Series EF1GH/1 **SET~1**

प्रश्न–पत्र कोड
Q.P. Code **65/1/1**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS
*

निर्धारित समय : 3 घण्टे *अधिकतम अंक : 80*
Time allowed : 3 hours *Maximum Marks : 80*

नोट / NOTE :

(i) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं ।***
*Please check that this question paper contains **23** printed pages.*

(ii) ***प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।***
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं ।***
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) ***इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।***
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :

(i) *इस प्रश्न-पत्र में **38** प्रश्न हैं । **सभी** प्रश्न **अनिवार्य** हैं ।*

(ii) *यह प्रश्न-पत्र **पाँच** खण्डों में विभाजित है – **क, ख, ग, घ** एवं **ङ** ।*

(iii) ***खण्ड क** में प्रश्न संख्या **1** से **18** तक बहुविकल्पीय तथा प्रश्न संख्या **19** एवं **20** अभिकथन एवं तर्क आधारित **एक-एक** अंक के प्रश्न हैं ।*

(iv) ***खण्ड ख** में प्रश्न संख्या **21** से **25** तक अति लघु-उत्तरीय (VSA) प्रकार के **दो-दो** अंकों के प्रश्न हैं ।*

(v) ***खण्ड ग** में प्रश्न संख्या **26** से **31** तक लघु-उत्तरीय (SA) प्रकार के **तीन-तीन** अंकों के प्रश्न हैं ।*

(vi) ***खण्ड घ** में प्रश्न संख्या **32** से **35** तक दीर्घ-उत्तरीय (LA) प्रकार के **पाँच-पाँच** अंकों के प्रश्न हैं ।*

(vii) ***खण्ड ङ** में प्रश्न संख्या **36** से **38** प्रकरण अध्ययन आधारित **चार-चार** अंकों के प्रश्न हैं ।*

(viii) *प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है । यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है ।*

(ix) *कैल्कुलेटर का उपयोग **वर्जित** है ।*

खण्ड क

इस खण्ड में बहुविकल्पीय प्रश्न हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है ।

1. यदि किसी वर्ग आव्यूह A के लिए $A^2 - 3A + I = O$ है तथा $A^{-1} = xA + yI$ है, तो x + y का मान है :

(a) -2 (b) 2

(c) 3 (d) -3

2. यदि एक 2×2 आव्यूह A के लिए $|A| = 2$ है, तो $|4A^{-1}|$ बराबर है :

(a) 4 (b) 2

(c) 8 (d) $\frac{1}{32}$

3. माना A एक ऐसा 3×3 आव्यूह है कि $|\text{adj } A| = 64$ है । तो $|A|$ बराबर है :

(a) केवल 8 (b) केवल -8

(c) 64 (d) 8 अथवा -8

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If for a square matrix A, $A^2 - 3A + I = O$ and $A^{-1} = xA + yI$, then the value of $x + y$ is :

(a) -2 (b) 2

(c) 3 (d) -3

2. If $|A| = 2$, where A is a 2×2 matrix, then $|4A^{-1}|$ equals :

(a) 4 (b) 2

(c) 8 (d) $\frac{1}{32}$

3. Let A be a 3×3 matrix such that $|\text{adj } A| = 64$. Then $|A|$ is equal to :

(a) 8 only (b) -8 only

(c) 64 (d) 8 or -8

4. यदि $A = \begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$ है तथा $2A + B$ एक शून्य आव्यूह है, तो B बराबर है :

(a) $\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 8 \\ 10 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & -8 \\ -10 & -3 \end{bmatrix}$

5. यदि $\frac{d}{dx}(f(x)) = \log x$ है, तो f(x) बराबर है :

(a) $-\frac{1}{x} + C$ (b) $x(\log x - 1) + C$

(c) $x(\log x + x) + C$ (d) $\frac{1}{x} + C$

6. $\int_0^{\frac{\pi}{6}} \sec^2(x - \frac{\pi}{6})\, dx$ बराबर है :

(a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) $-\sqrt{3}$

7. अवकल समीकरण $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = \sin y$ की कोटि तथा घात का योगफल है :

(a) 5 (b) 2

(c) 3 (d) 4

8. p का वह मान जिसके लिए सदिश $2\hat{i} + p\hat{j} + \hat{k}$ तथा $-4\hat{i} - 6\hat{j} + 26\hat{k}$ परस्पर लंबवत हैं, है :

(a) 3 (b) -3

(c) $-\frac{17}{3}$ (d) $\frac{17}{3}$

4. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$ and 2A + B is a null matrix, then B is equal to :

(a) $\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 8 \\ 10 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} -5 & -8 \\ -10 & -3 \end{bmatrix}$

5. If $\frac{d}{dx}(f(x)) = \log x$, then f(x) equals :

(a) $-\frac{1}{x} + C$

(b) $x(\log x - 1) + C$

(c) $x(\log x + x) + C$

(d) $\frac{1}{x} + C$

6. $\int_0^{\frac{\pi}{6}} \sec^2\left(x - \frac{\pi}{6}\right) dx$ is equal to :

(a) $\frac{1}{\sqrt{3}}$

(b) $-\frac{1}{\sqrt{3}}$

(c) $\sqrt{3}$

(d) $-\sqrt{3}$

7. The sum of the order and the degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = \sin y$ is :

(a) 5

(b) 2

(c) 3

(d) 4

8. The value of p for which the vectors $2\hat{i} + p\hat{j} + \hat{k}$ and $-4\hat{i} - 6\hat{j} + 26\hat{k}$ are perpendicular to each other, is :

(a) 3

(b) -3

(c) $-\frac{17}{3}$

(d) $\frac{17}{3}$

9. $(\hat{i} \times \hat{j}) \cdot \hat{j} + (\hat{j} \times \hat{i}) \cdot \hat{k}$ का मान है :

(a) 2 (b) 0

(c) 1 (d) -1

10. यदि $\vec{a} + \vec{b} = \hat{i}$ तथा $\vec{a} = 2\hat{i} - 2\hat{j} + 2\hat{k}$ है, तो $|\vec{b}|$ बराबर है :

(a) $\sqrt{14}$ (b) 3

(c) $\sqrt{12}$ (d) $\sqrt{17}$

11. रेखा $\frac{x-1}{2} = \frac{1-y}{3} = \frac{2z-1}{12}$ के दिक्-कोसाइन हैं :

(a) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$ (b) $\frac{2}{\sqrt{157}}, -\frac{3}{\sqrt{157}}, \frac{12}{\sqrt{157}}$

(c) $\frac{2}{7}, -\frac{3}{7}, -\frac{6}{7}$ (d) $\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}$

12. यदि $P\left(\frac{A}{B}\right) = 0{\cdot}3$, $P(A) = 0{\cdot}4$ तथा $P(B) = 0{\cdot}8$ है, तो $P\left(\frac{B}{A}\right)$ बराबर है :

(a) 0·6 (b) 0·3

(c) 0·06 (d) 0·4

13. k का वह मान जिसके लिए $f(x) = \begin{cases} 3x+5, & x \geq 2 \\ kx^2, & x < 2 \end{cases}$ एक संतत फलन है, है :

(a) $-\frac{11}{4}$ (b) $\frac{4}{11}$

(c) 11 (d) $\frac{11}{4}$

14. यदि $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ तथा $(3I + 4A)(3I - 4A) = x^2 I$ है, तो x का/के मान है/हैं :

(a) $\pm\sqrt{7}$ (b) 0

(c) ± 5 (d) 25

9. The value of $(\hat{i} \times \hat{j}) \cdot \hat{j} + (\hat{j} \times \hat{i}) \cdot \hat{k}$ is :

(a) 2 (b) 0

(c) 1 (d) -1

10. If $\vec{a} + \vec{b} = \hat{i}$ and $\vec{a} = 2\hat{i} - 2\hat{j} + 2\hat{k}$, then $|\vec{b}|$ equals :

(a) $\sqrt{14}$ (b) 3

(c) $\sqrt{12}$ (d) $\sqrt{17}$

11. Direction cosines of the line $\dfrac{x-1}{2} = \dfrac{1-y}{3} = \dfrac{2z-1}{12}$ are :

(a) $\dfrac{2}{7}, \dfrac{3}{7}, \dfrac{6}{7}$ (b) $\dfrac{2}{\sqrt{157}}, -\dfrac{3}{\sqrt{157}}, \dfrac{12}{\sqrt{157}}$

(c) $\dfrac{2}{7}, -\dfrac{3}{7}, -\dfrac{6}{7}$ (d) $\dfrac{2}{7}, -\dfrac{3}{7}, \dfrac{6}{7}$

12. If $P\left(\dfrac{A}{B}\right) = 0{\cdot}3$, $P(A) = 0{\cdot}4$ and $P(B) = 0{\cdot}8$, then $P\left(\dfrac{B}{A}\right)$ is equal to :

(a) 0·6 (b) 0·3

(c) 0·06 (d) 0·4

13. The value of k for which $f(x) = \begin{cases} 3x + 5, & x \geq 2 \\ kx^2, & x < 2 \end{cases}$ is a continuous function, is :

(a) $-\dfrac{11}{4}$ (b) $\dfrac{4}{11}$

(c) 11 (d) $\dfrac{11}{4}$

14. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $(3I + 4A)(3I - 4A) = x^2 I$, then the value(s) x is/are :

(a) $\pm\sqrt{7}$ (b) 0

(c) ± 5 (d) 25

15. अवकल समीकरण $x\,dy - (1 + x^2)\,dx = dx$ का व्यापक हल है :

(a) $y = 2x + \frac{x^3}{3} + C$ (b) $y = 2 \log x + \frac{x^3}{3} + C$

(c) $y = \frac{x^2}{2} + C$ (d) $y = 2 \log x + \frac{x^2}{2} + C$

16. यदि $f(x) = a(x - \cos x)$, $\mathbb{R}$ में निरंतर ह्रासमान है, तो 'a' निम्न में से किस में स्थित है ?

(a) $\{0\}$ (b) $(0, \infty)$

(c) $(-\infty, 0)$ (d) $(-\infty, \infty)$

17. किसी रैखिक प्रोग्रामन समस्या के आलेखीय निरूपण में सुसंगत क्षेत्र के शीर्ष बिन्दु (2, 72), (15, 20) तथा (40, 15) हैं । यदि $z = 18x + 9y$ उद्देश्य फलन है, तो :

(a) z, (2, 72) पर अधिकतम तथा (15, 20) पर न्यूनतम है ।

(b) z, (15, 20) पर अधिकतम तथा (40, 15) पर न्यूनतम है ।

(c) z, (40, 15) पर अधिकतम तथा (15, 20) पर न्यूनतम है ।

(d) z, (40, 15) पर अधिकतम तथा (2, 72) पर न्यूनतम है ।

18. व्यवरोधों $x - y \geq 0,\ 2y \leq x + 2,\ x \geq 0,\ y \geq 0$ द्वारा बने सुसंगत क्षेत्र के शीर्ष बिन्दुओं की संख्या है :

(a) 2 (b) 3

(c) 4 (d) 5

*प्रश्न संख्या **19** और **20** अभिकथन एवं तर्क आधारित प्रश्न हैं और प्रत्येक प्रश्न का 1 अंक है । दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है । इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (a), (b), (c) और (d) में से चुनकर दीजिए ।*

(a) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है ।

(b) अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या ***नहीं*** करता है ।

(c) अभिकथन (A) सही है तथा तर्क (R) ग़लत है ।

(d) अभिकथन (A) ग़लत है तथा तर्क (R) सही है ।

15. The general solution of the differential equation $x\,dy - (1 + x^2)\,dx = dx$ is :

(a) $y = 2x + \frac{x^3}{3} + C$
(b) $y = 2 \log x + \frac{x^3}{3} + C$
(c) $y = \frac{x^2}{2} + C$
(d) $y = 2 \log x + \frac{x^2}{2} + C$

16. If $f(x) = a(x - \cos x)$ is strictly decreasing in $\mathbb{R}$, then 'a' belongs to

(a) $\{0\}$
(b) $(0, \infty)$
(c) $(-\infty, 0)$
(d) $(-\infty, \infty)$

17. The corner points of the feasible region in the graphical representation of a linear programming problem are (2, 72), (15, 20) and (40, 15). If $z = 18x + 9y$ be the objective function, then :

(a) z is maximum at (2, 72), minimum at (15, 20)
(b) z is maximum at (15, 20), minimum at (40, 15)
(c) z is maximum at (40, 15), minimum at (15, 20)
(d) z is maximum at (40, 15), minimum at (2, 72)

18. The number of corner points of the feasible region determined by the constraints $x - y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$ is :

(a) 2
(b) 3
(c) 4
(d) 5

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).
(c) Assertion (A) is true and Reason (R) is false.
(d) Assertion (A) is false and Reason (R) is true.

19. *अभिकथन (A) :* फलन $f(x) = 2 \sin^{-1} x + \frac{3\pi}{2}$, जहाँ $x \in [-1, 1]$ का परिसर $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$ है ।

तर्क (R) : $\sin^{-1}(x)$ की मुख्य मान शाखा का परिसर $[0, \pi]$ है ।

20. *अभिकथन (A) :* बिन्दुओं (1, 2, 3) तथा (3, – 1, 3) से होकर जाने वाली रेखा का समीकरण $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-3}{0}$ है ।

तर्क (R) : बिन्दुओं (x_1, y_1, z_1) तथा (x_2, y_2, z_2) से होकर जाने वाली रेखा का समीकरण $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$ है ।

खण्ड ख

इस खण्ड में अति लघु-उत्तरीय (VSA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं ।

21. (क) $f(x) = 2x$ द्वारा परिभाषित फलन $f : A \rightarrow B$, एकैकी और आच्छादक दोनों है । यदि $A = \{1, 2, 3, 4\}$ है, तो समुच्चय B ज्ञात कीजिए ।

अथवा

(ख) मान ज्ञात कीजिए :

$$\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}\left(\cos \frac{3\pi}{4}\right) + \tan^{-1}(1)$$

22. परिमाण $3\sqrt{3}$ के वह सभी सदिश ज्ञात कीजिए जो सदिश $\hat{i} + \hat{j} + \hat{k}$ के संरेख हों ।

23. (क) नीचे दी गई आकृति में दर्शाए गए बिन्दुओं A, B तथा C के स्थिति सदिश क्रमशः $\vec{a}$, $\vec{b}$ तथा $\vec{c}$ हैं ।

यदि $\overrightarrow{AC} = \frac{5}{4}\overrightarrow{AB}$ है, तो $\vec{c}$ को $\vec{a}$ तथा $\vec{b}$ के पदों में व्यक्त कीजिए ।

अथवा

19. *Assertion (A) :* The range of the function $f(x) = 2 \sin^{-1} x + \frac{3\pi}{2}$, where $x \in [-1, 1]$, is $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$.

Reason (R) : The range of the principal value branch of $\sin^{-1}(x)$ is $[0, \pi]$.

20. *Assertion (A) :* Equation of a line passing through the points (1, 2, 3) and (3, – 1, 3) is $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-3}{0}$.

Reason (R) : Equation of a line passing through points (x_1, y_1, z_1), (x_2, y_2, z_2) is given by $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) A function $f : A \to B$ defined as $f(x) = 2x$ is both one-one and onto. If $A = \{1, 2, 3, 4\}$, then find the set B.

OR

(b) Evaluate :

$$\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}\left(\cos \frac{3\pi}{4}\right) + \tan^{-1}(1)$$

22. Find all the vectors of magnitude $3\sqrt{3}$ which are collinear to vector $\hat{i} + \hat{j} + \hat{k}$.

23. (a) Position vectors of the points A, B and C as shown in the figure below are $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.

If $\vec{AC} = \frac{5}{4}\vec{AB}$, express $\vec{c}$ in terms of $\vec{a}$ and $\vec{b}$.

OR

(ख) ज्ञात कीजिए कि क्या रेखाएँ जिनके समीकरण $x = 2\lambda + 2$, $y = 7\lambda + 1$, $z = -3\lambda - 3$ तथा $x = -\mu - 2$, $y = 2\mu + 8$, $z = 4\mu + 5$ हैं, परस्पर लंबवत हैं या नहीं ।

24. यदि $y = (x + \sqrt{x^2 - 1})^2$ है, तो दर्शाइए कि $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2$.

25. दर्शाइए कि फलन $f(x) = \frac{16 \sin x}{4 + \cos x} - x$, $\left(\frac{\pi}{2}, \pi\right)$ में निरंतर ह्रासमान है ।

खण्ड ग

इस खण्ड में लघु-उत्तरीय (SA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं ।

26. मान ज्ञात कीजिए :

$$\int_0^{\frac{\pi}{2}} [\log (\sin x) - \log (2 \cos x)]\, dx$$

27. ज्ञात कीजिए :

$$\int \frac{1}{\sqrt{x}(\sqrt{x} + 1)(\sqrt{x} + 2)}\, dx$$

28. (क) अवकल समीकरण $\frac{dy}{dx} + \sec^2 x \cdot y = \tan x \cdot \sec^2 x$ का विशिष्ट हल ज्ञात कीजिए, दिया गया है कि $y(0) = 0$.

अथवा

(ख) अवकल समीकरण $x\, dy - y\, dx - \sqrt{x^2 + y^2}\, dx = 0$ को हल कीजिए ।

(b) Check whether the lines given by equations $x = 2\lambda + 2$, $y = 7\lambda + 1$, $z = -3\lambda - 3$ and $x = -\mu - 2$, $y = 2\mu + 8$, $z = 4\mu + 5$ are perpendicular to each other or not.

24. If $y = (x + \sqrt{x^2 - 1})^2$, then show that $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2$.

25. Show that the function $f(x) = \frac{16 \sin x}{4 + \cos x} - x$, is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Evaluate :

$$\int_0^{\frac{\pi}{2}} [\log (\sin x) - \log (2 \cos x)]\, dx.$$

27. Find :

$$\int \frac{1}{\sqrt{x}(\sqrt{x} + 1)(\sqrt{x} + 2)}\, dx$$

28. (a) Find the particular solution of the differential equation $\frac{dy}{dx} + \sec^2 x \,.\, y = \tan x \,.\, \sec^2 x$, given that $y(0) = 0$.

OR

(b) Solve the differential equation given by
$x\, dy - y\, dx - \sqrt{x^2 + y^2}\, dx = 0.$

29. निम्न रैखिक प्रोग्रामन समस्या को आलेख द्वारा हल कीजिए :

$z = 6x + 3y$ का निम्नलिखित व्यवरोधों के अंतर्गत,

अधिकतम मान ज्ञात कीजिए :

$$4x + y \geq 80,$$
$$3x + 2y \leq 150,$$
$$x + 5y \geq 115,$$
$$x \geq 0, y \geq 0.$$

30. (क) किसी यादृच्छिक चर X का प्रायिकता बंटन नीचे दिया गया है :

X	1	2	3
P(X)	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k}{6}$

(i) k का मान ज्ञात कीजिए ।

(ii) ज्ञात कीजिए : $P(1 \leq X < 3)$

(iii) X का माध्य E(X) ज्ञात कीजिए ।

अथवा

(ख) A तथा B ऐसी स्वतंत्र घटनाएँ हैं कि $P(A \cap \bar{B}) = \frac{1}{4}$ तथा $P(\bar{A} \cap B) = \frac{1}{6}$ है । P(A) तथा P(B) ज्ञात कीजिए ।

31. (क) मान ज्ञात कीजिए :

$$\int_0^{\frac{\pi}{2}} e^x \sin x \, dx$$

अथवा

(ख) ज्ञात कीजिए :

$$\int \frac{1}{\cos(x-a)\ \cos(x-b)} \, dx$$

29. Solve graphically the following linear programming problem :

Maximise $z = 6x + 3y$,

subject to the constraints

$$4x + y \geq 80,$$
$$3x + 2y \leq 150,$$
$$x + 5y \geq 115,$$
$$x \geq 0, y \geq 0.$$

30. (a) The probability distribution of a random variable X is given below :

X	1	2	3
P(X)	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k}{6}$

(i) Find the value of k.

(ii) Find $P(1 \leq X < 3)$.

(iii) Find E(X), the mean of X.

OR

(b) A and B are independent events such that $P(A \cap \bar{B}) = \frac{1}{4}$ and $P(\bar{A} \cap B) = \frac{1}{6}$. Find P(A) and P(B).

31. (a) Evaluate :

$$\int_0^{\frac{\pi}{2}} e^x \sin x \, dx$$

OR

(b) Find :

$$\int \frac{1}{\cos(x-a)\ \cos(x-b)} \, dx$$

खण्ड घ

इस खण्ड में दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं ।

32. एक संबंध R, वास्तविक संख्याओं के समुच्चय $\mathbb{R}$ पर इस प्रकार परिभाषित है कि $R = \{(x, y) : x \cdot y$ एक अपरिमेय संख्या है$\}$ । जाँच कीजिए कि क्या R, स्वतुल्य, सममित या संक्रामक है या नहीं ।

33. (क) यदि $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ तथा $B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ है, तो $(AB)^{-1}$ ज्ञात कीजिए ।

अथवा

(ख) आव्यूह विधि द्वारा निम्न समीकरण निकाय को हल कीजिए :

$$x + 2y + 3z = 6$$
$$2x - y + z = 2$$
$$3x + 2y - 2z = 3$$

34. (क) उस रेखा के सदिश तथा कार्तीय समीकरण ज्ञात कीजिए, जो बिन्दु $(1, 2, -4)$ से होकर जाती है तथा बिन्दुओं $A(3, 3, -5)$ तथा $B(1, 0, -11)$ को मिलाने वाली रेखा के समांतर है । अत: इन दो रेखाओं के बीच की दूरी ज्ञात कीजिए ।

अथवा

(ख) बिन्दुओं $A(1, 2, 3)$ तथा $B(3, 5, 9)$ से होकर जाने वाली रेखा के समीकरण ज्ञात कीजिए । अत: इस रेखा पर उन बिन्दुओं के निर्देशांक ज्ञात कीजिए, जो बिन्दु B से 14 इकाई की दूरी पर हैं ।

35. समाकलन के प्रयोग से वक्र $x^2 = y$, $y = x + 2$ तथा x-अक्ष द्वारा घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए ।

SECTION D

This section comprises long answer type questions (LA) of 5 marks each.

32. A relation R is defined on a set of real numbers $\mathbb{R}$ as

$$R = \{(x, y) : x \, . \, y \text{ is an irrational number}\}.$$

Check whether R is reflexive, symmetric and transitive or not.

33. (a) If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$, find $(AB)^{-1}$.

OR

(b) Solve the following system of equations by matrix method :

$$x + 2y + 3z = 6$$

$$2x - y + z = 2$$

$$3x + 2y - 2z = 3$$

34. (a) Find the vector and the Cartesian equations of a line passing through the point $(1, 2, -4)$ and parallel to the line joining the points $A(3, 3, -5)$ and $B(1, 0, -11)$. Hence, find the distance between the two lines.

OR

(b) Find the equations of the line passing through the points $A(1, 2, 3)$ and $B(3, 5, 9)$. Hence, find the coordinates of the points on this line which are at a distance of 14 units from point B.

35. Find the area of the region bounded by the curves $x^2 = y$, $y = x + 2$ and x-axis, using integration.

खण्ड ङ

इस खण्ड में 3 प्रकरण अध्ययन आधारित प्रश्न हैं जिनमें प्रत्येक के 4 अंक हैं ।

प्रकरण अध्ययन – 1

36. योग के कई प्रकार हैं जिनमें योग के विभिन्न प्रकार के आसन, मनन-प्रार्थना, प्राणायाम इत्यादि हैं जैसा कि चित्र में दर्शाया गया है ।

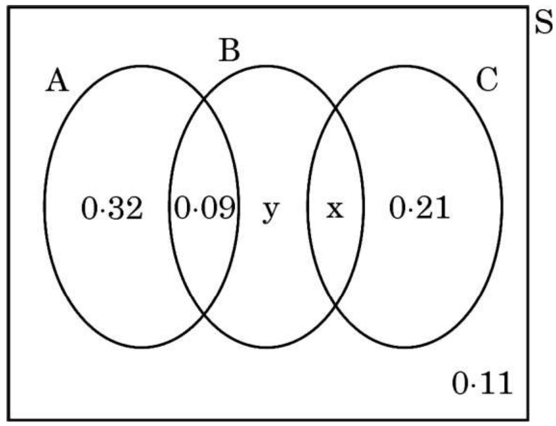
SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below :

नीचे दी गई वेन-आकृति में, एक सोसाइटी के लोगों द्वारा किए गए तीन विभिन्न प्रकार के योग A, B तथा C की प्रायिकताओं को दर्शाया गया है । यह भी दिया गया है कि एक सदस्य द्वारा C प्रकार के योग करने की प्रायिकता 0·44 है ।



उपर्युक्त सूचनाओं के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

(i) x का मान ज्ञात कीजिए । *1*

(ii) y का मान ज्ञात कीजिए । *1*

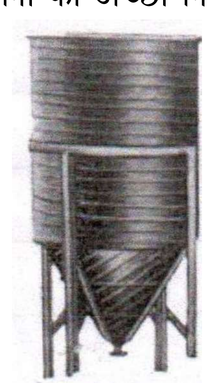
(iii) (क) $P\left(\frac{C}{B}\right)$ ज्ञात कीजिए । *2*

अथवा

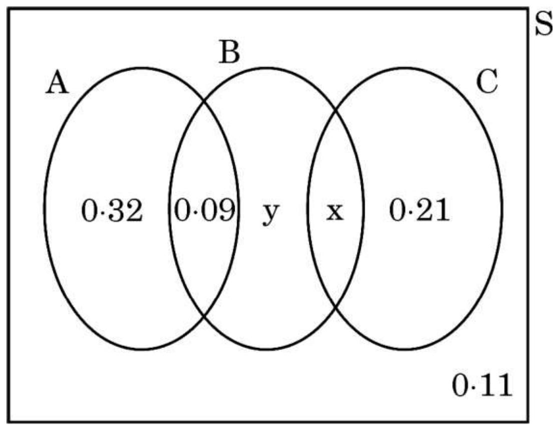
(iii) (ख) प्रायिकता ज्ञात कीजिए कि सोसाइटी का एक यादृच्छया चुना गया सदस्य A या B प्रकार का योग करता है परन्तु C प्रकार का नहीं । *2*

प्रकरण अध्ययन – 2

37. निम्न आकृति में दर्शाए गए टैंक, जो बेलन तथा शंकु को जोड़कर बने हैं, एक सीधे आधार वाले टैंक से पानी की अच्छी निकासी देते हैं ।



The Venn diagram below represents the probabilities of three different types of Yoga, A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0·44.



On the basis of the above information, answer the following questions :

(i) Find the value of x. *1*

(ii) Find the value of y. *1*

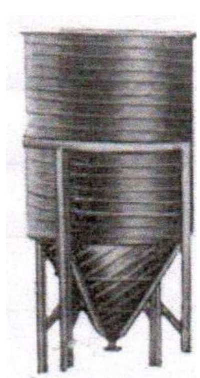
(iii) (a) Find $P\left(\frac{C}{B}\right)$. *2*

OR

(iii) (b) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C. *2*

Case Study – 2

37. A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank.



एक ऐसा टैंक, जिसका शंक्वाकार भाग पानी से भरा है, में एक नल लगाया गया तथा नल से $2 \text{ cm}^3/\text{s}$ की समान दर से पानी टपक रहा है । शंक्वाकार टैंक का अर्ध-शीर्ष कोण $45°$ है ।

उपर्युक्त सूचनाओं के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

(i) टैंक में पानी के आयतन को त्रिज्या r के पदों में व्यक्त कीजिए । *1*

(ii) उस समय जब $r = 2\sqrt{2}$ cm है, त्रिज्या के बदलने की दर ज्ञात कीजिए । *1*

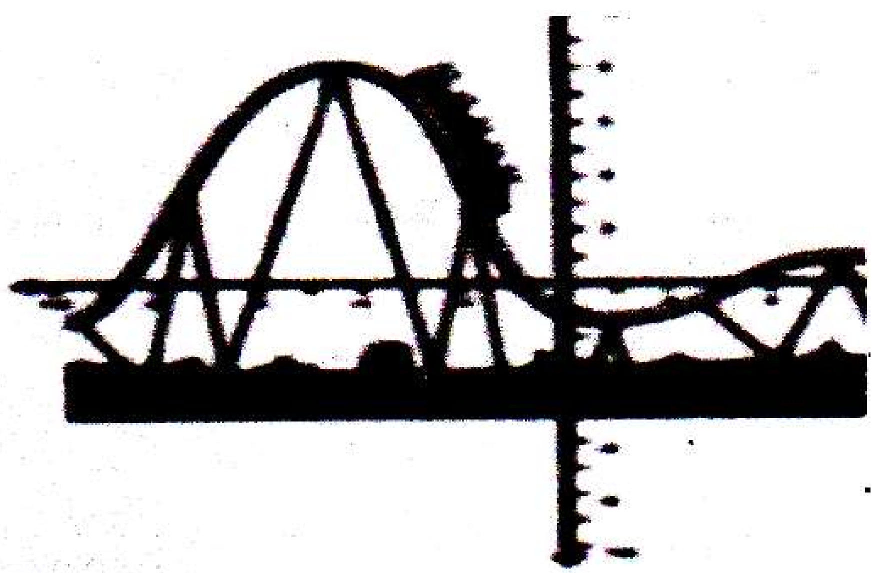
(iii) (क) उस समय जब $r = 2\sqrt{2}$ cm है, शंक्वाकार टैंक के गीले तल के घटने की दर ज्ञात कीजिए । *2*

अथवा

(iii) (ख) जब तिर्यक ऊँचाई 4 cm है, उस समय ऊँचाई 'h' के बदलने की दर ज्ञात कीजिए । *2*

प्रकरण अध्ययन – 3

38. एक रोलर-कोस्टर द्वारा तय किया गया पथ बहुपद $f(x) = a(x + 9)(x + 1)(x - 3)$ द्वारा प्रदत्त है । यदि यह रोलर-कोस्टर y-अक्ष को बिंदु $(0, -1)$ पर मिलता है, तो निम्न के उत्तर दीजिए :



(i) 'a' का मान ज्ञात कीजिए । *2*

(ii) $x = 1$ पर $f''(x)$ ज्ञात कीजिए । *2*

A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of 2 cm^3/s. The semi-vertical angle of the conical tank is 45°.

On the basis of given information, answer the following questions :

(i) Find the volume of water in the tank in terms of its radius r. *1*

(ii) Find rate of change of radius at an instant when $r = 2\sqrt{2}$ cm. *1*

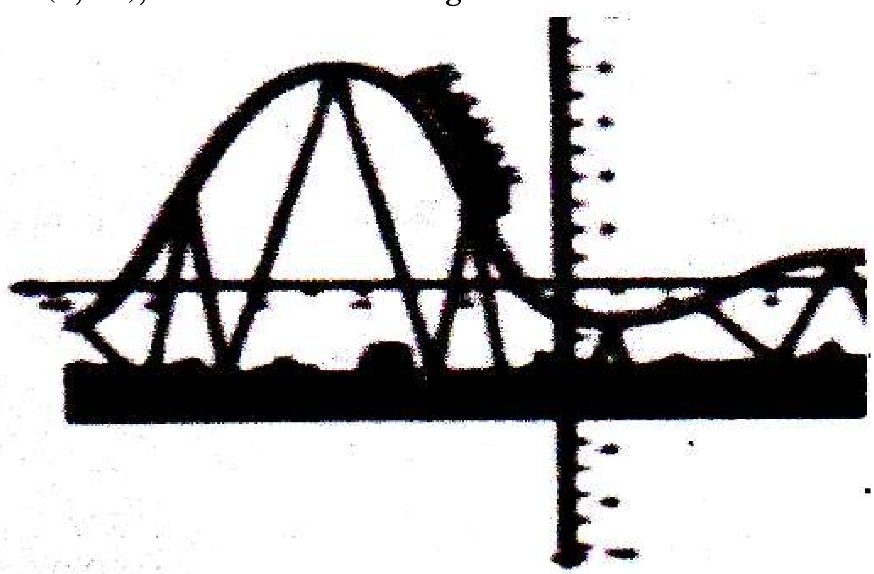
(iii) (a) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $r = 2\sqrt{2}$ cm. *2*

OR

(iii) (b) Find the rate of change of height 'h' at an instant when slant height is 4 cm. *2*

Case Study – 3

38. The equation of the path traced by a roller-coaster is given by the polynomial $f(x) = a(x + 9)(x + 1)(x - 3)$. If the roller-coaster crosses y-axis at a point $(0, -1)$, answer the following :



(i) Find the value of 'a'. *2*

(ii) Find $f''(x)$ at $x = 1$. *2*

Marking Scheme

Strictly Confidential

(For Internal and Restricted use only)

Senior School Certificate Examination, 2023

MATHEMATICS PAPER CODE 65/1/1

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc. may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓) while evaluating which gives the impression that answer is correct, and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totalled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other**

	answer scored out with a note "Extra Question".
10	**In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks __________ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) must be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.

14	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totalling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totalling on the title page. ● Wrong totalling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
16	Any unassessed portion, non-carrying over of marks to the title page, or totalling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totalled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

EXPECTED ANSWER/VALUE POINTS

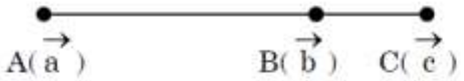
SECTION A

Q.No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION-A **(Question nos. 1 to 18 are Multiple Choice Questions carrying 1 mark each)**	
1	**1.** If for a square matrix A, $A^2 - 3A + I = O$ and $A^{-1} = xA + yI$, then the value of x + y is : (a) -2 (b) 2 (c) 3 (d) -3	
Ans	(b) 2	**1**
2.	**2.** If $\|A\| = 2$, where A is a 2×2 matrix, then $\|4A^{-1}\|$ equals : (a) 4 (b) 2 (c) 8 (d) $\frac{1}{32}$	
Ans	(c) 8	**1**
3.	**3.** Let A be a 3×3 matrix such that $\|\text{adj } A\| = 64$. Then $\|A\|$ is equal to : (a) 8 only (b) -8 only (c) 64 (d) 8 or -8	
Ans	(d) 8 or – 8	**1**
4.	**4.** If $A = \begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$ and 2A + B is a null matrix, then B is equal to : (a) $\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & 8 \\ 10 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & -8 \\ -10 & -3 \end{bmatrix}$	
Ans	(b) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$	**1**
5.	**5.** If $\frac{d}{dx}(f(x)) = \log x$, then f(x) equals : (a) $-\frac{1}{x} + C$ (b) $x(\log x - 1) + C$ (c) $x(\log x + x) + C$ (d) $\frac{1}{x} + C$	
Ans	(b) x(logx – 1) + C	**1**

6.	**6.** $\int_0^{\frac{\pi}{6}} \sec^2(x - \frac{\pi}{6})\, dx$ is equal to : (a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $-\sqrt{3}$	
Ans	(a) $\frac{1}{\sqrt{3}}$	**1**
7.	**7.** The sum of the order and the degree of the differential equat $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = \sin y$ is : (a) 5 (b) 2 (c) 3 (d) 4	
Ans	(c) 3	**1**
8.	**8.** The value of p for which the vectors $2\hat{i} + p\hat{j} + \hat{k}$ and $-4\hat{i} - 6\hat{j} + 26\hat{k}$ are perpendicular to each other, is : (a) 3 (b) -3 (c) $-\frac{17}{3}$ (d) $\frac{17}{3}$	
Ans	(a) 3	**1**
9.	**9.** The value of $(\hat{i} \times \hat{j}) \cdot \hat{j} + (\hat{j} \times \hat{i}) \cdot \hat{k}$ is : (a) 2 (b) 0 (c) 1 (d) -1	
Ans	(d) *-1*	**1**
10.	**10.** If $\vec{a} + \vec{b} = \hat{i}$ and $\vec{a} = 2\hat{i} - 2\hat{j} + 2\hat{k}$, then $\|\vec{b}\|$ equals : (a) $\sqrt{14}$ (b) 3 (c) $\sqrt{12}$ (d) $\sqrt{17}$	
Ans	(b) 3	**1**
11.	**11.** Direction cosines of the line $\frac{x-1}{2} = \frac{1-y}{3} = \frac{2z-1}{12}$ are : (a) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$ (b) $\frac{2}{\sqrt{157}}, -\frac{3}{\sqrt{157}}, \frac{12}{\sqrt{157}}$ (c) $\frac{2}{7}, -\frac{3}{7}, -\frac{6}{7}$ (d) $\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}$	
Ans	(d) $\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}$	**1**

12.	**12.** If $P\left(\frac{A}{B}\right) = 0{\cdot}3$, $P(A) = 0{\cdot}4$ and $P(B) = 0{\cdot}8$, then $P\left(\frac{B}{A}\right)$ is equal to : (a) 0·6 (b) 0·3 (c) 0·06 (d) 0·4	
Ans	(a) 0.6	**1**
13.	**13.** The value of k for which $f(x) = \begin{cases} 3x+5, & x \geq 2 \\ kx^2, & x < 2 \end{cases}$ is a continuous function, is : (a) $-\frac{11}{4}$ (b) $\frac{4}{11}$ (c) 11 (d) $\frac{11}{4}$	
Ans	(d) $\frac{11}{4}$	**1**
14.	**14.** If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $(3I + 4A)(3I - 4A) = x^2 I$, then the value(s) x is/are : (a) $\pm\sqrt{7}$ (b) 0 (c) ± 5 (d) 25	
Ans	(c) ± 5	**1**
15.	**15.** The general solution of the differential equation $x\,dy - (1 + x^2)\,dx = dx$ is : (a) $y = 2x + \frac{x^3}{3} + C$ (b) $y = 2\log x + \frac{x^3}{3} + C$ (c) $y = \frac{x^2}{2} + C$ (d) $y = 2\log x + \frac{x^2}{2} + C$	
Ans	(d) $y = 2\log x + \frac{x^2}{2} + C$	**1**
16.	**16.** If $f(x) = a(x - \cos x)$ is strictly decreasing in $\mathbb{R}$, then 'a' belongs to (a) {0} (b) $(0, \infty)$ (c) $(-\infty, 0)$ (d) $(-\infty, \infty)$	
Ans	(c) $(-\infty, 0)$	**1**

17.	**17.** The corner points of the feasible region in the graphical representation of a linear programming problem are (2, 72), (15, 20) and (40, 15). If $z = 18x + 9y$ be the objective function, then : (a) z is maximum at (2, 72), minimum at (15, 20) (b) z is maximum at (15, 20), minimum at (40, 15) (c) z is maximum at (40, 15), minimum at (15, 20) (d) z is maximum at (40, 15), minimum at (2, 72)	
Ans	(c) z is maximum at (40, 15) and minimum at (15, 20)	**1**
18.	**18.** The number of corner points of the feasible region determined by the constraints $x - y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$ is : (a) 2 (b) 3 (c) 4 (d) 5	
Ans	(a) *2*	**1**
(Question Nos. 19 & 20 are Assertion-Reason based questions of 1 mark each)		
19.	**19.** *Assertion (A) :* The range of the function $f(x) = 2\sin^{-1}x + \frac{3\pi}{2}$, where $x \in [-1, 1]$, is $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$. *Reason (R) :* The range of the principal value branch of $\sin^{-1}(x)$ is $[0, \pi]$.	
Ans	(c) Assertion is True, Reason is False	**1**
20.	**20.** *Assertion (A) :* Equation of a line passing through the points (1, 2, 3) and (3, – 1, 3) is $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-3}{0}$. *Reason (R) :* Equation of a line passing through points (x_1, y_1, z_1), (x_2, y_2, z_2) is given by $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.	
Ans	(d) Assertion is False, Reason is True	**1**
SECTION-B **(Question nos. 21 to 25 are very short Answer type questions carrying 2 marks each)**		
21.	**21.** (a) A function $f : A \to B$ defined as $f(x) = 2x$ is both one-one and onto. If $A = \{1, 2, 3, 4\}$, then find the set B. **OR** (b) Evaluate : $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}\left(\cos\frac{3\pi}{4}\right) + \tan^{-1}(1)$	
Ans	(a) $f(1) = 2, f(2) = 4, f(3) = 6, f(4) = 8$ $\therefore B = \{2, 4, 6, 8\}$ **OR** (b) Required value $= \frac{\pi}{4} + \frac{3\pi}{4} + \frac{\pi}{4}$	$1\frac{1}{2}$ ½ $1\frac{1}{2}$

	$= \frac{5\pi}{4}$	$\frac{1}{2}$
22.	**22.** Find all the vectors of magnitude $3\sqrt{3}$ which are collinear to vector $\hat{i} + \hat{j} + \hat{k}$.	
Ans	Unit vector along $\hat{i} + \hat{j} + \hat{k}$ is $\frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}$ Required vectors are $3\hat{i} + 3\hat{j} + 3\hat{k}$ and $-3\hat{i} - 3\hat{j} - 3\hat{k}$	**1** $\frac{1}{2} + \frac{1}{2}$
23.	**23.** (a) Position vectors of the points A, B and C as shown in the figure below are $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.  If $\vec{AC} = \frac{5}{4}\vec{AB}$, express $\vec{c}$ in terms of $\vec{a}$ and $\vec{b}$. **OR** (b) Check whether the lines given by equations $x = 2\lambda + 2$, $y = 7\lambda + 1$, $z = -3\lambda - 3$ and $x = -\mu - 2$, $y = 2\mu + 8$, $z = 4\mu + 5$ are perpendicular to each other or not.	
Ans	(a) According to question, $\vec{c} - \vec{a} = \frac{5}{4}(\vec{b} - \vec{a})$ $\therefore \vec{c} = \frac{5\vec{b}}{4} - \frac{\vec{a}}{4}$ OR (b) D.r.s. of lines are < 2, 7, – 3 > and < – 1, 2, 4 > Now $2.-1 + 7\cdot 2 + -3\cdot 4 = 0$ $\therefore$ given lines are perpendicular	1½ ½ 1 1
24.	**24.** If $y = (x + \sqrt{x^2 - 1})^2$, then show that $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2$.	
Ans	$\frac{dy}{dx} = 2\left(x + \sqrt{x^2 - 1}\right)\left(1 + \frac{x}{\sqrt{x^2 - 1}}\right) = \frac{2\left(x + \sqrt{x^2 - 1}\right)^2}{\sqrt{x^2 - 1}}$ $\sqrt{x^2 - 1}\,\frac{dy}{dx} = 2y$ $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2$	1½ ½

25.	**25.** Show that the function $f(x) = \frac{16 \sin x}{4 + \cos x} - x$, is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.	
Ans	$f'(x) = \frac{16[4 + \cos x] \cos x + 16 \sin^2 x}{(4 + \cos x)^2} - 1$ $= \frac{\cos x\,(56 - \cos x)}{(4 + \cos x)^2}$ in $(\frac{\pi}{2}, \pi)$, $\cos x < 0 \Rightarrow f'(x) < 0$ $\therefore$ f(x) in strictly decreasing in $(\frac{\pi}{2}, \pi)$	**1** **½** **½**
	SECTION-C **(Question nos. 26 to 31 are short Answer type questions carrying 3 marks each)**	
26.	**26.** Evaluate : $\int_0^{\frac{\pi}{2}} [\log(\sin x) - \log(2 \cos x)]\, dx.$	
Ans	Let I = $\int_0^{\pi/2} [\log \sin x - \log(2\cos x)]dx = \int_0^{\pi/2} \log\left(\frac{\tan x}{2}\right)dx$ Using property $\int_0^a f(x)\,dx = \int_0^a f(a-x)dx$ We get, I = $\int_0^{\pi/2} \log\left(\frac{\cot x}{2}\right)dx$ $\therefore\ 2I = \int_0^{\pi/2} \log\left(\frac{\tan x}{2} \times \frac{\cot x}{2}\right)dx = \int_0^{\pi/2} \log\left(\frac{1}{4}\right)dx$ $2I = \log\left(\frac{1}{4}\right) x \Big\vert_0^{\pi/2} = \frac{\pi}{2} \log \frac{1}{4}$ $I = \frac{\pi}{4}\, log\, \frac{1}{4}$ OR $-\frac{\pi}{2}\, log\ 2$	**½** **1** **½** **½** **½**
27.	**27.** Find : $\int \frac{1}{\sqrt{x}(\sqrt{x} + 1)(\sqrt{x} + 2)} dx$	
Ans	Let I = $\int \frac{dx}{\sqrt{x}\,(\sqrt{x} + 1)(\sqrt{x} + 2)}$	

	Let $\sqrt{x} = t$, $\frac{1}{2\sqrt{x}} dx = dt$ $\therefore I = 2\int \frac{dt}{(t+1)(t+2)}$ $= 2\int\left(\frac{1}{t+1} - \frac{1}{t+2}\right)dt$ $= 2[\log\|t+1\| - \log\|t+2\|] + C$ $= 2[\log(\sqrt{x}+1) - \log(\sqrt{x}+2)] + C$ or $2\log\left(\frac{\sqrt{x}+1}{\sqrt{x}+2}\right) + C$	½ **1** **1** ½
28.	**28.** (a) Find the particular solution of the differential equation $\frac{dy}{dx} + \sec^2 x \cdot y = \tan x \cdot \sec^2 x$, given that $y(0) = 0$. **OR** (b) Solve the differential equation given by $x\,dy - y\,dx - \sqrt{x^2+y^2}\,dx = 0$.	
Ans	**Let** (a) Integrating factor = $e^{\int \sec^2 x\,dx} = e^{\tan x}$ Solution is $ye^{\tan x} = \int \tan x \sec^2 x\, e^{\tan x} dx + C$ Let $\tan x = t$ $\sec^2 x\,dx = dt$ $\therefore \int e^{\tan x} \tan x \sec^2 x\,dx = \int e^t\, t\,dt = e^t(t-1)$ $\therefore ye^{\tan x} = e^{\tan x}(\tan x - 1) + C$ $y(0) = 0$ gives $C = 1$ Particular solution is $ye^{\tan x} = e^{\tan x}(\tan x - 1) + 1$or $y = \tan x - 1 + e^{-\tan x}$ OR (b)Given differential equation can be written as $\frac{dy}{dx} = \frac{y}{x} + \sqrt{1+\left(\frac{y}{x}\right)^2}$ --------- (i) Let $y = vx \Rightarrow \frac{dv}{dx} = v + x\frac{dv}{dx}$ substituting in (i) We get $v + x\frac{dv}{dx} = v + \sqrt{1+v^2}$ $\Rightarrow \frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$ Integrating both sides, we get $\log\|\sqrt{1+v^2} + v\| = \log\|x\| + \log C$ $y + \sqrt{x^2+y^2} = C x^2$	½ ½ ½ ½ ½ ½ ½ ½ ½ **1** ½

29.

29. Solve graphically the following linear programming problem :

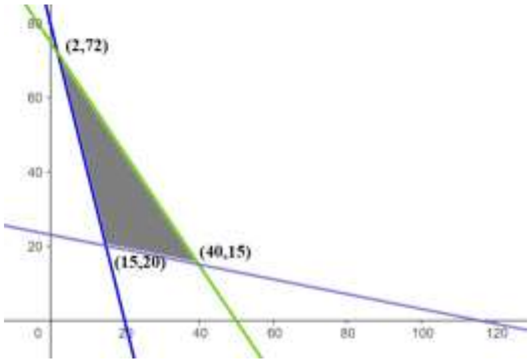
Maximise z = 6x + 3y,

subject to the constraints

$$4x + y \geq 80,$$
$$3x + 2y \leq 150,$$
$$x + 5y \geq 115,$$
$$x \geq 0,\ y \geq 0.$$

Ans

Correct Graph — **2**



Corner points	Value of Z	
(2, 72)	(12 + 216 = 228)	
(15, 20)	(90 + 60 = 150)	1
(40, 15)	(240 + 45 = 285) Maximum	

30.

30. (a) The probability distribution of a random variable X is given below :

X	1	2	3
P(X)	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k}{6}$

(i) Find the value of k.

(ii) Find $P(1 \leq X < 3)$.

(iii) Find E(X), the mean of X.

OR

(b) A and B are independent events such that $P(A \cap \overline{B}) = \frac{1}{4}$ and $P(\overline{A} \cap B) = \frac{1}{6}$. Find P(A) and P(B).

Ans

(a) (i) $\frac{k}{2} + \frac{k}{3} + \frac{k}{6} = 1$ — **1**

Gives k = 1 — ½

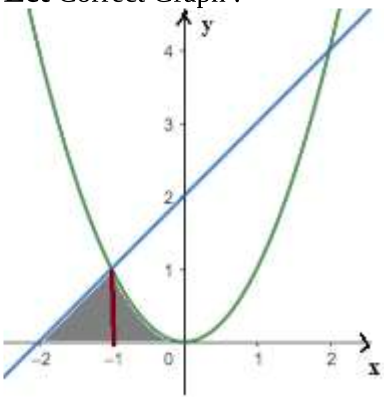
(ii) $P(1 \leq X < 3) = \frac{5k}{6} = \frac{5}{6}$ — ½

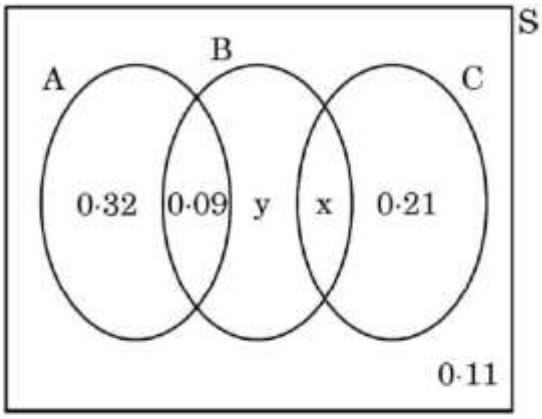
(iii) $E(X) = \sum p_i x_i = \frac{k}{2} + \frac{2k}{3} + \frac{k}{2} = \frac{5k}{3}$ — ½

	E(X) = $\frac{5}{3}$	½
	OR	
	(b)$P(A)\,P(\bar{B}) = \frac{1}{4}$ $\quad P(\bar{A})\,P(B) = \frac{1}{6}$	½
	Let P(A) = x $\quad$ P(B) = y	
	$x(1-y) = \frac{1}{4}$, $\ (1-x)y = \frac{1}{6} \Rightarrow x - y = \frac{1}{12}$	
	eliminating y, we get $12x^2 - 13x + 3 = 0$	1
	gives $x = \frac{1}{3}, \frac{3}{4}$	1
	$\left.\begin{array}{l} P(A) = \frac{1}{3} \Rightarrow P(B) = \frac{1}{4} \\ P(A) = \frac{3}{4} \Rightarrow P(B) = \frac{2}{3} \end{array}\right\}$	½
31.	**31.** (a) **Evaluate :** $$\int_0^{\frac{\pi}{2}} e^x \sin x \, dx$$ **OR** (b) **Find :** $$\int \frac{1}{\cos(x-a)\ \cos(x-b)}\ dx$$	
Ans	(a)Let $I = \int e^x \sin x \, dx$	
	$= e^x \sin x - \int \cos x \, e^x \, dx$	**1**
	$= e^x \sin x - \cos x \, e^x - I$	½
	$\therefore\ I = \frac{1}{2} e^x (\sin x - \cos x)$	½
	$\therefore \int_0^{\pi/2} e^x \sin x \, dx = \frac{1}{2} e^{\pi/2} + \frac{1}{2}$ or $\frac{1}{2}(e^{\pi/2} + 1)$	**1**
	OR	
	(b)Let $I = \int \frac{1}{\cos(x-a)\cos(x-b)} dx$	
	$= \frac{1}{\sin(a-b)} \int \frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)} dx$	1
	$= \frac{1}{\sin(a-b)} \left[\int \frac{\sin(x-b)\cos(x-a)}{\cos(x-a)\cos(x-b)} - \frac{\cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)} \right] dx$	½
	$= \frac{1}{\sin(a-b)} \left[\int [\tan(x-b) - \tan(x-a)] dx \right]$	½

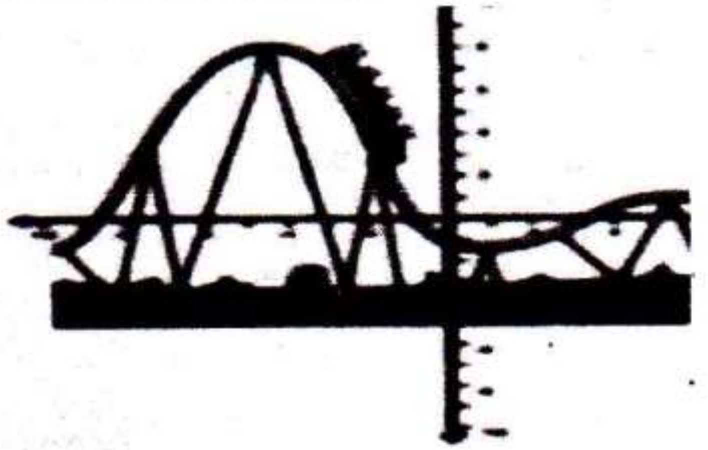
	$= \dfrac{1}{\sin(a-b)}[\log\lvert\sec(x-b)\rvert - \log\lvert\sec(x-a)\rvert] + C$	1
	SECTION-D **(Question nos. 32 to 35 are Long Answer type questions carrying 5 marks each)**	
32.	**32.** A relation R is defined on a set of real numbers $\mathbb{R}$ as $R = \{(x, y) : x \,.\, y \text{ is an irrational number}\}$. Check whether R is reflexive, symmetric and transitive or not.	
Ans	For reflexive $(1, 1) \notin R$ as 1^2 is rational (or any other counter example) R is not reflexive For symmetric Let $(x, y) \in R$ $\therefore$ x.y is an irrational number $\therefore$ (y.x) is an irrational number $\therefore$ $(y, x) \in R$ $\therefore$ R is symmetric For Transitive $(1, \sqrt{2}) \in R$, $(\sqrt{2}, 2) \in R$ (or any other counter example) but $(1, 2) \notin R$ $\therefore$ R is not transitive	**1 ½** **1 ½** **2**
33.	**33.** (a) If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$, find $(AB)^{-1}$. **OR** (b) Solve the following system of equations by matrix method : $x + 2y + 3z = 6$ $2x - y + z = 2$ $3x + 2y - 2z = 3$	
Ans	(a) $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ $(AB)^{-1} = B^{-1}A^{-1}$ $\lvert A\rvert = 1(3) - 2(-1) - 2(2) = 3 + 2 - 4 = 1 \neq 0$ **adj(A)** $= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ $\mathbf{A^{-1}} = \dfrac{1}{1}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ $\therefore B^{-1}A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$	½ 1 2 ½

	$= \begin{bmatrix} 10 & 7 & 21 \\ -49 & -34 & -103 \\ 17 & 12 & 36 \end{bmatrix}$ **OR** (b)Given system is $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}$ $A . X = B \Rightarrow X = A^{-1}B$ $\lvert A \rvert = 35 \neq 0$ $A_{11} = 0 \quad A_{12} = 7 \quad A_{13} = 7$ $A_{21} = 10 \quad A_{22} = -11 \quad A_{23} = 4$ $A_{31} = 5 \quad A_{32} = 5 \quad A_{33} = -5$ $\therefore A^{-1} = \frac{1}{35}\begin{bmatrix} 0 & 10 & 5 \\ 7 & -11 & 5 \\ 7 & 4 & -5 \end{bmatrix}$ $\Rightarrow X = \frac{1}{35}\begin{bmatrix} 0 & 10 & 5 \\ 7 & -11 & 5 \\ 7 & 4 & -5 \end{bmatrix}\begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix} = \frac{1}{35}\begin{bmatrix} 35 \\ 35 \\ 35 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\therefore x = 1 \quad y = 1 \quad z = 1$	1 ½ 1 1½ 1 1
34.	**34.** (a) Find the vector and the Cartesian equations of a line passing through the point (1, 2, – 4) and parallel to the line joining the points A(3, 3, – 5) and B(1, 0, – 11). Hence, find the distance between the two lines. **OR** (b) Find the equations of the line passing through the points A(1, 2, 3) and B(3, 5, 9). Hence, find the coordinates of the points on this line which are at a distance of 14 units from point B.	
Ans	(a) Vector equation of required line through (1, 2, – 4) is $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and cartesian equation: $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ Equation of line through A(3, 3, – 5) and B(1, 0, – 11) is $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$ Distance between parallel lines is given by $d = \frac{\lvert(\vec{a_2} - \vec{a}_1)\times\vec{b}\rvert}{\lvert\vec{b}\rvert}$ Here $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$, $\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$, $\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$ $(\vec{a}_2 - \vec{a}_1) = 2\hat{i} + \hat{j} - \hat{k}$ $(\vec{a}_2 - \vec{a}_1) \times \vec{b} = 9\hat{i} - 14\hat{j} + 4\hat{k}$	**1** **1** **½** **½** **1**

	$\therefore$ d = $\frac{\sqrt{293}}{7}$ **OR** (b)Equation of line AB is $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{6}$ Let coordinates of required point on AB be $(2\lambda + 1, 3\lambda + 2, 6\lambda + 3)$ for some λ According to Question $(2\lambda - 2)^2 + (3\lambda - 3)^2 + (6\lambda - 6)^2 = 14^2$ gives $\lambda^2 - 2\lambda - 3 = 0$ Solving we get $\lambda = 3$ and -1 $\therefore$ required points are (7, 11, 21) and (– 1, – 1, – 3)	**1** **1** **1** **1** **1** **1**
35.	**35.** Find the area of the region bounded by the curves $x^2 = y$, $y = x + 2$ and x-axis, using integration.	
Ans	**Let** Correct Graph :  x coordinates of point of intersection are – 1, 2 Required area = $\int_{-2}^{-1} (x+2)\,dx + \int_{-1}^{0} x^2\,dx$ $= \frac{(x+2)^2}{2}\Big\|_{-2}^{-1} + \frac{x^3}{3}\Big\|_{-1}^{0}$ $= \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$	**1 ½** ½ **1½** **1** ½
SECTION-E **(Question nos. 36 to 38 are source based/case based/passage based/integrated units of assessment questions carrying 4 marks each)**		

36.	The Venn diagram below represents the probabilities of three different types of Yoga, A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0·44.  On the basis of the above information, answer the following questions : (i) Find the value of x. (ii) Find the value of y. (iii) (a) Find $P\left(\frac{C}{B}\right)$. **OR** (iii) (b) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C.	
Ans	(i) $x + 0{\cdot}21 = 0{\cdot}44 \Rightarrow x = 0{\cdot}23$ (ii) $0{\cdot}41 + y + 0{\cdot}44 + 0.11 = 1 \Rightarrow y = 0{\cdot}04$ (iii) (a) $P\left(\frac{C}{B}\right) = \frac{P(C \cap B)}{P(B)}$ $P(B) = 0{\cdot}09 + 0{\cdot}04 + 0{\cdot}23 = 0{\cdot}36$ $P\left(\frac{C}{B}\right) = \frac{0{\cdot}23}{0{\cdot}36} = \frac{23}{36}$ **OR** (iii) (b) P(A or B but not C) $= 0{\cdot}32 + 0.09 + 0.04$ $= 0{\cdot}45$	1 1 1 1 1 ½ ½

37.	**Case Study – 2** **37.** A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank.  A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of 2 cm^3/s. The semi-vertical angle of the conical tank is 45°. On the basis of given information, answer the following questions : (i) Find the volume of water in the tank in terms of its radius r. (ii) Find rate of change of radius at an instant when r = $2\sqrt{2}$ cm. (iii) (a) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius r = $2\sqrt{2}$ cm. **OR** (iii) (b) Find the rate of change of height 'h' at an instant when slant height is 4 cm.	
Ans	(i) $v = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3$ [as θ = 45° gives r = h] (ii) $\frac{dv}{dt} = \pi r^2 \frac{dr}{dt}$ $\Rightarrow \left(\frac{dr}{dt}\right)_{r=2\sqrt{2}} = -\frac{1}{4\pi}$ cm/sec (iii)(a) $C = \pi r l = \pi r\sqrt{2}\, r = \sqrt{2}\,\pi r^2$ $\frac{dC}{dt} = \sqrt{2}\,\pi\, 2r\, \frac{dr}{dt}$ $\left(\frac{dC}{dt}\right)_{r=2\sqrt{2}} = -2$ cm^2/sec OR (iii)(b) $l^2 = h^2 + r^2$ $l = 4 \Rightarrow r = h = 2\sqrt{2}$ $h = r \Rightarrow \frac{dh}{dt} = \frac{dr}{dt} = -\frac{1}{4\pi}$ cm/sec	1 ½ ½ 1 ½ ½ 1 1

38.	**Case Study – 3** **38.** The equation of the path traced by a roller-coaster is given by the polynomial $f(x) = a(x + 9)(x + 1)(x - 3)$. If the roller-coaster crosses y-axis at a point $(0, -1)$, answer the following :  (i) Find the value of 'a'. (ii) Find $f''(x)$ at $x = 1$.	
Ans	(i) $-1 = a(-27) \Rightarrow a = \frac{1}{27}$	**1 + 1**
	(ii) $f(x) = \frac{1}{27}(x + 9)(x + 1)(x - 3)$	½
	$= \frac{1}{27}(x^3 + 7x^2 - 21x - 27)$	
	$f'(x) = \frac{1}{27}(3x^2 + 14x - 21)$	½
	$f''(x) = \frac{6x + 14}{27}$	½
	$f''(1) = \frac{20}{27}$	½