JEE Mains (Dropper)

Sample Paper - IV

DURATION : 180 Minutes **M. MARKS : 300**

ANSWER KEY

PHYSICS

1.	(4)
2.	(3)
3.	(1)
4.	(4)
5.	(1)
6.	(2)
7.	(1)
8.	(4)
9.	(3)
10.	(1)
11.	(3)
12.	(3)
13.	(3)
14.	(1)
15.	(3)
16.	(2)
17.	(2)
18.	(2)
19.	(3)
20.	(1)
21.	(12)
22.	(3)
23.	(0.312)
24.	(8)
25.	(500)
26.	(1.06)
27.	(31.2)
28.	(0.5)
29.	(11)
30.	(4)

CHEMISTRY

31.	(3)
32.	(3)
33.	(2)
34.	(3)
35.	(4)
36.	(1)
37.	(3)
38.	(1)
39.	(2)
40.	(4)
41.	(4)
42.	(4)
43.	(2)
44.	(3)
45.	(1)
46.	(2)
47.	(2)
48.	(4)
49.	(3)
50.	(2)
51.	(2)
52.	(40)
53.	(4)
54.	(50)
55.	(3155)
56.	(1125)
57.	(18)
58.	(316)
59.	(82)
60.	(4)

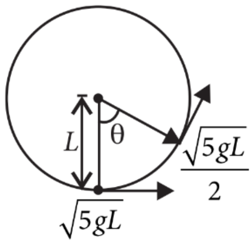
MATHEMATICS

61.	(1)
62.	(1)
63.	(4)
64.	(3)
65.	(2)
66.	(1)
67.	(2)
68.	(1)
69.	(1)
70.	(3)
71.	(2)
72.	(1)
73.	(2)
74.	(4)
75.	(3)
76.	(3)
77.	(1)
78.	(2)
79.	(3)
80.	(1)
81.	(18)
82.	(135)
83.	(2)
84.	(4)
85.	(21)
86.	(19)
87.	(56)
88.	(481)
89.	(15)
90.	(1250)

PHYSICS

1. **(4)**

According to conservation of energy,



$$\frac{1}{2}Mv^2 = \frac{1}{2}M\left(\frac{v}{2}\right)^2 + Mgh$$

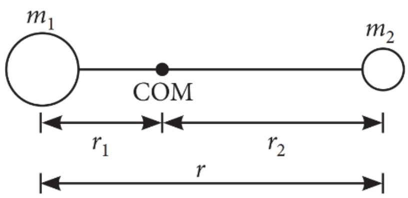
$$\frac{1}{2}M\,5gL = \frac{1}{2}M\,\frac{5gL}{4} + MgL(1-\cos\theta)$$

or $\cos\theta = -\frac{7}{8}$ or $\theta = \cos^{-1}\left(\frac{-7}{8}\right) = 151°$

$\therefore\ \frac{3\pi}{4} < \theta < \pi$

2. **(3)**

A diatomic molecule consists of two atoms of masses m_1 and m_2 at a distance r apart. Let r_1 and r_2 be the distances of the atoms from the centre of mass.



The moment of inertia of this molecule about an axis passing through its centre of mass and perpendicular to a line joining the atoms is

$I = m_1r_1^2 + m_2r_2^2$

As $m_1r_1 = m_2r_2$ or $r_1 = \frac{m_2}{m_1}r_2$

$\therefore\ r_1\frac{m_2}{m_1}(r - r_1);\ r_1 = \frac{m_2 r}{m_1 + m_2}$

Similarly, $r_2 = \frac{m_1 r}{m_1 + m_2}$

Therefore, the moment of inertia can be written as

$$I = m_1\left(\frac{m_2 r}{m_1+m_2}\right)^2 + m_2\left(\frac{m_1 r}{m_1+m_2}\right)^2$$

$$\frac{m_1m_2}{m_1+m_2}r^2 \qquad \ldots \text{(i)}$$

According to Bohr's quantisation condition

$$L = \frac{nh}{2\pi} \text{ or } L^2 = \frac{n^2h^2}{4\pi^2} \quad \ldots \text{(ii)}$$

Rotational energy, $E = \frac{L^2}{2I}$

$$E = \frac{n^2h^2}{8\pi^2 I} = \frac{n^2h^2(m_1+m_2)}{2m_1m_2r^2}$$ (Using (i) and (ii))

3. **(1)**

In pure semiconductor electron-hole pair = $7 \times 10^{15}\ \text{m}^{-3}$

Initially total charge carrier $n_{\text{initial}} = n_h + n_e$

$= 14 \times 10^{15}$

After doping donor impurity

$N_D = \frac{5\times10^{28}}{10^7} = 5\times10^{21}$ and $n_e = \frac{N_D}{2} = 2.5\times10^{21}$

So, $n_{\text{final}} = n_h + n_e$

$\Rightarrow n_{\text{final}} \approx n_e \approx 2.5 \times 10^{21}$

$(\because n_e >> n_h)$

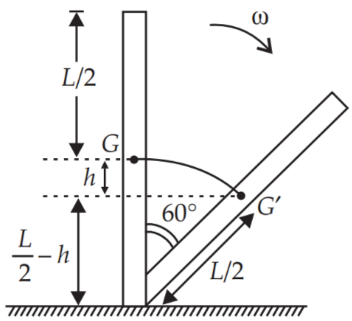
$$\text{Factor} = \frac{n_{\text{final}} - n_{\text{initial}}}{n_{\text{initial}}} = \frac{2.5\times10^{21} - 14\times10^{15}}{14\times10^{15}}$$

$$\approx \frac{2.5\times10^{21}}{14\times10^{15}} = 1.8\times10^5$$

4. **(4)**

The fall of centre of gravity h is given by

$$\frac{\left(\frac{L}{2}-h\right)}{\left(\frac{L}{2}\right)} = \cos 60° \text{ or } h = \frac{L}{2}(1 - \cos 60°)$$



$\therefore$ Decrease in potential energy

$= Mgh = Mg\frac{L}{2}(1-\cos 60°)$

Kinetic energy of rotation $= \frac{1}{2}I\omega^2$

$= \frac{1}{2}\times\frac{ML^2}{3}\omega^2$

[$I = \frac{ML^2}{3}$ (because rod is rotating about an axis passing through its one end)]

According to law of conservation of energy,

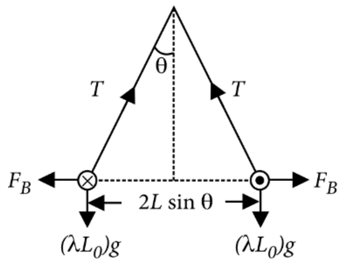
$$Mg\frac{L}{2}(1-\cos 60°) = \frac{ML^2}{6}\omega^2$$

$$\therefore\ \omega = \sqrt{\frac{6g}{L}}\sin 30° = \sqrt{\frac{6g}{L}}\left(\frac{1}{2}\right) = \sqrt{\frac{3g}{2L}}$$

5. (1)

The force per unit length between current carrying parallel wires is

$$\frac{dF}{dL} = \frac{\mu_0 I_1 I_2}{2\pi d}$$



If two wires carry current in opposite directions the magnetic force is repulsive, due to which the parallel wires have moved out so that equilibrium is reached. Figure shows free body diagram of each wire. In equilibrium,

$\Sigma F_y = 0,\ 2T\cos\theta = (\lambda L_0)g$ … (i)

$\Sigma F_z = 0,\ 2T\sin\theta = F_B$ … (ii)

Now dividing eqn. (ii) by eqn. (i) we get

$$\tan\theta = \frac{F_B}{L_0\lambda g}$$

where, the magnetic force,

$$F_B = \left(\frac{dF}{dL}\right)\times L_0 = \frac{\mu_0 I^2}{4\pi\sin\theta}\frac{L_0}{L}$$

For small θ, $\tan\theta \simeq \sin\theta \simeq \theta$

$$\therefore\ \theta = I\sqrt{\frac{\mu_0}{4\pi\lambda g L}}$$

6. (2)

The maximum value of the induced current,

$$I_{\max} = \frac{\varepsilon_{\max}}{R} = \frac{ABN\omega}{R}$$

Given, $A = \pi r^2 = 3.14(8\times10^{-2}\ m)^2$,

$B = 3\times10^{-2}$ T, $N = 20$, $\omega = 50$ rad s^{-1} and

$R = 10\ \Omega$.

$$I_{\max} = \frac{3.14(8\times10^{-2}\,m)^{-2}(3\times10^{-2}\,T)\times20(50\,rad\,s^{-1})}{10\Omega}$$

The average power loss in the form of heat,

$$P_{av} = \frac{1}{2}I_{\max}^2 R = \frac{1}{2}(0.0603A)^2(10\Omega)$$

= 0.018 W = 18 mW.

7. (1)

Slope of $v - t$ graph = Acceleration.

$$\alpha = \frac{v_0}{t_1},\ \beta = \frac{v_0}{t_2}$$

$$\therefore\ \frac{\beta}{\alpha} = \frac{t_1}{t_2}$$

Displacement = Area under $v - t$ graph

$$\therefore\ x = \frac{1}{2}t_1\times v_0 \text{ and } y = \frac{1}{2}t_2\times v_0$$

Hence, $\dfrac{x}{y} = \dfrac{t_1}{t_2} = \dfrac{\beta}{\alpha}$

8. (4)

Bulk modulus, $B = \dfrac{\text{Normal stress}}{\text{Volumetric strain}}$

$$P = \frac{N}{A} = \frac{N}{(2\pi a)b}$$

Volumetric strain = $\dfrac{2\pi a\,\Delta a\times b}{\pi a^2\times b} = \dfrac{2\Delta a}{a}$

$$\therefore\ B = \frac{N}{2\pi ab}\times\frac{a}{2\Delta a}$$

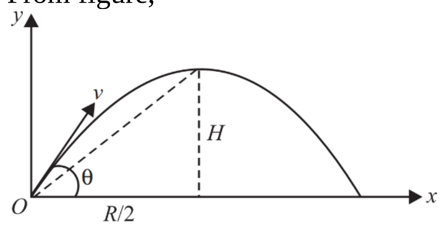
$N = 4\pi b\,\Delta a\times B$

∴ Required force = Frictional force

$= \mu N = (4\pi\mu Bb)\Delta a$

9. (3)

From figure,



Average velocity, $v_{av} = \dfrac{\sqrt{H^2 + (R^2/4)}}{T/2}$ … (i)

Here, $H = \dfrac{v^2\sin^2\theta}{2g}$;

$R = \dfrac{v^2\sin2\theta}{g}$ and $T = \dfrac{2v\sin\theta}{g}$

Putting these values in equation (i), we get,

$$v_{av} = \frac{v}{2}\sqrt{1+3\cos^2\theta}$$

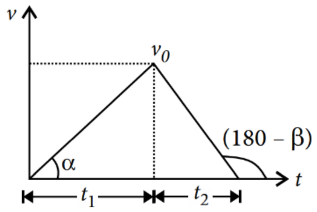
10. (1)

The current in the circuit at any instant is, $I = a + b\sin\omega t$.

Hence, the effective value, I_{eff}, of the current in the circuit will be given by

$$I_{eff}^2 = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt}\right] = \frac{1}{T}\int_0^T I^2 dt = \frac{1}{T}\int_0^T [a + b\sin\omega t]^2 dt$$

⇒



$$I_{eff}^2 = \frac{1}{T}\int_0^T [a^2 + 2ab\sin\omega t + b^2\sin\omega t]dt \ \ldots \text{(i)}$$

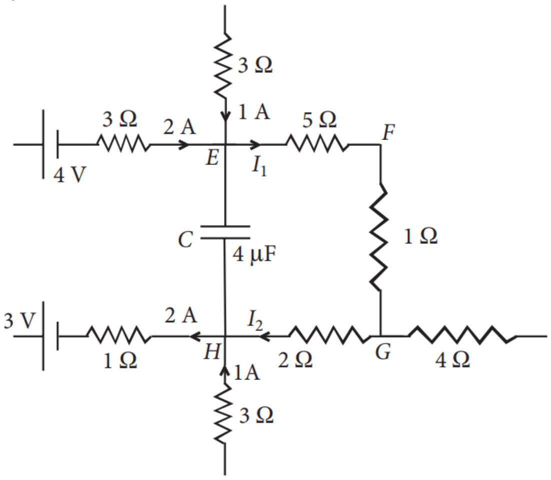
Now, we have

$$\Rightarrow\ I_{eff}^2 = \frac{1}{T}\int_0^T \sin\omega t\,dt = 0 \text{ and } \frac{1}{T}\int_0^T \sin^2\omega t\,dt = \frac{1}{2}$$

Substituting these values in equation (i), we get

$$I_{eff} = \sqrt{\left[a^2 + \frac{b^2}{2}\right]}$$

11. (3)



At steady state, no current flows through capacitor *C*.
According to Kirchhoff's law,
(i) at junction *E*, $I_1 = 3A$
(ii) at junction *H*, $I_2 = 1A$
(iii) Potential difference across capacitor,

$$V_E - V_H = 6I_1 + 2I_2$$

or $V = (6\times3)+(2\times1) = 18+2 = 20V$

$\therefore$ Energy = $\frac{1}{2}CV^2$

or $U = \frac{1}{2}\times(4\times10^{-6})\times(20)^2 = 8\times10^{-4}\,J$

12. (3)

When the like poles are tied together, the net magnetic moment is ($m_1 + m_2$) and the moment of inertia is ($I_1 + I_2$).

$\therefore$ The time period $T_1 = 2\pi\sqrt{\frac{I_1+I_2}{(m_1+m_2)B}}$.

When the unlike poles are tied together, the net magnetic moment is ($m_1 - m_2$), while the moment of inertia (being a scalar quantity) remains unchanged.

$\therefore$ The time period $T_2 = 2\pi\sqrt{\frac{I_1+I_2}{(m_1+m_2)B}}$.

Thus,

$$\frac{T_2^2}{T_1^2} = \frac{(m_1+m_2)}{(m_1=m_2)} \Rightarrow \frac{m_1}{m_2} = \frac{T_2^2+T_1^2}{T_2^2-T_1^2} = \frac{\upsilon_1^2+\upsilon_2^2}{\upsilon_1^2-\upsilon_2^2}.$$

Given, $\upsilon_1 = 12$ per minute and $\upsilon_2 = 4$ per minute.

$$\therefore \quad \frac{m_1}{m_2} = \frac{12^2+4^2}{12^2-4^2} = \frac{144+16}{144-16} = \frac{160}{128} = \frac{5}{4}.$$

13. (3)

The angular velocity is given as

$$\omega^2 = \omega_0^2 + 2\alpha(\theta-\theta_0)$$

When the fan is switch off $\theta = 2\pi n, \theta_0 = 0, \omega = \frac{\omega_0}{4}$

$$\left(\frac{\omega_0}{4}\right)^2 = \omega_0^2 - 2\alpha(2\pi n) \Rightarrow 2\alpha(2\pi n) = \frac{15}{16}\omega_0^2$$

$$2\pi n = \frac{15}{16}\left(\frac{\omega_0^2}{2\alpha}\right)$$

When the fan comes to rest

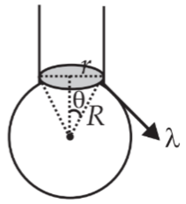
$$0 = \omega_0^2 - 2\alpha(2\pi n') \Rightarrow 2\pi n' = \left(\frac{\omega_0^2}{2\alpha}\right)$$

or $n' = \frac{16}{15}n$

14. (1)

The vertical force due to the surface tension on the drop

$= T2\pi r\sin\theta$



$$= T2\pi r\frac{r}{R} = \frac{2\pi r^2T}{R}$$

When the drop detaches from the dropper, then

$$\frac{2\pi r^2T}{R} = mg = \frac{4}{3}\pi R^3\rho g \quad \text{or} \quad R^4 = \frac{3}{2}\frac{r^2T}{\rho g}$$

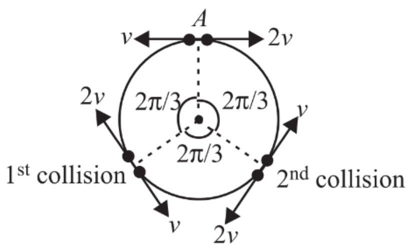
Or $R = \left(\frac{3}{2}\frac{r^2T}{\rho g}\right)^{1/4}$

Substituting the given values, we get

$$R = \left(\frac{3\times(5\times10^{-4})^2\times0.11}{2\times10^3\times10}\right)^{1/4} = 1.4\times10^{-3}m$$

15. (3)

At first collision, particle having speed $2v$ will rotate by 240° $\left(\text{or } \frac{4\pi}{3}\right)$ while other particle having speed v will rotate by 120° $\left(\text{or } \frac{2\pi}{3}\right)$. At first collision, they will exchange their velocities. Now as shown in figure, after two collisions they will again reach at point *A*.



16. **(2)**

Friction force = μmg = 0.2 × 5 × 10 = 10 N. Effective force F = applied force – frictional force = 25 – 10 = 15 N. Kinetic energy = work done by force F in pulling the body through a distance

S (= 10 m) = 15 × 10 = 150 J.

17. **(2)**

Kinetic energy of neutron = 0.0327 eV

or $K = 0.0327 \times 1.6 \times 10^{-19}$ J

or $\frac{1}{2}mv^2 = 0.0327 \times 1.6 \times 10^{-19}$

or $v^2 = \frac{2 \times 0.0327 \times 1.6 \times 10^{-19}}{1.675 \times 10^{-27}}$

or $v^2 = 0.0625 \times 10^8; \; v = 0.25 \times 10^4 \, ms^{-1}$

$\therefore$ Time taken = $\frac{\text{distance}}{\text{velocity}} = \frac{10}{0.25 \times 10^4}$

or $t = 4 \times 10^{-3}\, s$

$\therefore$ Fraction that decays = $\frac{N}{N_0} = \left(1 - e^{-\lambda t}\right)$

$= 1 - \left\{ e^{-\left(\frac{0.693}{700} \times 4 \times 10^{-3}\right)} \right\} = 3.9 \times 10^{-6}$

18. **(2)**

Here,

$c_m(t) = 30 \sin(300\pi t) + 10(\cos(200\pi t) - \cos(400\pi t))$

Compare this equation with standard equation of amplitude modulated wave,

$$c_m(t) = A_c \sin \omega_c t - \frac{\mu A_c}{2}\cos(\omega_c + \omega_m)t + \frac{\mu A_c}{2}\cos(\omega_c - \omega_m)t$$

$A_c = 30V, \omega_c = 300\pi \Rightarrow 2\pi\upsilon_c = 300\pi$

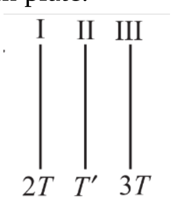
$\Rightarrow \; \upsilon_c = 150$ Hz

$\omega_c - \omega_m = 200\pi \Rightarrow \upsilon_c - \upsilon_m = 100$ Hz

$\frac{\mu A_c}{2} = 10, A_c = 30 \quad \therefore \mu = \frac{10}{15} = \frac{2}{3}$

19. **(3)**

Let T' be the temperature of the middle plate (II) and A be area of each plate.



Under steady state, the rate of energy received by the middle plate is equal to rate of energy emitted by it.

i.e. $\sigma A(3T')^4 - \sigma A(T')^4 = \sigma A(T')^4 - \sigma A(2T')^4$

or $\sigma A[(3T')^4 - (T')^4] = \sigma A[(T')^4 - (2T')^4]$

or $(3T')^4 - (T')^4 = (T')^4 - (2T')^4$

or $2(T')^4 = T^4(3^4 + 2^4) = T^4(81 + 16) = 97T^4$ or

$T'^4 = \frac{97}{2}T^4$ or $T' = \left(\frac{97}{2}\right)^{1/4} T$

20. **(1)**

As rms value of current,

$I_{rms} = V_{rms}/Z$

So, net impedance across LCR circuit,

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(100)^2 + (\omega L - X_C)^2}$

$= \sqrt{(100)^2 + \left[\left(100 \times \pi \times \frac{1}{\pi}\right) - (X_C)\right]^2}$

$\left(\frac{220}{2.2}\right)^2 = (100)^2 + ((100) - (x_C))^2 \Rightarrow X_C = 100\Omega$

As, $\tan\theta = \frac{X_C}{R} = \frac{100}{100} = 1$

$\Rightarrow \; \theta = \tan^{-1}(1)$ or $\theta = 45°$

Power factor, $\cos\theta = \frac{1}{\sqrt{2}}$.

21. **(12)**

Using mirror formula,

$v = \frac{uf}{u - f} = \frac{(-15) \times (-10)}{-15 + 10} = -30$ cm,

$m = -\frac{v}{u} = -2$

$\therefore A'B' = C'D' = 2 \times AB = 2 \times 1 = 2$ mm

Now for longitudinal magnification,

$\frac{B'C'}{BC} = \frac{A'D'}{AD} = \frac{v^2}{u^2} = 4 \Rightarrow = 4$ mm

$\therefore$ Perimeter length = 2 + 2 + 4 + 4 = 12 mm

22. **(3)**

Flux through an area with half angle θ as shown is

$\phi_E = \frac{Q}{2\varepsilon_0}(1 - \cos\theta)$

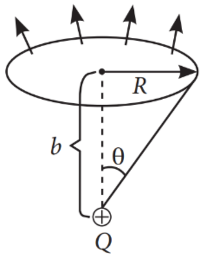
Here, $\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{1 + \tan^2\theta}}$

$= \frac{1}{\sqrt{1 + R^2/b^2}} = \frac{1}{\sqrt{\left(1 + \frac{3b^2}{b^2}\right)}} = \frac{1}{2}$

$\therefore$ Flux through disc, $\phi_1 = \frac{Q}{4\varepsilon_0}$

So, flux which is not passing through the disc

$\phi_2 = \frac{Q}{\varepsilon_0} - \frac{Q}{4\varepsilon_0} = \frac{3}{4}\frac{Q}{\varepsilon_0}$



Hence, ratio is $\frac{\phi_2}{\phi_1} = \frac{\frac{3}{4}\frac{Q}{\varepsilon_0}}{\frac{1}{4}\frac{Q}{\varepsilon_0}} = 3$

23. **(0.312)**

Here, Distance between the slits, d = 2 mm = 2×10^{-3} m Distance of the screen from the slits, D = 1.2 m Wavelengths,

$\lambda_1 = 6500$ Å $= 6500 \times 10^{-10} = 6.5 \times 10^{-7}$ m

and $\lambda_2 = 5200$ Å $= 5200 \times 10^{-10} = 5.2 \times 10^{-7}$ m

Distance of n^{th} bright fringe from the centre bright fringe is

$$x_n = \frac{n\lambda D}{d} \quad \ldots \text{(i)}$$

If x_4 and x'_4 be the distances of the fourth bright fringes of wavelengths λ_1 and λ_2 respectively, then from eqn. (i)

$$x_4 = \frac{4\lambda_1 D}{d} \text{ and } x'_4 = \frac{4\lambda_2 D}{d}$$

Thus, the separation between them is

$$\Delta x = x_4 - x'_4 = \frac{4\lambda_1 D}{d} - \frac{4\lambda_2 D}{d} = \frac{4D(\lambda_1 - \lambda_2)}{d}$$

$$= \frac{4(1.2\,\text{m})(6.5\times10^{-7}\,\text{m} - 5.2\times10^{-7}\,\text{m})}{(2\times10^{-3}\,\text{m})}$$

$= 3.12 \times 10^{-4}$ m $= 0.312 \times 10^{-3}$ m = 0.312 mm

24. **(8)**

Here,

radius of the sphere, r = 10 cm = 10×10^{-2} m

work function, ϕ_0 = 2.4 eV

energy of a photon, $E = h\upsilon$ = 4.2 eV

According to Einstein's photoelectric equation

$h\upsilon = \phi_0 + eV_s$

eV_s = 4.2 eV – 2.4 eV = 1.8 eV

$\therefore\ V_s$ = 1.8 V

The sphere will stop emitting photoelectrons, when the potential on its surface becomes 1.8 V. Let N be the number of photoelectrons emitted from the sphere. Then,

charge on the sphere, $Q = Ne$

$$V_s = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Ne}{r}$$

$$N = \frac{V_s \times r}{\frac{1}{4\pi\varepsilon_0}\times e} = \frac{1.8\times r}{\frac{1}{4\pi\varepsilon_0}\times e} = \frac{1.8\times10\times10^{-2}}{9\times10^{9}\times1.6\times10^{-19}}$$

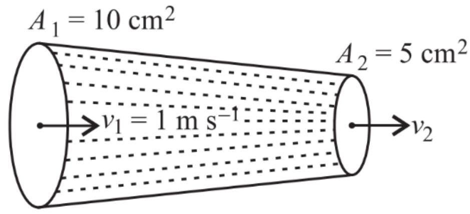
$$= \frac{18}{16}\times\frac{1}{9}\times10^{9} = 1.25\times10^{8}$$

25. **(500)**

According to equation of continuity,

$A_1v_1 = A_2v_2$

$$\therefore\ v_2 = \frac{A_1v_1}{A_2} = \frac{10\,\text{cm}^2\times1\,\text{ms}^{-1}}{5\,\text{cm}^2} = 2\,\text{ms}^{-1}$$



For a horizontal pipe, according to Bernoulli's theorem

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

or $P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2)$

$$= 2000 + \frac{1}{2}\times10^3\times(1^2 - 2^2)$$

($\because$ Density of water, $r = 10^3$ kg m^{-3})

$$= 2000 - \frac{1}{2}\times10^3\times3 = 2000 - 1500 = 500 \text{ Pa}$$

26. **(1.06)**

Here $qE = qvB\sin\phi$ or $v = \frac{E}{B\sin\phi}$

$v' = v\cos\phi$ = velocity along the field

$v'' = v\sin\phi$ = velocity perpendicular to field

By the dynamics of circular motion

$$qv'' = \frac{mv''^2}{r} \text{ or } qB = m\omega \text{ or } T = \frac{2\pi m}{qB}$$

$$\therefore\ p = T\times v\cos\phi = \frac{2\pi m}{qB}v\cos\phi$$

$$= \frac{2\pi m}{qB}\left(\frac{E}{B\sin\phi}\right)\cos\phi$$

or $p = \frac{2\pi mE}{qB^2}\cot\phi$

$$= \frac{2\pi\times1.67\times10^{-27}}{1.6\times10^{-19}\times40^2\times10^{-6}}\cot 60°$$

$\approx$ 1.06 m

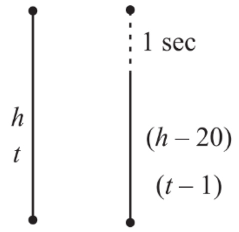
27. **(31.2)**

For first ball, $h = \frac{1}{2}gt^2$ … (i)

For second ball, $(h - 20) = \frac{1}{2}g(t-1)^2$ … (ii)

$$h - 20 = \frac{1}{2}h(t-1)^2$$

or $\frac{gt^2}{2} - 20 = \frac{g}{2}(t^2 - 2t + 1)$ (using (i))



or $\frac{gt^2}{2} - 20 = \frac{gt^2}{2} - gt + \frac{g}{2}$

or $gt = \frac{g}{2} + 20$ or $10t = \frac{10}{2} + 20 = 25$

or $t = \frac{25}{10} = 2.5$ … (iii)

$\because h = \frac{1}{2}gt^2$ $\therefore h = \frac{1}{2} \times 10 \times (2.5)^2$

or $h = \frac{10 \times 6.25}{2}$ or h = 31.25 m = 31.2 m

28. **(0.5)**

$\frac{\Delta V}{V} = \frac{\Delta(\pi r^2 L)}{\pi r^2 L} = \frac{r^2 \Delta L + 2r L \Delta r}{r^2 L}$

or $\frac{\Delta V}{V} = \frac{\Delta L}{L} + 2\frac{\Delta r}{r}$; But $\Delta V = 0$ (given)

$\therefore\ 0 = \frac{\Delta L}{L} + 2\frac{\Delta r}{r}$ or $\frac{\Delta L}{L} = -2\frac{\Delta r}{r}$

Poisson's ratio, $\sigma = -\frac{\Delta r / r}{\Delta L / L} = \frac{\Delta r / r}{2\Delta r / r} = \frac{1}{2} = 0.5$

29. **(11)**

$\beta = \frac{I_C}{I_B}$

$I_B = \frac{I_C}{\beta} = \frac{2.5}{100} = 0.0125$ mA

Applying Kirchhoff 's law to base emitter loop,

$V_{CE} = V_C - I_C R_C$

$= 20 - (2.5 \times 10^{-3}) \times (5 \times 10^3) = 7.5$ V

$V_{BE} = V_B - I_B R_B$

$= 20 - (0.0125 \times 10^{-3}) \times (120 \times 10^3) = 18.5$ V

$\therefore\ V_{BC} = (V_{BE} - V_{CE}) = (18.5 - 7.5) = 11$ V

30. **(4)**

On immersing the apparatus in a liquid of $\mu = 1.5$, wavelength $\lambda' = \frac{\lambda}{1.5} = \frac{\lambda}{3/2}$

As fringe width $\propto$ wavelength,

$\therefore$ fringe width $\beta' = \frac{\beta}{3/2}$

The distance between central maximum and tenth maximum is 3 cm in vacuum. When immersed in liquid, this distance would reduce to $\frac{3}{3/2}$ cm = 2 cm.

As the position of central maximum does not change, therefore, y-coordinate of central maximum will remain 2 cm and that of tenth maximum would become 4 cm.

CHEMISTRY

31. **(3)**

$CH_3-CH(CH_3)-CH_2-CH_2-C(=O)-Cl \xrightarrow{\text{alc. } NH_3}$

$CH_3-CH(CH_3)-CH_2-CH_2-C(=O)-NH_2$

$CH_3-CH(CH_3)-CH_2-CH_2-C(=O)-NH_2 \xrightarrow[\text{Hoffmann's Bromamide Degradation}]{NaOH+Br_2}$

$CH_3-CH(CH_3)-CH_2-CH_2-NH_2$

$CH_3-CH(CH_3)-CH_2-CH_2-NH_2 \xrightarrow{NaNO_2,\ HCl}$

$CH_3-CH(CH_3)-CH_2-CH_2-\overset{+}{N}_2Cl^-$

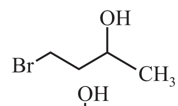
Diazonium salt

$CH_3-CH(CH_3)-CH_2-CH_2-\overset{+}{N}_2Cl^- \xrightarrow{H_2O}$

$CH_3-CH(CH_3)-CH_2-CH_2-OH$

32. **(3)**

$CH_3-C\equiv C-H + NaNH_2 \longrightarrow CH_3-C\equiv C^-\ Na^+$



$CH_3-C\equiv C-(CH_2)_2-CH(OH)-CH_3$

$CH_3-C\equiv C-(CH_2)_2-CH(OH)-CH_3 \xrightarrow[\text{Catalytic Reduction}]{H_2/Pd-C}$

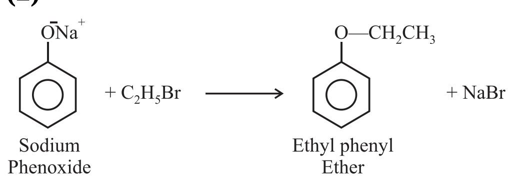
$CH_3-CH_2-CH_2-CH_2-CH_2-CH(OH)-CH_3$

$\xrightarrow[\text{oxidation}]{CrO_3} CH_3-CH_2-CH_2-CH_2-CH_2-C(=O)-CH_3$

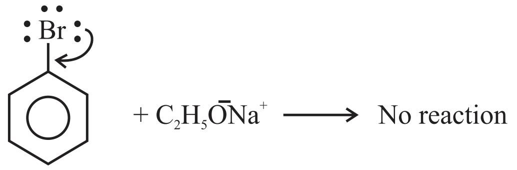
(Ketone)

CrO_3 mild oxidizing agent.

33. **(2)**

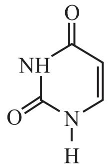


Due to double bond character it does not involve in substitution reaction



⇒ Assertion is correct but reason is not correct.

34. **(3)**



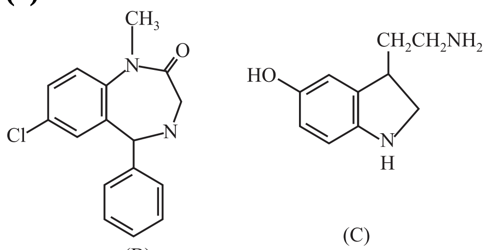
This is isomeric form of Uracil, which is present in RNA

35. **(4)**

In stratosphere CFC get broken down by powerful UV radiation releasing $Cl^{\bullet}$

$$\underset{(A)}{CF_2Cl_2(g)} \xrightarrow{UV} Cl^{\bullet}(g) + C^{\bullet}F_2Cl(g)$$

36. **(1)**



(B) (C)

B and C are tranquilizers

37. **(3)**

Number of water molecule

Gypsum: $CaSO_4.2H_2O$ ⇒ 2

Plaster of paris: $CaSO_4.\frac{1}{2}H_2O$ ⇒ 0.5

Dead burnt plaster: $CaSO_4$ ⇒ 0

38. **(1)**

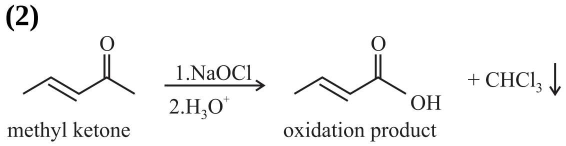
Unit of $\frac{an^2}{V^2} = atm$

Unit of a $\frac{atm \times \text{unit of } V^2}{\text{unit of } n^2}$

$= \frac{atm \times (dm^3)^2}{mol^2}$

$= atm\, dm^6 mol^{-2}$

39. **(2)**

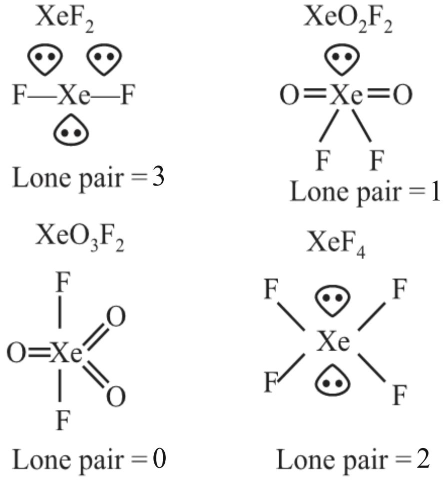


⇒ This is chloroform reaction

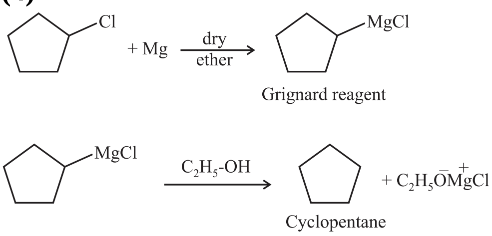
40 **(4)**

The bond dissociation energy of D_2 is greater than H_2, therefore D_2 reacts slower than H_2.

41. **(4)**



42. **(4)**



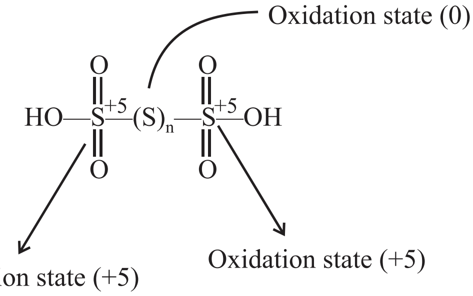
43. **(2)**

V_2O_3 – Basic

CrO – Basic

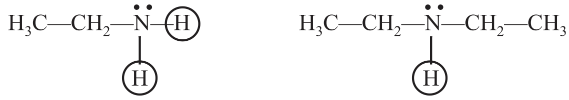
Metal oxides are generally basic in nature but if more electronegative element are connected through central atom then deficiency of electron occurs on central atom and it behaves as acidic in nature.

44. **(3)**



45. **(1)**

The intermolecular association which is H-bonding is more prominent in case of primary amine as compared to secondary amines due to the availability of two hydrogen atom.



Primary Amine Secondary Amine

46. **(2)**

Type of back bonding:

BF_3	BCl_3	BBr_3
$(2P\pi - 2P\pi)$	$(2P\pi - 3P\pi)$	$(2P\pi - 4P\pi)$
BI_3		
$(2P\pi - 5P\pi)$		

Therefore, order of back bonding strength

$\Rightarrow BF_3 > BCl_3 > BBr_3 > BI_3$

47. **(2)**

Tyndall effect is more effectively shown by lyophobic colloidal solution.

48. **(4)**

MnO → Anti ferromagnetism

O_2 → Paramagnetism

NaCl → Diamagnetism

Fe_3O_4 → Ferrimagnetism

49. **(3)**

$FeCl_3 + K_4[Fe(CN)_6]$ (Potassium ferrocyanide)
(excess)
↓
$Fe_4[Fe(CN)_6]_3$
(colloidal species)

Prussian Blue coloured colloidal complex

50. **(2)**

Liquation is applicable for the refining of low melting point metals.

In liquation low melting point metal, melt at first and impurities take more time for melting therefore we can separate metal from their impurities.

51. **(2)**

$MCl_3.2L \Rightarrow$ octahedral complex

$MCl_3.2L \xrightarrow{AgNO_3(excess)} 1$ mole of AgCl

It means one Cl^- ion present in ionization sphere.

$\therefore [MCl_2L_2]Cl$

For octahedral complex co-ordination numbers is 6

$\therefore$ L is bidentate ligand i.e., its denticity is 2.

52. **(40)**

Mass of organic compound = 0.2 gram

$$n_{AgBr} = \frac{0.188}{188} \text{mol}$$

$n_{Br} = n_{AgBr} = 0.001$ mole

mass of Br = 0.001×80 g = 0.08 g

Mass % of Br $= \frac{0.08}{0.2} \times 100 = 40\%$

53. **(4)**

$2K_2Cr_2O_7 + 8H_2SO_4 + 3C_2H_6O \rightarrow 2Cr_2(SO_4)_3 + 3C_2H_4O_2 + 2K_2SO_4 + 11H_2O$

Given that, rate of appearance of $Cr_2(SO_4)_3$

$$= + \frac{d[Cr_2(SO_4)_3]}{dt} = 2.67 \text{ mol/min}$$

$$\underset{\left(\frac{\text{Rate of disappearance of } C_2H_6O}{3}\right)}{\text{LHS}} = \underset{\left(\frac{\text{Rate of appearance of } Cr_2(SO_4)_3}{2}\right)}{\text{RHS}}$$

Rate of disappearance of $C_2H_6O = \frac{2.67 \times 3}{2}$ mol/min

Rate of disappearance of C_2H_6O = 4.005 mol/min

Ans. = 4 (nearest integer)

54. **(50)**

$\Delta E = q + w$ for cyclic process; $\Delta E = 0$

o = q + (–50)

q = 50 kJ/cycle

55. **(3155)**

$mvr = \frac{nh}{2\pi}$ (angular momentum)

$KE = \frac{n^2h^2}{8\pi^2mr^2}$ (Bohr's kinetic energy)

$= \frac{4h^2}{8\pi^2m(4a_0)^2} (n = 2)$

$= \left(\frac{4}{8\pi^2 \times 16}\right)\frac{h^2}{ma_0^2}$

$K.E = \frac{1}{32\pi^2}\frac{h^2}{ma_0^2}$

Comparing with $\frac{h^2}{xma_0^2}$

$x = 32\pi^2 = 315.50$

10x = 3155

56. **(1125)**

Dumas method,

Moles of N in N, N-dimethylaminopentane ($C_7H_{17}N$)

$= \frac{57.5}{115} = 0.5$ mole

$$C_7H_{17}N + \frac{45}{2}CuO \rightarrow 7CO_2 + \frac{17}{2}H_2O + \frac{1}{2}N_2 + \frac{45}{2}Cu$$

$$\frac{n_{CuO}}{\left(\frac{45}{2}\right)} = \frac{n_{C_7H_{17}N}}{1}$$

$n_{CuO} = \left(\frac{45}{2}\right) \times 0.5$ mol

= 11.25 mol

= 1125×10^{-2} mol

Ans. = 1125

57. (18)

$\underset{(z-93)}{Np} \Rightarrow 1s^2 2s^2 2p^6 3s^2 4s^2 3d^{10} 4p^6 5s^2$

$4d^{10} 5p^6 6s^2 4f^{14} 5d^{10} 6p^6 7s^2 5f^4 6d^1$

Total number of f-electrons = 14 + 4 = 18 electrons

Ans. = 18

58. (316)

Let molarity of $KMnO_4$ = x

$\underset{n=5}{KMnO_4} + \underset{n=1}{FeSO_4} \longrightarrow \underset{\text{Ferric sulphate}}{Fe_2(SO_4)_3} + Mn^{2+}$

Equivalent of $KMnO_4$ = equivalent of $FeSO_4$

5 × x × 10 = 1 × 0.1 × 10

x = 0.02 M

strength = (0.02 × 158) = 3.16 g/L

= 316×10^{-2} g/L

Ans. = 316

59. (82)

Millimoles of HCl = 200 × 0.2 = 40

Millimoles of NaOH = 300 × 0.1 = 30

Heat released $= \frac{30}{1000} \times 57.1 \times 1000\,J = 1713\,J$

$\begin{bmatrix} \rho = \frac{m}{v} \\ m = \rho v \end{bmatrix}$

Mass of solution = 500 × 1g = 500 g

Specific heat of water = 4.18 $Jg^{-1}\,K^{-1}$

$\Delta T = \frac{q}{mc}$

$= \frac{1713}{500 \times 4.18}\,^{o}C$

= 0.8196°C

= 81.96×10^{-2}°C ≈ 82×10^{-2}°C

60. (4)

$Ag^+(aq) + 2NH_3(aq)$

$Ag^+(aq) + 2NH_3(aq) \rightleftharpoons Ag(NH_3)_2^+(aq)$

$t = 0 \quad 0.8M \quad \left(\frac{a}{2}\right)M$

$t = \infty \quad 5 \times 10^{-8}M \quad \left(\frac{a}{2} - 1.6\right)M \quad 0.8M$

$k_f = \frac{[Ag(NH_3)_2^+]}{[Ag^+][NH_3]^2}$

$10^8 = \frac{0.8}{(5 \times 10^{-8})\left(\frac{a}{2} - 1.6\right)^2}$

$\frac{0.8}{5} = \left(\frac{a}{2} - 1.6\right)^2$

$\left(\frac{a}{2} - 1.6\right) = 0.4$

$\frac{a}{2} = 0.4 + 1.6$

$\frac{a}{2} = 2$

Concentration of NH_3 added $= \frac{a}{2} = 2M$

Volume of solution = 2L

Moles of NH_3 added = 2 × 2 mol = 4 mol.

MATHEMATICS

61. (1)

$(2+\alpha)^4 = \left(\frac{3}{2} + i\frac{\sqrt{3}}{2}\right)^4$

$= \left[\sqrt{3}e^{i\left(\frac{\pi}{6}\right)}\right]^4$

$= 9\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$= \frac{-9}{2} + \frac{9\sqrt{3i}}{2}$

$\Rightarrow 0 + 9\left(\frac{-1 + i\sqrt{3}}{2}\right)$

$\therefore a = 0, b = 9$

62. (1)

Contrapositive of $p \to q$ is ~ is $q \to \sim p$.

63. (4)

${}^{n+5}C_{r-1} : {}^{n+5}C_r : {}^{n+5}C_{r+1}$

$= 5 : 10 : 4$

$2\left({}^{n+5}C_{r-1}\right) = {}^{n+5}C_r \Rightarrow 3r = n + 6$

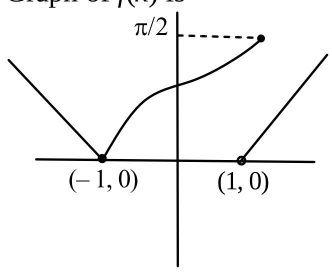
$7\left({}^{n+5}C_r\right) = 5\left({}^{n+5}C_{r+1}\right) \Rightarrow 12r = 18 + 5n$

Solving: $n = 6$, $r = 4$

$\therefore$ Largest coefficient is ${}^{n+5}C_{r+1} = {}^{11}C_5 = 462$

64. (3)

Graph of $f(x)$ is



65. **(2)**

Here, $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 3 & 2 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = \frac{9}{2}$

Also, $\begin{vmatrix} 1 & 1 & 2 \\ 2 & 4 & 6 \\ 3 & 2 & \mu \end{vmatrix} = 0 \Rightarrow \mu = 5$

66. **(1)**

Given $3\alpha^2 - 10\alpha + 27\lambda = 0$ …..(i)

$3\alpha^2 - 3\alpha + 6\lambda = 0$ …..(ii)

Subtract $-7\alpha + 21\lambda = 0$

$3\lambda = 0$

By (ii) $9\lambda^2 - 3\lambda + 2\lambda = 0 \quad \Rightarrow \lambda = 0, \frac{1}{9}$

$\therefore \alpha = \frac{1}{3}, \beta = \frac{2}{3}, \alpha = \frac{1}{3}, \gamma = 3$

$\therefore \frac{\beta\gamma}{\lambda} = \frac{\frac{2}{3}.}{\frac{1}{9}} = 18$

67. **(2)**

By family of circle, passing through intersection of given circle will be member of $S + \lambda S' = 0$ family $(\lambda \neq 1)$

$\left(x^2 + y^2 - 6x\right) + \lambda\left(x^2 + y^2 - 4y\right) = 0$

$(\lambda + 1)x^2 + (\lambda + 1)y^2 - 6x - 4\lambda y = 0$

$x^2 + y^2 - \frac{6}{\lambda + 1} \times - \frac{4\lambda}{\lambda + 1} y = 0$

Centre $\left(\frac{3}{\lambda + 1}, \frac{2\lambda}{\lambda + 1}\right)$

Centre lies on $2x - 3y + 12 = 0$

$2\left(\frac{3}{\lambda + 1}\right) - 3\left(\frac{2\lambda}{\lambda + 1}\right) + 12 = 0$

$6\lambda + 18 = 0 \Rightarrow \lambda = -3$

Circle $x^2 + y^2 - 3x - 6y = 0$

68. **(1)**

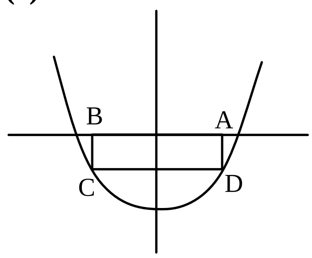
$\int_{\pi/6}^{\pi/3} \left(\frac{\frac{d}{dx}\left(\tan^4 x\right)}{2} \cdot \sin'3x + \tan'x \cdot \frac{\frac{d}{dx}\left(\sin^4 3x\right)}{2} \right)$

$= \frac{1}{2}\int_{\pi/6}^{\pi/3} \frac{d}{dx}\left(\tan^4 x \cdot \sin^4 3x\right) dx$

$= \frac{1}{2}\left[\tan^4 x \cdot \sin^4 3x\right]_{\pi/6}^{\pi/3}$

$= \frac{1}{2}\left[(\sqrt{3})^4.0 - \frac{1}{(\sqrt{3})^4}\right] = -\frac{1}{18}$

69. **(1)**



$A(\alpha, 0), B(-\beta, 0)$

$\Rightarrow D\left(\alpha, \alpha^2 - 1\right)$

Area (ABCD) = (AB) (AD)

$\Rightarrow S = (2\alpha)\left(1 - \alpha^2\right) = 2\alpha - 2\alpha^2$

$\frac{ds}{d\alpha} = 2 - 6\alpha^2$

Put $\frac{ds}{d\alpha} = 0 \Rightarrow 2 - 6\alpha^2 = 0$

$\alpha^2 = \frac{1}{3} \qquad \Rightarrow \alpha = \frac{1}{\sqrt{3}}$

Area $= 2\alpha - 2\alpha^2 = \frac{2}{\sqrt{3}} - \frac{2}{3\sqrt{3}} = \frac{4}{3\sqrt{3}}$

70. **(4)**

Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$Ax_1 = B_1$

$a_1 + a_2 + a_3 = 1$

$b_1 + b_2 + b_3 = 0$

$c_1 + c_2 + c_3 = 0$

Similar $2a_3 + a_3 = 0$ and $a_3 = 0$

$2b_2 + b_3 = 2 \qquad b_3 = 0$

$2c_2 + c_3 = 0 \qquad c_3 = 2$

$\therefore a_2 = 0, b_2 = 1, c_2 = -1$

$a_1 = 1, b_1 = -1, c_1 = -1$

$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix} \therefore |A| = 2$

71. **(2)**

$\frac{dy}{dx} - \frac{y - 3x}{l\,\text{n}(y - 3x)} + 3 = 0$

$\frac{dy}{dx} + 3 = \frac{y + 3x}{l\,\text{n}(y + 3x)}$

$\frac{d}{dx}(y + 3x) = \frac{y + 3x}{l\,\text{n}(y + 3x)}$

$\int \frac{l\,\text{n}(y + 3x)}{(y + 3x)} d(y + 3x) = \int dx$

Let $\ell\text{n}(y + 3x) = t$

$$\frac{1}{(y+3x)}d(y+3x)=dt$$

$$\int t\,dt=\int dx$$

$$\frac{t^2}{2}=x+c$$

$$\frac{(l\,n(y+3x))^2}{2}=x+c$$

72. **(1)**

Since AM two positive quantities ≥ their G. M.

$$\frac{2^{\sin x}+2^{\cos x}}{2}\ge\sqrt{2^{\sin x}\cdot 2^{\cos x}}$$

$$=\sqrt{2^{\sin x+\cos x}}=\sqrt{2^{\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)}}$$

$$\ge\sqrt{2^{-\sqrt{2}}}\Rightarrow 2^{\sin x}+2^{\cos x}\ge 2.2^{1//\sqrt{2}}=2^{1-1/\sqrt{2}}$$

73. **(2)**

Applying L – Hospital Rule

$$\lim_{t\to x}\frac{2tf^2(x)-x^2\left(2f(t)f'(t)\right)}{1}$$

$$\therefore 2\times f^2(x)-x^2\left(2f(x)f'(x)\right)=0$$

$$\Rightarrow f(x)-xf'(x)=0$$

$$\Rightarrow\frac{f('x)}{f(x)}=\frac{1}{x}\Rightarrow\ln f(x)=\ln x+C$$

At $x=1, c=1$

$\therefore l\,n\,f(x)=l\,n\,x+1$

When $f(x)=1$

then $l\,n\,x=-1$

$$x=\frac{1}{e}$$

74. **(4)**

Equation PQ

$$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6}=\lambda$$

Let $Q=(2\lambda+1, 3\lambda-2, -6\lambda+3)$

Q lies on $x-y+z=5$

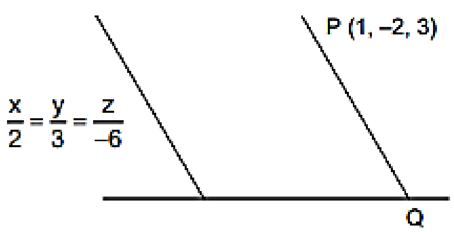
$=(2\lambda+1)-(3\lambda-2)+(-6\lambda+3)=5$

$$\Rightarrow\lambda=-\frac{1}{7}$$

$$Q=\left(\frac{5}{7},\frac{-17}{7},\frac{15}{7}\right)$$

$$\therefore PQ=\sqrt{\left(\frac{2}{7}\right)^2+\left(\frac{3}{7}\right)^2+\left(\frac{6}{7}\right)^2}$$

$PQ=1$



75. **(3)**

sum 6 → (1, 5), (5, 1), (3, 3), (2, 4), (4, 2)

sum 6 → (1, 6), (6, 1), (5, 2), (2, 5), (3, 4), (4, 3)

$$=P(A)+P(\bar{A})\cdot P(\bar{B})\cdot P(A)+P(\bar{A})P(\bar{B})P(\bar{A})\cdot P(\bar{B})\cdot P(A)$$

+ …….

This is infinite G.P. with common ratio

$P(\bar{A})\times P(\bar{B})$

Probability of A wins $=\dfrac{P(A)}{1-P(\bar{A})P(\bar{B})}$

$$=\frac{\frac{5}{36}}{1-\frac{31}{36}\cdot\frac{30}{36}}=\frac{30}{61}$$

76. **(3)**

$\tan 30^\circ=\dfrac{x}{y}=\dfrac{1}{\sqrt{3}}\Rightarrow y=\sqrt{3}x$ …..(i)

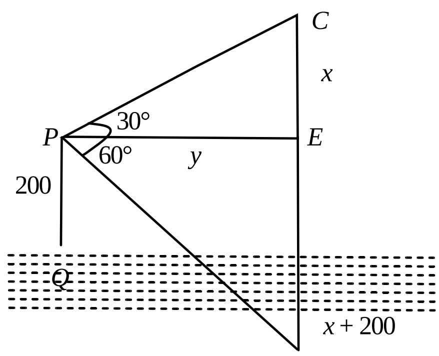
and $\tan 60^\circ=\dfrac{x+400}{y}\Rightarrow\sqrt{3}y$

$=x+400$ …..(ii)

Solving (i) and (ii), we get

$2x=400, x=200$

$\sin 30^\circ=\dfrac{x}{PC}=\dfrac{200}{PC}\Rightarrow PC=400$



77. **(1)**

Given: $\dfrac{a}{e}=4$ and $\dfrac{1}{4}=1-\dfrac{b^2}{a^2}$

Solving: $a=2, b=\sqrt{3}$

Parametric coordinate is

$\left(2\cos\theta,\sqrt{3}\sin\theta\right)=(1,\beta)$

$\therefore\theta=60^\circ$

$\therefore$ Equation of normal is $ax\sec\theta-$ by

$\operatorname{cosec}\theta=a^2-b^2$

$\Rightarrow 4x-2y=1$

78. **(2)**

Let number of elements in T is n.

$$50\times\frac{10}{20}=n\times\frac{5}{6}\Rightarrow n=30$$

79. **(2)**

Any point (*x*, *y*) on perpendicular bisector equidistant from *p* and *q*

$\therefore (x-1)^2+(y-4)^2=(x-k)^2+(y-3)^2$

At $x = 0$, $y = -4$

$k^2 = 16$

80. **(1)**

Given,

$300 = 1 + (N-1)d$

$\Rightarrow (N-1)d = 299$

$\therefore (N, d) = (24, 13)$ is the only possible pair

$\therefore a_{20} + 1 + 19(13) = 248$ and , $S_{20} = \frac{1+248}{2}\times 20$

$= 2490$

81. **(18)**

Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$\hat{i}\times(\vec{a}\times\hat{i}) = (\hat{i}.\hat{i})\vec{a} - (\vec{a}\cdot\hat{i}) = yj + zk$

Similarly

$j\times(\vec{a}\times j) = x\hat{i} + zk \text{ and } k\times(\vec{a}\times k) = x\hat{i} + yk$

$|\hat{i}\times(\vec{a}\times\hat{i})|^2 + |\hat{j}\times(\vec{a}\times\hat{j})|^2 + |\hat{k}\times(\vec{a}\times\hat{k})|^2$

. $= |y\hat{j} + z\hat{k}|^2 + |x\hat{i} + z\hat{k}|^2 + |x\hat{i} + y\hat{j}|^2 = 2|a|^2$

$= 2(9) = 18$

82. **(135)**

Ways of selecting correct questions $= {}^6C_4 = 15$

Ways of doing them correct = 1

Ways of doing remaining 2 questions incorrect

$= 3^2 = 9$

$\therefore$ No. of ways $= 15\times 1\times 9 = 135$

83. **(2)**

Let $P(2\cos\theta, 2\sin\theta)$

$\therefore Q(-2\cos\theta, -2\sin\theta)$

Given line $x + y - 2 = 0$

$\therefore \alpha = \frac{|2\cos\theta + 2\sin\theta - 2|}{\sqrt{2}}$

$\beta = \frac{|-2\cos\theta - 2\sin\theta - 2|}{\sqrt{2}}$

$\therefore \alpha\beta = \sqrt{2}(\cos\theta + \sin\theta - 1)\cdot\sqrt{2}(\cos\theta + \sin\theta + 1)$

$= 2\left|\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta - 1\right| = 2|\sin 2\theta|$

$\max|\sin\theta| = 1$

$\therefore$ maximum $\alpha\beta = 2$

84. **(4)**

x_i	5	15	25
f_i	2	x	2

$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{10 + 15x + 50}{4 + x} = \frac{60 + 15x}{4 + x} = 15$

$\sigma^2 = 50 = \frac{\sum f_i x_i^2}{\sum f_i} - (\bar{x})^2$

$50 = \frac{50 + 225x + 1250}{4 + x} - (15)^2$

$50 = \frac{1300 + 225x}{4 + x} - 225$

$\Rightarrow 275(4 + x) = 1300 + 225x$

$\Rightarrow 50x = 200 \Rightarrow x = 4$

85. **(21)**

Clearly, $\int_0^n \{x\}dx = \frac{n}{2}$

$\int_0^n [x]dx = 1 + 2 + 3\ldots n - 1 \quad = \frac{n(n-1)}{2}$

$\therefore \left(\frac{n(n-1)}{2}\right)^2 = \frac{n}{2}\{10n(n-1)\}$

Solving, $n = 21$

86. **(19)**

$\sum_{k=0}^{10}\left(2^2 + 3k\right)^{10}C_k = 4\sum_{k=0}^{10}{}^{10}C_k + 3\times 10\sum_{k=0}^{10}{}^{9}C_{k-1}$

$= 4\times 2^{10} + 30\times 2^9 \;= 19\times 2^{10}$

So, $\alpha = 0, \beta = 19 \Rightarrow \alpha + \beta = 19$

87. **(56)**

Given equation

$\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$

$\Rightarrow 1 - 2\sin^2\theta\cos^2\theta - \sin\theta\cos\theta = 0$

$\Rightarrow 2 - (\sin 2\theta)^2 - \sin 2\theta = 0$

$\Rightarrow (\sin 2\theta)^2 + (\sin 2\theta) - 2 = 0$

$\Rightarrow (\sin 2\theta + 2)(\sin 2\theta - 1) = 0$

$\Rightarrow \sin 2\theta = 1 (as\ \sin 2\theta = -2 \text{ is not possible})$

$\Rightarrow 2\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}$

$\Rightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$

$\Rightarrow S = \frac{\pi}{4} + \frac{5\pi}{4} + \frac{9\pi}{4} + \frac{13\pi}{4} = 7\pi \quad \Rightarrow \frac{8S}{\pi} = 56$

88. **(481)**

$f(x) = \sin\left(\cos^{-1}\left(\frac{1-\left(2^x\right)^2}{1+\left(2^x\right)^2}\right)\right) = \sin\left(2\tan^{-1}2^x\right)$

$= \sin\left(\sin^{-1}\frac{2\times 2^x}{1+\left(2^x\right)^2}\right)$

$\therefore f(x) = \frac{2\times 2^x}{1 + 4^x}$

$$\Rightarrow f'(x) = 2\frac{\left(1+4^x\right)\left(2^x \log_e 2\right) - 2^x \cdot 4^x \cdot \log_e 4}{\left(1+4^x\right)^2}$$

$$= \frac{20\log_e 2 - 32\log_e 2}{25} = \frac{-12}{25}\log_e 2 \quad = -\frac{b}{a}\log_e 2$$

So, $a = 25$, $b = 12$

$$\Rightarrow \left|a^2 - b^2\right| = 25^2 - 12^2 = 625 - 144 = 481$$

89. **(15)**

Area $\Delta = \frac{1}{2}bc\sin A = \frac{1}{2}\times 5\times 12\times \sin A = 30$

$\therefore \sin A = 1$ So, $A = 90°$

$\therefore BC = 13$

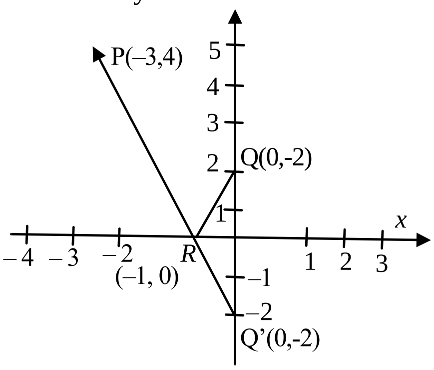
$BC = 2R = 13$

$$r = \frac{\Delta}{S} = \frac{30}{15} = 2$$

So, $2R + r = 15$

90. **(1250)**

For minimum time, PR is incident ray and RQ must reflected ray.



$$\therefore 50\left(PR^2 + RQ^2\right) = 50(20+5) = 1250$$

PW Web/App - https://smart.link/7wwosivoicgd4

Library- https://smart.link/sdfez8ejd80if