JEE

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Maths

Determinants

DPP: 3

Q1 Find the number of distinct real roots of the equation

$$\begin{vmatrix} x^2 & (x-1)^2 & (x-2)^2 \\ (x-1)^2 & (x-2)^2 & (x-3)^2 \\ (x-2)^2 & (x-3)^2 & (x-4)^2 \end{vmatrix} = 0.$$

(A) 6
(B) 3
(C) 2
(D) 0

Q2 The value of the determinant

$$\begin{vmatrix} 2 & \tan A \cot B + \cot A \tan B & \tan A \cot C + \cot A \tan C \\ \tan B \cot A + \tan A \cot B & 2 & \tan B \cot C + \cot B \tan C \\ \tan C \cot A + \cot C \tan A & \tan C \cot B + \cot C \tan B & 2 \end{vmatrix}$$ is

(A) 1

(B) 2

(C) 0

(D) None of these

Q3 If

$$f(x) = \begin{vmatrix} 4x-4 & (x-2)^2 & x^3 \\ 8x-4\sqrt{2} & (x-2\sqrt{2})^2 & (x+1)^3 \\ 12x-4\sqrt{3} & (x-2\sqrt{3})^2 & (x-1)^3 \end{vmatrix}$$

then the coefficient of x in $f(x)$ is

(A) $64(5-\sqrt{2}-\sqrt{3})$
(B) $64(5+\sqrt{2}-\sqrt{3})$
(C) $64(5+\sqrt{2}+\sqrt{3})$
(D) None of these

Q4

If $\begin{vmatrix} a & l & m \\ l & b & n \\ m & n & c \end{vmatrix} \begin{vmatrix} bc-n^2 & mn-lc & \ln-bm \\ mn-lc & ac-m^2 & lm-an \\ l\,n-bm & lm-an & ab-l^2 \end{vmatrix} = 64$

then the value of $\begin{vmatrix} 2a+3l & 3l+5m & 5m+4a \\ 2l+3b & 3b+5n & 5n+4l \\ 2m+3n & 3n+5c & 5c+4m \end{vmatrix}$ is

equals to

(A) 120

(B) 240

(C) 360

(D) 480

Q5 If $x > m$, $y > n$, $z > r$ $(x, y, z > 0)$ such that $\begin{vmatrix} x & n & r \\ m & y & r \\ m & n & z \end{vmatrix} = 0$. The greatest value of $\frac{xyz}{(x-m)(y-n)(z-r)}$ is

(A) 27
(B) 8/27
(C) 64/27
(D) 128/27

Q6 If $a, b, c, d > 0$; $x \in R$ and $(a^2 + b^2 + c^2)x^2 - 2(ab + bc + cd)x + b^2 + c^2 + d^2 \leq 0$, then $\begin{vmatrix} 33 & 14 & \log a \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix} =$

(A) 1
(B) –1
(C) 0
(D) None of these

Q7 If $Y = SX$, $Z = tX$ all the variable being differentiable functions of x and lower suffices

denotes the derivative with respect to x and
$$\begin{vmatrix} X & Y & Z \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix} \div \begin{vmatrix} S_1 & t_1 \\ S_2 & t_2 \end{vmatrix} = X^n$$, then $n =$

(A) 1 (B) 2
(C) 3 (D) 4

Q8 If $f(x) = ax^2 + bx + c$ where $a, b, c \in R$ and the equation $f(x) - x = 0$ has imaginary roots α and β and γ and δ be the roots of $f(f(x)) - x = 0$,

then $\begin{vmatrix} 2 & \alpha & \delta \\ \beta & 0 & \alpha \\ \gamma & \beta & 1 \end{vmatrix}$ is

(A) 0
(B) Purely real
(C) Purely imaginary
(D) None of these

Q9 Let $x > 0$, $y > 0$, $z > 0$ are respectively the 2nd, 3rd, 4th, terms of a G.P. and
$$\Delta = \begin{vmatrix} x^k & x^{k+1} & x^{k+2} \\ y^k & y^{k+1} & y^{k+2} \\ z^k & z^{k+1} & z^{k+2} \end{vmatrix}$$
$= (r-1)^2\left(1 - \frac{1}{r^2}\right)$

(where r is the common ratio) then

(A) $k = -1$ (B) $k = 1$
(C) $k = 0$ (D) None of these

Q10 The number of positive integral solution of the equation $\begin{vmatrix} x^3+1 & x^2y & x^2z \\ xy^2 & y^3+1 & y^2z \\ xz^2 & yz^2 & z^3+1 \end{vmatrix} = 11$ is

(A) 0 (B) 3
(C) 6 (D) 12

Q11 If

$$\begin{vmatrix} a^2+\lambda^2 & ab+c\lambda & ca-b\lambda \\ ab-c\lambda & b^2+\lambda^2 & bc+a\lambda \\ ca+b\lambda & bc-a\lambda & c^2+\lambda^2 \end{vmatrix} \begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix}$$

$= \left(1 + a^2 + b^2 + c^2\right)^3$, then the value of λ is

(A) 8 (B) 27
(C) 1 (D) -1

Q12 The value of $\sum_{r=2}^{n}(-2)^r \begin{vmatrix} {}^{n-2}C_{r-2} & {}^{n-2}C_{r-1} & {}^{n-2}C_r \\ -3 & 1 & 1 \\ 2 & -1 & 0 \end{vmatrix} (n > 2)$ is

(A) $2n - 1 + (-1)^n$
(B) $2n + 1 + (-1)^{n-1}$
(C) $2n - 3 + (-1)^n$
(D) none of these

Q13 Let n be a positive integer and
$$\Delta_r = \begin{vmatrix} r^2+r & r+1 & r-2 \\ 2r^2+3r-1 & 3r & 3r-3 \\ r^2+2r+3 & 2r-1 & 2r-1 \end{vmatrix}$$ and $\sum_{r=1}^{n} \Delta_r = an^2 + bn + c$, then the value of $a + b + c$ is

(A) 13 (B) 12
(C) 11 (D) 10

Q14 If
$$\Delta(x) = \begin{vmatrix} \tan x & \tan(x+h) & \tan(x+2h) \\ \tan(x+2h) & \tan x & \tan(x+h) \\ \tan(x+h) & \tan(x+2h) & \tan x \end{vmatrix},$$
, then the value of $\lim_{h\to 0} \frac{\Delta(\pi/3)}{\sqrt{3}h^2}$ is

(A) 144 (B) 81
(C) 64 (D) 36

Q15 Let $\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = Ax^4 + Bx^3 + Cx^2 + Dx + E$

Then the value of $5A + 4B + 3C + 2D + E$ is equal to

(A) zero
(B) -16
(C) 16
(D) -11

Q16 If $f(x)$
$$= \begin{vmatrix} \cos(x+\alpha) & \cos(x+\beta) & \cos(x+\gamma) \\ \sin(x+\alpha) & \sin(x+\beta) & \sin(x+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta) \end{vmatrix}$$
then $f(\theta) - 2f(\phi) + f(\psi)$ is equal to

(A) 0
(B) $\alpha - \beta$
(C) $\alpha + \beta + \gamma$
(D) $\alpha + \beta - \gamma$

Q17 If the matrix M_r is given by

$M_r = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix}$, $r = 1, 2, 3, ...$ then

the value of

$\det(M_1) + \det(M_2) + \ldots + \det(M_{2008})$ is

(A) 2007
(B) 2008
(C) $(2008)^2$
(D) $(2007)^2$

Q18 If $A = [a_{ij}]_{4\times 4}$, such that

$a_{ij} = \begin{cases} 2, & \text{when } i = j \\ 0, & \text{when } i \neq j \end{cases}$, then

$\left\{ \frac{\det(\text{adj}(\text{adj } A))}{7} \right\}$ is (where $\{\cdot\}$ represents fractional part function)

(A) $\frac{1}{7}$
(B) $\frac{2}{7}$
(C) $\frac{3}{7}$
(D) none of these

Q19 Let $\Delta_1 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$, $\Delta_1 \neq 0$

$\Delta_2 = \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix}$ where b_{ij} is cofactor of

$a_{ij} \forall i, j = 1, 2, 3$

and $\Delta_3 = \begin{vmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{vmatrix}$ where c_{ij} is

cofactor of $b_{ij} \forall i, j = 1, 2, 3$.

then which one of the following is always correct.

(A) $\Delta_1, \Delta_2, \Delta_3$ are in A.P.
(B) $\Delta_1, \Delta_2, \Delta_3$ are in G.P.
(C) $\Delta_1^2 = \frac{\Delta_3}{\Delta_2}$
(D) $\Delta_1 = \frac{\Delta_2}{\sqrt{\Delta_3}}$

Q20 Three digit numbers $x17$, $3y6$ and $12z$ where x, y, z are integers from 0 to 9 , are divisible by a fixed constant k. Then the determinant

$\begin{vmatrix} x & 3 & 1 \\ 7 & 6 & z \\ 1 & y & 2 \end{vmatrix}$ must be divisible by

(A) k
(B) k^2
(C) k^3
(D) None

Q21 Let $S = \{\sqrt{n} : 1 \leq n \leq 50$ and n is odd $\}$.Let

$a \in S$ and $A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$.If

$\sum_{a\in S} \det(\text{adj } A) = 100\lambda$ then λ is equal to

(A) 218
(B) 221
(C) 663
(D) 1717

Q22 Let $A = \begin{pmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{pmatrix}$

where $[t]$ denotes the greatest integer less than or equal to t. If $\det(A) = 192$, then the set of values of x is the interval.

(A) $[60, 61)$
(B) $[68, 69)$
(C) $[62, 63)$
(D) $[65, 66)$

Q23 For $\alpha, \beta \in \mathbb{R}$ and a natural number n, let

$A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2} + \alpha \\ 2r & 2 & n^2 - \beta \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$. Then $2A_{10} - A_8$

is

(A) 0
(B) $4\alpha + 2\beta$
(C) $2n$
(D) $2\alpha + 4\beta$

Q24 If the system of equations

$(a-t)x + by + cz = 0$
$bx + (c-t)y + az = 0$ has non-trivial
$cx + ay + (b-t)z = 0$

solution, then product of all possible values of t is

(A)

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

(B) $a+b+c$

(C) $a^2+b^2+c^2$

(D) 1

Q25 If the system of linear equations.

$8x+y+4z=-2$

$x+y+z=0$

$\lambda x-3y=\mu$

has infinitely many solutions, then the distance of the point $\left(\lambda, \mu, -\frac{1}{2}\right)$ from the plane $8x+y+4z+2=0$

(A) $3\sqrt{5}$

(B) 4

(C) $\frac{26}{9}$

(D) $\frac{10}{3}$

Answer Key

Q1	(D)
Q2	(C)
Q3	(D)
Q4	(C)
Q5	(B)
Q6	(C)
Q7	(C)
Q8	(B)
Q9	(A)
Q10	(B)
Q11	(C)
Q12	(A)
Q13	(B)
Q14	(A)
Q15	(D)
Q16	(A)
Q17	(C)
Q18	(A)
Q19	(C)
Q20	(A)
Q21	(B)
Q22	(C)
Q23	(B)
Q24	(A)
Q25	(D)