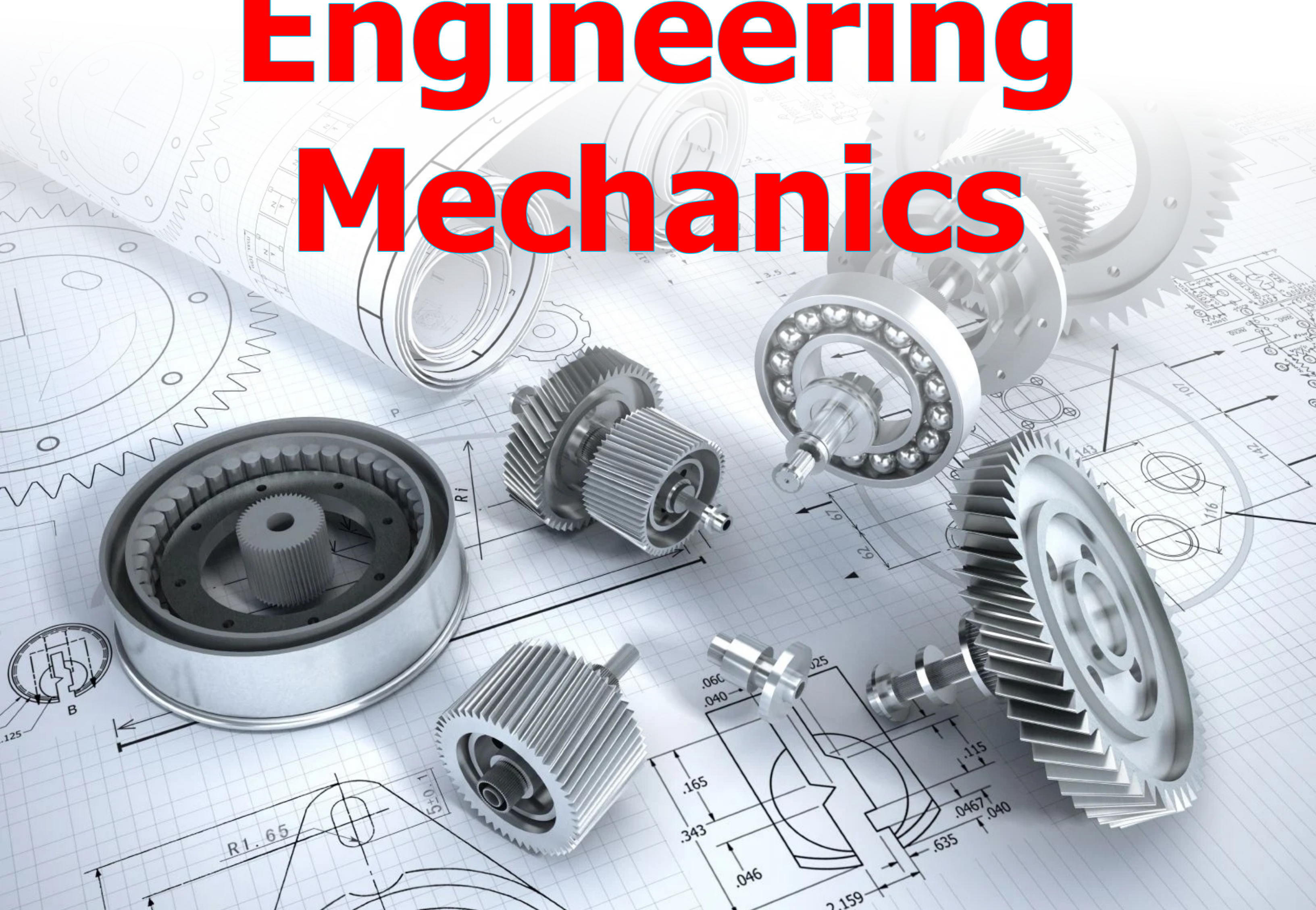
Engineering Mechanics



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ENGINEERING MECHANICS

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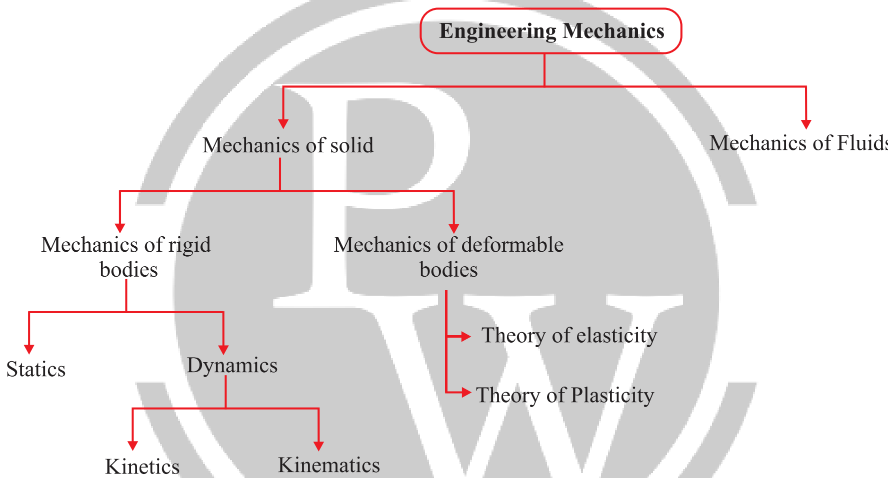
1 FUNDAMENTAL OF ENGINEERING MECHANICS

1.1. Fundamental Concepts of Mechanics

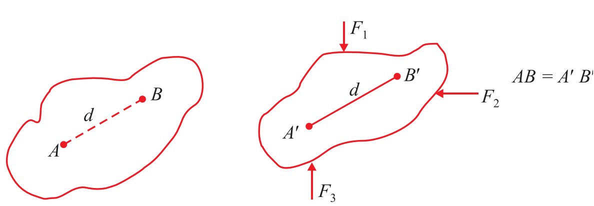
- Mechanics is a branch of physical science which deals with effects of force on objects.
- Application of law of mechanics to field problem is termed as Engineering mechanics.



- Static only-In Gate Syllabus.

1.1.1. Basic Terminologies in Mechanics

- **Length:** Used to locate the position of point of space and thereby describe the size of physical system.
- **Space:** The geometrical region in which study of body is involved is called space.
- **Time:** It is a measure of succession of event. As static is time independent, this quantity plays an important role in dynamics.
- **Mass:** Measure of inertia of body, which is its resistance to a change in velocity.
- **Particle:** It is a concentrated point mass of negligible dimension. Such body cannot exist theoretically. e.g., a bomber aeroplane is a particle for a gunner operating from ground.
- **Rigid body:** A body is said to be rigid, if the relative position of any two points in it do not change under the action of forces.



1.1.2. Newton's Law of Motion

(1) First Law:

A particle remains at rest or continues to move with uniform velocity if there is no unbalanced force acting on it.

(2) Second Law:

The rate of change of momentum of a body is directly proportional to impressed force and it takes place in the direction of force acting on it.

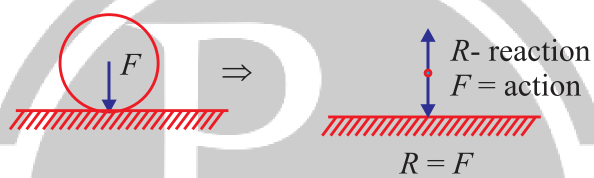
$$\vec{F} = \frac{\overrightarrow{dp}}{dt} = \frac{\vec{d}\,(mV)}{dt} = \frac{md\vec{V}}{dt} + \vec{V}.\frac{dm}{dt}$$

If mass 'm' is constant $\Rightarrow \vec{F} = m\vec{a}$

$$\frac{dm}{dt} = 0$$

(3) Third Law:

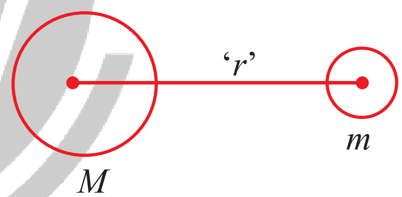
The force of action and reaction between interacting bodies are equal in magnitude, opposite in direction and collinear (they lie on the same plane)



1.1.3. Newton's Law of Gravitational Attraction:

Gravitational attraction between any two particles is formulated as

$$F = G\frac{mM}{r^2}$$



F = Force of gravity between two particles

G = Universal constant of gravitation

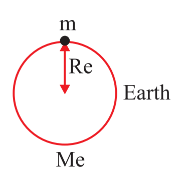
G = 6.67 × 10^{-11} Nm^2/kg^2

Weight:

When particle is located at surface of earth, the force of gravity between them is called weight.

$$W = G\frac{mM_e}{R_e^2}$$

$\Rightarrow$ $$\boxed{W = mg} = \frac{GMe}{R_e^2} = g = 9.81\,\text{m/s}^2$$



- For most engineering calculation 'g' is determined at sea level and at latitude of 45° (standard location)

  "The most effective way of learning the principles of engineering mechanics is to solve problem"

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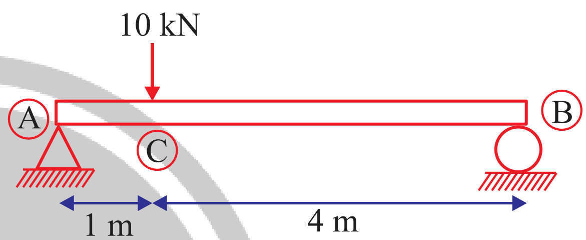
2 VECTORS

2.1. Force Vectors

In general force is considered as "Push" or "pull" exerted by one body on another.

A force is completely characterized by

(1) Magnitude
(2) Direction
(3) Point of Application
(4) Line of action



Eg.,

(1) Magnitude $\Rightarrow$ 10 kN
(2) Direction $\Rightarrow$ Downward
(3) Point of Application $\Rightarrow$At C which is 1 m from left support, i.e., point A.
(4) Line of action $\Rightarrow$ Vertical

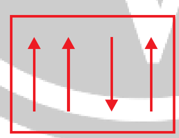
2.1.1. System of Force

When numerous amount of force are acting on a body simultaneously. They will form a system called force system

(1) Collinear

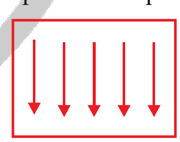
- Line of action of all force act along the same line

(2) Coplanar parallel

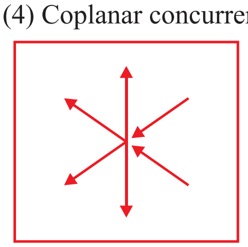


- All force are parallel to each other and lie in a single plane

(3) Coplanar like parallel

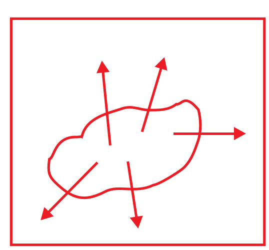


(4) Coplanar concurrent



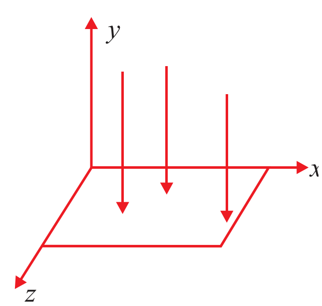
Line of action passing through a single point and force lie in same plane

(5) Coplanar non-concurrent

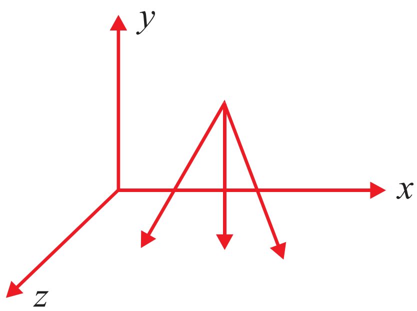


All force do not meet at a point but lies in a same plane.

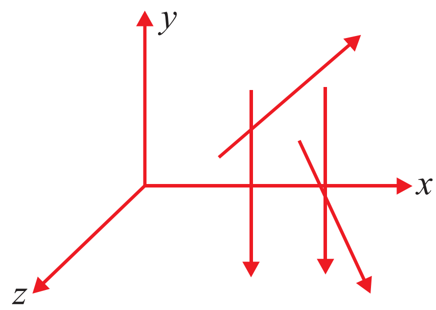
(6) non coplanar parallel



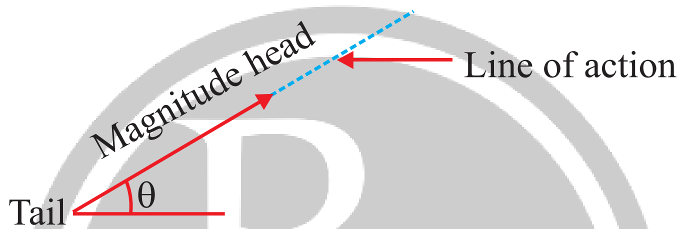
(7) non-coplanar concurrent



(8) Non Coplanar non concurrent

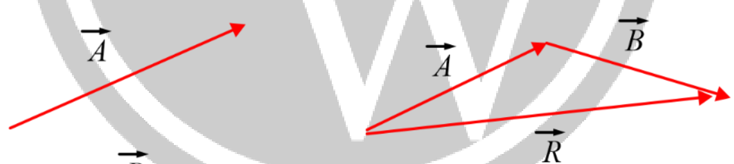


- **Scalar:** A scalar is any positive or negative physical quantity that can be completely specified by its magnitude. e.g., Mass, Time etc.
- **Vector:** A vector is any physical quantity that can be completely specified by its magnitude as well as direction. e.g., Force position, momentum, etc.



1.2.2. Vector Operations

(1) Vector Addition:

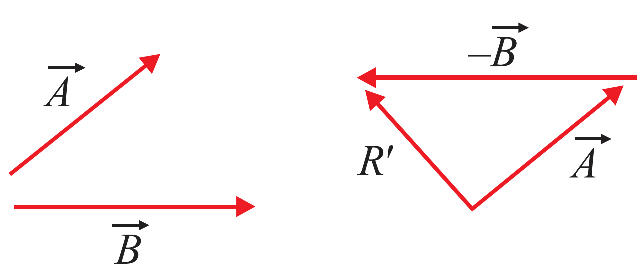


(Commutative)

$$\vec{R} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$$

(1) Add given vector "Head to tail" fashion
(2) Resultant vector is extend from tail of first to head of last vector

(2) Vector Subtraction:



$$R' = \vec{A} + (-\vec{B}) \neq -\vec{A} + \vec{B}$$ (not commutative)

2.1.3. Resultant Force:

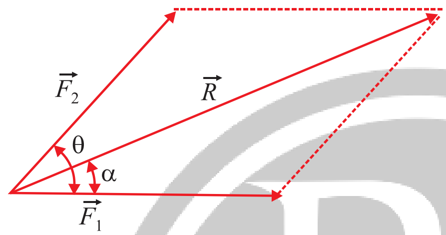
If number of forces are acting on a particle simultaneously then it is possible to find out a single-force which could replace them i.e., which could produce the same effect as produced by all given forces. This single called as resultant force.

Process of finding out resultant force is called composition or compounding of force.

Analytical method (most feasible)

- Parallelogram Law of Force
- Method of resolution

Parallelogram Law of Force:

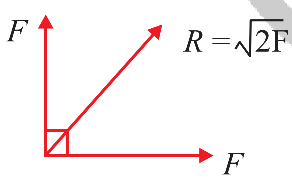


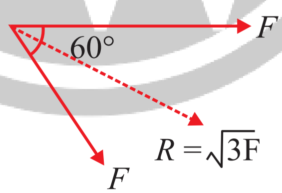
$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

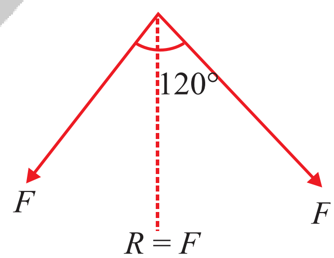
$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

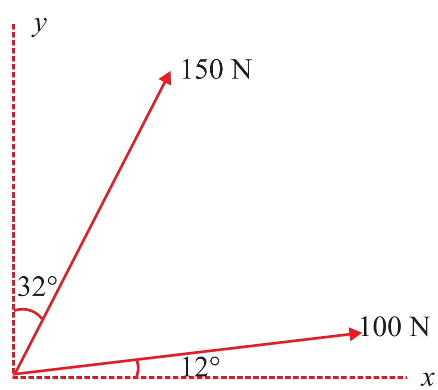
- $\theta = 90^\circ \quad R = \sqrt{F_1^2 + F_2^2}$
- If $\theta = 60^\circ \quad R = \sqrt{F_1^2 + F_2^2 + F_1F_2}$
- If $\theta = 0^\circ \quad R = F_1 + F_2$
- If $\theta = 180^\circ \quad R = F_1 - F_2$
- $F_1 = F_2 \quad R = 2F\cos\left(\frac{\theta}{2}\right)$

Remember A:









The magnitude and direction measured from horizontal of the resultant force is

$$F_1 = 100\ N\ F_2 = 150\ N\ \theta = 90 - 32 - 12 = 46°$$

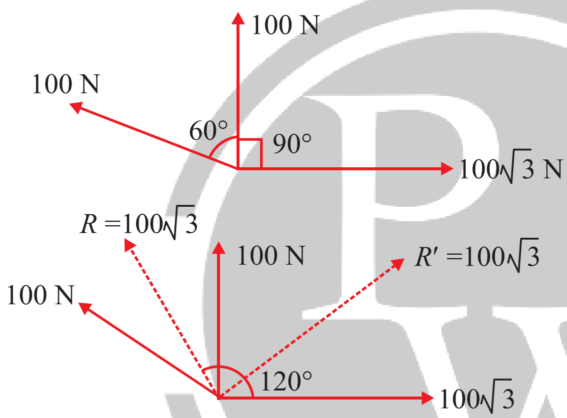
$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{100^2 + 150^2 + 2\times100\times150\times\cos 46} = 231\ \text{N}$$

$$\tan\alpha = \frac{F_2\sin\theta}{F_1F_2\cos\theta} = \frac{150\times\sin 46}{150+100\times\cos 46} = 0.431$$

$\Rightarrow$ $$\alpha = 26.18° \ \Rightarrow\ \phi = 12 + 26.18 = 38°18°$$

- The magnitude and direction measured from horizontal are:



$\Rightarrow$ Resultant $= 100\sqrt{3}$ N

Angle = 60°

- If two force F_1 and F_2 act at 60° their resultant is $\sqrt{13}$N and if they act at right angle their resultant is $\sqrt{10}$ N Find F_1 and F_2

(I) When $\theta = 90°$ $R = \sqrt{F_1^2 + F_2^2} \Rightarrow \sqrt{10} = \sqrt{F_1^2 + F_2^2}$

$\therefore$ $F_1^2 + F_2^2 = 10$

(II) When $\theta = 60°$ $R = \sqrt{F_1^2 + F_2^2 + F_1F_2} \Rightarrow \sqrt{13} = \sqrt{F_1^2 + F_2^2 + F_1F_2}$

$$13 = F_1^2 + F_2^2 + F_1F_2$$

$$F_1F_2 = 13 - 10 = 3$$

$$(F_1 + F_2)^2 = F_1^2 + F_2^2 + 2F_1F_2 = 10 + 2\times3 = 16$$

$\Rightarrow F_1 = 3\ \text{kN},\quad F_2 = 1\ \text{kN}$

$\Rightarrow F_1 + F_2 = 4$ ….(1)

$(F_1 - F_2)^2 = F_1^2 + F_2^2 - 2F_1F_2 = 10.2\times3 = 4$

$\Rightarrow F_1 - F_2 = 2$ ….(2)

Methods of Resolution:

Steps:

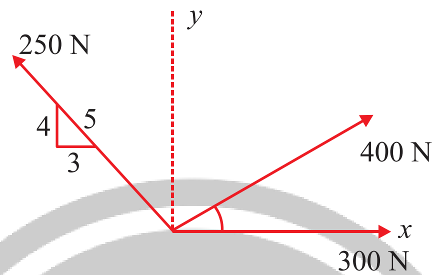
(1) Resolve the forces horizontally and find algebraic sum of all horizontal component (ΣF_x)

(2) Resolve the forces vertically and find algebraic sum of all vertical component (ΣF_y)

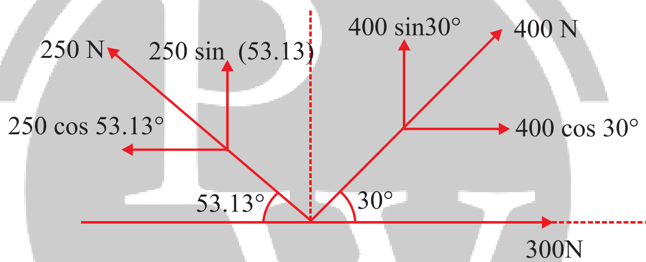
(3) Resultant $R\sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

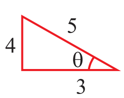
(4) Resultant force will be inclined at an angle θ with horizontal

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}$$



Determine the magnitude and direction of resultant force





$$\Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°$$

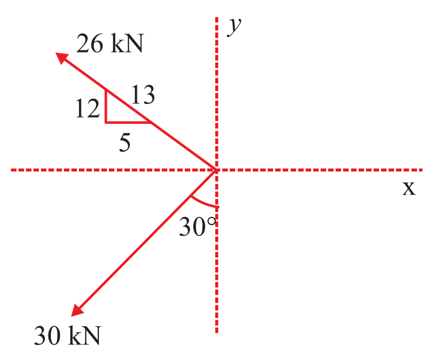
$$\Sigma F_x = 400\cos 30 + 300 - 250\cos 53.13 = 496.4 \text{ N}$$

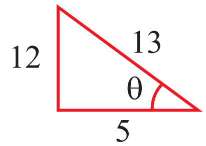
$$\Sigma F_y = 400\sin 30 + 250\sin(53.13) = 400 \text{ N}$$

$$R = \sqrt{496.4^2 + 400^2} = 637.5 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{400}{496.4}\right) = 38.86°$$

Find the direction of resultant force measured counter clockwise from positive x-axis.



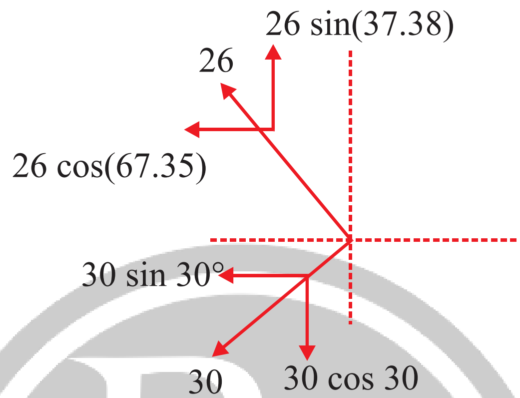


$$\theta = \tan^{-1}\left(\frac{12}{3}\right) = 67.38^\circ$$

$$\Sigma F_x = -26\cos(67.38) - 30\sin 30 = -25\ \text{kN}$$

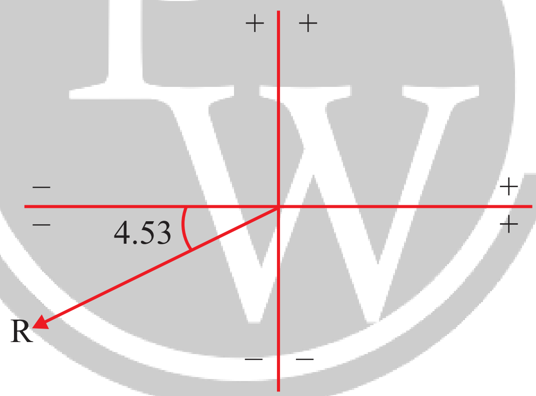
$$\Sigma F_y = 26\sin(67.38) - 30\cos 30 = -1.98\ \text{kN}$$

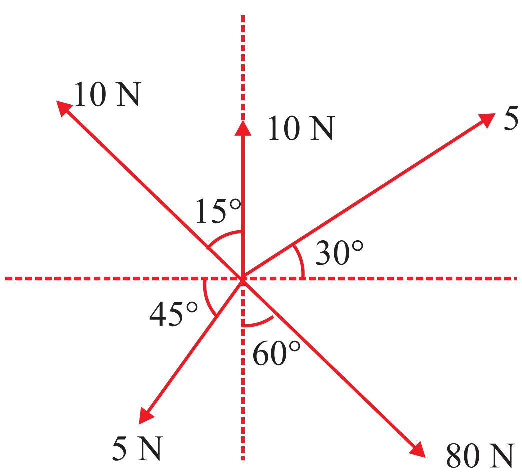
$$\theta = \tan^{-1}\left(\frac{-1.98}{-25}\right) = 4.53^\circ$$

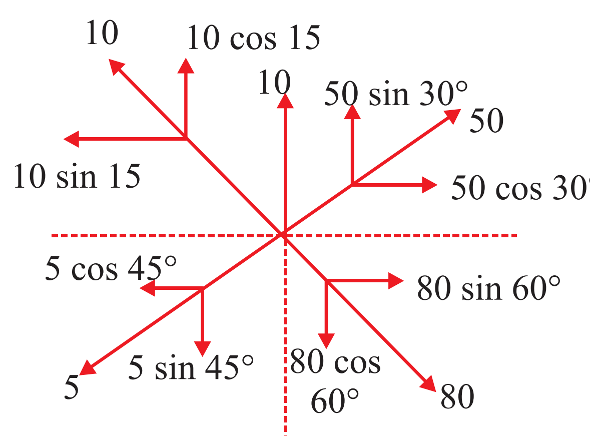


- As both F_x and F_y are –ve $\Rightarrow$ 3rd quadrant.

$\Rightarrow\ \theta = 180 + 4.53 = 184.53^\circ$



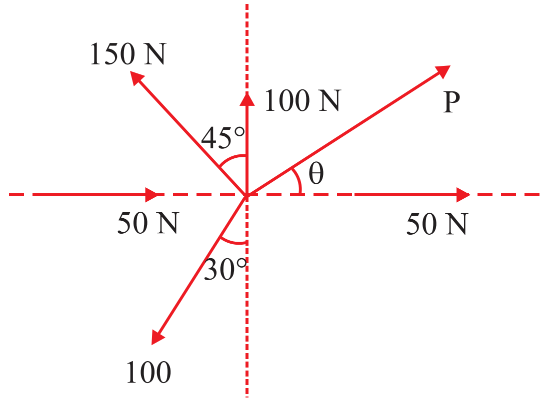




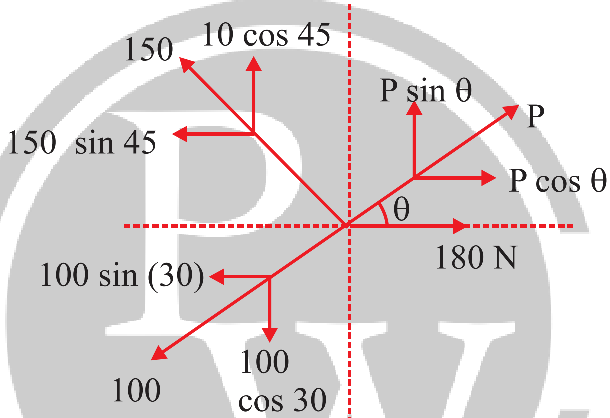
$$\Sigma F_x = 50\cos 30 + 80\sin 60 - 5\cos 45 - 10\sin 15 = 106.46\ \text{N}$$

$$\Sigma F_y = 50\sin 30 + 80\cos 60 - 5\sin 45 - 10\cos 15 = 51.80 \text{ N}$$

$$R = \sqrt{106.46^2 + 51.80^2} = 118.39 \text{ N}$$



A system of force has resultant 80 N along positive *x*-axis find magnitude and inclination of unknown force *P*



$$\Sigma F_x = 180 + P\cos\theta - 150\sin 45 - 100\sin 30$$

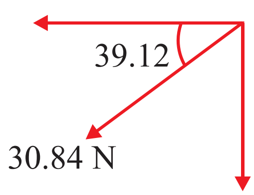
$$= P\cos\theta + 23.93 = 0 \Rightarrow P\cos\theta = -23.93$$

$$\Sigma F_y = P\sin\theta + 150\cos 45 - 100\cos 30$$

$$= P\sin\theta + 19.46 = 0 \Rightarrow P\sin\theta = -19.46$$

$$\tan\theta = \left(\frac{-19.46}{-23.93}\right) \Rightarrow \theta = 39.12°$$

$$P = \frac{-23.93}{\cos(39.12)} = -30.84 \text{ N}$$



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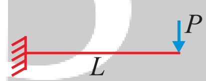
3 MOMENT

3.1. Introduction

3.1.1. Moment:

- Turning effect of force.
- Moment of a force is equal to the product of Force and the perpendicular distance of the point about which moment is required and line of action of force.

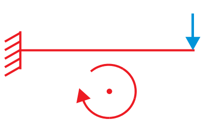
  Moment at 'O' = $P \times L$



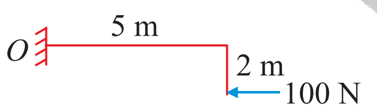
Unit: KN-m

Types of Moment:

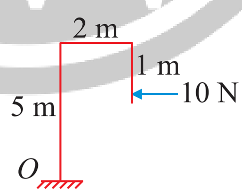
(1) Clockwise moment (2) Anticlockwise moment



➢ Determine the moment about point O

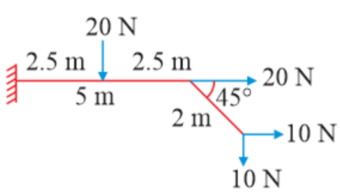


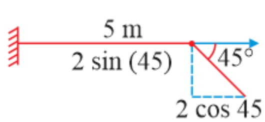
$M_o = 100 \times 2 = 200$ N.m (clockwise)



$M_o = 10 \times 4 = 40$ N.m (anticlockwise)

➢ Find moment of point O



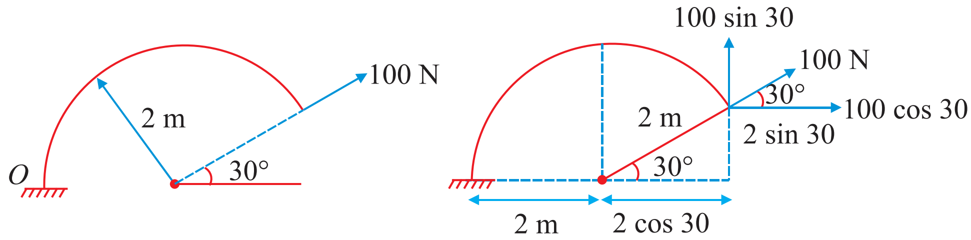


Assuming c/w +ve

$$M_o = 10 \times (5 + 2\cos 45) - 10 \times 2\sin 45 + 20(0) + 20 \times 2.5$$

$= 100$ N-m (clockwise)

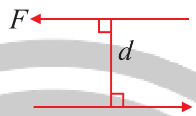
➢ Determine moment at point O



$$M_o = 100\sin 30°(2 + 2\cos 30°) - 100\cos 30°\,(2\sin 30°)$$
$$= 86.6\ \text{N-m}$$

3.1.2. Couple

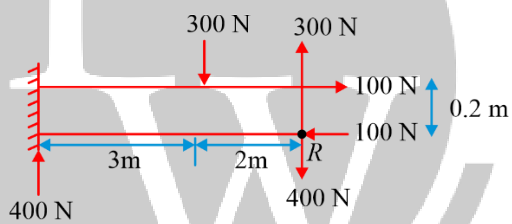
- It is defined as two parallel forces that have the same magnitude and opposite direction and are separated by perpendicular distance d.
- Resultant of force is zero
- Only effect is to produce rotation



Moment of couple

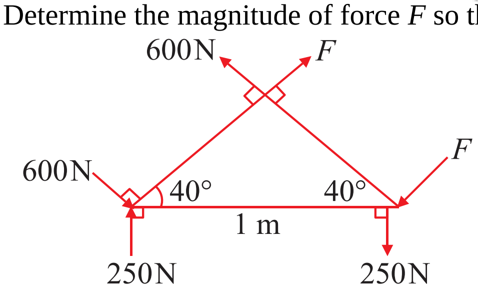
$$M = Fd$$

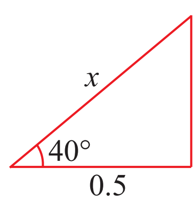
➢ Determine the resultant couple moment acting on beam



$$M_R = -300 \times 2 + 400 \times 5 + 100 \times 0.2$$
$$= 1420\ \text{N-m}$$

➢ Determine the magnitude of force F so that the resultant couple moment is 400 N.m clockwise.





$\cos 40 = 0.5/x$

$\Rightarrow \quad x = 0.652$

$M_R = F \times 0.652 - 600 \times 0.652 + 250 \times 1$

$= 204.400$ Nm

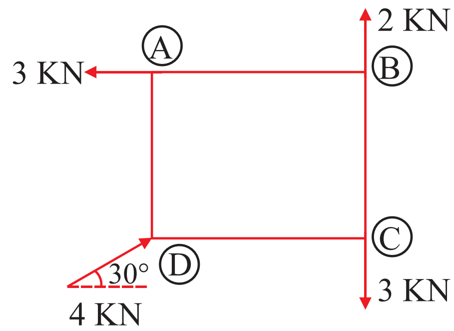
$\Rightarrow \quad F = 530.06$ N

3.1.3. Varignon's Principle of Moments

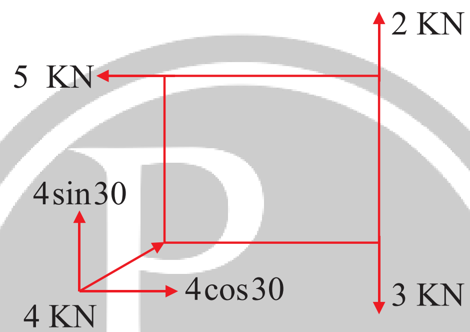
- It is the number of coplanar force are acting simultaneously on a particle, the algebraic sum of moments of all forces about any point is equal to the moment of their resultant force about the same point"

Application:

Position of resultant force for non-concurrent force system.



Four coplanar forces are acting on a square body having it side 1 m. Determine the position of single resultant force from point A.



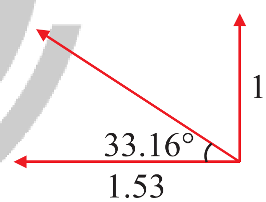
$$\sum fx = 4\cos 30 - 5 = -1.53\,\text{KN}$$

$$\sum fy = 2 + 4\sin 30 - 3 = 1\,\text{KN}$$

$$\Rightarrow R = \sqrt{1.53^2 + 1^2} = 1.83\,\text{KN}$$

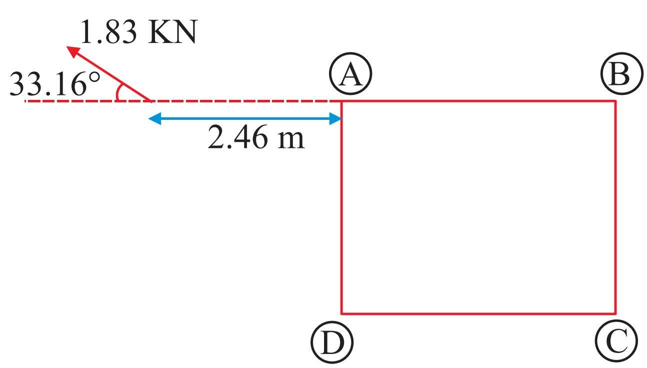
$$\tan\theta = \left(\frac{1}{1.53}\right)$$

$$\theta = 33.16°$$

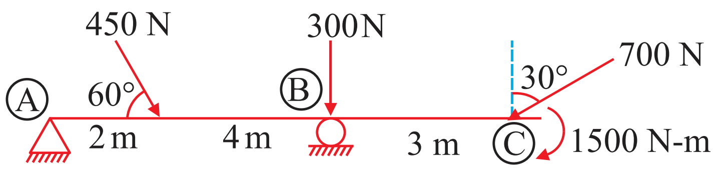


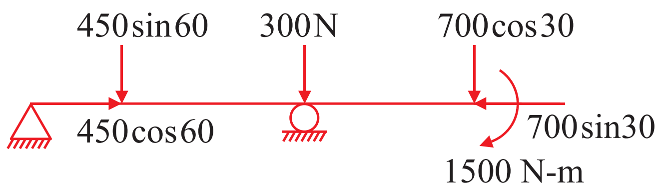
By Vangon's Theorem

$$x = \frac{\Sigma M_A}{R_y} = \frac{-2\times 1 + 3\times 1 - 4\cos 30\times 1}{1} = -2.46\,\text{m (Left of } A)$$



Replace the loading acting on beam by single resultant force. Specify where the force acts measured from *B*.



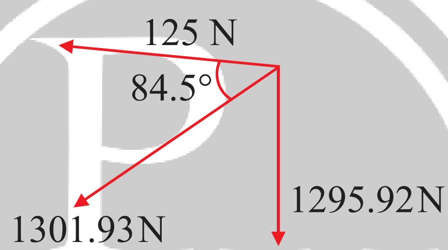


$$\sum fy = 450\sin 60 + 300\text{ N} + 700\cos 30 = 1295.92\text{ N}(\downarrow)$$

$$\sum fx = 450\cos 60 - 700\sin 30 = -125\text{ KN}$$

$$R = \sqrt{(1295.92)^2 + (125)^2} = 1301.93\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{1295.92}{125}\right) = 84.5°$$

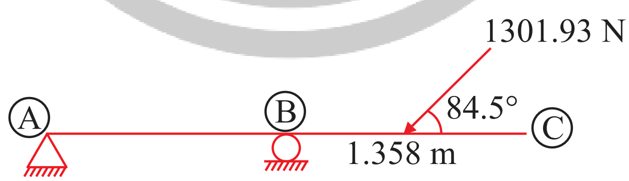


$$\sum M_B = 450\sin 60\times 4 - 3\times 700\cos 30 - 1500 = -1759.8\text{ N-m}$$

By Varignon's principle

$$Ryd = \Sigma M_B$$

$$\Rightarrow \quad d = \frac{1759.8}{1500} = 1.358\text{ m}$$



❑❑❑

4 EQUILIBRIUM

4.1. Introduction

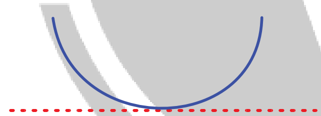
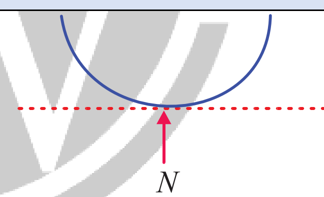
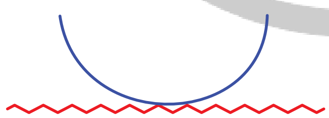
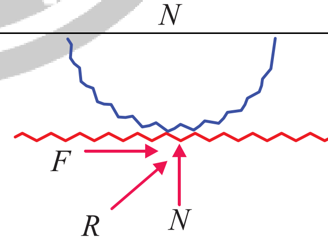
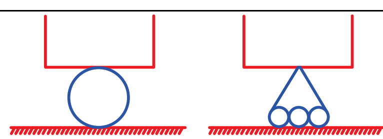
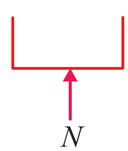
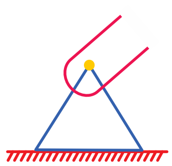
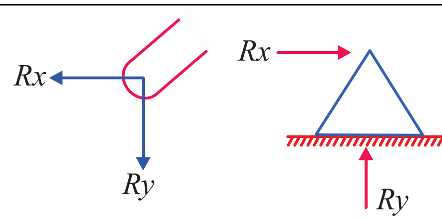
(a) Equilibrium of a Particle

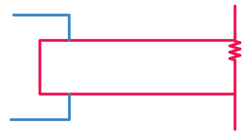
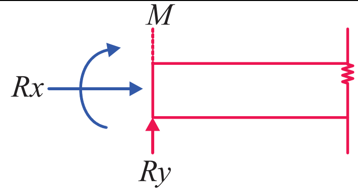
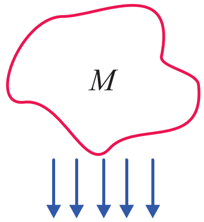
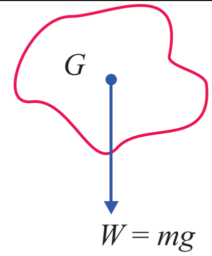
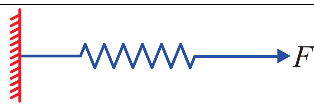
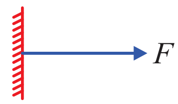
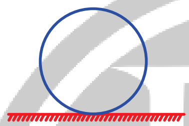
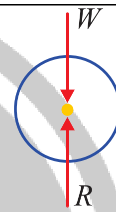
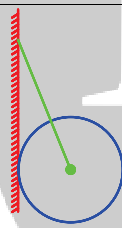
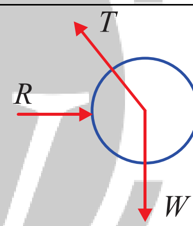
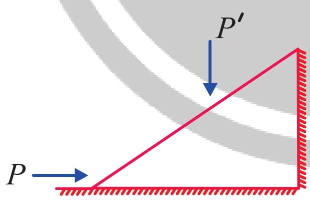
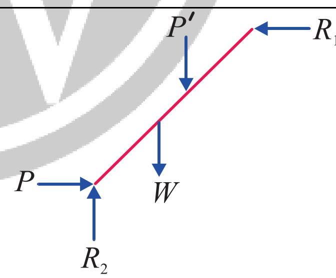
A particle is said to be in equilibrium, if both resultant force '*R*' and the resultant couple '*M*' both are zero for 2D bodies.

$$\sum Fx = 0 \quad \sum Fy = 0 \quad \sum M = 0$$

(b) Free body diagrams (FBD):

A diagram of a body in which the body under consideration is freed from all contact surface and is shown with all the force on it (including self weight and reaction from other contact surface) is called free body diagram.

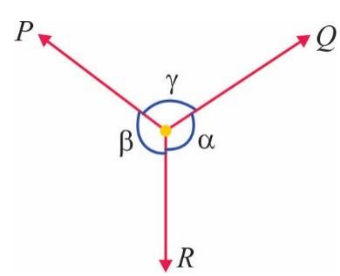
	Reacting Bodies	FBD
1	 smooth surface	
2	 rough surface	
3	 roller support	
4	 pin connection	

5	 build in/fixed support	
6	 Gravitational attraction	
7	 spring force	
8	 ball	
9		
10	 Ladder	

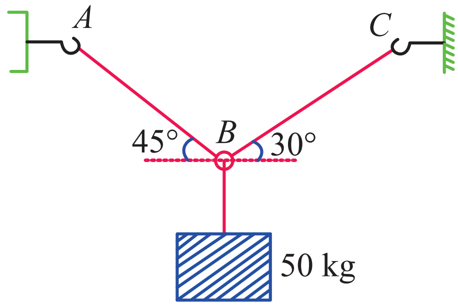
(c) Lami's Theorem:

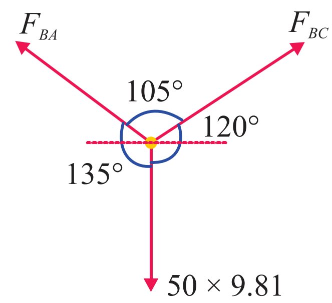
"If three coplanar force acting at a point be in equilibrium, then each force is proportional to sine of the angle between other two"

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$



- Determine tension in cable AB and BC.



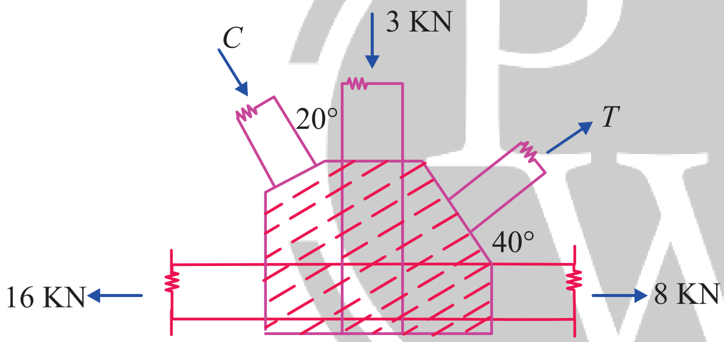


By Lami's Theorem

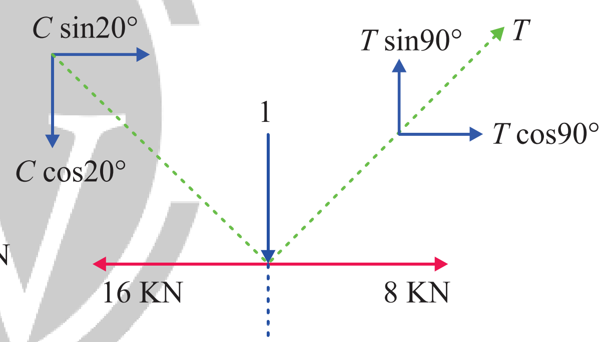
$$\frac{F_{BA}}{\sin 120°} = \frac{F_{BC}}{\sin 135°} = \frac{50 \times 9.81}{\sin 105°} = 507.8$$

$\Rightarrow$ $F_{BA} = 439.77\ N$ and $F_{BC} = 395.06\ N$

- Determine magnitude of C and T



Bridge truss joint



$$\sum Fx = -16 + 8 + C\sin 20° + T\cos 40° = 0$$

$$T\cos 40° + C\sin 20° = 8 \quad \text{...(i)}$$

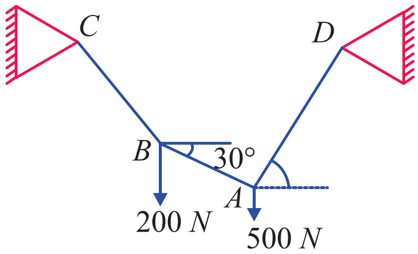
$$\sum Fy = -3 - C\cos 20° + T\sin 40° = 0$$

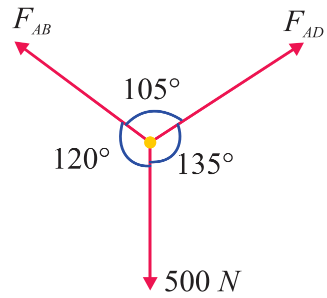
$$T\sin 40° - C\cos 20° = 3 \quad \text{...(ii)}$$

$\Rightarrow$ C = 3.03 KN

T = 9.09 KN

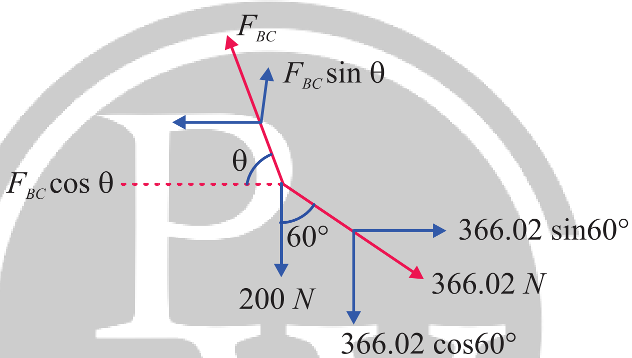
- Identify each force





$$\frac{F_{AB}}{\sin 135°} = \frac{F_{AD}}{\sin 120°} = \frac{500}{\sin(105)}$$

$\Rightarrow$ $F_{AB} = 366.02$ N $F_{AD} = 448.28\ N$



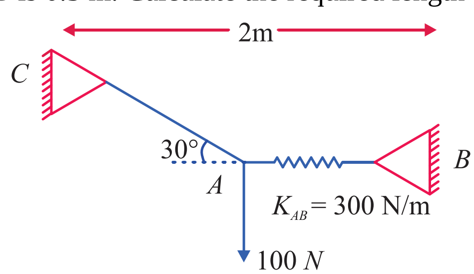
$$\sum Fy = 0$$

$\Rightarrow$ $F_{BC} \sin\theta = 200 + 366.02 \cos 60 = 383.01$

$\Rightarrow$ $F_{BC}^2 \cos^2\theta + F_{BC}^2 \sin^2\theta = 316.98^2 + 383.01^2$

$\Rightarrow$ $F_{BC} = 497.16\ N$

- If undeformed length of spring *AB* is 0.5 m. Calculate the required length of chord *AC*.

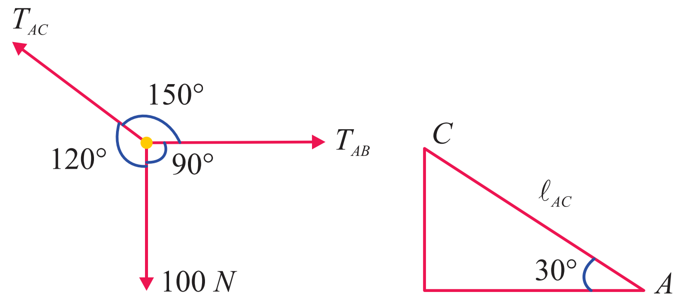


$$\frac{100}{\sin 150°} = \frac{T_{AC}}{\sin 90°} = \frac{T_{AB}}{\sin 120°}$$

$\Rightarrow$ $T_{AC} = 200\ N$ $T_{AB} = 173.2\ N$

$$T_{AB} = K_{AB} \times \ell'_{AB} \Rightarrow \ell'_{AB} = \frac{173.2}{300} = 0.577\ \text{m}$$

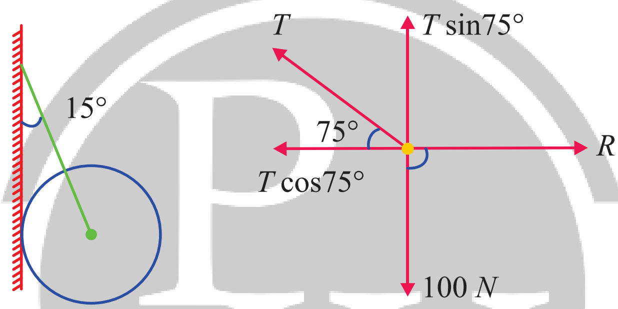
⇒ Stretched length = 0.5 + 0.577 = 1.077 m



⇒ ℓ_{AC} = cos30° + 1.077 = 2

⇒ ℓ_{AC} = 1.065 m

- A sphere weighing 100 N is tied to a smooth wall by a string as shown in figure. Find the tension T in the string and reaction R from the wall.



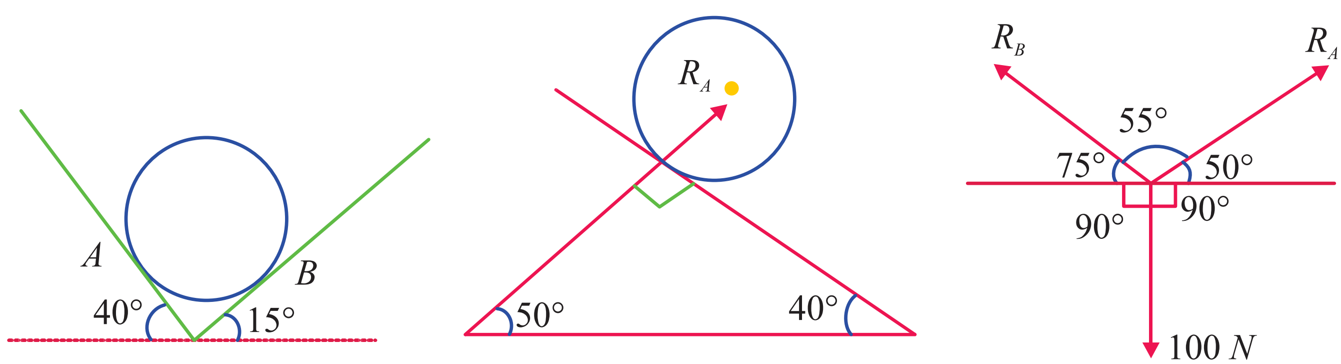
⇒ $R = T\cos75°$ $\sum Fx = 0$

$100 = T\sin75°$ $\sum Fy = 0$

⇒ $T = 103.52\ N$

$R = 26.79\ N$

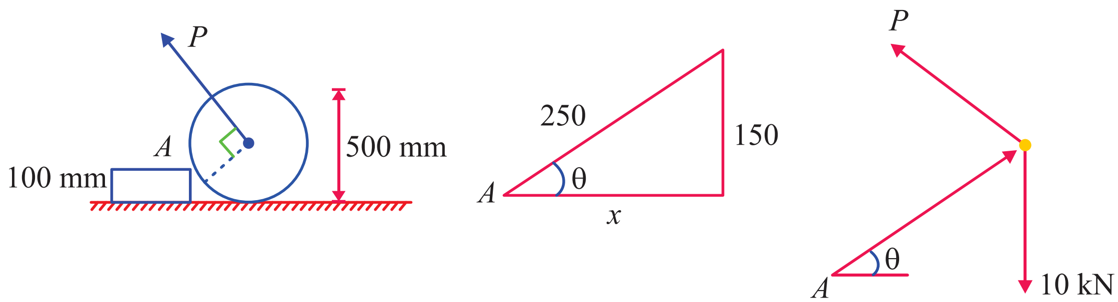
- A smooth circular cylinder of radius 1.5 m is laying in triangular groove as shown in figure. Find the reaction at contact surface. Assume surface is frictionless and cylinder weight is 100 N.



$$\frac{R_B}{\sin(140)} = \frac{R_A}{\sin(165)} = \frac{100}{\sin 55}$$

$\Rightarrow$ $R_A = 31.6\ N$ $R_B = 78.5\ N$

- A uniform wheel of 500 mm weighing 10 kN rest against a rigid rectangular block of 100 mm height. Find the least pull, through centre of wheel require to just turn the wheel over the corner *A* of block.

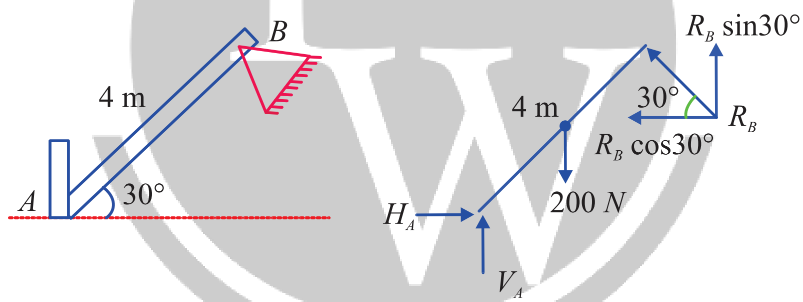


$$x = \sqrt{250^2 - 150^2} = 200$$

$$\sum M_A = 0 \Rightarrow P \times 250 = 10 \times 200$$

$\Rightarrow$ $P_{min} = 8$ kN

- Find reaction acting on the bar of weight 200 *N*.



$$\sum M_A = 0$$

$R_B \times 4 + 200 \times 2\cos 30° = 0$

$R_B = 86.6\ N$

$$\sum fx = 0$$

$R_B \sin 30° – 200 + V_A = 0$

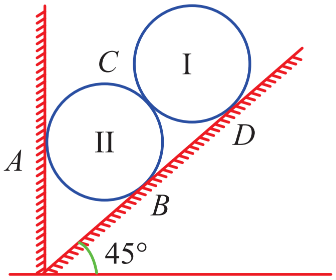
$V_A = 200 – 86.6 \sin 30° = 156.7\ N$

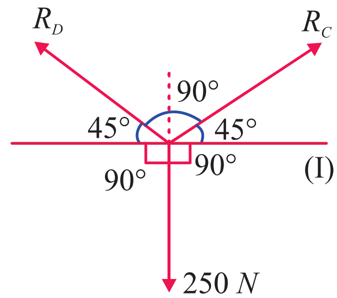
$$\sum fy = 0$$

$R_B \cos 30° = H_A$

$H_A = 75\ N$

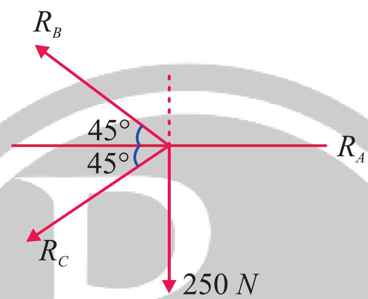
- If two identical cylinder each weighing 250 N then find the reaction at point A. (Assume all point of contact are smooth)





$$\frac{R_D}{\sin 135°} = \frac{R_C}{\sin 135°} = \frac{250}{\sin 90°}$$

$\Rightarrow$ $R_C = 176.77\ N$



$$\sum Fx = 0$$

$\Rightarrow$ $R_A - R_B \cos 45° - R_C \cos 45° = 0$

$\Rightarrow$ $R_A - R_B \cos 45° = 425$ …(i)

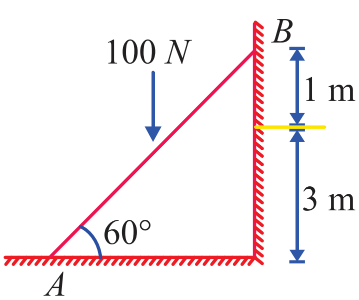
$$\sum Fy = 0$$

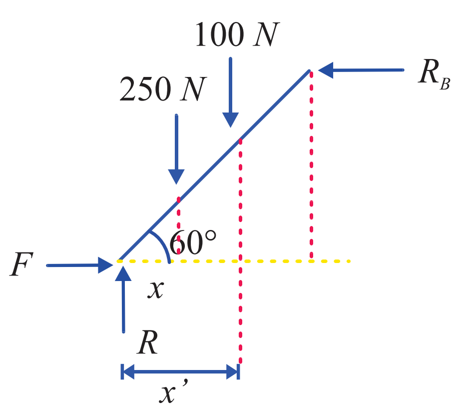
$$-250 - 276.77 \sin 45° + R_B \sin 45° = 0$$

$$R_B = 530.32\ N$$

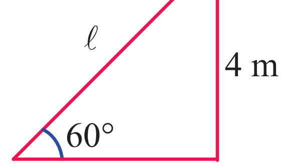
From (i) $R_A = 500\ N$

- A ladder weighting 250 N is to be kept in the position as shown in figure. Determine the horizontal force required at floor level to prevent it from slipping.





$x' = 2.31 \cos 60°$
$= 1.15$ m



$$\Rightarrow \quad \sin 60° = \frac{4}{\ell}$$

$$\Rightarrow \quad \ell = 4.62 \text{ m}$$

$$x'' = \frac{3}{\tan 60°} = \sqrt{3} \text{ m}$$

$$\sum M_A = 0$$

$$-R_B \times 4 + 100 \times \sqrt{3} + 250 \times 1.15 = 0$$

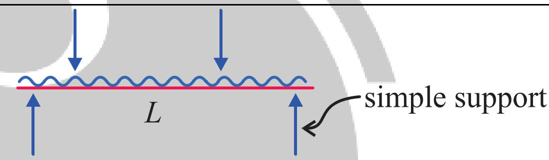
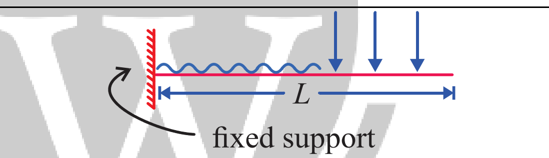
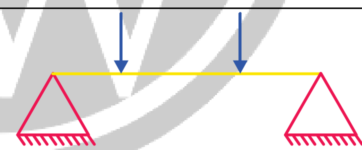
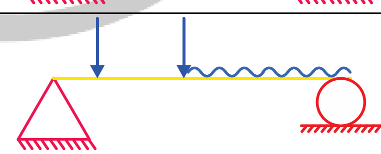
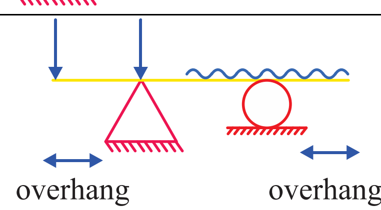
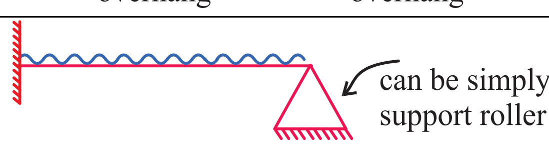
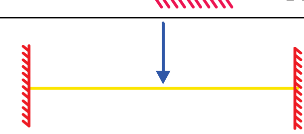
$$\Rightarrow \quad R_B = 115.18 \, N$$

$$\sum Fx = 0$$

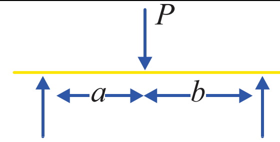
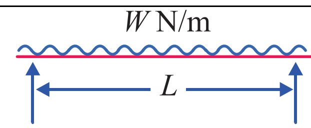
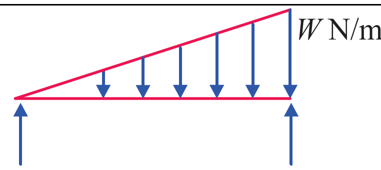
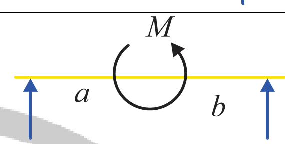
$$\Rightarrow \quad F = R_B = 115.18 \, N$$

(d) Application to Beam problem

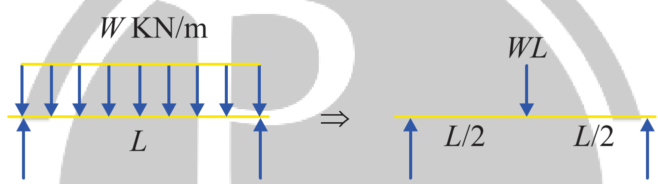
Type of Beam

1	Simply supported	
2	Cantilever	
3	Both end hinge	
4	One end hinge other end rotter	
5	Overhanging	
6	Propped cantilever	
7	Fixed	

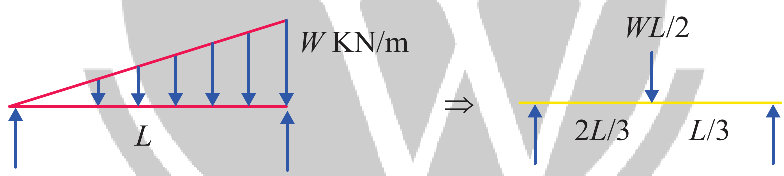
Type of Loading

1	Concentrated point load	
2	Uniformly distributed load (UDL)	
3	Uniformly varying load (UVL)	
4	External moment	

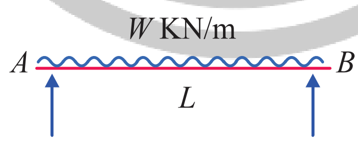
Conversion of load:



"Equivalent concentrated point load = Area of loading diagram which act at center of gravity of loading diagram"



Find the reaction at both end.



$$\sum M_A = 0$$

$\Rightarrow$ $$-R_B \times 1 + (W \times L) \times \frac{L}{2} = 0$$

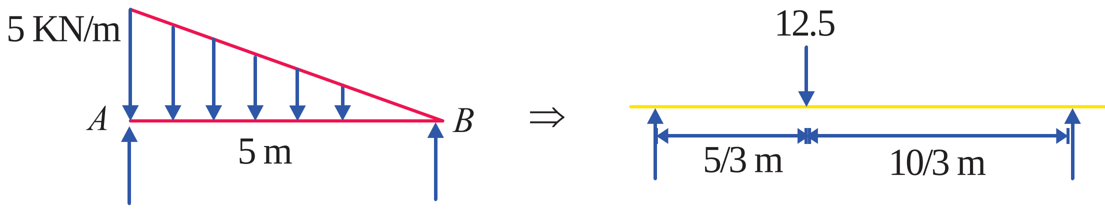
$$R_B = \frac{WL}{2}$$

$$\sum fy = 0$$

$\Rightarrow$ $$R_A + R_B = WL$$

⇒ $R_A = \frac{WL}{2}$

⇒ $\frac{1}{2} \times 5 \times 5 = 12.5 \text{ KN}$



$\sum M_A = 0$

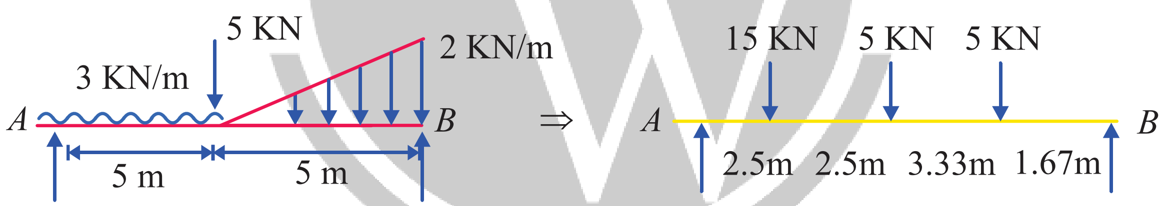
$-R_B \times 5 + 12.5 \times \frac{5}{3} = 0$

⇒ $R_B = 4.167$ KN

$\sum fy = 0$

$R_A + R_B = 12.5$

$R_A = 12.5 - 4.167 = 8.33$ KN



$\sum M_B = 0$

$R_A \times 10 - 15 \times 7.5 - 5 \times 2.5$

$-5 \times 1.67 = 0$

⇒ $R_A = 13.335$ KN

$\sum fy = 0$

$R_A + R_B = 15 + 5 + 5 = 25$

⇒ $R_A = 11.665$ KN

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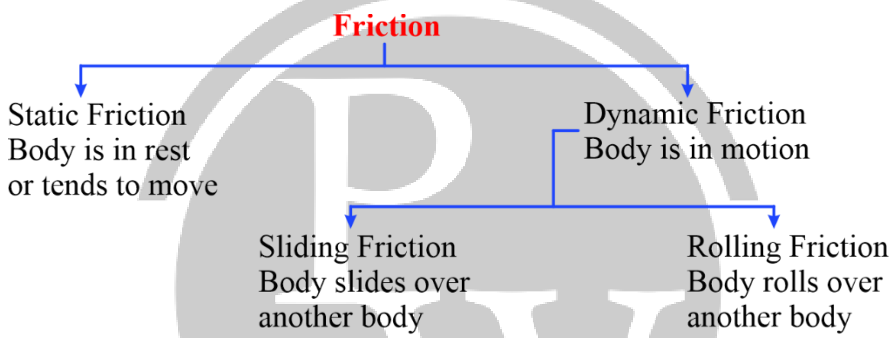
5 FRICTION

5.1. Introduction

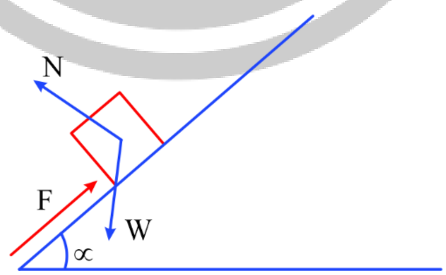
(a) Friction:

- It is a force that resist the movement of two contacting surface that slide relative to one another.



(b) Angle of Friction:

- Let the angle (α) be gradually increased, till the body just starts sliding down the plane. This angle at which a body just begins to slide down the plane, is called angle of friction/ angle of repose.



(c) Limiting Friction:

- When a body is at verge of moving, the force of friction acting at that time is called limiting friction or impending motion.

Mathematically $F_s = \mu_s N$

$= F > F_s$ (slipping occurs)

$F_s \rightarrow$ limiting frictional force

$\mu_s \rightarrow$ Coeff. of friction

$F < F_s$ (Motion not impending)

.

(d) Coefficient of Friction: -

- Ratio of limiting friction to normal reaction.

$$\mu = \frac{F}{N} = \tan\theta$$

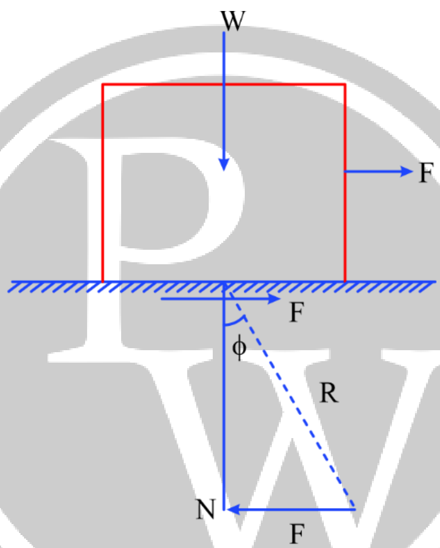
θ = Angle of friction

(e) Angle of Friction: -

- Ratio of limiting friction to normal reaction.

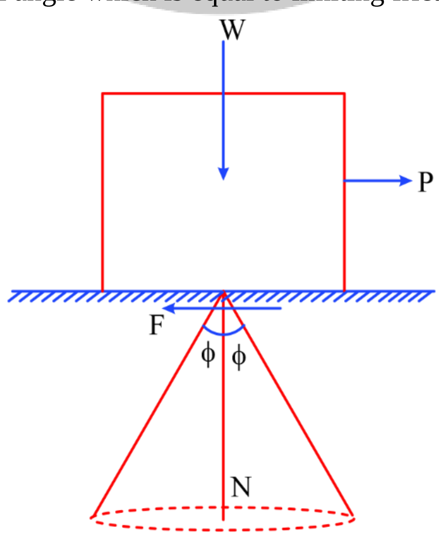
$$\tan\theta = \frac{F}{n}$$

$$\tan\theta = \mu$$



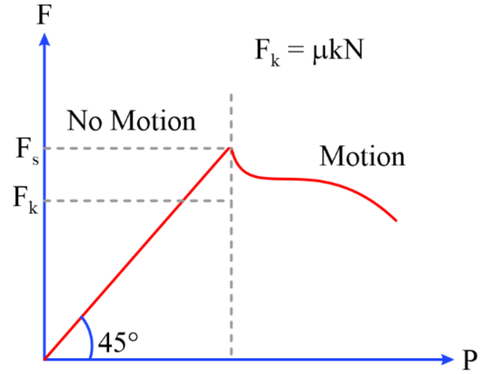
(f) Cone of Friction: -

- The inverted cone with semi central angle which is equal to limiting friction angle is called cone of friction.



(g) Kinetic Friction: -

- After slipping occurs, a condition of kinetic friction accompanies the ensuring motion.



$F_k = \mu k N$

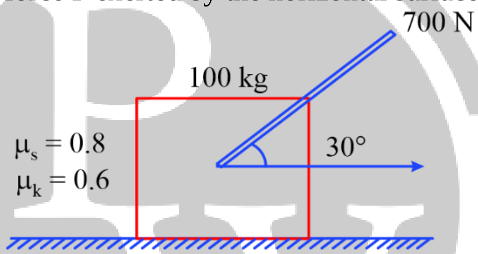
$\mu_k \rightarrow$ Coeff. Of kinetic friction.

$\mu_s \geq \mu_k$

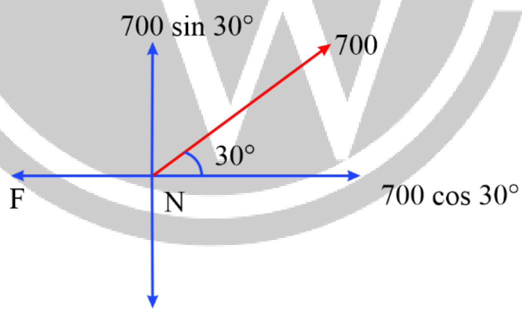
$\mu = (0 \text{ to } 1)$

$\mu > 1 \rightarrow$ Silicon rubber or acrylic rubber coated surface.

- Determine the magnitude of frictional force F exerted by the horizontal surface.



- Assume equilibrium



$$\sum F_x = 0$$

$$700 \cos 30° - F = 0$$

$$F = 606 \text{ N}$$

$$\sum F_y = 0$$

$$\text{N} - 981 + 700 \sin 30° = 0$$

$$\Rightarrow \quad N = 631 \text{ N}$$

$$F_{max} = \mu_s N = 0.8 \times 631 = 505 < F = 606 \text{ N}$$

Motion occurs

$$F = \mu_k N = 0.6 \times 631 = 379 \text{ N}$$

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6 TRUSSES

6.1. Trusses

A truss is structure composed of slender member joined together at end points by bolting/Rivelting/welding. Ends of the member are pointed to a common plate called Gusset plate

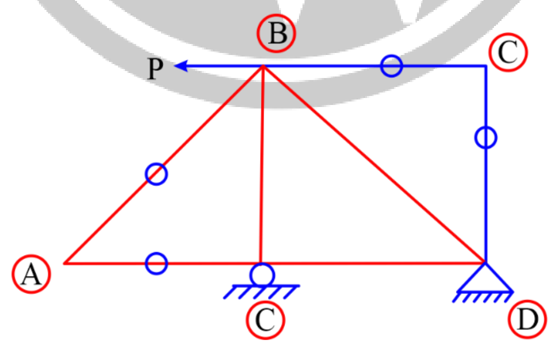
6.1.1. Assumption

- Member joint together by smooth pin
- All loading are applied to joint
- Self Weight of member is negligible.

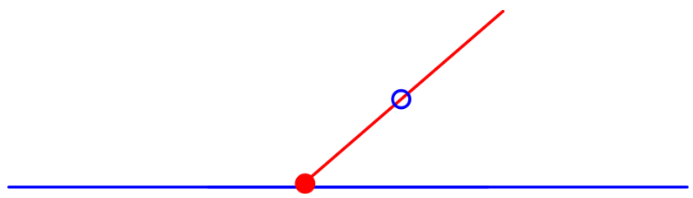
6.1.2. Zero Force Member

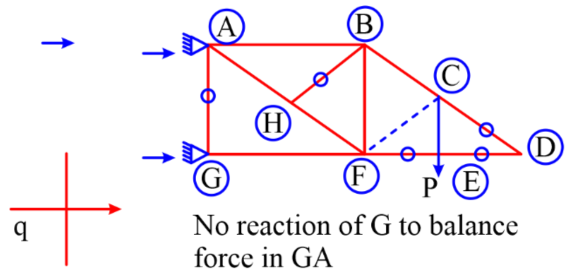
They are provided to reduce effective length of other members.

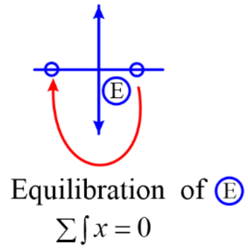
- If two non – collinear member exist at a truss joint and no external force or support reaction is applied at joint, the member must be zero force member.

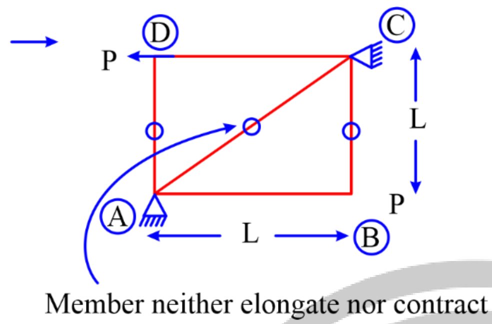


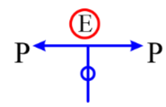
- If 3 member joint at a point and out of 2 are collinear and also no external load acts at joint, then the third member is zero force member.

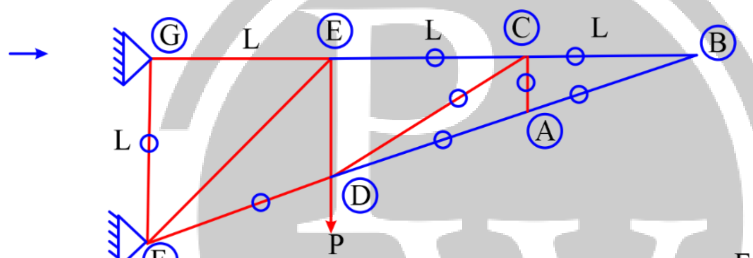


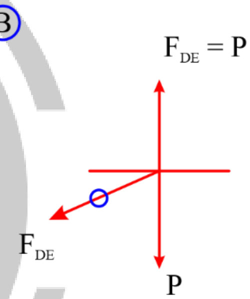


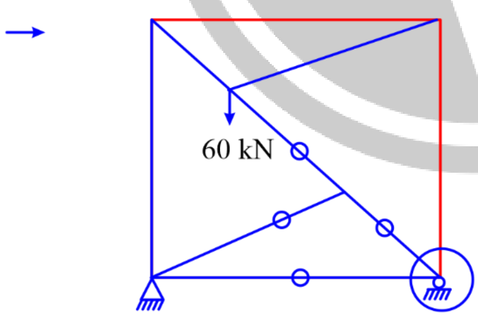


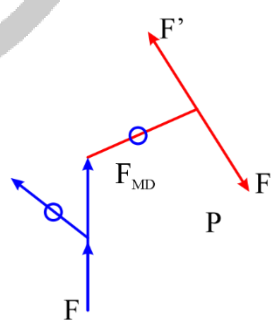












1. Method of joint:-

- Equilibrium of each joint is considered
- Not applicable if number of unknown forces at the joint is more than 2
- Start from there where 2 member forces are unknown.

Sign Convention

Ques. Calculate forces in all members given truss.

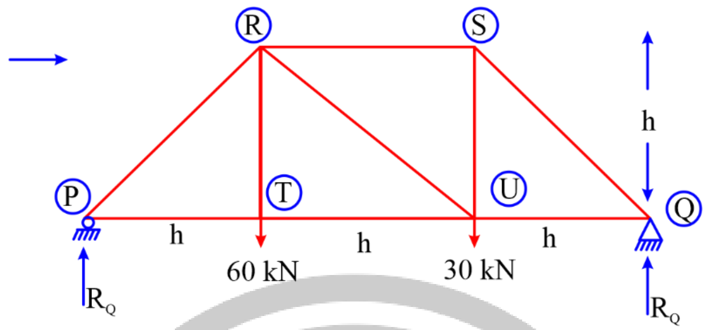
$$R_P + R_Q = 90$$

$$\Sigma M_P = 0$$

$$R_Q \times 3h = 30 \times 2h + 60 \times h$$

$$R_Q = 40 \text{ kN}$$

$$\Rightarrow R_P = 50 \text{ kN}$$



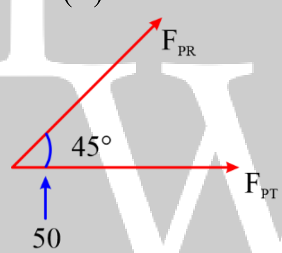
$$\sum f_x = 0 \Rightarrow F\eta + F_{PR}\cos 45 = 0$$

$$\sum f_x = 0 \Rightarrow F_{PR}\sin 45 + 50 = 0$$

$\Rightarrow$

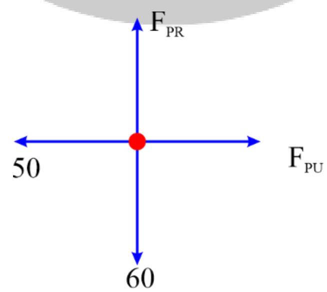
$$F_{PR} = -70.71 kN(c)$$

$$F_{PR} = 50 kN(T)$$



$$\sum f_x = 0 \quad F_{ru} = +50 kN(T)$$

$$\sum f_y = 0 \quad F_{rR} = +60 kN(T)$$



$$\sum f_x = 0$$

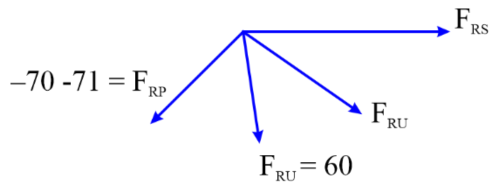
$$F_{RS} + F_{RU,}\cos 45, = F_{RP}\cos 45$$

$$F_{RS} = -40 kN$$

$$\sum f_y = 0$$

$$F_{RP}\sin 45 + 60 + F_{RU}\sin 45 = 0$$

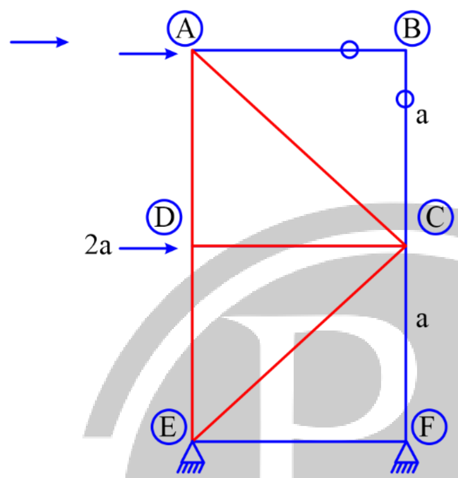
$$\Rightarrow F_{RU} = -14.14\ kN$$

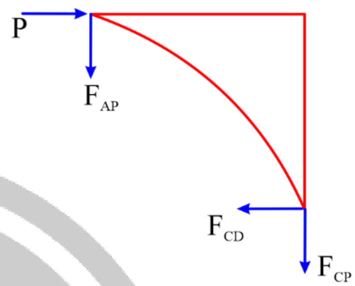


2. Method of Section

- Used only if force in few member is required.
- Section line should not cut more than 3 member and whole truss is cut into two portion.

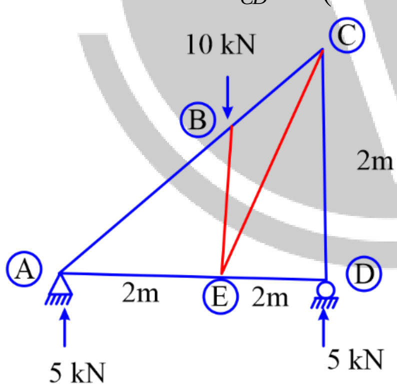
Find force in CD?

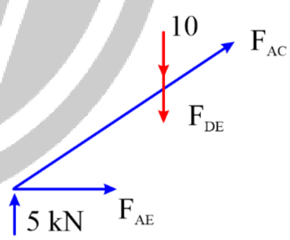




$$\sum f_x = 0$$

→ $$F_{CD} = P(\text{Tensile})$$

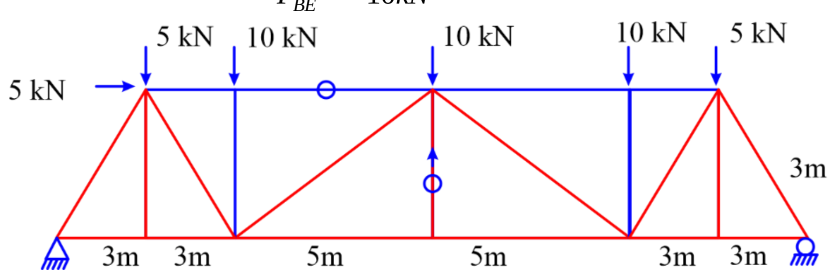




$$\sum M_n = 0$$

$$-10 \times .F_{CE} \times 2 = 0$$

$$F_{BE} = -10kN$$

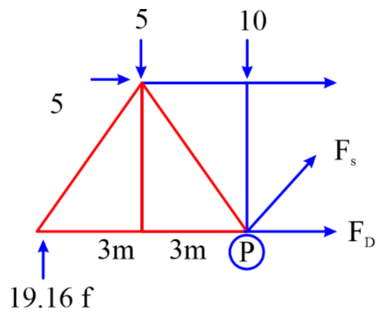


$$\sum M_1 = 0$$

$$R_2 \times 18 = 5\times15 + 10\times12 + 10\times9 + 10\times6 + 5\times3 + 5\times3$$

$\Rightarrow$ R_2 = 20.833 kN

R_1 = 19.1667 kN



$\Sigma M_P = 0$

$+ F_c \times 3 + 5 \times 3 + 19.167 \times 6 = 5 \times 3$

F_c = –38.334 kN

F_A = 0

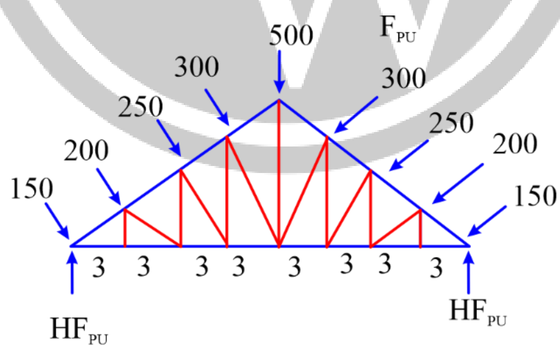
F_y = 0

F_B cos 45 + 19.167 = 5 + 10 → considering vertical equilibrium

→ F_B = - 5.89 kN

3. Graphical Method

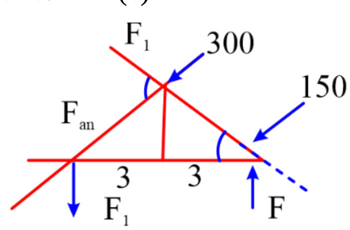
4. Tension coeff Method.



Most of force acting at joint ΣM joint = 0

$$F_{ac} \times \sin 30 + 300 \times 3 \cos 30 = 0 \times 6$$

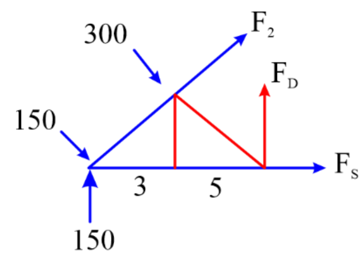
$$F_{ac} = -346.41 \text{ N (c)}$$



$$\Sigma M_J = 0$$

$$F_{bb} \times 6 - 300 \times 300 \times \frac{3}{\cos 30} = 0$$

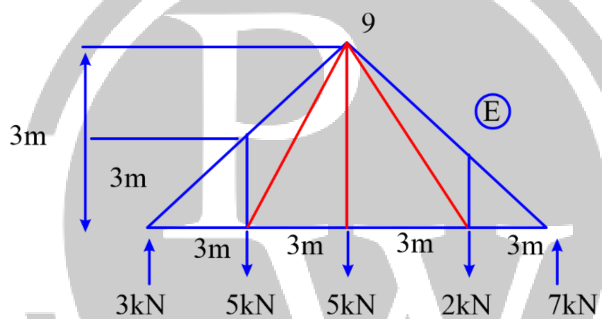
$$F_{bb} = 173.2 \text{ (T)}$$

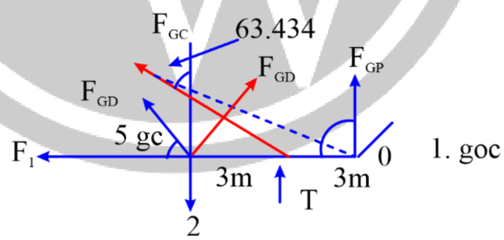


$$\cos 30 = \frac{3}{x}$$

$$x = \frac{3}{\cos 30}$$

Find F_{GF} and F_{GD}.





$$\Sigma M_D = 0$$

$$F_{GF} \sin (26.6) \times 6 + 7 \times 3 = 0$$

$$F_{GF} = -7.83 \text{ kN (c)}$$

$$\Sigma M_Q = 0$$

$$F_{GD} \sin 56.3 \times 6 + 7 \times 3 - 2 \times 6 = 0$$

$$F_{GD} = -1.8 \text{ kN (c)}$$

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7 CENTER OF MASS

7.1. Introduction

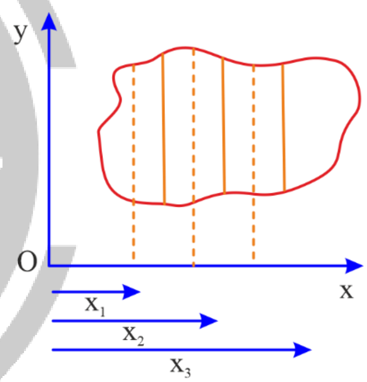
- Center of mass is defined as a point where the whole mass of the body appeared to be concentrated.
- Consider a body of mass 'M' whose center of mass is required to found out. Dividing the body into small masses of known center of mass m_1, m_2, m_3, … and centroid co-ordinates (x_1, y_1), (x_2, y_2), (x_3, y_3) $\bar{x} \rightarrow$ co-ordinate of center of mass of body.

  By principle of moments

$$M\bar{x} = m_1x_1 + m_2x_2 + m_3x_3$$

$$\bar{x} = \frac{\sum mx}{M}$$

$$= \frac{\int xdm}{M}$$



Similarly; $\bar{y} = \frac{\sum my}{M} = \frac{\int ydm}{M}$

7.1.1. Locate the centroid of Circular Arc

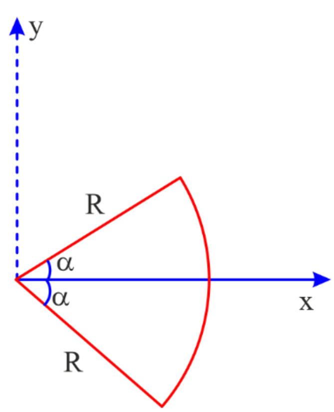
Symmetry $\Rightarrow \bar{y} = 0$

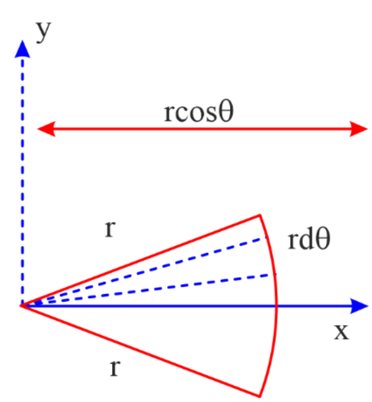
$$dL = rd\theta$$

$$\Rightarrow \quad (2dr)\bar{a} = \int_{-\alpha}^{\alpha} (r\cos\theta) rd\theta$$

$$2\alpha r\bar{x} = 2r^2 \sin\alpha$$

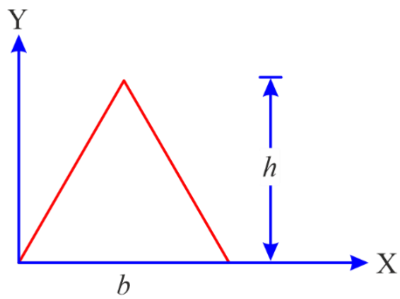
$$\boxed{\bar{x} = \frac{r\sin\alpha}{\alpha}}$$

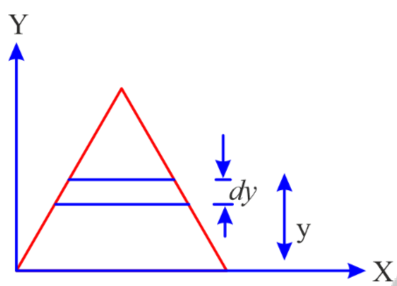




Find centroid from base.

$$A\bar{y} = \int y_c dA$$





$$\frac{bh}{2}\bar{y} = \int_0^h y\frac{b(h-y)}{h}dy = \frac{bh^2}{6}$$

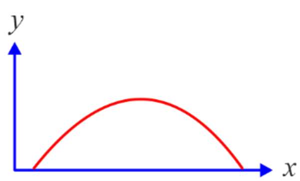
$$\boxed{\bar{y} = \frac{h}{3}}$$

7.1.2. Basic Shape

Area = bd $\bar{x} = b/2$ $\bar{y} = d/2$

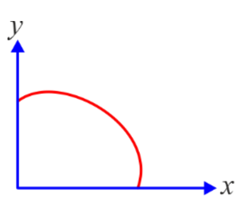
Area = $\frac{1}{2}hb$ $\bar{x}\, b/3$ $\bar{y} = h/3$

Area = πr^2 $\bar{x} = r$ $\bar{y} = r$



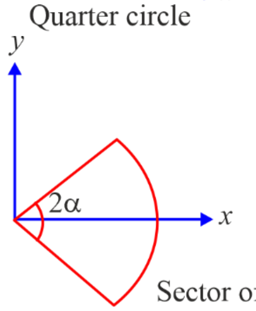
Semicircle

Area = $\frac{\pi r^2}{2}$ $\bar{x} = r$ $\bar{y} = \frac{4r}{3\pi}$



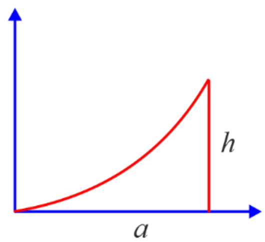
Quarter circle

Area = $\frac{\pi r^2}{2}$ $\bar{x} = \frac{4r}{3\pi}$ $\bar{y} = \frac{4r}{3\pi}$



Sector of circle

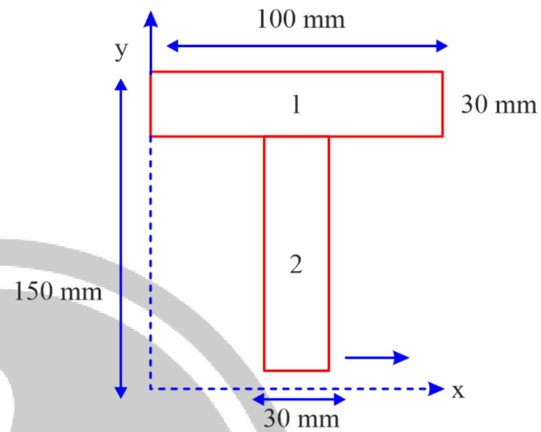
Area = αR^2 $\bar{x} = \frac{2R\sin\alpha}{3\alpha}$ $\bar{y} = 0$



Parabolic spandrel

$$\text{Area} = \frac{ah}{3} \qquad \bar{x} = \frac{3a}{4} \qquad \bar{y} = \frac{3h}{10}$$

Find centroid of 100 mm × 150 mm × 30 mm T section



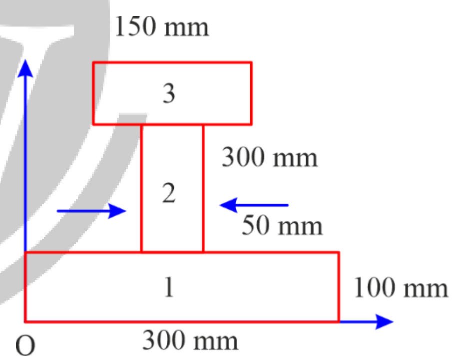
$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2}$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2}$$

$$= \frac{(100\times30)\times135 + 30\times120\times60}{100\times30 + 30\times120}$$

$$\bar{y} = 94.1mm$$

Find position of center of gravity from point 0



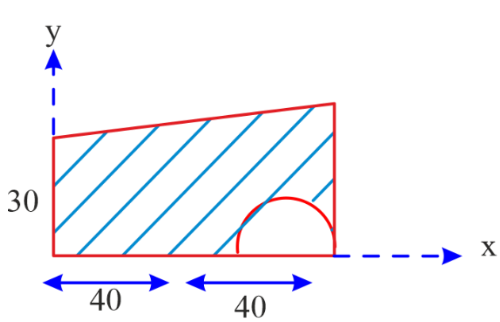
$$\bar{x} = \frac{100\times300\times150 + 50\times300\times150 + 150\times50\times150}{100\times300 + 50\times300 + 150\times50}$$

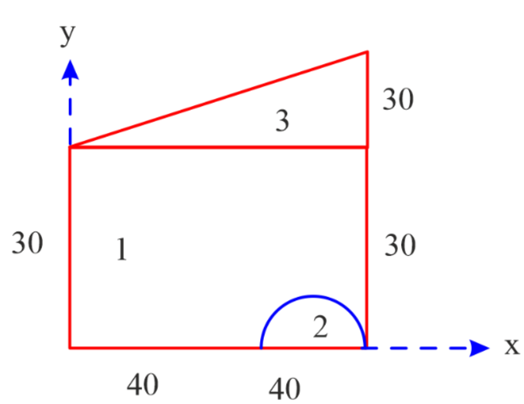
= 150 mm

$$\bar{y} = \frac{100\times300\times50 + 50\times300\times250 + 150\times50\times425}{100\times300 + 50\times300 + 150\times50}$$

= 160.71 mm

Find Centroid





$$\bar{x} = \frac{a_1 x_1 + a_3 x_3 - a_2 x_2}{a_1 + a_3 - a_2}$$

$$= \frac{30 \times 80 \times 40 + \frac{1}{2} \times 80 \times 30 \times \frac{80 \times 2}{3} - \frac{\pi \times 20^2}{2}(40 + 20)}{30 \times 80 + \frac{1}{2} \times 80 \times 30 - \pi \times 20^2}$$

$= 41.15$ mm

$$\bar{y} = \frac{30 \times 80 \times 15 + \frac{1}{2} \times 80 \times 30 \times \left(30 + \frac{30}{3}\right) - \frac{\pi \times 20^2}{2}\left(\frac{4 \times 20}{3\pi}\right)}{30 \times 80 + \frac{1}{2} \times 80 \times 30 - \frac{\pi \times 20^2}{2}}$$

$= 26.47$ mm