# JEE Mains (Dropper)

## Sample Paper - III

**DURATION : 180 Minutes** **M. MARKS : 300**

## ANSWER KEY

| PHYSICS | | CHEMISTRY | | MATHEMATICS | |
|---|---|---|---|---|---|
| 1. | (1) | 31. | (2) | 61. | (3) |
| 2. | (1) | 32. | (1) | 62. | (1) |
| 3. | (4) | 33. | (3) | 63. | (4) |
| 4. | (3) | 34. | (4) | 64. | (3) |
| 5. | (3) | 35. | (3) | 65. | (1) |
| 6. | (4) | 36. | (4) | 66. | (3) |
| 7. | (3) | 37. | (4) | 67. | (3) |
| 8. | (3) | 38. | (4) | 68. | (4) |
| 9. | (2) | 39. | (1) | 69. | (1) |
| 10. | (4) | 40. | (3) | 70. | (4) |
| 11. | (1) | 41. | (4) | 71. | (2) |
| 12. | (1) | 42. | (3) | 72. | (2) |
| 13. | (3) | 43. | (4) | 73. | (3) |
| 14. | (2) | 44. | (4) | 74. | (1) |
| 15. | (1) | 45. | (4) | 75. | (4) |
| 16. | (3) | 46. | (3) | 76. | (3) |
| 17. | (4) | 47. | (1) | 77. | (4) |
| 18. | (1) | 48. | (2) | 78. | (3) |
| 19. | (4) | 49. | (2) | 79. | (4) |
| 20. | (4) | 50. | (1) | 80. | (4) |
| 21. | (1) | 51. | (6) | 81. | (4) |
| 22. | (2) | 52. | (32) | 82. | (54) |
| 23. | (5) | 53. | (3) | 83. | (8) |
| 24. | (2) | 54. | (8) | 84. | (5) |
| 25. | (6) | 55. | (4) | 85. | (39) |
| 26. | (1) | 56. | (64) | 86. | (5) |
| 27. | (20) | 57. | (3) | 87. | (9) |
| 28. | (25) | 58. | (6) | 88. | (5) |
| 29. | (5) | 59. | (4) | 89. | (7) |
| 30. | (2) | 60. | (3) | 90. | (1) |

# PHYSICS

**1. (1)**

By Newton's second law

$F_2 - F_1 = ma$

$\therefore F_1 = F_2 - ma$

$= 5 - 1 \times = 3N$

For rotational equilibrium, taking moment of forces about centre of mass, we get

$$F_1 \times \frac{1}{2} - F_2\left(\frac{1}{2} - y\right) = 0$$

$$3 \times \frac{1}{2} - 5\left(\frac{1}{2} - 0.2\right) = 0$$

$\therefore l = 1m$

**2. (1)**

Electric field on surface of a uniformly charged sphere is given by $\frac{Q}{4\pi\varepsilon_0 R^3} = \frac{\rho R}{3\varepsilon_0}$

Electric field at outside point is given by

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{\rho R^3}{3\varepsilon_0 r^2}$$

$$\left|\vec{E}\right| = \frac{\rho r_0}{3\varepsilon_0} - \frac{\rho\left(\frac{r_0}{2}\right)^3}{3\varepsilon_0\left(\frac{3r_0}{2}\right)^2} = \frac{17\rho r_0}{54\varepsilon_0} \text{ left}$$

**3. (4)**

For a positive line charge or charged wire with uniform density $\lambda$, electric field at distance $x$ is

$$E = \frac{2k\lambda}{x} = \frac{\lambda}{2\pi\varepsilon_0 x} \quad ...(i)$$

So, force on charge q which is at a distance $r_0$ due to this line charge is

$$F = qE = \frac{2kq\lambda}{x} \quad ...(ii) \text{ [using Eq. (i)]}$$

Now, work done when charge is pushed by field by a small displacement $dx$ is

$$dW = F.dx = \frac{2kq\lambda}{x} \cdot dx \text{ [Using Eq. (ii)]}$$

$\therefore$ Total work done by field of wire in taking charge $q$ from distance $r_0$ to distance $r$ will be

$$W = \int_{r_0}^{r} dW = \int_{r_0}^{r} \frac{2kq\lambda}{x} \cdot dx$$

$$= 2kq\lambda[\log x]_{r_0}^{r} = 2kq\lambda(\log r - \log r_0)$$

$$= 2kq\lambda \log\left|\frac{r}{r_0}\right| \quad ...(iii)$$

As we known, from work-kinetic energy theorem,

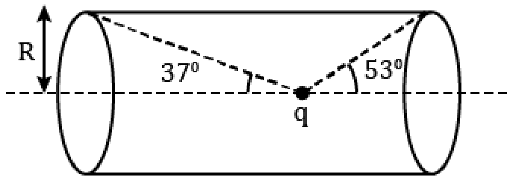
$K_{final} - K_{initial} = W$

$$\Rightarrow \frac{1}{2}mv^2 - 0 = 2kq\lambda \log\left|\frac{r}{r_0}\right| \quad \text{[Using Eq. (iii)]}$$

$$\Rightarrow v = \left(\frac{4kq\lambda}{m}\log\left|\frac{r}{r_0}\right|\right)^{1/2}$$

$$\therefore v \propto \left(\log\left|\frac{r}{r_0}\right|\right)^{1/2}$$

**4. (3)**




$$\phi = \frac{q}{\epsilon_0}\left(\frac{2\pi(1-\cos 37°)}{4\pi}\right) - \frac{q}{\epsilon_0}\left(\frac{2\pi(1-\cos 53°)}{4\pi}\right)$$

$$\phi = \frac{q}{\epsilon_0}\left[1 - \frac{1}{2}\left(1 - \frac{4}{5}\right) - \frac{1}{2}\left(1 - \frac{3}{5}\right)\right]$$

$$\frac{q}{\epsilon_0}\left[1 - \frac{1}{10} - \frac{2}{10}\right] = \frac{7q}{10\epsilon_0}$$

**5. (3)**

$K_i + U_i = K_f + U_f$

$$\frac{1}{2}m(kv_e)^2 - \frac{GMm}{R} = 0 - \frac{GMm}{r}$$

$$\therefore \frac{1}{2}k^2(2gR) - \frac{GM}{R} = -\frac{GM}{r}$$

$$\text{or } \frac{GM}{r}(1-k^2) = \frac{GM}{r}\left(\text{as } gR = \frac{GM}{R}\right)$$

$$\text{so, } r = \frac{R}{1-k^2}$$

**6. (4)**

$$g = \frac{GM}{R^2} \quad \Rightarrow \therefore GM = gR^2$$

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+x}} = \sqrt{\frac{gR^2}{R+x}}$$

**7. (3)**

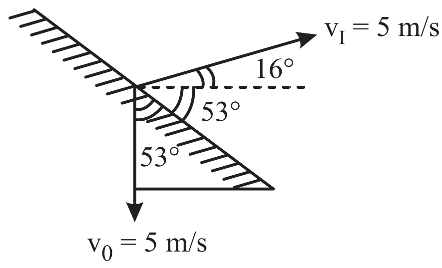
$$a_{LHS} = a_1 = \frac{\text{Net pulling force}}{\text{Total mass}} = \frac{3\text{ mg}}{4m} = \frac{3}{4}g$$

$$a_{RHS} = a_2 = \frac{\text{Net pulling force}}{\text{Total mass}} = \frac{2\text{ mg}}{3m} = \frac{2}{3}g$$

The relative acceleration is therefore:

$$\frac{3}{4}g + \frac{2}{3}g = \frac{17}{12}g$$

**8. (3)**




$$v_I = (5\cos 16°)\hat{i} + (5\sin 16°)\hat{j}$$

$$= (4.8\hat{i} + 1.4\hat{j})\text{m/s}$$

**9.** **(2)**

$v_{OM} = v_0 - v_M = (-\hat{i} - 3\hat{k})$

$\therefore \quad v_{IM} = (-\hat{i} + 3\hat{k}) = v_I - v_M$

or $\quad v_I = (-\hat{i} + 3\hat{k}) + v_M$

$= (3\hat{i} + 4\hat{j} + 11\hat{k})$

**10.** **(4)**

$\delta_a = \left(\frac{3}{2} - 1\right) \times A = \frac{A}{2}$

$\delta_W = \left(\frac{3/2}{4/3} - 1\right) A = \frac{A}{8}$

$\frac{\delta_{air}}{\delta_{water}} = \frac{4}{1}$

**11.** **(1)**

As refractive index for $z > 0$ and $z \le 0$ is different, *xy*-plane should be boundary between two media.

Angle of incidence,

$\cos i = \left| \frac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}} \right| = \frac{1}{2}$

$\therefore \quad i = 60°$

From Snell's law,

$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = \frac{\sqrt{3}}{\sqrt{2}}$

$\sin r = \frac{\sqrt{2}}{\sqrt{3}} \times \sin 60°$

$= \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{2}} = 45°$

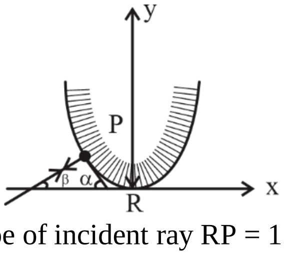
$\Rightarrow \quad r = 45°$

**12.** **(1)**

Point of intersection of two curves. $y = \frac{x^2}{4}$ and

y = x + 3 is, (–2, 1)

$\frac{dy}{dx} = \frac{x}{2}$ i.e. $\left(\frac{dy}{dx}\right)_{x=-2} = \frac{-2}{2} = -1$




tanβ = slope of incident ray RP = 1 or β = 45°

and $\tan\alpha = \left| \left(\frac{dy}{dx}\right)_{x=-2} \right| = 1 \Rightarrow \alpha = 45°$

From geometry we can show that ∠i = ∠r = 0° i.e., the ray is incident normally.

Hence, the desired unit vector:

$A = \frac{1}{\sqrt{2}}\left(-\hat{i} - \hat{j}\right)$

**13.** **(3)**

Let the focal length of each piece be f

Then $\frac{1}{f_1} = \frac{1}{f} + \frac{1}{f}$

$\frac{1}{f_2} = \frac{1}{f} + \frac{1}{f}$

$\Rightarrow f_1 = f_2$

For the third arrangement the liquid forms a concave lens which has a diverging effect.

So $f_3 > f_1 = f_2$

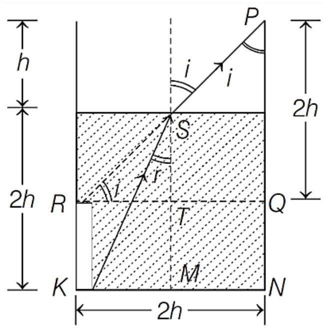
**14.** **(2)**

$PQ = QR = 2h \qquad \Rightarrow \angle i = 45°$

$\therefore \; ST = RT = h = KM = MN$

So, *KS* =

$\therefore \; \sin r = \frac{h}{h\sqrt{5}} = \frac{1}{\sqrt{5}}$




$\therefore \; \frac{\sin i}{\sin r} = \frac{\sin 45°}{1/\sqrt{5}} = \sqrt{\frac{5}{2}}$

**15.** **(1)**

$\therefore \quad \frac{E_1}{(2)^2} - E_1 = 40.8 \text{ eV}$

or $\quad -\frac{3}{4} E_1 = 40.8 \text{ eV}$

or $\quad E_1 = -54.4 \text{ eV}$

$|E_1| = 54.4 \text{ eV}$

**16.** **(3)**

$i = qf = q\left(\frac{v}{2\pi r}\right)$

or $\quad i \propto \frac{v}{r} \propto \frac{(1/n)}{(n)^2}$ or $i \propto \frac{1}{n^3}$

$\frac{i_1}{i_2} = \left(\frac{n_2}{n_1}\right)^3 = (2)^3 = 8$

**17.** **(4)**

$E_n \propto \frac{1}{n^2}$ and $L_n \propto n$

**18.** **(1)**

$K = \frac{p^2}{2m}$

In collision, momentum $p$ remains constant.

$\therefore \quad K \propto \frac{1}{\text{mass}}$

After collision, mass has doubled. So, kinetic energy will remain $\frac{K}{2}$. Hence, loss is also $\frac{K}{2}$

Now, $\frac{K}{2}$ = minimum excitation energy required.

= 10.2 eV

$\Rightarrow$ K = 20.4 eV

**19.** **(4)**

Energy released = Final binding energy – initial binding energy

= 110 × 8.2 + 90 × 8.2 – 200 × 7.4

= 160 MeV

**20.** **(4)**

When the rate production = rate of disintegration, number of nuclei or maximum.

$\therefore \lambda N = A$

or $\frac{\ln 2}{T} N = A$ or $N = \frac{AT}{\ln 2}$ = maximum

**21.** **(1)**

$W_{NA} + W_{NG} + \frac{1}{2} W_G = m_B V^2 - 0$

$W_{NA} + 0 + 0 = \frac{1}{2}(2)(1)^2 - 0 = 1$

**22.** **(2)**

$a_t = \frac{F}{m}\sin\theta$

$\frac{Rd^2(2\theta)}{dt^2} = \frac{F}{m}\sin\theta$

$\frac{d^2\theta}{dt^2} = \frac{F\sin\theta}{2mR}$ …… (i)

$a_c = \frac{F}{m}\cos\theta$

$R\left[\frac{d}{dt}(2\theta)\right]^2 = \frac{F}{m}\cos\theta$ …… (ii)

$$\frac{\left(\frac{d^2\theta}{dt^2}\right)}{\left(\frac{d\theta}{dt}\right)^2} = 2\tan\theta = 2$$

**23.** **(5)**

$\Delta Q_{AB} = nC_P\Delta T = \frac{\gamma}{\gamma - 1} nR\Delta T$

$= \frac{\gamma}{\gamma - 1}[3P_0V_0 - P_0V_0]$

$= 2PV_0 \times \frac{\gamma}{\gamma - 1}$

$\Delta Q_{AC} = \Delta U + \Delta W$

$= \frac{nR}{\gamma - 1}\Delta T + \frac{1}{2} \times 3V_0[P_0 + 4P_0]$

$= \frac{[16P_0V_0 - P_0V_0[}{\gamma - 1} + \frac{15P_0V_0}{2}$

$56 = 2P_0V_0 \times \frac{\gamma}{\gamma - 1}$

$360 = 15P_0V_0\left[\frac{\gamma + 1}{2(\gamma - 1)}\right]$

$\frac{360}{56} = \frac{15}{4}\frac{(\gamma + 1)}{\gamma}$

$12\gamma = 7\gamma + 7$

$\gamma = \frac{7}{5} = 1 + \frac{2}{f} \Rightarrow f = 5$

**24.** **(2)**

From Gauss theorem

$E \propto \frac{q}{r^2}$ ($q$ = charge enclosed)

$\frac{E_2}{E_1} = \frac{q_2}{q_1} \times \frac{r_1^2}{r_2^2}$

or $$8 = \frac{\int_0^R (4\pi r^2)kr^a dr}{\int_0^{R/2} (4\pi r^2)kr^a dr} \times \frac{(R/2)^2}{(R)^2}$$

Solving this equation we get,

$a = 2$.

**25.** **(6)**

$X_{BF} = \mu\ell_1 + \ell_2$

$\frac{dx_{BF}}{dt} = \frac{\mu d\ell_1}{dt} + \frac{d\ell_2}{dt}$

$V_{BF} = \frac{4}{3}(-8 + 2) + (4 - 2)$

$= \frac{4}{3} \times (-6) + 2 = -8 + 2 = -6 \text{ ms}^{-1}$

**26.** **(1)**

at plane surface

Image of object is $= -\frac{xR}{12} \times 4$ distance

at current surface

$u = -\left[\frac{xR}{12}4 + R\right]$

V = ∞ ROC = –R

$\frac{\mu_r - \mu_i}{R} = \frac{\mu_r}{v} - \frac{\mu_i}{u}$

$\frac{1 - 4}{-R} = \frac{-4}{-\left[\frac{xR4}{12} + R\right]}$

$\Rightarrow$ x = 1

**27.** **(20)**

According to FBD – for vertical equilibrium $f_{net}$ = F sin 30° – mg = 50 – 30 = 20 N in upward direction

As block has tendency to slip up the wall, Hence friction on it will act downwards.

N = F cos 30° = $50\sqrt{3}$ N

But the limiting friction is,

$\mu N = \frac{1}{4}\left(50\sqrt{3}\right) = \frac{25\sqrt{3}}{2}$ N = 21.65 N

For no slipping $f$ must be 20 N, which is less than maximum value of friction.

**28.** **(25)**

Given $a_A = 2\alpha = 5\ m/s^2$

$\Rightarrow \alpha = 2.5\ rad/s^2$

$\Rightarrow a_B = 1.(\alpha) = 2.5\ m/s^2$

$\frac{N}{10} = 2.5$

$N = 2.5 \times 10 = 25$

**29.** **(5)**

P = F.V = constant

$F = \frac{P}{V}$

$F \propto \frac{1}{V}$ as V ↑, F ↓

When

Net force on block becomes zero i.e. its maximum velocity

$P = (\mu mg)\ V_{max};\ V_{max} = \frac{100}{1\times2\times10} = 5$ m/s

**30.** **(2)**

## CHEMISTRY

**31.** **(2)**

Apply Kolharausch's law.

**32.** **(1)**

$\Delta T_f = iK_f\ m$

**33.** **(3)**

$$\underset{\substack{\text{c.c.p}\\(4)}}{\overset{\text{Cu}}{\downarrow}} \qquad \underset{\substack{\text{edge centre}\\ \frac{1}{4}\times 12}}{\overset{\text{Ag}}{\downarrow}} \qquad \underset{\substack{\text{body centre}\\ 1}}{\overset{\text{Au}}{\downarrow}}$$

$Cu_4Ag_3Au$

**34.** **(4)**

Conceptual

**35.** **(3)**

$r = K[A]^n$ …..(i)

$2r = K[4A]^n = K[A]^n 2^{2n}$ …..(ii)

(ii) $= 2 = 2^{2n}$

(i) $2n = 1,\ n = \frac{1}{2}$

**36.** **(4)**

Fact.

**37.** **(4)**

$3^2 y$

**38.** **(4)**

If central atom does not have any lone pair then required geometry will be obtained.

**39.** **(1)**

**40** **(3)**

Crowding

**41.** **(4)**

Inert pair effect.

**42.** **(3)**

Conceptual

**43.** **(4)**

Fact

**44.** **(4)**

Fact.

**45.** **(4)**

Non terminal alkyne does not give tollen's test.

**46.** **(3)**

$$CH_3CCl_3 \xrightarrow[\text{hydrolysis}]{\text{Alkaline}} CH_3-\overset{\overset{\large O}{\|}}{C}-OH$$

$$\xrightarrow[AgNO_3]{NH_4OH} CH_3COOAg \xrightarrow{Br_2} CH_3-Br + AgBr + CO_2$$

**47.** **(1)**

Hoffman bromamide degradation reaction followed by elimination.

**48.** **(2)**

Glycine simplest alpha amino acid.

**49.** **(2)**

alkoxy mercuration de mercuration

**50.** **(1)**
Crowding & +I retards nucleophilic addition.

**51.** **(6)**
$hcp : z = 6$

**52.** **(32)**

$$\underset{(r)}{30} \longrightarrow \underset{(2r)}{40} \longrightarrow \underset{(2\times2r)}{50} \longrightarrow \underset{(2\times2\times2r)}{60} \longrightarrow \underset{(2^4 r)}{70} \longrightarrow \underset{(2^5 r)}{80}$$

**53.** **(3)**
Spherical node: $n - l - 1$; 4s $4 - 0 - 1 = 3$

**54.** **(8)**

$$2H_2 + O_2 \longrightarrow 2H_2O$$

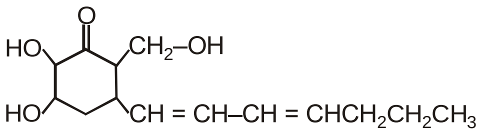
$$eq. = \frac{m.w.}{4} = \frac{32}{4} = 8$$

**55.** **(4)**

$$[\overset{(I)}{Fe}(H_2O)_5 NO]SO_4, K_3[\overset{(III)}{Fe}(CN)_6], [\overset{(0)}{Fe}(CO)_5]$$

$1 + 3 = 4$

**56.** **(64)**
$2^n = \text{stereoisomer}$




**57.** **(3)**

$$COCl_3 \cdot 5NH_3 \rightarrow SFL\,(NH_3)$$

$$[Co(NH_3)_5Cl]Cl_2$$

**58.** **(6)**
Nitrogen molecule is diatomic containing a triple bond between two N atoms, $\ddot{N} \equiv \ddot{N}$. Therefore, nitrogen molecule is formed by sharing six electrons.

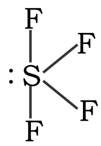
**59.** **(4)**

$$CH_3 - \overset{\overset{\displaystyle O}{\|}}{C} - H \qquad CH_3 - CH_2 - \overset{\overset{\displaystyle O}{\|}}{C} - H$$

self (1)      self (1)

Cross=2
Total=4

**60.** **(3)**




3 atoms in a plane

## MATHEMATICS

**61.** **(3)**

$$488 = \frac{n}{2}\left[2\left(\frac{100}{5}\right) + (n-1)\left(\frac{2}{5}\right)\right]$$

$$488 = \frac{n}{2}(101 - n)$$

$$\Rightarrow n^2 - 101n + 2440 = 0$$

$$\Rightarrow n = 61 \text{ or } 40$$

For $n = 40 \Rightarrow T_n > 0$

For $n = 61 \Rightarrow T_n < 0$

$$T_n = \frac{100}{5} + (61-1)\left(-\frac{2}{5}\right) = -4$$

**62.** **(1)**

$$|z_1 - 1| = \text{Re}(z_1)$$

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$$(x_1 - 1)^2 + y_1^2 = x_1^2$$

$$y_1^2 - 2x_1 + 1 = 0 \qquad \ldots(1)$$

$$|z_2 - 1| = \text{Re}(z_2)$$

$$(x_2 - 1)^2 + y_2^2 = x_2^2$$

$$y_2^2 - 2x_2 + 1 = 0 \qquad \ldots(2)$$

$$y_1^2 - y_2^2 - 2(x_1 - x_2) = 0$$

$$(y_1 - y_2)(y_1 + y_2) = 2(x_1 - x_2)$$

$$y_1 + y_2 = 2\left(\frac{x_1 - x_2}{y_1 - y_2}\right) \qquad \ldots(3)$$

$$\arg(z_1 - z_2) = \frac{\pi}{6}$$

$$\tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \frac{\pi}{6}$$

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{1}{\sqrt{3}} \qquad \ldots(4)$$

$$\therefore y_1 + y_2 = 2\sqrt{3}$$

$$\Rightarrow \text{lm}(z_1 + z_2) = 2\sqrt{3}$$

**63.** **(3)**

$$a\cos\theta = b\cos\left(\theta + \frac{2\pi}{3}\right) = c\cos\left(\theta + \frac{4\pi}{3}\right) = k$$

$$a = \frac{k}{\cos\theta},\ b = \frac{k}{\cos\left(\theta + \frac{2\pi}{3}\right)},$$

$$c=\frac{k}{\cos\left(\theta+\frac{4\pi}{3}\right)}$$

$ab + bc + ca$

$$=k^2\frac{\left[\cos\left(\theta+\frac{4\pi}{3}\right)+\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)\right]}{\cos\left(\theta+\frac{4\pi}{3}\right)\cos\theta\cos\left(\theta+\frac{2\pi}{3}\right)}$$

$$=k^2\left[\frac{\cos\theta+2\cos(\theta+\pi).\cos\left(\frac{\pi}{3}\right)}{\cos\theta.\cos\left(\theta+\frac{2\pi}{3}\right).\cos\left(\theta+\frac{4\pi}{3}\right)}\right]$$

$$=k^2\left[\frac{\cos\theta-2\cos\theta.\frac{1}{2}}{\cos.\theta\cos\left(2+\frac{2\pi}{3}\right).\cos\left(\theta+\frac{4\pi}{3}\right)}\right]=0$$

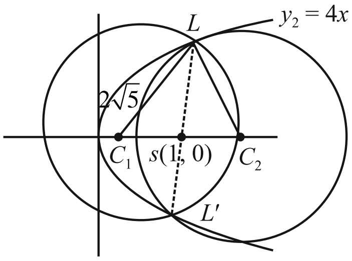
$$\cos\phi=\frac{\left(a\hat{i}+b\hat{j}+c\hat{k}\right).\left(b\hat{i}+c\hat{j}+a\hat{k}\right)}{\sqrt{a^2+b^2+c^2}.\sqrt{b^2+c^2+a^2}}$$

$\Rightarrow ab + bc + ca = 0$

$\Rightarrow \phi=\frac{\pi}{2}$

**64. (3)**

$C_1C_2=2C_1S=2\sqrt{20-4}=8$




**65. (1)**

For $R_1$ let $a=1+\sqrt{2},\ b=1-\sqrt{2},\ c=8^{1/4}$

$aR_1b \Rightarrow a^2+b^2=\left(1+\sqrt{2}\right)^2+\left(1-\sqrt{2}\right)^2$

$= 6 \in Q$

$aR_1c \Rightarrow b^2+c^2=\left(1-\sqrt{2}\right)^2+\left(8^{1/4}\right)$

$= 3 \in Q$

$aR_1c \Rightarrow a^2+c^2=\left(1+\sqrt{2}\right)^2+\left(8^{1/4}\right)$

$=3+4\sqrt{2}\notin Q$

$\therefore$ $R_1$ is not transitive.

For $R_2$ let $a=1+\sqrt{2},\ b=\sqrt{2},\ c=1-\sqrt{2}$

$aR_2B \Rightarrow a^2+b^2=\left(1+\sqrt{2}\right)^2+\left(\sqrt{2}\right)^2$

$=5+2\sqrt{2}\notin Q$

$bR_2b \Rightarrow b^2+c^2=\left(\sqrt{2}\right)^2+\left(1-\sqrt{2}\right)^2$

$=5-2\sqrt{2}\notin Q$

$aR_2c \Rightarrow a^2+c^2=\left(1+\sqrt{2}\right)^2+\left(1-\sqrt{2}\right)^2$

$= 6 \in Q$

$\therefore$ $R^2$ is not transitive.

**66. (3)**

$$\frac{k}{6}=\int_0^{\frac{1}{2}}\frac{x^2}{\left(1-x^2\right)^{3/2}}dx$$

Put $x=\sin\theta;\ dx=\cos\theta d\theta$

$$\Rightarrow \frac{k}{6}=\int_0^{\frac{\pi}{6}}\frac{\sin^2\theta}{\left(1-\sin^2\theta\right)^{3/2}}.\cos\theta d\theta$$

$$\Rightarrow \frac{k}{6}=\int_0^{\frac{\pi}{6}}\frac{\sin^2\theta}{\cos^3\theta}.\cos\theta d\theta$$

$$\Rightarrow \frac{k}{6}=\int_0^{\frac{\pi}{6}}\tan^2\theta d\theta=\int_0^{\frac{\pi}{6}}\left(\sec^2\theta-1\right)d\theta$$

$$\Rightarrow \frac{k}{6}=\left(\tan\theta-\theta\right)_0^{\pi/6}=\left(\frac{1}{\sqrt{3}}-\frac{\pi}{6}\right)=\frac{2\sqrt{3}-\pi}{6}$$

$\Rightarrow k=2\sqrt{3}-\pi$

**67. (3)**

$x^3dy+xydx=2ydx+x^2dy$

$\Rightarrow \left(x^3-x^2\right)dy=(2-x)ydx$

$$\Rightarrow \frac{dy}{y}=\frac{2-x}{x^2(x-1)}dx$$

$$\Rightarrow \int\frac{dy}{y}=\int\frac{2-x}{x^2(x-1)}dx \quad \ldots\text{(i)}$$

Let $\frac{2-x}{x^2(x-1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-1}$

$\Rightarrow 2-x=A(x-1)+B(x-1)+Cx^2$

$\Rightarrow C=1,\ B=-2$ and $A=-1$

$$\Rightarrow \int\frac{dy}{y}=\int\left\{\frac{-1}{x}-\frac{2}{x^2}+\frac{1}{x-1}\right\}dx$$

$$\Rightarrow \ell ny=-\ell nx+\frac{2}{x}+\ell n|x-1|+C$$

$\therefore$ $y(B) = e$

$\Rightarrow 1=-\ell n2+1+0+C$

$\Rightarrow C=-\ell n2$

$$\Rightarrow \ell ny=-\ell nx+\frac{2}{x}+\ell n|x-1|+\ell n2$$

at $x = 4$

$$\Rightarrow \ell ny(4)=-\ell n4+\frac{1}{2}+\ell n3+\ell n2$$

$$\Rightarrow \ell ny(4)=-\ell n\left(\frac{3}{2}\right)+\frac{1}{2}=\ell n\left(\frac{3}{2}e^{1/2}\right)$$

$$\Rightarrow y(4)=\frac{3}{2}e^{1/2}$$

**68. (4)**

Total = $9(10^4)$

Fav. Way = ${}^9C_2\,(2^5 - 2) + {}^9C_1\,(2^4 - 1)$

$= 36(30) + 9(15) = 1080 + 135$

$$\text{Probability} = \frac{36\times30+9\times15}{9\times10^4} = \frac{4\times30+15}{10^4}$$

$$= \frac{135}{10^4}$$

**69. (1)**

$$S = 6a^2 \Rightarrow \frac{ds}{dt} = 12a.\frac{da}{dt} = 3.6$$

$$\Rightarrow 12(10)\frac{da}{dt} = 3.6$$

$$\Rightarrow \frac{da}{dt} = 0.03$$

$$V = a^3 \Rightarrow \frac{dv}{dt} = 3a^2.\frac{da}{dt}$$

$$= 3(10)^2.\left(\frac{3}{10}\right) = 9$$

**70. (4)**

$f(0)f(1) \le 0$

$$\Rightarrow 2\left(\lambda^2 + 1 - 4\lambda + 2\right) \le 0$$

$$\Rightarrow 2\left(\lambda^2 - 4\lambda + 3\right) \le 0$$

$$(\lambda-1)(\lambda-3) \le 0$$

$$\Rightarrow \lambda \in [1,\ 3]$$

But at $\lambda = 1$, both roots are 1 so $\lambda \ne 1$

**71. (2)**

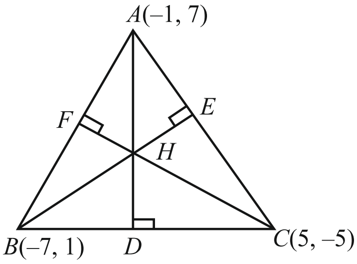
$$m_{BC} = \frac{6}{-12} = -\frac{1}{2}$$

$\therefore$ Equation of $AD$ is $y - 7 = 2(x + 1)$

$y = 2x + 9$ …(1)

$$m_{AC} = \frac{12}{-6} = -2$$

$\therefore$ Equation of $BE$ is $y - 1 = \frac{1}{2}(x+7)\,y$




$$y = \frac{x}{2} + \frac{9}{2} \quad …(2)$$

by (1) and (2)

$$2x + 9 = \frac{x+9}{2}$$

$\Rightarrow 4x + 18 = x + 9$

$\Rightarrow 3x = 9 \Rightarrow x = -3$

$\therefore\ y = 3$

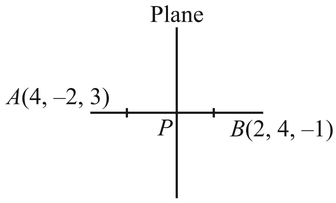
**72. (2)**

$(p \wedge q)$ should be TRUE and $(-q \vee r)$ should be FALSE.

**73. (3)**

Mid point $P \equiv (3, 1, 1)$

Normal of plane is along the line AB.




D.R.'s of normal = 4 – 2, –2 –4, 3 – 1 (–1)

= 2, –6, 4 = 1, –3, 2

Plane $\rightarrow 1(x - 3) - 3(y - 1) + 2(z - 1) = 0$

$\Rightarrow x - 3y + 2z - 2 = 0$

**74. (1)**

$$e_1 = \sqrt{1-\frac{b^2}{25}};\ e_2 = \sqrt{1+\frac{b^2}{16}}$$

$$e_1e_2 = 1$$

$$\Rightarrow (e_1e_2)^2 = 1$$

$$\Rightarrow \left(1-\frac{b^2}{25}\right)\left(1+\frac{b^2}{16}\right) = 1$$

$$\Rightarrow 1 + \frac{b^2}{16} - \frac{b^2}{25} - \frac{b^4}{25\times16} = 1$$

$$\Rightarrow \frac{9}{16.25}b^2 - \frac{b^4}{25.16} = 0$$

$\Rightarrow b^2 = 9$

$$e_1 = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$$

$$e_2 = \sqrt{1+\frac{9}{16}} = \frac{5}{4}$$

$$\alpha = 2(5)(e_1) = 8$$

$$\beta = 2(4)(e_2) = 10$$

$$(\alpha,\ \beta) = (8,\ 10)$$

**75. (4)**

$$|\text{adj } A| = |A|^2 = 9$$

$$\Rightarrow |A| = \pm3 = \lambda \Rightarrow |\lambda| = 3$$

$$\Rightarrow |B| = |\text{adj } A|^2 = 81$$

$$\Rightarrow \left|\left(B^{-1}\right)^T\right| = \left|B^{-1}\right| = |B|^{-1} = \frac{1}{|B|} = \frac{1}{81} = \mu$$

**76.** **(3)**

$$I=\int \sin^{-1}\sqrt{\frac{x}{1+x}}dx$$

Put $x=\tan^2\theta$

$dx=2\tan\theta\sec^2\theta d\theta$

$$\therefore I=\int \sin^{-1}\sqrt{\frac{\tan^2\theta}{1+\tan^2\theta}}\ 2\tan\theta\sec^2\theta d\theta$$

$$=2\int\theta\left(\tan\theta\sec^2\theta\right)dx$$

$$=\theta\tan^2\theta-\int\left(\sec^2\theta-1\right)d\theta$$

$$=\theta\left(1+\tan^2-1\right)-\tan\theta+c$$

$$=\tan^{-1}\sqrt{x}(1+x)-\sqrt{x}+c$$

**77.** **(4)**

$$S.D.=\sqrt{\frac{\sum_{i=1}^{10}(x_i-p)^2}{10}-\left(\frac{\sum_{i=1}^{10}(x_i-p)}{10}\right)^2}$$

$$\sqrt{\frac{9}{10}-\left(\frac{3}{10}\right)^2}=\frac{9}{10}$$

**78.** **(3)**

$$T_{r+1}={}^9C_r\left(\frac{3x^2}{2}\right)^{9-r}\left(-\frac{1}{3x}\right)^r$$

$$={}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{18-3r}$$

for the term independent of $x$ put $r=6$

$$\Rightarrow T_7={}^9C_6\left(\frac{3}{2}\right)^3\left(-\frac{1}{3}\right)^6$$

$$={}^9C_3\left(\frac{1}{6}\right)^3=\frac{9\times8\times7}{3\times2\times1}\left(\frac{1}{6}\right)^3=\left(\frac{7}{18}\right)$$

**79.** **(4)**

$$f'(x)=k.x(x+1)(x-1)=k\left(x^3-x\right)$$

$$\Rightarrow f(x)=k\left(\frac{x^4}{4}-\frac{x^2}{2}\right)+C$$

$\Rightarrow f(0)=C$

$\Rightarrow f(x)=f(0)$

$$\Rightarrow k\frac{\left(x^4-2x^2\right)}{4}+C=C$$

$$\Rightarrow x^2\left(x^2-2\right)=0$$

$$\Rightarrow x=\left\{0,\ \sqrt{2}-\sqrt{2}\right\}$$

**80.** **(4)**

$$\lim_{x\to a}\frac{\frac{1}{3}(a+2x)^{-2/3}.2-\frac{1}{3}.(3x)^{-2/3}.3}{\frac{1}{3}(3a+x)^{-2/3}.-\frac{1}{3}(4x)^{-2/3}.4}$$

$$=\frac{\frac{1}{3}(3a)^{-2/3}.(2-3)}{\frac{1}{3}(4a)^{-2/3}.(1-4)}=\frac{3^{-2/3}}{4^{-2/3}}.\frac{1}{3}$$

$$=\frac{2^{4/3}}{9^{1/3}}.\frac{1}{3}=\frac{2}{3}.\left(\frac{2}{9}\right)^{1/3}$$

**81.** **(04)**

For (1, 2) of $y^2=4x\Rightarrow t=1,\ a=1$

normal $\Rightarrow tx+y=2at+at^3$

$\Rightarrow x+y=3$ intersect $x$ – axis at (3, 0)

$$y=e^x\Rightarrow\frac{dy}{dx}=e^x$$

tangent $\Rightarrow y-e^c=e^c(x-c)$

at (3, 0) $\Rightarrow 0-e^c=e^c(3-c)\Rightarrow c=4$

**82.** **(54)**

Let $xyz$ be the three digit number

$x+y+z=10,\ x\ge1,\ y\ge0,\ z\ge0$

$x-1=t\quad\Rightarrow\quad x=1+t\quad x-1\ge0$

$t\ge0$

$t+y+z=10-1$

$t+y+z=9,\quad 0\le t,\ y,\ z\le9$

total number of non-negative integral solution = ${}^{9+3-1}C_{3-1}={}^{11}C_2=\frac{11.10}{2}=55$

But for $t=9$, $x=10$, so required number or integers $=55-1=54$

**83.** **(08)**

$x-2y+5z=0\qquad\ldots$(i)

$-2x+4y+z=0\qquad\ldots$(ii)

$-7x+14y+9z=0\qquad\ldots$(iii)

2 × (i) + (ii) $\Rightarrow z=0$

$\Rightarrow x=2y$

$\Rightarrow 15\le x^2+y^2+z^2\le150$

$\Rightarrow 15\le 4y^2+y^2\le150$

$\Rightarrow 3\le y^2\le30$

$\Rightarrow y=\pm2,\ \pm3,\ \pm4,\ \pm5$

$\Rightarrow$ 8 solutions

**84. (5)**

Normal of plane = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$

$\vec{n} = -\hat{i} + \hat{j} + \hat{k}$

D.R.'s = –1, 1, 1

Plane $\Rightarrow -1(x-1) + 1(y-0) + 1(z-0) = 0$

$\Rightarrow x - y - z - 1 = 0$

If $(x, y, z)$ is foot of perpendicular of $M$ (1, 0, 1) on the plane then

$\Rightarrow \frac{x-1}{1} = \frac{y-0}{-1} = \frac{z-1}{0} = \frac{-(1-0-1-1)}{3}$

$x = \frac{4}{3},\ y = -\frac{1}{3},\ z = \frac{2}{3}$

$\alpha + \beta + \gamma = \frac{4}{3} - \frac{1}{3} + \frac{2}{3} = \frac{5}{2}$

**85. (39)**

$3, A_1, A_2, A_3, \ldots A_m, 243$

$d = \frac{243-3}{m+1} = \frac{240}{m+1}$

$3, G_1, G_2, G_3, 243$

$r = \left(\frac{243}{3}\right)^{\frac{1}{3+1}} = (81)^{1/4} = 3$

$G_2 = A_4$

$\Rightarrow 3(3)^2 = 3 + 4\left(\frac{240}{m+1}\right)$

$\Rightarrow 27 = 3 + \frac{960}{m+1}$

$\Rightarrow m + 1 = 40$

$\Rightarrow m = 39$

**86.** (5)

$f : [-1, 1] \to R$

$f(x) = ax^2 + bx + c$

$f(-1) = a - b + c = 2$

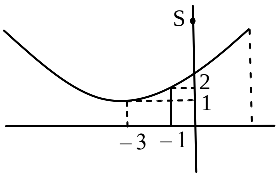
$f'(-1) = -2a + b = 1$

$f''(x) = 2a$

$\Rightarrow$ Max. value $f''(x) = 2a = \frac{1}{2}$

$\Rightarrow a = \frac{1}{4}; b = \frac{3}{2}; c = \frac{13}{4}$

$\therefore f(x) = \frac{x^2}{4} + \frac{3}{2}x + \frac{13}{4}$




For $x \in [-1, 1] \Rightarrow 2 \le f(x) \le 5$

$\therefore$ Least value of $\alpha$ is 5

**87. (9)**

Let the equation of normal is $Y - y = -\frac{1}{m}(X - x)$,

where $m = \frac{dy}{dx}$ As it passes through $(a, b)$

$b - y = -\frac{1}{m}(a - x) = -\frac{dx}{dy}(a - x)$

$\Rightarrow (b - y)dy = (x - a)dx$

by $-\frac{y^2}{2} = \frac{x^2}{2} - ax + c$

It passes through (3, – 3) & $(4, -2\sqrt{2})$

$\therefore -3b - \frac{9}{2} = \frac{9}{2} - 3a + c$

$\Rightarrow -6b - 9 = 9 - 6a + 2c$

$\Rightarrow 3a - 3b - c = 9$

Also $-2\sqrt{2}b - 4 = 8 - 4a + c$

$4a - 2\sqrt{2}b - c = 12$

Also $a - 2\sqrt{2}\mathrm{b} = 3$ ……..(iv) (given)

(ii) – (iii) $\Rightarrow -a + (2\sqrt{2} - 3)b = -3$

(iv) + (v)

$\Rightarrow \mathrm{b} = 0, \quad \mathrm{a} = 3$

$\therefore a^2 + b^2 + ab = 9$

**88. (5)**

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > b)$

Given $2a - 2b = 10 \Rightarrow a - b = 5$

$c = 5\sqrt{3}$

$c^2 = a^2 - b^2 = 75$

$\Rightarrow (a - b)(a + b) = 45$

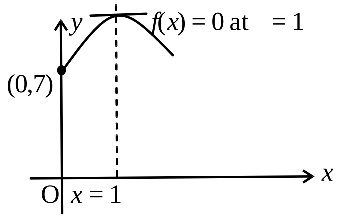
$a + b = 15$

$\therefore\ a = 10, b = 5$

Length of $LR = \frac{2b^2}{a} = \frac{7 \times 25}{10} = 5$

**89. (7)**

$f(x) = x^3 - 3(a-2)x^2 + 3ax + 7, f(0) = 7$




$\Rightarrow f'(x) = 3x^2 - 6(a-2)x + 3a$

$f'(1) = 0$

$\Rightarrow 1 - 2a + 4 + a = 0$

$\Rightarrow a = 5$

Then, $f(x) = x^3 - 9x^2 + 15x + 7$

Now, $\dfrac{f(x)-14}{(x-1)^2}=0$

$$\Rightarrow \frac{x^3-9x^2+15x+7-14}{(x-1)^2}=0$$

$$\Rightarrow \frac{(x-1)^2(x-7)}{(x-1)^2}=0 \Rightarrow x=7$$

**90.** **(1)**

$A = XB$

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}=\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{3}a_1 \\ \sqrt{3}a_2 \end{bmatrix}=\begin{bmatrix} b_1-b_2 \\ b_1+kb_2 \end{bmatrix}$$

$\mathrm{b}_1-\mathrm{b}_2=\sqrt{3}\mathrm{a}_1$........(i)

$\mathrm{b}_1+\mathrm{kb}_2=\sqrt{3}\mathrm{a}_2$ ........(ii)

Given $a_1^2+a_2^2=\frac{2}{3}\left(b_1^2+b_2^2\right)$

$(1)^2+(2)^2$

$\left(b_1+b_2\right)^2+\left(b_1+kb_2\right)^2=3\left(a_1^2+a_2^2\right)$

$$a_1^2+a_2^2=\frac{2}{3}b_1^2+\frac{\left(1+k^2\right)}{3}b_2^2+\frac{2}{3}b_1b_2(k-1)$$

Given, $a_1^2+a_2^2=\frac{2}{3}b_1^2+\frac{2}{3}b_2^2$

On comparing we get

$$\frac{\mathrm{k}^2+1}{3}=\frac{2}{3}\Rightarrow \mathrm{k}^2+1=2$$

$$\Rightarrow k=\pm 1 \;\&\; \frac{2}{3}(\mathrm{k}-1)=0\Rightarrow \mathrm{k}=1$$

From both we get $k = 1$