CBSE Class 9 Maths Notes Chapter 7: In CBSE Class 9 Maths, Chapter 7 is all about triangles. Triangles are basic shapes with three sides, and this chapter teaches us about their different types and properties. We learn about triangles with equal sides, called equilateral triangles, and those with two equal sides, known as isosceles triangles.

The chapter also covers the Pythagorean theorem, which helps us find the sides of right-angled triangles. We learn about triangle congruence and similarity, which are important for comparing and analyzing different triangles. By studying this chapter, we can understand triangles better and solve problems involving them more easily.

CBSE Class 9 Maths Notes Chapter 7 Triangles Overview

The Class 9 Maths Notes for Chapter 7, "Triangles," are made by subject experts of Physics Wallah. They're designed to make learning easy and clear for students. This chapter teaches about triangles, explaining their properties and features step by step. It covers everything from the basic definition of triangles to more advanced ideas like the Pythagoras' theorem.

With these notes, students can understand different types of triangles and learn how to solve problems using theorems. It's a great resource to help students improve their math skills and succeed in their studies.

CBSE Class 9 Maths Notes Chapter 7 PDF

You can access the CBSE Class 9 Maths Notes for Chapter 7 "Triangles" in PDF format using the provided link.

These notes provide comprehensive explanations and examples to help you grasp the concepts of triangles effectively.

CBSE Class 9 Maths Notes Chapter 7 PDF

CBSE Class 9 Maths Notes Chapter 7 Triangles

Inyroduction

Understanding the concepts covered in the "Triangles" chapter of Class 9 Mathematics is crucial as they form the foundation for higher education levels. Students are encouraged to grasp these concepts thoroughly as they play a significant role in various mathematical applications. The chapter aims to teach students the following key concepts:

1. Triangle congruence: Understanding when two triangles are congruent, meaning they have the same shape and size.

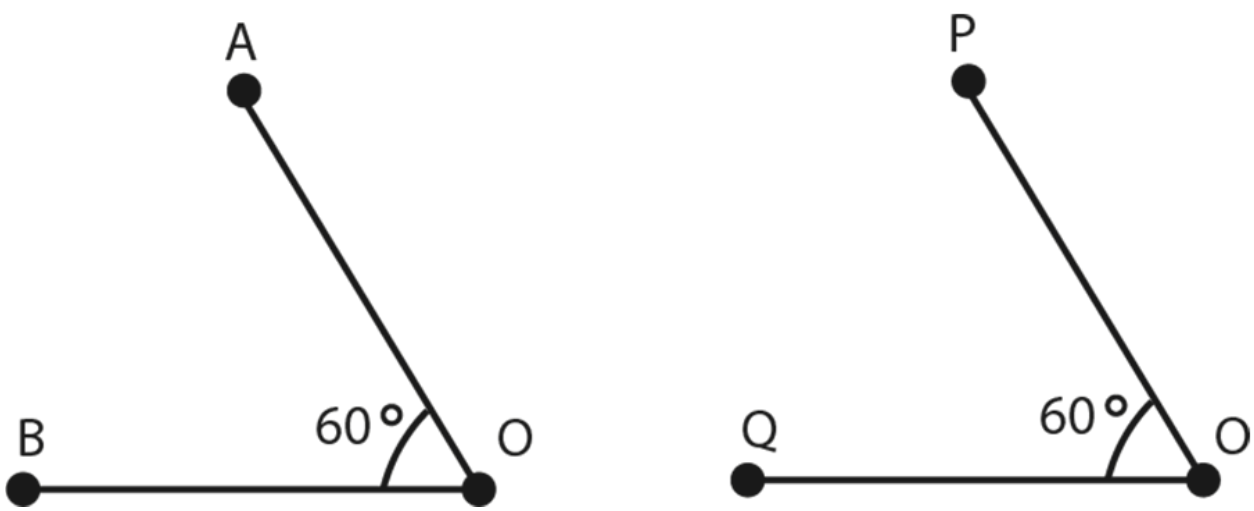
2. Congruence rules: Exploring different criteria for triangle congruence, such as Side-Angle-Side (SAS), Angle-Side-Angle (ASA), Side-Side-Side (SSS), and Right Angle-Hypotenuse-Side (RHS).
3. Triangle properties: Learning about the properties of triangles, including angles, sides, and interior angles' sum.
4. Triangle inequalities: Understanding the conditions that determine whether a set of given side lengths can form a triangle or not.

Congruent Triangles

Congruent geometric figures share the same shape and size. To verify if two plane figures are congruent, one can superimpose them and compare their alignment. For instance, if the shaded portion of one figure aligns perfectly with the unshaded portion of another, they are considered congruent.

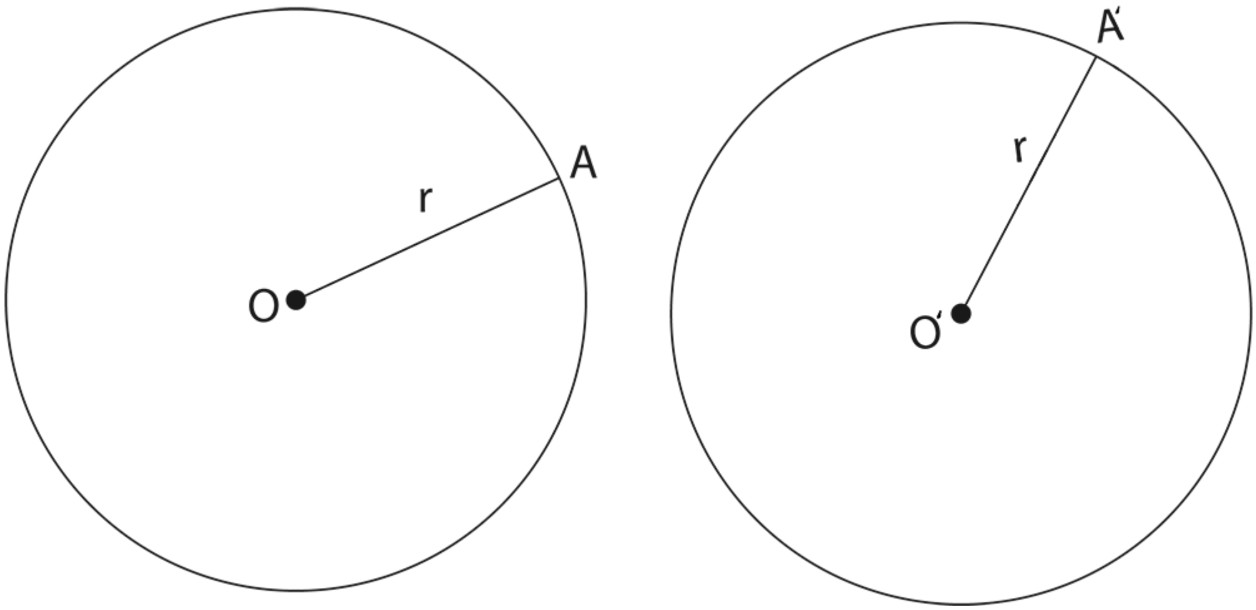
When comparing two line segments or angles, if they have the same length or measure respectively, they are congruent. This is denoted as AB ≅ CD for line segments and ∠AOB ≅ ∠POQ for angles, where AOB and POQ are angle names.

For example, in Figure line segments AB and CD are congruent because they have the same length.



In Figure angles a and b, as well as angles m and n, are congruent because they share the same measure.Angles m and n are considered congruent as they are vertically opposite angles.

Congruent Circles



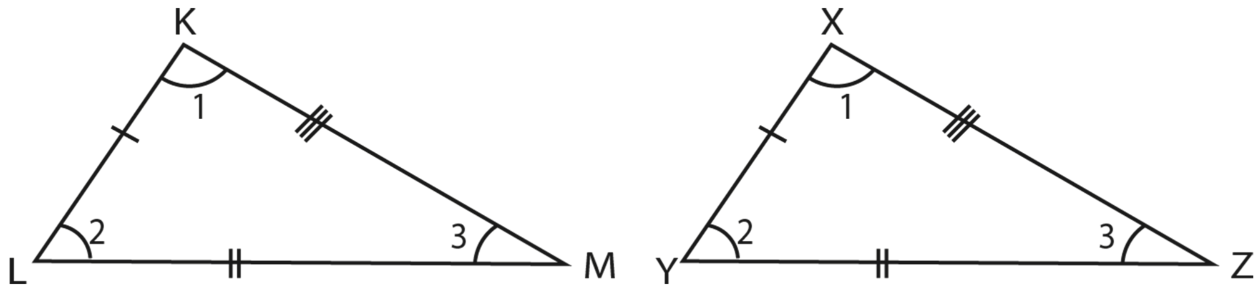
Two circles are said to be congruent if they have the same radius.

In the above figure, both the circles have the same radius

OA = O ′ A ′ = rOA = O ′ A ′ = r

Congruent Triangles

Congruent triangles are those with identical shapes and sizes. While the angles determine the shape, the sides determine the size of a triangle. Therefore, the congruency of triangles relies on both angles and sides.



Consider triangles KLM and XYZ. When you align one on top of the other, they match perfectly, indicating congruency. Both triangles have congruent angles and sides: KL ≅ XY, LM ≅ YZ, and KM ≅ XZ.

For two triangles to be congruent, all three angles and three sides of one triangle must match the corresponding angles and sides of the other triangle.

Triangles' Postulates of Congruency

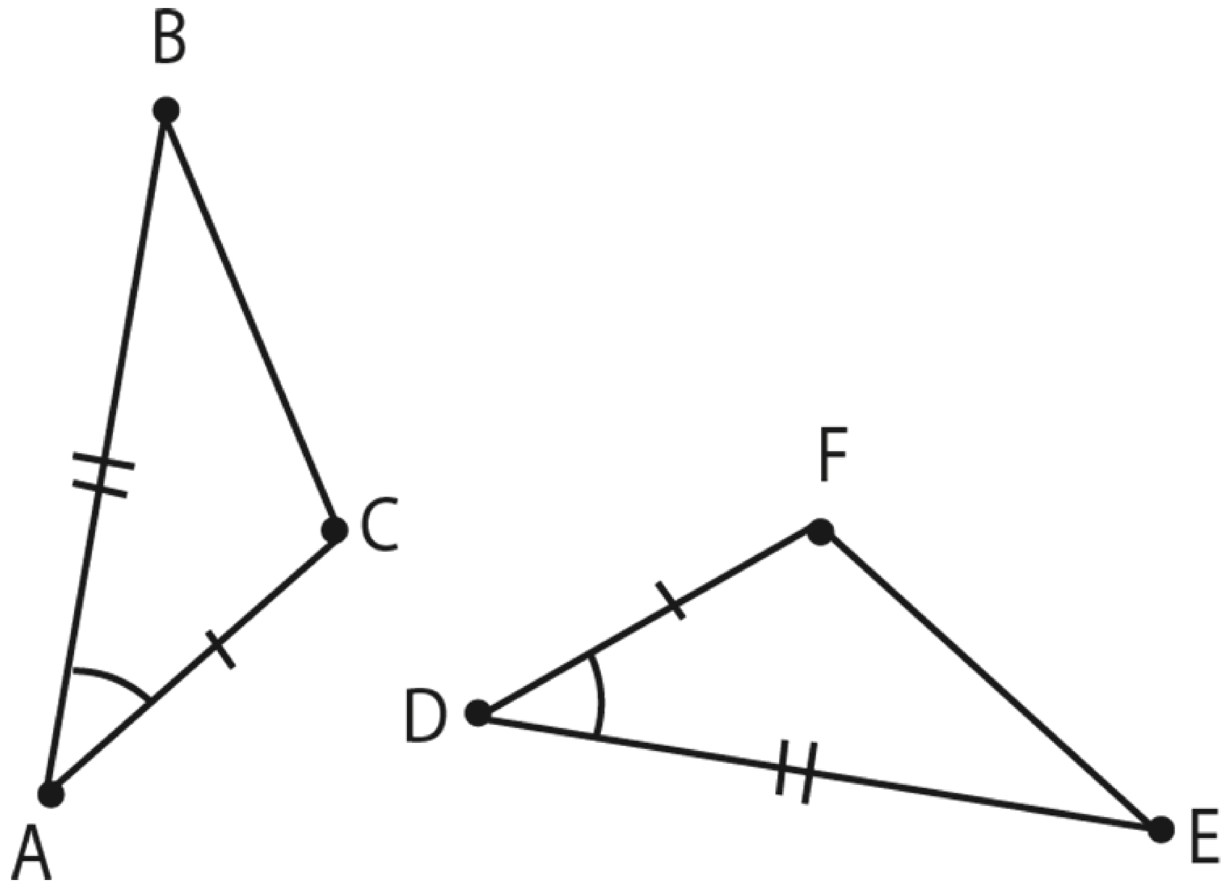
The definition of congruent triangles states that two triangles are congruent if and only if all their sides and angles match. This means that if all the sides and angles of one triangle are equal to the corresponding sides and angles of another triangle, then the triangles are congruent.

Sufficient Condition for Triangle Congruence

In the activity comparing triangles ΔABC and ΔDEF, we observed that when two sides and the included angle of one triangle match those of another, the triangles are congruent. This principle is known as the Side-Angle-Side (SAS) congruency condition. It simplifies the process of determining triangle congruence by requiring only three conditions to be met instead of all six.

Theorem 9

"Two triangles are congruent if any two sides and the included angle of one triangle are equal to the corresponding sides and the included angle of the other triangle".



In the two given triangles, ΔABCΔABC and ΔDEFΔDEF, AB = DE AB = DE

AC = DFAC = DF and

∠BAC = ∠EDF ∠BAC = ∠EDF .

To prove: ΔABC ≅ ΔDEFΔABC ≅ ΔDEF

Proof:

If we rotate and drag ΔABCΔABC on ΔDEFΔDEF, such the vertices B falls on the vertex of the other triangle E and place BC along EF, we will find that, since AB = DE AB = DE , C falls on F.

Also, ∠B = ∠E∠B = ∠E.

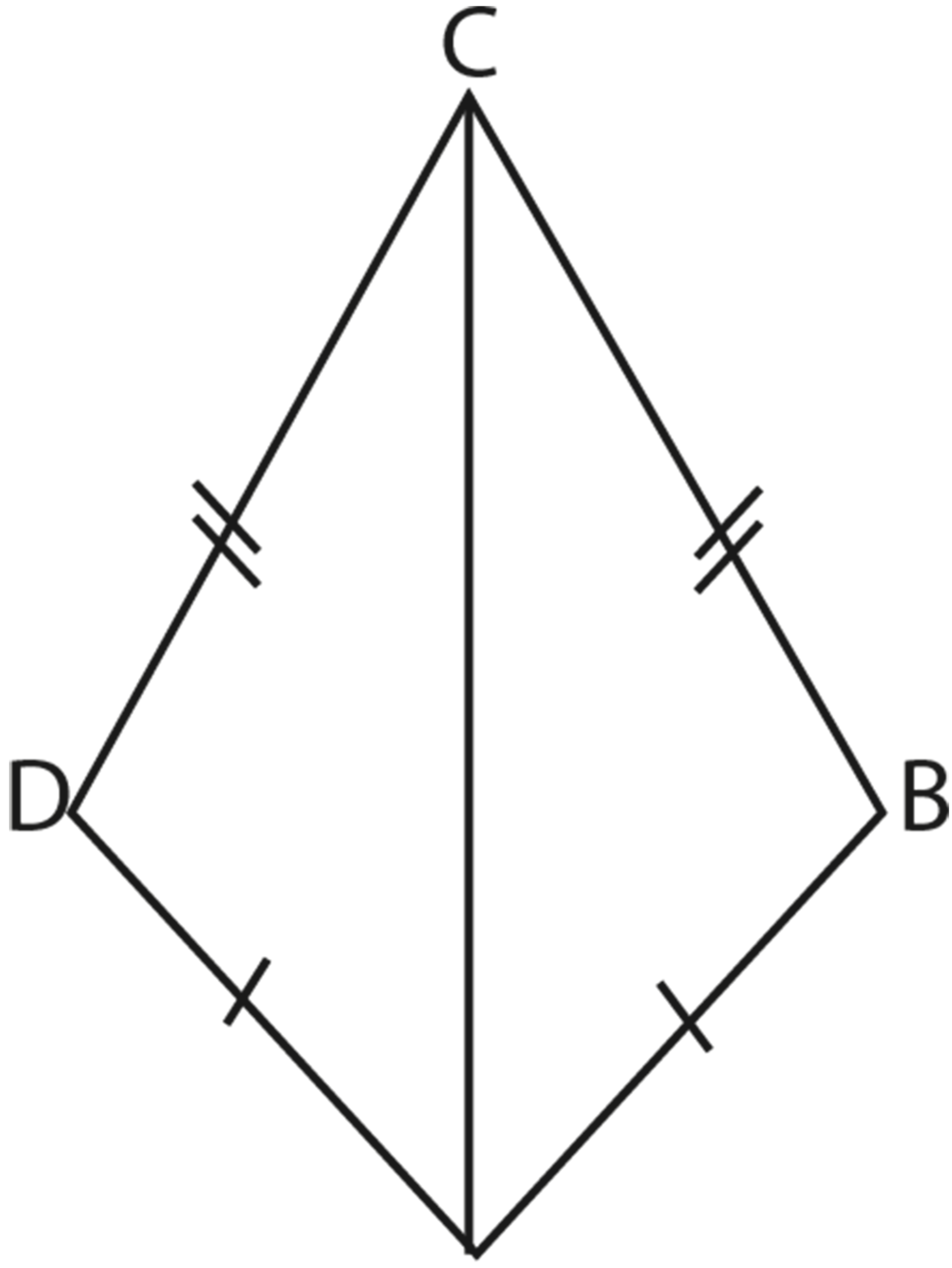
AB falls on DE, A will coincide with the vertex D and C with F.

So, AC coincides with DF.

∴ΔABC∴ΔABC coincides with ΔDEFΔDEF.

∴ΔABC ≅ ΔDEF

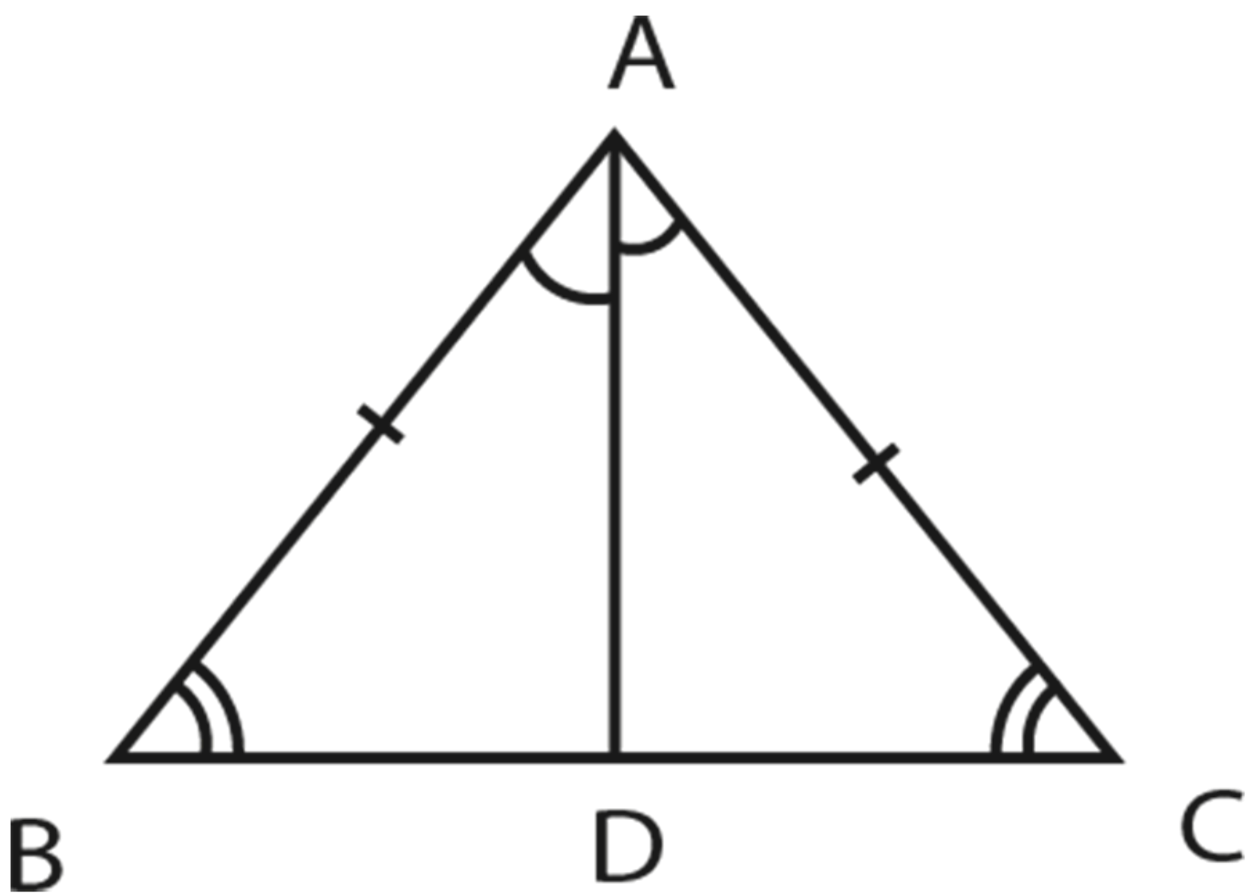
Application of SAS Congruency


C
D
B
A

In this figure we cannot apply SAS as the given data is not sufficient. On the basis of angles and sides, the SAS congruency criterion defines the relationship between two triangles. As a result, here we cannot say if AC is the angle bisector of $\angle A \angle A$, hence we cannot determine if the two triangles that may be shown are congruent.

Theorem 10

"Angles opposite to equal sides are equal".



In $\Delta ABC \Delta ABC$, $AB = AC AB = AC$

To prove:

$\angle ABC = \angle ACB \angle ABC = \angle ACB$

Construction: Draw the angle bisector of A, AD.

Proof:

Comparing both the triangles,

Given that $AB = ACAB = AC$

AD is the common side.

$\angle BAD = \angle DAC \angle BAD = \angle DAC$ (as AD is the angle bisector)

$\therefore \Delta BAD = \Delta DAC \therefore \Delta BAD = \Delta DAC$ (by SAS congruency rule)

$\therefore \angle ABD = \angle ACD \therefore \angle ABD = \angle ACD$ (by CPCT)

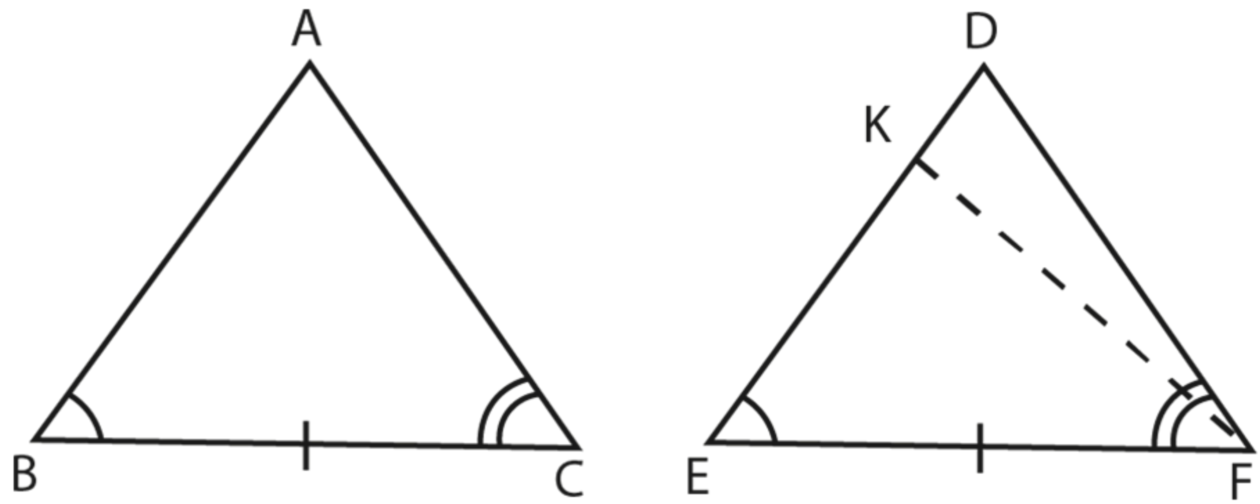
$\therefore \angle B = \angle C \therefore \angle B = \angle C$

Hence, proved.

ASA Congruence Condition

The Angle-Side-Angle (ASA) congruence condition states that two triangles are congruent if two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the other triangle. This condition simplifies the process of determining triangle congruence by requiring the equality of two angles and the side between them, thereby ensuring that the triangles have the same shape and size.

Theorem 11



Given:

In ΔABCΔABC and ΔDEFΔDEF,

∠B = ∠E∠B = ∠E and ∠C = ∠F∠C = ∠F

BC = EFBC = EF.

To prove:

ΔABC ≅ ΔDEFΔABC ≅ ΔDEF

Proof:

There are three possibilities

Case I:

AB = DEAB = DE

Case II:

AB DEAB DE

Case III:

AB DEAB DE

Case I: In addition to data, if

$AB = DE AB = DE$

then

$\Delta ABC \cong \Delta DEF \Delta ABC \cong \Delta DEF$ (by SAS congruence postulate)

Case II: If

$AB \quad DE AB \quad DE$

and let K is any point on DE such that

$EK = AB EK = AB$

.

Join KF.

Now compare triangles ABC and KCF.

$BC = EF BC = EF$

(given)

$\angle B = \angle E \angle B = \angle E$(given)

Let

$AB = EK AB = EK$
$\Delta ABC \cong \Delta KEF \Delta ABC \cong \Delta KEF$

(SAS criterion)

Hence,

$\angle ABC = \angle KEF \angle ABC = \angle KEF$

But,

$\angle ABC = \angle DEF \angle ABC = \angle DEF$

(given)

Hence, K coincides with D.

Therefore, AB must be equal to DE.

Case III: If

AB DEAB DE

, then a similar argument applies.

AB must be equal to DE.

Hence the only possibility is that AB must be equal to DE and from SAS congruence condition

ΔABC ≅ ΔDEFΔABC ≅ ΔDEF

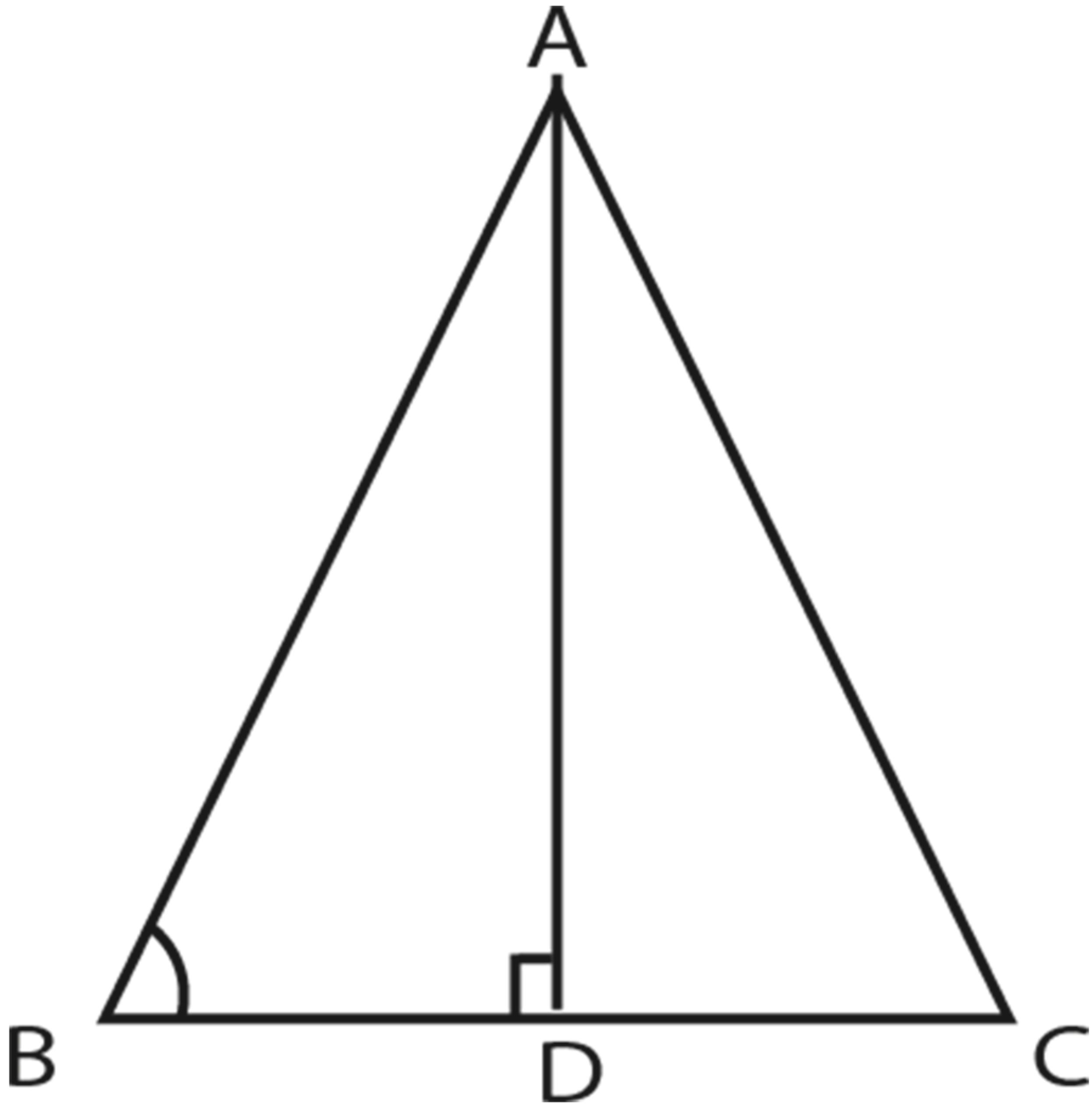
Hence the theorem is proved.

Theorem 12

"In a triangle the sides opposite to equal angles are equal".

This theorem can also be stated as

"The sides opposite to equal angles of a triangle are equal".



Given:

In

ΔABC, ∠B = ∠CΔABC, ∠B = ∠C

To prove:

$\overline{AB} = \overline{AC}$ $\overline{AB} = \overline{AC}$

Construction:

Draw $\overline{AD} \perp \overline{BC}$ $\overline{AD} \perp \overline{BC}$

Proof:

Construct two right angle triangles, ADB and ADC, right angled at D.

Here,

$\Delta ABC,\ \angle B = \angle C$ $\Delta ABC,\ \angle B = \angle C$

$\angle ADB = \angle ADC = 90^\circ$ $\angle ADB = \angle ADC = 90^\circ$ (from the construction)

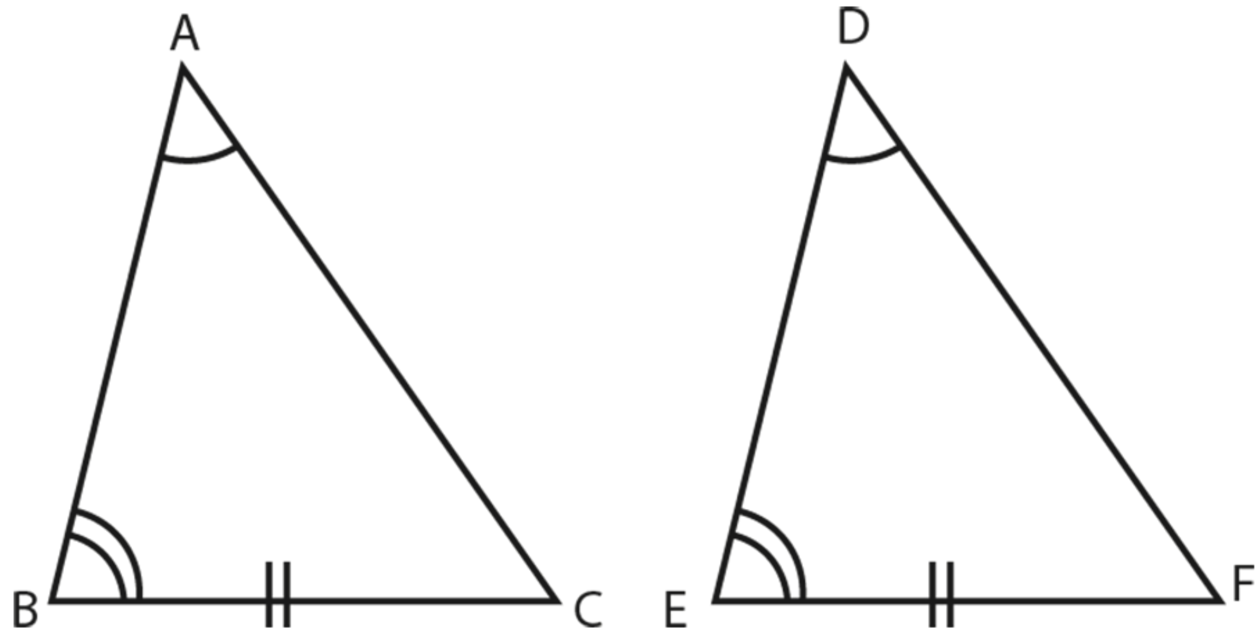
$\overline{AD}$ $\overline{AD}$ is common for both the triangles.

$\therefore \Delta ADB \cong \Delta ADC$ $\therefore \Delta ADB \cong \Delta ADC$ (by ASA postulate)

$\overline{AB} = \overline{AC}$ $\overline{AB} = \overline{AC}$ (corresponding sides)

AAS Congruence Condition

The Angle-Angle-Side (AAS) congruence condition states that two triangles are congruent if two angles and a non-included side of one triangle are equal to the corresponding two angles and the side of the other triangle. This condition provides another method for determining triangle congruence by requiring the equality of two angles and a side not included between them, ensuring that the triangles have the same shape and size.



Given:

In triangles ABC and DEF,

$BC = EFBC = EF$

(non-included sides)

$\angle B = \angle E\angle B = \angle E$
$\angle A = \angle D\angle A = \angle D$

To prove:

$\Delta ABC \cong \Delta DEF\Delta ABC \cong \Delta DEF$

Proof:

$\angle B = \angle E\angle B = \angle E$

(given)

$\angle A = \angle D\angle A = \angle D$

(given)

Now, adding both,

$\angle A + \angle B = \angle E + \angle D$ $\angle A + \angle B = \angle E + \angle D$

…(1)

Since,

$\angle A + \angle B + \angle C = \angle E + \angle D + \angle F = 180^\circ$ $\angle A + \angle B + \angle C = \angle E + \angle D + \angle F = 180^\circ$

, considering (1) we can say that,

$\angle C = \angle F$ $\angle C = \angle F$

…(2)

Now in triangle ABC and DEF,

$\angle B = \angle E$ $\angle B = \angle E$

(given)

$\angle C = \angle F$ $\angle C = \angle F$

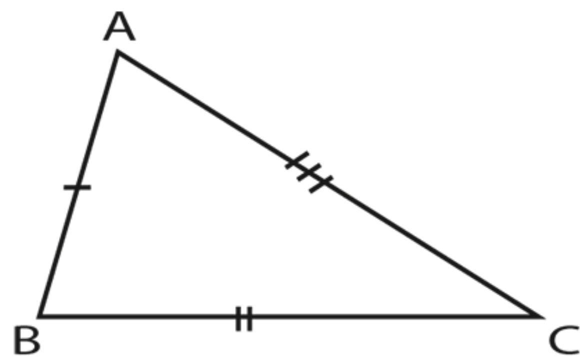
(proved in (2))

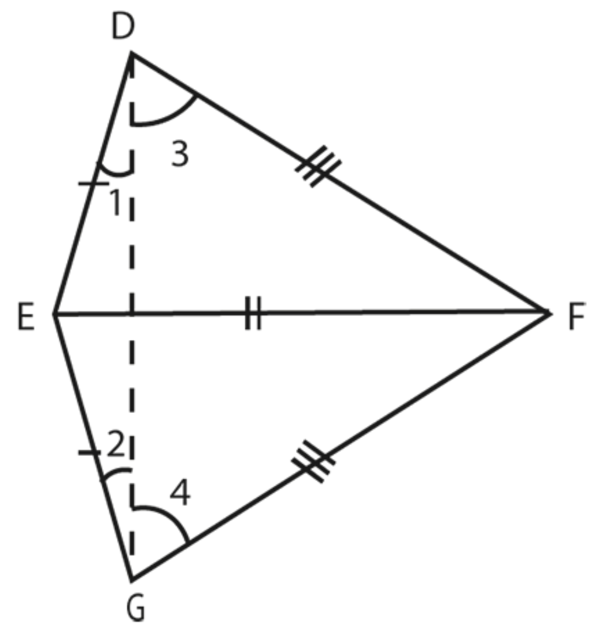
$BC = EF$ $BC = EF$ (given)

$\Delta ABC \cong \Delta DEF$ $\Delta ABC \cong \Delta DEF$ (by SAS congruency)

SSS Congruence Condition

"Two triangles are congruent if the three sides of one triangle are equal to the corresponding three sides of the other triangle".





Given:

In triangles ABC and DEF,

AB = DEAB = DE
BC = EFBC = EF
AC = DFAC = DF

Note:

Let BC and EF be the longest sides of triangles ABC and DEF respectively.

To prove:

ΔABC ≅ ΔDEFΔABC ≅ ΔDEF

Construction: If BC is the longest side, draw EG such that EG = ABEG = AB and ∠GEF = ∠ABC∠GEF = ∠ABC.

Join GF and DG.

Proof:

In triangles ABC and GEF,

AB = GEAB = GE

(by construction)

$BC = EF$BC = EF

(given)

$\angle ABC = \angle GEF$∠ABC = ∠GEF (by construction)

$\therefore \Delta ABC = \Delta GEF$∴ΔABC = ΔGEF (SAS congruence condition)

$\angle BAC = \angle EGF$∠BAC = ∠EGF (by CPCT)

and $AC = GF$AC = GF (by construction)

But $AB = DE$AB = DE (given)

$\therefore DE = GE$∴DE = GE

Similarly, $DF = GF$DF = GF

In ΔEDGΔEDG,

$DE = GE$DE = GE (Proved)

$\therefore \angle 1 = \angle 2$∴∠1=∠2…(1) (angles opposite equal sides)

In ΔEGFΔEGF,

$DF = GF$DF = GF (Proved)

$\therefore \angle 3 = \angle 4$∴∠3=∠4…(2) (angles opposite equal sides)

By adding (1) and (2), we get,

$\therefore \angle 1 + \angle 3 = \angle 4 + \angle 2$∴∠1+∠3=∠4+∠2

$\angle EDF = \angle EGF$∠EDF = ∠EGF but we have proved that $\angle BAC = \angle EGF$∠BAC = ∠EGF

Therefore, $\angle EDF = \angle BAC$ ∠EDF = ∠BAC

Now in triangles ABC and DEF,

$AB = DE$AB = DE

(given)

AC = DFAC = DF

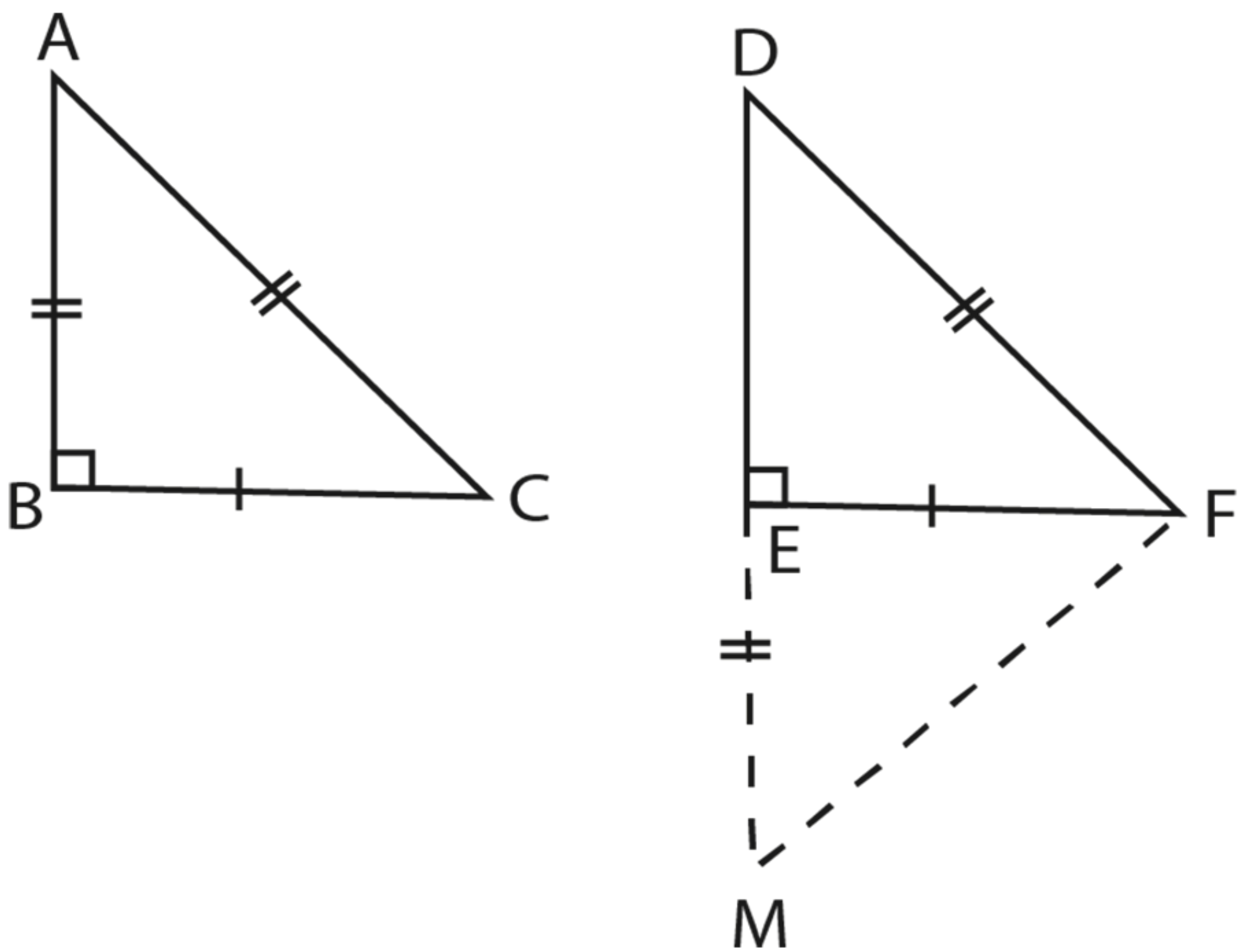
(given)

∠EDF = ∠BAC ∠EDF = ∠BAC (proved)

ΔABC ≅ ΔDEFΔABC ≅ ΔDEF (by SAS congruency)

Theorem of RHS (Right Angle Hypotenuse Side) Congruence

If the hypotenuse and one side of one triangle are equal to the hypotenuse and corresponding side of the other triangle, the two triangles are congruent.



Given:

ABC and DEF are two right-angled triangles such that

1. $\angle B = \angle E = 90^\circ$
2. Hypotenuse $\overline{AC} = $ Hypotenuse $\overline{DF}$ and
3. Side $\overline{BC} = $ Side $\overline{EF}$

To prove:

$\Delta ABC \cong \Delta DEF$

Construction

Produce DE to M so that

$EM = AB$

. Join MF.

Proof:

In triangles ABC and MEF,

$EM = AB$

(construction)

$BC = EF$

(given)

$\angle ABC = \angle MEF = 90^\circ$

Therefore, $\Delta ABC \cong \Delta MEF$ (SAS congruency)

Hence, $\angle A = \angle M$…(1) (by CPCT)

$\overline{AC} = \overline{MF}$…(2) (by CPCT)

Also, $AC = DF$ … (3) (given)

From (1) and (3), $DF = MF$

Therefore, $\angle D = \angle M$...(4) (Angles opposite to equal sides of

ΔDFM

)

From (2) and (4),

$\angle A = \angle D \angle A = \angle D$...(5)

Now compare triangles ABC and DEF,

$\angle A = \angle D \angle A = \angle D$ (from 5)

$\angle B = \angle E = 90^\circ \angle B = \angle E = 90^\circ$ (given)

$\therefore \angle C = \angle F \therefore \angle C = \angle F$...(6)

Compare triangles ABC and DEF,

$\overline{BC} = \overline{EF} \overline{BC} = \overline{EF}$ (given)

$\overline{AC} = \overline{DF} \overline{AC} = \overline{DF}$ (given)

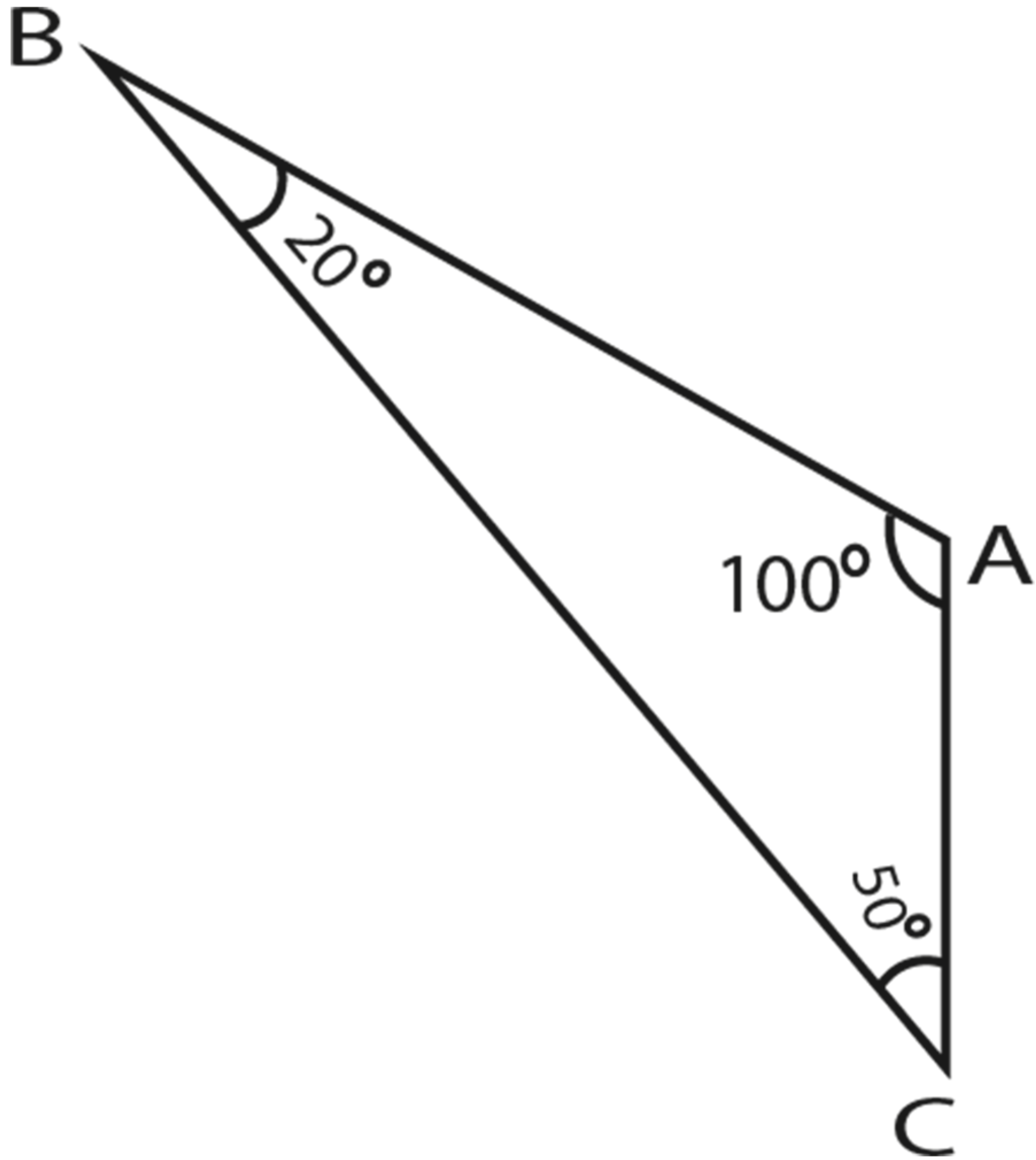
$\angle C = \angle F \angle C = \angle F$ (from 6)

Therefore, $\Delta ABC \cong \Delta DEF \Delta ABC \cong \Delta DEF$ (by SAS congruency)

Inequality of Angles

Here we can see that, both these angles are not equal, as $135^\circ < 160^\circ 135^\circ < 160^\circ$

Inequality in a Triangle



Construct a triangle ABC as shown in the figure.

Observe that in triangle ABC,
AC‾‾‾‾‾‾‾‾AC‾ is the smallest side (2 cm)

B is the angle opposite to $\overline{AC}$ and $\angle B = 20^\circ$

$\overline{BC}$ is the greatest side (6 cm)

A is the angle opposite to $\overline{BC}$ and $\angle A = 100^\circ$

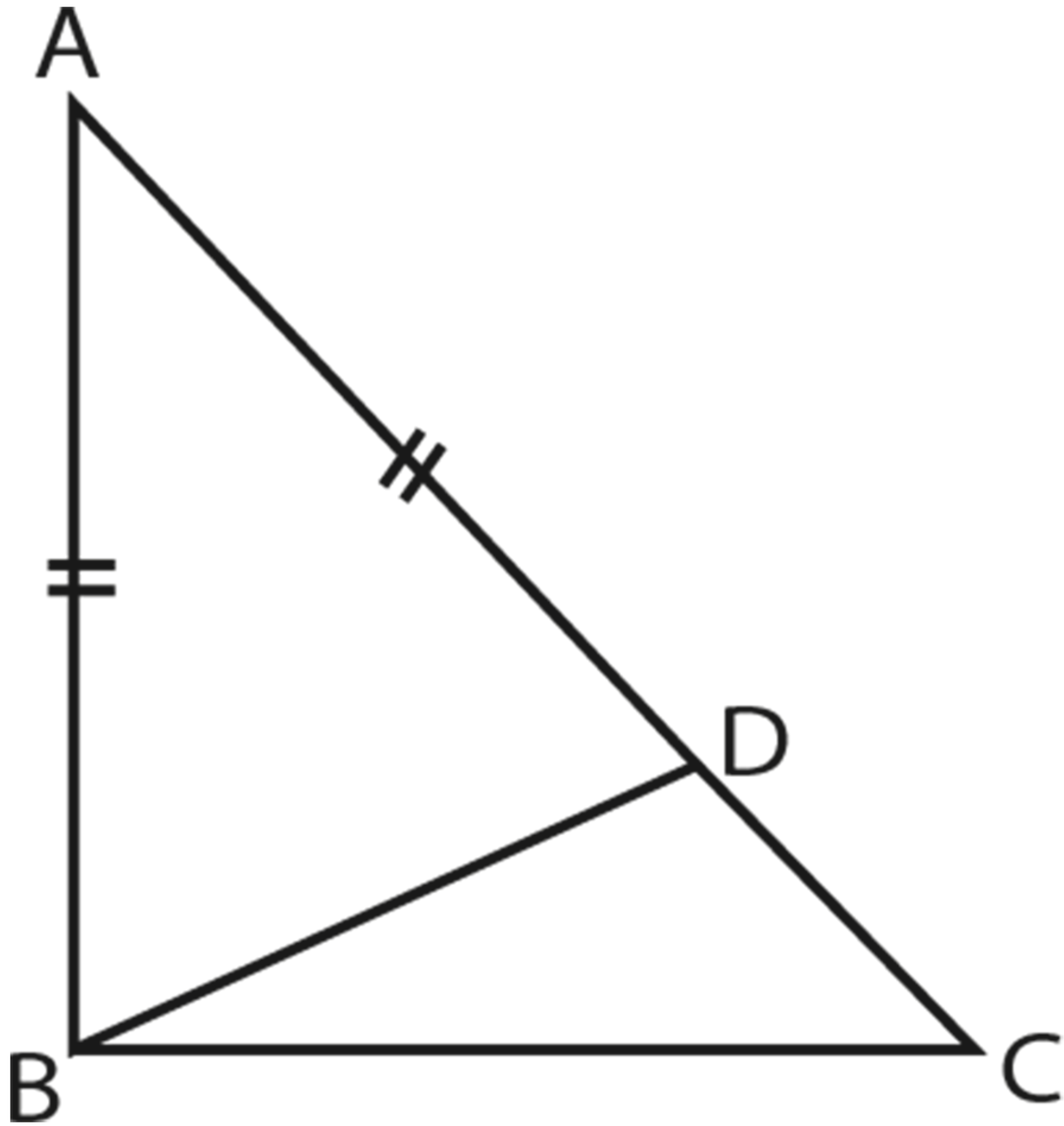
From the measurements made above of side and angle opposite to it, we can write the relation in the form of a statement.

"If two sides of a triangle are unequal then the longer side has the greater angle opposite to it”.

Theorem on Inequalities

Theorem 1:

If two sides of a triangle are unequal, the longer side has the greater angle opposite to it.



Read the statement and draw a triangle as per data.

Draw

ΔABCΔABC

, such that

AC ABAC AB

.

Data:

AC ABAC AB

To Prove:

$\angle ABC = \angle ACB \angle ABC = \angle ACB$

Construction:

Take a point D on $\overline{AC}$ $\overline{AC}$ such that $\overline{AB} = \overline{AD}$ $\overline{AB} = \overline{AD}$. Join B to D.

Proof:

In $\Delta ABC \Delta ABC$, $\overline{AB} = \overline{AD}$ $\overline{AB} = \overline{AD}$ (by construction)

$\therefore \angle ABD = \angle ADB \therefore \angle ABD = \angle ADB$…(1)

but $\angle ADB \angle ADB$ is the exterior angle with reference to $\Delta DBC \Delta DBC$.

$\angle ADB \;\; \angle DCB \angle ADB \;\; \angle DCB$ … (2)

From relation (1) and (2) we can write $\angle ABD \;\; \angle DCB \angle ABD \;\; \angle DCB$

But $\angle ABD \angle ABD$ is a part of $\angle ACB \angle ACB$.

$\therefore \angle ACB \;\; \angle DCB \therefore \angle ACB \;\; \angle DCB$ or $\therefore \angle ABC \;\; \angle ACB \therefore \angle ABC \;\; \angle ACB$

Hence, proved.

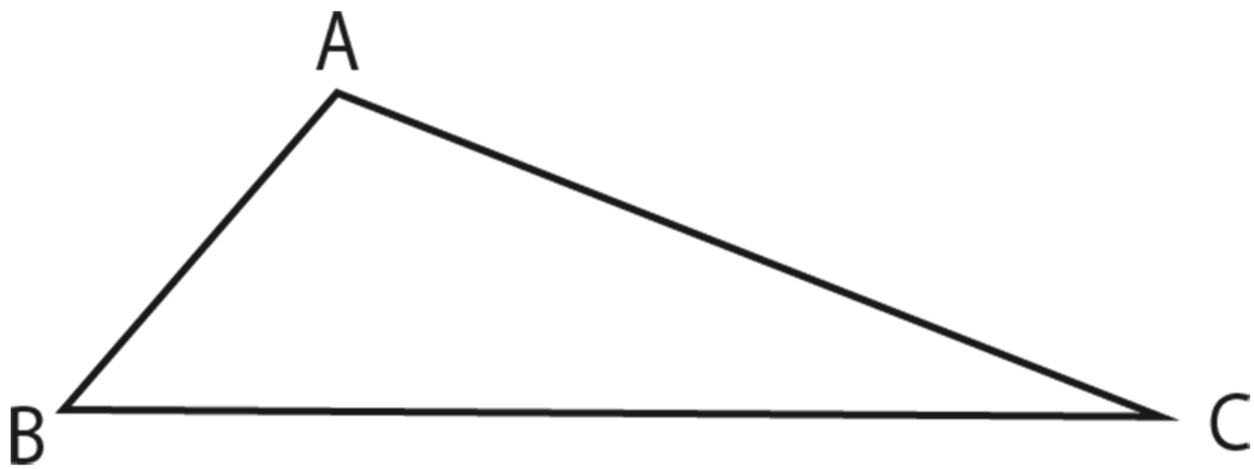
Angle Side Relation

Theorem 2:

In a triangle, if two angles are unequal, the side opposite to greater angle is longer than the side opposite to the smaller angle.

Theorem 3

In a triangle, the greater angle has the longer side opposite to it.



Given:

In ΔABCΔABC, ∠ABC ∠ACB∠ABC ∠ACB

To prove:

AC ABAC AB

Proof:

In

ΔABCΔABC

, AB and AC are two line segments. So the following are the three possibilities of which exactly one must be true.

1. either
   AB = ACAB = AC
   , then ∠B = ∠C∠B = ∠C which is contrary to the hypothesis.

∴AB ≠ AC∴AB ≠ AC

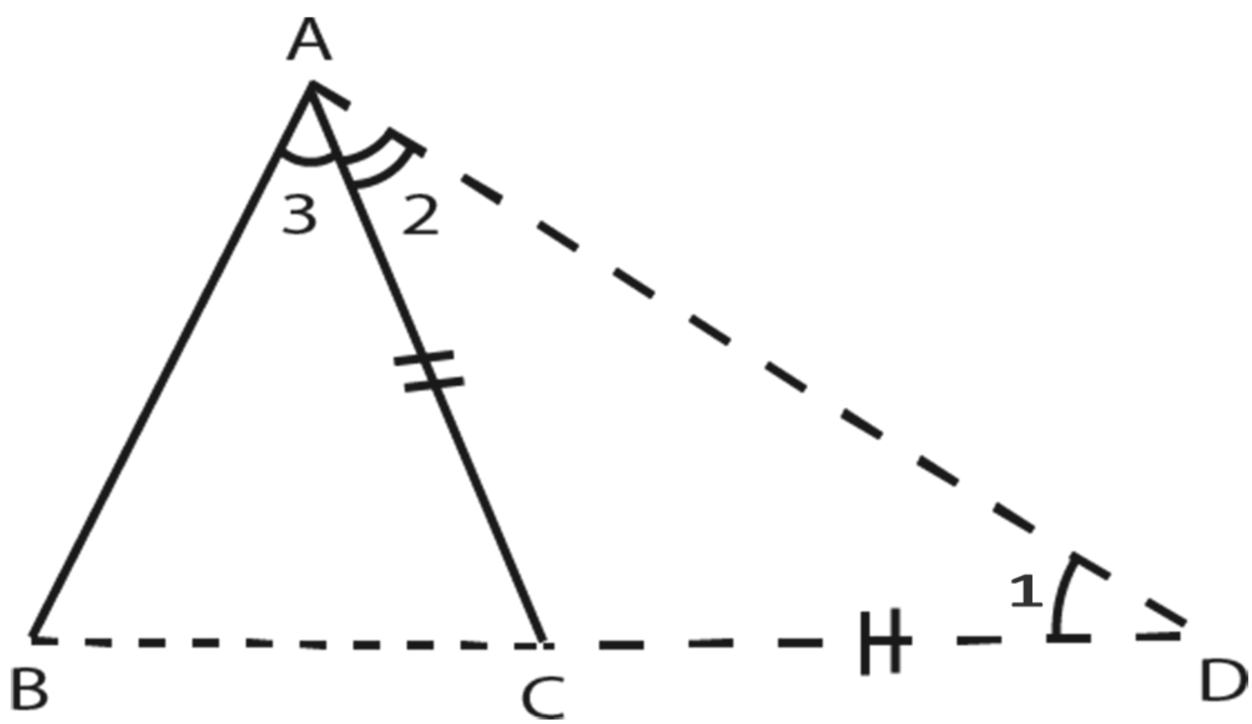
2. AB ACAB AC
   , then ∠B ∠C∠B ∠C which is contrary to the hypothesis.
3. AB ACAB AC
   , this is the only condition we are left with, so

AB ACAB AC
must be true.

Hence, proved.

Theorem 4

Prove that in any triangle the sum of the lengths of any two sides of a triangle is greater than the length of its third side.
Draw Δ ABC Δ ABC.



Data:

ABC is a triangle.

To prove:

AB¯¯¯¯¯¯¯¯+AC¯¯¯¯¯¯¯¯>BC¯¯¯¯¯¯¯¯AB¯+AC¯>BC¯

AB¯¯¯¯¯¯¯¯+BC¯¯¯¯¯¯¯¯>AC¯¯¯¯¯¯¯¯AB¯+BC¯>AC¯

$\overline{AC}+\overline{BC}>\overline{BA}$ $\overline{AC}+\overline{BC}>\overline{BA}$

Construction:

Produce $\overline{BC}$ $\overline{BC}$ to D such that

$\overline{AC} = \overline{CD}$ $\overline{AC} = \overline{CD}$

. Join A to D.

Proof:

$\overline{AC} = \overline{CD}$ $\overline{AC} = \overline{CD}$

(by construction)

$\angle 1=\angle 2 \angle 1=\angle 2$…(1)

From the figure,

$\angle BAD = \angle 2+\angle 3 \angle BAD = \angle 2+\angle 3$ …(2)

$\angle BAD \ \angle 2 \angle BAD \ \angle 2$

$\therefore \angle BAD \ \angle 1 \therefore \angle BAD \ \angle 1$ (proved from 1)

$\overline{AB}$ $\overline{AB}$ is opposite to $\angle 1 \angle 1$ and $\overline{BD}$ $\overline{BD}$ is opposite to $\angle BAD$ $\angle BAD$.

In ΔBAD ΔBAD , $\angle 1 \ \angle 2+\angle 3 \angle 1 \ \angle 2+\angle 3$

$BD \ \ AB \ BD \ \ AB$

(side opposite to greater angle is greater)

From figure

$BD = BC + CD BD = BC + CD$
$\therefore BC + CD AB \therefore BC + CD AB$
$\therefore BC + AC AB \therefore BC + AC AB$

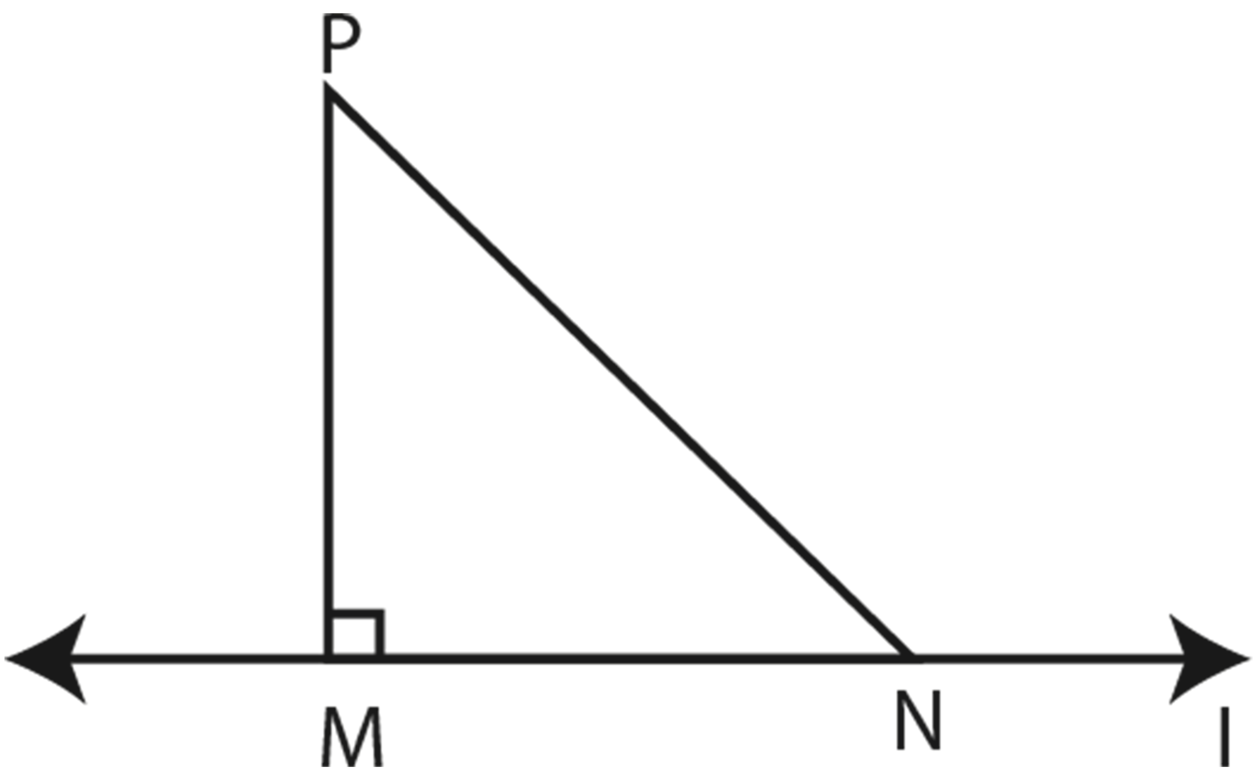
Since, $CD = AC CD = AC$, hence, sum of two sides of a triangle is greater than the third side.

Similarly, $\overline{AB}+\overline{BC}>\overline{AC}$ and $\overline{AC}+\overline{BC}>\overline{BA}$.

Hence, proved.

Theorem 5

Of all the line segments that can be drawn to a given line, from a point not lying on it, the perpendicular line segment is the shortest.



Given:

l is a line and P is a point not lying on it.

$PM \perp l$

. N is any point on l other than M.

To prove:

PM PN

Proof:

In Δ PMN Δ PMN, $\angle M \angle M$ is the right angle.

$\therefore \angle N \therefore \angle N$ is an acute angle, from angle sum property.

$\because \angle M \ \angle N \because \angle M \ \angle N$

PN PMPN PM (side opposite to greater angle)

$\therefore$PMPN$\therefore$PMPN.

Benefits of CBSE Class 9 Maths Notes Chapter 7 Triangles

- **Concept Clarity**: These notes provide clear explanations of various concepts related to triangles, making it easier for students to understand the fundamentals.
- **Structured Learning**: The notes follow a structured format, covering each topic comprehensively, ensuring that students don't miss out on any essential concepts.
- **Exam Preparation**: The notes include important formulas, theorems, and properties related to triangles, which are crucial for exam preparation. Students can use these notes for quick revision before exams.
- **Visual Aid:** Diagrams and illustrations included in the notes help students visualize geometric concepts and understand them better.