Part-A: 3-Mark Questions

1. The wavefunction of a hydrogen atom is given by the following superposition of energy eigenfunctions $\psi_{nlm}(\vec{r})$ (n, l, m are the usual quantum numbers):

$$\psi(\vec{r}) = \frac{\sqrt{2}}{\sqrt{7}}\psi_{100}(\vec{r}) - \frac{3}{\sqrt{14}}\psi_{210}(\vec{r}) + \frac{1}{\sqrt{14}}\psi_{322}(\vec{r}).$$

The ratio of expectation value of the energy to the ground state energy and the expectation value of L^2 are, respectively:

(A) $\frac{229}{504}$ and $\frac{12\hbar^2}{7}$

(B) $\frac{101}{504}$ and $\frac{12\hbar^2}{7}$

(C) $\frac{101}{504}$ and $\hbar^2$

(D) $\frac{229}{504}$ and $\hbar^2$

2. An ideal gas with adiabatic exponent γ undergoes a process in which its pressure P is related to its volume V by the relation $P = P_0 - \alpha V$, where P_0 and α are positive constants. The volume starts from being very close to zero and increases monotonically to P_0/α. At what value of the volume during the process does the gas have maximum entropy?

(A) $\frac{P_0}{\alpha(1+\gamma)}$

(B) $\frac{\gamma P_0}{\alpha(1-\gamma)}$

(C) $\frac{\gamma P_0}{\alpha(1+\gamma)}$

(D) $\frac{P_0}{\alpha(1-\gamma)}$

3. The H_2 molecule has a reduced mass $M = 8.35 \times 10^{-28}$ kg and an equilibrium internuclear distance $R = 0.742 \times 10^{-10}$ m. The rotational energy in terms of the rotational quantum number J is:

(A) $E_{\text{rot}}(J) = 7J(J-1)$ meV

(B) $E_{\text{rot}}(J) = \frac{5}{2}J(J+1)$ meV

(C) $E_{\text{rot}}(J) = 7J(J+1)$ meV

(D) $E_{\text{rot}}(J) = \frac{5}{2}J(J-1)$ meV

4. The Hamiltonian of a quantum particle of mass m confined to a ring of unit radius is:

$$H = \frac{\hbar^2}{2m}\left(-i\frac{\partial}{\partial\theta} - \alpha\right)^2,$$

where θ is the angular coordinate, α is a constant. The energy eigenvalues and eigenfunctions of the particle are (n is an integer):

(A) $\psi_n(\theta) = \frac{e^{in\theta}}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}(n-\alpha)^2$
(B) $\psi_n(\theta) = \frac{\sin(n\theta)}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}(n-\alpha)^2$
(C) $\psi_n(\theta) = \frac{\cos(n\theta)}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}(n-\alpha)^2$
(D) $\psi_n(\theta) = \frac{e^{in\theta}}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}(n+\alpha)^2$

5. Consider a quantum particle of mass m in one dimension in an infinite potential well, *i.e.*, $V(x) = 0$ for $-a/2 < x < a/2$, and $V(x) = \infty$ for $|x| \geq a/2$. A small perturbation, $V'(x) = 2\epsilon|x|/a$, is added. The change in the ground state energy to $O(\epsilon)$ is:

(A) $\frac{\epsilon}{2\pi^2}(\pi^2 - 4)$

(B) $\frac{\epsilon}{2\pi^2}(\pi^2 + 4)$

(C) $\frac{\epsilon\pi^2}{2}(\pi^2 + 4)$

(D) $\frac{\epsilon\pi^2}{2}(\pi^2 - 4)$

6. A spin-1 particle is in a state $|\psi\rangle$ described by the column matrix $(1/\sqrt{10})\{2, \sqrt{2}, 2i\}$ in the S_z basis. What is the probability that a measurement of operator S_z will yield the result $\hbar$ for the state $S_x|\psi\rangle$?

(A) 1/2

(B) 1/3

(C) 1/4

(D) 1/6

7. The energy of a particle is given by $E = |p| + |q|$, where p and q are the generalized momentum and coordinate, respectively. All the states with $E \leq E_0$ are equally probable and states with $E > E_0$ are inaccessible. The probability density of finding the particle at coordinate q, with $q > 0$ is:

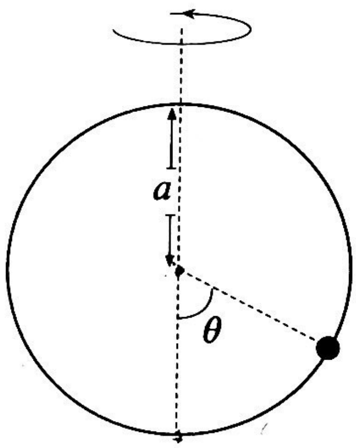
(A) $(E_0 + q)/E_0^2$

(B) q/E_0^2

(C) $(E_0 - q)/E_0^2$

(D) $1/E_0$

8. A hoop of radius a rotates with constant angular velocity ω about the vertical axis as shown in the figure. A bead of mass m can slide on the hoop without friction. If $g < \omega^2 a$, at what angle θ apart from 0 and π is the bead stationary (*i.e.*, $\frac{d\theta}{dt} = \frac{d^2\theta}{dt^2} = 0$)?



(A) $\tan\theta = \pi g/\omega^2 a$

(B) $\sin\theta = g/\omega^2 a$

(C) $\cos\theta = g/\omega^2 a$

(D) $\tan\theta = g/\pi\omega^2 a$

9. A gas of N molecules of mass m is confined in a cube of volume $V = L^3$ at temperature T. The box is in a uniform gravitational field $-g\hat{z}$. Assume that the potential energy of a molecule is $U = mgz$, where $z \in [0, L]$ is the vertical coordinate inside the box. The pressure $P(z)$ at height z is:

(A) $P(z) = \frac{N}{V}\frac{mgL}{2}\frac{\exp\left(-\frac{mg(z-L/2)}{k_BT}\right)}{\sinh\left(\frac{mgL}{2k_BT}\right)}$

(B) $P(z) = \frac{N}{V}\frac{mgL}{2}\frac{\exp\left(-\frac{mg(z-L/2)}{k_BT}\right)}{\cosh\left(\frac{mgL}{2k_BT}\right)}$

(C) $P(z) = \frac{k_BTN}{V}$

(D) $P(z) = \frac{N}{V}\, mgz$

10. A point charge q of mass m is released from rest at a distance d from an infinite grounded conducting plane (ignore gravity). How long does it take for the charge to hit the plane?

(A) $\frac{\sqrt{2\pi^3\epsilon_0 md^3}}{q}$

(B) $\frac{\sqrt{2\pi^3\epsilon_0 md}}{q}$

(C) $\frac{\sqrt{\pi^3\epsilon_0 md^3}}{q}$

(D) $\frac{\sqrt{\pi^3\epsilon_0 md}}{q}$

11. The strength of magnetic field at the center of a regular hexagon with sides of length a carrying a steady current I is:

(A) $\frac{\mu_0 I}{\sqrt{3}\pi a}$

(B) $\frac{\sqrt{6}\mu_0 I}{\pi a}$

(C) $\frac{3\mu_0 I}{\pi a}$

(D) $\frac{\sqrt{3}\mu_0 I}{\pi a}$

12. The maximum relativistic kinetic energy of β particles from a radioactive nucleus is equal to the rest mass energy of the particle. A magnetic field is applied perpendicular to the beam of β particles, which bends it to a circle of radius R. The field is given by:

(A) $3m_0c/eR$

(B) $\sqrt{2}m_0c/eR$

(C) $\sqrt{3}m_0c/eR$

(D) $\sqrt{3}m_0c/2eR$

13. Light takes approximately 8 minutes to travel from the Sun to the Earth. Suppose in the frame of the Sun an event occurs at $t = 0$ at the Sun and another event occurs on Earth at $t = 1$ minute. The velocity of the inertial frame in which both these events are simultaneous is:

(A) $c/8$ with the velocity vector pointing from Earth to Sun

(B) $c/8$ with the velocity vector pointing from Sun to Earth

(C) The events can never be simultaneous - no such frame exists

(D) $c\sqrt{1-(\frac{1}{8})^2}$ with velocity vector pointing from Sun to Earth

14. The central force which results in the orbit $r = a(1 + \cos\theta)$ for a particle is proportional to:

(A) r

(B) r^2

(C) r^{-2}

(D) None of the above

15. A spin-1/2 particle in a uniform external magnetic field has energy eigenstates $|1\rangle$ and $|2\rangle$. The system is prepared in ket-state $(|1\rangle + |2\rangle)/\sqrt{2}$ at time $t = 0$. It evolves to the state described by the ket $(|1\rangle - |2\rangle)/\sqrt{2}$ in time T. The minimum energy difference between two levels is:

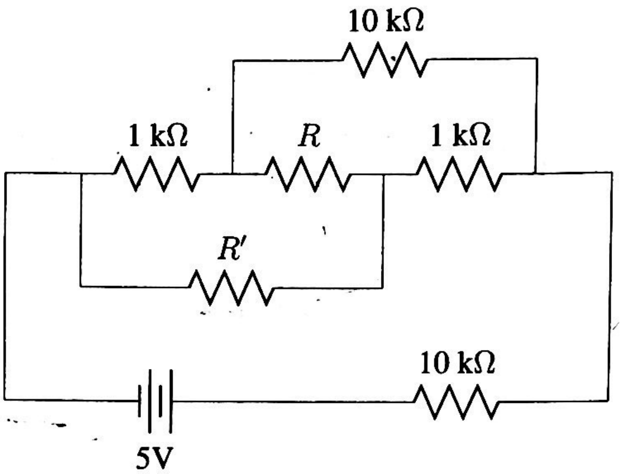
(A) $h/6T$

(B) $h/4T$

(C) $h/2T$

(D) h/T

16. It is found that when the resistance R indicated in the figure below is changed from 1 kΩ to 10 kΩ, the current flowing through the resistance R' does not change. What is the value of the resistor R'?



(A) 5 kΩ

(B) 100 Ω

(C) 10 kΩ

(D) 1 kΩ

17. The sum of the infinite series $1 - 1/3 + 1/5 - 1/7 + ...$ is:

(A) 2π

(B) π

(C) $\pi/2$

(D) $\pi/4$

18. A spherical shell of radius R carries a constant surface charge density σ and is rotating about one of its diameters with an angular velocity ω. The magnitude of the magnetic moment of the shell is:

(A) $4\pi\sigma\omega R^4$

(B) $4\pi\sigma\omega R^4/3$

(C) $4\pi\sigma\omega R^4/15$

(D) $4\pi\sigma\omega R^4/9$

19. Given a matrix $M = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, which of the following represents $\cos(\pi M/6)$?

(A) $\frac{1}{2}\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

(B) $\frac{\sqrt{3}}{4}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$

(C) $\frac{\sqrt{3}}{4}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

(D) $\frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$

20. Consider N non-interacting electrons ($N \sim N_A$) in a box of sides L_x, L_y, L_z. Assuming that the dispersion relation is $\epsilon(k) = C\,k^4$ where C is a constant, the ratio of the ground state energy per particle to the Fermi energy is:

(A) 3/7

(B) 7/3

(C) 3/5

(D) 5/7

21. A transistor in common base configuration has ratio of collector current to emitter current β and ratio of collector to base current α. Which of the following is true?

(A) $\beta = \alpha/(\alpha + 1)$

(B) $\beta = (\alpha + 1)/\alpha$

(C) $\beta = \alpha/(\alpha - 1)$

(D) $\beta = (\alpha - 1)/\alpha$

22. You receive on average 5 emails per day during a 365-days year. The number of days on average on which you do not receive any emails in that year are:

(A) More than 5

(B) More than 2

(C) 1

(D) None of the above

23. If $Y_{xy} = \frac{1}{\sqrt{2}}(Y_{2,2} - Y_{2,-2})$ where $Y_{l,m}$ are spherical harmonics, then which of the following is true?

(A) Y_{xy} is an eigenfunction of both L^2 and L_z

(B) Y_{xy} is an eigenfunction of L^2 but not L_z

(C) Y_{xy} is an eigenfunction of L_z but not L^2

(D) Y_{xy} is not an eigenfunction of either L^2 or L_z

24. A two dimensional box in a uniform magnetic field B contains $N/2$ localised spin-$1/2$ particles with magnetic moment μ, and $N/2$ free spinless particles which do not interact with each other. The average energy of the system at a temperature T is:

(A) $3NkT - \frac{1}{2}N\mu B \sinh(\mu B/k_B T)$

(B) $NkT - \frac{1}{2}N\mu B \tanh(\mu B/k_B T)$

(C) $\frac{1}{2}NkT - \frac{1}{2}N\mu B \tanh(\mu B/k_B T)$

(D) $\frac{3}{2}NkT + \frac{1}{2}N\mu B \cosh(\mu B/k_B T)$

25. The value of the integral $\int_0^\infty \frac{\ln x}{(x^2+1)} dx$ is:

(A) $\pi^2/4$

(B) $\pi^2/2$

(C) π^2

(D) 0

Part-B: 1-Mark Questions

26. An ideal gas has a specific heat ratio $C_P/C_V = 2$. Starting at a temperature T_1 the gas undergoes an isothermal compression to increase its density by a factor of two. After this an adiabatic compression increases its pressure by a factor of two. The temperature of the gas at the end of the second process would be:

(A) $T_1/2$

(B) $\sqrt{2}T_1$

(C) $2T_1$

(D) $T_1/\sqrt{2}$

27. The electric field $\vec{E} = E_0 \sin(\omega t - kz)\hat{x} + 2E_0 \sin(\omega t - kz + \pi/2)\hat{y}$ represents:

(A) a linearly polarized wave

(B) a right-hand circularly polarized wave

(C) a left-hand circularly polarized wave

(D) an elliptically polarized wave

28. If $\vec{k}$ is the wavevector of incident light ($|\vec{k}| = 2\pi/\lambda$, λ is the wavelength of light) and $\vec{G}$ is a reciprocal lattice vector, then the Bragg's law can be written as:

(A) $\vec{k} + \vec{G} = 0$

(B) $2\vec{k} \cdot \vec{G} + G^2 = 0$

(C) $2\vec{k} \cdot \vec{G} + k^2 = 0$

(D) $\vec{k} \cdot \vec{G} = 0$

29. The number of different Bravais lattices possible in two dimensions is:

(A) 2

(B) 3

(C) 5

(D) 6

30. An electron confined within a thin layer of semiconductor may be treated as a free particle inside an infinitely deep one-dimensional potential well. If the difference in energies between the first and the second energy levels is δE, then the thickness of the layer is:

(A) $\sqrt{\frac{3\hbar^2\pi^2}{2m\delta E}}$

(B) $\sqrt{\frac{2\hbar^2\pi^2}{3m\delta E}}$

(C) $\sqrt{\frac{\hbar^2\pi^2}{2m\delta E}}$

(D) $\sqrt{\frac{\hbar^2\pi^2}{m\delta E}}$

31. The *adjoint* of a differential operator $\frac{d}{dx}$ acting on a wavefunction $\psi(x)$ for a quantum mechanical system is:

(A) $\frac{d}{dx}$

(B) $-i\hbar\frac{d}{dx}$

(C) $-\frac{d}{dx}$

(D) $i\hbar\frac{d}{dx}$

32. In the ground state of hydrogen atom, the most probable distance of the electron from the nucleus, in units of Bohr radius a_0 is:

(A) 1/2

(B) 1

(C) 2

(D) 3/2

33. Circular fringes are obtained with a Michelson interferometer using 600nm laser light. What minimum displacement of one mirror will make the central fringe from bright to dark?

(A) 600 nm

(B) 300 nm

(C) 150 nm

(D) 120 Å

34. Given the condition $\nabla^2\Phi = 0$, the solution of the equation $\nabla^2\Psi = k\vec{\nabla}\Phi \cdot \vec{\nabla}\Phi$ is given by:

(A) $\Psi = k\Phi^2/2$

(B) $\Psi = k\Phi^2$

(C) $\Psi = k\Phi\ln\Phi$

(D) $\Psi = k\Phi\ln\Phi/2$

35. The output intensity I of radiation from a single mode of resonant cavity obeys

$$\frac{d}{dt}I = -\frac{\omega_0}{Q}I,$$

where Q is the quality factor of the cavity and ω_0 is the resonant frequency. The form of the frequency spectrum of the output is:

(A) Delta function

(B) Gaussian

(C) Lorentzian

(D) Exponential

36. For operators P and Q, the commutator $[P, Q^{-1}]$ is:

(A) $Q^{-1}[P,Q]\,Q^{-1}$

(B) $-Q^{-1}[P,Q]\,Q^{-1}$

(C) $Q^{-1}[P,Q]\,Q$

(D) $-Q\,[P,Q]\,Q^{-1}$

37. For a quantum mechanical harmonic oscillator with energies, $E_n = (n + 1/2)\hbar\omega$, where n = 0, 1, 2 ..., the partition function is:

(A) $\frac{e^{\hbar\omega/k_BT}}{e^{\hbar\omega/k_BT}-1}$

(B) $e^{\hbar\omega/2k_BT} - 1$

(C) $e^{\hbar\omega/2k_BT} + 1$

(D) $\frac{e^{\hbar\omega/2k_BT}}{e^{\hbar\omega/k_BT}-1}$

38. A semicircular piece of paper is folded to make a cone with the centre of the semicircle as the apex. The half-angle of the resulting cone would be:

(A) 90°

(B) 60°

(C) 45°

(D) 30°

39. A spin 1/2 particle is in a state $(|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$, where $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstates of S_z operator. The expectation value of the spin angular momentum measured along x direction is:

(A) $\hbar$

(B) $-\hbar$

(C) 0

(D) $\hbar/2$

40. The half-life of a radioactive nuclear source is 9 days. The fraction of nuclei which are left undecayed after 3 days is:

(A) 7/8

(B) 1/3

(C) 5/6

(D) $1/2^{1/3}$

41. If the Rydberg constant of an atom of finite nuclear mass is αR_∞, where R_∞ is the Rydberg constant corresponding to an infinite nuclear mass, the ratio of the electronic to nuclear mass of the atom is:

(A) $(1-\alpha)/\alpha$

(B) $(\alpha-1)/\alpha$

(C) $(1-\alpha)$

(D) $1/\alpha$

42. A gas contains particles of type A with fraction 0.8, and particles of type B with fraction 0.2. The probability that among 3 randomly chosen particles at least one is of type A is:

(A) 0.8

(B) 0.25

(C) 0.33

(D) 0.992

43. A cylindrical shell of mass m has an outer radius b and an inner radius a. The moment of inertia of the shell about the axis of the cylinder is:

(A) $\frac{1}{2}m(b^2 - a^2)$

(B) $\frac{1}{2}m(b^2 + a^2)$

(C) $m(b^2 + a^2)$

(D) $m(b^2 - a^2)$

44. If the direction with respect to a right-handed cartesian coordinate system of the ket vector $|z, +\rangle$ is $(0, 0, 1)$, then the direction of the ket vector obtained by application of rotations: $\exp(-i\sigma_z\pi/2)\exp(i\sigma_y\pi/4)$, on the ket $|z, +\rangle$ is (σ_y, σ_z are the Pauli matrices):

(A) $(0, 1, 0)$
(B) $(1, 0, 0)$
(C) $(1, 1, 0)/\sqrt{2}$
(D) $(1, 1, 1)/\sqrt{3}$

45. Suppose yz plane forms the boundary between two linear dielectric media I and II with dielectric constant $\epsilon_I = 3$ and $\epsilon_{II} = 4$, respectively. If the electric field in region I at the interface is given by $\vec{E}_I = 4\hat{x} + 3\hat{y} + 5\hat{z}$, then the electric field $\vec{E}_{II}$ at the interface in region II is:

(A) $4\hat{x} + 3\hat{y} + 5\hat{z}$

(B) $4\hat{x} + 0.75\hat{y} - 1.25\hat{z}$

(C) $-3\hat{x} + 3\hat{y} + 5\hat{z}$

(D) $3\hat{x} + 3\hat{y} + 5\hat{z}$

46. How much force does light from a 1.8 W laser exert when it is totally absorbed by an object?

(A) 6.0×10^{-9} N

(B) 0.6×10^{-9} N

(C) 6.0×10^{-8} N

(D) 4.8×10^{-9} N

47. Self inductance per unit length of a long solenoid of radius R with n turns per unit length is:

(A) $\mu_0 \pi R^2 n^2$

(B) $2\mu_0 \pi R^2 n$

(C) $2\mu_0 \pi R^2 n^2$

(D) $\mu_0 \pi R^2 n$

48. In Millikan's oil-drop experiment an oil drop of radius r, mass m and charge $q = 6\pi\eta r(v_1 + v_2)/E$ is moving upwards with a terminal velocity v_2 due to an applied electric field of magnitude E, where η is the coefficient of viscosity. The acceleration due to gravity is given by:

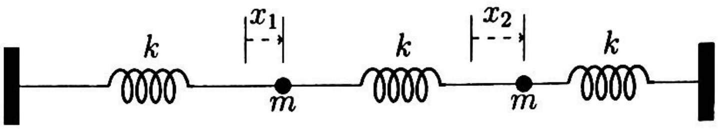
(A) $g = 6\pi\eta r v_1/m$

(B) $g = 3\pi\eta r v_1/m$

(C) $g = 6\pi\eta r v_2/m$

(D) $g = 3\pi\eta r v_2/m$

49. For the coupled system shown in the figure, the normal coordinates are $x_1 + x_2$ and $x_1 - x_2$, corresponding to the normal frequencies ω_0 and $\sqrt{3}\omega_0$, respectively.



At $t = 0$, the displacements are $x_1 = A,\ x_2 = 0$, and the velocities are $v_1 = v_2 = 0$. The displacement of the second particle at time t is given by:

(A) $x_2(t) = \frac{A}{2}\left(\cos(\omega_0 t) + \cos\left(\sqrt{3}\omega_0 t\right)\right)$

(B) $x_2(t) = \frac{A}{2}\left(\cos(\omega_0 t) - \cos\left(\sqrt{3}\omega_0 t\right)\right)$

(C) $x_2(t) = \frac{A}{2}\left(\sin(\omega_0 t) - \sin\left(\sqrt{3}\omega_0 t\right)\right)$

(D) $x_2(t) = \frac{A}{2}\left(\sin(\omega_0 t) - \frac{1}{\sqrt{3}}\sin\left(\sqrt{3}\omega_0 t\right)\right)$

50. The mean value of random variable x with probability density $p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp[-(x^2 + \mu x)/(2\sigma^2)]$, is:

(A) 0

(B) $\mu/2$

(C)' $-\mu/2$

(D) σ