Date: 07/06/2022

SET No. 1

Question Paper Code

65/5/1

Time: 2 hrs.

Class-XII

Max. Marks: 40

MATHEMATICS

Term-II

(CBSE-2022)

GENERAL INSTRUCTIONS

Read the following instructions carefully and strictly follow them :

(i) This question paper contains **THREE Sections - A, B** and **C**.

(ii) Each section is compulsory.

(iii) **Section A** - has 6 short-answer type-I questions of **2** marks each.

(iv) **Section B** - has 4 short-answer type-II questions of **3** marks each.

(v) **Section C** - has 4 long-answer type questions of **4** marks each.

(vi) There is an internal choice in some questions.

(vii) Question 14 is a case study based question with **two** subparts of **2** marks each.

SECTION-A

Question numbers 1 to 6 carry 2 marks each.

1. Find : $\int \frac{dx}{\sqrt{4x - x^2}}$ [2]

Sol. $I = \int \frac{dx}{\sqrt{4x - x^2}}$

$= \int \frac{dx}{\sqrt{4 - 4 + 4x - x^2}}$

$= \int \frac{dx}{\sqrt{4 - (x - 2)^2}}$

$= \sin^{-1}\left(\frac{x - 2}{2}\right) + C$, where C is integration constant. $\left\{\because \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a} + C\right.$

2. Find the general solution of the following differential equation: [2]

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$$

Sol. $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

$\Rightarrow$ $e^y dy = (e^x + x^2)dx$.

$\Rightarrow$ $e^y = e^x + \frac{x^3}{3} + C$, where C is constant of integration.

3. Let X be a random variable which assumes values x_1, x_2, x_3, x_4 such that [2]

$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$

Find the probability distribution of X.

Sol. $\because$ $2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4) = \lambda$ (say)

So, $P(X = x_1) = \frac{\lambda}{2}$, $P(X = x_2) = \frac{\lambda}{3}$

$P(X = x_3) = \lambda$ and $P(X = x_4) = \frac{\lambda}{5}$

$\because$ $\sum_{i=1}^{4} P(X = x_i) = 1 \Rightarrow \frac{\lambda}{2} + \frac{\lambda}{3} + \lambda + \frac{\lambda}{5} = 1 \Rightarrow \lambda = \frac{30}{61}$

Probability distribution of X

X	x_1	x_2	x_3	x_4
$P(X = x_i)$	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

4. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $a \cdot b = 1$ and $\vec{a} \times b = \hat{j} - \hat{k}$, then find $|b|$. [2]

Sol. $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$

$\because \quad \left|\vec{a} \times \vec{b}\right|^2 = |\vec{a}|^2 \cdot \left|\vec{b}\right|^2 - \left(\vec{a} \cdot \vec{b}\right)^2$

$\Rightarrow \quad \left|\hat{j} - \hat{k}\right|^2 = \left|\hat{i} + \hat{j} + \hat{k}\right|^2 \cdot \left|\vec{b}\right|^2 - (1)^2$

$\Rightarrow \quad (1^2 + 1^2) = (1^2 + 1^2 + 1^2)\left|\vec{b}\right|^2 - 1$

$\Rightarrow \quad 2 = 3\left|\vec{b}\right|^2 - 1$

$\Rightarrow \quad \left|\vec{b}\right| = 1$

5. If a line makes an angle α, β, γ with the coordinate axes, then find the value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$. [2]

Sol. If a line makes angles α, β, γ with the coordinate axes then we know that

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ …(i)

Now, $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$

$= (2\cos^2\alpha - 1) + (2\cos^2\beta - 1) + (2\cos^2\gamma - 1)$

$= 2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) - 3$

$= 2 \times 1 - 3 = -1$

6. (a) Events A and B are such that [2]

$P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\bar{A} \cup \bar{B}) = \frac{1}{4}$

Find whether the events A and B are independent or not.

OR

(b) A box B_1 contains 1 white ball and 3 red balls. Another box B_2 contains 2 white balls and 3 red balls. If one ball is drawn at random from each of the boxes B_1 and B_2, then find the probability that the two balls drawn are of the same colour. [2]

Sol. (a) $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\bar{A} \cup \bar{B}) = \frac{1}{4}$

$\because \quad P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B})$

$\Rightarrow \quad \frac{1}{4} = P(\overline{A \cap B})$

$\Rightarrow \quad \frac{1}{4} = 1 - P(A \cap B)$

$\Rightarrow \quad P(A \cap B) = \frac{3}{4}$ ….(i)

Also, $P(A) \cdot P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24}$ ...(ii)

From (i) and (ii), $P(A \cap B) \neq P(A) \cdot P(B)$

Hence A and B are not independent events

OR

(b) $B_1 \equiv$ 1 White + 3 Red balls.

$B_2 \equiv$ 2 White + 3 Red balls.

Let A be the event that the balls drawn from B_1 and B_2 are of the same colour.

Event A can occur when both balls are red or both are white

$$P(A) = \frac{1}{4} \times \frac{2}{5} + \frac{3}{4} \times \frac{3}{5} = \frac{11}{20}$$

SECTION-B

Question numbers 7 to 10 carry 3 marks each.

7. Evaluate: $\int_0^{\pi/4} \frac{dx}{1+\tan x}$. [3]

Sol. $\int_0^{\pi/4} \frac{dx}{1+\tan x}$

$= \int_0^{\pi/4} \frac{\cos x}{\sin x + \cos x} dx$

$= \frac{1}{2} \int_0^{\pi/4} \frac{(\sin x + \cos x) - (\sin x - \cos x)}{\sin x + \cos x} dx$

$= \frac{1}{2} \int_0^{\pi/4} \left(1 + \frac{\cos x - \sin x}{\cos x + \sin x}\right) dx$

$= \frac{1}{2} [x + \ln(\sin x + \cos x]_0^{\pi/4}$

$= \frac{1}{2}\left[\frac{\pi}{4} + \ln\left(\sqrt{2}\right)\right] = \frac{\pi}{8} + \frac{1}{4}\ln 2$

8. (a) If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} + \vec{b}| = |\vec{b}|$, then prove that $\left(\vec{a} + 2\vec{b}\right)$ is perpendicular to $\vec{a}$. [3]

OR

(b) If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then prove that $\sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$. [3]

Sol. (a) $\because \quad |\vec{a} + \vec{b}|^2 = |\vec{b}|^2$

$\Rightarrow \quad \left(\vec{a} + \vec{b}\right) \cdot \left(\vec{a} + \vec{b}\right) = \vec{b} \cdot \vec{b}$

$\Rightarrow \quad \vec{a} \cdot \vec{a} + 2\vec{a} \cdot \vec{b} = 0$

$\Rightarrow \quad \vec{a} \cdot (\vec{a} + 2\vec{b}) = 0$

Hence $\vec{a} \perp (\vec{a} + 2\vec{b})$

OR

(b) $\because \quad |\vec{a}| = |\vec{b}| = 1$

Now, $|\vec{a}-\vec{b}|^2 = (\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})$

$= \vec{a}\cdot\vec{a}+\vec{b}\cdot\vec{b}-2\vec{a}\cdot\vec{b}$

$= 1 + 1 - 2\cos\theta$

$= 2(1 - \cos\theta)$

$= 4\sin^2\frac{\theta}{2}$

So $\sin\frac{\theta}{2} = \frac{1}{2}|\vec{a}-\vec{b}|$

9. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=10$ and $\vec{r}\cdot\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$ and passing through the points (–2, 3, 1). [3]

Sol. Equation of plane through the line of intersection of two given planes is,

$P \equiv \vec{r}\cdot(\hat{i}+\hat{j}+\hat{k}+\lambda(2\hat{i}+3\hat{j}-\hat{k}) = 10-4\lambda \qquad \ldots(i)$

$\because$ Plane P passes through $(-2\hat{i}+3\hat{j}+\hat{k})$, so

$(-2\hat{i}+3\hat{j}+\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k}+\lambda(2\hat{i}+3\hat{j}-\hat{k}) = 10-4\lambda$

$\Rightarrow \quad -2+3+1+\lambda(-4+9-1) = 10-4\lambda$

$\Rightarrow \quad 8\lambda = 8 \Rightarrow \lambda = 1$

Putting the value of λ in (i)

Hence, $P \equiv \vec{r}\cdot(3\hat{i}+4\hat{j}) = 6$

10. (a) Find: $\int e^x \cdot \sin 2x\, dx$ [3]

OR

(b) Find: $\int \frac{2x}{(x^2+1)(x^2+2)}dx$ [3]

Sol. (a) Let $I = \int \underbrace{e^x}_{II}\cdot\underbrace{\sin 2x}_{I}\, dx = \sin 2x \cdot e^x - \int \underbrace{2\cos 2x}_{I}\cdot\underbrace{e^x}_{II}\, dx$

$= e^x \cdot \sin 2x - 2\left[\cos 2x \cdot e^x + \int 2\sin 2x \cdot e^x dx\right]$

$\Rightarrow \quad I = e^x(\sin 2x - 2\cos 2x) - 4I$

$\Rightarrow \quad I = \frac{1}{5}e^x(\sin 2x - 2\cos 2x) + c$

where c is constant of integration.

OR

(b) $\int \frac{2x}{(x^2+1)(x^2+2)}dx$

Let $x^2+1=t$

$\Rightarrow \quad 2xdx = dt$

$\Rightarrow \quad \int \frac{dt}{t(t+1)}$

$= \int \left(\frac{1}{t} - \frac{1}{t+1}\right)dt$

$= \ln|t| - \ln|t+1| + c$

$= \ln\left|\frac{t}{t+1}\right| + c$

$= \ln\left|\frac{x^2+1}{x^2+2}\right| + c$, where c is constant of integration.

SECTION-C

Question numbers 11 to 14 carry 4 marks each.

11. Three persons A, B and C apply for a job of manager in a private company. Chances of their selection are in the ratio 1 : 2 : 4. The probability that A, B and C can introduce changes to increase the profits of a company are 0.8, 0.5 and 0.3 respectively. If increase in the profit does not take place, find the probability that it is due to the appointment of A. [4]

Sol. Let probability of A, B, C being selected be $P(A)$, $P(B)$ and $P(C)$ respectively

$\therefore \quad P(A) = \frac{1}{7}$, $P(B) = \frac{2}{7}$ and $P(C) = \frac{4}{7}$

Let P = Profit does not take place

also $P\left(\frac{\bar{P}}{A}\right) = 0.8$, $P\left(\frac{\bar{P}}{B}\right) = 0.5$, $P\left(\frac{\bar{P}}{C}\right) = 0.3$

i.e. $P\left(\frac{P}{A}\right) = 0.2$, $P\left(\frac{P}{B}\right) = 0.5$ and $P\left(\frac{P}{C}\right) = 0.7$

Using Bayes' theorem

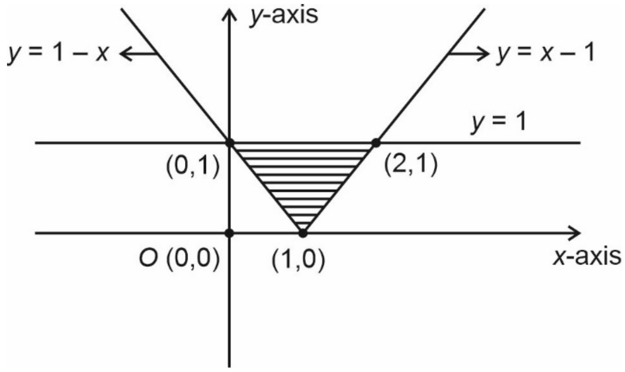
$$\therefore \quad P\left(\frac{A}{P}\right) = \frac{P\left(\frac{P}{A}\right)\cdot P(A)}{P\left(\frac{P}{A}\right)\cdot P(A) + P\left(\frac{P}{B}\right)\cdot P(B) + P\left(\frac{P}{C}\right)\cdot P(C)}$$

$$= \frac{0.2 \times \frac{1}{7}}{\left(0.2\times\frac{1}{7}\right) + \left(0.5\times\frac{2}{7}\right) + \left(0.7\times\frac{4}{7}\right)}$$

$$= \frac{0.2}{0.2+1+2.8} = \frac{0.2}{4} = \frac{1}{20}$$

12. Find the area bounded by the curves $y = |x - 1|$ and $y = 1$, using integration. [4]

Sol. Given curves are $y = |x - 1|$ and $y = 1$



$\therefore$ So, Area of shaded region (required area)

$$= \int_0^1 |x_2 - x_1| dy = \int_0^1 [(y+1) - (1-y)] dy$$

$$= \int_0^1 2y\, dy = \left[y^2\right]_0^1$$

$= 1 - 0$

$= 1$ sq. unit

13. (a) Solve the following differential equation: [4]

$$(y - \sin^2 x)dx + \tan x\, dy = 0$$

OR

(b) Find the general solution of the differential equation: [4]

$$(x^3 + y^3)dy = x^2 y dx$$

Sol. (a) $(y - \sin^2 x)\, dx + \tan x\, dy = 0$

$$y - \sin^2 x + \tan x \frac{dy}{dx} = 0$$

i.e. $\tan x \dfrac{dy}{dx} + y = \sin^2 x$

i.e. $\dfrac{dy}{dx} + y \cot x = \dfrac{1}{2}\sin 2x$

I.F. $= e^{\int \cot x\, dx} = e^{\ln \sin x} = \sin x$

Solution of linear Differential equation can be written as

$$y.(\sin x) = \int \frac{1}{2} \sin 2x \sin x\, dx$$

$$= \int \sin^2 x. \cos x\, dx$$

Let $\sin x = t$, $\cos x\, dx = dt$

$\Rightarrow \quad y.\sin x = \int t^2 dt = \frac{t^3}{3} + c$, where c is integration constant

$$\Rightarrow \quad \boxed{y.\sin x = \frac{(\sin x)^3}{3} + c}$$

OR

(b) $\frac{dy}{dx} = \frac{x^2 y}{x^3 + y^3}$

Dividing Numerator and Denominator by x^3

$$\frac{dy}{dx} = \frac{y/x}{1 + \left(y/x\right)^3}$$

Let $y = vx$

$$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{v}{1+v^3}$$

i.e. $\quad x\frac{dv}{dx} = \frac{v}{1+v^3} - v$

i.e. $\quad x\frac{dv}{dx} = v\left(\frac{-v^3}{1+v^3}\right)$

i.e. $\quad x\frac{dv}{dx} = \frac{-v^4}{1+v^3}$

i.e. $\quad \frac{\left(1+v^3\right)dv}{v^4} = \frac{-dx}{x}$

$\Rightarrow \quad \left(\frac{v^{-3}}{-3} + \ln v\right) = -\ln x - \ln c$, where c is arbitrary constant

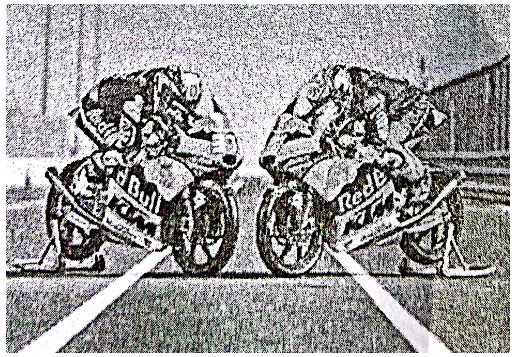
$$\Rightarrow \quad \frac{v^{-3}}{3} = \ln(v \times c)$$

Putting $v = \frac{y}{x}$

$$\boxed{\frac{1}{3}\frac{x^3}{y^3} = \ln(c\,y)}$$

Case Study Based Question

14. Two motorcycles *A* and *B* are running at the speed more than the allowed speed on the roads represented by the lines $\vec{r} = \lambda\left(\hat{i} + 2\hat{j} - \hat{k}\right)$ and $\vec{r} = \left(3\hat{i} + 3\hat{j}\right) + \mu\left(2\hat{i} + \hat{j} + \hat{k}\right)$ respectively. [2 × 2 = 4]



Based on the above information, answer the following questions:

(a) Find the shortest distance between the given lines. [2]

(b) Find the point at which the motorcycles may collide. [2]

Sol. The equation of road 1 is $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$

The equation of road 2 is $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$

For finding the lines are intersecting, skew or parallel

$$\vec{r} = \lambda\hat{i} + 2\lambda\hat{j} - \lambda\hat{k}$$

$$\vec{r} = (3 + 2\mu)\hat{i} + (3 + \mu)\hat{j} + \mu\hat{k}$$

equating

$\lambda = 3 + 2\mu$ ... (1)

$2\lambda = 3 + \mu$ ... (2)

$-\lambda = \mu$ ... (3)

Solving (1) and (2) we get $\lambda = 1$ and $\mu = -1$.

$\lambda = 1$ and $\mu = -1$ satisfies equation (3).

∴ Lines are intersecting.

Now, (a) Shortest distance b/w two roads = 0 {∵ roads are intersecting}

(b) Motor bike will collide at the point of intersection of the roads i.e. (1, 2, –1)

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