GATE WALLAH

ESE-2024

MAIN EXAM DETAILED SOLUTION

MECHANICAL ENGINEERING

PAPER-I

EXAM DATE - 23 JUNE 2024

9 : 00 AM TO 12 : 00 AM

SECTION-A

Q.1.

(a)

(i) Explain with examples the concept of Rheology of fluids.

(ii) Clearly mention the basic difference between the Euler's equations of motion and the Navier-Stokes equations.

Sol. 1. (a)

(i) **Rheology:** Rheology is the branch of fluid mechanics deal with the study of non-Newtonian fluid. Fluids which do not obey the Newton's law of viscosity are known as Rheological fluid or non-Newtonian fluid.

For Newtonian Fluid

$$\tau = \mu \frac{du}{dy}$$

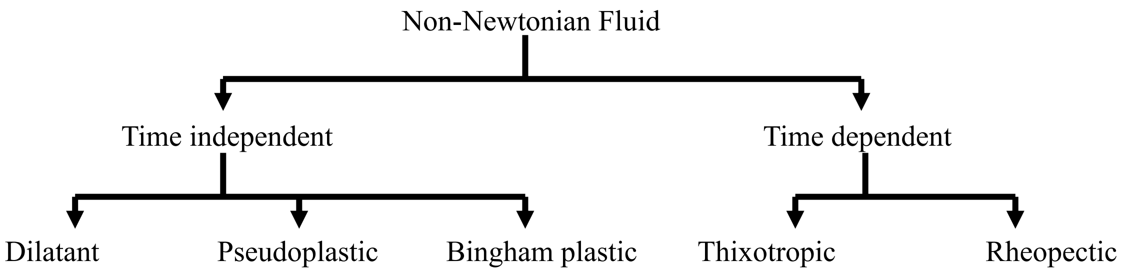
Where μ is viscosity

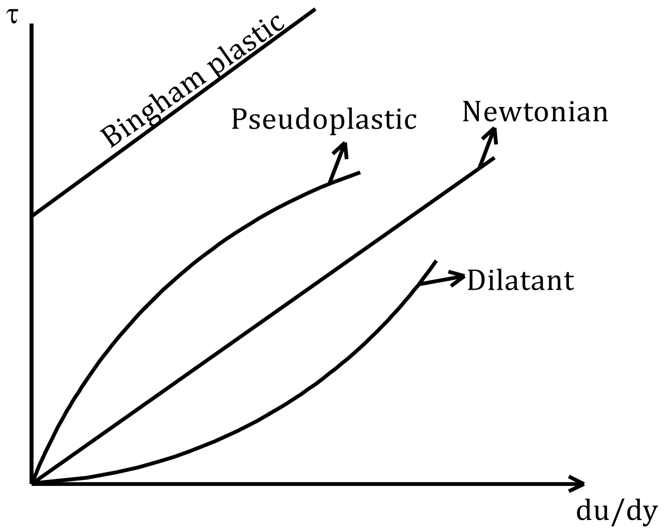
For Newtonian fluid $\mu = f(T)$ only as long as temperature is constant viscosity of Newtonian fluid is also constant, for Newtonian fluid viscosity do not vary with rate of deformation.

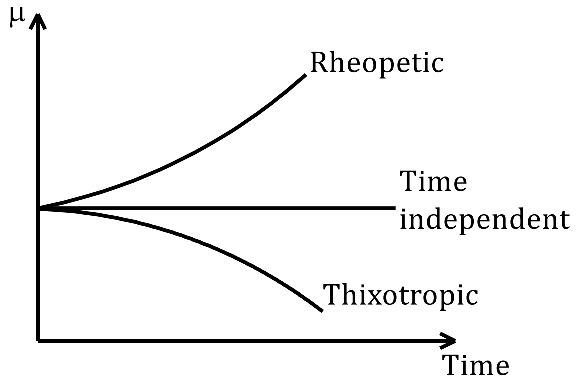
The general relationship between shear stress (τ) and velocity gradient du/dy for non-Newtonian fluid is given as

$$\tau = A\left(\frac{du}{dy}\right)^n + B$$

The viscosity of non-Newtonian fluid depends on temperature, time and rate of deformation.







Sol. 1. (a)

(ii) The various forces that may influence the motion of a fluid are due to gravity pressure, viscosity, turbulence, surface tension and compressibility.

$$F_{net} = F_g + F_p + F_v + F_t + F_s + F_e$$

In case of Navier-Stoke equation of motion forces due to turbulence, surface tension and compressibility are neglected.

Therefore, for Navier-Stoke equation of motion

$$F_{net} = F_g + F_p + F_v$$

It is useful in analysis of viscous flow

In case of Euler's equation of motion only pressure force and gravity force are considered.

So for Euler's equation of motion

$$F_{net} = F_g + F_p$$

Euler equation is useful in analysis of non-viscous flow (ideal fluid flow)

(b) Explain the concept of mixing length introduced by Prandtl and state the relationship that exists between the turbulent shearing stress and the mixing length.

Sol. 1. (b)

Prandtl mixing length theory

We know the Reynolds shear stress can be represented by

$$\bar{\tau} = \rho \overline{\nu_x \nu_y}$$

In 1952 L Prandtl gave mixing length hypothesis, by means of which the turbulent shear stress can be expressed in terms of measurable quantities related to the average flow characteristics.

According to Prandtl mixing length is that distance in the transverse direction which must be covered by a lump of fluid particles travelling with its original mean velocity in order to make the difference between its velocity and the velocity of new layer equal to the mean transverse fluctuation in turbulent flow.

Prandtl has further indicated that the velocity fluctuation in the direction may be related to the mixing length l by the following expression.

$$\nu_x = l\frac{\partial \bar{v}}{\partial y}$$

$$\nu_y = l\frac{\partial \bar{v}}{\partial y}$$

So Renold shear stress can be written as

$$\bar{\tau} = \rho \nu_x \nu_y$$

$$\bar{\tau} = \rho l^2 \left(\frac{\partial \bar{v}}{\partial y}\right)^2$$

Total shear stress can be given as sum of viscous shear stress and turbulent shear stress.

So, $\bar{\tau} = \mu\frac{\partial \bar{v}}{\partial y} + \rho l^2 \left(\frac{\partial \bar{v}}{\partial y}\right)^2$

However viscous shear stress is very less as compared to turbulent shear stress.

(c) A stationary mass of gas is compressed without friction from an initial state of 0.45 m³ and 0.12 MPa to a final state of 0.15 m³ and 0.12 MPa, the pressure remaining constant during the process. There is a transfer of 57.6 kJ of heat from the gas during the process. How much does the internal energy of the gas change?

Sol. 1. (c)

Given

$P_1 = 0.12$ MPa $= 120$ kPa

$V_1 = 0.45$ m^3

$P_2 = 0.12$ MPa $= 120$ kPa

$V_2 = 0.15$ m^3

$P_1 = P_2 = P = 120$ kPa

$Q = 57.6$ kJ (From the gas)

1st law of TD

$$\int \delta Q = \int dU + \int \delta W$$

$$-57.6 = (U_2 - U_1) + P(V_2 - V_1)$$

$$-57.6 = (U_2 - U_1) + 120(0.15 - 0.45)$$

$$U_2 - U_1 = -57.6 + 36$$

$$= -21.6$$

So internal energy of gas is decrease by 21.6 kJ.

(d) What is condensation? Explain the terms filmwise condensation and dropwise condensation. Which one is more preferred? Justify.

Sol. 1. (d)

Condensation

When a saturated vapor come into contact with a surface having temperature below the saturation temperature corresponding to that vapor pressure then condensation of vapour occurs.

$T_w < T_{sat}$

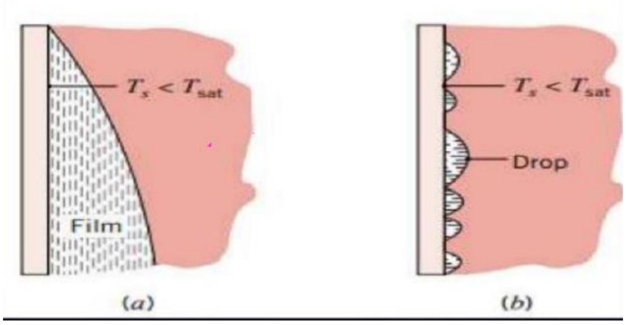
$\Rightarrow$ Condensation of vapour occurs

vertical wall (T_W)

Saturated vapour (T_{sat})

Types of Condensation

- Film wise condensation or Film Condensation (fig a)
- Dropwise condensation or Drop Condensation (fig b)



Film wise condensation or Film Condensation:-

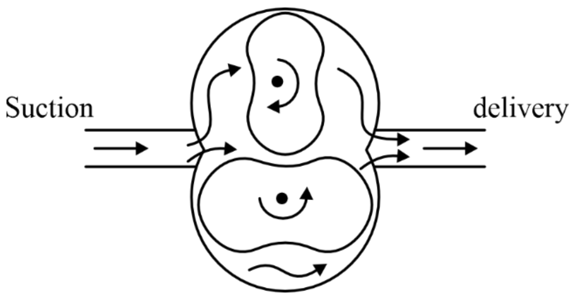
- In a Film condensation a film of liquid condensate is formed on the surface and this liquid condensate film wets the surface. On a wettable cooling surface film condensation occurs.

Dropwise condensation or Drop Condensation :-

- In dropwise condensation, vapor condensate in the form of drops on the surface. The rate of heat transfer is many times larger as compared to film condensation. Dropwise condensation occurs on a non-wettable cooling surface where the liquid condensate drops do not spread.
- Dropwise condensation is much desirable because of its higher heat transfer rate. However, it is hardly occurring on a cooling surface.

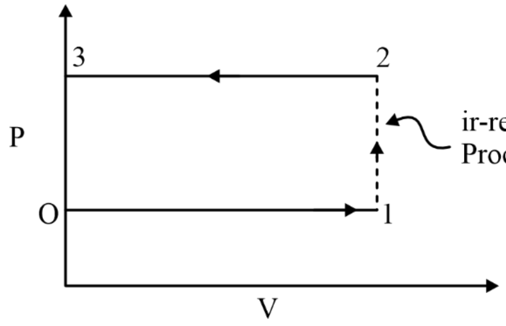
(e) Explain the working of roots blower supercharger with diagram and mention its advantages.

Sol. 1. (e)

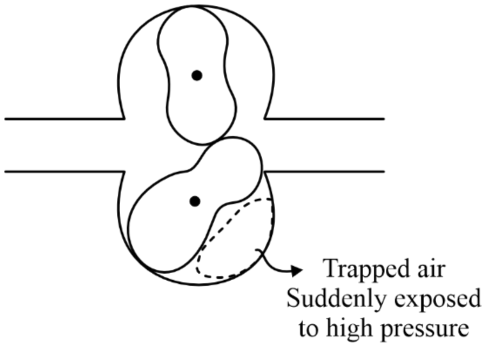


Roots blower work by meshing of two lobes rotating in counter directions.

- Fluid in trapped in pockets of lobe and casing and delivered to discharge side.
- Used for medium air pressure increase of pressure ratio 1.1 → 2



Actual pressure rise happens when air is compressed when lobe pocket opens to discharge pressure.



Advantage

1. Very less moving parts
2. Rotational operation and hence smooth operation.
3. Dynamically balanced so no vibrations
4. very cheep

Disadvantages

1. Effeciencies are low
2. Pressure developed are low

Q.2. **(a)**

(i) The velocity distribution in the boundary layer over a high spillway face was found to have the following form:

$$\frac{u}{U_\infty} = \eta^{0.22}, \text{ where } \eta = y/\delta$$

The free stream velocity U_∞ at a certain section was observed to be 20 m/s and boundary layer thickness of 5 cm was estimated from the velocity distribution measured at the section. The discharge passing over the spillway was 5 m³/s per metre length of spillway.

Determine

(I) Displacement thickness,

(II) Momentum thickness,

(III) Energy thickness, and

(IV) Loss of energy up to the section under consideration.

Sol. 2. **(a)**

(i) Given velocity profile is $\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{0.22}$

$U_\infty = 20$ m/s; $\delta = 5$ cm; Q = 5 m^3/s per meter length.

I) Displacement Thickness (δ^*)

$$\delta^* = \int_0^\delta \left(1-\frac{u}{U_\infty}\right)dy = \int_0^\delta \left(1-\left(\frac{y}{\delta}\right)^{0.22}\right)dy = y - \frac{y^{1.22}}{1.22\,\delta}\Big|_0^\delta$$

$$\Rightarrow \quad \delta^* = \left(1-\frac{1}{1.22}\right)\delta = \frac{0.22}{1.22}\delta = \frac{0.22}{1.22}\times 5 = 0.9016 \text{ cm}$$

∴ Displacement thickness = $\delta^* = 0.9016$ cm

II) Momentum Thickness (θ):

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1-\frac{u}{U_\infty}\right)dy = \int_0^\delta \left(\left(\frac{y}{\delta}\right)^{0.22} - \left(\frac{y}{\delta}\right)^{0.44}\right)dy$$

$$= \left(\frac{y^{1.22}}{1.22\,\delta^{0.22}} - \frac{y^{1.44}}{1.44\,\delta^{0.44}}\right)\Big|_0^\delta = \left(\frac{1}{1.22}-\frac{1}{1.44}\right)\delta$$

$$= 0.125227 \times 5 = 0.6261 \text{ cm}$$

Momentum thickness = $\theta = 0.6261$ cm

III) Energy thickness (δ^{**}):

$$\delta^{**} = \int_0^\delta \frac{u}{U_\infty}\left(1-\frac{u^2}{U_\infty^2}\right)dy = \int_0^\delta \left(\left(\frac{y}{\delta}\right)^{0.22} - \left(\frac{y}{\delta}\right)^{0.66}\right)dy$$

$$= \left(\frac{y^{1.22}}{1.22\,\delta^{0.22}} - \frac{y^{1.66}}{1.66\,\delta^{0.66}}\right)\Big|_0^\delta = \left(\frac{1}{1.22}-\frac{1}{1.66}\right)\delta$$

$$= \left(\frac{1}{1.22}-\frac{1}{1.66}\right)\times 5 = 1.0863 \text{ cm}$$

∴ Energy thickness = $\delta^{**} = 1.0863$ cm

IV) Loss of Energy:

Loss of Energy = Energy flow through energy thickness of flow

$$= \frac{1}{2}\times(\rho\times\delta^{**}\times U_\infty)\times U_\infty^2 = \frac{1}{2}\rho\delta^{**}U_\infty^3$$

$$= \frac{1}{2}\times 10^3\times(1.0863\times 10^{-2})\times(20)^3 = 43.44 \text{ kW}$$

(ii) A plate 4 m long and 20 cm wide is immersed in a fluid of density 1.2 kg/m^3 and kinematic viscosity 10^{-4} m^2/s. The fluid is moving with a velocity of 5 m/s. Calculate,
(I) Boundary layer thickness, and
(II) Drag force on both sides of the plate.
Assume Blasius's solution.

Sol. 2. (a)

(ii) Given length of the plate = L = 4m
Width of the Plate = W = 20 cm = 0.2 m
$\rho = 1.2$ kg/m^3; $\nu = 10^{-4}$ m^2/s
Velocity of the fluid (free stream) = $U_\infty = 5$ m/s

(I) Boundary layer thickness:

By Blasius solution: $\delta = \frac{5.0\ L}{\sqrt{R_{e_L}}} = \frac{5.0\ L}{\sqrt{\frac{U_\infty \cdot L}{\nu}}}$

$$\Rightarrow \delta = \frac{5 \times 4}{\sqrt{\frac{5 \times 4}{10^{-4}}}} = \frac{20 \times 10^{-2}}{\sqrt{5} \times 2} = 0.044721 \text{ m}$$

$$\Rightarrow \boxed{\delta = 4.4721 \text{ cm}}$$

(II) Drag force on both sides of plate:

$$F_D = 2 \times F_D\big|_{\text{single side}}$$

$$= 2 \times \frac{1}{2} \times \rho A \times U_\infty^2 \times C_D$$

$$= \rho \times (L \times W) \times U_\infty^2 \times \frac{1.328}{\sqrt{R_{e_L}}} \quad \left(\because C_D = \frac{1.328}{\sqrt{R_{e_L}}}\right)$$

$$= 1.2 \times (4 \times 0.2) \times 25 \times \frac{1.328 \times 10^{-2}}{\sqrt{5 \times 4}}$$

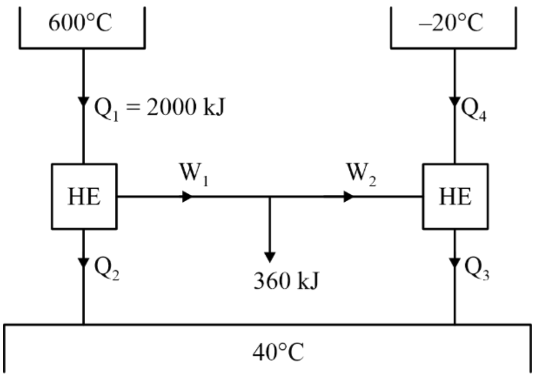
= 0.07126 N

Drag force acting on plate on both sides = 0.07126 N.

(b) A reversible heat engine operates between two reservoirs at temperatures of 600°C and 40°C. The engine drives a reversible refrigerator which operates between reservoirs at temperatures of 40°C and −20°C. The heat transfer to the heat engine is 2000 kJ and net work output of the combined engine refrigerator plant is 360 kJ.

(i) Evaluate the heat transfer to the refrigerant and the net heat transfer to the reservoir at 40°C.

(ii) Evaluate heat transfer to the refrigerant and the net heat transfer to the reservoir when the efficiency of the heat engine and COP of the refrigerator are each 40% of their maximum possible values.



Sol. 2. (b)

(i)

$$\eta_{ME} = 1 - \frac{313}{873} \Rightarrow 0.6414$$

Q_1 = 2000 kJ

$W_1 = \eta_1 Q_1$ = 1282.93 kJ

$Q_2 = Q_1 - W_1 = 717.06$ kJ

For refrigerator

$$COP_R = \frac{253}{313-253} = \frac{253}{60} = 4.2166$$

hence $Q_A = COP_R(W_1 - 360)$

4.2166 (1283.93 – 360)

= 3895.9 kJ

Hence heat flow to reservoir at 40°C.

= 2000 + 3895.9 – 360

$Q_{res} = 5535.90$ kJ

Hence heat transfer to refrigerator

$Q_4 = 3895.9$ kJ

and heat transfer to Res

$Q_{res} = 5535.90$ kJ

If η_{HE} & COP_R are 40% of ideal cycle

$\eta_{HE} = 0.4 \times 0.6414 \Rightarrow 0.25656$

$W_1 = 2000 \times 0.25656 \Rightarrow 573.12$ kJ

$W_{res} = 513.12 - 360 \Rightarrow 153.12$ kJ

$COP_R = 4.2166 \times 0.4 \Rightarrow 1.6866$ kJ

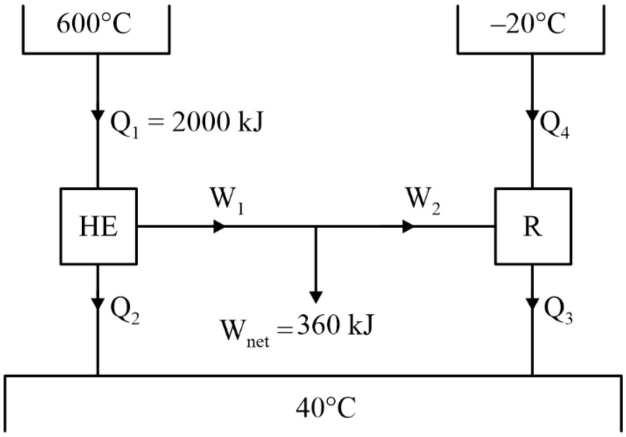
$Q_4 = 153.1 \times 1.686 = 258.22$ kJ

Heat engine to reservoir = 2000 – 360 + 258.22

$Q_{res} = 1898.22$ kJ

Sol. 2. (b)

(ii)



Given

For reverible ehat engine

$$\eta = 1 - \frac{T_L}{T_h} = 1 - \frac{Q_2}{Q_1}$$

$$1 - \frac{273+40}{273+600} = 1 - \frac{Q_2}{2000\text{ kJ}}$$

$Q_2 = 717$ kJ

$w_1 = Q_1 - Q_2 = 1283$ kJ

$W_{net} = W_1 - W_2$

$W_2 = 1283 - 360$

$W_2 = 923$ kJ

For reversible refrigerator

$$(COP)_R = \frac{T_L}{T_h - T_k} = \frac{Q_4}{W_2}$$

$$\frac{273-20}{60} = \frac{Q_4}{923}$$

$Q_4 = 3891.98$ kJ $= 38921$ kJ

$Q_3 = Q_4 + W_2$

$= 4815$ kJ

$$\eta_{HE} = 0.4\left(1 - \frac{273+40}{273+600}\right) = 1 - \frac{Q_2}{2000}$$

$Q_2 = 1486.82$ kJ

$W_1 = Q_1 - Q_2$

$= 513.173$ kJ

$W_{net} = W_1 - W_2$

$W_2 = 153.173$ kJ

$$(COP)_R = 0.4\left[\frac{273-20}{60}\right] = \frac{Q_4}{W_2}$$

$Q_4 = 258.35$ kJ

$Q_3 = Q_4 + W_2 \quad = 411.52$ kJ

heat transfer to the refrigerator

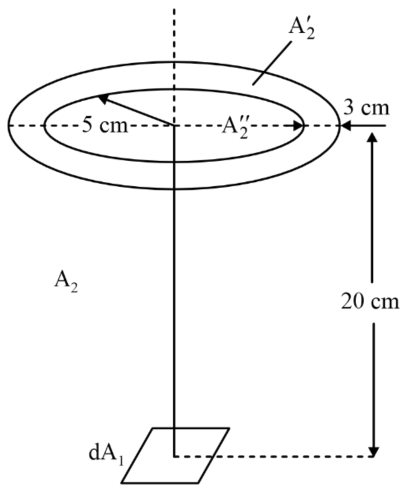
$= Q_4 = 258.35$ kJ

Heat transfer to the reservoir (40°C)

$= Q_2 + Q_3 = 1486.82 + 411.52 = 1898.34$ kJ

(c) Write down the properties of a black body. How do we define spectral emissivity of a real surface? Consider a small plane surface area dA_1 placed parallel to a circular ring A'_2 of inner radius 5 cm and width 3 cm as shown in the figure.

Calculate the fraction of the radiation emitted by the surface dA_1 that is intercepted by the ring (A'_2) and also the fraction that passes through the hole (area = A''_2) in the ring if the surfaces are placed 20 cm apart. Assume dA_1 is a very small (differential) surface element.



Sol. 2. (c)

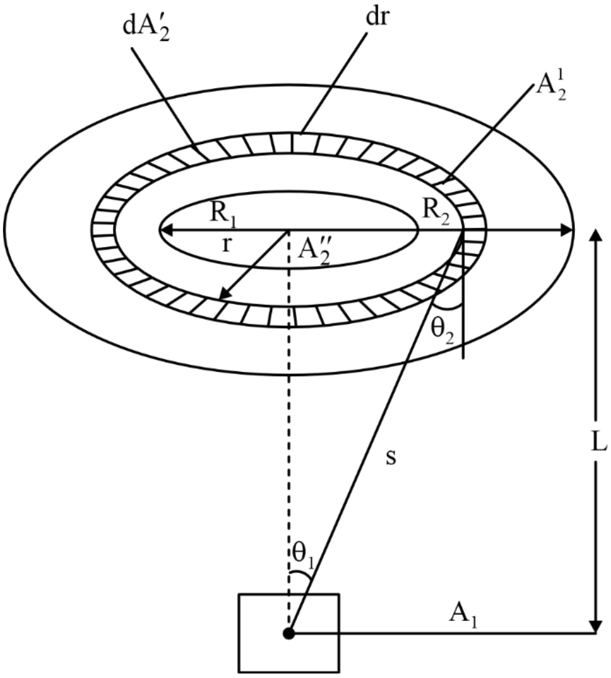
$$F_{12'} = \frac{1}{A_1}\int_{A_1}\int_{A_2'} \frac{\cos\theta_1 \cos\theta_2}{\pi s^2}\, dA_1 dA_2$$

$$= \frac{1}{A_1}\int_{A_1} dA_1 \int_{A_2'} \frac{\cos\theta_1 \cos\theta_2}{\pi s^2}\, dA_2$$

$$F_{12'} = \int_{A_2'} \frac{\cos\theta_1 \cos\theta_2}{\pi s^2}\, dA_2$$

$$F_{12'} = \int_{A_2'} \frac{\cos\theta_1 \cos\theta_2}{\pi s^2}\, dA_2'$$

$[\theta_1 = \theta_2 = \theta]$



$$F_{12'} = \int_{A_2'} \frac{\cos^2\theta}{\pi s^2}\, dA_2'$$

From geometry $s^2 = r^2 + L^2$

$$\cos\theta = \frac{L}{s}$$

$$\cos\theta = \frac{L}{\sqrt{r^2 + L^2}}$$

$$dA_2' = 2\pi r dr$$

$$F_{12'} = \int_{R_1}^{R_2} \frac{L^2}{r^2 + L^2} \cdot \frac{1}{\pi\left(r^2 + L^2\right)} \cdot 2\pi r dr$$

$$= \int_{R_1}^{R_2} \frac{L^2}{\left(r^2 + L^2\right)^2} \cdot 2r dr$$

Let $r^2 + L^2 = t$, $2r dr = dt$

$$= \int_{R_1^2+L^2}^{R_2^2+L^2} \frac{L^2}{t^2} dt = L^2 \left[\frac{t^{-2+1}}{-2+1}\right]_{R_1^2+L^2}^{R_2^2+L^2}$$

$$= -L^2 \left[\frac{1}{t}\right]_{R_1^2+L^2}^{R_2^2+L^2} = -L^2 \left[\frac{1}{R_2^2+L_2^2} - \frac{1}{R_1^2+L^2}\right]$$

$$= -L^2 \left[\frac{R_1^2+L^2-R_2^2-L^2}{(R_1^2+L^2)(R_2^2+L^2)}\right]$$

$$= L^2 \left[\frac{R_2^2-R_1^2}{(R_1^2+L^2)(R_2^2+L^2)}\right]$$

$$R_{12'} = \frac{L^2(R_2^2-R_1^2)}{(R_1^2+L^2)(R_2^2+L^2)}$$

$$R_{12'} = \frac{0.2^2\left[.08^2-0.05^2\right]}{(.05^2+0.2^2)(.08^2+0.2^2)}$$

$F_{12'}' = 0.07910$

Black body: Properties of black body.

1. Black body is perfect absorber (absorptivity =1)
2. Black body is perfect emitter (emissivity = 1)
3. Emissivity of black body does not depend on wavelength and temperature.
4. Black body is diffuse body that means radiation intensity is independent of direction.

Spectral emissivity (ε_λ): spectral emissive power of a non-black body at temperature T to spectral emissive power of a black body at same temperature T is called spectral emissivity

$$\varepsilon_\lambda = \frac{E_\lambda(T)}{E_{b\lambda}(T)}$$

$E_\lambda(T) \Rightarrow$ spectral emissive power of non-black body.

$E_\lambda(T) \Rightarrow$ spectral emissive power of black body.

Q.3. (a)

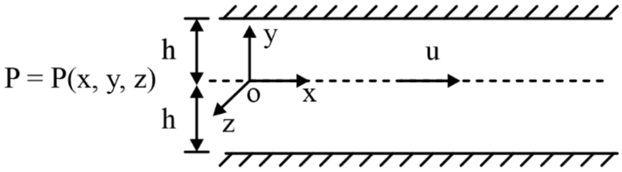
Prove that the velocity distribution for viscous flow between two parallel plates, when both plates are fixed across a section is parabolic in nature. Also, prove that the maximum velocity is equal to one and a half times the average velocity.

Sol. 3. (a)

Plane-Poiseuille Flow: The flow between two fixed plates.

Continuity Equation: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

$$\Rightarrow \frac{\partial u}{\partial x} = 0$$



$u \neq 0,\ v = 0,\ w = 0$

Momentum Equation

x-direction: $\frac{\partial u}{\partial t} + u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y} + w \cdot \frac{\partial u}{\partial z} = x_x - \frac{1}{\rho} \cdot \frac{\partial P}{\partial x} + \frac{\mu}{\rho}\left\{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right\}$

y-direction: $\frac{\partial v}{\partial t} + u \cdot \frac{\partial v}{\partial x} + v \cdot \frac{\partial v}{\partial y} + w \cdot \frac{\partial v}{\partial z} = x_y - \frac{1}{\rho} \cdot \frac{\partial P}{\partial y} + \frac{\mu}{\rho} \cdot \left\{\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right\}$

z-direction: $\frac{\partial w}{\partial t} + u \cdot \frac{\partial w}{\partial x} + v \cdot \frac{\partial w}{\partial y} + w \cdot \frac{\partial w}{\partial z} = x_z - \frac{1}{\rho} \cdot \frac{\partial P}{\partial z} + \frac{\mu}{\rho} \cdot \left\{\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right\}$

x_z = Neglected

$$\frac{\partial u}{\partial x} = 0;\quad \frac{-1}{\rho} \cdot \frac{\partial P}{\partial x} + \frac{\mu}{\rho} \cdot \frac{\partial^2 u}{\partial y^2} = 0,\ \underbrace{\frac{\partial P}{\partial y} = 0;\ \frac{\partial P}{\partial z} = 0}_{P=P(x)}$$

$$\Rightarrow \quad \frac{-dP}{dx} + \mu \cdot \frac{d^2 u}{dy^2} = 0$$

$$\Rightarrow \quad \mu \cdot \frac{d^2 u}{dy^2} = \frac{dP}{dx} = \text{constant}$$

On Integrating,

$$\Rightarrow \quad \mu \cdot \frac{du}{dy} = \frac{dPy}{dx} + C_1$$

Integrating once again

$$\Rightarrow \quad \mu \cdot u = \frac{dP}{dx} \cdot \frac{y^2}{2} + C_1 y + C_2$$

$$\Rightarrow \quad \mu \cdot u = \left(\frac{dP}{dx}\right) \cdot \frac{y^2}{2} + C_1 y + C_2$$

At $\quad y = -h;\ u = 0$

$\quad\ \ y = h;\ u = 0$

$$0 = \left(\frac{dP}{dx}\right) \cdot \frac{h^2}{2} - C_1 h + C_2$$

$$0 = \left(\frac{dP}{dx}\right) \cdot \frac{h^2}{2} + C_1 h + C_2$$

$$\underline{-\quad -\qquad\quad -\qquad -}$$

$$0 = -2C_1 h$$

$$\Rightarrow \quad C_1 = 0$$

$$\Rightarrow \quad \mu \cdot u = \left(\frac{dP}{dx}\right) \cdot \frac{y^2}{2} + C_2$$

$$\Rightarrow \quad 0 = \left(\frac{dP}{dx}\right)\cdot\frac{h^2}{2} + C_2$$

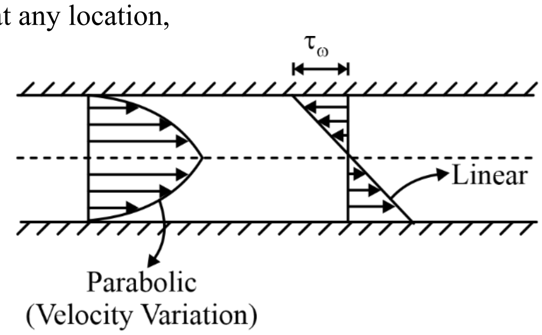
$$\Rightarrow \quad C_2 = \frac{-h^2}{2}\cdot\left(\frac{dP}{dx}\right)$$

$\therefore$ Velocity profile is given by

$$\mu\cdot u = \left(\frac{dP}{dx}\right)\frac{y^2}{2} + \left(\frac{dP}{dx}\right)\cdot\left(\frac{-h^2}{2}\right)$$

$$\Rightarrow \quad u = \frac{h^2}{2\mu}\cdot\left(\frac{-dP}{dx}\right)\left(1-\frac{y^2}{h^2}\right) \rightarrow \text{Parabolic in nature.}$$

Shear stress at any location,



$$\tau = \mu\cdot\frac{du}{dy}$$

$$\Rightarrow \quad \tau = \mu\cdot\left\{\frac{h^2}{2\mu}\cdot\left(\frac{-dP}{dx}\right)\cdot\left(\frac{-2y}{h^2}\right)\right\}$$

$$\Rightarrow \quad \tau = \left(\frac{dP}{dx}\right)\cdot y \rightarrow \text{Linear}$$

$$\tau_w = \tau|_{y=h} = \left(\frac{dP}{dx}\right)\cdot h$$

$$Q = \int_{y=-h}^{y=h}\frac{h^2}{2\mu}\cdot\left(\frac{-dP}{dx}\right)\cdot\left\{1-\frac{y^2}{h^2}\right\}\cdot(dy\times 1)$$

$$\Rightarrow \quad Q = 2\times\int_{y=0}^{y=h}\frac{h^2}{2\mu}\cdot\left(\frac{-dP}{dx}\right)\left\{1-\frac{y^2}{h^2}\right\}dy$$

$$\Rightarrow \quad Q = \frac{h^2}{\mu}\left(\frac{-dP}{dx}\right)\times\int_{y=0}^{y=h}\left(1-\frac{y^2}{h^2}\right)dy$$

$$\Rightarrow \quad Q = \frac{h^2}{\mu}\left(\frac{-dP}{dx}\right)\left\{y-\frac{y^3}{3h^2}\Big|_0^h\right\}$$

$$\Rightarrow \quad Q = \frac{h^2}{\mu}\left(\frac{-dP}{dx}\right)\left\{\frac{2h}{3}\right\}$$

$$Q = \frac{2h^3}{3\mu}\cdot\left(\frac{-dP}{dx}\right)$$

Average velocity of flow

$$= U_{avg} = \frac{Q}{A} = \frac{2h^3}{3\mu}\cdot\left(\frac{-dP}{dx}\right)\times\frac{1}{(2h\times1)}$$

$$\Rightarrow \quad U_{avg} = \frac{h^2}{3\mu}\cdot\left(\frac{-dP}{dx}\right)$$

$$u_{max} = u|_{y=0} = \frac{h^2}{2\mu}\cdot\left(\frac{-dP}{dx}\right)$$

$$\frac{u_{max}}{u_{avg}} = \frac{\left(\frac{h^2}{2\mu}\right)\cdot\left(\frac{-dP}{dx}\right)}{\left(\frac{h^2}{3\mu}\right)\cdot\left(\frac{-dP}{dx}\right)}$$

$$\Rightarrow \frac{u_{max}}{u_{avg}} = \frac{3}{2}$$

(b)

A four-cylinder petrol engine has an output of 52 kW at 2000 rpm. A Morse test is carried out and the brake torque readings are 177, 170, 168 and 174 N-m respectively. For normal running at this speed the specific fuel consumption is 0.364 kg/kWhr. The calorific value of fuel is 44,200 kJ/kg. Calculate the mechanical and brake thermal efficiency of the engine.

Sol. 3. (b)

P = 52 kW

N = 2000 rpm

When Morse key test is performed

BP, when cylinder is cut off = 177 N-m

Brake torque in normal running

$$T = \frac{P}{W} = \frac{52000}{\frac{2\pi\times2000}{60}} \Rightarrow 248.28 \text{ N-m}$$

Hence

$IP_1 = 248.28 - 177 \Rightarrow 71.28$ N-m

$IP_2 = 248.28 - 170 \Rightarrow 78.28$ N-m

$IP_3 = 248.28 - 168 \Rightarrow 80.28$ N-m

$IP_4 = 248.28 - 174 \Rightarrow 74.28$ N-m

hence torque developed by all cylinder

$IP_1 + IP_2 + IP_3 + IP_4 = 71.28 + 78.28 + 80.28 + 74.28$

$\Rightarrow 304.12$ N-m

Frictional torque = 304.12 – 248.28 = 55.84 nozzle

$$\eta_m = \frac{248.28}{304.12} \Rightarrow 0.816 \Rightarrow 81.6\%$$

$$\boxed{\eta_m = 81.6\%}$$

sfc = 0.364 kg/kWhr

Fuel consumption per sec

$$\frac{0.364}{3600}\times52 \Rightarrow 0.00525 \text{ kg/s}$$

Energy produced by fuel per

$Q = 0.00525 \times 44200$
$= 232.39$ kJ/s

$$\eta_{BT} = \frac{520}{232.39} = 0.223 \Rightarrow 22.3\%$$

$$\boxed{\begin{aligned}\eta_m &= 81.6\% \\ \eta_{BT} &= 22.3\%\end{aligned}}$$

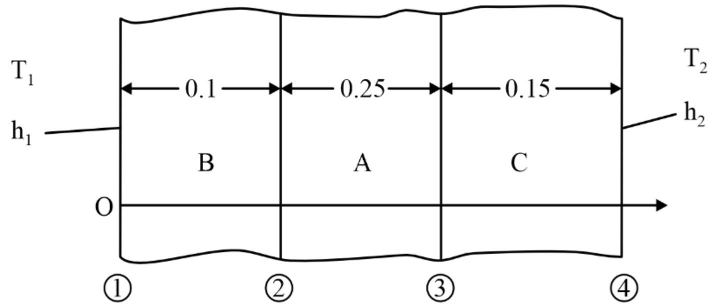
(c)

A heat generating slab A (thickness = 0.25 m, thermal conductivity K_A = 15 W/m°C) is sandwiched between two other slabs B (thickness = 0.1 m, K_B = 10 W/m°C) and C (thickness = 0.15 m, K_C = 30 W/m°C) as shown in the figure. There is no heat generation in slab B or C. The temperature distribution in slab A is known to be $T_A = 90 + 4500\,x - 11000\,x^2$, where T is in °C and x is the distance in metres from left surface of B. The wall B is in contact with a fluid at temperature T_1 = 40°C, the wall heat transfer coefficient being h_1. Similarly, the free surface of C loses heat to a medium at temperature 35°C, and the surface heat transfer coefficient is h_2. Assume steady state condition.

(i) Calculate the temperature at the surfaces of slab A. What is the maximum temperature in A and where does it occur?

(ii) Determine the temperature gradient at both the surfaces of each of the slabs A, B and C.

(iii) Find the temperature profiles in slabs B and C. Also, calculate the values of the heat transfer coefficients h_1 and h_2.



Sol. 3. (c)

$$K_A = 15\frac{W}{m°C}$$

$$K_B = 10\frac{W}{m°C}$$

$$K_C = 30\frac{W}{m°C}$$

$T_A = 90 + 4500\,x - 11000\,x^2$

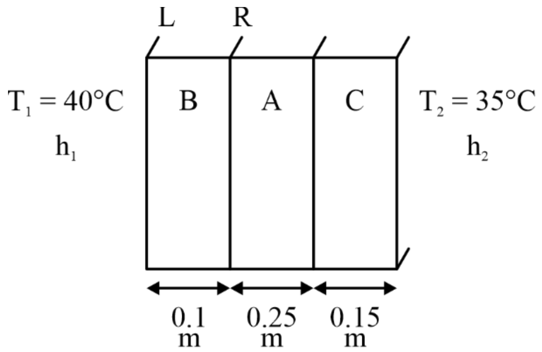
(i) At x = 0.1 m

$T_A = 90 + 4500 \times 0.1 - 1000 \times 0.1^2$

$= 430°C$

at x = 0.35 m

$T_A = 317.5°C$



Max temperature and location of Max temperature

$$\frac{dT_A}{dx} = 0, \frac{d}{dx}, \left[90 + 4500x - 11000x^2\right] = 0$$

$0 + 4500 - 22000\,x = 0$

$x = 0.2045$ m

$(T_A)_{max} = 90 + 4500 \times (0.2045) - 11000 \times (0.2045)^2$

$= 550.227°C$

At the interface of wall B and A

$$-K_B \left.\frac{dT}{dx}\right|_{x=0.1} = -K_A \left.\frac{dT}{dx}\right|_{x=0.1}$$

$$-K_B \times \frac{430 - T_{B_L}}{0.1} = -K_A\left[4500 - 22000 \times (0.1)\right]$$

$$10 \times \frac{430 - T_{B_L}}{0.1} = 15 \times \left[4500 - 2200\right]$$

$$\boxed{T_{B_L} = 85°C}$$

at the interface of wall A and C

$$-K_A \left.\frac{dT}{dx}\right|_{x=0.35} = -K_C \left.\frac{dT}{dx}\right|_{x=0.35}$$

$$15 \times (4500 - 22000 \times 0.35) = 30 \times \frac{T_{C_R} - 317.5}{0.15}$$

$$-240 = T_{C_R} - 317.5$$

$$\boxed{T_{C_R} = 77.5°C}$$

(ii) $$\left(\frac{dT}{dx}\right)_B = \frac{430 - 85}{0.1} = 3450\ \text{K/m}$$

$$\left(\frac{dT}{dx}\right)_C = \frac{317.5 - 775}{0.15} = 1600\,\text{K/m}$$

$$\left(\frac{dT}{dx}\right)_A = 4500 - 22000\,x$$

$$\left.\left(\frac{dT}{dx}\right)_A\right|_{x=0.1} = 2300\ \text{K/m}$$

$$\left.\left(\frac{dT}{dx}\right)_A\right|_{x=0.35} = -3200\,K/m$$

(iii) For h_1:

$$\frac{430-85}{(R_{th})_B} = \frac{85-40}{1/h_1A}$$

$$\frac{430-85}{\frac{0.1}{10A}} = \frac{85-40}{1/h_1A}$$

$$\boxed{h_1 = 766.67\frac{W}{m^2K}}$$

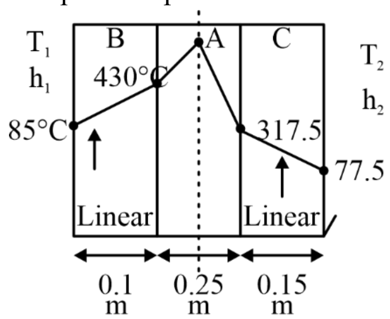
For h_2:

$$\frac{317.5-77.5}{(R_{th})_C} = (77.5-35)\times h_2A$$

$$\frac{(317.5-77.5)\times 30\times A}{0.15} = (77.5-35)\times h_2A$$

$$\boxed{h_2 = 1129.411\frac{W}{m^2K}}$$

Temperature profile



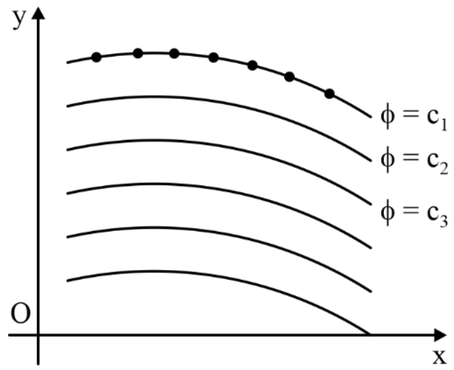
Q.4. **(a)**

(i) Show that the streamlines and equipotential lines form a net of mutually perpendicular lines.

Sol. 4. **(a)**

(i) For irrotational flow, velocity potential satisfies Laplace equation.

For a 2-D Irrotational flow,



$\phi(x, y)$

$$u = \frac{\partial\phi}{\partial x}; v = \frac{\partial\phi}{\partial y}$$

$$d\phi = \left(\frac{\partial\phi}{\partial x}\right)dx + \left(\frac{\partial\phi}{\partial y}\right)dy$$

$\Rightarrow$ $0 = u \times dx + v \times dy$

$\Rightarrow$ $-u \times dx = v \times dy$

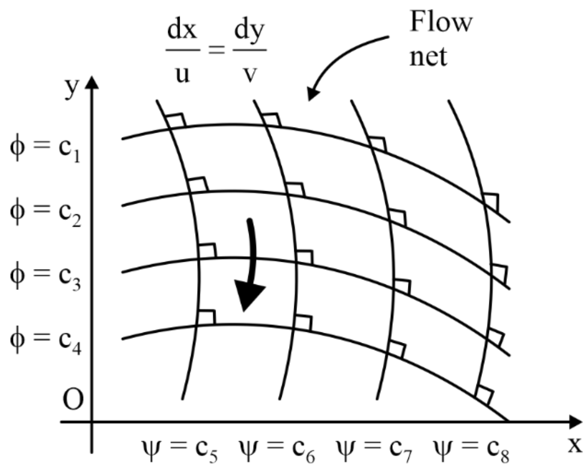
Along an equipotential line.

$$\phi = c \Rightarrow d\phi = 0$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{\phi=c} = \frac{-u}{v}$$

Slope of a streamline, (Lines along which stream function is constant)

$$\left.\frac{dy}{dx}\right|_{\psi=c} = \frac{v}{u}$$



Consider

$$\left.\frac{dy}{dx}\right|_{\phi=c} \times \left.\frac{dy}{dx}\right|_{\psi=c} = \frac{-u}{v} \times \frac{v}{u} = -1$$

$\Rightarrow$ Equipotential lines and stream lines intersect each other perpendicularly.

(ii) Why do we use fins in heat exchanging devices? How do we define fin efficiency? Give a few examples of different finned surfaces with sketches.

Sol. 4. (a)

(ii) Fins are extended surfaces which are used to enhance the heat transfer between the surface and surrounding fluid.

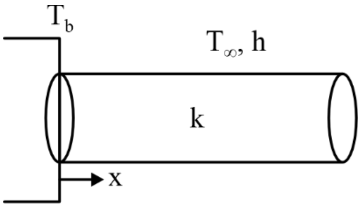
Fin efficiency: It is ratio of actual heat transfer rate through the fin $\left(\dot{Q}_{act}\right)$ to maximum rate of heat transfer $\left(\dot{Q}_{max}\right)$ if entire fin surface is maintained at the base temperature.

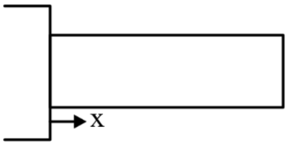
$$\eta = \frac{\dot{Q}_{act}}{\dot{Q}_{max}}$$

For ∞ long fin

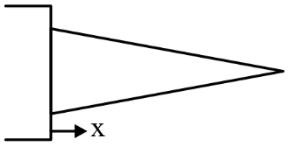
$$\eta = \frac{\sqrt{hPkA_c}\,(T_b - T_\infty)}{hA_c\left[T_b - T_\infty\right]}$$

$$\eta = \frac{1}{mL}$$

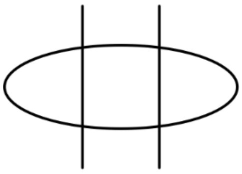




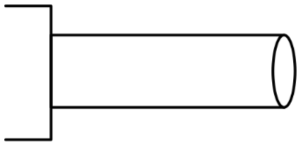
Straight uniform
x section fin



Straight non-uniform
x section fin



Annular fin



Pin fin

(b) What is a polytropic process?
Give an expression between temperature and volume between two states of a polytropic process.
Find out entropy change between the two states of an ideal gas.
A mass of 0.25 kg of an ideal gas has a pressure of 300 kPa, a temperature of 90°C and a volume of 0.07 m³. The gas undergoes an irreversible adiabatic process to a final pressure of 300 kPa and final volume of 0.10 m³, during which the work done on the gas is 25 kJ. Evaluate Cp and C, of the gas and the increase in entropy of the gas.

Sol. 4. (b)
In thermodynamics any process can be represented by equation
PV^K = Constant
Where K = $-\infty$ to $+\infty$
K is depends on process
A polytropic process is a thermodynamic process that follows that relation
PV^n = constant
Where n $\rightarrow$ polytropic index
$1 < n < \gamma$ $\{\gamma$ = adiabatic index$\}$
In polytropic process heat transfer
Also take place along with work transfer
In such a polytropic process work transfer is more than heat transfer and this excess work come's from internal energy. So internal energy decreases and temperature is also decreases even we are supplying heat that's why the specific heat of a polytropic process is always negative.
For polytropic process between two states equation can be given as
$P_1V_1^n = P_2V_2^n$

It can be written as

$P_1V_1V_1^{n-1} = P_2V_2V_2^{n-1}$ For ideal

$mRT_1V_1^{n-1} = mRT_2V_2^{n-1}$ $\{PV = mRT\}$

$$\boxed{T_1V_1^{n-1} = T_2V_2^{n-1}}$$

Equation of polytropic process between two states in terms of temperature & volume

Given

$m = 0.25$ kg

$P_1 = 300$ kPa

$T_1 = 90°C = 363$ K

$V_1 = 0.07\ m^3$

$P_2 = 0.10\ m^3$

$V_2 = 0.10\ m^3$

$W = 25$ kJ

At initial state

$P_1V_1 = mRT_1$

$300 \times 0.07 = 0.25\ R \times 3.63$

$R = 0.2314$ kJ/kgK

Process is constant pressure

$V \propto T$

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}$$

$$\frac{0.1}{0.07} = \frac{T_2}{363}$$

$T_2 = 518.57$ K

1st law of thermodynamics

$dQ = dU + dW$

$0 = mC_V(T_2 - T_1) - 25$

$C_V = 0.6428$ kJ/kgK

We know

For ideal gas

$C_P - C_V = R$

$C_P = 0.2314 + 0.6428$

$= 0.8742$ kJ/kgK

Increase in entropy

For ideal gas

$$ds = m\left[Cp\ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}\right]$$

$$= 0.25\left[0.8742\ln\frac{518.57}{363} - 0.2314\ln\frac{300}{300}\right]$$

$$\boxed{ds = 0.078\ kJ/K}$$

(c) What is catalytic converter package? Show the arrangement of catalytic converter package with the help of a diagram for HC, CO and NO_x. Explain its functioning also.

Sol. 4. (c)

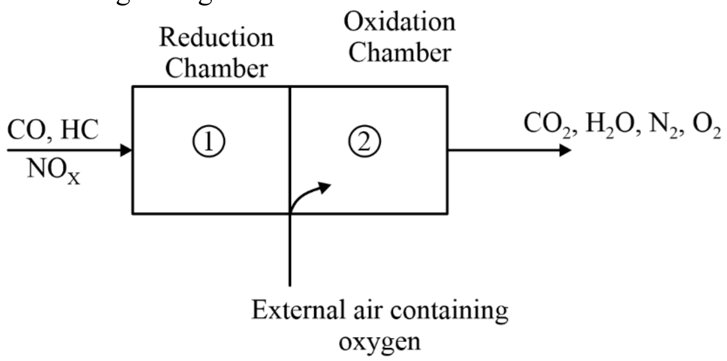
Catalytic converter package consists of two sections employing 3-way catalytic converters. This is used to prevent pollution for the IC engine exhaust.

IC engine exhaust have following pollutions

- CO
- HC
- NO_x

These three can be converted to CO_2, H_2O and N_2 & O_2 with help of 3-way catalytic converters.

It consists of following arrangement.



1. Reduction chamber : NO_x is converted into N_2 and O_2

$NO_x \rightarrow N_2 + O_2$

This process is achieved with help of catalyst, Rhodium,

2. Oxidation chamber, here unburnt hydro carbon and CO are converted into CO_2 and H_2O.

Oxygen liberated from reduction of NO_x is used for oxidation. Also, additional air is provided from outside for complete oxidation of HC and CO.

Here catalyst Palladium and Platinum are used. Since Platinum is very costly Palladium is used often.

Q.5. (a)

(i) What is the significance of specific speed of a centrifugal pump?

(ii) A centrifugal pump operates at its optimal efficiency and delivers 3 cubic metre per second over a height of 22 m. The pump has a 36 cm diameter impeller and rotates at 3250 rpm. Compute the specific speed of the pump

(1) in terms of discharge, and

(2) in terms of power if maximum efficiency of the pump is 80%.

Sol. 5. (a)

(i)

The significance of the specific speed (ns) of a centrifugal pump includes the following aspects:

Classification of Impeller Design: Specific speed helps in categorizing the impeller designs. Different values of specific speed indicate different types of impeller geometries, which are optimized for varying flow rates and head conditions.

Performance Prediction: It provides insight into the pump's performance characteristics. By knowing the specific speed, engineers can predict the efficiency, head, and flow rate behavior of the pump.

Design Optimization: Specific speed aids in selecting the most efficient pump for a given application. Pumps with similar specific speeds have similar hydraulic characteristics, which helps in identifying the best pump design for the required operational conditions.
Comparative Analysis: It allows for the comparison of different pumps, regardless of their size or operating conditions. This is particularly useful in standardizing pump performance and making informed decisions about pump selection.
Standardization and Interchangeability: Specific speed helps in creating standards for pump designs, facilitating the interchangeability and compatibility of pumps and components within the same category.
Cavitation Assessment: It is also used to assess the susceptibility of the pump to cavitation. Pumps with higher specific speeds are generally more prone to cavitation, which can inform design and operational adjustments to mitigate this issue.

Sol. 5. **(a)**

(ii)

$Q = 3\ m^3/s$; $H = 22$ m; $D_2 = 0.36$ m; $N = 3250$ rpm

(1) Specific speed of pump in terms of discharge

$$\text{Specific speed, } N_s = \frac{N\sqrt{Q}}{H^{3/4}} = \frac{3250\times\sqrt{3}}{(22)^{3/4}}$$

$$N_s = 554.14$$

(2) specific speed of pump in terms of power if maximum efficiency of the pump is 80%.

$$\eta_{mano} = \frac{P}{\rho g Q H_m}$$

$$\therefore \quad H_m = \frac{P}{\rho g Q \eta_{mano}}$$

$$\therefore \text{Specific Speed, } N_s = \frac{3250\times\sqrt{3}}{\left(\frac{P}{10^3\times9.81\times3\times0.8}\right)^{3/4}} = \frac{10699270.69}{P^{3/4}}$$

(b) A shell and coil type of evaporator is to be designed for a refrigerator. Coil is bare tube of copper. Refrigerant flows inside the tube and shell side water in stagnant condition. Coil is maintained at −5°C. Heat - transfer coefficient on water side is 4100 W/m²K. The load on the evaporator is 2.8 kW. The LMTD is 18°C. The tube side heat transfer coefficient is given by

$$h_i = 0.555\left[\frac{9.81(\rho_f-\rho_g)k_f^3.h_{f_g}}{\mu_f.\text{evaporator temp}\times D_i}\right]$$

The properties of refrigerant are as given below at –5°C.

Dynamic viscosity = 0.000191 kg/ms
Density of liquid = 1136 kg/m³
Density of vapour = 14.43 kg/m³
Tube inner diameter Di = 0.005715 m and thickness of tube is 0.001905 m.
Thermal conductivity of refrigerant = 0.0857 W/mK.
Thermal conductivity of tube is 400 W/mK.
Thermal conductivity of ice is 2.25 W/mK.
Latent heat of vapourization is 173.1 kJ/kg.

Find the tube length required.

Sol. 5. **(b)**

(**)

(c) Air at DBT 35°C and WBT 23°C is passed over a coil and air comes out of the coil at DBT 25°C and WBT 20°C. Show the procedure on skeletal psychrometric chart.

(i) Mention the name of the process taking place.

(ii) Find the coil surface temperature.

(iii) Find the bypass factor of the coil.

Sol. 5. (c)

From Psychrometric chart

Inlet

DBT = 35°C

WBT = 25°C

$\omega = 0.016$ kg/kg of d.a.

$\phi = 45\%$

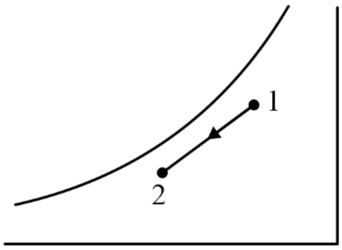
Outlet

DBT = 25°C

WBT = 20°C

$\phi = 55\%$

Dew point temperature = 17.5°C



Process is cooling and dehumidification

Coil surface parts is equal to dew points temperature of 17.5°C

$$\text{BPF} = \frac{25-17.5}{35-17.5} = 0.428$$

(d)

(i) What are the performance parameters of cooling towers? Define them.

(ii) What are the main advantages and disadvantages of mechanical draught cooling towers?

Sol. 5. (d)

(i)

Performance parameters of cooling towers are

1. Water flow rate through tower expressed in kg/s or ton/minutes
2. Cooling water temperature exiting cooling tower
3. Cooling capacity usually measured in MW or KW
4. Air flow rate which depends in size of cooling tower

Cooling towers work on principle of heat transfer from hot water to cold air. Saturated air from atmosphere having humidity less than 100% is passed through water thus absorving heat and evaporating a partial quantity of cooling water. This partial water evaporating absorb latent heat from water and thus cools the water, principle is similar to desert cooler cooling capacity is defined by

$\dot{Q} = \dot{m}C\,\Delta T$

ΔT is temperature difference between incoming hot water and outgoing cold water

$\dot{m} \rightarrow$ mass flow of water

Sol. 5. (d)

(ii) Advantages

1. Presence of positive pressure gradient ensures high heat transfer coefficient
2. Better efficiency as compared to natural drought
3. Better air flow distribution throughout the tower with help of forced drought fan.
4. Very adaptable to variable heat loads and plant operations
5. Smaller in size and compact

Disadvantages

1. Incorporation of fan requires electricity consumption thus reducing power output of plant.
2. Can be used for small power plants only.
3. Operation of fan cause noise
4. High power plant dependency on operability of fan.
5. Difficult to carryout maintenance on fan
6. Additional cost at time and installation of power plant
7. Complexity in integration and operation.

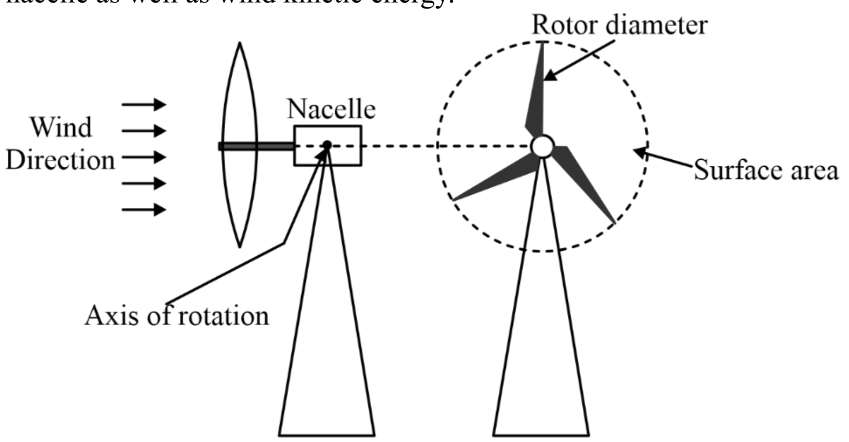
(e) With the help of sketches, define Horizontal Axis Wind Turbine (HAWT) and Vertical Axis Wind Turbine (VAWT). Compare HAWT and VAWT in respect of

(i) Efficiency
(ii) Space requirement
(iii) Cost
(iv) Design and installation
(v) Noise
(vi) Self-starting

Sol. 5. (e)

Horizontal Axis Wind Turbine (HAWT):

The Rotor blades are fitted on the main shaft in a horizontal hub. This direction of wind is parallel to the axis of rotation of rotor blades. The horizontal hub is connected to a gearbox and generator, which are located inside the nacelle. The nacelle houses the electrical components and is mounted at the top of the tower. There is a supporting tower to withstand the rotor and nacelle as well as wind kinetic energy.



Vertical Axis Wind Turbine (VAWT):

The Rotor blades are fitted on the main shaft in a vertical hub. This direction of wind is perpendicular to the axis of rotation of rotor blades. The main shaft is connected to a gearbox

and generator. There is a supporting wire to withstand the rotor as well as wind kinetic energy..
The main attraction of VAWT are:
(i) It can accept wind from any direction, eliminating the need of yaw control.
(ii) The gearbox, generator, etc, are located at the ground, thus eliminating the heavy nacelle at the top of tower.
(iii) The inspection and maintenance also get easier.
(iv) It also reduces overall cost.

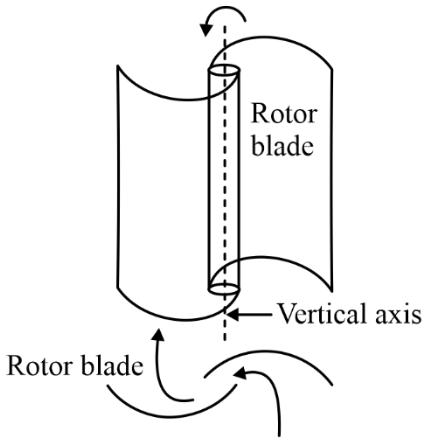


Figure: Savonious vertical-axis rotor

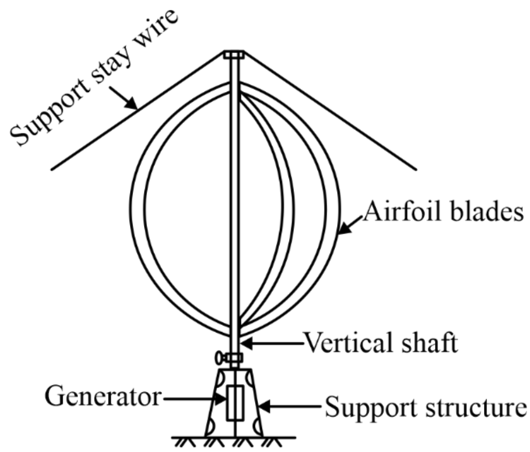


Figure: Darrieus rotor

Rotor diameters on modern turbines can be more than 80 meters. The turbines rotor diameter determines its swept area. The swept area is the area through which the rotors of a wind turbine rotate. Larger swept areas usually translate to higher output machines. Machine capacity can range anywhere from a few hundred kilowatts to 5 megawatts. The amount of power produced is a direct result of the wind speed.
Horizontal-axis wind turbines (HAWTs) dominate global use, chiefly known for their efficiency and high power output. These turbines feature a rotor with aerodynamically designed blades, which are parallel to the ground, allowing them to capture wind energy effectively.

	VAWT	**HAWT**
Efficiency	Lower practical efficiency due to aerodynamic drag, higher under ideal conditions	Generally higher efficiency, capturing 40% - 50% of wind energy
Design and Installation	Simpler, less costly, easier installation	More complex, costly due to need for precise alignment and substantial structural support
Noise Levels	High	Low
Cost	Less expensive because their design and installation is quite simple	More expensive due their complex design and installation
Self-start	Many of VAWT requires some self-start mechanism (as they are generally unable to produce sufficient aerodynamic starting torque)	It does not requires any self-starting mechanism.
Space requirement	Low	High

Q.6. **(a)**

(i) Explain the working of counterflow Ranque-Hilsch tube refrigeration system with the help of a sketch. Also define the following terms:

(I) Cold mass flow ratio

(II) Normalised temperature drop

(III) Cold orifice diameter ratio

(IV) Isentropic efficiency

(ii) Explain how the critical temperature of a refrigerant affects the performance of a refrigeration system represented on T-s diagram.

Sol. 6. **(a)**

(i)

A vortex tube workd by feedig high pressure air into a chamber which causes air to swing and split into hot and cold streams. Tube is considered adiabatic.

Energy Balance

$\dot{m}_h c_p (T_h - T_f) = \dot{m}_c c_p (T_f - T_c)$

Where,

T_f is temperature of air supplied

T_h is temperature at which air exits from hot side.

T_c is temperature at which air exits in cold side

(1) Cold mass flow rate is defined as $\dfrac{\dot{m}_c}{\dot{m}_{in}}$

(2) Normalised temperature drop: It is defined as ratio of temperature drop to inlet temperature

$$\frac{\Delta T_c}{T_{in}} = \frac{T_c - T_{in}}{T_{in}}$$

(3) Cold orifice diameter ratio : It is defined as ratio of cold orifice diameter to diameter of vortex tube.

(4) Isentropic efficiency is defined as

$$\eta_{isen} = \frac{T_f - T_c}{\left(T_f - T_c\right)_{isentropic}}$$

Sol. 6. **(a)**

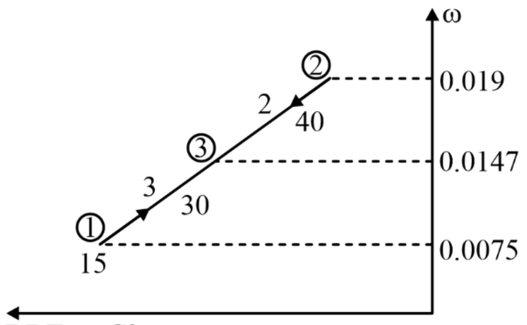
(ii)

Critical Temperature and Pressure

For high COP, in general, the critical temperature should be very high so that the condenser temperature line on the p-h diagram is far moved from the critical point. This ensures reasonable refrigerating effect which becomes very small if the state of the liquid before expansion is near the critical point. Also, the critical pressure should be low so as to result in low condensing pressure, except for carbon dioixide for which the critical temperature is 31°C for most of the common refrigerants, critical temperature is much above the normal condensing temperature.

(b) In an air conditioning system two streams are mixed adiabatically. One stream is at DBT 15°C and WBT 12°C and the flow rate is 20 m³/min. The second stream volume flow rate is 30 m³/min. After mixing the two streams the condition of air is found to be 30°C DBT and 23°C WBT. Find the 2nd stream condition before mixing. Also, find the DBT, WBT, enthalpy, mass flow rate, volume flow rate, specific humidity, and relative humidity of the 2nd stream before mixing. On the skeletal chart, show the procedure to get the 2nd stream condition.

Sol. 6. (b)



Since the volume flow rate of cold stream is 20 m^3/min and hot stream is 30 m^3/min.

Final state of air can be drawn in ratio as shown.

Corresponding to state (2)

$\omega_2 = 0.019$ kg/kg of dA

DBT = 40°C

WBT = 28°C

Enthalpy = 90 kJ/kg of dry air

Volume flow rate of air=30m^3 /min

Corresponding to state 1

DBT = 15°C

WBT = 12°C

Volume flow rate of air=20m^3 /min

v_1=0.825 m^3/kg

$$m_1 = \frac{20}{0.825} = 24.24\,\text{kg/min}$$

h_1=34 kJ/Kg of DA

For final stream the condition is

DBT = 30°C

WBT = 23°C

$v_3 = 0.85$ m^3/min

$h_3 = 68$ kJ/kg

volume flow rate V_3= 50 kg/min

$$m_3 = \frac{50}{0.85} = 58.82\,\text{kg/min}$$

Hence using energy balance for both steam

$m_1h_1 + m_2h_2 = m_3h_3$

$$24.24\times 34 + m_2\times 90 = 58.82\times 68$$

$$m_2 = 35.28\,\text{kg/min}$$

Relative humidity, ϕ =39%

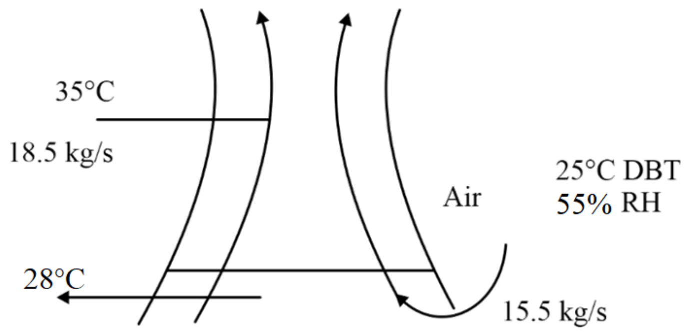
(c) In a direct contact counterflow splash-type filled cooling tower of a thermal power station, air enters at the bottom and leaves at the top of the cooling tower. Water is sprayed from the top of the cooling tower. The water enters at 35°C and leaves at 28°C. The ambient conditions are 25°C DBT and 55% RH. Water and air specific heats are 4-1867 and 1 kJ/kg K and mass flow rates are 18-5 and 15.5 kg/s respectively. Density of water is 1000 kg/m³. Find the following:

(i) Range
(ii) Approach
(iii) Cooling capacity of tower
(iv) Evaporation loss

Evaporation loss (m³/hr) is given by Evaporation loss = 0.00085 x 1.8 x circulation rate (m³/hr) x δT

δT difference in water entry and exit temperature.

Sol. 6. (c)



1. Approach = $T_{cool} - T_{WBT}$
   for air, WBT = 18.5°C
   Hence
   Approach = 2δ – 18.5 = 95°C
2. Range = $T_{HOT} - T_{cool}$
   = 35 – 28 = 7°C

Cooling capacity = $\dot{Q} = \dot{m}C\,\Delta T$

$= 18.5 \times 4.1867 \times 7$

CC = 542.17 kW

Evaporation loss

$$0.00085 \times 1.8 \times \frac{18.5}{1000} \times 3600 \times 7$$

Evaporation = 0.7132 m^3/hr

Q.7. (a)

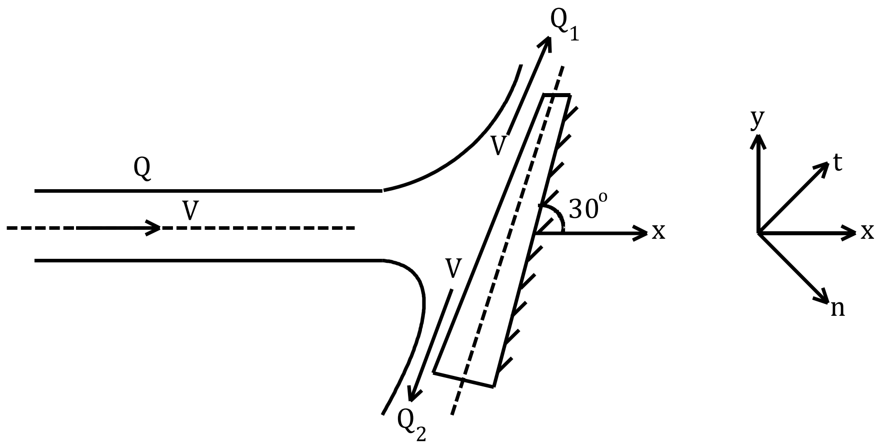
(i) A jet of velocity of 20 m/s strikes a flat plate inclined at 30° with the axis of the jet. If the cross-sectional area of the jet is 20 cm², find the force exerted by the jet on the plane. Also calculate the components of the force in the direction normal to the jet. Find also the ratio in which the discharge gets divided after striking the plate. Take density of water as 1000 kg/m³.

(ii) What is tidal range?

A simple single-basin type tidal power plant has a basin area of 20 km². The tide has a range of 8 m. The turbine stops operation when the head on it falls below 2 m. Determine the average power generated during one filling/emptying process in MW if the turbine generator efficiency is 70%. Assume specific gravity of sea water as 1.025.

Sol. 7. (a)

(i)



For given configuration

From continuity; $Q = Q_1 + Q_2$ ______ (1)

Net force acting in Tangential direction = 0

(for a smooth plate, the fluid just glides)

$\Rightarrow \Sigma F_t = \Sigma(\dot{m}V_t)_{out} - \Sigma(\dot{m}V_t)_{in} = 0$

$\Rightarrow \rho Q_1 V + \rho Q_2(-V) - \rho(Q)(V\cos\theta) = 0$

$\Rightarrow Q_1 - Q_2 = Q\cos\theta$ _______ (2)

Solving (1) & (2) $\Rightarrow \frac{Q_1}{Q_2} = \frac{\frac{Q}{2}(1+\cos\theta)}{\frac{Q}{2}(1-\cos\theta)} = \frac{1+\left(\frac{\sqrt{3}}{2}\right)}{1-\left(\frac{\sqrt{3}}{2}\right)} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$

$\therefore \frac{Q_1}{Q_2} = 13.928$

Since force exerted by fluid on plate = $\sqrt{F_n^2 + F_t^2}$

Since $F_t = 0$

$\Rightarrow |F| = F_n|_{\text{on plate}} = \Sigma(\dot{m}V_n)_{in} - \cancelto{0}{\Sigma(\dot{m}V_n)_{out}}$

(outlet is purely tangential)

$= \rho Q(V\sin\theta)$

$= \rho A V^2 \sin\theta = 10^3 \times (20\times10^{-4}) \times (20)^2 \times \frac{1}{2}$

= 400 N

∴ Force exerted by jet on plate = 400 N (in n-direction)

0

Force acting in the direction Normal to jet $= \Sigma(\dot{m}V_y)_{in} - \Sigma(\dot{m}V_y)_{out}$

$$= -\{\rho Q_1 V \sin\theta + \rho Q_2(-V\sin\theta)\}$$

$$= -\{\rho V \sin\theta(Q_1 - Q_2)\}$$

$$= -\rho QV \sin\theta \cos\theta$$

$$= -\rho AV^2 \sin\theta \,.\, \cos\theta$$

∴ Force acting in direction normal to jet

$$= 10^3 \times 20 \times 10^{-4} \times (20)^2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = -346.41\text{ N}$$

'–' sign denotes downward direction.

Force acting in direction of jet $= F_x = \rho AV^2 \sin^2\theta$

$$= 10^3 \times 20 \times 10^{-4} \times 20^2 \times \frac{1}{4}$$

$$= 200\text{ N}$$

∴ Force acting in jet direction = 200 N.

Sol. 7. (a)

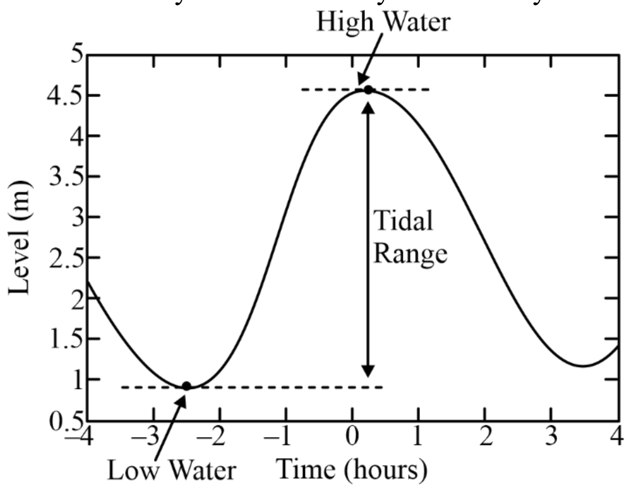
(ii)

Tidal range: Tidal range is the vertical difference in height between consecutive high and low waters over a tidal cycle (as shown in figure). The range of the tide varies between locations and also varies over a range of time scales. The tidal range R is defined as:

R = Water elevation at high tide – Water elevation at low tide

R = [Rmax-Rmin]

Tidal cycle: Time duration between two high or two low tides is called as one tidal cycle. There are two tidal cycle in a lunar day. One tidal cycle is 12 hr 25 min



Energy potential, $E_f = \rho Ag \int_2^8 z\,dz = \frac{1}{2}\rho Ag(8^2 - 2^2)$

Where, $A = 20 \times 10^6\text{ m}^2$; $\rho = 1025\text{ kg/m}^3$; g = 9.81

Average power, $P = \frac{W}{\text{Time}} = \frac{1}{2} \times 20 \times 10^3 \times 1025 \times 9.81 \times (8^2 - 2^2)$

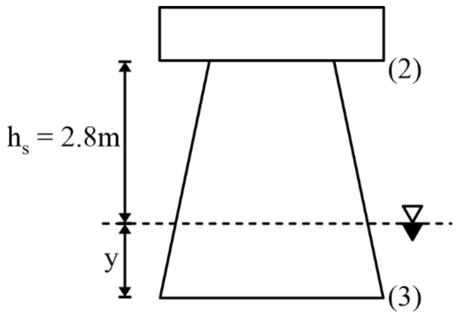
= 6033.15 MW

Turbine generator efficiency is 70%.

∴ $P_{avg} = 6033.15 \times 0.7 = 4223.205$ MW

(b) The draft tube of a Kaplan turbine has inlet diameter 2.5 m and inlet is set at 2.8 m above the tail race. When the turbine develops 2200 metric horsepower under a net head of 6.2 m, it is found that the vacuum gauge fitted at the inlet to the draft tube indicates a negative head of 4 m. If the turbine efficiency is 88%, calculate the draft tube efficiency. Further, if the turbine output is reduced to 50% with the same head, speed and draft tube efficiency, calculate the reading of the vacuum gauge. Assume, atmospheric pressure is 10.3 m of water and specific weight of water is 1000 kg/m³.3

Sol. 7. (b)



Given: $d_2 = 2.5$ m, $kP_1 = 2200$ metric horsepower $= 2200\times 735.499$ W $= 1618.0987$ kW,
$H = 6.2$ m, $\eta_{overall} = 0.88$, $\eta_{DT} = ?$, $\frac{p_2}{\rho g} = -4$ m,

$W_P = \frac{kP}{\eta_0}$, $\rho QgH = \left(\frac{kP}{\eta_0}\right)$

$Q = \left(\frac{kP}{\eta_0}\right)\times\left(\frac{1}{\rho gH}\right) = 30.23162\ m^3/s$

$a_2 = \frac{\pi}{4}\times(2.5)^2 = 4.9087\ m^2$

$V_2 = 6.1587$ m/s

$\frac{V_2^2}{2g} = 1.9332$ m

Applying hydrostatic law between free surface and section (3)

$\frac{p_3}{\rho g} = y$

Applying energy balance between 2 and 3

$$\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_s + y = \frac{p_2}{\rho g} + \frac{V_3^2}{2g} + h_f$$

$$\frac{V_3^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_s = -4 + 1.9332 + 2.8 = 0.7332\ m$$

$$\eta_{draft\ tube} = \frac{\frac{V_2^2}{2g} - \frac{V_3^2}{2g}}{\frac{V_2^2}{2g}} = 0.6207$$

$$kP_2 = \frac{1}{2}kP_1 = 809.04935\ kW$$

Assuming same overall efficiency of turbine,

$$WP_2 = \frac{kP_2}{\eta_0}$$

$Q = 15.1158\ m^3/s$

$$V_2 = \frac{Q}{a_2} = 3.0794\ m/s$$

$$\frac{V_2^2}{2g} = 0.4833\ m$$

$$\eta_{DT} = \frac{\frac{V_2^2}{2g} - \frac{V_3^2}{2g}}{\frac{V_2^2}{2g}} = 1 - \frac{\left(\frac{V_3^2}{2g}\right)}{\left(\frac{V_2^2}{2g}\right)}$$

$$\frac{V_3^2}{2g} = \frac{V_2^2}{2g}(1 - \eta_{DT}) = 0.1833 \text{ m}$$

Applying energy balance,

$$\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_s + y = \frac{P_3}{\rho g} + \frac{V_3^2}{2g} + h_L$$

$$\frac{P_2}{\rho g} = \frac{V_3^2}{2g} - \frac{V_2^2}{2g} - h_s = -3.09998 \text{ m}$$

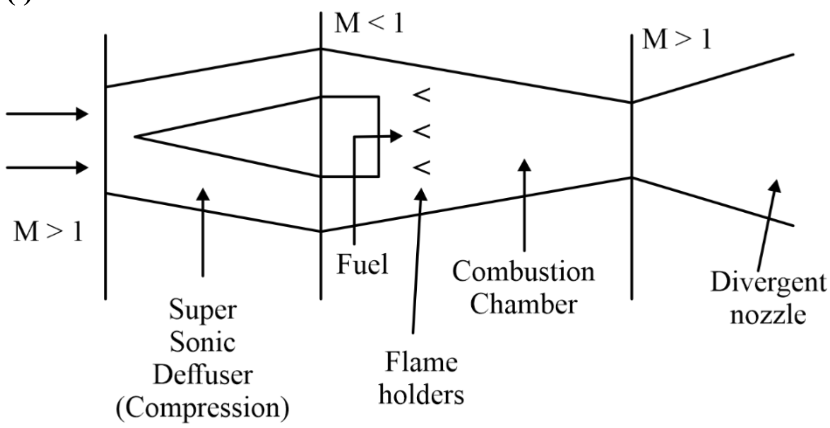
(c)

(i) Explain with the aid of illustrative sketch the working of a Ramjet engine. What are its applications?

(ii) Mention the various industrial wastes and by-products used as boiler fuels. Briefly explain them.

Sol. 7. (c)

(i)



Ramjet engine is the aircraft engines for suspersonic aircraft where compression of air takes place due to supersonic diffuser. This is a converging passage. Air is compressed and its velocity reduced in supersonic diffuser. Now high pressure air enters combution chamber where fuel is injected and combustion takes place.

High pressure high temp gases are expended in convergent-divergent nozzle.

Sol. 7. (c)

(ii)

The industrial by-products used for boiler fuels are as follows

(1) Heavy oil – The fuel oil left at the bottom of fractional distillation of petroleum is most commonly used in boilers.

(2) Oxygenetated vegetable oils and used lubricating oils coming out of machineries are also used as boiler fuels.

(3) Wood, molasses paper products and agricultural wastes are also used as boiler fuels.

(4) Coal refuses – waste products of coal mining, physical coal cleaning and preparation operations.

Q.8. **(a)**

A flat plate collector is of size 2 m length and 1 m width with one glass cover and 0º slope with horizontal. The gap between absorber plate and glass cover is 0.05 m and absorber plate to back plate is 0.015 m. Ambient air is passed between absorber plate and back plate at the rate of m=0.1 kg/s. The solar radiation is I = 800 W/m²,

Transmittance-Absorptance Product is T_A = 0.8, Overall loss coefficient U_l = 9.65 W/m² K, Ambient temperature T_a = 300 K, Specific heat of air C_p = 1006.4 J/kg K, natural convection heat transfer coefficient is 12.08 W/m² K.

Find the plate temperature T_p and heat removal factor F_R. Use the correlations given below:

(i) Plate temperature

$$T_p = T_a + \left[\frac{I \times T_A}{U_L}\right]\left(1 - G\xi\frac{G_p}{U_L}\right)$$

where

$$\xi = 1 - \exp\left[\left(\frac{-U_L}{G\,C_p}\right)\left(1 + \frac{U_L}{h}\right)^{-1}\right]$$

Heat removal factor is given by

$$F_R = \left(G\frac{C_p}{U_L}\right)\left[1 - \exp\left(\frac{-F'U_L}{G\,C_p}\right)\right]$$

$$\text{where } F' = \left(1 + \frac{U_L}{h}\right)^{-1}$$

where G is the mass flow rate per unit area of absorber plate area (kg/s/m²).

(ii) A company wanted 2 hours of buffer storage for a 1.5 MW solar thermal power plant that operates between 230°C and 380°C. Estimate the amount of material that is needed if Lithium Nitrate is used. Lithium Nitrate has the properties at melting point 252°C, latent heat 530 kJ/kg, specific heat of solid 2.02 and liquid 2.041 kJ/kg K respectively. Density of solid 2310 kg/m³ and liquid 1776 kg/m³, energy density 261 kWh/m² K and thermal conductivity 1.35 W/mK.

Sol. 8. **(a)**

(i)

Plate temperature, $T_P = T_a + \left[\frac{I \times T_A}{U_L}\right]\left(1 - G\xi\frac{C_P}{U_L}\right)$

Where, $\xi = 1 - \exp\left[\left(\frac{-U_L}{GC_P}\right)\left(1 + \frac{U_L}{h}\right)^{-1}\right]$

Now, $A = 2 \times 1 = 2\text{m}^2$, $\dot{m} = 0.1$ kg/s, J = 800 W/m²

$T_A = 0.8$; $U_L = 9.65$ W/m²-K; $T_a = 300$ K

$C_P = 1006.4$ J/kgK; h = 12.08 W/m²K

$$\therefore \quad \xi = 1 - \exp\left[\left(\frac{-9.65 \times 2}{0.1 \times 1006.4}\right)\left(1 + \frac{9.65}{12.08}\right)^{-1}\right]$$

$$\xi = 0.1011 \text{ and } G = \frac{M}{A} = \frac{0.1}{2} = 0.05\ \text{kg/s.m}^2$$

$$\therefore \quad T_P = 300 + \left[\frac{800 \times 0.8}{9.65}\right]\left[1 - \frac{0.05 \times 0.1011 \times 1006.4}{9.65}\right]$$

$$= 300 + 31.35 = 331.35 \text{ K}$$

Now, $F' = \left(1 + \frac{U_L}{h}\right)^{-1} = \left(1 + \frac{9.65}{12.08}\right)^{-1}$

$F' = 0.5559$

$\therefore$ Heat removal factor, $F_R = \left(G\frac{C_P}{U_L}\right)\left[1 - \exp\left(\frac{-F'U_L}{GC_P}\right)\right]$

$\therefore$ $F_R = \left(\frac{0.05\times1006.4}{9.65}\right)\left(1 - \exp\left(\frac{-0.5559\times9.65}{0.05\times1006.4}\right)\right)$

$F_R = 5.2145 \times 0.1011$

$\therefore$ $F_R = 0.5273$

Sol. 8. (a)

(ii)

Total energy required = 1.5 × 2 × 3600 = 10800 MJ

Now, energy that can be obtained, is given by

$$E_a = m[C_{Ps}(252 - 230) + 530 + C_{pl}(380 - 252)]$$
$$E_a = m[(2.02 \times 22)] + 530 + (2.041 \times 128)]$$
$$= m \times 835.688 \text{ kJ}$$

The total energy required = Energy obtained by the lithium nitrate

$10800 \times 10^3 = m \times 835.688$

$m = \frac{10800\times10^3}{835.688} = 12923.48$ kg

(b) The percentage composition of a solid fuel used in a boiler of a power station is as follows:

Carbon 90%, Hydrogen 3.5%, Oxygen 3%, Nitrogen 1%, Sulphur 1% and the remaining being ash.

Determine the excess air supplied for the combustion of coal if the volumetric analysis of dry flue gases shows the following composition:

CO_2: 10%, CO: 1%, N_2: 82%, O_2:7%

Take oxygen as 23% in the air by mass.

Sol. 8. (b)

C – 90%

H –3.5%

O – 3%

N – 1%

S – 1%

Exhaust gases

CO_2 – 10%

CO – 1%

N_2 – 82%

O_2 – 7%

Let's take 1 kg of fuel

$90\%C \Rightarrow 900 \text{ gm} = \frac{900}{12}$ moles of C

	% by mass	Mass/kg	Moles
C	90%	0.9	75
H	35%	0.035kg	17.5
O_2	3%	0.03 kg	0.9375
N	1%	0.01	0.3571
S	1%	0.01	0.1565

$$75C + 17.5H_2 + 0.9375O_2 + 0.3571N_2 + 0.1562S + x\left(O_2 + \frac{79}{21}N_2\right)$$

$\Rightarrow A\ CO_2 \quad + \ B\ CO \quad + \ C\ N_2 \quad + D\ O_2$

We know for exhaust

A = 10%	A : B = 10 : 1
B – 1%	
C = 82%	
O2 – 7%	

By stoichiometric balance

$A + B = 75$

$\frac{A}{B} = 10 \qquad \Rightarrow \frac{A = 68.2}{B = 6.82}$

$\frac{A}{C} = \frac{10}{82} \Rightarrow C \Rightarrow \frac{82}{10} \times 68.2 = 559.24$

Hence $\frac{79}{21} \times x + 0.3571 = 559.24$

$\Rightarrow x = 148.56$

For stoichiometric

$75C + 17.5\ H_2 + 0.9375O_2 + 0.3571N_2 + 0.1562S + yO_2 + CO_2 + N_2 + H_2O$

Oxygen requirement

$$y = 75 + \frac{17.5}{2} - 0.9375$$

$y \Rightarrow 82.81$

Hence excess air supplied

$$\frac{148.56 - 82.81}{82.81} = 79.34\%$$

(c) A single stage, single acting air compressor 30 cm bore and 40 cm stroke runs at 200 rpm. The suction pressure is 1 bar at 15°C and delivery pressure is 5 bar. Determine the indicated mean effective pressure and the ideal power required to run it, when

(i) Compression is isothermal

(ii) Compression follows the law $PV^{1.25}$ = Constant,

(iii) Compression is reversible adiabatic ($\gamma = 1.4$), and

(iv) Compression is irreversible adiabatic ($n = 1.5$).

Neglect clearance.

Determine the isothermal efficiency for (ii), (iii) and (iv). Assume isentropic or reversible adiabatic index, $\gamma = 1.4$ and $R = 0.287$ kJ/kg K

Sol. 8. (c)

$$V_{swept} = \frac{\pi d^2}{4} \cdot L = 28274.33\ cm^3$$

$$V_{swept/sec} = 0.094248\ m^3 \Rightarrow \dot{m} = 0.114\ kg/s$$

For (1) Rn isothermal compression

$$W_C = mRT\ln\left(\frac{P_2}{P_1}\right) = 15.168 \text{ kW}$$

(ii) $PV^{1.25}$ = constant

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$W_C = \frac{n}{n-1} mR\ (T_2 - T_1)$$

If $n \neq 1$

$$W_C = mRT\ \ln\left(\frac{P_2}{P_1}\right) \text{ for } n = 1$$

$$P_{mcp} = \frac{W_C}{V_S}$$

P_{ween}	T_2	$T_2 - T_1$	W_C	η_{iso}	P_{mean}
Isothermal	288	0	15.168	100%	1.60 bar
$PV^{1.25} = C$	397.3	109.36	17.87 kW	84%	1.89 bar
$PV^{r} = C$	456.14	168.14	19.25 kW	78%	2.04 bar
$PV^{1.5} = C$	492.47	204.47	20.0 kW	75%	2.12 bar

❑❑❑

THANK YOU!