

NCERT Solutions for Class 9 Maths Chapter 9: NCERT Solutions for Class 9 Maths Chapter 9 Areas of Parallelograms and Triangles are designed to help students grasp the concepts of calculating areas of these geometric shapes.

These solutions are prepared by subject experts from Physics Wallah provide detailed explanations and step-by-step guidance for solving problems related to areas of parallelograms and triangles.

By studying these solutions students can improve their understanding of the principles and formulas involved in determining areas, which are crucial for their exams.

The solutions also help in building a strong foundation in geometry, aiding in better performance in assessments. For easy access and thorough preparation students can refer to the PDF link provided below.

NCERT Solutions for Class 9 Maths Chapter 9 Areas Of Parallelograms And Triangles Overview

These solutions provided by subject experts from Physics Wallah provide a detailed overview of NCERT Solutions for Class 9 Maths Chapter 9 Areas of Parallelograms and Triangles.

With clear explanations and step-by-step guidance, these solutions help students grasp the concepts more effectively. By following these expert-prepared solutions students can enhance their understanding and problem-solving skills ensuring they are well-prepared for their exams.

NCERT Solutions for Class 9 Maths Chapter 9 Areas Of Parallelograms And Triangles PDF

Here we have provided NCERT Solutions for Class 9 Maths Chapter 9 Areas of Parallelograms and Triangles to assist students in their exam preparation.

By using these solutions students can practice effectively clarify their doubts, and improve their understanding of the topic which will be beneficial for achieving better results in their exams. The PDF link is available below:

NCERT Solutions for Class 9 Maths Chapter 9 Areas of Parallelograms and Triangles PDF

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1. Which of the following figures lie on the same base and in-between the same parallels? In such a case, write the common base and the two parallels.

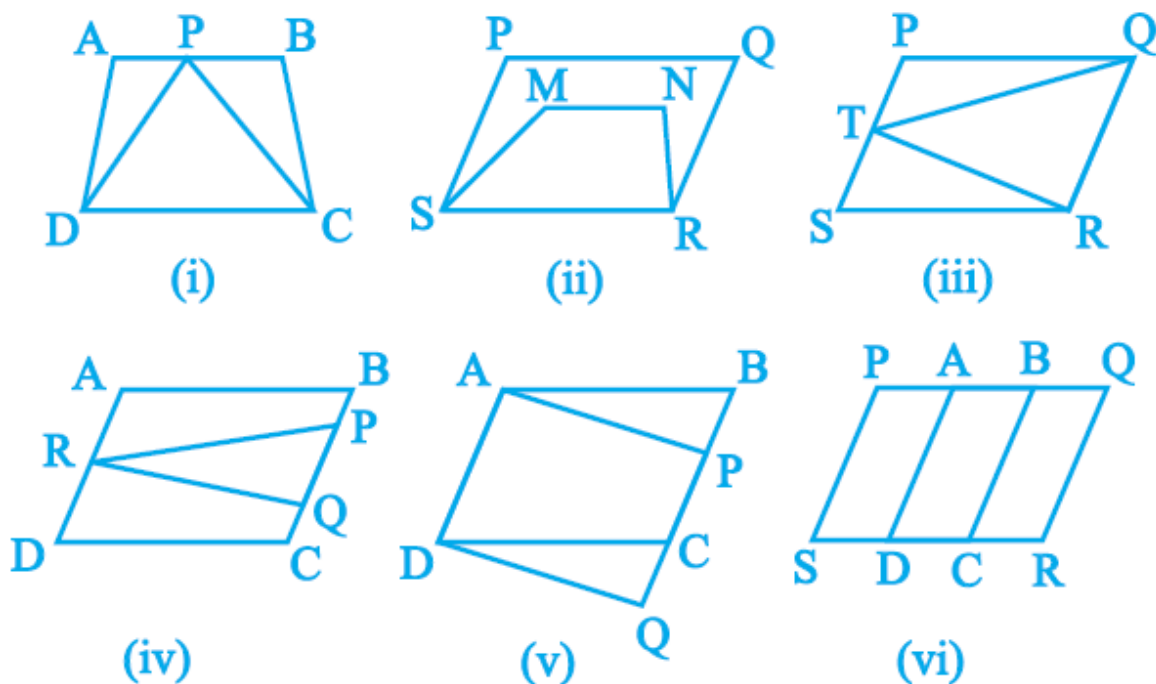


Fig. 9.8

Solution:

(i) Trapezium ABCD and $\triangle PDC$ lie on the same DC and in-between the same parallel lines AB and DC.

(ii) Parallelogram PQRS and trapezium SMNR lie on the same base SR but not in-between the same parallel lines.

(iii) Parallelogram PQRS and $\triangle TRQ$ lie on the same base QR and in-between the same parallel lines QR and PS.

(iv) Parallelogram ABCD and $\triangle PQR$ do not lie on the same base but in-between the same parallel lines BC and AD.

(v) Quadrilateral ABQD and trapezium APCD lie on the same base AD and in-between the same parallel lines AD and BQ.

(vi) Parallelogram PQRS and parallelogram ABCD do not lie on the same base SR but in-between the same parallel lines SR and PQ.

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1. In Fig. 9.15, ABCD is a parallelogram, $AE \perp DC$ and $CF \perp AD$. If $AB = 16$ cm, $AE = 8$ cm and $CF = 10$ cm, find AD.

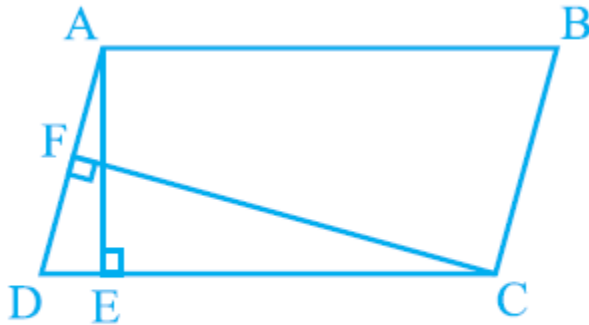


Fig. 9.15

Solution:

Given,

$AB = CD = 16$ cm (Opposite sides of a parallelogram.)

$CF = 10$ cm and $AE = 8$ cm

Now,

Area of parallelogram = Base \times Altitude

$$= CD \times AE = AD \times CF$$

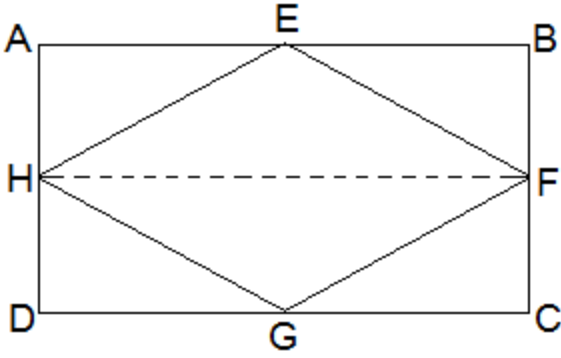
$$\Rightarrow 16 \times 8 = AD \times 10$$

$$\Rightarrow AD = 128/10 \text{ cm}$$

$$\Rightarrow AD = 12.8 \text{ cm}$$

2. If E, F, G and H are, respectively, the mid-points of the sides of a parallelogram ABCD show that $\text{ar} (EFGH) = \frac{1}{2} \text{ar} (ABCD)$.

Solution:



Given,

E, F, G and H are the mid-points of the sides of a parallelogram ABCD, respectively.

To prove,

$$\text{ar}(EFGH) = \frac{1}{2} \text{ar}(ABCD)$$

Construction,

H and F are joined.

Proof,

$AD \parallel BC$ and $AD = BC$ (Opposite sides of a parallelogram.)

$$\Rightarrow \frac{1}{2} AD = \frac{1}{2} BC$$

Also,

$AH \parallel BF$ and $DH \parallel CF$

$$\Rightarrow AH = BF \text{ and } DH = CF \text{ (H and F are mid-points.)}$$

\therefore , ABFH and HFCD are parallelograms.

Now,

We know that $\triangle EFH$ and parallelogram ABFH lie on the same FH, the common base and in-between the same parallel lines AB and HF.

$$\therefore \text{area of } \triangle EFH = \frac{1}{2} \text{area of ABFH} \text{ — (i)}$$

$$\text{And, area of } \triangle GHF = \frac{1}{2} \text{area of HFCD} \text{ — (ii)}$$

Adding (i) and (ii),

Area of $\triangle EFH$ + area of $\triangle GHF$ = $\frac{1}{2}$ area of $ABFH$ + $\frac{1}{2}$ area of $HFCD$

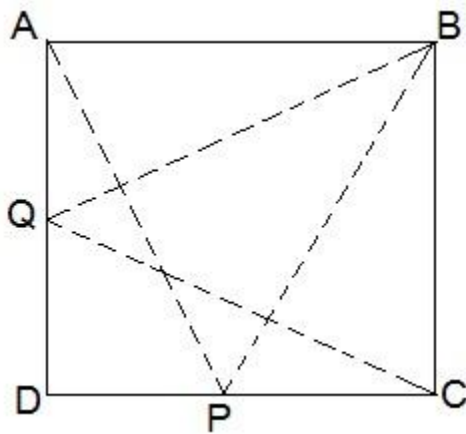
\Rightarrow area of $EFGH$ = area of $ABFH$

$\therefore \text{ar}(EFGH) = \frac{1}{2} \text{ar}(ABCD)$

3. P and Q are any two points lying on the sides DC and AD, respectively, of a parallelogram ABCD.

Show that $\text{ar}(\triangle APB) = \text{ar}(\triangle BQC)$.

Solution:



$\triangle APB$ and parallelogram $ABCD$ lie on the same base AB and in-between the same parallel AB and DC .

$\text{ar}(\triangle APB) = \frac{1}{2} \text{ar}(\text{parallelogram } ABCD)$ — (i)

Similarly,

$\text{ar}(\triangle BQC) = \frac{1}{2} \text{ar}(\text{parallelogram } ABCD)$ — (ii)

From (i) and (ii), we have

$\text{ar}(\triangle APB) = \text{ar}(\triangle BQC)$

4. In Fig. 9.16, P is a point in the interior of a parallelogram ABCD. Show that

(i) $\text{ar}(\triangle APB) + \text{ar}(\triangle PCD) = \frac{1}{2} \text{ar}(ABCD)$

(ii) $\text{ar}(\triangle APD) + \text{ar}(\triangle PBC) = \text{ar}(\triangle APB) + \text{ar}(\triangle PCD)$

[Hint: Through P, draw a line parallel to AB.]

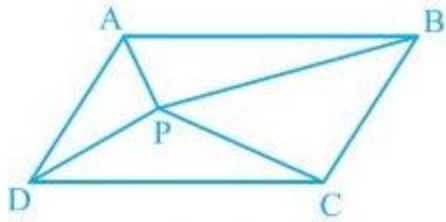
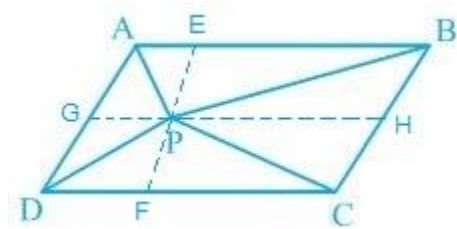


Fig. 9.16

Solution:



(i) A line GH is drawn parallel to AB passing through P.

In a parallelogram,

$AB \parallel GH$ (by construction) — (i)

\therefore ,

$AD \parallel BC \Rightarrow AG \parallel BH$ — (ii)

From equations (i) and (ii),

ABHG is a parallelogram.

Now,

$\triangle APB$ and parallelogram ABHG are lying on the same base AB and in-between the same parallel lines AB and GH.

$\therefore \text{ar}(\triangle APB) = \frac{1}{2} \text{ar}(\text{ABHG})$ — (iii)

also,

$\triangle PCD$ and parallelogram CDGH are lying on the same base CD and in-between the same parallel lines CD and GH.

$\therefore \text{ar}(\triangle PCD) = \frac{1}{2} \text{ar}(\text{CDGH})$ — (iv)

Adding equations (iii) and (iv),

$$\text{ar}(\triangle APB) + \text{ar}(\triangle PCD) = \frac{1}{2} [\text{ar}(\triangle ABHG) + \text{ar}(\triangle CDGH)]$$

$$\Rightarrow \text{ar}(\triangle APB) + \text{ar}(\triangle PCD) = \frac{1}{2} \text{ar}(\triangle ABCD)$$

(ii) A line EF is drawn parallel to AD passing through P.

In the parallelogram,

$$AD \parallel EF \text{ (by construction) — (i)}$$

\therefore ,

$$AB \parallel CD \Rightarrow AE \parallel DF \text{ — (ii)}$$

From equations (i) and (ii),

AEDF is a parallelogram.

Now,

$\triangle APD$ and parallelogram AEDF are lying on the same base AD and in-between the same parallel lines AD and EF.

$$\therefore \text{ar}(\triangle APD) = \frac{1}{2} \text{ar}(\triangle AEDF) \text{ — (iii)}$$

also,

$\triangle PBC$ and parallelogram BCFE are lying on the same base BC and in-between the same parallel lines BC and EF.

$$\therefore \text{ar}(\triangle PBC) = \frac{1}{2} \text{ar}(\triangle BCFE) \text{ — (iv)}$$

Adding equations (iii) and (iv),

$$\text{ar}(\triangle APD) + \text{ar}(\triangle PBC) = \frac{1}{2} \{\text{ar}(\triangle AEDF) + \text{ar}(\triangle BCFE)\}$$

$$\Rightarrow \text{ar}(\triangle APD) + \text{ar}(\triangle PBC) = \text{ar}(\triangle APB) + \text{ar}(\triangle PCD)$$

5. In Fig. 9.17, PQRS and ABRS are parallelograms, and X is any point on side BR. Show that

(i) $\text{ar}(\triangle PQRS) = \text{ar}(\triangle ABRS)$

(ii) $\text{ar}(\triangle AXS) = \frac{1}{2} \text{ar}(\triangle PQRS)$

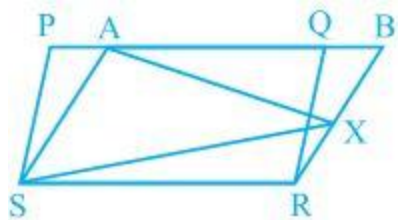


Fig. 9.17

Solution:

(i) Parallelogram PQRS and ABRS lie on the same base SR and in-between the same parallel lines SR and PB.

$$\therefore \text{ar}(PQRS) = \text{ar}(ABRS) \text{ — (i)}$$

(ii) ΔAXS and parallelogram ABRS are lying on the same base AS and in-between the same parallel lines AS and BR.

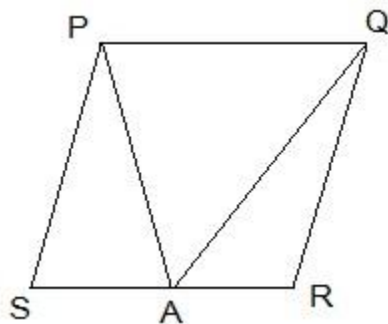
$$\therefore \text{ar}(\Delta AXS) = \frac{1}{2} \text{ar}(ABRS) \text{ — (ii)}$$

From (i) and (ii), we find that

$$\text{ar}(\Delta AXS) = \frac{1}{2} \text{ar}(PQRS)$$

6. A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts are the fields divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?

Solution:



The field is divided into three parts, each in a triangular shape.

Let ΔPSA , ΔPAQ and ΔQAR be the triangles.

Area of $(\Delta PSA + \Delta PAQ + \Delta QAR) = \text{Area of PQRS}$ — (i)

Area of $\Delta PAQ = \frac{1}{2}$ area of PQRS — (ii)

Here, the triangle and parallelogram are on the same base and in-between the same parallel lines.

From (i) and (ii),

Area of $\Delta PSA + \text{Area of } \Delta QAR = \frac{1}{2}$ area of PQRS — (iii)

From (ii) and (iii), we can conclude that

The farmer must sow wheat or pulses in ΔPAQ or either in both ΔPSA and ΔQAR .

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1. In Fig.9.23, E is any point on the median AD of a ΔABC . Show that $\text{ar}(\Delta ABE) = \text{ar}(\Delta ACE)$.

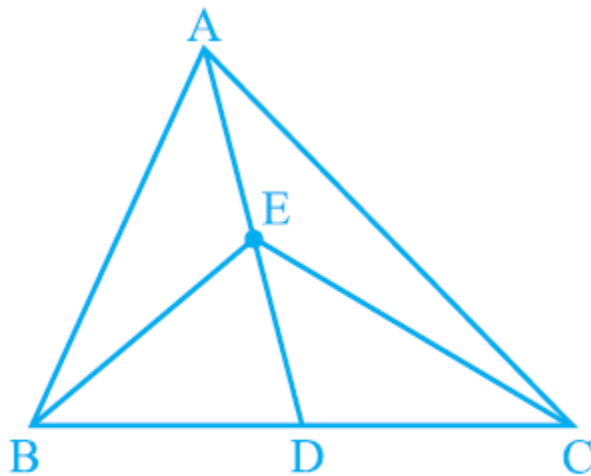


Fig. 9.23

Solution:

Given,

AD is the median of ΔABC . \therefore , it will divide ΔABC into two triangles of equal area.

$\therefore \text{ar}(\Delta ABD) = \text{ar}(\Delta ACD)$ — (i)

also,

ED is the median of $\triangle ABC$.

$$\therefore \text{ar}(\triangle EBD) = \text{ar}(\triangle ECD) \text{ — (ii)}$$

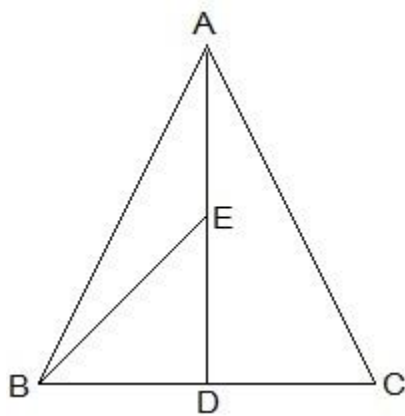
Subtracting (ii) from (i),

$$\text{ar}(\triangle ABD) - \text{ar}(\triangle EBD) = \text{ar}(\triangle ACD) - \text{ar}(\triangle ECD)$$

$$\Rightarrow \text{ar}(\triangle ABE) = \text{ar}(\triangle ACE)$$

2. In a triangle ABC, E is the mid-point of median AD. Show that $\text{ar}(\triangle BED) = \frac{1}{4} \text{ar}(\triangle ABC)$.

Solution:



$$\text{ar}(\triangle BED) = \frac{1}{2} \times BD \times DE$$

Since E is the mid-point of AD,

$$AE = DE$$

Since AD is the median on side BC of triangle ABC,

$$BD = DC$$

$$DE = \frac{1}{2} AD \text{ — (i)}$$

$$BD = \frac{1}{2} BC \text{ — (ii)}$$

From (i) and (ii), we get

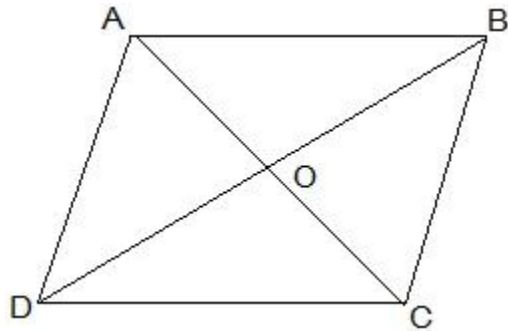
$$\text{ar}(\triangle BED) = \frac{1}{2} \times \frac{1}{2} BC \times \frac{1}{2} AD$$

$$\Rightarrow \text{ar}(\triangle BED) = \frac{1}{2} \times \frac{1}{2} \text{ar}(\triangle ABC)$$

$$\Rightarrow \text{ar}(\triangle BED) = \frac{1}{4} \text{ar}(\triangle ABC)$$

3. Show that the diagonals of a parallelogram divide it into four triangles of equal area.

Solution:



O is the midpoint of AC and BD. (Diagonals bisect each other.)

In $\triangle ABC$, BO is the median.

$$\therefore \text{ar}(\triangle AOB) = \text{ar}(\triangle BOC) \text{ — (i)}$$

also,

In $\triangle BCD$, CO is the median.

$$\therefore \text{ar}(\triangle BOC) = \text{ar}(\triangle COD) \text{ — (ii)}$$

In $\triangle ACD$, OD is the median.

$$\therefore \text{ar}(\triangle AOD) = \text{ar}(\triangle COD) \text{ — (iii)}$$

In $\triangle ABD$, AO is the median.

$$\therefore \text{ar}(\triangle AOD) = \text{ar}(\triangle AOB) \text{ — (iv)}$$

From equations (i), (ii), (iii) and (iv), we get

$$\text{ar}(\triangle BOC) = \text{ar}(\triangle COD) = \text{ar}(\triangle AOD) = \text{ar}(\triangle AOB)$$

Hence, we get that the diagonals of a parallelogram divide it into four triangles of equal area.

4. In Fig. 9.24, $\triangle ABC$ and $\triangle ABD$ are two triangles on the same base AB. If the line-segment CD is bisected by AB at O, show that $\text{ar}(\triangle ABC) = \text{ar}(\triangle ABD)$.

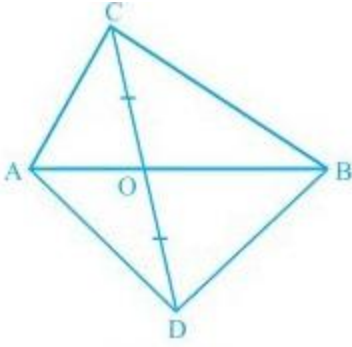


Fig. 9.24

Solution:

In $\triangle ABC$, AO is the median. (CD is bisected by AB at O .)

$$\therefore \text{ar}(\triangle AOC) = \text{ar}(\triangle AOD) \text{ — (i)}$$

also,

$\triangle BCD$, BO is the median. (CD is bisected by AB at O .)

$$\therefore \text{ar}(\triangle BOC) = \text{ar}(\triangle BOD) \text{ — (ii)}$$

Adding (i) and (ii),

We get

$$\text{ar}(\triangle AOC) + \text{ar}(\triangle BOC) = \text{ar}(\triangle AOD) + \text{ar}(\triangle BOD)$$

$$\Rightarrow \text{ar}(\triangle ABC) = \text{ar}(\triangle ABD)$$

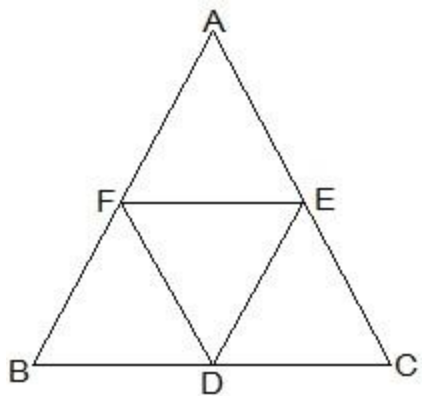
5. D, E and F are, respectively, the mid-points of the sides BC, CA and AB of a $\triangle ABC$. Show that

(i) BDEF is a parallelogram.

(ii) $\text{ar}(\triangle DEF) = \frac{1}{4} \text{ar}(\triangle ABC)$

(iii) $\text{ar}(\text{BDEF}) = \frac{1}{2} \text{ar}(\triangle ABC)$

Solution:



(i) In $\triangle ABC$,

$EF \parallel BC$ and $EF = \frac{1}{2} BC$ (by the mid-point theorem.)

also,

$BD = \frac{1}{2} BC$ (D is the mid-point.)

So, $BD = EF$

also,

BF and DE are parallel and equal to each other.

\therefore , the pair of opposite sides are equal in length and parallel to each other.

\therefore BDEF is a parallelogram.

(ii) Proceeding from the result of (i),

BDEF, DCEF, and AFDE are parallelograms.

A diagonal of a parallelogram divides it into two triangles of equal area.

$\therefore \text{ar}(\triangle BFD) = \text{ar}(\triangle DEF)$ (For parallelogram BDEF) — (i)

also,

$\text{ar}(\triangle AFE) = \text{ar}(\triangle DEF)$ (For parallelogram DCEF) — (ii)

$\text{ar}(\triangle CDE) = \text{ar}(\triangle DEF)$ (For parallelogram AFDE) — (iii)

From (i), (ii) and (iii)

$\text{ar}(\triangle BFD) = \text{ar}(\triangle AFE) = \text{ar}(\triangle CDE) = \text{ar}(\triangle DEF)$

$$\Rightarrow \text{ar}(\triangle BFD) + \text{ar}(\triangle AFE) + \text{ar}(\triangle CDE) + \text{ar}(\triangle DEF) = \text{ar}(\triangle ABC)$$

$$\Rightarrow 4 \text{ar}(\triangle DEF) = \text{ar}(\triangle ABC)$$

$$\Rightarrow \text{ar}(\triangle DEF) = \frac{1}{4} \text{ar}(\triangle ABC)$$

$$\text{(iii) Area (parallelogram BDEF)} = \text{ar}(\triangle DEF) + \text{ar}(\triangle BDE)$$

$$\Rightarrow \text{ar}(\text{parallelogram BDEF}) = \text{ar}(\triangle DEF) + \text{ar}(\triangle DEF)$$

$$\Rightarrow \text{ar}(\text{parallelogram BDEF}) = 2 \times \text{ar}(\triangle DEF)$$

$$\Rightarrow \text{ar}(\text{parallelogram BDEF}) = 2 \times \frac{1}{4} \text{ar}(\triangle ABC)$$

$$\Rightarrow \text{ar}(\text{parallelogram BDEF}) = \frac{1}{2} \text{ar}(\triangle ABC)$$

6. In Fig. 9.25, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD.

If AB = CD, then show that

(i) $\text{ar}(\triangle DOC) = \text{ar}(\triangle AOB)$

(ii) $\text{ar}(\triangle DCB) = \text{ar}(\triangle ACB)$

(iii) DA || CB or ABCD is a parallelogram.

[Hint: From D and B, draw perpendiculars to AC.]

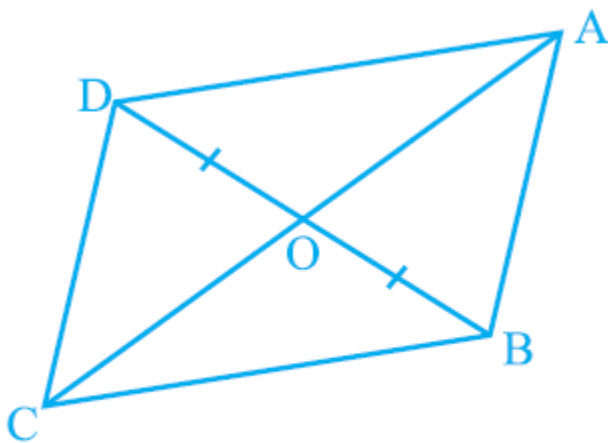


Fig. 9.25

Solution:

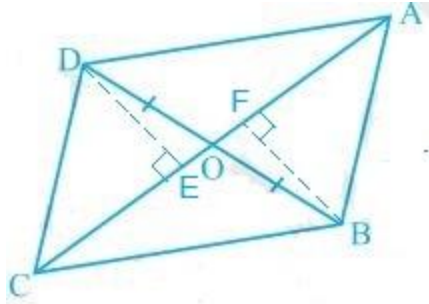


Fig. 9.25

Given,

$OB = OD$ and $AB = CD$

Construction,

$DE \perp AC$ and $BF \perp AC$ are drawn.

Proof:

(i) In $\triangle DOE$ and $\triangle BOF$,

$\angle DEO = \angle BFO$ (Perpendiculars)

$\angle DOE = \angle BOF$ (Vertically opposite angles)

$OD = OB$ (Given)

$\therefore \triangle DOE \cong \triangle BOF$ by AAS congruence condition.

$\therefore DE = BF$ (By CPCT) — (i)

also, $\text{ar}(\triangle DOE) = \text{ar}(\triangle BOF)$ (Congruent triangles) — (ii)

Now,

In $\triangle DEC$ and $\triangle BFA$,

$\angle DEC = \angle BFA$ (Perpendiculars)

$CD = AB$ (Given)

$DE = BF$ (From i)

$\therefore \triangle DEC \cong \triangle BFA$ by RHS congruence condition.

$\therefore \text{ar}(\triangle DEC) = \text{ar}(\triangle BFA)$ (Congruent triangles) — (iii)

Adding (ii) and (iii),

$$\text{ar}(\triangle DOE) + \text{ar}(\triangle DEC) = \text{ar}(\triangle BOF) + \text{ar}(\triangle BFA)$$

$$\Rightarrow \text{ar}(\triangle DOC) = \text{ar}(\triangle AOB)$$

$$(ii) \text{ar}(\triangle DOC) = \text{ar}(\triangle AOB)$$

Adding $\text{ar}(\triangle OCB)$ in LHS and RHS, we get

$$\Rightarrow \text{ar}(\triangle DOC) + \text{ar}(\triangle OCB) = \text{ar}(\triangle AOB) + \text{ar}(\triangle OCB)$$

$$\Rightarrow \text{ar}(\triangle DCB) = \text{ar}(\triangle ACB)$$

(iii) When two triangles have same base and equal areas, the triangles will be in between the same parallel lines,

$$\text{ar}(\triangle DCB) = \text{ar}(\triangle ACB).$$

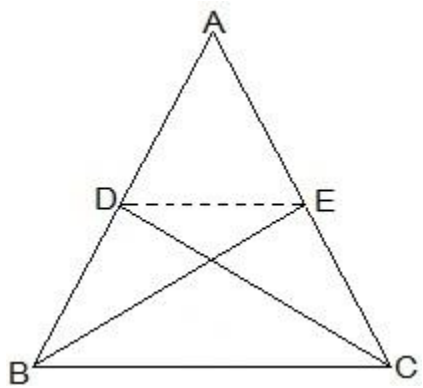
$$DA \parallel BC \text{ — (iv)}$$

For quadrilateral ABCD, one pair of opposite sides are equal ($AB = CD$), and the other pair of opposite sides are parallel.

\therefore , ABCD is parallelogram.

7. D and E are points on sides AB and AC, respectively, of $\triangle ABC$ such that $\text{ar}(\triangle DBC) = \text{ar}(\triangle EBC)$. Prove that $DE \parallel BC$.

Solution:



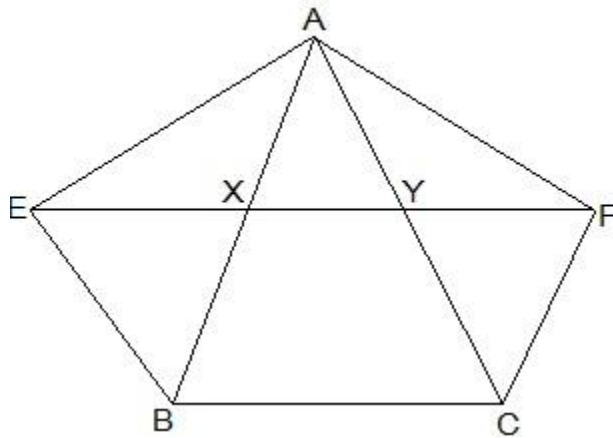
$\triangle DBC$ and $\triangle EBC$ are on the same base BC and also have equal areas.

\therefore , they will lie between the same parallel lines.

\therefore , $DE \parallel BC$

8. XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and F respectively, show that $\text{ar}(\triangle ABE) = \text{ar}(\triangle ACF)$

Solution:



Given,

$XY \parallel BC$, $BE \parallel AC$ and $CF \parallel AB$

To show,

$\text{ar}(\triangle ABE) = \text{ar}(\triangle ACF)$

Proof:

BCYE is a || gm as $\triangle ABE$ and ||gm BCYE are on the same base BE and between the same parallel lines BE and AC.

$$\therefore \text{ar}(\triangle ABE) = \frac{1}{2} \text{ar}(\text{BCYE}) \dots (1)$$

Now,

$CF \parallel AB$ and $XY \parallel BC$

$\Rightarrow CF \parallel AB$ and $XF \parallel BC$

$\Rightarrow BCFX$ is a || gm

As $\triangle ACF$ and || gm BCFX are on the same base CF and in-between the same parallel AB and FC.

$$\therefore \text{ar}(\triangle ACF) = \frac{1}{2} \text{ar}(\text{BCFX}) \dots (2)$$

But,

\parallel gm BCFX and \parallel gm BCYE are on the same base BC and between the same parallels BC and EF.

$$\therefore, \text{ar}(\text{BCFX}) = \text{ar}(\text{BCYE}) \dots (3)$$

From (1), (2) and (3), we get

$$\text{ar}(\triangle ABE) = \text{ar}(\triangle ACF)$$

$$\Rightarrow \text{ar}(\text{BEYC}) = \text{ar}(\text{BXFC})$$

As the parallelograms are on the same base BC and in-between the same parallels EF and BC—(iii)

Also,

$\triangle AEB$ and \parallel gm BEYC are on the same base BE and in-between the same parallels BE and AC.

$$\Rightarrow \text{ar}(\triangle AEB) = \frac{1}{2} \text{ar}(\text{BEYC}) \text{ — (iv)}$$

Similarly,

$\triangle ACF$ and \parallel gm BXFC on the same base CF and between the same parallels CF and AB.

$$\Rightarrow \text{ar}(\triangle ACF) = \frac{1}{2} \text{ar}(\text{BXFC}) \text{ — (v)}$$

From (iii), (iv) and (v),

$$\text{ar}(\triangle ABE) = \text{ar}(\triangle ACF)$$

9. The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q, and then parallelogram PBQR is completed (see Fig. 9.26). Show that

$$\text{ar}(\text{ABCD}) = \text{ar}(\text{PBQR}).$$

[Hint: Join AC and PQ. Now compare $\text{ar}(\text{ACQ})$ and $\text{ar}(\text{APQ})$.]

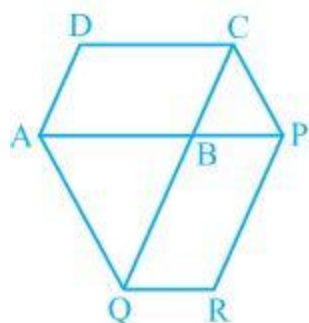
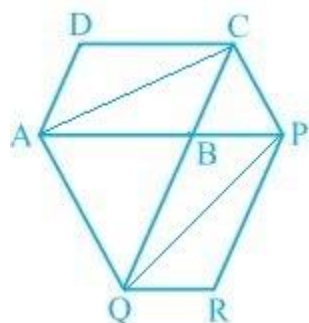


Fig. 9.26

Solution:



AC and PQ are joined.

$\text{Ar}(\triangle ACQ) = \text{ar}(\triangle APQ)$ (On the same base AQ and between the same parallel lines AQ and CP)

$$\Rightarrow \text{ar}(\triangle ACQ) - \text{ar}(\triangle ABQ) = \text{ar}(\triangle APQ) - \text{ar}(\triangle ABQ)$$

$$\Rightarrow \text{ar}(\triangle ABC) = \text{ar}(\triangle QBP) \text{ — (i)}$$

AC and QP are diagonals ABCD and PBQR.

$$\therefore \text{ar}(\triangle ABC) = \frac{1}{2} \text{ar}(\triangle ABCD) \text{ — (ii)}$$

$$\text{ar}(\triangle QBP) = \frac{1}{2} \text{ar}(\triangle PBQR) \text{ — (iii)}$$

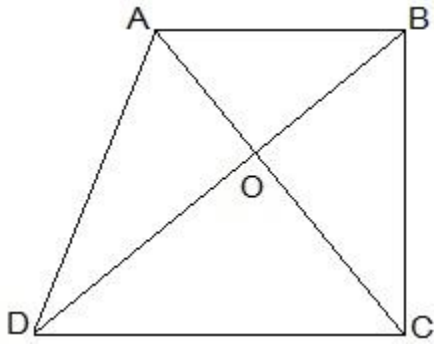
From (ii) and (iii),

$$\frac{1}{2} \text{ar}(\triangle ABCD) = \frac{1}{2} \text{ar}(\triangle PBQR)$$

$$\Rightarrow \text{ar}(\triangle ABCD) = \text{ar}(\triangle PBQR)$$

10. Diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$ intersect each other at O. Prove that $\text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$.

Solution:



$\triangle DAC$ and $\triangle DBC$ lie on the same base DC and between the same parallels AB and CD .

$$\text{Ar}(\triangle DAC) = \text{ar}(\triangle DBC)$$

$$\Rightarrow \text{ar}(\triangle DAC) - \text{ar}(\triangle DOC) = \text{ar}(\triangle DBC) - \text{ar}(\triangle DOC)$$

$$\Rightarrow \text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$$

11. In Fig. 9.27, $ABCDE$ is a pentagon. A line through B parallel to AC meets DC produced at F .

Show that

(i) $\text{ar}(\triangle ACB) = \text{ar}(\triangle ACF)$

(ii) $\text{ar}(\text{AEDF}) = \text{ar}(\text{ABCDE})$

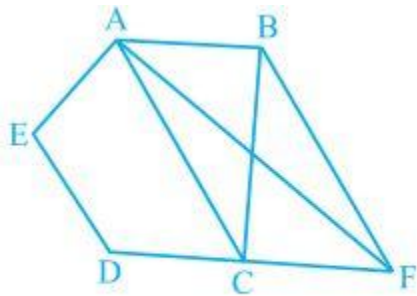


Fig. 9.27

Solution:

1. $\triangle ACB$ and $\triangle ACF$ lie on the same base AC and between the same parallels AC and BF .

$$\therefore \text{ar}(\triangle ACB) = \text{ar}(\triangle ACF)$$

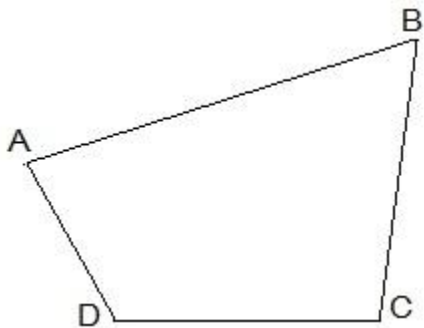
1. $\text{ar}(\triangle ACB) = \text{ar}(\triangle ACF)$

$$\Rightarrow \text{ar}(\triangle ACB) + \text{ar}(\triangle CDE) = \text{ar}(\triangle ACF) + \text{ar}(\triangle CDE)$$

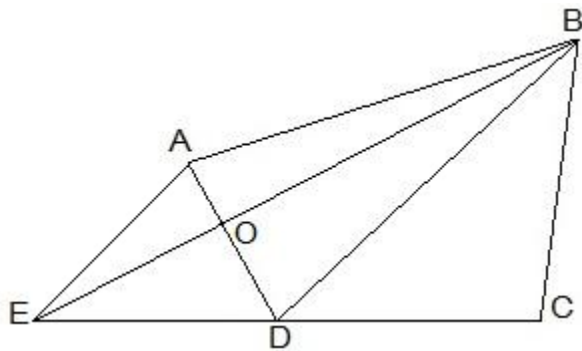
$$\Rightarrow \text{ar}(ABCD) = \text{ar}(AEFD)$$

12. A villager Itwaari has a plot of land in the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given an equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

Solution:



Let ABCD be the plot of the land in the shape of a quadrilateral.



To construct,

Join the diagonal BD.

Draw AE parallel to BD.

Join BE, which intersected AD at O.

We get

$\triangle BCE$ is the shape of the original field.

$\triangle AOB$ is the area for constructing a health centre.

$\triangle DEO$ is the land joined to the plot.

To prove:

$$\text{ar}(\triangle DEO) = \text{ar}(\triangle AOB)$$

Proof:

$\triangle DEB$ and $\triangle DAB$ lie on the same base BD , in-between the same parallels BD and AE .

$$\text{Ar}(\triangle DEB) = \text{ar}(\triangle DAB)$$

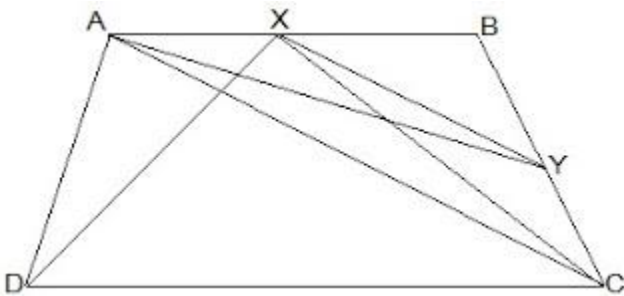
$$\Rightarrow \text{ar}(\triangle DEB) - \text{ar}(\triangle DOB) = \text{ar}(\triangle DAB) - \text{ar}(\triangle DOB)$$

$$\Rightarrow \text{ar}(\triangle DEO) = \text{ar}(\triangle AOB)$$

13. ABCD is a trapezium with $AB \parallel DC$. A line parallel to AC intersects AB at X and BC at Y . Prove that $\text{ar}(\triangle ADX) = \text{ar}(\triangle ACY)$.

[Hint: Join CX .]

Solution:



Given,

ABCD is a trapezium with $AB \parallel DC$.

$XY \parallel AC$

Construction,

Join CX

To prove,

$$\text{ar}(\triangle ADX) = \text{ar}(\triangle ACY)$$

Proof:

$\text{ar}(\triangle ADX) = \text{ar}(\triangle AXC)$ — (i) (Since they are on the same base AX and in-between the same parallels AB and CD)

Also,

$\text{ar}(\triangle AXC) = \text{ar}(\triangle ACY)$ — (ii) (Since they are on the same base AC and in-between the same parallels XY and AC.)

(i) and (ii),

$$\text{ar}(\triangle ADX) = \text{ar}(\triangle ACY)$$

14. In Fig.9.28, $AP \parallel BQ \parallel CR$. Prove that $\text{ar}(\triangle AQC) = \text{ar}(\triangle PBR)$.

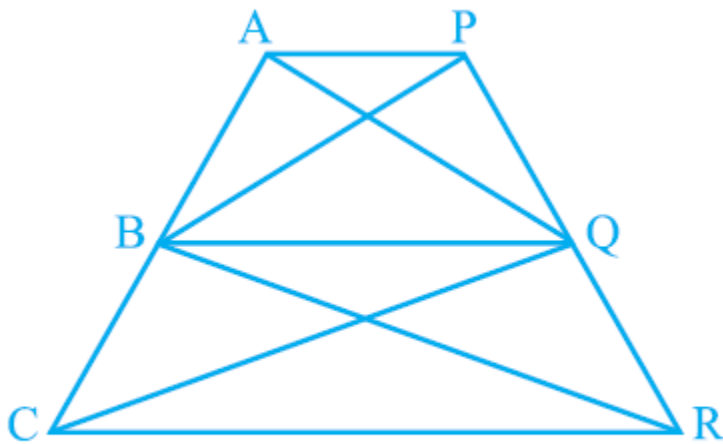


Fig. 9.28

Solution:

Given,

$$AP \parallel BQ \parallel CR$$

To prove,

$$\text{ar}(\triangle AQC) = \text{ar}(\triangle PBR)$$

Proof:

$\text{ar}(\triangle AQB) = \text{ar}(\triangle PBQ)$ — (i) (Since they are on the same base BQ and between the same parallels AP and BQ.)

also,

$\text{ar}(\triangle BQC) = \text{ar}(\triangle BQR)$ — (ii) (Since they are on the same base BQ and between the same parallels BQ and CR.)

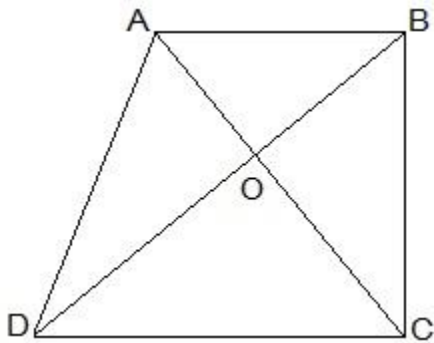
Adding (i) and (ii),

$$\text{ar}(\triangle AQB) + \text{ar}(\triangle BQC) = \text{ar}(\triangle PBQ) + \text{ar}(\triangle BQR)$$

$$\Rightarrow \text{ar}(\triangle AQC) = \text{ar}(\triangle PBR)$$

15. Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that $\text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$. Prove that ABCD is a trapezium.

Solution:



Given,

$$\text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$$

To prove,

ABCD is a trapezium.

Proof:

$$\text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$$

$$\Rightarrow \text{ar}(\triangle AOD) + \text{ar}(\triangle AOB) = \text{ar}(\triangle BOC) + \text{ar}(\triangle AOB)$$

$$\Rightarrow \text{ar}(\triangle ADB) = \text{ar}(\triangle ACB)$$

Areas of $\triangle ADB$ and $\triangle ACB$ are equal. \therefore , they must lie between the same parallel lines.

\therefore , $AB \parallel CD$

\therefore , ABCD is a trapezium.

16. In Fig.9.29, $\text{ar}(\triangle DRC) = \text{ar}(\triangle DPC)$ and $\text{ar}(\triangle BDP) = \text{ar}(\triangle ARC)$. Show that both the quadrilaterals ABCD and DCPR are trapeziums.

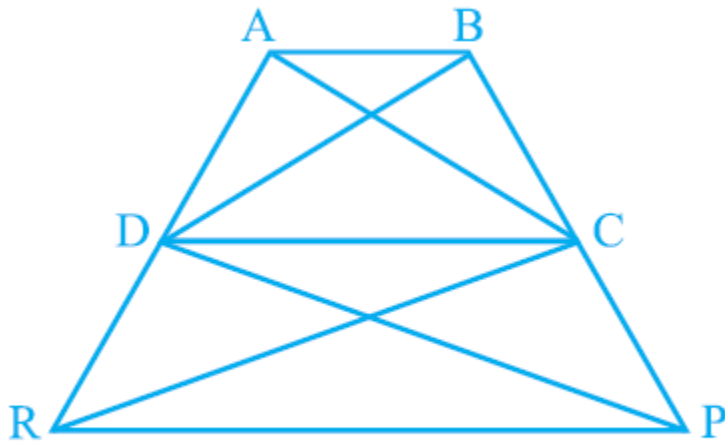


Fig. 9.29

Solution:

Given,

$$\text{ar}(\triangle DRC) = \text{ar}(\triangle DPC)$$

$$\text{ar}(\triangle BDP) = \text{ar}(\triangle ARC)$$

To prove,

ABCD and DCPR are trapeziums.

Proof:

$$\text{ar}(\triangle BDP) = \text{ar}(\triangle ARC)$$

$$\Rightarrow \text{ar}(\triangle BDP) - \text{ar}(\triangle DPC) = \text{ar}(\triangle DRC)$$

$$\Rightarrow \text{ar}(\triangle BDC) = \text{ar}(\triangle ADC)$$

\therefore , $\text{ar}(\triangle BDC)$ and $\text{ar}(\triangle ADC)$ are lying in-between the same parallel lines.

$$\therefore, AB \parallel CD$$

ABCD is a trapezium.

Similarly,

$$\text{ar}(\triangle DRC) = \text{ar}(\triangle DPC).$$

\therefore , $\text{ar}(\triangle DRC)$ and $\text{ar}(\triangle DPC)$ are lying in-between the same parallel lines.

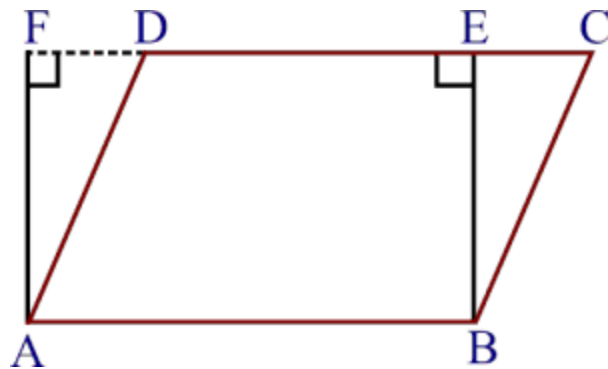
\therefore , $DC \parallel PR$

\therefore , $DCPR$ is a trapezium.

NCERT Solutions for Class 9 Maths Chapter 9 Areas Of Parallelograms And Triangles Page: 164

1. Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.

Solution:



Given,

\parallel gm ABCD and a rectangle ABEF have the same base AB and equal areas.

To prove,

The perimeter of \parallel gm ABCD is greater than the perimeter of rectangle ABEF.

Proof,

We know that the opposite sides of a \parallel gm and rectangle are equal.

, $AB = DC$ [As ABCD is a \parallel gm]

and, $AB = EF$ [As ABEF is a rectangle]

, $DC = EF \dots (i)$

Adding AB on both sides, we get

$$\Rightarrow AB + DC = AB + EF \dots (ii)$$

We know that the perpendicular segment is the shortest of all the segments that can be drawn to a given line from a point not lying on it.

$$, BE < BC \text{ and } AF < AD$$

$$\Rightarrow BC > BE \text{ and } AD > AF$$

$$\Rightarrow BC + AD > BE + AF \dots (iii)$$

Adding (ii) and (iii), we get

$$AB + DC + BC + AD > AB + EF + BE + AF$$

$$\Rightarrow AB + BC + CD + DA > AB + BE + EF + FA$$

$$\Rightarrow \text{perimeter of } \parallel \text{ gm } ABCD > \text{perimeter of rectangle } ABEF.$$

The perimeter of the parallelogram is greater than that of the rectangle.

Hence, proved.

2. In Fig. 9.30, D and E are two points on BC such that $BD = DE = EC$.

Show that $\text{ar} (ABD) = \text{ar} (ADE) = \text{ar} (AEC)$.

Can you now answer the question that you have left in the ‘Introduction’ of this chapter, whether the field of Budhia has been actually divided into three parts of equal area?

[Remark: Note that by taking $BD = DE = EC$, the triangle ABC is divided into three triangles – ABD, ADE and AEC – of equal areas. In the same way, by dividing BC into n equal parts and joining the points of division so obtained to the opposite vertex of BC, you can divide DABC into n triangles of equal areas.]

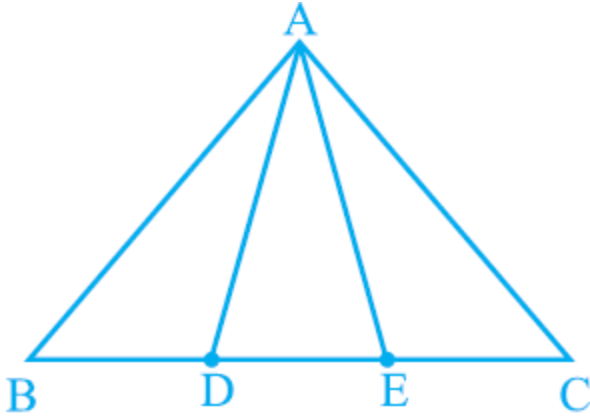


Fig. 9.30

Solution:

Given,

$$BD = DE = EC$$

To prove,

$$\text{ar}(\triangle ABD) = \text{ar}(\triangle ADE) = \text{ar}(\triangle AEC)$$

Proof,

In $(\triangle ABE)$, AD is median [since, $BD = DE$, given]

We know that the median of a triangle divides it into two parts of equal areas.

$$\therefore \text{ar}(\triangle ABD) = \text{ar}(\triangle AED) \text{ ---(i)}$$

Similarly,

In $(\triangle ADC)$, AE is median [Since $DE = EC$ is given]

$$\therefore \text{ar}(\triangle ADE) = \text{ar}(\triangle AEC) \text{ ---(ii)}$$

From the equation (i) and (ii), we get

$$\text{ar}(\triangle ABD) = \text{ar}(\triangle ADE) = \text{ar}(\triangle AEC)$$

3. In Fig. 9.31, ABCD, DCFE and ABFE are parallelograms. Show that $\text{ar}(\triangle ADE) = \text{ar}(\triangle BCF)$.

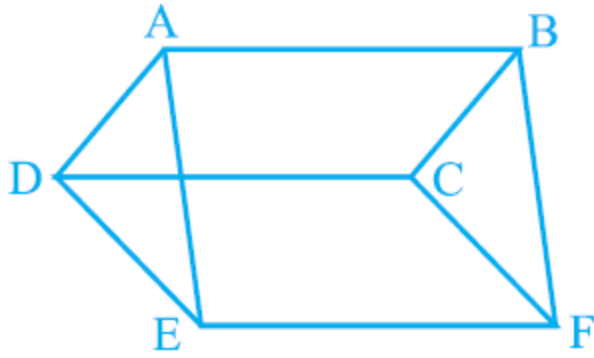


Fig. 9.31

Solution:

Given,

ABCD, DCFE and ABFE are parallelograms

To prove,

$$\text{ar}(\triangle ADE) = \text{ar}(\triangle BCF)$$

Proof,

In $\triangle ADE$ and $\triangle BCF$,

$AD = BC$ [Since they are the opposite sides of the parallelogram ABCD.]

$DE = CF$ [Since they are the opposite sides of the parallelogram DCFE.]

$AE = BF$ [Since they are the opposite sides of the parallelogram ABFE.]

, $\triangle ADE \cong \triangle BCF$ [Using the SSS Congruence theorem.]

, $\text{ar}(\triangle ADE) = \text{ar}(\triangle BCF)$ [By CPCT]

4. In Fig. 9.32, ABCD is a parallelogram and BC is produced to a point Q such that $AD = CQ$. If AQ intersects DC at P, show that $\text{ar}(\triangle BPC) = \text{ar}(\triangle DPQ)$.

[Hint: Join AC.]

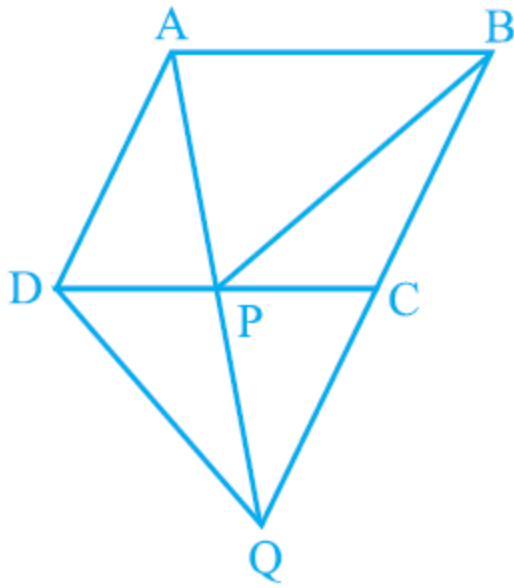


Fig. 9.32

Solution:

Given:

ABCD is a parallelogram

$AD = CQ$

To prove:

$\text{ar}(\triangle BPC) = \text{ar}(\triangle DPQ)$

Proof:

In $\triangle ADP$ and $\triangle QCP$,

$\angle APD = \angle QPC$ [Vertically Opposite Angles]

$\angle ADP = \angle QCP$ [Alternate Angles]

$AD = CQ$ [given]

, $\triangle ABO \cong \triangle ACD$ [AAS congruency]

, $DP = CP$ [CPCT]

In $\triangle CDQ$, QP is median. [Since, $DP = CP$]

The median of a triangle divides it into two parts of equal areas.

$$, \text{ar}(\triangle DPQ) = \text{ar}(\triangle QPC) \text{ ---(i)}$$

In $\triangle PBQ$, PC is the median. [Since, $AD = CQ$ and $AD = BC \Rightarrow BC = QC$]

The median of a triangle divides it into two parts of equal areas.

$$, \text{ar}(\triangle QPC) = \text{ar}(\triangle BPC) \text{ ---(ii)}$$

From the equation (i) and (ii), we get

$$\text{ar}(\triangle BPC) = \text{ar}(\triangle DPQ)$$

5. In Fig.9.33, ABC and BDE are two equilateral triangles such that D is the mid-point of BC . If AE intersects BC at F , show that

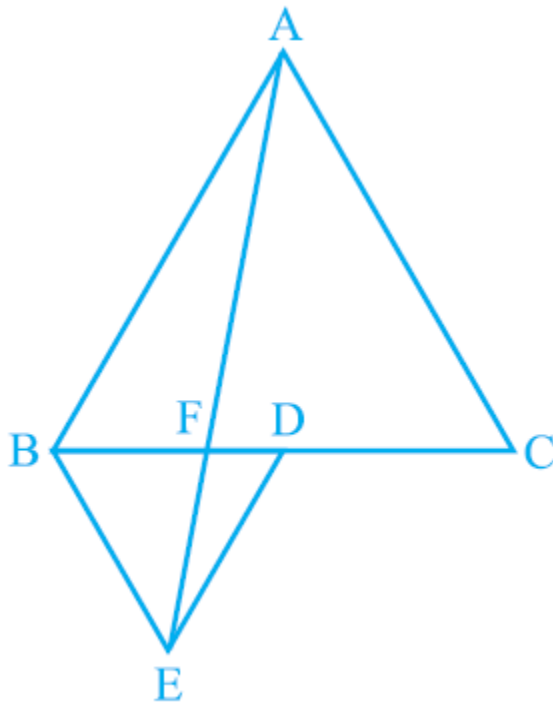


Fig. 9.33

(i) $\text{ar}(\triangle BDE) = \frac{1}{4} \text{ar}(\triangle ABC)$

(ii) $\text{ar}(\triangle BDE) = \frac{1}{2} \text{ar}(\triangle BAE)$

(iii) $\text{ar}(\triangle ABC) = 2 \text{ar}(\triangle BEC)$

(iv) $\text{ar}(\text{BFE}) = \text{ar}(\text{AFD})$

(v) $\text{ar}(\text{BFE}) = 2 \text{ar}(\text{FED})$

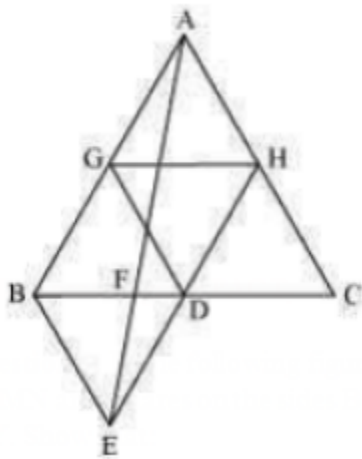
(vi) $\text{ar}(\text{FED}) = \frac{1}{8} \text{ar}(\text{AFC})$

Solution:

(i) Assume that G and H are the mid-points of the sides AB and AC, respectively.

Join the mid-points with line-segment GH. Here, GH is parallel to third side.

BC will be half of the length of BC by the mid-point theorem.



$\therefore GH = \frac{1}{2} BC$ and $GH \parallel BD$

$\therefore GH = BD = DC$ and $GH \parallel BD$ (D is the mid-point of BC)

Similarly,

$GD = HC = HA$

$HD = AG = BG$

$\triangle ABC$ is divided into 4 equal equilateral triangles $\triangle BGD$, $\triangle AGH$, $\triangle DHC$ and $\triangle GHD$

We can say that

$\triangle BGD = \frac{1}{4} \triangle ABC$

Considering $\triangle BDG$ and $\triangle BDE$,

$BD = BD$ (Common base)

Since both triangles are equilateral triangle, we can say that

$$BG = BE$$

$$DG = DE$$

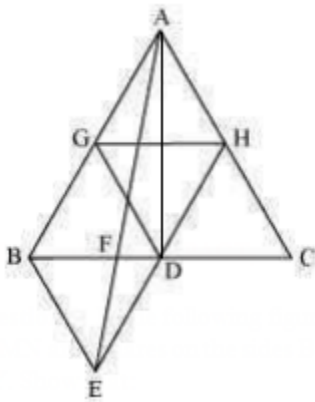
, $\triangle BDG \cong \triangle BDE$ [By SSS congruency]

, $\text{area}(\triangle BDG) = \text{area}(\triangle BDE)$

$$\text{ar}(\triangle BDE) = \frac{1}{4} \text{ar}(\triangle ABC)$$

Hence, proved.

(ii)



$$\text{ar}(\triangle BDE) = \text{ar}(\triangle AED) \text{ (Common base DE and } DE \parallel AB)$$

$$\text{ar}(\triangle BDE) - \text{ar}(\triangle FED) = \text{ar}(\triangle AED) - \text{ar}(\triangle FED)$$

$$\text{ar}(\triangle BEF) = \text{ar}(\triangle AFD) \dots (i)$$

Now,

$$\text{ar}(\triangle ABD) = \text{ar}(\triangle ABF) + \text{ar}(\triangle AFD)$$

$$\text{ar}(\triangle ABD) = \text{ar}(\triangle ABF) + \text{ar}(\triangle BEF) \text{ [From equation (i)]}$$

$$\text{ar}(\triangle ABD) = \text{ar}(\triangle ABE) \dots (ii)$$

AD is the median of $\triangle ABC$.

$$\text{ar}(\triangle ABD) = \frac{1}{2} \text{ar}(\triangle ABC)$$

$$= \frac{4}{2} \text{ar}(\triangle BDE)$$

$$= 2 \text{ ar } (\triangle BDE) \dots (iii)$$

From (ii) and (iii), we obtain

$$2 \text{ ar } (\triangle BDE) = \text{ ar } (\triangle ABE)$$

$$\text{ ar } (BDE) = \frac{1}{2} \text{ ar } (BAE)$$

Hence, proved.

$$(iii) \text{ ar}(\triangle ABE) = \text{ ar}(\triangle BEC) \text{ [Common base BE and BE } \parallel \text{ AC]}$$

$$\text{ ar}(\triangle ABF) + \text{ ar}(\triangle BEF) = \text{ ar}(\triangle BEC)$$

From eqⁿ (i), we get,

$$\text{ ar}(\triangle ABF) + \text{ ar}(\triangle AFD) = \text{ ar}(\triangle BEC)$$

$$\text{ ar}(\triangle ABD) = \text{ ar}(\triangle BEC)$$

$$\frac{1}{2} \text{ ar}(\triangle ABC) = \text{ ar}(\triangle BEC)$$

$$\text{ ar}(\triangle ABC) = 2 \text{ ar}(\triangle BEC)$$

Hence, proved.

(iv) $\triangle BDE$ and $\triangle AED$ lie on the same base (DE) and are in-between the parallel lines DE and AB.

$$\therefore \text{ ar } (\triangle BDE) = \text{ ar } (\triangle AED)$$

Subtracting $\text{ ar}(\triangle FED)$ from L.H.S and R.H.S,

We get

$$\therefore \text{ ar } (\triangle BDE) - \text{ ar } (\triangle FED) = \text{ ar } (\triangle AED) - \text{ ar } (\triangle FED)$$

$$\therefore \text{ ar } (\triangle BFE) = \text{ ar}(\triangle AFD)$$

Hence, proved.

(v) Assume that h is the height of vertex E, corresponding to the side BD in $\triangle BDE$.

Also, assume that H is the height of vertex A, corresponding to the side BC in $\triangle ABC$.

While solving Question (i),

We saw that

$$\text{ar}(\triangle BDE) = \frac{1}{4} \text{ar}(\triangle ABC)$$

While solving Question (iv),

We saw that

$$\text{ar}(\triangle BFE) = \text{ar}(\triangle AFD)$$

$$\therefore \text{ar}(\triangle BFE) = \text{ar}(\triangle AFD)$$

$$= 2 \text{ar}(\triangle FED)$$

$$\text{Hence, ar}(\triangle BFE) = 2 \text{ar}(\triangle FED)$$

Hence, proved.

$$(vi) \text{ar}(\triangle AFC) = \text{ar}(\triangle AFD) + \text{ar}(\triangle ADC)$$

$$= 2 \text{ar}(\triangle FED) + \frac{1}{2} \text{ar}(\triangle ABC) \text{ [using (v)]}$$

$$= 2 \text{ar}(\triangle FED) + \frac{1}{2} [4 \text{ar}(\triangle BDE)] \text{ [Using the result of Question (i)]}$$

$$= 2 \text{ar}(\triangle FED) + 2 \text{ar}(\triangle BDE)$$

$\triangle BDE$ and $\triangle AED$ are on the same base and between same parallels.

$$= 2 \text{ar}(\triangle FED) + 2 \text{ar}(\triangle AED)$$

$$= 2 \text{ar}(\triangle FED) + 2 [\text{ar}(\triangle AFD) + \text{ar}(\triangle FED)]$$

$$= 2 \text{ar}(\triangle FED) + 2 \text{ar}(\triangle AFD) + 2 \text{ar}(\triangle FED) \text{ [From question (viii)]}$$

$$= 4 \text{ar}(\triangle FED) + 2 \text{ar}(\triangle AFD)$$

$$\Rightarrow \text{ar}(\triangle AFC) = 8 \text{ar}(\triangle FED)$$

$$\Rightarrow \text{ar}(\triangle FED) = \frac{1}{8} \text{ar}(\triangle AFC)$$

Hence, proved.

6. Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that

$$\text{ar}(\triangle APB) \times \text{ar}(\triangle CPD) = \text{ar}(\triangle APD) \times \text{ar}(\triangle BPC).$$

[Hint: From A and C, draw perpendiculars to BD.]

Solution:

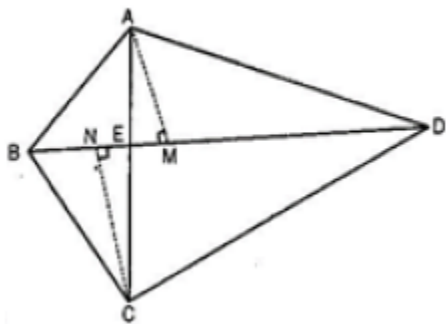
Given:

The diagonal AC and BD of the quadrilateral ABCD intersect each other at point E.

Construction:

From A, draw AM perpendicular to BD.

From C, draw CN perpendicular to BD.



To prove,

$$\text{ar}(\triangle AED) \times \text{ar}(\triangle BEC) = \text{ar}(\triangle ABE) \times \text{ar}(\triangle CDE)$$

Proof,

$$\text{ar}(\triangle ABE) = \frac{1}{2} \times BE \times AM \dots\dots\dots (i)$$

$$\text{ar}(\triangle AED) = \frac{1}{2} \times DE \times AM \dots\dots\dots (ii)$$

Dividing eq. ii by i , we get

$$\frac{\text{ar}(\triangle AED)}{\text{ar}(\triangle ABE)} = \frac{\frac{1}{2} \times DE \times AM}{\frac{1}{2} \times BE \times AM}$$

$$\text{ar}(\triangle AED)/\text{ar}(\triangle ABE) = DE/BE \dots\dots\dots (iii)$$

Similarly,

$$\text{ar}(\triangle CDE)/\text{ar}(\triangle BEC) = DE/BE \dots\dots\dots (iv)$$

From eq. (iii) and (iv), we get

$$\text{ar}(\triangle AED)/\text{ar}(\triangle ABE) = \text{ar}(\triangle CDE)/\text{ar}(\triangle BEC)$$

$$, \text{ar}(\Delta AED) \times \text{ar}(\Delta BEC) = \text{ar}(\Delta ABE) \times \text{ar}(\Delta CDE)$$

Hence, proved.

7. P and Q are, respectively, the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP. Show that

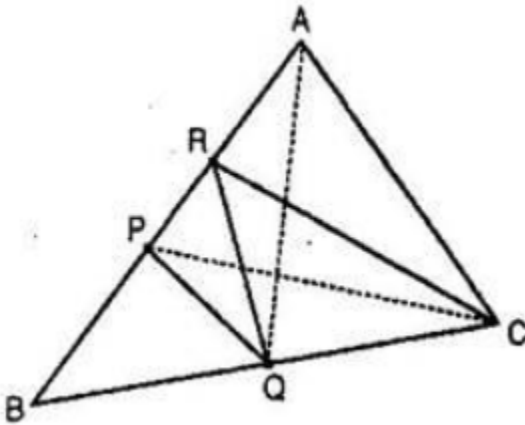
(i) $\text{ar}(\text{PRQ}) = \frac{1}{2} \text{ar}(\text{ARC})$

(ii) $\text{ar}(\text{RQC}) = \frac{3}{8} \text{ar}(\text{ABC})$

(iii) $\text{ar}(\text{PBQ}) = \text{ar}(\text{ARC})$

Solution:

(i)



We know that the median divides the triangle into two triangles of equal area.

PC is the median of ABC.

$$\text{Ar}(\Delta BPC) = \text{ar}(\Delta APC) \dots\dots\dots(i)$$

RC is the median of APC.

$$\text{Ar}(\Delta ARC) = \frac{1}{2} \text{ar}(\Delta APC) \dots\dots\dots(ii)$$

PQ is the median of BPC.

$$\text{Ar}(\Delta PQC) = \frac{1}{2} \text{ar}(\Delta BPC) \dots\dots\dots(iii)$$

From eq. (i) and (iii), we get

$$\text{ar}(\Delta PQC) = \frac{1}{2} \text{ar}(\Delta APC) \dots\dots\dots(iv)$$

From eq. (ii) and (iv), we get

$$\text{ar } (\Delta PQC) = \text{ar } (\Delta ARC) \dots\dots\dots(v)$$

P and Q are the mid-points of AB and BC, respectively [given]

$$PQ \parallel AC$$

$$\text{and, } PA = \frac{1}{2} AC$$

Triangles between the same parallel are equal in area, and we get

$$\text{ar } (\Delta APQ) = \text{ar } (\Delta PQC) \dots\dots\dots(vi)$$

From eq. (v) and (vi), we obtain

$$\text{ar } (\Delta APQ) = \text{ar } (\Delta ARC) \dots\dots\dots(vii)$$

R is the mid-point of AP.

, RQ is the median of APQ.

$$\text{Ar } (\Delta PRQ) = \frac{1}{2} \text{ar } (\Delta APQ) \dots\dots\dots(viii)$$

From (vii) and (viii), we get

$$\text{ar } (\Delta PRQ) = \frac{1}{2} \text{ar } (\Delta ARC)$$

Hence, proved.

(ii) PQ is the median of ΔBPC .

$$\text{ar } (\Delta PQC) = \frac{1}{2} \text{ar } (\Delta BPC)$$

$$= \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \text{ar } (\Delta ABC)$$

$$= \frac{1}{4} \text{ar } (\Delta ABC) \dots\dots\dots(ix)$$

Also,

$$\text{ar } (\Delta PRC) = \frac{1}{2} \text{ar } (\Delta APC) \text{ [From (iv)]}$$

$$\text{ar } (\Delta PRC) = \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \text{ar } (\Delta ABC)$$

$$= \frac{1}{4} \text{ar } (\Delta ABC) \dots\dots\dots(x)$$

Add eq. (ix) and (x), we get

$$\text{ar}(\Delta PQC) + \text{ar}(\Delta PRC) = (1/4) \times (1/4) \text{ar}(\Delta ABC)$$

$$\text{ar}(\text{quad. PQCR}) = 1/4 \text{ar}(\Delta ABC) \dots\dots\dots(\text{xi})$$

Subtracting $\text{ar}(\Delta PRQ)$ from L.H.S and R.H.S,

$$\text{ar}(\text{quad. PQCR}) - \text{ar}(\Delta PRQ) = 1/2 \text{ar}(\Delta ABC) - \text{ar}(\Delta PRQ)$$

$$\text{ar}(\Delta RQC) = 1/2 \text{ar}(\Delta ABC) - 1/2 \text{ar}(\Delta ARC) \text{ [From result (i)]}$$

$$\text{ar}(\Delta ARC) = 1/2 \text{ar}(\Delta ABC) - (1/2) \times (1/2) \text{ar}(\Delta APC)$$

$$\text{ar}(\Delta RQC) = 1/2 \text{ar}(\Delta ABC) - (1/4) \text{ar}(\Delta APC)$$

$$\text{ar}(\Delta RQC) = 1/2 \text{ar}(\Delta ABC) - (1/4) \times (1/2) \text{ar}(\Delta ABC) \text{ [As, PC is median of } \Delta ABC \text{]}$$

$$\text{ar}(\Delta RQC) = 1/2 \text{ar}(\Delta ABC) - (1/8) \text{ar}(\Delta ABC)$$

$$\text{ar}(\Delta RQC) = [(1/2) - (1/8)] \text{ar}(\Delta ABC)$$

$$\text{ar}(\Delta RQC) = (3/8) \text{ar}(\Delta ABC)$$

$$\text{(iii) ar}(\Delta PRQ) = 1/2 \text{ar}(\Delta ARC) \text{ [From result (i)]}$$

$$2 \text{ar}(\Delta PRQ) = \text{ar}(\Delta ARC) \dots\dots\dots(\text{xii})$$

$$\text{ar}(\Delta PRQ) = 1/2 \text{ar}(\Delta APQ) \text{ [RQ is the median of APQ] } \dots\dots\dots(\text{xiii})$$

But, we know that

$$\text{ar}(\Delta APQ) = \text{ar}(\Delta PQC) \text{ [From the reason mentioned in eq. (vi)] } \dots\dots\dots(\text{xiv})$$

From eq. (xiii) and (xiv), we get

$$\text{ar}(\Delta PRQ) = 1/2 \text{ar}(\Delta PQC) \dots\dots\dots(\text{xv})$$

At the same time,

$$\text{ar}(\Delta BPQ) = \text{ar}(\Delta PQC) \text{ [PQ is the median of } \Delta BPC \text{]} \dots\dots\dots(\text{xvi})$$

From eq. (xv) and (xvi), we get

$$\text{ar}(\Delta PRQ) = 1/2 \text{ar}(\Delta BPQ) \dots\dots\dots(\text{xvii})$$

From eq. (xii) and (xvii), we get

$$2 \times (1/2) \text{ar}(\Delta BPQ) = \text{ar}(\Delta ARC)$$

$$\Rightarrow \text{ar}(\triangle BPQ) = \text{ar}(\triangle ARC)$$

Hence, proved.

8. In Fig. 9.34, $\triangle ABC$ is a right triangle right angled at A . $BCED$, $ACFG$ and $ABMN$ are squares on the sides BC , CA and AB , respectively. Line segment $AX \perp DE$ meets BC at Y . Show that:

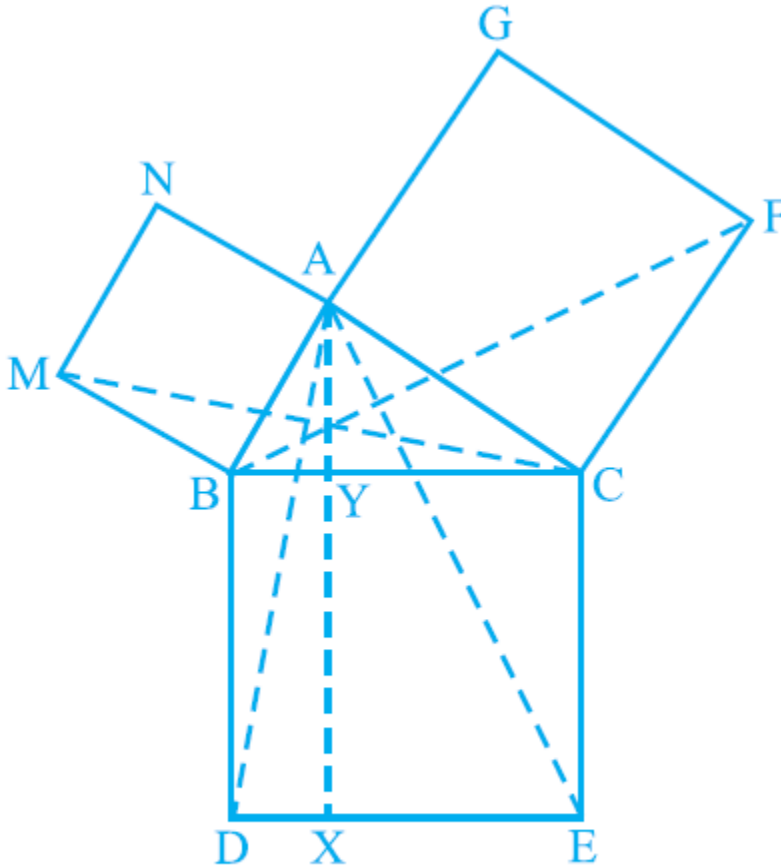


Fig. 9.34

(i) $\triangle MBC \cong \triangle ABD$

(ii) $\text{ar}(\triangle BYXD) = 2\text{ar}(\triangle MBC)$

(iii) $\text{ar}(\triangle BYXD) = \text{ar}(\triangle ABMN)$

(iv) $\triangle FCB \cong \triangle ACE$

(v) $\text{ar}(\triangle CYXE) = 2\text{ar}(\triangle FCB)$

(vi) $\text{ar}(\text{CYXE}) = \text{ar}(\text{ACFG})$

(vii) $\text{ar}(\text{BCED}) = \text{ar}(\text{ABMN}) + \text{ar}(\text{ACFG})$

Note: Result (vii) is the famous Theorem of Pythagoras. You shall learn a simpler proof of this theorem in Class X.

Solution:

(i) We know that each angle of a square is 90° . Hence, $\angle \text{ABM} = \angle \text{DBC} = 90^\circ$

$$\therefore \angle \text{ABM} + \angle \text{ABC} = \angle \text{DBC} + \angle \text{ABC}$$

$$\therefore \angle \text{MBC} = \angle \text{ABD}$$

In $\triangle \text{MBC}$ and $\triangle \text{ABD}$,

$$\angle \text{MBC} = \angle \text{ABD} \text{ (Proved above)}$$

$$\text{MB} = \text{AB} \text{ (Sides of square ABMN)}$$

$$\text{BC} = \text{BD} \text{ (Sides of square BCED)}$$

$$\therefore \triangle \text{MBC} \cong \triangle \text{ABD} \text{ (SAS congruency)}$$

(ii) We have

$$\triangle \text{MBC} \cong \triangle \text{ABD}$$

$$\therefore \text{ar}(\triangle \text{MBC}) = \text{ar}(\triangle \text{ABD}) \dots \text{(i)}$$

It is given that $\text{AX} \perp \text{DE}$ and $\text{BD} \perp \text{DE}$ (Adjacent sides of square BDEC)

$$\therefore \text{BD} \parallel \text{AX} \text{ (Two lines perpendicular to same line are parallel to each other.)}$$

$\triangle \text{ABD}$ and parallelogram BYXD are on the same base BD and between the same parallels BD and AX .

$$\text{Area}(\triangle \text{YXD}) = 2 \text{ Area}(\triangle \text{MBC}) \text{ [From equation (i)]} \dots \text{(ii)}$$

(iii) $\triangle \text{MBC}$ and parallelogram ABMN are lying on the same base MB and between the same parallels MB and NC .

$$2 \text{ ar}(\triangle \text{MBC}) = \text{ar}(\text{ABMN})$$

$$\text{ar}(\triangle \text{YXD}) = \text{ar}(\text{ABMN}) \text{ [From equation (ii)]} \dots \text{(iii)}$$

(iv) We know that each angle of a square is 90° .

$$\therefore \angle FCA = \angle BCE = 90^\circ$$

$$\therefore \angle FCA + \angle ACB = \angle BCE + \angle ACB$$

$$\therefore \angle FCB = \angle ACE$$

In $\triangle FCB$ and $\triangle ACE$,

$$\angle FCB = \angle ACE$$

$$FC = AC \text{ (Sides of square ACFG)}$$

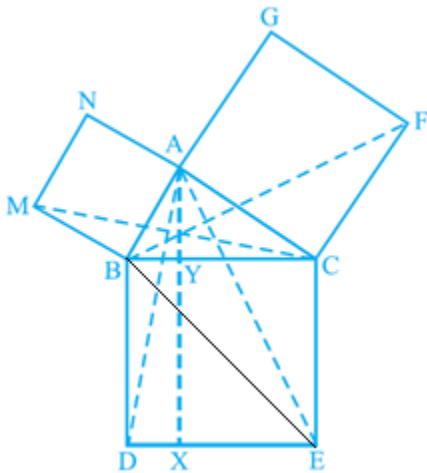
$$CB = CE \text{ (Sides of square BCED)}$$

$$\triangle FCB \cong \triangle ACE \text{ (SAS congruency)}$$

(v) $AX \perp DE$ and $CE \perp DE$ (Adjacent sides of square BDEC) [given]

Hence,

$CE \parallel AX$ (Two lines perpendicular to the same line are parallel to each other.)



Consider $\triangle ACE$ and parallelogram $CYXE$.

$\triangle ACE$ and parallelogram $CYXE$ are on the same base CE and between the same parallels CE and AX .

$$\therefore \text{ar}(\triangle YXE) = 2\text{ar}(\triangle ACE) \dots \text{(iv)}$$

We had proved that

$$\therefore \triangle FCB \cong \triangle ACE$$

$$\text{ar}(\triangle FCB) \approx \text{ar}(\triangle ACE) \dots (v)$$

From equations (iv) and (v), we get

$$\text{ar}(\triangle CYXE) = 2 \text{ar}(\triangle FCB) \dots (vi)$$

(vi) Consider BFCB and parallelogram ACFG.

BFCB and parallelogram ACFG lie on the same base CF and between the same parallels CF and BG.

$$\therefore \text{ar}(\triangle ACFG) = 2 \text{ar}(\triangle FCB)$$

$$\therefore \text{ar}(\triangle ACFG) = \text{ar}(\triangle CYXE) \text{ [From equation (vi)]} \dots (vii)$$

(vii) From the figure, we can observe that

$$\text{ar}(\triangle BCED) = \text{ar}(\triangle BYXD) + \text{ar}(\triangle CYXE)$$

$$\therefore \text{ar}(\triangle BCED) = \text{ar}(\triangle ABMN) + \text{ar}(\triangle ACFG) \text{ [From equations (iii) and (vii)]}$$

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Clear Conceptual Understanding: These solutions provide detailed explanations for each problem helping students understand the underlying concepts of areas of parallelograms and triangles.

Step-by-Step Guidance: The step-by-step approach ensures that students follow the correct method to solve problems. This helps in avoiding mistakes and understanding the logical progression of solving geometrical problems.

Exam Preparation: Using these solutions students can practice various types of questions that are likely to appear in exams. This practice enhances their problem-solving skills and boosts their confidence during exams.

Time Management: Regular practice with these solutions helps students manage their time effectively during exams. They learn to solve questions quickly and accurately, which is important for completing the exam within the given time frame.

Self-Assessment: These solutions allow students to check their answers and understand their mistakes. This self-assessment helps in identifying weak areas and focusing on them for improvement.

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