \cancel{\beta^2} - 2\alpha^2\beta^2 + 2\alpha\beta\sqrt{(1-\alpha^2)(1-\beta^2)} = \cancel{\alpha^2} + \cancel{\beta^2} - \alpha\beta.$$

$$+2\alpha\beta + 2\sqrt{(1-\alpha^2)(1-\beta^2)} = 1$$

$\sin^{-1}\alpha = A$, $\sin^{-1}\beta = B$, $\sin^{-1}\gamma = C$

$\sin A = \alpha$, $\sin B = \beta$, $\sin C = \gamma$.

$$(\alpha+\beta+\gamma)(\alpha+\beta-\gamma) = 3\alpha\beta$$

$$(\alpha+\beta)^2 - \gamma^2 = 3\alpha\beta$$

$$\alpha^2+\beta^2+2\alpha\beta-\gamma^2 = 3\alpha\beta$$

$$\alpha^2+\beta^2-\alpha\beta = \gamma^2.$$

$$(1-2\alpha\beta) = 2\sqrt{(1-\alpha^2)(1-\beta^2)}$$

$$(1-2\alpha\beta)^2 = 4(1-\alpha^2)(1-\beta^2)$$

$$1+\cancel{4\alpha^2\beta^2}-4\alpha\beta = 4(1-\alpha^2-\beta^2+\cancel{\alpha^2\beta^2})$$

$$1-4\alpha\beta = 4-4(\alpha^2+\beta^2)$$

$$4(\alpha^2+\beta^2)-4\alpha\beta = 3$$

$$4(\alpha^2+\beta^2-\alpha\beta) = 3$$

$$4r^2 = 3$$

$$r^2 = 3/4$$

$$r = \sqrt{3}/2$$

JEE MAIN 2024 LIVE PAPER DISCUSSION

(LB)

If one of the diameter of the circle $x^2 + y^2 - 10x + 4y + 13 = 0$ is a chord of another circle and whose centre is the point of intersection of the lines $2x + 3y = 12$ and $3x - 2y = 5$, then the radius of the circle is

(A) 6

(B) 3

(C) $\sqrt{6}$

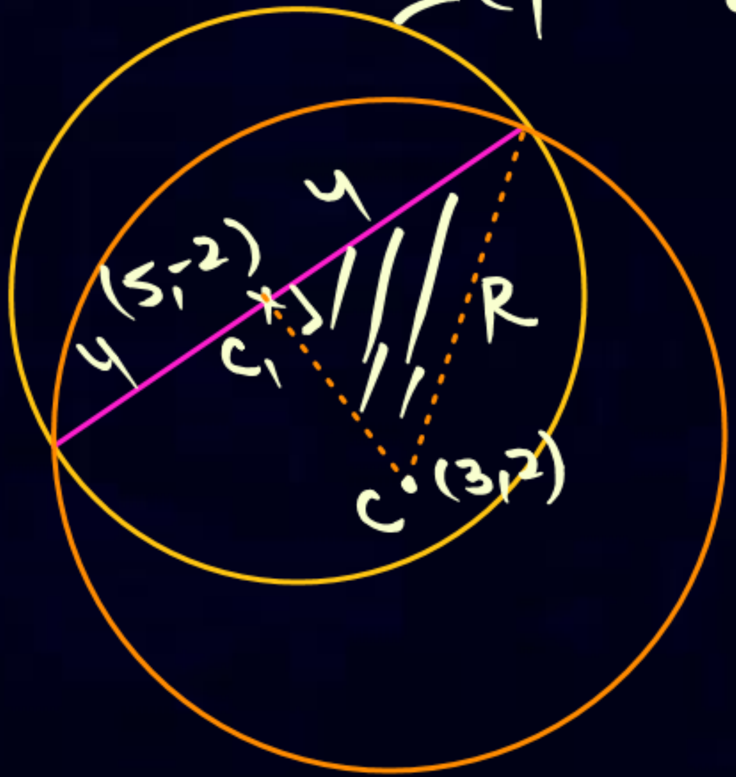
(D) 4

$2x+3y=12 \quad \times 2$

$3x-2y=5 \quad \times 3$

$13x = 39$

$x=3,\ y=2 \Rightarrow C(3,2)$




$r_1 = \sqrt{25+4-13} = \sqrt{16} = 4$

$C_1C = \sqrt{2^2+4^2} = \sqrt{20}$

$R^2 = 16 + (C_1C)^2$

$R^2 = 36$

$R = 6.$

$$\lim_{x\to 0}\frac{e^{|2\sin x|}-2|\sin x|-1}{x^2} \text{ is}$$

(A) Does not exist

(B) 2

(C) 1

(D) -1

RHL

$$\lim_{x\to 0^+}\frac{e^{2\sin x}-2\sin x-1}{x^2}$$

$$\lim_{x\to 0^+}\frac{\left(\cancel{1}+\cancel{2\sin x}+\frac{(2\sin x)^2}{2!}+\dots\infty\right)-\cancel{2\sin x}-\cancel{1}}{x^2}$$

$$=\lim_{x\to 0^+}\frac{4\sin^2 x}{x^2}\left(\frac{1}{2!}+\frac{2\sin x}{3!}+\dots\infty\right)=2$$

LHL

$$\lim_{x\to 0^-}\frac{e^{-2\sin x}+2\sin x-1}{x^2}$$

$$=\lim_{x\to 0^-}\frac{\left(\cancel{1}-\cancel{2\sin x}+\frac{(2\sin x)^2}{2!}-\frac{(2\sin x)^3}{3!}+\dots\infty\right)+\cancel{2\sin x}-\cancel{1}}{x^2}$$

$$=\lim_{x\to 0^-}\frac{4\sin^2 x}{x^2}\left(\frac{1}{2!}-\frac{2\sin x}{3!}+\dots\infty\right)$$

$=2.$

Find number of different words from "DISTRIBUTION" taking 4 at a time.

D S R B U O N 3I 2T

| Category | Selection | Arrangements | No. of words |
|---|---|---|---|
| 4 Diff | ${}^9C_4$ | $4!$ | ${}^9C_4 \times 4!$ |
| 2 Alike + 2 Diff | ${}^2C_1 \times {}^8C_2$ | $\frac{4!}{2!}$ | ${}^2C_1 \times {}^8C_2 \times \frac{4!}{2!}$ |
| 2 Alike + 2 other Alike | ${}^2C_2$ | $\frac{4!}{2!\cdot 2!}$ | $\frac{4!}{2!\cdot 2!} \times {}^2C_2$ |
| 3 Alike + 1 diff | ${}^1C_1 \times {}^8C_1$ | $\frac{4!}{3!}$ | ${}^1C_1 \times {}^8C_1 \times \frac{4!}{3!}$ |

# JEE MAIN 2024 LIVE ((•)) PAPER DISCUSSION

Let $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}, \vec{b} = 4\hat{i} + \hat{j} + 7\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 4\hat{k}$ be 3 vectors. If a vector $\vec{p}$ satisfies $\vec{p} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{p} \cdot \vec{a} = 0$, then $\vec{p} \cdot (\hat{i} - \hat{j} - \hat{k})$ is equal to ______

(A) ~~32~~

(B) 23

(C) 16

(D) 61

$\bar{p} \times \bar{b} = \bar{c} \times \bar{b}$

$\bar{p} \times \bar{b} - \bar{c} \times \bar{b} = \bar{0}$

$(\bar{p} - \bar{c}) \times \bar{b} = \bar{0}$

$\cancel{\bar{p} - \bar{c} = 0}$ $\quad$ $\cancel{\bar{b} = 0}$ $\quad$ $\bar{p} - \bar{c} \parallel \bar{b}$

$\cancel{\bar{p} = \bar{c}}$

$\Downarrow$

$\bar{p} - \bar{c} = \lambda \bar{b}$

$\bar{p} = \bar{c} + \lambda \bar{b}$

Dot with $\bar{a}$

$\bar{a} \cdot \bar{p} = \bar{a} \cdot \bar{c} + \lambda \bar{a} \cdot \bar{b}$

$0 = -8 + \lambda(-1) \Rightarrow \lambda = -8$

$\bar{a} \cdot \bar{c} = 3 - 3 - 8 = -8$

$\bar{a} \cdot \bar{b} = 12 + 1 - 14 = -1$

$\bar{p} = \bar{c} - 8\bar{b}$

$= i - 3j + 4k - 8(4i + j + 7k)$

$= -31i - 11j - 52k.$

$$\bar{p}\cdot(i-j-k) = -31+11+52 = \underline{\underline{32}}.$$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ has foci $(\pm 5, 0)$ and latus rectum $=\sqrt{50}$, find square of eccentricity of $\frac{x^2}{a^2}-\frac{y^2}{a^2b^2}=1$.

$ae=5$

$\frac{2b^2}{a}=\sqrt{50}$

$H: \frac{x^2}{a^2}-\frac{y^2}{a^2b^2}=1$

$a^2e^2=25$

$a^2\left(1-\frac{b^2}{a^2}\right)=25$

$a^2-b^2=25$ — (1)

$a^2-\frac{\sqrt{50}\,a}{2}=25$

$2a^2-5\sqrt{2}a-50=0 \Rightarrow a=\frac{5\sqrt{2}\pm\sqrt{50+400}}{4}=\frac{5\sqrt{2}\pm 15\sqrt{2}}{4}=5\sqrt{2}$

$e_H^2=1+\frac{a^2b^2}{a^2}$

$e_H^2=1+b^2$

$\boxed{e_H^2=1+5^2=26}$

$a=\cancel{-\frac{5\sqrt{2}}{2}}$

$a^2-b^2=25$

$25/2-25=b^2$

$b^2=-ve$

$$a^2 - b^2 = 25$$

$$(5\sqrt{2})^2 - b^2 = 25$$

$$b^2 = 25$$

$$b = \underline{\underline{5}}$$

The solution of differential equation $y\frac{dx}{dy} = x(\log_e x - \log_e y + 1)$ , $x > 0$, $y > 0$ and passing through $(e, 1)$ is

$$y\frac{dx}{dy} = x(\ln x - \ln y + 1)$$

(A) $\left|\log_e\left(\frac{y}{x}\right)\right| = y^2$

$$\frac{1}{x}\frac{dx}{dy} = \frac{\ln x - \ln y + 1}{y}$$

put $\ln x = z$

$$\frac{1}{x}\frac{dx}{dy} = \frac{dz}{dy}$$

(B) $2\left|\log_e\left(\frac{x}{y}\right)\right| = y$

$$\frac{dz}{dy} = \frac{z}{y} + \frac{1-\ln y}{y}$$

(C) $\left|\log_e\left(\frac{y}{x}\right)\right| = x$

$$\frac{dz}{dy} - \frac{z}{y} = \frac{1-\ln y}{y} \rightsquigarrow \text{LDE}$$

$$IF = e^{-\int \frac{1}{y}dy} = e^{-\ln y} = 1/y$$

(D) $\left|\log_e\left(\frac{x}{y}\right)\right| = y$

$$z(1/y) = \int \frac{1-\ln y}{y^2}\,dy + C.$$

$$\frac{z}{y} = \int \frac{1}{y^2}\,dy - \int \overset{\mathrm{I}}{\ln y}\cdot\overset{\mathrm{II}}{\frac{1}{y^2}}\,dy + c$$

$$\frac{z}{y} = -\frac{1}{y} - \left[-\frac{1}{y}\cdot\ln y + \int \frac{1}{y}\cdot\frac{1}{y}\,dy\right] + c$$

$$\frac{\ln x}{y} = \cancel{-\frac{1}{y}} - \left[-\frac{\ln y}{y} \cancel{- \frac{1}{y}}\right] + c$$

$$\frac{\ln x}{y} = \frac{\ln y}{y} + c$$

$$\ln x = \ln y + cy.$$

$$\ln(x/y) = cy. \quad \text{passes } (e,1)$$

$$\ln e = c \Rightarrow c = 1 \longrightarrow y = \ln(x/y)$$

$$y = |\ln(x/y)|$$

Find sum of coefficient of $x^3$ and $x^{-13}$ in $(1+x)(1-x^2)\left(1+\frac{3}{x^2}+\frac{3}{x}+\frac{1}{x^3}\right)^5$.

$$(1+x)(1-x^2)\frac{(x^3+3x+3x^2+1)^5}{x^{15}} = \frac{(1+x)(1-x^2)(1+x)^{15}}{x^{15}} = \frac{(1+x)^{16}(1-x^2)}{x^{15}}$$

coeff of $x^3$ in $\frac{1}{x^{15}}(1+x)^{16}(1-x^2)$ = coeff of $x^{18}$ in $(1-x^2)(1+x)^{16}$

$G.T = {}^{16}C_r x^r$

$r = 16.$

$\alpha = -1 \cdot {}^{16}C_{16} = -1.$

coeff of $x^{-13}$ in $\frac{1}{x^{15}}(1-x^2)(1+x)^{16}$ = coeff of $x^2$ in $(1-x^2)(1+x)^{16}$.

= coeff of $x^2$ in $\left(1\cdot(1+x)^{16} - x^2(1+x)^{16}\right)$

$\beta = {}^{16}C_2 - {}^{16}C_0 = \frac{16\cdot 15}{2} - 1 = 120 - 1$

$\alpha + \beta = 118$

A set $A$ is given $\{1, 2, 3, 4\}$ and a relation $R$ is defined on set $A$ such that $R = \{(1, 1), (1, 2), (1, 4)\}$ and $R$ is subset of $S$, where $S$ is equivalence relation, find minimum number of elements in $S$.

Reflexive : $(1,1)\,(1,2)\,(1,4)\,(2,2)\,(3,3)\,(4,4)$

Symmt : $(1,1)\,(1,2)\,(1,4)\,(2,2)\,(3,3)\,(4,4)\,(2,1)\,(4,1)$

Transitive : $(1,1)\,(1,2)\,(1,4)\,(2,2)\,(3,3)\,(4,4)\,(2,1)\,(4,1)$

$(2,1)\,(1,4)$

$(2,4) \in R$

$(1,1)\,(1,2)\,(1,4)\,(2,2)\,(3,3)\,(4,4)\,(2,1)\,(4,1)\,(2,4),\,(4,2)$

7 elements + 3 elements

# JEE MAIN 2024 LIVE PAPER DISCUSSION

Let $S = \left[y^2 \le 4x, x < 4, \left[\frac{xy(x-1)(x-2)}{(x-3)(x-4)}\right] < 0, x \ne 3\right]$. Find area of region $S$.

$$\frac{xy(x-1)(x-2)}{(x-3)(x-4)} < 0$$

if $y > 0$

$$\frac{x(x-1)(x-2)}{(x-3)(x-4)} < 0$$

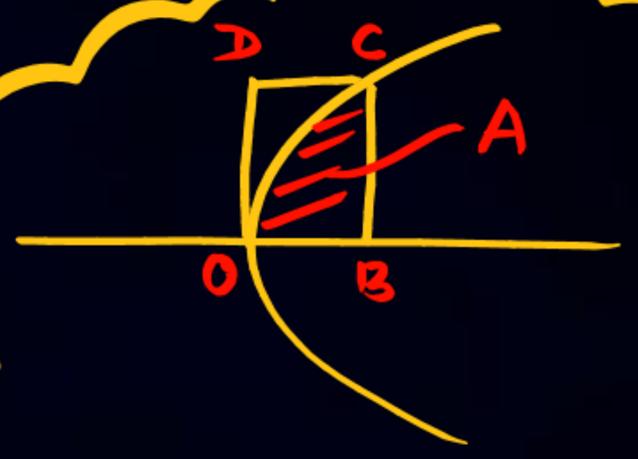
$-\ \ 0\ \ +\ \ 1\ \ -\ \ 2\ \ +\ \ 3\ \ -\ \ 4\ \ +$

$x \in (-\infty, 0) \cup (1,2) \cup (3,4)$

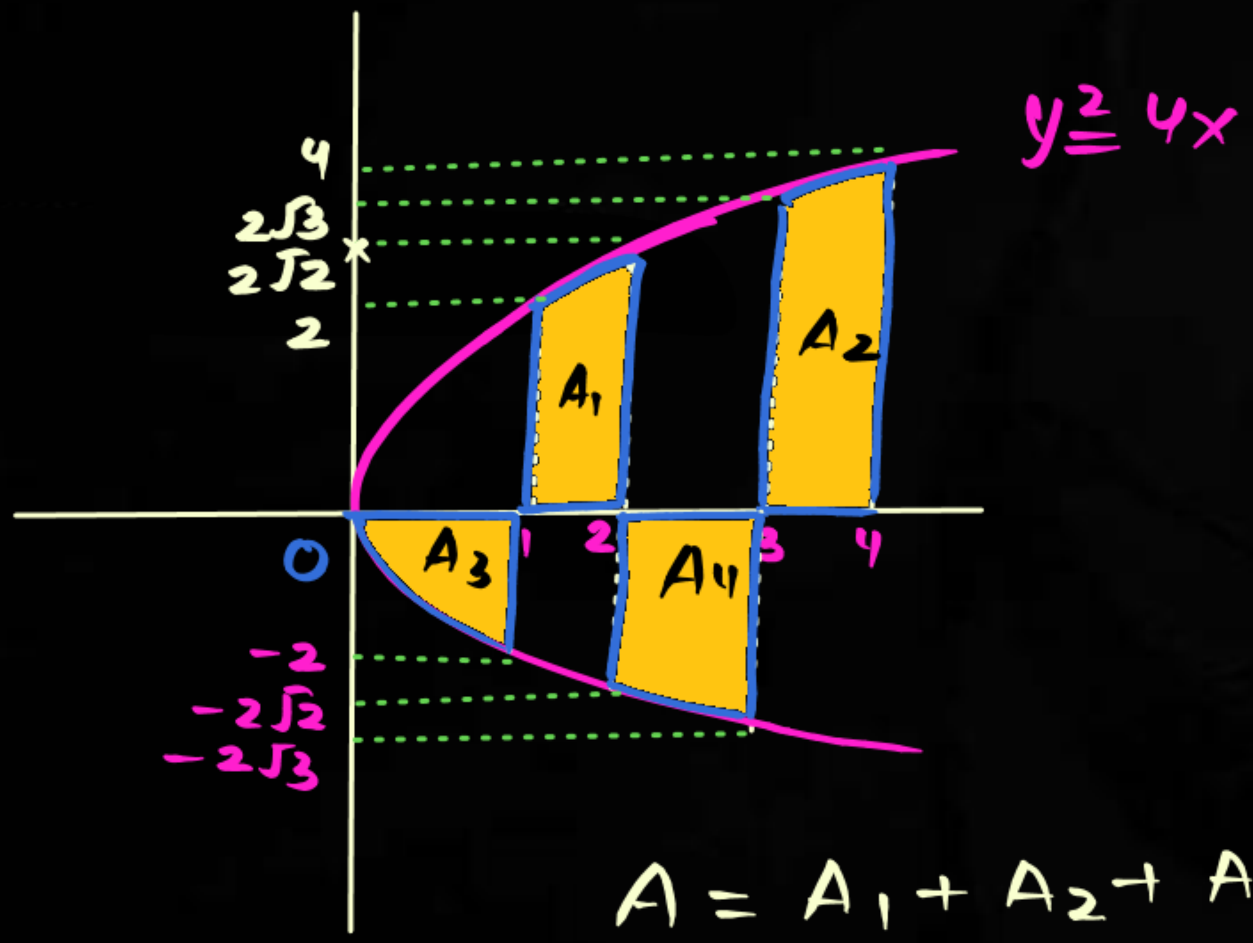
if $y < 0$

$$\frac{x(x-1)(x-2)}{(x-3)(x-4)} > 0$$

$x \in (0,1) \cup (2,3)$




$A = \frac{2}{3}(\square OBCD)$




$$A = A_1 + A_2 + A_3 + A_4$$

$$A = \frac{2}{3}\left(4\sqrt{2} - 2\right) + \frac{2}{3}\left(16 - 6\sqrt{3}\right) + \frac{2}{3}(2) + \frac{2}{3}\left(6\sqrt{3} - 4\sqrt{2}\right)$$

$$\boxed{A = 32/3}$$

Let $\vec{p} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{p} \cdot \vec{a} = 0$, find $\vec{p} \cdot (\hat{\imath} - \hat{\jmath} - \hat{k})$. (repeat)

Where,

$\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$,

$\vec{b} = 2\hat{\imath} - 4\hat{\jmath} - \hat{k}$

$\vec{c} = \hat{\jmath} + 2\hat{k}$

A basket has 15 good apples and 3 rotten apples. Two apples are drawn from this basket. Let number of rotten apples drawn be the random variable $x$. find variance

15 G  3 R

| $X$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(X)$ | $\frac{{}^{15}C_2}{{}^{18}C_2}$ | $\frac{{}^{15}C_1 \cdot {}^{3}C_1}{{}^{18}C_2}$ | $\frac{{}^{3}C_2}{{}^{18}C_2}$ |

$= \frac{\cancel{15}^{5} \times \cancel{14}^{7}}{{}_{3}\cancel{18} \times 17} = \frac{35}{51}$

$= \frac{\cancel{15} \times \cancel{3}}{\cancel{18} \times 17} \times 2 = 5/17$

$= \frac{3 \times 2}{18 \times 17} = \frac{1}{51}$

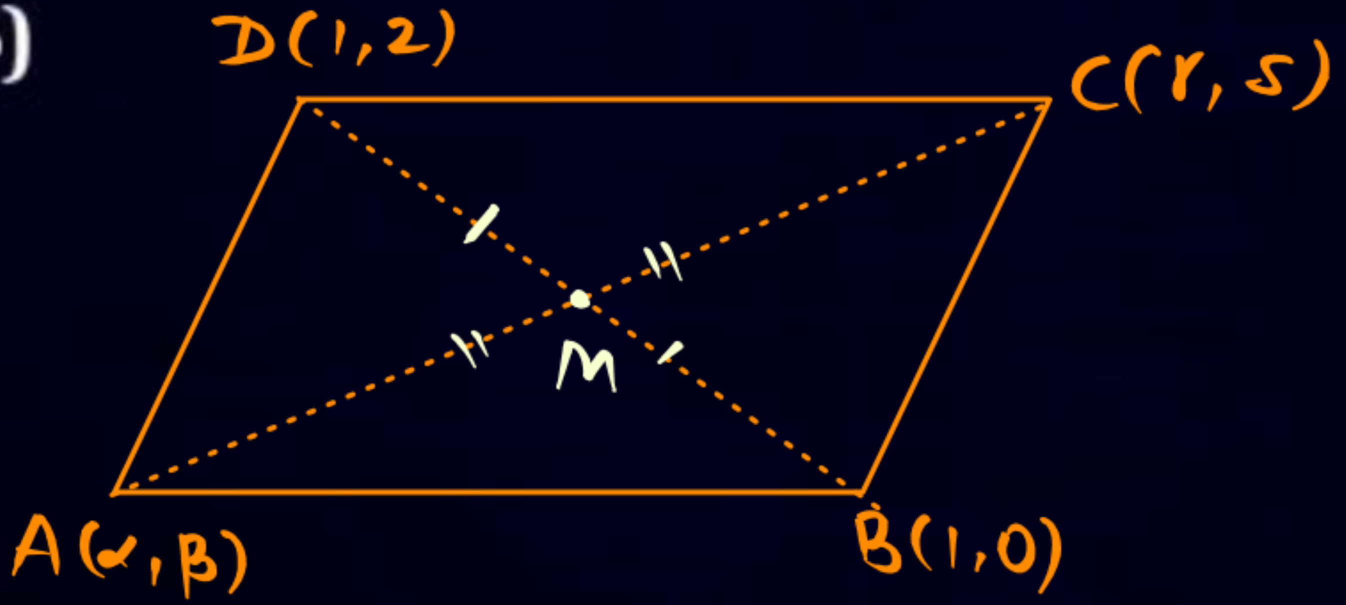
| $x_i$ | $x_i^2$ | $p_i$ | $p_i x_i$ | $p_i x_i^2$ |
|---|---|---|---|---|
| 0 | 0 | $\frac{35}{51}$ | 0 | 0 |
| 1 | 1 | $\frac{5}{17}$ | $\frac{5}{17}$ | $\frac{5}{17}$ |
| 2 | 4 | $\frac{1}{51}$ | $\frac{2}{51}$ | $\frac{4}{51}$ |

$$\sigma^2 = \sum p_i x_i^2 - \sum p_i x_i$$

$$= \frac{5}{17} + \frac{4}{51} - \left(\frac{5}{17} + \frac{2}{51}\right)$$

$$= \frac{2}{51}$$

A parallelogram $ABCD$ is given such that vertices are $A(\alpha, \beta)$, $B(1, 0)$, $C(\gamma, \delta)$, $D(1, 2)$ and a line $3y = 2x + 1$ passes through $A$ and $C$. Find the value of $(\alpha + \beta + \gamma + \delta)$




$$\frac{\alpha+\gamma}{2} = \frac{1+1}{2} \Rightarrow \alpha+\gamma = 2$$

$$\frac{\beta+\delta}{2} = \frac{2+0}{2} \Rightarrow \beta+\delta = 2$$

$$\alpha+\beta+\gamma+\delta = 4$$

$|\vec{a}| = 1, |\vec{b}| = 4, \vec{c} = 2\vec{a} \times \vec{b} - 3\vec{b}$. If α is angle between $\vec{b}$ & $\vec{c}$ find 192 $\sin^2\alpha$. Given that $\vec{a} \cdot \vec{b} = 2$.

$\bar{c} = 2(\bar{a} \times \bar{b}) - 3\bar{b}$ $\quad |\bar{a}| = 1, |\bar{b}| = 4 \quad , \bar{a} \cdot \bar{b} = 2$

squaring

$c^2 = 4(\bar{a} \times \bar{b})^2 + 9b^2 - 12\, \bar{b} \cdot (\bar{a} \times \bar{b})$

$c^2 = 4(\bar{a} \times \bar{b})^2 + 9b^2$

$c^2 = 4(a^2b^2 - (\bar{a} \cdot \bar{b})^2) + 9b^2$

$c^2 = 4(16 - 2^2) + 9 \cdot 16.$

$= 4(12) + 144 = 144 + 48 = 192 \Rightarrow c^2 = 192.$

$3\bar{b} + \bar{c} = 2(\bar{a} \times \bar{b})$

$9b^2 + c^2 + 6\bar{b} \cdot \bar{c} = 4(\bar{a} \times \bar{b})^2 = 48.$

$$9b^2+c^2+6\bar{b}\cdot\bar{c}=48.$$

$$144+192+6\bar{b}\cdot\bar{c}=48$$

$$144+144=-6\bar{b}\cdot\bar{c}$$

$$\bar{b}\cdot\bar{c}=\frac{288}{-6.}=-48.$$

$$bc\cos\alpha=-48.$$

$$\cancel{4}\sqrt{192}\cos\alpha=-\cancel{48}.$$

$$\sqrt{192}\cos\alpha=-12$$

$$192\cos^2\alpha=144.$$

$$192-192\sin^2\alpha=144$$

$$192\sin^2\alpha=\underline{48}$$

$f(x) = \int_{-1}^{x} (e^t - 1)^{11}(2t-1)^{15}(t-3)^{18}(2t-10)^{19}\,dt$. If $p$ is number of maxima of $f(x)$ and $q$ is number of minima, then find value of $p^2 + 2q = ?$

$f'(x) = (e^x - 1)^{11}(2x-1)^{15}(x-3)^{18}(2x-10)^{19}$

$(e^x-1)^{11} \downarrow x = 0$

$(2x-1)^{15} \downarrow x = \frac{1}{2}$

$(x-3)^{18}$: $x = 3$ inflexion

$(2x-10)^{19}$: $x = 5$.

<u>if $x>0$</u>

$f'(x) = (e^x-1)(2x-1)^{15}(2x-10)^{19}(x-3)^{18}$

$+ \quad \tfrac{1}{2} \quad - \quad 5 \quad +$

Inc $(0, \frac{1}{2}) \cup (5, \infty)$

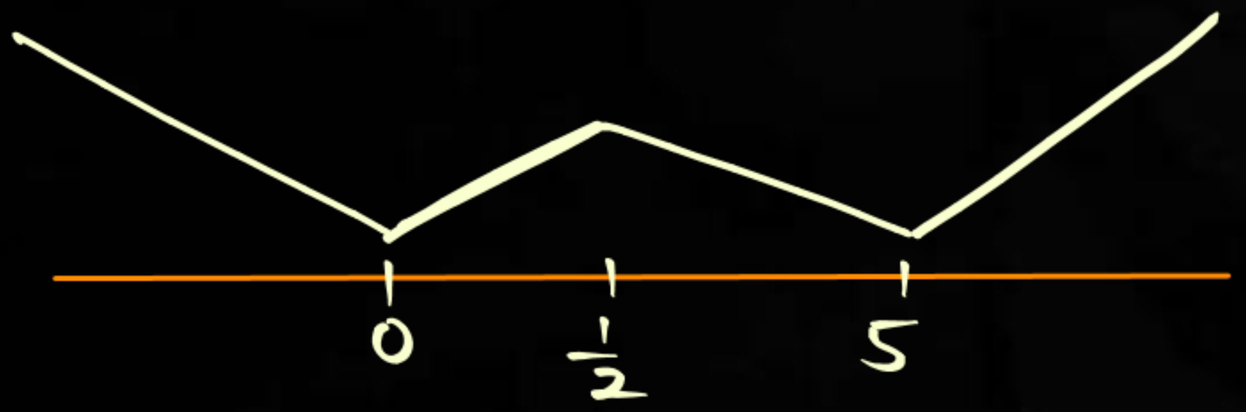
dec $(1/2, 5)$

<u>If $x<0$</u>

$f'(x) = (e^x-1)(2x-1)^{15}(2x-10)^{19}(x-3)^{18}$

$- \quad 1/2 \quad + \quad 5 \quad -$

$(-\infty, 0) \downarrow$




MAX at $x = 1/2$

MIN at $x = 0, 5$.

$p = 1, q = 2$

$p^2 + 2q = 1 + 4 = \underline{5}$.

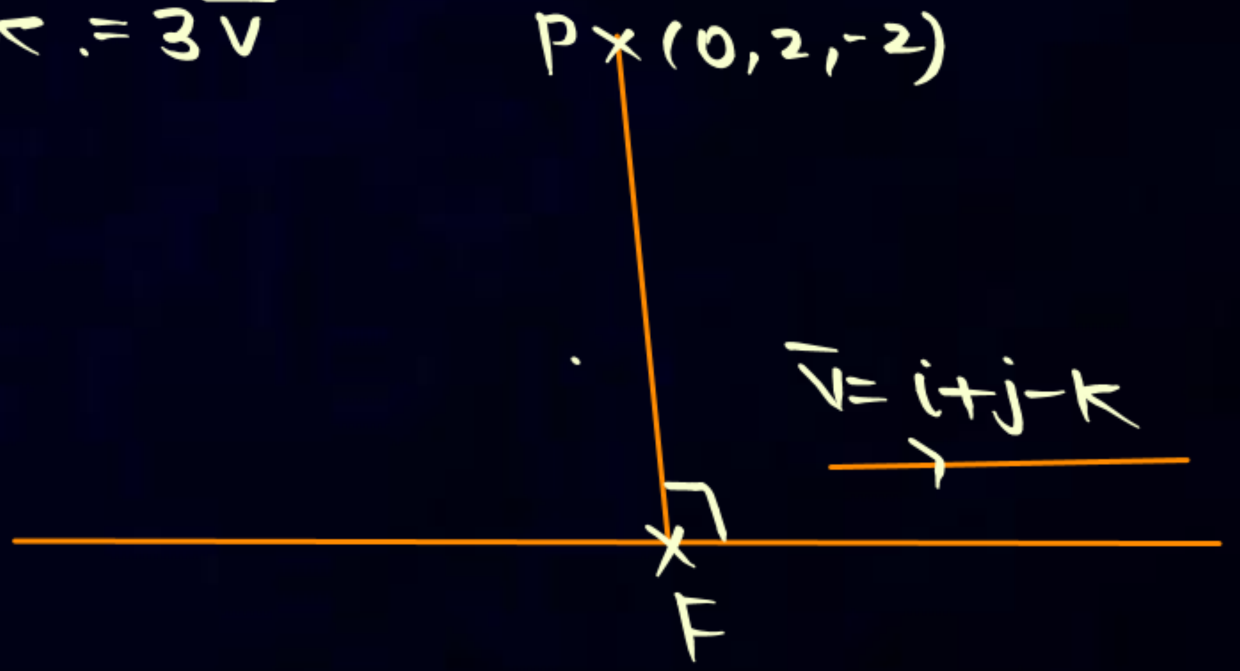
A line perpendicular to the lines

$\vec{r} = \hat{i} - \hat{j} + \hat{k} + \lambda(\underbrace{\hat{i} + \hat{j} + 2\hat{k}}_{\bar{b}_1})$

$\vec{r} = \hat{j} + \hat{k} + \mu(\underbrace{2\hat{i} - \hat{j} + \hat{k}}_{\bar{b}_2})$

passes through (5, 3, –2). Find the perpendicular distance of (0, 2, –2) from this line.

$\bar{b} = \bar{b}_1 \times \bar{b}_2$ $\quad \bar{b} = \begin{vmatrix} i & j & k \\ 1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix} = 3i + 3j - 3k = 3\bar{v}$




$L: \bar{r} = 5i + 3j - 2k + \lambda(i + j - k)$

$F(5+\lambda, 3+\lambda, -\lambda-2)$

$\overline{PF} = (5+\lambda)i + (1+\lambda)j - \lambda k$

$\overline{PF} \cdot \bar{v} = 0 \Rightarrow 5 + \lambda + 1 + \lambda + \lambda = 0$

$\lambda = -2 \Rightarrow \overline{PF} = 3i - j + 2k \quad |\overline{PF}| = \sqrt{9+1+4} = \sqrt{14}$

# JEE MAIN 2024 LIVE PAPER DISCUSSION

Let $a$ be the sum of all coefficients in the expansion of

$$(1-2x+2x^2)^{2023}(3-4x+2x^3)^{2024} \text{ and } b=\lim_{x\to 0}\frac{\int_0^x \frac{\log(1+t)}{t^{2024}+1}dt}{x^2}.$$

If the equation $cx^2+dx+e=0$ and $2bx^2+ax+4=0$ has a common root, where $c, d, e \in R$. Find $d:c:e$.

$$a=(1-2+2)^{2023}(3-4+2)^{2024}=1 \quad , \quad b=\lim_{x\to 0}\frac{\int_0^x \frac{\ln(1+t)}{t^{2024}+1}dt}{x^2} \left(\frac{0}{0}\right)$$

$$a=1, b=1/2$$

$$=\lim_{x\to 0}\frac{\frac{\ln(1+x)}{1+x^{2024}}}{2x}$$

$$=\lim_{x\to 0}\frac{\ln(1+x)}{2x(1+x^{2024})}=1/2$$

$\alpha + i\beta$

$\alpha - i\beta$ $\quad cx^2 + dx + e = 0$ have a common root

$\alpha + i\beta$ $\quad x^2 + x + 4 = 0$

$\alpha - i\beta$ $\quad D < 0$ both roots imag

$\Downarrow$

Both roots common

$$\frac{c}{1} = \frac{d}{1} = \frac{e}{4}$$

$$c : d : e = 1 : 1 : 4$$

If the system of linear equation $x - 2y + z = -4$; $2x + \alpha y + 3z = 5$ and $3x - y + \beta z = 3$ has infinitely many solutions, then $12\alpha + 13\beta$ is equal to

(A) 52

(B) 55

(C) 58

(D) 60

$x - 2y + z = -4 \quad \times 2$

$2x + \alpha y + 3z = 5$

$(-4-\alpha)y - z = -13$

$2x + \alpha y + 3z = 5 \quad \times 3$

$3x - y + \beta z = 3 \quad \times 2$

$(3\alpha + 2)y + (9 - 2\beta)z = 9$

$\times (9-2\beta)$

Add

$y\big(\underbrace{3\alpha + 2 + (-4-\alpha)(9-2\beta)}_{=0}\big) = \underbrace{9 - 13(9-2\beta)}_{=0}$

for $\infty$ many soln: $9 - 13(9-2\beta) = 0$

$-108 + 26\beta = 0 \Rightarrow \beta = 54/13$

$$\Rightarrow (3\alpha+2)-(\alpha+4)(9-2\beta)=0$$

$$3\alpha+2-(\alpha+4)\left(9-\frac{108}{13}\right)=0$$

$$3\alpha+2-(\alpha+4)\frac{9}{13}=0$$

$$3\alpha-\frac{9\alpha}{13}+2-\frac{36}{13}=0$$

$$30\alpha+26-36=0$$

$$\alpha=1/3, \quad \beta=\frac{54}{13}.$$

$$12\alpha+13\beta=4+54=58.$$

Let $y = y(x)$ be the solution of differential equation $\frac{dy}{dx} = \frac{(\tan x)+y}{\sin x(\sec x - \sin x \tan x)}$, $x \in \left(0, \frac{\pi}{2}\right)$ satisfy the condition $y\left(\frac{\pi}{4}\right) = 2$, then $y\left(\frac{\pi}{3}\right)$ is.

$$\frac{dy}{dx} = \frac{\sin x + y\cos x}{\sin x \cdot \cos x(\sec x - \sin x \tan x)}$$

$$\frac{dy}{dx} = \frac{\sin x + y\cos x}{\sin x(1-\sin^2 x)} = \frac{\sin x + y\cos x}{\sin x \cdot \cos^2 x}$$

$$\frac{dy}{dx} - \frac{y\cos x}{\sin x \cdot \cos^2 x} = \sec^2 x.$$

$$I.F = e^{-\int \frac{\cos x}{\sin x \cos^2 x}dx} = e^{-\int \frac{\sec^2 x}{\tan x}dx} = e^{-\ln \tan x} = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

$$e^{-\int \frac{1}{\sin x \cdot \cos x}}$$
$$= e^{-2\int \operatorname{cosec} 2x \, dx}$$
$$= e^{-\ln|\operatorname{cosec} 2x - \cot 2x|}$$
$$= \frac{1}{\operatorname{cosec} 2x - \cot 2x}.$$
$$= \frac{\sin 2x}{1-\cos 2x} = \frac{2\sin x \cdot \cos x}{2\sin^2 x}$$

$$y \cdot \frac{1}{\tan x} = \int \frac{\sec^2 x}{\tan x}\,dx + C$$

$$\frac{y}{\tan x} = \ln|\tan x| + C$$

$$\underbrace{x = \frac{\pi}{4},\ y = 2}$$

$$2 = 0 + C \Rightarrow C = 2$$

$$\frac{y}{\tan x} = \ln|\tan x| + 2$$

$$\underline{x = \pi/3} \quad \frac{y}{\sqrt{3}} = \ln\sqrt{3} + 2$$

$$y = 2\sqrt{3} + \sqrt{3}\ln\sqrt{3}.$$

# THANK YOU

vector

3D → 1) Foot of $\perp$ar

2) $\perp$ar dist

3) S.D b/w skew lines