

ICSE Class 10 Maths Selina Solutions Chapter 20: For ICSE Class 10, this ICSE Class 10 Maths Selina Solutions Chapter 20 is among the most crucial ones to focus on. This chapter covers several important subjects, including converting solids, combining solids, and solving many miscellaneous issues.

ICSE Class 10 Maths Selina Solutions Chapter 20 covers the surface area and volume of cylinders, cones, and spheres. The goal of the Selina Solutions for Class 10 Mathematics, which are provided by our knowledgeable professors here, is to answer students' questions at their convenience. Additionally, it gives pupils pointers on how to successfully solve difficulties, enhancing their ability to solve problems, which is crucial when it comes to exams.

ICSE Class 10 Maths Selina Solutions Chapter 20 Overview

ICSE Class 10 Maths Selina Solutions Chapter 20 focuses on three important geometric solids: cylinders, cones, and spheres. These three-dimensional shapes are fundamental in geometry and have practical applications in various fields, from engineering to everyday objects.

The ICSE Class 10 Maths Selina Solutions Chapter 20 begins by introducing cylinders, which are characterized by their two circular bases connected by a curved surface. It covers topics such as calculating the total surface area and volume of cylinders using specific formulas and understanding their properties.

Moving on to cones, the chapter explains their structure where a circular base tapers to a single vertex. It details how to find the total surface area and volume of cones, similar to cylinders but with different geometric formulas.

Finally, the ICSE Class 10 Maths Selina Solutions Chapter 20 explores spheres, perfectly round three-dimensional objects defined by a curved surface with every point equidistant from its center. It covers methods to calculate the surface area and volume of spheres using their respective formulas.

ICSE Class 10 Maths Selina Solutions Chapter 20

Below we have provided ICSE Class 10 Maths Selina Solutions Chapter 20. Students are advised to thoroughly practice the questions of ICSE Class 10 Maths Selina Solutions Chapter 20.

1. The height of a circular cylinder is 20 cm and the radius of its base is 7 cm. Find:

(i) the volume

(ii) the total surface area.

Solution:

Given, a circular cylinder whose

Height, $h = 20$ cm and base radius, $r = 7$ cm

$$\begin{aligned} \text{(i) Volume of cylinder} &= \pi r^2 h = \frac{22}{7} \times 7^2 \times 20 \text{ cm}^3 \\ &= 3080 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{(ii) Total surface area of a cylinder} &= 2\pi r (h + r) \\ &= 2 \times \frac{22}{7} \times 7(20 + 7) \text{ cm}^2 \\ &= 2 \times 22 \times 27 \text{ cm}^2 \\ &= 1188 \text{ cm}^2 \end{aligned}$$

2. The inner radius of a pipe is 2.1 cm. How much water can 12 m of this pipe hold?

Solution:

Given,

The inner radius of the pipe = 2.1 cm

Length of the pipe = 12 m = 1200 cm

$$\begin{aligned} \text{Volume of the pipe} &= \pi r^2 h = \frac{22}{7} \times 2.1^2 \times 1200 \\ &= 16632 \text{ cm}^3 \end{aligned}$$

3. A cylinder of circumference 8 cm and length 21 cm rolls without sliding for $4\frac{1}{2}$ seconds at the rate of 9 complete rounds per second. Find:

(i) distance travelled by the cylinder in $4\frac{1}{2}$ seconds, and

(ii) the area covered by the cylinder in $4\frac{1}{2}$ seconds

Solution:

Given,

Base circumference of cylinder (c) = 8 cm

$$\text{So, the radius} = \frac{c}{2\pi} = \frac{8 \times 7}{2 \times 22} = \frac{14}{11} \text{ cm}$$

Length of the cylinder (h) = 21 cm

(i) If distance covered in one revolution is 8 cm, then distance covered in 9 revolutions = $9 \times 8 = 72$ cm or distance covered in 1 second = 72 cm.

Thus, distance covered in $4\frac{1}{2}$ seconds = $72 \times (9/2) = 324$ cm

(ii) Curved surface area = $2\pi rh$

$$= 2 \times \frac{22}{7} \times \frac{14}{11} \times 21$$

$$= 168 \text{ cm}^2$$

So, the area covered in one revolution = 168 cm^2

Then,

The area covered in 9 revolutions = $168 \times 9 = 1512 \text{ cm}^2$

Which is also the area covered in 1 second = 1512 cm^2

Therefore, the area covered in $4\frac{1}{2}$ seconds = $1512 \times 9/2 = 6804 \text{ cm}^2$

4. How many cubic meters of earth must be dug out to make a well 28 m deep and 2.8 m in diameter? Also, find the cost of plastering its inner surface at Rs 4.50 per sq meter.

Solution:

Given,

Radius of the well = $2.8/2 = 1.4$ m

Depth of the well = 28 m

Hence, the volume of earth dug out = $\pi r^2 h$

$$= \left(\frac{22}{7}\right) \times 1.4 \times 1.4 \times 28$$

$$= 17248/100 = 172.48 \text{ m}^3$$

Area of curved surface = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 1.4 \times 28$$

$$= 246.40 \text{ m}^2$$

Now, the cost of plastering at the rate of Rs 4.50 per sq m

$$= \text{Rs } 246.40 \times 4.50$$

$$= \text{Rs } 1108.80$$

5. What length of solid cylinder 2 cm in diameter must be taken to recast into a hollow cylinder of external diameter 20 cm, 0.25 cm thick and 15 cm long?

Solution:

Given,

External diameter of hollow cylinder = 20 cm

So, it's radius = 10 cm = R

Thickness = 0.25 cm

Hence, the internal radius = $(10 - 0.25) = 9.75 \text{ cm} = r$

Length of cylinder (h) = 15 cm

Now,

$$\text{Volume} = \pi h (R^2 - r^2) = \pi \times 15(10^2 - 9.75^2) = 15 \pi (100 - 95.0625)$$

$$= 15 \pi \times 4.9375 \text{ cm}^3$$

Now,

Diameter of the solid cylinder = 2 cm

so, radius (r) = 1 cm

Let h be the length of the solid cylinder,

$$\text{Volume} = \pi r^2 h = \pi 1^2 h = \pi h \text{ cm}^3$$

Then, according to given condition in the question

$$\pi h = 15 \pi \times 4.9375$$

$$h = 15 \times 4.9375$$

$$h = 74.0625$$

Thus, the length of solid cylinder = 74.0625 cm

6. A cylinder has a diameter of 20 cm. The area of curved surface is 100 sq cm. Find:

(i) the height of the cylinder correct to one decimal place.

(ii) the volume of the cylinder correct to one decimal place.

Solution:

Given,

The diameter of the cylinder = 20 cm

So, the radius (r) = 10 cm

And the curved surface area = 100 cm^2

Height = h cm

(i) Curved surface area = $2\pi rh$

So,

$$2\pi rh = 100 \text{ cm}^2$$

$$2 \times \frac{22}{7} \times 10 \times h = 100$$

$$h = (100 \times 7) / (22 \times 10 \times 2) = 35/22$$

$$h = 1.6 \text{ cm}$$

(ii) Volume of the cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 10 \times 10 \times 1.6$$

$$= 502.9 \text{ cm}^3$$

7. A metal pipe has a bore (inner diameter) of 5 cm. The pipe is 5 mm thick all round. Find the weight, in kilogram, of 2 metres of the pipe if 1 cm^3 of the metal weights 7.7 g.

Solution:

Given,

Inner radius of the pipe = r = $5/2 = 2.5 \text{ cm}$

External radius of the pipe = R = Inner radius of the pipe + Thickness of the pipe

$$= 2.5 \text{ cm} + 0.5 \text{ cm} = 3 \text{ cm}$$

Length of the pipe = h = 2m = 200 cm

Volume of the pipe = External Volume – Internal Volume

$$= \pi R^2 h - \pi r^2 h$$

$$= \pi (R^2 - r^2) h$$

$$= \pi (R + r)(R - r) h$$

$$= \frac{22}{7} (3 - 2.5)(3 + 2.5) \times 200$$

$$= \frac{22}{7} (0.5) (5.5) \times 200 = 1728.6 \text{ cm}^3$$

Since, 1 cm³ of the metal weight 7.7g,

Hence, weight of the pipe = (1728.6 x 7.7)g = (1728.6 x 7.7)/ 1000 kg = 13.31 kg

8. A cylindrical container with diameter of base 42 cm contains sufficient water to submerge a rectangular solid of iron with dimensions 22 cm x 14 cm x 10.5 cm. Find the rise in level of the water when the solid is submerged.

Solution:

Given,

Diameter of cylindrical container = 42 cm

So, it's radius (r) = 21 cm

Dimensions of rectangular solid = 22cm x 14cm x 10.5cm

Hence,

The volume of solid = 22 x 14 x 10.5 cm³ (i)

Let the height of water = h

So, the volume of water in the container will be = $\pi r^2 h$

$$= \frac{22}{7} \times 21 \times 21 \times h \text{ cm}^3 \text{ (ii)}$$

According to the question, from (i) and (ii) we have

$$\frac{22}{7} \times 21 \times 21 \times h = 22 \times 14 \times 10.5$$

$$22 \times 3 \times 21 \times h = 22 \times 14 \times 10.5$$

$$h = (22 \times 14 \times 10.5) / (22 \times 3 \times 21)$$

$$= 7/3 = 2.33 \text{ cm}$$

Therefore, the water level will be raised to a level of 2.33 cm when the solid is submerged.

9. A cylindrical container with internal radius of its base 10 cm, contains water up to a height of 7 cm. Find the area of wetted surface of the cylinder.

Solution:

Given,

Internal radius of the cylindrical container = 10 cm = r

Height of water = 7 cm = h

So, the surface area of the wetted surface = $2\pi rh + \pi r^2 = \pi r(2h + r) = 22/7 \times 10 \times (2 \times 7 + 10)$

$$= 220/7 \times 24$$

$$= 754.29 \text{ cm}^2$$

10. Find the total surface area of an open pipe of length 50 cm, external diameter 20 cm and internal diameter 6 cm.

Solution:

Given,

Length of the open pipe = 50 cm

Its external diameter = 20 cm

So, it's external radius (R) = 10 cm

And, its internal diameter = 6 cm

So, it's internal radius (r) = 3 cm

Then.

Surface area of pipe open from both sides = $2\pi Rh + 2\pi rh = 2\pi h(R + r)$

$$= 2 \times 22/7 \times 50 \times (10 + 3)$$

$$= 4085.71 \text{ cm}^2$$

Area of upper and lower part = $2\pi(R^2 - r^2)$

$$= 2 \times 22/7 \times (10^2 - 3^2)$$

$$= 2 \times 22/7 \times 91$$

$$= 572 \text{ cm}^2$$

$$\text{Hence, the total surface area} = 4085.71 + 572 = 4657.71 \text{ cm}^2$$

11. The height and the radius of the base of a cylinder are in the ratio 3:1. If its volume is $1029\pi \text{ cm}^3$; find its total surface area.

Solution:

Given,

The ratio between height and radius of a cylinder = 3:1

$$\text{Volume} = 1029\pi \text{ cm}^3 \dots\dots(i)$$

Let the radius of the base = r

Then, it's height will be = $3r$

Hence,

$$\text{Volume} = \pi r^2 h = \pi \times r^2 \times 3r = 3\pi r^3 \dots\dots(ii)$$

Equating (i) and (ii), we get

$$3\pi r^3 = 1029\pi$$

$$r^3 = 1029\pi / 3\pi$$

$$r^3 = 343$$

$$r = 7$$

Thus, radius = 7 cm and height = $3 \times 7 = 21$ cm

Now,

$$\text{Total surface area} = 2\pi r(h + r) = 2 \times 22/7 \times 7 \times (21 + 7)$$

$$= 2 \times 22/7 \times 7 \times 28$$

$$= 44 \times 28$$

$$= 1232 \text{ cm}^2$$

12. The radius of a solid right circular cylinder increases by 20% and its height decreases by 20%. Find the percentage change in its volume.

Solution:

Let the radius of a solid right cylinder (r) = 100 cm

And, let the height of a solid right circular cylinder (h) = 100 cm

So, the volume (original) of a solid right circular cylinder = $\pi r^2 h$

$$= \pi \times (100)^2 \times 100$$

$$= 10000 \pi \text{ cm}^3$$

Now, the new radius = $r' = 120$ cm

New height = $h' = 80$ cm

So, the volume (new) of a solid right circular cylinder = $\pi r'^2 h' = \pi \times (120)^2 \times 80$

$$= 1152000 \pi \text{ cm}^3$$

Thus, the increase in volume = New volume – Original volume

$$= 1152000 \pi \text{ cm}^3 - 1000000 \pi \text{ cm}^3$$

$$= 152000 \pi \text{ cm}^3$$

Therefore, percentage change in volume = Increase in volume/Original volume $\times 100\%$

$$= 152000 \pi \text{ cm}^3 / 1000000 \pi \text{ cm}^3 \times 100\%$$

$$= 15.2 \%$$

13. The radius of a solid right circular cylinder decreases by 20% and its height increases by 10%. Find the percentage change in its:

(i) volume (ii) curved surface area

Solution:

Let the original dimensions of the solid right cylinder be radius (r) and height (h) in cm.

Then its volume = $\pi r^2 h \text{ cm}^3$

And, curved surface area = $2\pi rh$

Now, after the changes the new dimensions are:

New radius (r') = $r - 0.2r = 0.8r$ and

New height (h') = $h + 0.1h = 1.1h$

So,

The new volume = $\pi r'^2 h'$ cm^3

$$= \pi (0.8r)^2 (1.1h) \text{ cm}^3$$

$$= 0.704 \pi r^2 h \text{ cm}^3$$

And, the new curved surface area = $2\pi r' h' = 2\pi (0.8r)(1.1h)$

$$= (0.88) 2\pi r h$$

(i) Percentage change in its volume = Decrease in volume/ original volume x 100 %

$$= (\text{Original volume} - \text{new volume}) / \text{original volume} \times 100 \%$$

$$= (\pi r^2 h - 0.704 \pi r^2 h) / \pi r^2 h \times 100$$

$$= 0.296 \times 100 = 29.6 \%$$

(ii) Percentage change in its curved surface area = Decrease in CSA/ original CSA x 100 %

$$= (\text{Original CSA} - \text{new CSA}) / \text{original CSA} \times 100 \%$$

$$= (2\pi r h - (0.88) 2\pi r h) / 2\pi r h \times 100$$

$$= 0.12 \times 100 = 12 \%$$

14. Find the minimum length in cm and correct to nearest whole number of the thin metal sheet required to make a hollow and closed cylindrical box of diameter 20 cm and height 35 cm. Given that the width of the metal sheet is 1 m. Also, find the cost of the sheet at the rate of Rs. 56 per m.

Find the area of metal sheet required, if 10% of it is wasted in cutting, overlapping, etc.

Solution:

Given,

Height of the cylinder box = $h = 35$ cm

Base radius of the cylinder box = $r = 10$ cm

Width of metal sheet = 1m = 100 cm

Area of metal sheet required = total surface area of the box

$$\text{Length} \times \text{width} = 2\pi r(r + h)$$

$$\text{Length} \times 100 = 2 \times \frac{22}{7} \times 10(10 + 35)$$

$$\text{Length} \times 100 = 22 \times \frac{22}{7} \times 10 \times 45$$

$$\text{Length} = (2 \times 22 \times 10 \times 45) / (100 \times 7) = 28.28 \text{ cm} = 28 \text{ cm (correcting to the nearest whole number)}$$

Thus,

$$\text{Area of metal sheet} = \text{length} \times \text{width} = 28 \times 100 = 2800 \text{ cm}^2 = 0.28 \text{ m}^2$$

$$\text{So, the cost of the sheet at the rate of Rs 56 per m}^2 = \text{Rs } (56 \times 0.28) = \text{Rs } 15.68$$

Let the total sheet required be x.

$$\text{Then, } x - 10\% \text{ of } x = 2800 \text{ cm}^2$$

$$x - \frac{10}{100} \times x = 2800 \text{ cm}^2$$

$$9x = 2800$$

$$x = 3111 \text{ cm}^2$$

Therefore, a metal sheet of area 3111 cm² is required.

ICSE Class 10 Maths Selina Solutions Chapter 20

Exercise 20(B)

1. Find the volume of a cone whose slant height is 17 cm and radius of base is 8 cm.

Solution:

Given,

$$\text{Slant height of the cone (l)} = 17 \text{ cm}$$

$$\text{Base radius (r)} = 8 \text{ cm}$$

We know that,

$$l^2 = r^2 + h^2$$

$$h^2 = l^2 - r^2 = 17^2 - 8^2 = 289 - 64 = 225$$

$$h = 15 \text{ cm}$$

$$\text{Now, the volume of cone} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 8 \times 8 \times 15 \text{ cm}^3$$

$$= 7040/7 \text{ cm}^3$$

$$= 1005.71 \text{ cm}^3$$

2. The curved surface area of a cone is 12320 cm^2 . If the radius of its base is 56 cm , find its height.

Solution:

Given,

$$\text{Curved surface area of the cone} = 12320 \text{ cm}^2$$

$$\text{Radius of the base} = 56 \text{ cm}$$

Let the slant height be 'l'

Then,

$$\text{Curved surface area} = \pi r l = 12320 \text{ cm}^2$$

$$\frac{22}{7} \times 56 \times l = 12320 \text{ cm}^2$$

$$l = (12320 \times 7) / (22 \times 56)$$

$$l = 70 \text{ cm}$$

We know that,

$$l^2 = h^2 + r^2$$

$$70^2 = h^2 + 56^2$$

$$h^2 = 4900 - 3136 = 1764$$

$$\text{Thus, } h = 42 \text{ cm}$$

3. The circumference of the base of a 12 m high conical tent is 66 m . Find the volume of the air contained in it.

Solution:

Given,

Circumference of the base (c) = 66 m

Height of the conical tent (h) = 12 m

Radius = $c/2\pi = 66/2\pi = (33 \times 7)/22 = 21/2 = 10.5$ m

Thus, the volume of the cone tent = $\frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{21}{2}\right)^2 \times 12$$

$$= 1386 \text{ m}^3$$

Therefore, the volume of air contained is 1386 m^3 .

4. The radius and height of a right circular cone are in the ratio 5:12 and its volume is 2512 cubic cm. Find the radius and slant height of the cone. (Take $\pi = 3.14$)

Solution:

Given,

The ratio between radius and height = 5: 12

Volume of the right circular cone = 2512 cm^3

Let its radius (r) = 5x, its height (h) = 12x and slant height = l

We know that,

$$l^2 = r^2 + h^2 = (5x)^2 + (12x)^2 = 25x^2 + 144x^2$$

$$l^2 = 169x^2$$

$$l = 13x$$

Now, the volume = $\frac{1}{3} \pi r^2 h$

$$\frac{1}{3} \pi r^2 h = 2512$$

$$\frac{1}{3} \times (3.14) \times (5x)^2 \times (12x) = 2512$$

$$\frac{1}{3} \times (3.14) \times (300x^3) = 2512$$

$$x^3 = (2512 \times 3) / (3.14 \times 300) = 8$$

$$x = 2$$

Thus,

$$\text{Radius} = 5x = 5 \times 2 = 10 \text{ cm}$$

$$\text{Height} = 12x = 12 \times 2 = 24 \text{ cm}$$

$$\text{Slant height} = 13x = 13 \times 2 = 26 \text{ cm}$$

5. Two right circular cones x and y are made, x having three times the radius of y and y having half the volume of x. Calculate the ratio between the heights of x and y.

Solution:

From the question,

Let radius of cone y = r

So, radius of cone x = 3r

Let volume of cone y = V

Then, volume of cone x = 2V

Let h_1 be the height of x and h_2 be the height of y.

Now,

$$\text{Volume of cone x} = \frac{1}{3} \pi (3r)^2 h_1 = \frac{1}{3} \pi 9r^2 h_1 = 3\pi r^2 h_1$$

$$\text{Volume of cone y} = \frac{1}{3} \pi r^2 h_2$$

So,

$$2V/V = 3\pi r^2 h_1 / (\frac{1}{3} \pi r^2 h_2)$$

$$2 = 3h_1 \times 3 / h_2 = 9h_1 / h_2$$

$$h_1/h_2 = 2/9$$

$$\text{Hence, } h_1 : h_2 = 2 : 9$$

6. The diameters of two cones are equal. If their slant heights are in the ratio 5:4, find the ratio of their curved surface areas.

Solution:

Let radius of each cone = r

Given that, ratio between their slant heights = 5: 4

Let slant height of the first cone = $5x$

And slant height of second cone = $4x$

So, curved surface area of the first cone = $\pi r l = \pi r \times (5x) = 5\pi r x$

And, the curved surface area of the second cone = $\pi r \times (4x) = 4\pi r x$

Therefore,

The ratio between them = $5\pi r x : 4\pi r x = 5: 4$

7. There are two cones. The curved surface area of one is twice that of the other. The slant height of the latter is twice that of the former. Find the ratio of their radii.

Solution:

Let slant height of the first cone = l

So, the slant height of the second cone = $2l$

Radius of the first cone = r_1

And, the radius of the second cone = r_2

Now,

Curved surface area of first cone = $\pi r_1 l$

Curved surface area of second cone = $\pi r_2 (2l) = 2\pi r_2 l$

According to given condition, we have

$$\pi r_1 l = 2(2\pi r_2 l)$$

$$r_1 = 4r_2$$

$$r_1 / r_2 = 4/1$$

Thus, $r_1 : r_2 = 4: 1$

ICSE Class 10 Maths Selina Solutions Chapter 20

Exercise 20(C)

1. The surface area of a sphere is 2464 cm^2 , find its volume.

Solution:

Given,

$$\text{Surface area of the sphere} = 2464 \text{ cm}^2$$

Let the radius of the sphere be r .

$$\text{Then, surface area of the sphere} = 4\pi r^2$$

$$4\pi r^2 = 2464$$

$$4 \times \frac{22}{7} \times r^2 = 2464$$

$$r^2 = (2464 \times 7) / (4 \times 22) = 196$$

$$r = 14 \text{ cm}$$

$$\text{So, volume} = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14$$

$$= 11498.67 \text{ cm}^3$$

2. The volume of a sphere is 38808 cm^3 ; find its diameter and the surface area.

Solution:

Given,

$$\text{Volume of the sphere} = 38808 \text{ cm}^3$$

Let the radius of the sphere = r

$$\frac{4}{3} \pi r^3 = 38808$$

$$\frac{4}{3} \times \left(\frac{22}{7}\right) \times r^3 = 38808$$

$$r^3 = (38808 \times 7 \times 3) / (4 \times 22) = 9261$$

$$r = 21 \text{ cm}$$

$$\text{So, the diameter} = 2r = 21 \times 2 = 42 \text{ cm}$$

And,

$$\text{Surface area} = 4\pi r^2 = 4 \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2 = 5544 \text{ cm}^2$$

3. A spherical ball of lead has been melted and made into identical smaller balls with radius equal to half the radius of the original one. How many such balls can be made?

Solution:

Let the radius of the spherical ball = r

So, the volume = $\frac{4}{3} \pi r^3$

And, the radius of smaller ball = $r/2$

Volume of smaller ball = $\frac{4}{3} \pi (r/2)^3 = \frac{4}{3} \pi r^3/8 = \pi r^3/6$

Thus, the number of balls made out of the given ball = $(\frac{4}{3} \pi r^3) / (\pi r^3/6)$

= $\frac{4}{3} \times 6 = 8$

4. How many balls each of radius 1 cm can be made by melting a bigger ball whose diameter is 8 cm.

Solution:

Given,

Diameter of bigger ball = 8 cm

So, radius of bigger ball = 4 cm

Volume = $\frac{4}{3} \pi r^3 = \frac{4}{3} \pi 4^3 = \frac{265\pi}{3} \text{ cm}^3$

Radius of small ball = 1 cm

Then, volume = $\frac{4}{3} \pi r^3 = \frac{4}{3} \pi 1^3 = \frac{4\pi}{3} \text{ cm}^3$

Thus,

The number of balls = $(\frac{265\pi}{3}) / (\frac{4\pi}{3})$

= $\frac{265\pi}{3} \times \frac{3}{4\pi}$

= 64

5. 8 metallic sphere; each of radius 2 mm, are melted and cast into a single sphere. Calculate the radius of the new sphere.

Solution:

Radius of metallic sphere = 2mm = $\frac{1}{5} \text{ cm}$

$$\text{Volume} = \frac{4}{3} \pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{88}{(21 \times 125)} \text{ cm}^3$$

$$\text{Volume of 8 spheres} = (88 \times 8) / (21 \times 125) = 704 / (21 \times 125) \text{ cm}^3 \dots\dots (i)$$

Let the radius of new sphere = R

$$\text{Volume} = \frac{4}{3} \pi R^3 = \frac{4}{3} \times \frac{22}{7} \times R^3 \dots\dots (ii)$$

According to the question, equating (i) and (ii) we have

$$704 / (21 \times 125) = \frac{4}{3} \times \frac{22}{7} \times R^3$$

$$704 / (21 \times 125) = 88/21 \times R^3$$

$$R^3 = (704 \times 21) / (21 \times 125 \times 88) = 8/125$$

$$R = \sqrt[3]{8/125} = 2/5 = 0.4 \text{ cm} = 4 \text{ mm}$$

Therefore, the radius of the new sphere is 4 mm.

6. The volume of one sphere is 27 times that of another sphere. Calculate the ratio of their:

(i) radii

(ii) surface areas

Solution:

Given,

The volume of first sphere = 27 x volume of second sphere

Let the radius of the first sphere = r_1

And, radius of second sphere = r_2

(i) Then, according to the question we have

$$\frac{4}{3} \pi r_1^3 = 27 \left(\frac{4}{3} \pi r_2^3 \right)$$

$$r_1^3 / r_2^3 = 27$$

$$r_1 / r_2 = \sqrt[3]{27} = 3/1$$

Thus, $r_1 : r_2 = 3 : 1$

(ii) Surface area of the first sphere = $4 \pi r_1^2$

And the surface area of second sphere = $4 \pi r_2^2$

Ratio of their surface areas = $4 \pi r_1^2 / 4 \pi r_2^2 = r_1^2 / r_2^2 = 3^2 / 1^2 = 9$

Hence, the ratio = 9: 1

ICSE Class 10 Maths Selina Solutions Chapter 20

Exercise 20(D)

1. A solid sphere of radius 15 cm is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate the number of cones recast.

Solution:

Given,

Radius of the sphere = $R = 15$ cm

So, the volume of sphere melted = $\frac{4}{3} \pi R^3 = \frac{4}{3} \times \pi \times 15 \times 15 \times 15$

Radius of each cone recasted = $r = 2.5$ cm

And, height of each cone recasted = $h = 8$ cm

So, volume of each cone recasted = $\frac{1}{3} \pi r^2 h = \frac{1}{3} \times \pi \times 2.5 \times 2.5 \times 8$

Thus,

Number of cones recasted = Volume of sphere melted / Volume of each cone formed

$$= \frac{\frac{4}{3} \times \pi \times 15 \times 15 \times 15}{\frac{1}{3} \times \pi \times 2.5 \times 2.5 \times 8}$$

= 270

2. A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted into a cone of base diameter 8 cm. Find the height of the cone.

Solution:

Given,

External diameter of the hollow sphere = 8 cm

So, radius (R) = 4 cm

Internal diameter of the hollow sphere = 4 cm

So, radius (r) = 2 cm

Then, the volume of metal used in hollow sphere =

$$\frac{4}{3} \pi (R^3 - r^3) = \frac{4}{3} \times \frac{22}{7} \times (4^3 - 2^3) = \frac{88}{21} (64 - 8) = \frac{88}{21} \times 56 \text{ cm}^3 \dots (i)$$

Also given,

Diameter of cone = 8 cm

Therefore, radius = 4 cm

Let height of the cone = h

$$\text{Volume} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 4 \times 4 \times h = \frac{352}{21} h \dots (ii)$$

So, according to the question

From (i) and (ii), we have

$$\begin{aligned} \frac{352}{21} h &= \frac{88}{21} \times 56 \\ \Rightarrow h &= \frac{88 \times 56 \times 21}{21 \times 352} = 14 \text{ cm} \end{aligned}$$

$$h = (88 \times 56 \times 21) / (21 \times 352) = 14 \text{ cm}$$

Thus, the height of the cone = 14 cm

3. The radii of the internal and external surfaces of a metallic spherical shell are 3 cm and 5 cm respectively. It is melted and recast into a solid right circular cone of height 32 cm. find the diameter of the base of the cone.

Solution:

Given,

Height of the solid right circular cone = 32 cm

Internal radius metallic spherical shell = 3 cm

External radius spherical shell = 5 cm

Then, the volume of the spherical shell = $\frac{4}{3} \pi (5^3 - 3^3)$

$$= \frac{4}{3} \times \frac{22}{7} \times (125 - 27)$$

$$= \frac{4}{3} \times \frac{22}{7} \times 98$$

Volume of solid right circular cone = $\frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times r^2 \times 32$$

According to the question,

$$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 32 = \frac{4}{3} \times \frac{22}{7} \times 98$$

$$r^2 = (4 \times 98) / 32$$

$$r = 7/2 = 3.5 \text{ cm}$$

Therefore, diameter = $2r = 7 \text{ cm}$

4. Total volume of three identical cones is the same as that of a bigger cone whose height is 9 cm and diameter 40 cm. Find the radius of the base of each smaller cone, if height of each is 108 cm.

Solution:

Let the radius of the smaller cone be $r \text{ cm}$

Given,

Diameter of bigger cone = 40 cm

So, the radius = 20 cm and height = 9 cm

Volume of larger cone = $\frac{1}{3} \pi \times (20)^2 \times 9$

And,

The volume of the smaller cone = $\frac{1}{3} \pi \times r^2 \times 108$

From the question, we have

Volume of larger cone = 3 x Volume of smaller cone

$$\frac{1}{3} \pi \times (20)^2 \times 9 = 3 \times (\frac{1}{3} \pi \times r^2 \times 108)$$

$$r^2 = (20)^2 \times 9 / (108 \times 3)$$

$$r = 20/6 = 10/3 \text{ cm}$$

5. A solid rectangular block of metal 49 cm by 44 cm by 18 cm is melted and formed into a solid sphere. Calculate the radius of the sphere.

Solution:

$$\text{Volume of rectangular block} = 49 \times 44 \times 18 \text{ cm}^3 = 38808 \text{ cm}^3 \dots\dots (i)$$

Let r be the radius of sphere.

$$\text{So, the volume} = \frac{4}{3} \pi r^3 = \frac{4}{3} \times \frac{22}{7} \times r^3 = \frac{88}{21} r^3 \dots\dots (ii)$$

Then, according to the question

$$\frac{88}{21} r^3 = 38808$$

$$r^3 = (38808 \times 21) / 88 = 441 \times 21$$

$$= 9261$$

$$r = 21 \text{ cm}$$

Therefore, the radius of sphere = 21 cm

6. A hemi-spherical bowl of internal radius 9 cm is full of liquid. This liquid is to be filled into conical shaped small containers each of diameter 3 cm and height 4 cm. How many containers are necessary to empty the bowl?

Solution:

Given,

Radius of hemispherical bowl = 9 cm

$$\text{Volume} = \frac{2}{3} \pi r^3 = \frac{2}{3} \pi 9^3 = \frac{2}{3} \pi \times 729 = 486 \pi \text{ cm}^3$$

Diameter of each cylindrical bottle = 3 cm

So, the radius = 1.5 cm and height = 4 cm

$$\text{Volume of bottle} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (3/2)^2 \times 4 = 3 \pi$$

Thus,

$$\text{The number of bottles} = 486 \pi / 3 \pi$$

$$= 162$$

ICSE Class 10 Maths Selina Solutions Chapter 20

Exercise 20(E)

1. A cone of height 15 cm and diameter 7 cm is mounted on a hemisphere of same diameter. Determine the volume of the solid thus formed.

Solution:

Given,

Height of the cone = 15 cm

Diameter of the cone = 7 cm

So, its radius = 3.5 cm

Radius of the hemisphere = 3.5 cm

Now,

Volume of the solid = Volume of the cone + Volume of the hemisphere

$$= \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3$$

$$= \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 15 + \frac{2}{3} \times \frac{22}{7} \times (3.5)^3$$

$$= \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 (15 + 2 \times 3.5)$$

$$= \frac{847}{3}$$

$$= 282.33 \text{ cm}^3$$

2. A buoy is made in the form of a hemisphere surmounted by a right cone whose circular base coincides with the plane surface of the hemisphere. The radius of the base of the cone is 3.5 m and its volume is two-third of the hemisphere. Calculate the height of the cone and the surface area of the buoy, correct to two decimal places.

Solution:

Given,

Radius of the hemisphere part (r) = 3.5 m = $\frac{7}{2}$ m

Volume of hemisphere = $\frac{2}{3} \pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 539/6 \text{ m}^3$$

Volume of conical part = $\frac{2}{3} \times 539/6 \text{ m}^3$ [$\frac{2}{3}$ rd of the hemisphere]

Let height of the cone = h

Then,

$$\frac{1}{3} \pi r^2 h = (2 \times 539) / (3 \times 6)$$

$$\frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h = (2 \times 539) / (3 \times 6)$$

$$h = (2 \times 539 \times 2 \times 2 \times 7 \times 3) / (3 \times 6 \times 22 \times 7 \times 7)$$

$$h = 14/3 \text{ m} = 4.67 \text{ m}$$

Height of the cone = 4.67 m

Surface area of buoy = $2 \pi r^2 + \pi r l$

But, we know that

$$l = (r^2 + h^2)^{1/2}$$

$$l = ((7/2)^2 + (14/3)^2)^{1/2}$$

$$l = (1225/36)^{1/2} = 35/6 \text{ m}$$

Hence,

$$\text{Surface area} = 2\pi r^2 + \pi r l$$

$$= (2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}) + (\frac{22}{7} \times \frac{7}{2} \times \frac{35}{6})$$

$$= 77 + 385/6 = 847/6$$

$$= 141.17 \text{ m}^2 \text{ [Corrected to 2 decimal places]}$$

3. From a rectangular solid of metal 42 cm by 30 cm by 20 cm, a conical cavity of diameter 14 cm and depth 24 cm is drilled out. Find:

(i) the surface area of the remaining solid

(ii) the volume of remaining solid

(iii) the weight of the material drilled out if it weighs 7 gm per cm³.

Solution:

Given,

Dimensions of rectangular solid $l = 42$ cm, $b = 30$ cm and $h = 20$ cm

Conical cavity's diameter = 14 cm

So, its radius = 7 cm

Depth (height) = 24 cm

(i) Total surface area of cuboid = $2(lb + bh + lh)$

$$= 2(42 \times 30 + 30 \times 20 + 20 \times 42)$$

$$= 2(1260 + 600 + 840)$$

$$= 2(2700)$$

$$= 5400 \text{ cm}^2$$

Diameter of the cone = 14 cm

So, its radius = $14/2 = 7$ cm

$$\text{Area of circular base} = \pi r^2 = 22/7 \times 7 \times 7 = 154 \text{ cm}^2$$

We know that,

$$l = (7^2 + 24^2)^{1/2} = (49 + 576)^{1/2} = (625)^{1/2} = 25$$

$$\text{Area of curved surface area of cone} = \pi r l = 22/7 \times 7 \times 25 = 22 \times 25 = 550 \text{ cm}^2$$

$$\text{Surface area of remaining part} = 5400 + 550 - 154 = 5796 \text{ cm}^2$$

(ii) Volume of the rectangular solid = $(42 \times 30 \times 20) \text{ cm}^3 = 25200 \text{ cm}^3$

Radius of conical cavity (r) = 7 cm

Height (h) = 24 cm

$$\text{Volume of cone} = 1/3 \pi r^2 h$$

$$= 1/3 \times 22/7 \times 7 \times 7 \times 24$$

$$= 1232 \text{ cm}^3$$

(iii) Weight of material drilled out = $1232 \times 7\text{g} = 8624\text{g} = 8.624 \text{ kg}$

4. The cubical block of side 7 cm is surmounted by a hemisphere of the largest size. Find the surface area of the resulting solid.

Solution:

It's known that, the diameter of the largest hemisphere that can be placed on a face of a cube of side 7 cm will be 7 cm.

So, it's radius = $r = 7/2$ cm

It's curved surface area = $2 \pi r^2 = 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 77 \text{ cm}^2 \dots\dots (i)$

Now,

Surface area of the top of the resulting solid = Surface area of the top face of the cube – Area of the base of the hemisphere

$$\begin{aligned} &= (7 \times 7) - \left(\frac{22}{7} \times \frac{49}{4}\right) \\ &= 49 - \frac{77}{2} \\ &= \frac{98 - 77}{2} \\ &= \frac{21}{2} \\ &= 10.5 \text{ cm}^2 \dots\dots (ii) \end{aligned}$$

Surface area of the cube = $5 \times (\text{side})^2 = 5 \times 49 = 245 \text{ cm}^2 \dots\dots (iii)$

Hence,

Total area of resulting solid = $(i) + (ii) + (iii) = 245 + 10.5 + 77 = 332.5 \text{ cm}^2$

5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of the top which is open is 5 cm. It is filled with water up to the rim. When lead shots, each of which is a sphere of radius 0.5 cm, are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Solution:

Given,

Height of cone = 8 cm

Radius = 5 cm

Volume = $\frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 8 \text{ cm}^3$$

$$= \frac{4400}{21} \text{ cm}^3$$

Hence, the volume of water that flowed out = $\frac{1}{4} \times \frac{4400}{21} \text{ cm}^3$

$$= \frac{1100}{21} \text{ cm}^3$$

Also, given

Radius of each ball = 0.5 cm = $\frac{1}{2}$ cm

Volume of a ball = $\frac{4}{3} \pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \text{ cm}^3$$

$$= \frac{11}{21} \text{ cm}^3$$

Thus, the no. of balls = $\frac{1100}{21} \div \frac{11}{21} = 100$

ICSE Class 10 Maths Selina Solutions Chapter 20

Exercise 20(F)

1. From a solid right circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and same base are removed. Find the volume of the remaining solid.

Solution:

Given,

Height of the cylinder (h) = 10 cm

And radius of the base (r) = 6 cm

Volume of the cylinder = $\pi r^2 h$

Height of the cone = 10 cm

Radius of the base of cone = 6 cm

Volume of the cone = $\frac{1}{3} \pi r^2 h$

Volume of the remaining part = $\pi r^2 h - \frac{1}{3} \pi r^2 h$

$$= \frac{2}{3} \pi r^2 h$$

$$= \frac{2}{3} \times \frac{22}{7} \times 6 \times 6 \times 10$$

$$= \frac{5280}{7} \text{ cm}^3$$

$$= 754 \frac{(2/7)}{7} \text{ cm}^3$$

2. From a solid cylinder whose height is 16 cm and radius is 12 cm, a conical cavity of height 8 cm and of base radius 6 cm is hollowed out. Find the volume and total surface area of the remaining solid.

Solution:

Given,

Radius of solid cylinder (R) = 12 cm

And, Height (H) = 16 cm

$$\text{Volume} = \pi R^2 h = \frac{22}{7} \times 12 \times 12 \times 16$$

$$= \frac{50688}{7} \text{ cm}^3$$

Also given,

Radius of cone (r) = 6 cm, and height (h) = 8 cm.

$$\text{Volume} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 8$$

$$= \frac{2112}{7} \text{ cm}^3$$

$$(i) \text{ Volume of remaining solid} = \frac{50688}{7} - \frac{2112}{7}$$

$$= \frac{48576}{7}$$

$$= 6939.43 \text{ cm}^3$$

$$(ii) \text{ Slant height of cone } l = (h^2 + r^2)^{1/2}$$

$$= (6^2 + 8^2)^{1/2} = (36 + 64)^{1/2} = (100)^{1/2} = 10 \text{ cm}$$

Thus,

Total surface area of remaining solid = curved surface area of cylinder + curved surface area of cone + base area of cylinder + area of circular ring on upper side of cylinder

$$\begin{aligned}
&= 2\pi R h + \pi r l + \pi R^2 + \pi(R^2 - r^2) \\
&= (2 \times \frac{22}{7} \times 12 \times 16) + (\frac{22}{7} \times 6 \times 10) + (\frac{22}{7} \times 12 \times 12) + (\frac{22}{7}(12^2 - 6^2)) \\
&= \frac{8448}{7} + \frac{1320}{7} + \frac{3168}{7} + \frac{22}{7}(144 - 36) \\
&= \frac{8448}{7} + \frac{1320}{7} + \frac{3168}{7} + \frac{2376}{7} \\
&= \frac{15312}{7} \\
&= 2187.43 \text{ cm}^2
\end{aligned}$$

$$= 15312/7$$

$$= 2187.43 \text{ cm}^2$$

3. A circus tent is cylindrical to a height of 4 m and conical above it. If its diameter is 105 m and its slant height is 80 m, calculate the total area of canvas required. Also, find the total cost of canvas used at Rs 15 per meter if the width is 1.5 m

Solution:

Given,

Radius of the cylindrical part of the tent (r) = 105/2 m

Slant height (l) = 80 m

So, the total curved surface area of the tent = $2\pi r h + \pi r l$

$$= (2 \times \frac{22}{7} \times 105/2 \times 4) + (\frac{22}{7} \times 105/2 \times 80)$$

$$= 1320 + 13200$$

$$= 14520 \text{ m}^2$$

Width of canvas used = 1.5 m

Length of canvas = $14520/1.5 = 9680 \text{ m}$

Hence,

Total cost of canvas at the rate of Rs 15 per meter = $9680 \times 15 = \text{Rs } 145200$

4. A circus tent is cylindrical to a height of 8 m surmounted by a conical part. If total height of the tent is 13 m and the diameter of its base is 24 m; calculate:

(i) total surface area of the tent

(ii) area of canvas, required to make this tent allowing 10% of the canvas used for folds and stitching.

Solution:

Given,

Height of the cylindrical part = $H = 8$ m

Height of the conical part = $h = (13 - 8)$ m = 5 m

Diameter = 24 m

So, its radius = $24/2 = 12$ m

Then, slant height (l) = $(r^2 + h^2)^{1/2}$

$$= (12^2 + 5^2)^{1/2} = (144 + 25)^{1/2} = (169)^{1/2} = 13 \text{ m}$$

(i) Total surface area of the tent = $2\pi rh + \pi rl = \pi r(2h + l)$

$$= 22/7 \times 12 \times (2 \times 8 + 13)$$

$$= 264/7 (16 + 3)$$

$$= 264/7 \times 29$$

$$= 7656/7 \text{ m}^2$$

$$= 1093.71 \text{ m}^2$$

(ii) Area of canvas used in stitching = total area of canvas

$$\text{Total area of canvas} = 7656/7 + \text{total area of canvas}/10$$

$$\text{Total area of canvas} - \text{Total area of canvas}/10 = 7656/10$$

$$\text{Total area of canvas} (1 - 1/10) = 7656/7$$

$$\text{Total area of canvas} \times 9/10 = 7656/7$$

$$\text{Total area of canvas} = 7656/7 \times 10/9 = 76560/63$$

$$\text{Therefore, the total area of canvas} = 1215.23 \text{ m}^2$$

5. A cylindrical boiler, 2 m high, is 3.5 m in diameter. It has a hemispherical lid. Find the volume of its interior, including the part covered by the lid.

Solution:

Given,

Diameter of cylindrical boiler = 3.5 m

So, the radius (r) = $3.5/2 = 35/20 = 7/4$ m

Height (h) = 2m

Radius of hemisphere (R) = $7/4$ m

Total volume of the boiler = $\pi r^2 h + \frac{2}{3} \pi r^3$

$$= \pi r^2 (h + \frac{2}{3}r)$$

$$= \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} (2 + \frac{2}{3} \times \frac{7}{4})$$

$$= \frac{77}{8} (2 + \frac{7}{6})$$

$$= \frac{77}{8} \times \frac{19}{6} = \frac{1463}{48}$$

$$= 30.48 \text{ m}^3$$

6. A vessel is a hollow cylinder fitted with a hemispherical bottom of the same base. The depth of the cylindrical part is $4\frac{2}{3}$ m and the diameter of hemisphere is 3.5 m. Calculate the capacity and the internal surface area of the vessel.

Solution:

Given,

Diameter of the base = 3.5 m

So, its radius = $3.5/2 \text{ m} = 1.75 \text{ m} = \frac{7}{4} \text{ m}$

Height of cylindrical part = $4 + \frac{2}{3} = \frac{14}{3}$

(i) Capacity (volume) of the vessel = $\pi r^2 h + \frac{2}{3} \pi r^3 = \pi r^2 (h + \frac{2}{3}r)$

$$= \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} (\frac{14}{3} + \frac{2}{3} \times \frac{7}{4})$$

$$= \frac{77}{8} (\frac{14}{3} + \frac{7}{6})$$

$$= \frac{77}{8} (28 + \frac{7}{6})$$

$$= \frac{77}{8} \times \frac{35}{6}$$

$$= 2695/48$$

$$= 56.15 \text{ m}^3$$

$$(ii) \text{ Internal curved surface area} = 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$$

$$= 2 \times 22/7 \times 7/4(14/3 + 7/4)$$

$$= 11(56 + 21)/12$$

$$= 11 \times 77/12$$

$$= 847/12$$

$$= 70.58 \text{ m}^2$$

7. A wooden toy is in the shape of a cone mounted on a cylinder as shown alongside.

If the height of the cone is 24 cm, the total height of the toy is 60 cm and the radius of the base of the cone = twice the radius of the base of the cylinder = 10 cm; find the total surface area of the toy. [Take $\pi = 3.14$]

Solution:

Given,

Height of the cone (h) = 24 cm

Height of the cylinder (H) = 36 cm

Radius of the cone (r) = twice the radius of the cylinder = 10 cm

Radius of the cylinder (R) = 5 cm

Then, we know the slant height of the cone = $(r^2 + h^2)^{1/2} = (10^2 + 24^2)^{1/2} = (100 + 576)^{1/2}$

$$= (676)^{1/2} = 26 \text{ cm}$$

Now, the surface area of the toy = curved area of the conical point + curved area of the cylinder

$$= \pi rl + \pi r^2 + 2\pi RH$$

$$= \pi (rl + r^2 + 2RH)$$

$$= 3.14 (10 \times 26 + 10^2 + 2 \times 5 \times 36)$$

$$= 3.14 (260 + 100 + 360)$$

$$= 3.14 (720)$$

$$= 2260.8 \text{ cm}^2$$

ICSE Class 10 Maths Selina Solutions Chapter 20

Exercise 20(G)

1. What is the least number of solid metallic spheres, each of 6 cm diameter, that should be melted and recast to form a solid metal cone whose height is 45 cm and diameter is 12 cm?

Solution:

Given,

Diameter of solid metallic sphere = 6 cm

So, its radius = 3 cm

Height of solid metal cone = 45

Diameter of metal cone = 12 cm

So, its radius = 6 cm

Now,

Let the number of solid metallic spheres be 'n'

$$\text{Volume of 1 sphere} = \frac{4}{3} \pi (3)^3$$

$$\text{Volume of metallic cone} = \frac{1}{3} \pi 6^2 \times 45$$

$$\text{Hence, } n = \frac{\text{Volume of metal cone}}{\text{Volume of 1 sphere}}$$

$$n = \frac{(1/3 \pi 6^2 \times 45)}{(4/3 \pi (3)^3)}$$

$$= \frac{(6 \times 6 \times 45)}{(4 \times 3 \times 3 \times 3)}$$

$$n = 15$$

2. A largest sphere is to be carved out of a right circular cylinder of radius 7 cm and height 14 cm. Find the volume of the sphere. (Answer correct to the nearest integer)

Solution:

Radius of the largest sphere that can be formed inside the cylinder will be equal to the radius of the cylinder.

So, radius of the largest sphere = 7 cm

Volume of sphere = $\frac{4}{3} \pi 7^3$

= $\frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$

= $4312/3$

= 1437 cm^3

3. A right circular cylinder having diameter 12 cm and height 15 cm is full of ice-cream. The ice-cream is to be filled in identical cones of height 12 cm and diameter 6 cm having a hemi-spherical shape on the top. Find the number of cones required.

Solution:

Given,

Diameter of the cylinder = 12 cm

So, its radius = 6 cm

Height of the cylinder = 15 cm

Diameter of the cone = 6 cm

So, its radius = 3 cm

Height of the cone = 12 cm

Radius of the hemisphere = 3 cm

Now,

Let the number of cones be 'n'.

Volume of the cylinder = $\pi r^2 h = \pi 6^2 \times 15$

Volume of an ice-cream cone with ice-cream = Volume of cone + Volume of hemisphere

= $\frac{1}{3} \pi (3)^2 \times 12 + \frac{2}{3} \pi (3)^2$

= $36 \pi + 18 \pi$

= $54 \pi \text{ cm}^3$

Number of cones = Volume of cylinder/ Volume of ice-cream cone

$$= \pi 6^2 \times 15 / 54 \pi$$

$$= (36 \times 15) / 54$$

$$= 10$$

Therefore, the number of cones required = 10

4. A solid is in the form of a cone standing on a hemisphere with both their radii being equal to 8 cm and the height of cone is equal to its radius. Find in terms of π , the volume of the solid.

Solution:

Given,

Radius of both cone and hemisphere = 8 cm

And, height = 8 cm

Hence,

Volume of the solid = Volume of cone + volume of hemisphere

$$= \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3$$

$$= \frac{1}{3} \pi (8)^2 (8) + \frac{2}{3} \pi (8)^3$$

$$= \pi (8)^3$$

$$= 512 \pi \text{ cm}^3$$

5. The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 0.2 cm. Find the length of wire.

Solution:

Given,

Diameter of the sphere = 6 cm

So, its radius = 3 cm

$$\text{And, volume} = \frac{4}{3} \pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 3 \times 3 \times 3 = \frac{792}{7} \text{ cm}^3 \dots\dots (i)$$

Diameter of cylindrical wire = 0.2 cm

So, the radius of the wire = $0.2/2 = 0.1 = 1/10$ cm

Now, let length of wire = h

$$\text{Volume} = \pi r^2 h = 22/7 \times 1/10 \times 1/10 \times h \text{ cm}^3 = 22h/700 \text{ cm}^3 \dots\dots (ii)$$

According to the question, equating (i) and (ii) we have

$$22h/700 = 792/7$$

$$h = 792/7 \times 700/22$$

$$h = 3600 \text{ cm} = 36 \text{ m}$$

Therefore, length of the wire = 36 m

6. Determine the ratio of the volume of a cube to that of a sphere which will exactly fit inside the cube.

Solution:

Let edge of the cube = a

Then, volume of the cube = $a \times a \times a = a^3$

The sphere that exactly fits in the cube will have radius 'a/2'

So,

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (a/2)^3 = \frac{4}{3} \times \frac{22}{7} \times a^3/8 = \frac{11}{21} a^3$$

Now,

Volume of cube: Volume of sphere

$$= a^3 : \frac{11}{21} a^3$$

$$= 1 : \frac{11}{21}$$

$$= 21 : 11$$

7. An iron pole consisting of a cylindrical portion 110 cm high and of base diameter 12 cm is surmounted by a cone 9 cm high. Find the mass of the pole, given that 1 cm³ of iron has 8 gm of mass (approx). (Take $\pi = 355/113$)**Solution:**

Given,

Radius of the base of poles (r) = 6 cm

Height of the cylindrical part (h_1) = 110 cm

Height of the conical part (h_2) = 9 cm

Now,

Total volume of the iron pole = $\pi r^2 h_1 + \frac{1}{3} \pi r^2 h_2 = \pi r^2 (h_1 + \frac{1}{3} h_2)$

$$= 355/113 \times 6 \times 6 (110 + \frac{1}{3} \times 9)$$

$$= 355/113 \times 36 \times 113$$

$$= 12780 \text{ cm}^3$$

Weight of $1 \text{ cm}^3 = 8 \text{ gm}$ (Given)

Thus, the total weight = $12780 \times 8 = 102240 \text{ gm} = 102.24 \text{ kg}$

8. In the following diagram a rectangular platform with a semicircular end on one side is 22 meters long from one end to the other end. If the length of the half circumference is 11 meters, find the cost of constructing the platform, 1.5 meters high at the rate of Rs 4 per cubic meters.

Solution:

Given,

Length of the platform = 22 m

Circumference of semi-circle (c) = 11 m

So, its radius = $(c \times 2) / (2 \times \pi) = (11 \times 2) / 22 = 1 \text{ m}$

Thus, the breadth of the part = $1 \times 2 = 2 \text{ m}$

And, length = $22 - 2 = 20 \text{ m}$

Now, area of platform = $l \times b + \frac{1}{2} \pi r^2$

$$= 20 \times 2 + \frac{1}{2} \times 22/7 \times 1 \times 1 \text{ m}^2$$

$$= 40 + 11/7 \text{ m}^2$$

$$= 41.57 \text{ m}^2$$

Also given, height of the platform = 1.5 m

So, the volume = $41.57 \times 1.5 = 62.355 \text{ m}^3$

Rate of construction = Rs 4 per m^3

Therefore,

Total expenditure = Rs 4 x 223.125 = Rs 892.50

9. The cross-section of a tunnel is a square of side 7 m surmounted by a semicircle as shown in the following figure.

The tunnel is 80 m long. Calculate:

(i) its volume

(ii) the surface area of the tunnel (excluding the floor) and

(iii) its floor area

Solution:

Given,

Side of square (a) = 7m

And, the radius of semi-circle = $\frac{7}{2}$ m

Length of the tunnel = 80 m

Area of cross section of the front part = $a^2 + \frac{1}{2} \pi r^2$

$$= 7 \times 7 + \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 49 + \frac{77}{4} \text{ m}^2$$

$$= \frac{(196 + 77)}{4}$$

$$= \frac{273}{4} \text{ m}^2$$

(i) Thus, the volume of the tunnel = area of cross section x length of the tunnel

$$= \frac{273}{4} \times 80$$

$$= 5460 \text{ m}^3$$

(ii) Circumference of the front of tunnel (excluding the floor) = $2a + \frac{1}{2} \times 2\pi r$

$$= 2 \times 7 + \frac{22}{7} \times \frac{7}{2}$$

$$= 14 + 11 = 25 \text{ cm}$$

Hence, the surface area of the inner part of the tunnel = $25 \times 80 = 2000 \text{ m}^2$

(iii) Area of floor = $l \times b = 7 \times 80 = 560 \text{ m}^2$

10. A cylindrical water tank of diameter 2.8m and height 4.2m is being fed by a pipe of diameter 7 cm through which water flows at the rate of 4m/s. Calculate, in minutes, the time it takes to fill the tank.

Solution:

Given,

Diameter of cylindrical tank = 2.8 m

So, its radius = 1.4 m

Height = 4.2 m

Volume of water filled in it = $\pi r^2 h$

$$= \frac{22}{7} \times 1.4 \times 1.4 \times 4.2 \text{ m}^3$$

$$= 181.104/7 \text{ m}^3$$

$$= 25.872 \text{ m}^3 \dots\dots (i)$$

Diameter of the pipe = 7 cm

So, the radius (r) = 7/2 cm

Now, let the length of water in the pipe = h_1

Volume = $\pi r^2 h_1$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h_1$$

$$= 77/2 h_1 \text{ cm}^3 \dots\dots (ii)$$

According to the question, equating (i) and (ii) we have

$$77/2 h_1 \text{ cm}^3 = 25.872 \times 100^3 \text{ cm}^3 \text{ (Converted from m}^3 \text{ to cm}^3\text{)}$$

$$h_1 = \frac{25.872 \times 100^3 \times 2}{77} \text{ cm}$$

$$h_1 = \frac{25.872 \times 100^3 \times 2}{77 \times 100} \text{ m}$$

$$h_1 = 0.672 \times 100^2 \text{ m}$$

$$h_1 = 6720 \text{ m}$$

Thus, the time taken at the speed of 4 m per second = $6720/(4 \times 60) \text{ min} = 28 \text{ min}$

Benefits of ICSE Class 10 Maths Selina Solutions Chapter 20

Studying Chapter 20 on Cylinder, Cone, and Sphere from ICSE Class 10 Maths Selina Solutions offers several benefits to students:

Conceptual Clarity: The ICSE Class 10 Maths Selina Solutions Chapter 20 provides a clear understanding of geometric solids like cylinders, cones, and spheres, including their definitions, properties, and mathematical formulas for calculating surface area and volume. This clarity helps in building a strong foundation in geometry.

Practical Applications: These geometric shapes have numerous real-life applications in architecture, engineering, physics, and everyday objects. Understanding their properties and calculations prepares students to apply mathematical concepts to practical scenarios.

Problem-Solving Skills: By solving exercises and problems related to cylinders, cones, and spheres, students develop critical thinking and problem-solving skills. They learn how to approach complex geometry problems and apply the appropriate formulas to find solutions.

Preparation for Higher Studies: The concepts covered in this chapter are fundamental and form the basis for more advanced topics in geometry and mathematics. Mastering these concepts in Class 10 lays a solid groundwork for future studies in fields requiring geometric understanding.

Enhanced Mathematical Proficiency: Working through the exercises and understanding the theoretical concepts improves overall mathematical proficiency. It builds confidence in handling mathematical problems and equations related to three-dimensional shapes.