CHAPTER-5
CONTINUITY AND DIFFERENTIABILITY

Cheat Sheet

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CONTINUITY AND DIFFERENTIABILITY

Algebra of continuous functions

- Let f and g be two real continuous functions at a real number c. Then
  (i) $f + g$ is continuous at $x = c$.
  (ii) $f - g$ is continuous at $x = c$.
  (iii) $f \cdot g$ is continuous at $x = c$.
  (iv) $\left(\dfrac{f}{g}\right)$ is continuous at $x = c$, (provided $g(c) \neq 0$).
- If f is constant function i.e $f(x) = \lambda$ then the function $(\lambda \cdot g)$ defined by $(\lambda \cdot g)(x) = \lambda \cdot g(x)$ is also continuous.
- If f is constant function $f(x) = \lambda$, then the function $\dfrac{\lambda}{g}$ defined by $\dfrac{\lambda}{g}(x) = \dfrac{\lambda}{g(x)}$ is also continuous wherever $g(x) \neq 0$.

Differentiability

A function $y = f(x)$ is said to be differentiable at a point a, if at $x = a$ left hand derivative $f'(a^-)$ and right hand derivative $f'(a^+)$ both exist finitely and are equal. There common value is called derivative of $f(x)$ at $x = a$.

Right hand derivative at $x = a$.

$$f'(a^+) \equiv \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}, \ (h > 0)$$

Left hand derivative at $x = a$:

$$f'(a^-) \equiv \lim_{h\to 0} \frac{f(a-h) - f(a)}{-h}, \ (h > 0)$$

Thus $f(x)$ is differentiable at $x = a$ if $f'(a^-) = f'(a^+)$ at some fixed finite quantity.

Derivatives of inverse trigonometric functions

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}, \text{ for } -1 < x < 1$$

$$\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}, \text{ for } -1 < x < 1$$

$$\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}, \text{ for } |x| > 1$$

$$\frac{d}{dx}\text{cosec}^{-1} x = \frac{-1}{|x|\sqrt{x^2-1}}, \text{ for } |x| > 1$$

$$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}, \text{ for } x \in R$$

$$\frac{d}{dx}\cot^{-1} x = \frac{-1}{1+x^2}, \text{ for } x \in R$$

Continuity of a Function at a Point

A function $f(x)$ is said to be continuous at a point $x = a$ of its domain if and only if it satisfies the following three conditions:

$f(a)$ exists. ('a' lies in the domain of f)

$\lim_{x\to a} f(x)$ exist i.e. $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$

$\lim_{x\to a} f(x) = f(a)$

Algebra of Derivatives

If u, v are functions of x, then

$$\frac{d(u \pm v)}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx} \quad \text{(Product Rule)}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2} \quad \text{(Quotient Rule)}$$

Derivatives of composite functions

Given a composite function $y = f(x)$, i.e. a function represented by $y = f(u)$, $u = \phi(x)$ or $y = f[\phi(x)]$, then $y' = \dfrac{dy}{dx} = \dfrac{df}{du} \cdot \dfrac{du}{dx}$.

This is called the chain rule.

Derivatives of Functions in Parametric Forms

In order to find derivatives of function in such a form, we use chain rule.

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$$\frac{dy}{dx} = \frac{g'(t)}{f'(t)} \quad \left(as \ \frac{dy}{dt} = g'(t) \text{ and } \frac{dx}{dt} = f'(t)\right)$$

[Provided $f'(t) \neq 0$]

Derivatives of implicit functions

If a function is written in the form $f(x, y) = 0$, known as implicit form.

Working rule:

- Differentiate each term of $f(x, y) = 0$ with respect to x.
- Collect the terms containing $\dfrac{dy}{dx}$ on one side and the terms not involving dy/dx on the other side.
- Express dy/dx as a function of x or y or both.

Second Order Derivative

Let $y = f(x)$. Then $\dfrac{dy}{dx} = f'(x)$

Again on differentiating w.r.t x. Then, the left hand side becomes $\dfrac{d}{dx}\left(\dfrac{dy}{dx}\right)$ which is called the second order derivative of y w.r.t. x and denoted by $\dfrac{d^2y}{dx^2}$.

The second order derivative of $f(x)$ is denoted by $f''(x)$.

Logarithmic Differentiation

If differentiation of an expression or an equation is done after taking log on both sides, then it is called logarithmic differentiation.

If $y = [f(x)]^{g(x)}$ where $f(x)$ and $g(x)$ are functions of x. To find the derivative of this type of functions we proceed as follows: Let $y = [f(x)]^{g(x)}$. Taking logarithm on both sides, we have $\log y = g(x)$. log $f(x)$ and then we differentiate w.r.t. x.