LIVE

JEE MAIN 2024

ATTEMPT – 01 , 29TH JAN 2024 , SHIFT - 01

PAPER DISCUSSION

Mathematics

Let a die rolled till 2 is obtained. The probability that 2 obtain on even numbered toss is equal to

(A) $\frac{5}{11}$

(B) $\frac{5}{6}$

(C) $\frac{1}{11}$

(D) $\frac{6}{11}$

$P(2) = \frac{1}{6}$ (LB)

$P(\bar{2}) = \frac{5}{6}$

$P(2 \text{ on Even number toss})$

$= P(\bar{2}\,2) + P(\bar{2}\,\bar{2}\,\bar{2}\,2) + P(\bar{2}\,\bar{2}\,\bar{2}\,\bar{2}\,\bar{2}\,2) + \cdots \infty$

$= \frac{5}{6}\cdot\frac{1}{6} + (5/6)^3\cdot\frac{1}{6} + (5/6)^5\cdot\frac{1}{6} + \cdots \infty$

$= 1/6\left(5/6 + (5/6)^3 + (5/6)^5 + \cdots \infty\right)$

$= 1/6 \; \frac{5/6}{1-(5/6)^2} = \frac{5}{36}\cdot\frac{36}{36-25} = \frac{5}{11}$

If all the letters of the word 'GTWENTY' are arranged to form a new word and all these words are arranged in English dictionary, then find Rank of the word 'GTWENTY'

GTWENTY → alphabets E G N T T W Y (LB)

(A) ~~553~~

(B) 552

(C) 550

(D) 554

$\boxed{E} = \frac{6!}{2!} = 360$

$GE = \frac{5!}{2!} = 60$

$GN = \frac{5!}{2!} = 60$

$\boxed{GTE} = 4! = 24$

$\boxed{GTN} = 4! = 24$

$\boxed{GTT} = 4! = 24$

GTWENTY = 1

Rank of GTWENTY = 553

$$\begin{array}{r} 480 \\ 72 \\ 1 \\ \hline 553 \end{array}$$

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If $\frac{{}^{11}C_1}{2}+\frac{{}^{11}C_2}{3}+\ldots\ldots\frac{{}^{11}C_9}{10}=\frac{n}{m}$, then find the value of $m+n-8$

(A) 2033

(B) 2035

(C) 2050

(D) 2044

$${}^{n}C_r = \frac{n}{r}\cdot{}^{n-1}C_{r-1}$$

(L B)

$$\frac{1}{12}\left(\frac{12}{2}\cdot{}^{11}C_1+\frac{12}{3}\cdot{}^{11}C_2+\cdots+\frac{12}{10}\,{}^{11}C_9\right)$$

$$\frac{1}{12}\left({}^{12}C_2+{}^{12}C_3+\cdots+{}^{12}C_{10}\right)$$

$$\frac{1}{12}\left(\left({}^{12}C_0+{}^{12}C_1+{}^{12}C_2+{}^{12}C_3+\cdots+{}^{12}C_{10}+{}^{12}C_{11}+{}^{12}C_{12}\right)-{}^{12}C_0-{}^{12}C_1-{}^{12}C_{11}-{}^{12}C_{12}\right)$$

$$\frac{1}{12}\left(2^{12}-1-12-12-1\right)=\frac{1}{12}(4096-26)=\frac{4070}{12}=\frac{2035}{6}\begin{matrix}\leftarrow n\\ \leftarrow m\end{matrix}$$

$$n+m-8=2035+6-8=2033.$$

a, ar, ar^2 .... G.P. has 64 terms and $(S_n) = 7(S_n)_{odd}$. Find value of r.

(A) 6

(B) 8

(C) 4

(D) 12

$T_1 \quad T_3 \quad T_5$

$a, ar, ar^2, ar^3, \ldots, ar^{63}$

$2, 4, 8, 16$

(LB)

$S_n = \frac{a(r^{64}-1)}{r-1}$

$S_{odd} = a + ar^2 + ar^4 + \ldots$ up to 32 terms.

$S_{odd} = \frac{a((r^2)^{32}-1)}{r^2-1}$

$S_n = 7 S_{odd}$

$\frac{a(r^{64}-1)}{r-1} = \frac{7a(r^{64}-1)}{(r-1)(r+1)} \Rightarrow r+1 = 7$

$r = 6.$

Evaluate : $\lim_{x\to\frac{\pi}{2}} \frac{\int_{x^3}^{\left(\frac{\pi}{3}\right)^3} \cos t^{1/3}\,dt}{\left(x-\frac{\pi}{2}\right)^2}$. $\left(\frac{0}{0}\right)$

(A) $\frac{3\pi^2}{4}$

(B) $\frac{3\pi}{4}$

(C) $\cancel{\frac{3\pi^2}{8}}$

(D) $\frac{3\pi}{8}$

$l = \lim_{x\to\pi/2} \frac{0 - \cos x \cdot 3x^2}{2(x-\pi/2)}$

$= \lim_{x\to\pi/2} \frac{-\cos x}{x-\pi/2} \cdot \frac{3x^2}{2}$

$\lim_{x\to\pi/2} \frac{-\cos x}{x-\pi/2}$ $\frac{0}{0}$ (LB)

$\lim_{x\to\pi/2} \frac{\sin x}{1} = 1$

$l = 1 \cdot \frac{3}{2} \cdot (\pi/2)^2 = \frac{3\pi^2}{8}.$

In an AP $a_6 = 2$, then find common difference for which $a_1 \cdot a_4 \cdot a_5$ is greatest. (MB)

$a_6 = 2 \Rightarrow a + 5d = 2 \Rightarrow a = 2 - 5d$

(A) 8/5

(B) 12/5

(C) 4/2

(D) 6/2

$P = a_1 \cdot a_4 a_5 = a(a+3d)(a+4d) = (2-5d)(2-2d)(2-d)$

$= (4 - 14d + 10d^2)(2-d)$

$= 8 - 28d + 20d^2 - 4d + 14d^2 - 10d^3.$

$P = -10d^3 + 34d^2 - 32d + 8.$

$\frac{dP}{d(d)} = -30d^2 + 68d - 32 = -2(15d^2 - 34d + 16)$

for MAX, MIN $15d^2 - 34d + 16 = 0$

$15d^2 - 10d - 24d + 16 = 0$

$(5d-8)(3d-2) = 0$

5 3 2 2 2 2

$$\frac{dP}{d(d)} = -2(15d^2 - 34d + 16)$$
$$= -2\cdot 15\left(d - \frac{2}{3}\right)\left(d - \frac{8}{5}\right)$$

$$d = \frac{2}{3}, \frac{8}{5}$$

max at $d = 8/5$.

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(Easy Question)(LB)

If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$ & $(|2A|)^3 = 2^{21}$, then find $\alpha \cdot (\alpha, \beta \in I^+)$

$|KA| = K^n |A|$

If order of $A = n$

(A) 5

(B) 8

(C) 10

(D) 9

$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{vmatrix} = 1 \cdot (\alpha^2 - \beta^2)$

$(|2A|)^3 = (2^3|A|)^3 = 2^9 \cdot |A|^3 = 2^{21}$

$|A|^3 = 2^{12}$

$|A| = 2^4 = 16$

$\alpha^2 - \beta^2 = 16. \quad \alpha, \beta \in I^+$

$\alpha = 5, \beta = 3$

$$f(x)=\begin{cases}2+2x, & -3\le x\le 0\\ 1-\frac{x}{2}, & 0<x\le 1\end{cases},\quad g(x)=\begin{cases}-x, & -1\le x<0\\ x, & 0\le x\le 1\end{cases},$$

find the range of $fog(x)$ is

(A) $\left[\frac{1}{2},1\right)\cup\{2\}$

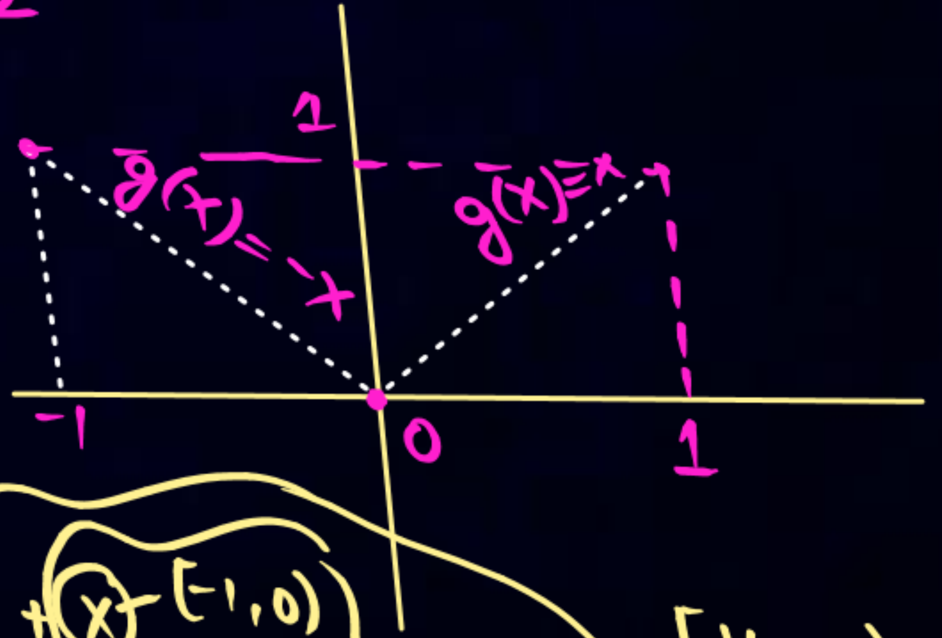
(B) $\left[\frac{1}{2},1\right]\cup\{2\}$

(C) $\left(\frac{1}{2},1\right)\cup\{2\}$

(D) $\left[\frac{1}{2},1\right)$

$$fog(x)=f(g(x))=\begin{cases}2+2g(x) & -3\le g(x)\le 0\\ 1-\frac{g(x)}{2} & 0<g(x)\le 1\end{cases}$$

$$f(g(x))=\begin{cases}2+0 & x=0\\ 1-\frac{x}{2} & 0<x\le 1\\ 1+\frac{x}{2} & -1\le x<0\end{cases}$$



$1-\frac{x}{2}$: $x \in (0,1] \to (0,1/2] \to [1/2,1)$

$1+\frac{x}{2}$: $x \in [-1,0) \to [-1/2,0) \to [1/2,1)$

Range of $f o g(x) = \left[\frac{1}{2}, 1\right) \cup \{2\}$

(Bouncer)

$$f(x)=\begin{cases}2+2x, & -3\le x\le 0\\ 1-\frac{x}{3}, & 0<x\le 1\end{cases},\; g(x)=\begin{cases}-x, & -1\le x<0\\ x, & 0\le x\le 1\end{cases},$$

find the range of $f o g(x)$ is

(A) $\left[\frac{1}{2},1\right)\cup\{2\}$

(B) $\left[\frac{1}{2},1\right]\cup\{2\}$

(C) $\left(\frac{1}{2},1\right)\cup\{2\}$

(D) $\left[\frac{1}{2},1\right)$

$$fog(x)=f(g(x))=\begin{cases}2+2g(x) & -3\le g(x)\le 0\\ 1-\frac{g(x)}{3} & 0<g(x)\le 1\end{cases}$$

$$f(g(x))=\begin{cases}2+0 & x=0\\ 1-\frac{x}{3} & 0<x\le 1\\ 1+x/3 & -1\le x<0\end{cases}$$

$1-\frac{x}{3}$: $x\in(0,1] \to \frac{x}{3}\in(0,1/3] \to 1-\frac{x}{3}\in[2/3,1)$

$1+\frac{x}{3}$: $x\in[-1,0) \to \frac{x}{3}\in[-1/3,0) \to 1+\frac{x}{3}\in[2/3,1)$

$$R_{fog} = \left[\frac{2}{3}, 1\right) \cup \{2\}$$

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(Difficult)

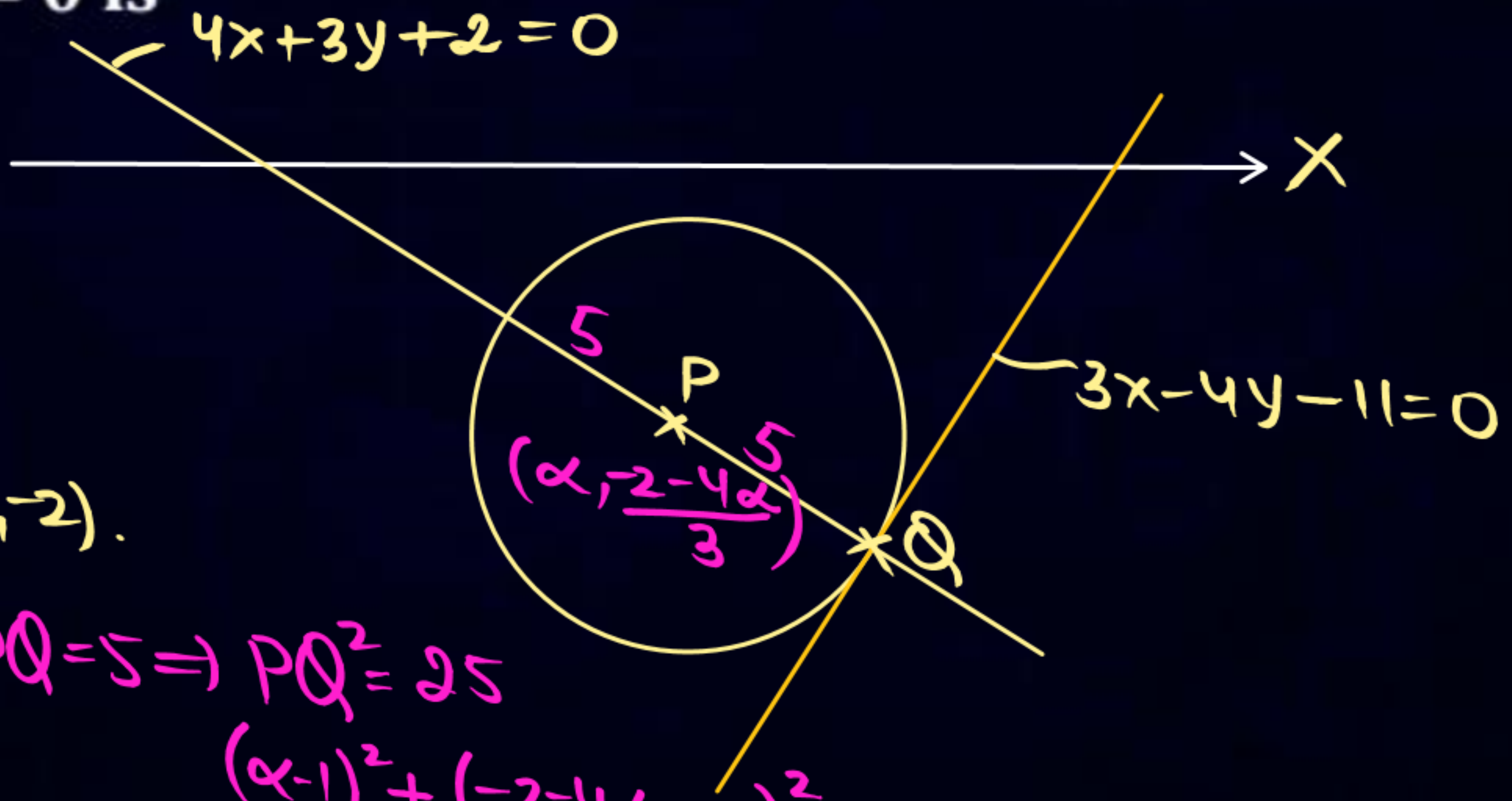
Let a circle C of radius 5 lie below the x-axis. The line $L_1 : 4x + 3y + 2 = 0$ passes through the centre P of the circle C and intersects the line $L_2 : 3x - 4y - 11 = 0$ at Q. The line L_2 touches C at the point Q. Then the distance of P from the line $5x - 12y + 51 = 0$ is

(Bouncer)

$$\begin{aligned} 4x+3y+2=0 \quad &\times 4 \\ 3x-4y-11=0 \quad &\times 3 \\ \hline 25x-25=0 \end{aligned}$$

$x=1$

$y=-2$ $\quad Q(1,-2)$.



$PQ=5 \Rightarrow PQ^2=25$

$(\alpha-1)^2+\left(\frac{-2-4\alpha}{3}+2\right)^2=25.$

$$(\alpha-1)^2+\left(\frac{4-4\alpha}{3}\right)^2=25$$

$$(\alpha-1)^2+\frac{16}{9}(\alpha-1)^2=25$$

$$(\alpha-1)^2\left(1+\frac{16}{9}\right)=25.$$

$$(\alpha-1)^2=9$$

$$\alpha-1=-3,3$$

$$\alpha=-2,4$$

$$P(-2,2) \text{ or } P(4,-6)$$

Distance of $P(4,-6)$ from $5x-12y+51=0$

$$p=\frac{20+72+51}{13}=\frac{143}{13}=11.$$

$\int_0^{\pi/2} \sin^2 x\,dx = \frac{\pi}{4}$

$\int_{-a}^{a} f(x)\,dx = \int_0^{a} (f(x)+f(-x))\,dx$

If $\int_{-\pi/2}^{\pi/2}\left(\frac{x^2\cos x}{1+\pi^x}+\frac{1+\sin^2 x}{1+e^{\sin 2023x}}\right)dx=\frac{\pi}{4}(\pi+\alpha)-2$, find α.

(A) ~~3~~

(B) 8

(C) 9

(D) None of these

$$I=\int_0^{\pi/2}\left(\frac{x^2\cos x}{1+\pi^x}+\frac{x^2\cos x}{1+\pi^{-x}}+\frac{1+\sin^2 x}{1+e^{\sin 2023x}}+\frac{1+\sin^2 x}{1+e^{-\sin 2023x}}\right)dx$$

$$=\int_0^{\pi/2} x^2\cos x\left(\frac{1}{1+\pi^x}+\frac{\pi^x}{\pi^x+1}\right)+(1+\sin^2 x)\left(\frac{1}{1+e^{\sin 2023x}}+\frac{e^{\sin 2023x}}{e^{2023x}+1}\right)$$

$$=\int_0^{\pi/2}(x^2\cos x+1+\sin^2 x)\,dx$$

$$=x^2\sin x\Big|_0^{\pi/2}-\int_0^{\pi/2}2x\sin x\,dx+\pi/2+\pi/4$$

$$I = \frac{\pi^2}{4} - 2\int_0^{\pi/2} x\sin x\,dx + \frac{\pi}{2} + \frac{\pi}{4}$$

$$= \frac{3\pi}{4} + \frac{\pi^2}{4} - 2\left[-x\cos x\Big|_0^{\pi/2} + \int_0^{\pi/2}\cos x\,dx\right]$$

$$= \frac{\pi^2}{4} + \frac{3\pi}{4} - 2\left[0+1\right]$$

$$I = \frac{\pi}{4}(\pi+3) - 2$$

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(Bouncer)

If α, β are the roots of $x^2 - x + 2 = 0$, such that $\text{Im}(\alpha) > (\text{Im}\,\beta)$, find $\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$

(A) 13

(B) 16

(C) 18

(D) None of these

$x^2 - x + 2 = 0 \begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}$

$\alpha^2 - \alpha + 2 = 0$

$\alpha^2 = \alpha - 2$ } squaring

$\alpha^4 = \alpha^2 + 4 - 4\alpha = (\alpha - 2) + 4 - 4\alpha = 2 - 3\alpha$

lly $\beta^4 = 2 - 3\beta$.

$\alpha^6 = (\alpha - 2)(2 - 3\alpha)$

$= 2\alpha - 3\alpha^2 - 4 + 6\alpha$

$= 8\alpha - 3\alpha^2 - 4$

$= 8\alpha - 3(\alpha - 2) - 4$

$= 5\alpha + 2$.

$\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$

$5\alpha + 2 + 2 - 3\alpha + 2 - 3\beta - 5(\alpha - 2)$

$16 - 3(\alpha + \beta) = 16 - 3 \cdot 1 = 13$

If $\frac{dy}{dx}-\left(\frac{\sin 2x}{1+\cos^2 x}\right)y=\frac{\sin x}{1+\cos^2 x}$ and $y(0)=0$, then $y\left(\frac{\pi}{2}\right)$ is equal to

(LB)

A -1

B 1

C 0

D 2

$1+\cos^2 x = t \quad -2\sin x\cos x\,dx = dt$

$IF = e^{-\int \frac{\sin 2x}{1+\cos^2 x}dx}$

$= e^{\int \frac{dt}{t}} = e^{\ln t} = e^{\ln(1+\cos^2 x)}$
$= (1+\cos^2 x)$

soln: $y\cdot(1+\cos^2 x) = \int \frac{\sin x}{\cancel{1+\cos^2 x}}\cdot\cancel{(1+\cos^2 x)}\,dx + C.$

$y\cdot(1+\cos^2 x) = \int \sin x\,dx + C$

$y(1+\cos^2 x) = -\cos x + C$

$x=0,\ y=0 \Rightarrow 0=-1+C \Rightarrow C=1$

$y(1+\cos^2 x) = 1-\cos x$ put $x=\pi/2$

$y\cdot 1 = 1 \Rightarrow y=1$

If : $|z + 1| = \alpha z + \beta(i + 1)$ and $z = \frac{1}{2} - 2i$, find $\alpha + \beta$. where $\underbrace{\alpha, \beta \in R}$ (LB)

(A) 3

(B) 6

(C) 9

(D) None of these

$$\left|\frac{1}{2} - 2i + 1\right| = \alpha\left(\frac{1}{2} - 2i\right) + \beta(i+1)$$

$$\left|\frac{3}{2} - 2i\right| = \frac{\alpha}{2} + \beta + i(\beta - 2\alpha)$$

$$\sqrt{(3/2)^2 + 2^2} = \alpha/2 + \beta + i(\beta - 2\alpha)$$

$$\frac{5}{2} = \alpha/2 + \beta + i(\beta - 2\alpha)$$

$$\alpha/2 + \beta = 5/2, \quad \beta = 2\alpha$$

$$\alpha + 2\beta = 5$$

$$5\alpha = 5.$$

$$\alpha = 1$$

$$\beta = 2$$

Given data 60, 60, 44, 58, 68, α, β, 56 has mean 58, variance = 66.2, then find $\alpha^2 + \beta^2$.

(LB)

$$\sigma^2 = \frac{\sum x_i^2}{N} - (\bar{x})^2 = 66.2$$

A) 7181.6

B) 7152.8

C) 7081.6

D) 7161.5

$$\frac{44^2+60^2+60^2+58^2+68^2+\alpha^2+\beta^2+56^2}{8} - 58^2 = 66.2.$$

$$\frac{1936+3600+3600+3364+4624+\alpha^2+\beta^2+3136}{8} - 3364 = 66.2$$

If $f(x)=\int \frac{\operatorname{cosec}x+\sin x}{\operatorname{cosec}x\sec x+\tan x\sin^2 x}dx$ & $\lim_{x\to\pi/2} f(x)=1$, then value of $f\left(\frac{\pi}{4}\right)=?$ (LB)

(A) ~~$1-\frac{1}{\sqrt{2}}\tan^{-1}\frac{1}{2}$~~

(B) $1-\frac{1}{\sqrt{2}}\tan^{-1}\frac{1}{\sqrt{2}}$

(C) $1-\frac{1}{2}\tan^{-1}\frac{1}{2}$

(D) $1-\frac{1}{2}\tan^{-1}\frac{1}{\sqrt{2}}$

$$\int \frac{\frac{1+\sin^2 x}{\sin x}}{\frac{1}{\sin x\cdot\cos x}+\frac{\sin^3 x}{\cos x}}dx=\int \frac{1+\sin^2 x}{\sin x}\cdot\frac{\sin x\cdot\cos x}{1+\sin^4 x}dx$$

$$=\int\left(\frac{1+\sin^2 x}{1+\sin^4 x}\right)\cos x\,dx \qquad \sin x=t$$

$$=\int\frac{1+t^2}{1+t^4}dx. =\int\frac{\frac{1}{t^2}+1}{t^2+1/t^2-2+2}dt.$$

$$=\int\frac{1/t^2+1}{(t-1/t)^2+2}dt \qquad t-1/t=v$$

$$(1+1/t^2)dt=dv$$

$$I=\int\frac{dv}{2+v^2}=\frac{1}{\sqrt{2}}\tan^{-1}v+C$$

$$=\frac{1}{\sqrt{2}}\tan^{-1}\frac{\left(t-\frac{1}{t}\right)}{\sqrt{2}}+C.$$

$$f(x)=I=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\left(\sin x-\frac{1}{\sin x}\right)\right)+C$$

$$\lim_{x\to\pi/2}f(x)=1\Rightarrow\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\cdot 0\right)+C=1$$

$$C=1$$

$$f(x)=1+\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\left(\sin x-\frac{1}{\sin x}\right)\right)$$

$$f(\pi/4)=1+\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}-\sqrt{2}\right)\right)$$

$$=1+\frac{1}{\sqrt{2}}\tan^{-1}(1/2-1)=1-\frac{1}{\sqrt{2}}\tan^{-1}1/2.$$

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$\vec{a}, \vec{b}, \vec{c}$ are three non collinear vectors. $\vec{a} + 6\vec{b}$ is collinear with $\vec{c}$, $\vec{b} + 5\vec{c}$ is collinear with $\vec{a}$. Then $\vec{a} + \alpha\vec{b} + \beta\vec{c} = 0$, then find $\alpha + \beta$. (LB)

A ~~36~~

B 98

C 37

D None of these

$\bar{a} + 6\bar{b} = \lambda\bar{c}$ — (1)

$\bar{b} + 5\bar{c} = \mu\bar{a}$ ×6.

$\bar{a} - 30\bar{c} = \lambda\bar{c} - 6\mu a$

$\underbrace{(1+6\mu)\bar{a} = (30+\lambda)\bar{c}}_{\text{non collinear.}}$

$\lambda = -30, \mu = -1/6$. put in (1)

$\bar{a} + 6\bar{b} = -30\bar{c}$

$\bar{a} + 6\bar{b} + 30\bar{c} = \bar{0}$

$$\int \frac{(\sin x-\cos x)\sin^2 x}{\sin x\cos^2 x+\tan x\sin^3 x}dx \text{ is equal to}$$

(LB)

A. ~~$\frac{\ln|\sin^3 x+\cos^3 x|}{3}+C$~~

B. $\frac{\ln|\sin^3 x-\cos^3 x|}{3}+C$

C. $\frac{\ln|\sin^3 x-\cos^3 x|}{6}+C$

D. None of these

$$\int \frac{(s-c)s^2}{sc^2+\frac{s^4}{c}}dx$$

$$\int \frac{(s-c)s^2c}{sc^3+s^4}dx$$

$$\int \frac{(s-c)sc}{s^3+c^3}dx=\int \frac{s^2c-sc^2}{s^3+c^3}dx=\int \frac{dt}{3t}$$

$$=\frac{1}{3}\ln|t|+C.$$

$s^3+c^3=t$

$(3s^2c-3c^2s)dx=dt$

$sc(s-c)dx=dt/3$

(MB)

$Z \times Z \to Z \times Z$

If relation $R : (a, b)R(c, d)$ is only if $ad - bc$ is divisible by 5, $(a, b, c, d \in Z)$, then R is

$((a,b),(c,d)) \in R \Leftrightarrow ad - bc = 5m,\ m \in I$

(A) Reflexive

(B) ~~Symmetric, Reflexive but not transitive~~

(C) ~~Reflexive, Transitive but not Symmetric~~

(D) Equivalence Relation

Reflexive:

$((a,b),(a,b)) \in R\ \forall a, b \in Z$

b'coz $ab - ba = 0 = 5 \cdot 0$

Symmt

Let $((a,b),(c,d)) \in R$

$ad - bc = 5m,\ m \in Z$

$bc - ad = 5(-m) = 5n,\ n \in Z$

$cb - da = 5n$

$\Rightarrow ((c,d),(a,b)) \in R$

Transitive.

$((1,3),(5,5)) \in R$ $\quad 1\cdot 5-3\cdot 5=-10$

$((5,5),(2,3)) \in R$ $\quad 5\cdot 3-5\cdot 2=5$

$\Big\}$ divisible by 5

But $((1,3)(2,3)) \notin R$

$1\cdot 3-3\cdot 2=-3$ is not a multiple of 5.

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(L B)

If $AA^T = I$, then find $\frac{1}{2}A\left[(A+A^T)^2 + (A-A^T)^2\right]$

~~(A)~~ $A^3 + A^T$

(B) $A^2 + A^T$

(C) $(A^T)^2 + A$

(D) $A^3 + I$

$AA^T = I \Rightarrow A$ is orthogonal.

$\Rightarrow A^TA = I \Rightarrow AA^T = I = A^TA$

$\frac{1}{2}A\left(A^2 + (A^T)^2 + \cancel{2AA^T} + A^2 + (A^T)^2 - \cancel{2AA^T}\right)$

$= \cancel{\frac{1}{2}}A\,\cancel{2}\left(A^2 + (A^T)^2\right)$

$= A^3 + A(A^T)^2$

$= A^3 + AA^TA^T$

$= A^3 + A^T$

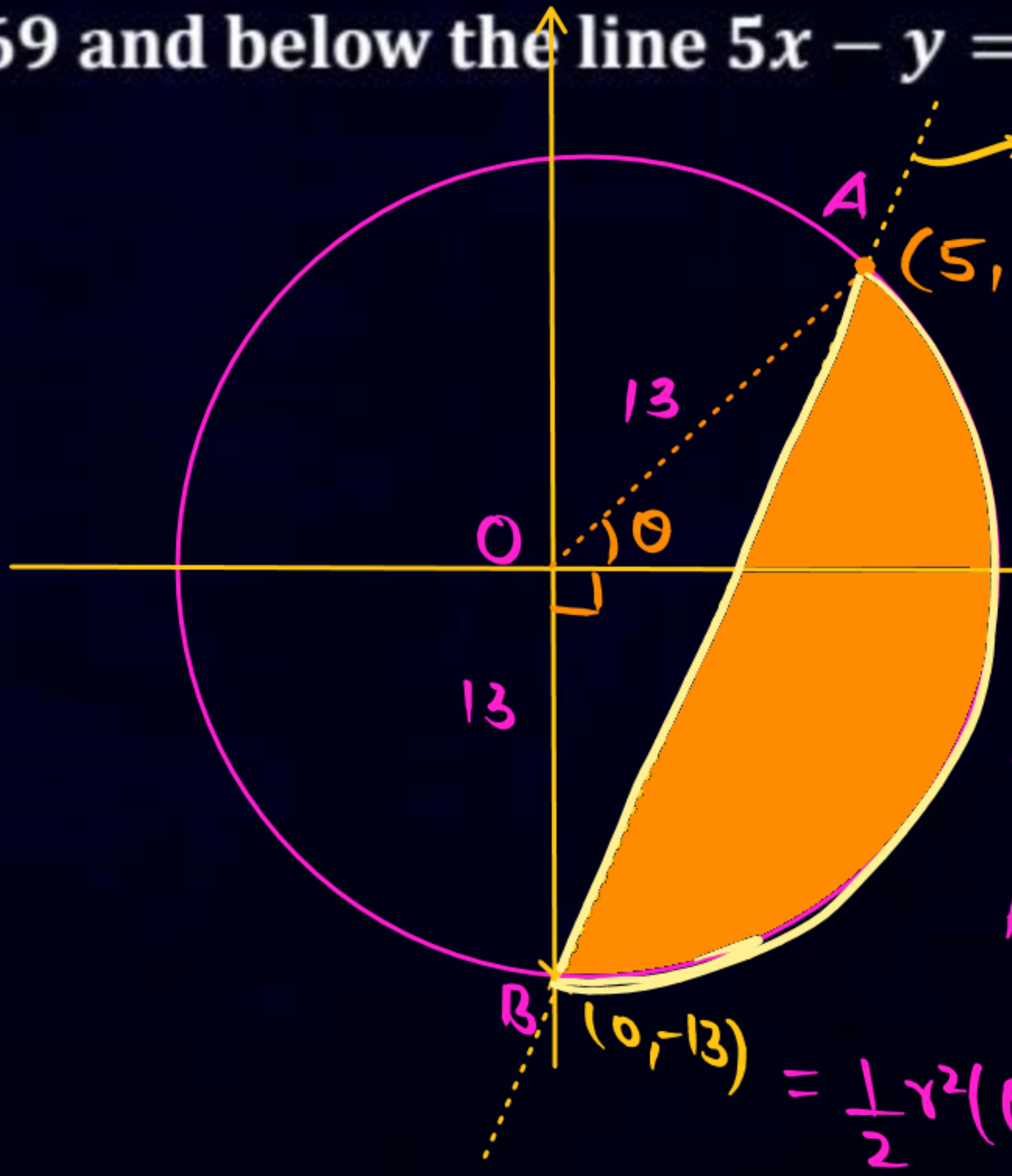
Area under the curve $x^2 + y^2 = 169$ and below the line $5x - y = 13$ is

(A) $\dfrac{169\pi}{4} - \dfrac{65}{2} + \dfrac{169}{2}\sin^{-1}\dfrac{12}{13}$

(B) $\dfrac{169\pi}{4} + \dfrac{65}{2} - \dfrac{169}{2}\sin^{-1}\dfrac{12}{13}$

(C) $\dfrac{169\pi}{4} - \dfrac{65}{2} + \dfrac{169}{2}\sin^{-1}\dfrac{13}{14}$

(D) $\dfrac{169\pi}{4} + \dfrac{65}{2} + \dfrac{169}{2}\sin^{-1}\dfrac{13}{14}$



$\tan\theta = \dfrac{12-0}{5-0} = \dfrac{12}{5}$

$\sin\theta = \dfrac{12}{13}.$

$\theta = \sin^{-1}\dfrac{12}{13}$

Shaded area
$=$ Area of sector AOB
$-$ Area Δ AOB.

$= \frac{1}{2}r^2(\theta + \pi/2) - 1/2\, r^2 \sin(90^\circ+\theta)$

$$A = \frac{1}{2} \cdot 13^2 \left(\sin^{-1}\frac{12}{13} + \frac{\pi}{2} \right) - \frac{1}{2} \cdot 13^2 \cdot \cos\theta$$

$$= \frac{169}{2} \cdot \sin^{-1}\frac{12}{13} + \frac{169\pi}{4} - \frac{1}{2} \cdot 169 \cdot \frac{5}{13}$$

$$= \frac{169}{2} \cdot \sin^{-1}\frac{12}{13} + \frac{169\pi}{4} - \frac{65}{2}.$$

(Bouncer)

$f(x) = 2^x - x^2 = 0$

m = number of solution such that $f(x)$ with x-axis

n = number of solutions such that $f'(x)$ with x-axis

Then $m + n = ?$ 5

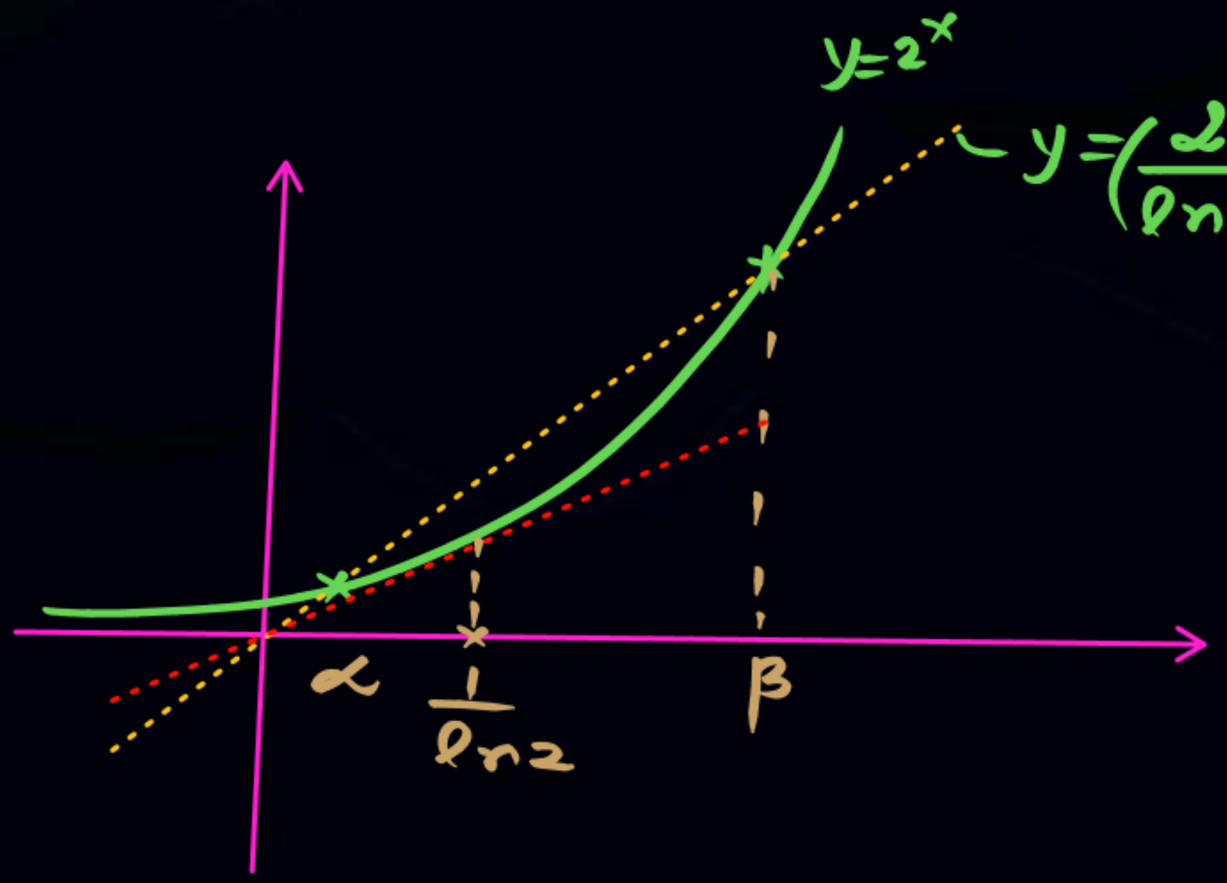
$f(x) = 2^x - x^2$ $\quad y = f'(x) = 2^x \cdot \ln 2 - 2x = \ln 2\left(2^x - \frac{2x}{\ln 2}\right) = 0$

$2^x = \frac{2x}{\ln 2}$

$y = 2^x$ $\qquad y = \left(\frac{2}{\ln 2}\right)x \rightsquigarrow$ St. line with slope $= \frac{2}{\ln 2}$ passing $(0,0)$

$m_{OP} = \left.\frac{dy}{dx}\right|_{AT\ P}$

$\frac{2^{x_1} - 0}{x_1 - 0} = 2^x \cdot \ln 2\Big|_{x = x_1} = 2^{x_1} \cdot \ln 2 \Rightarrow x_1 = \frac{1}{\ln 2}.$



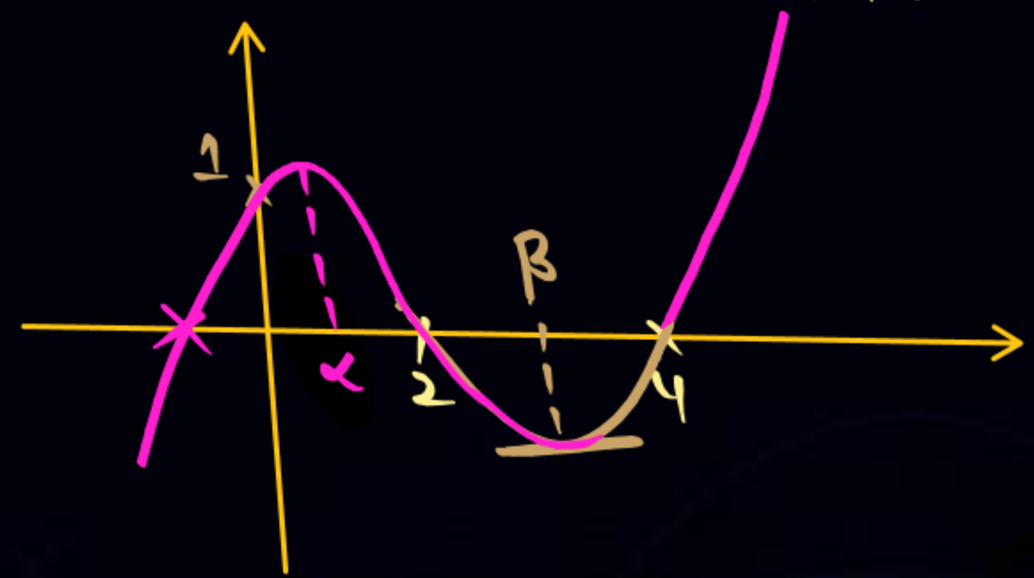
$2^x=\left(\frac{2}{\ln 2}\right)x \rightsquigarrow 2$ soln.

$n=2$

$\ln 2=0.693$

$f(x)=2^x-x^2=0$

$2^x=x^2 \longrightarrow x=2,4.$



$f(x)=2^x-x^2$

as $x\to\infty$ $\lim_{x\to\infty} f(x)=\lim_{x\to\infty} 2^x\left(1-\frac{x^2}{2^x}\right)$ ($\frac{x^2}{2^x}\to 0$)

$=\infty$

$\lim_{x\to-\infty} 2^x-x^2=2^{-\infty}-(-\infty)^2=-\infty.$

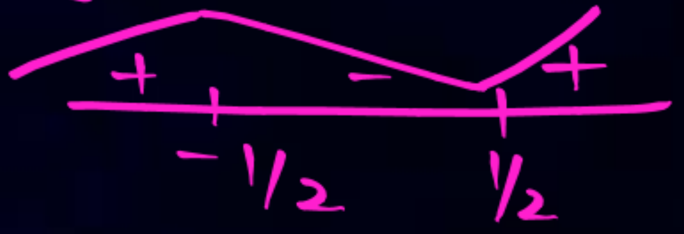
$m=3$

Considering : $f(x) = 4\sqrt{2}x^3 - 3\sqrt{2}x - 1$

S-1 : $f:\left[\frac{1}{2}, 1\right] \to R$, $f(x)$ intersects x-axis at 1 point. $\longrightarrow$ T

S-2 : $f(x)$ intersects x-axis at $x = \cos\frac{\pi}{12}$. — True

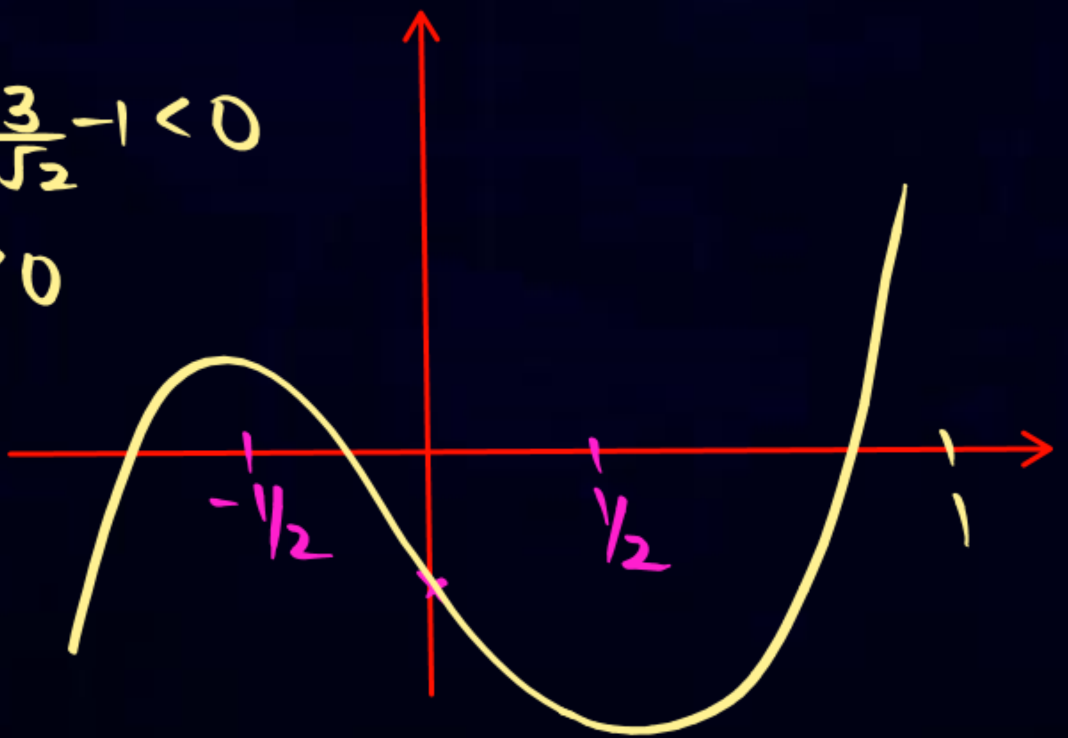
$f'(x) = 12\sqrt{2}x^2 - 3\sqrt{2} = 3\sqrt{2}(4x^2 - 1) = 3\sqrt{2}(2x-1)(2x+1)$



By IVT there is a root b/w 1/2 & 1

$f(1/2) = \frac{1}{\sqrt{2}} - \frac{3}{\sqrt{2}} - 1 < 0$

$f(1) = \sqrt{2} - 1 > 0$



$4\sqrt{2}x^3 - 3\sqrt{2}x = 1 \quad x \in (1/2, 1)$

let $x = \cos\theta$

$\sqrt{2}(4x^3 - 3x) = 1$

$4\cos^3\theta - 3\cos\theta = 1/\sqrt{2}$

$\cos 3\theta = \cos \pi/4$

$$3\theta = \frac{\pi}{4}$$

$$\theta = \frac{\pi}{12}$$

$$x = \cos\theta = \cos\frac{\pi}{12}.$$

JEE MAIN 2024 LIVE PAPER DISCUSSION

(LB)

If $f(x) = \dfrac{(2^x+2^{-x})(\tan x)\sqrt{\tan^{-1}(2x^2-3x+1)}}{(7x^2-3x+1)^3}$, then $f'(0)$ is equal to

A $\sqrt{\pi}$

B $\sqrt{\dfrac{\pi}{4}}$

C π

D $2\pi^{3/2}$

$$f(x) = (2^x+2^{-x})\sqrt{\tan^{-1}(2x^2-3x+1)}\,(7x^2-3x+1)^{-3}\cdot\tan x.$$

$$f'(x) = \frac{d}{dx}(2^x+2^{-x})\Big(\qquad\Big)\tan x$$

$$+\frac{d}{dx}\left(\sqrt{\tan^{-1}(2x^2-3x+1)}\right)\Big(\qquad\Big)\tan x$$

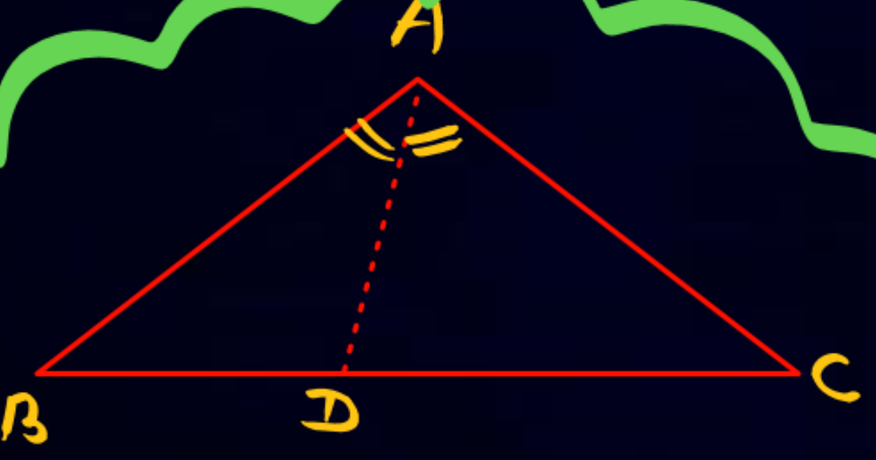
$$+\frac{d}{dx}(7x^2-3x+1)^{-3}\Big(\qquad\Big)\tan x + \frac{d}{dx}(\tan x)\cdot\sqrt{\tan^{-1}(2x^2-3x+1)}\,(7x^2-3x+1)^{-3}\cdot(2^x+2^{-x})$$

$$f'(0) = 0+0+0+\sec^2 0\cdot\sqrt{\tan^{-1}1}\cdot 1^{-3}\cdot(2)$$

$$f'(0) = 1 \cdot \sqrt{\frac{\pi}{4}} \cdot 2 = \sqrt{\pi}$$

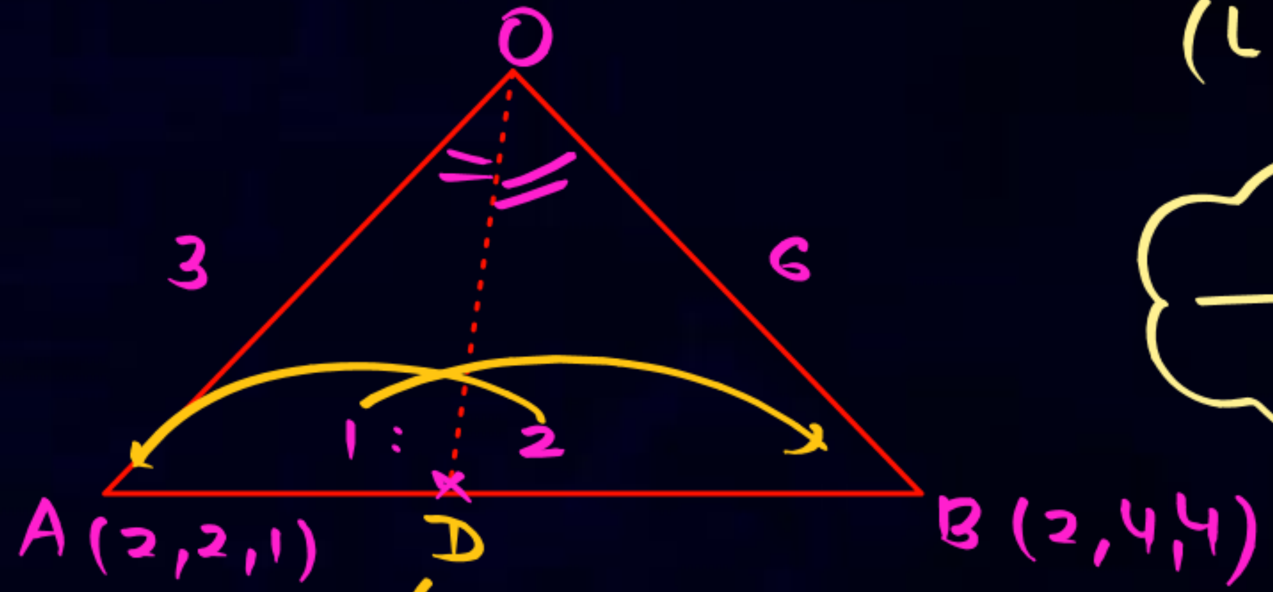
If $(A)\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$ & $B(\vec{b}) = 2\hat{i} + 4\hat{j} + 4\hat{k}$, find the length of angle bisector of $\angle AOB$ in triangle AOB.

(LB)



$$\frac{AB}{AC} = \frac{BD}{DC}$$

i.e $BD : DC = AB : AC$.



$$\frac{14-15}{24}$$

$$\left(\frac{2+4}{3}, \frac{4+4}{3}, \frac{4+2}{3}\right)$$

$$= (2, 8/3, 2)$$

length of angle Bisector $= OD = \sqrt{4 + \frac{64}{9} + 4}$

$$= \sqrt{8 + \frac{64}{9}} = \frac{\sqrt{136}}{3}$$

$$= \frac{2\sqrt{34}}{3}.$$

THANK YOU