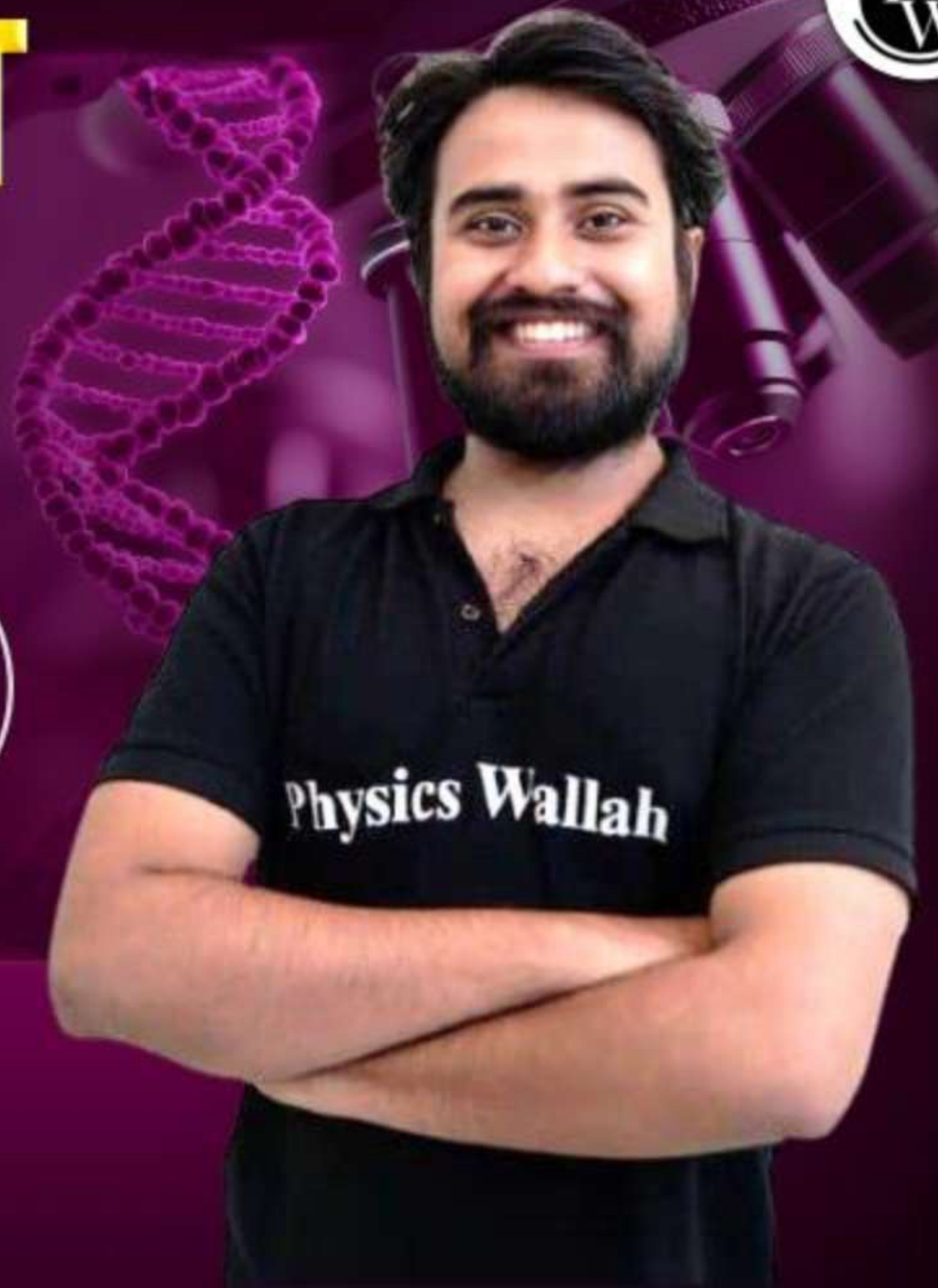
PRACHAND NEET

ONE SHOT

Physics

VECTORS

By - Aayudh Yashlaha Sir



Topics *to be covered*

1. Maths and Feel
2. Introduction
3. Magnitude and unit vector
4. Resolution
5. Addition of vectors
6. Multiplication of vectors
7. Theory and AR and PUPPY POINTS


PRACHAND SERIES
TELEGRAM CHANNEL
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NEET SYLLABUS

- Scalars and Vectors
- Vector. Addition and subtraction
- Scalar and Vector Products.
- Unit Vector
- Resolution of a Vector.

Part 1 – Maths and Horror Story

Maths to be used

$\cos(180 - \theta) = -\cos\theta$
$\cos(90 - \theta) = \sin\theta$

Common Kidnapping

Square Root

Fractions

Trigonometry Identities
$\cos(180-\theta) = -\cos\theta$
$\cos(90-\theta) = \sin\theta$
$1 + \cos(2\theta) = 2\cos^2\theta$
$1 - \cos(2\theta) = 2\sin^2\theta$

Trigonometric Values
Sin cos tan from 0^0 to 180^0

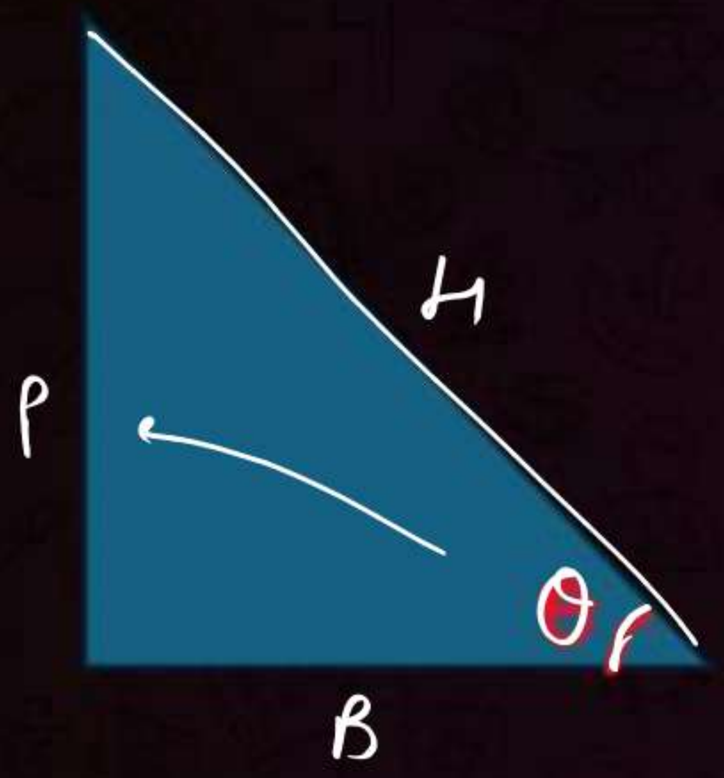
α	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$-$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

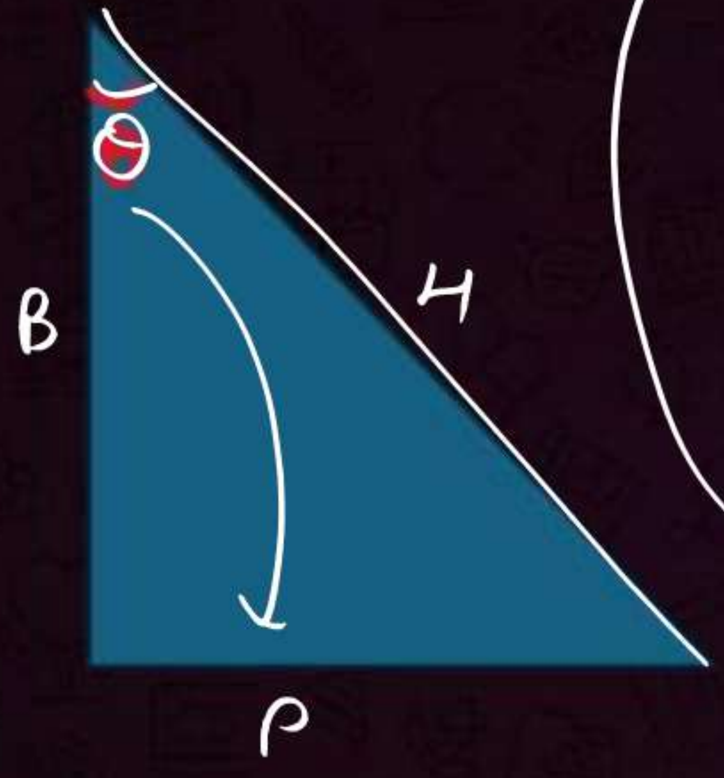
Geometry of Square, Rectangle, Parallelogram, Rhombus, Cube
Sides and Diagonals
Angle with diagonal

Pythagoras Theorem & Triplets

Pythagoras Theorem & Triplets

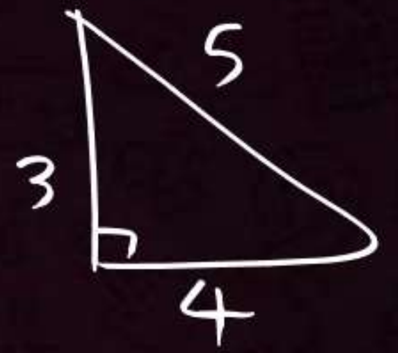
Pythagoras Theorem & Triplets





$$H^2 = P^2 + B^2$$

3, 4, 5

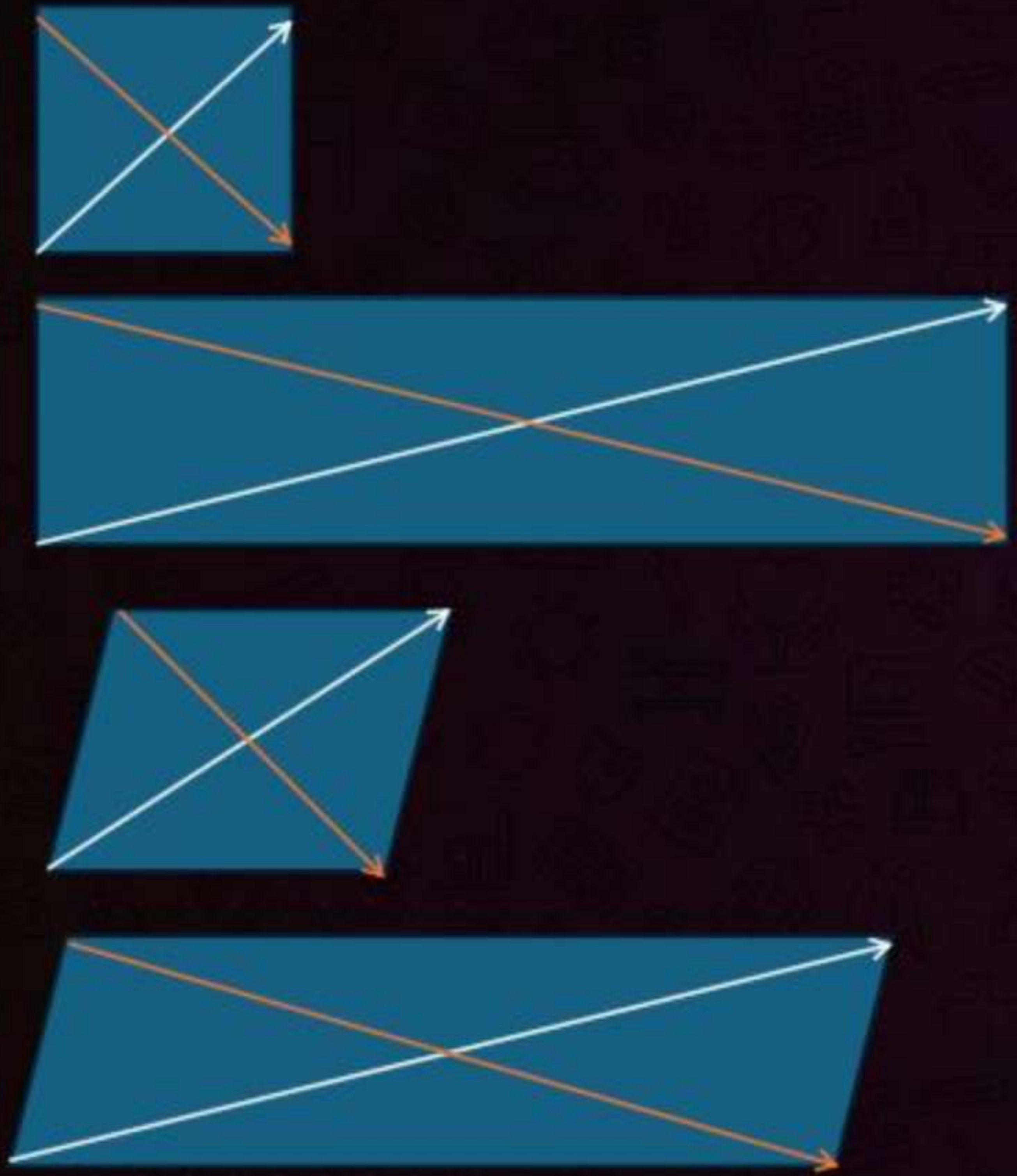


$3^2 = 9 \quad 4^2 = 16 \quad 5^2 = 25$

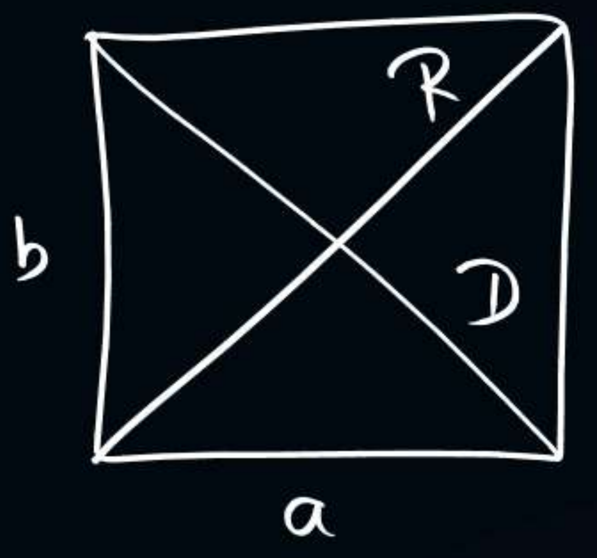
$3^2 + 4^2 = 9 + 16 = 25 = 5^2$

3, 4, 5 ↔	$x, x, \sqrt{2}x$
6, 8, 10	$1, \sqrt{3}, 2$
5, 12, 13	
7, 24, 25	

Geometry

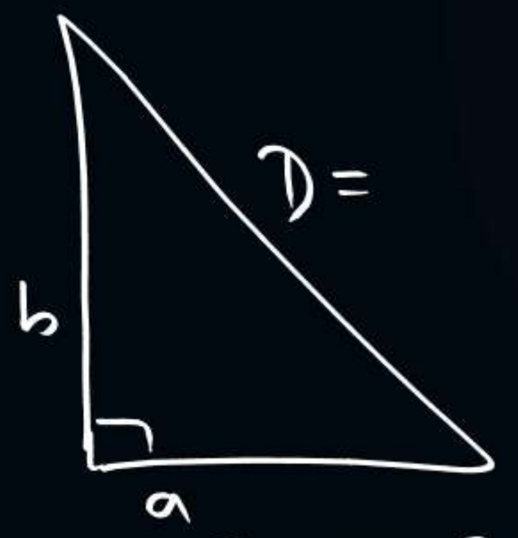


$2\sqrt{2} = \sqrt{8}$



$a = b$

$R = D$



$D^2 = a^2 + b^2$

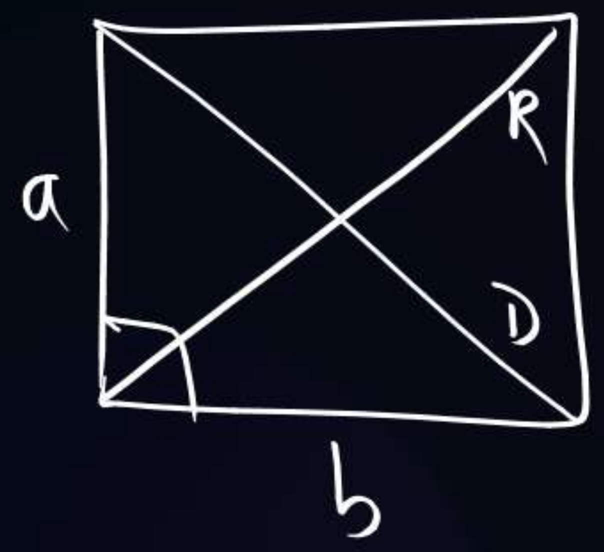
$D = \sqrt{a^2 + b^2}$

(a) a

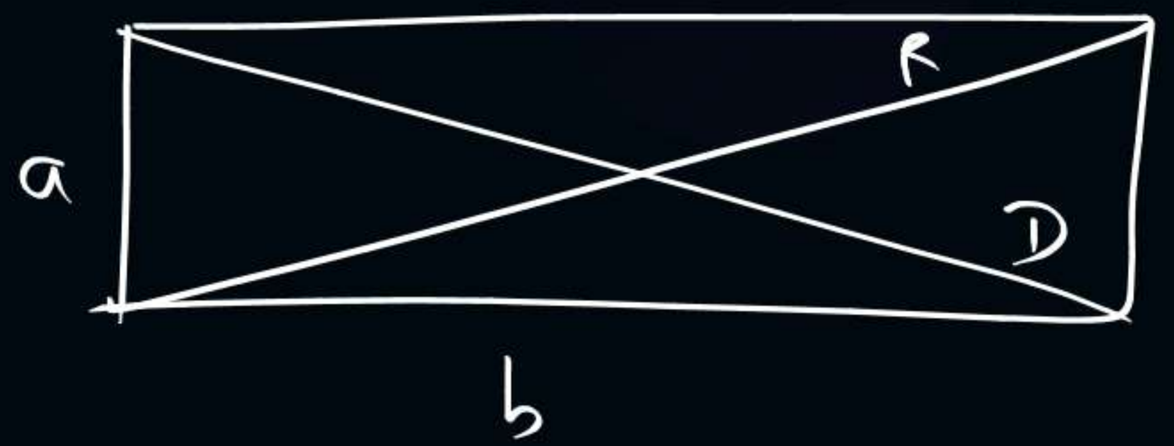
(b) b

(c) $\sqrt{a^2 + b^2}$ ✓

(c) $\sqrt{a^2 - b^2}$



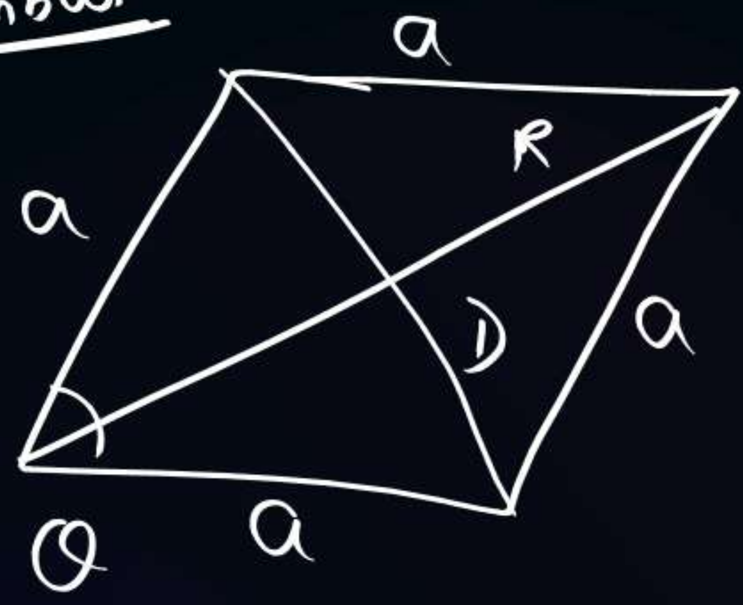
$R = D = \sqrt{a^2 + b^2}$ ✓



$a \neq b$

$$R = D = \sqrt{a^2 + b^2}$$

Rhombus

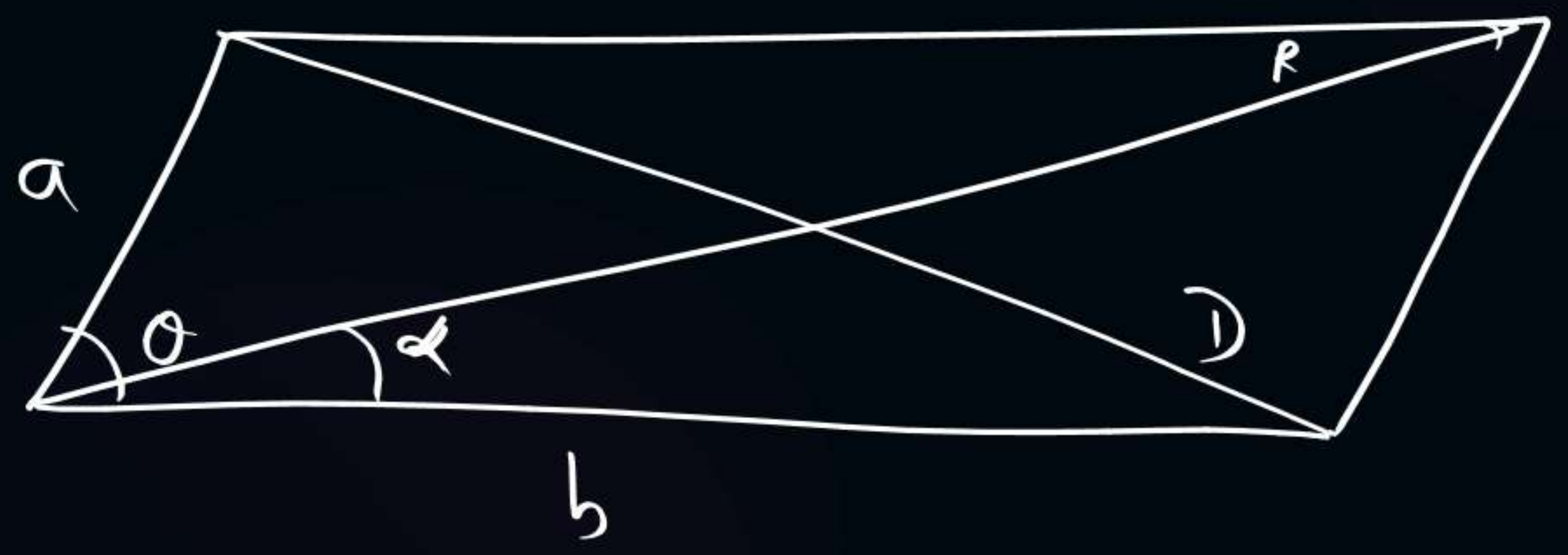


$$R = 2a\cos\left(\frac{\theta}{2}\right)$$

$$D = 2a\sin\left(\frac{\theta}{2}\right)$$

$R \perp D$

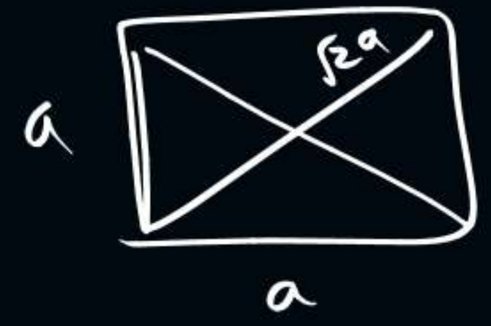
Parallelogram ($||^{gm}$)



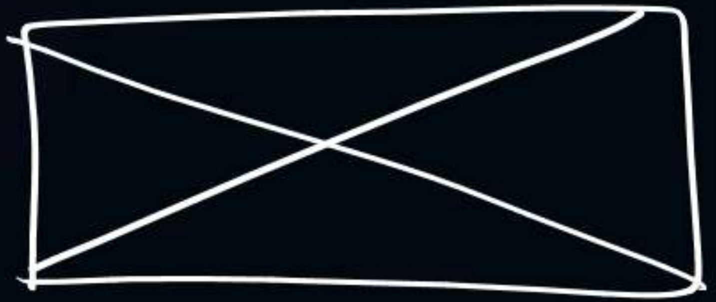
$$\tan\alpha = \frac{a\sin\theta}{b + a\cos\theta}$$

$$R = \sqrt{a^2 + b^2 + 2ab\cos\theta}$$

$$D = \sqrt{a^2 + b^2 - 2ab\cos\theta}$$

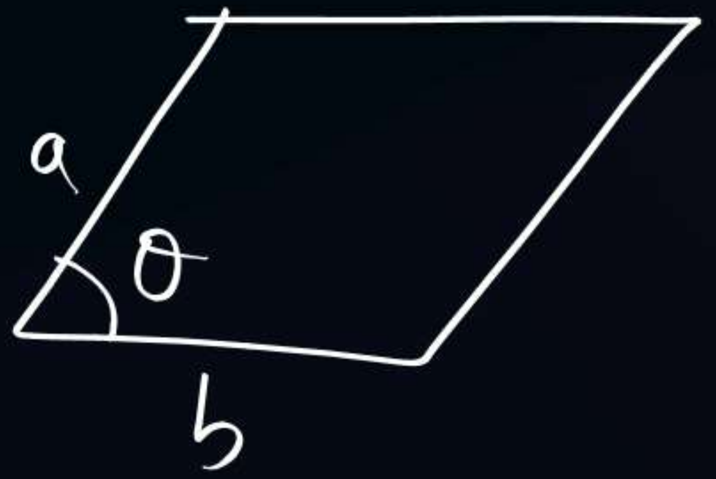


$R = D = \sqrt{2}\,a$



$R = D = \sqrt{a^2 + b^2}$

Note



$R = 2a\cos\left(\frac{\theta}{2}\right)$

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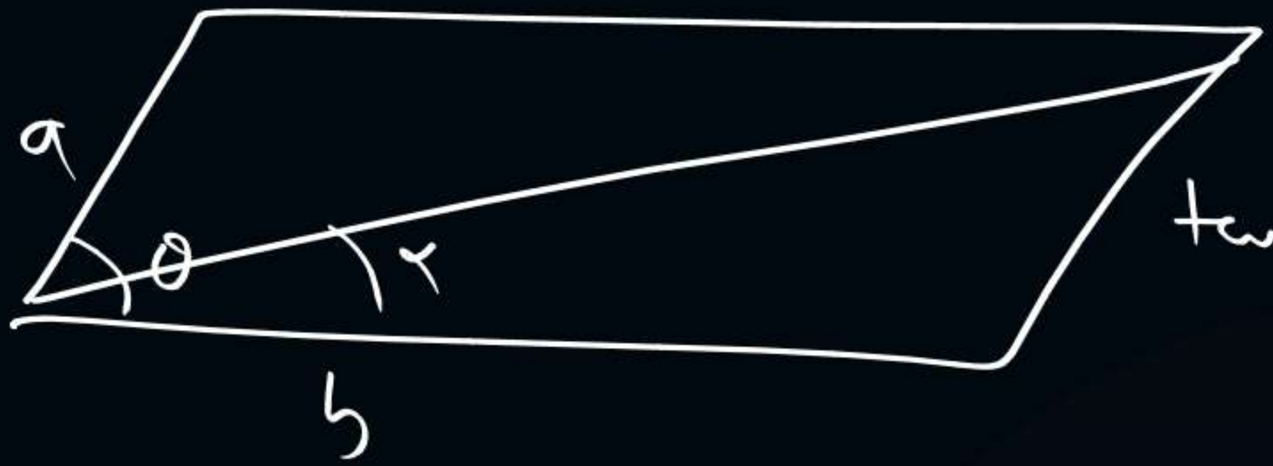

$\tan\alpha = \frac{a\sin\theta}{b + a\cos\theta}$

$R = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

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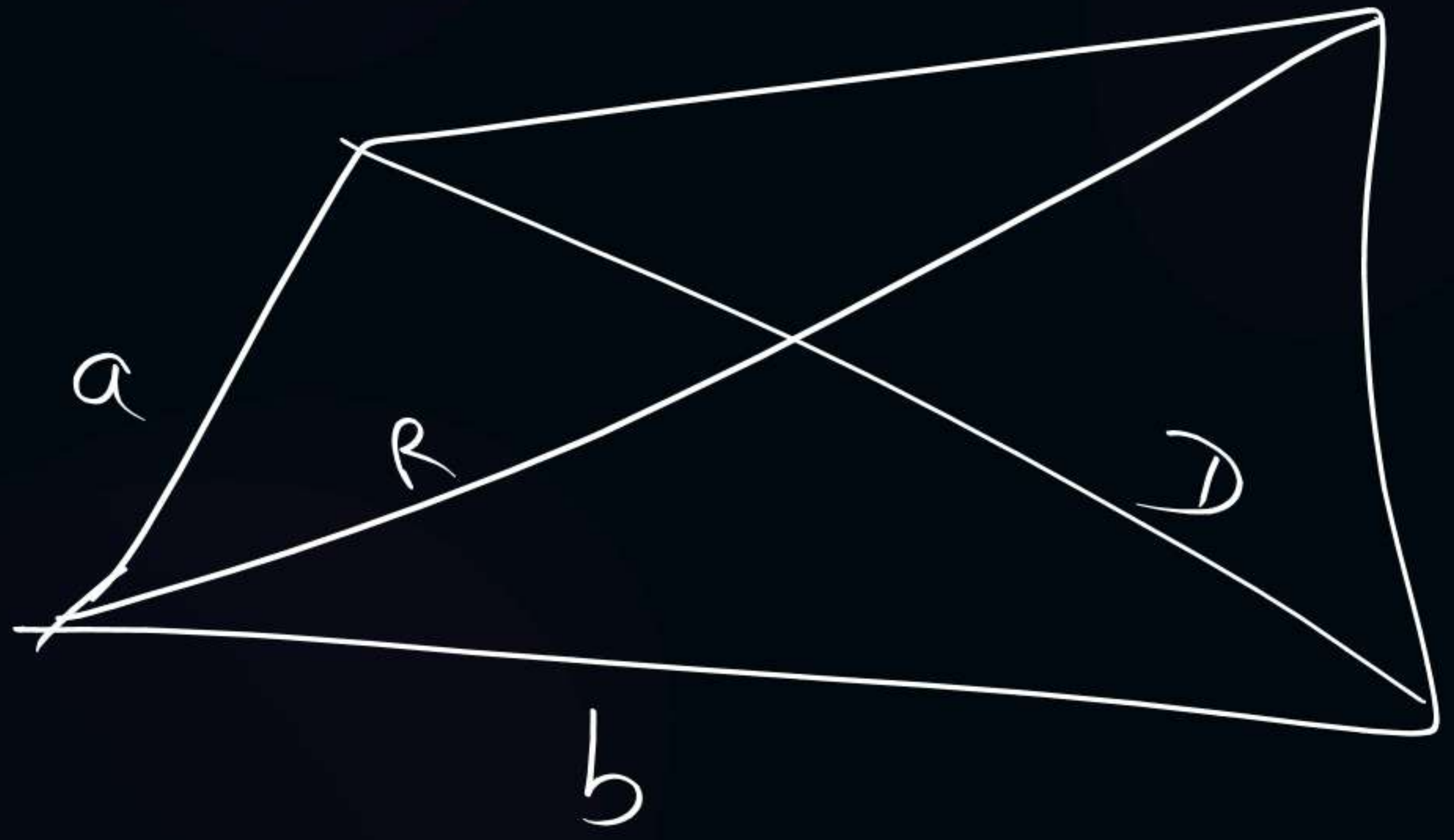
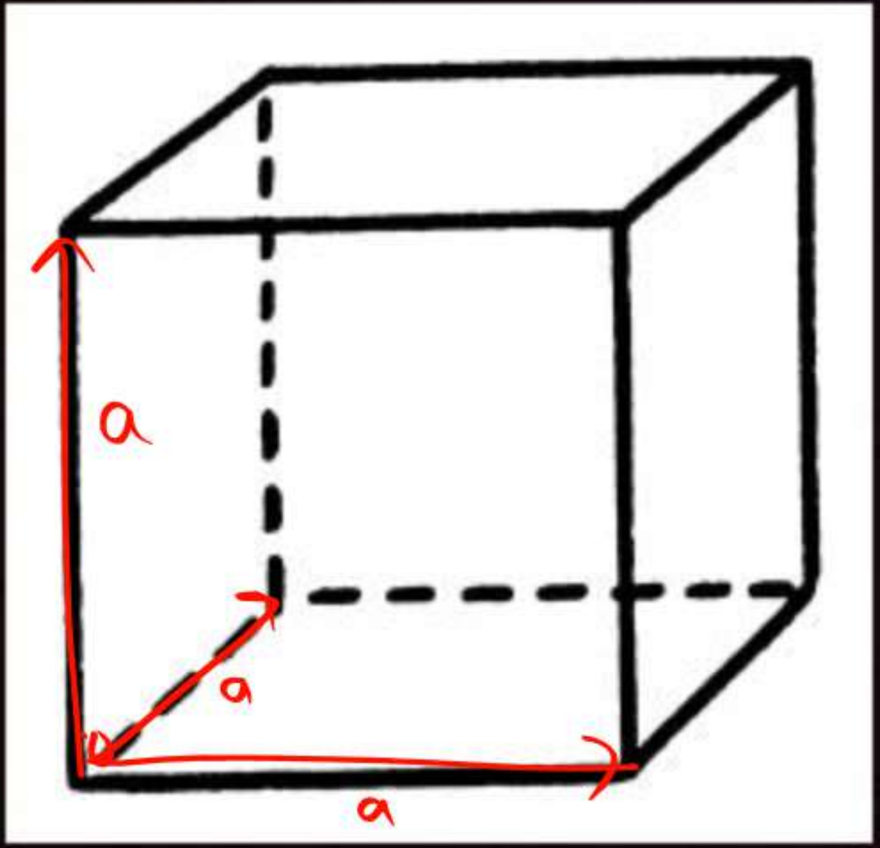
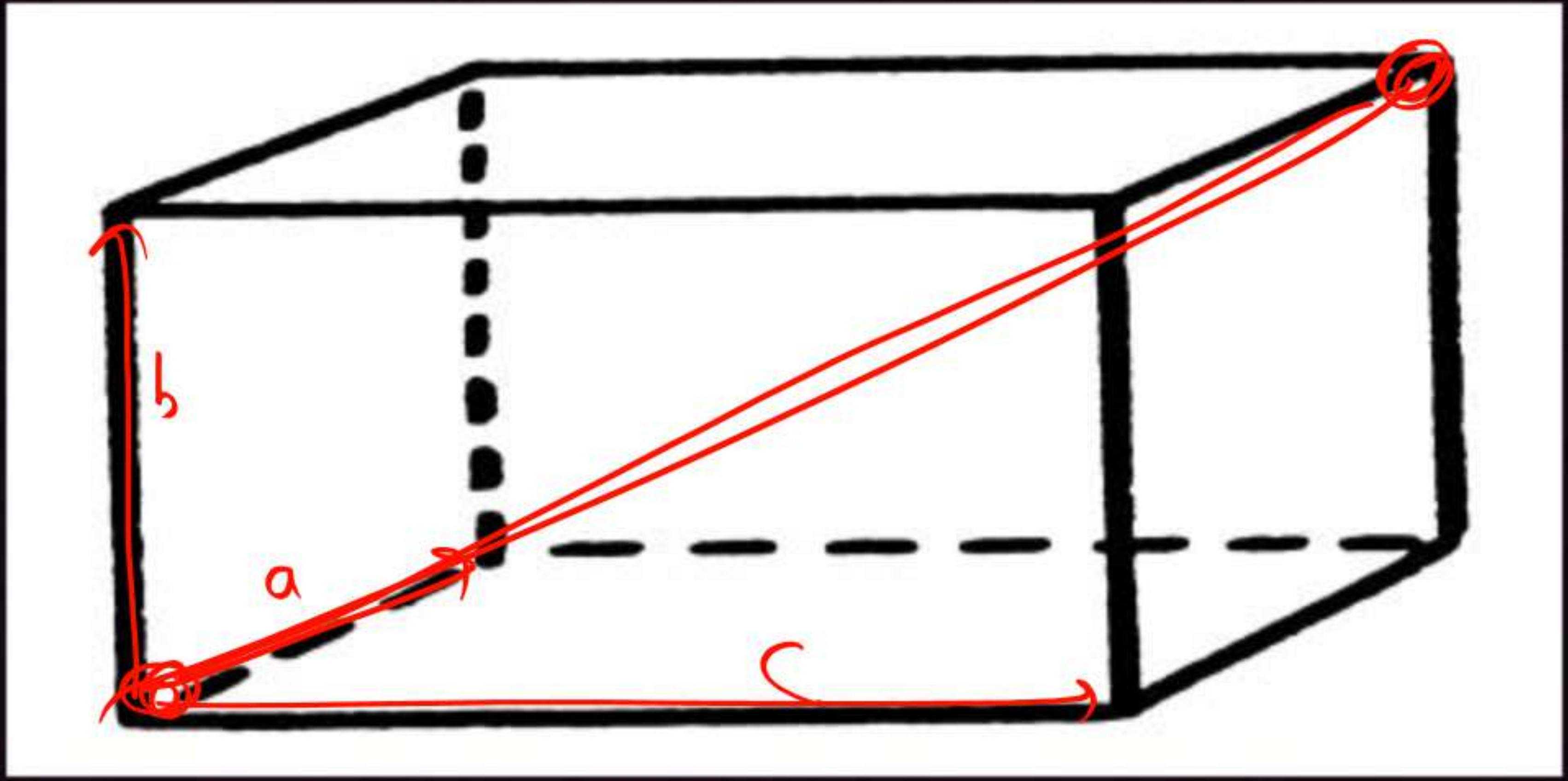

a
R
D
b

Cube & Cuboid





Diagonal $= \sqrt{a^2+a^2+a^2}$

$= \sqrt{3a^2}$

$= a\sqrt{3}$

Diagonal $= \sqrt{a^2+b^2+c^2}$

Vector Horror Story

Ganji Chudail

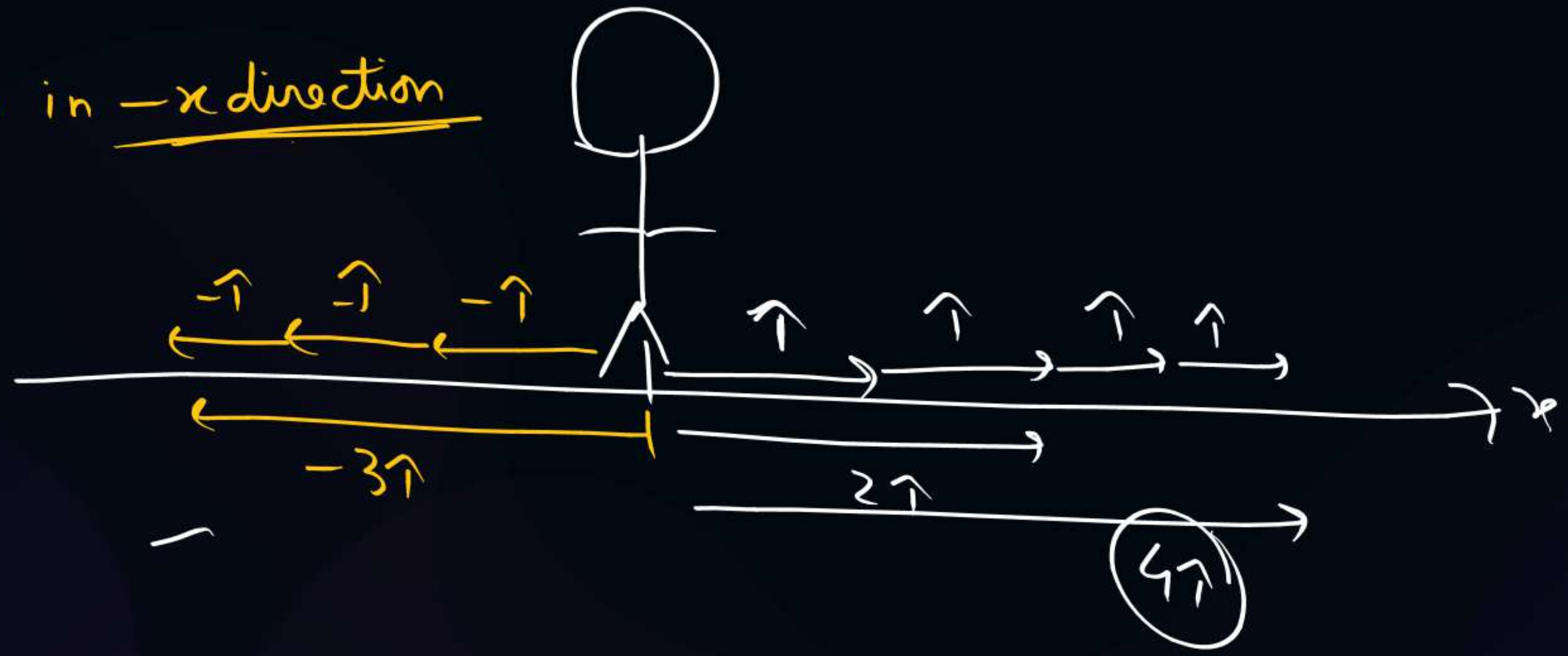
$\hat{i} \rightarrow$ Unit Vector along x direction

Q $5\hat{i} = 5m \longrightarrow$ in x direction

$-\hat{i} \rightarrow$ Unit Vector along $-x$ direction

Q $x = -3\hat{i}$

3m in $-x$ direction



x — $\hat{i}$

y — $\hat{j}$

z — $\hat{k}$

$\vec{A} = -\vec{B}$

Negative of vector matlab ulta vector

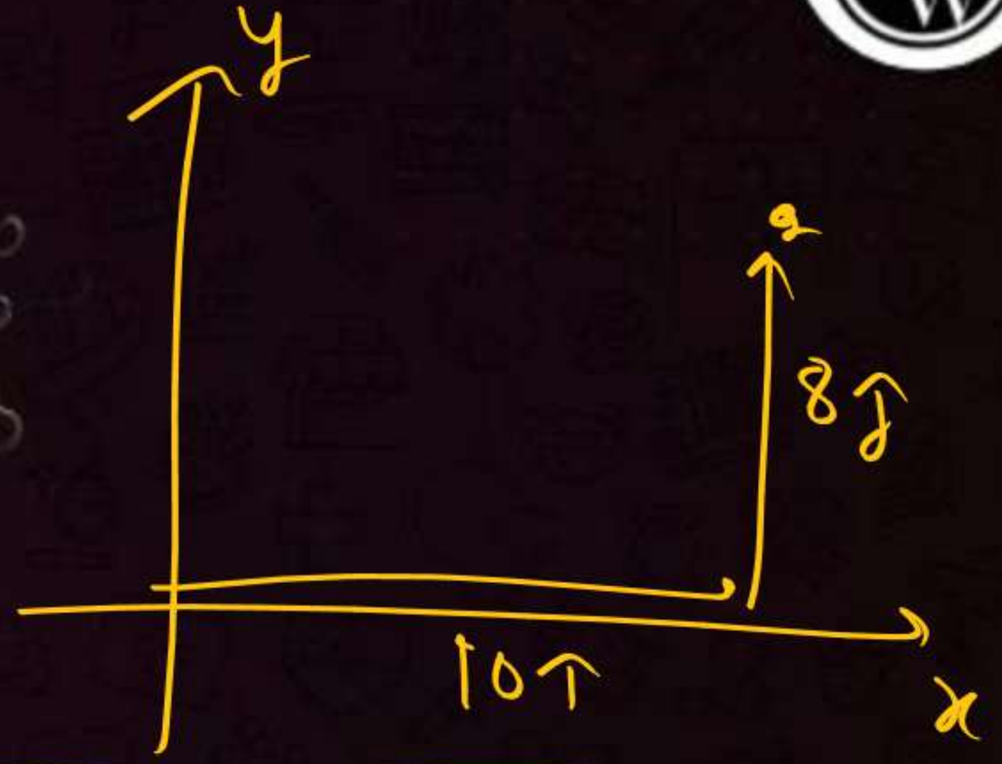
Bablu

$(x = -10\uparrow)$ $\quad$ $(x = 10\uparrow)$

$\vec{B}$ $\quad$ $\vec{A}$

10T

j

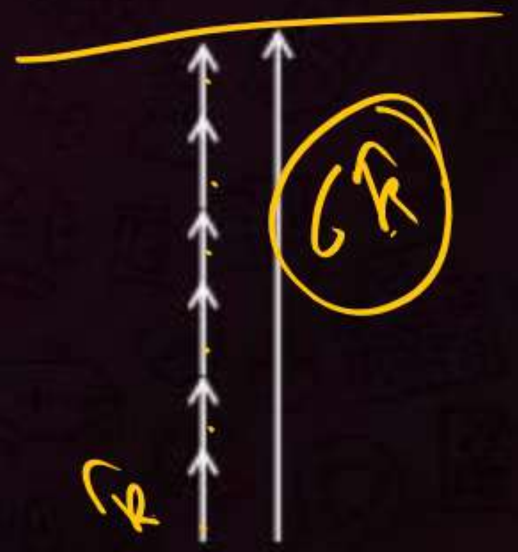


Displacement $= 10\hat{i} + 8\hat{j}$

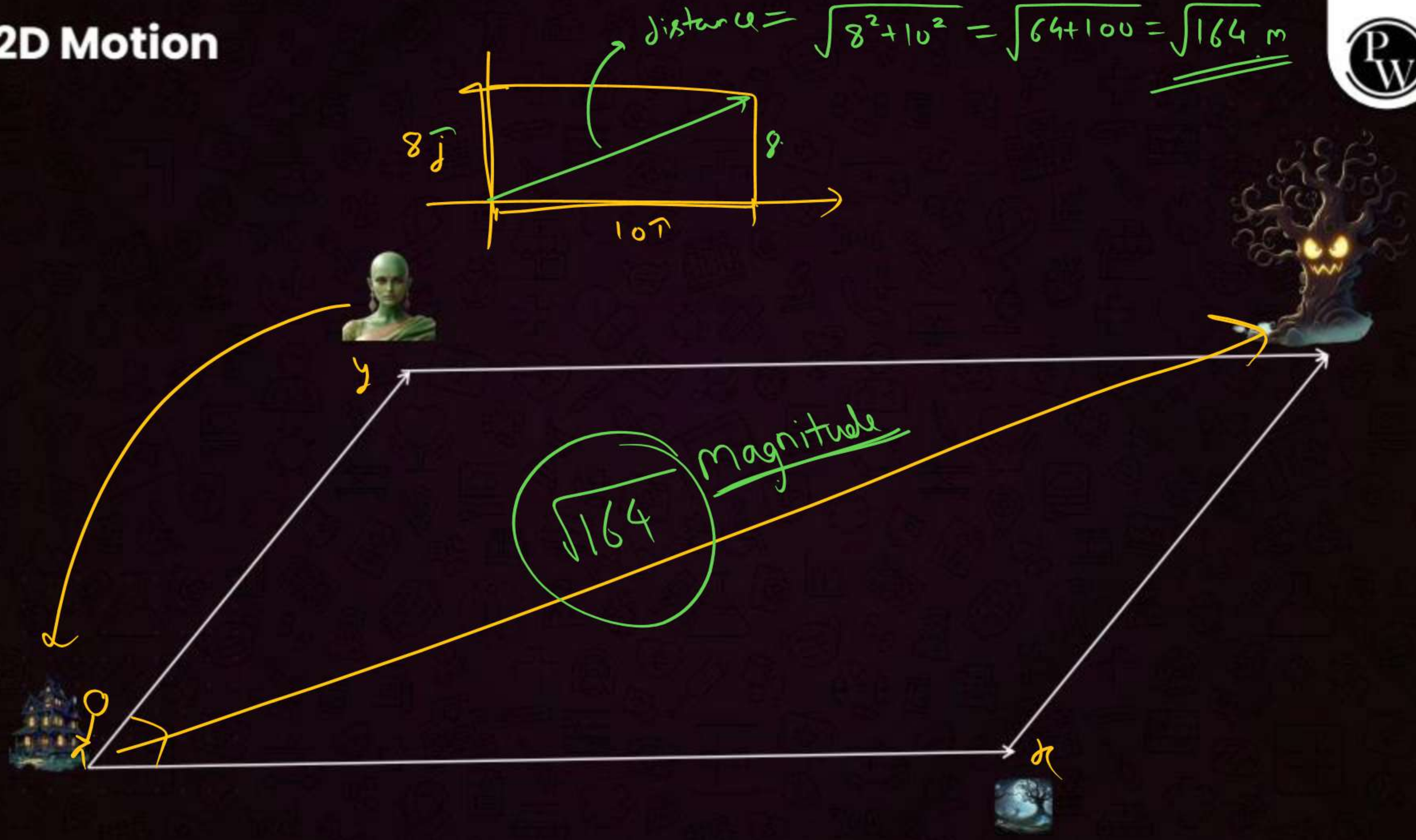
k

$z\ (\hat{k})$

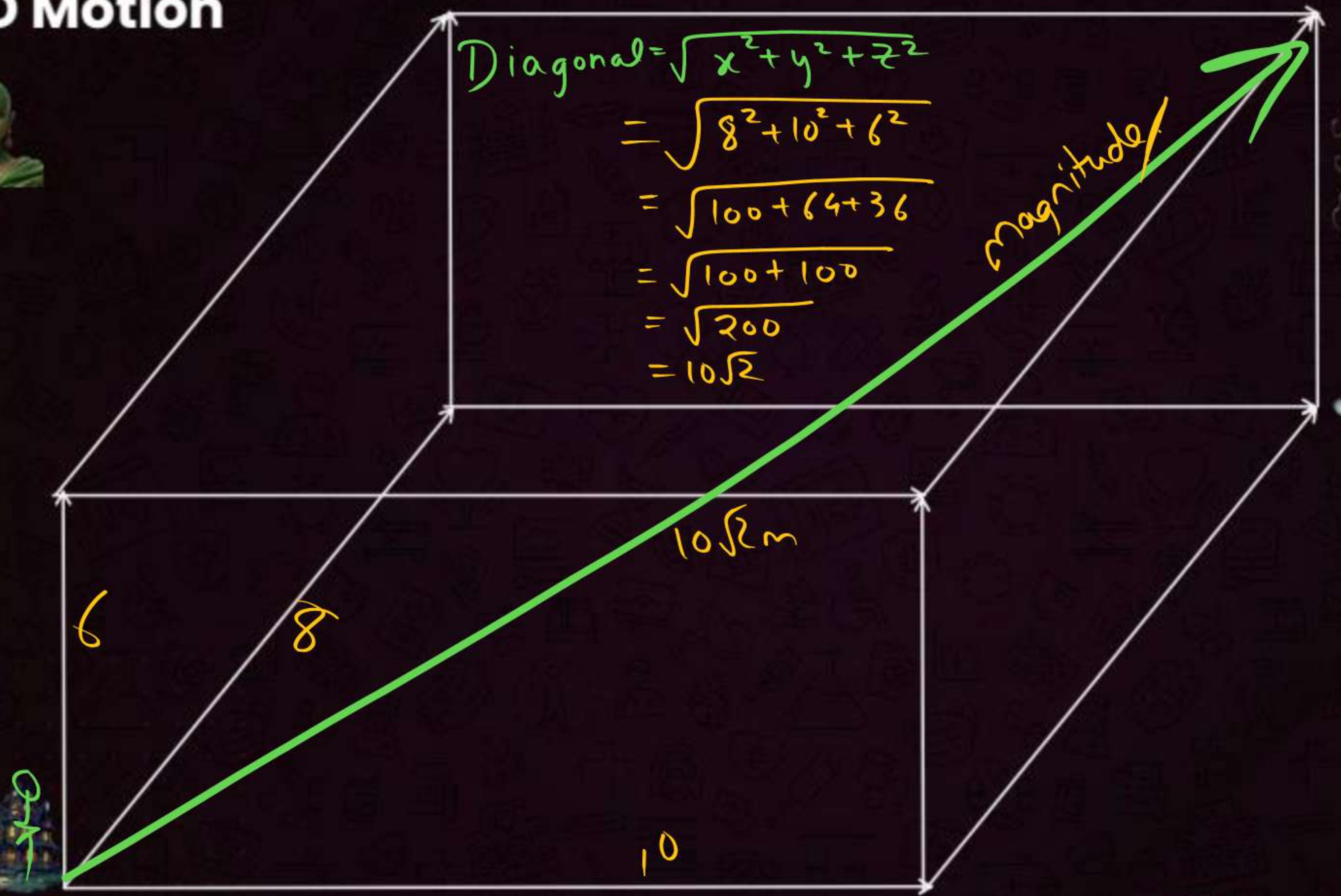
$$\vec{S} = 10\hat{i} + 8\hat{j} + 6\hat{k}$$



2D Motion



3D Motion


Diagonal = $\sqrt{x^2+y^2+z^2}$
$= \sqrt{8^2+10^2+6^2}$
$= \sqrt{100+64+36}$
$= \sqrt{100+100}$
$= \sqrt{200}$
$= 10\sqrt{2}$
magnitude
$10\sqrt{2}$ m
6
8
10

Part 2 – Introduction to Vectors

Physical Quantity → Measure

2.1 – Scalars vs Vectors

Scalars

- Magnitude

❤ Same value for observers with different orientations of the axes.

Example

- Mass
- Distance
- Speed
- Time
- Temperature
- Energy & Potential
- Area
- Volume
- Density
- Electric Charge

Vectors

- Magnitude
- Direction
- Must follow Vector Law of Addition

Example

- Displacement
- Velocity
- Acceleration
- Force
- Momentum
- Electric field
- Magnetic field
- Weight
- ~~Displacement~~
- Torque

2.2 – Special Quantities

- Current
- Moment of Inertia
- Stress
- Strain
- Pressure

→ Scalar / Vector / ~~Tensor~~

- Area

→ Scalar, Kabhi Kabhi vector assume

QUESTION - 01

Difficulty Level : Easy

Identify the vector quantity among the following:

PYQ - (1997)

(1) Distance

(2) Angular momentum

(3) Heat

(4) Energy

QUESTION - 02

Difficulty Level : Medium

Consider the quantities: pressure, power, energy, impulse, gravitational potential, electrical charge, temperature, area. Out of these, the only vector quantities are

(NCERT Exemplar)

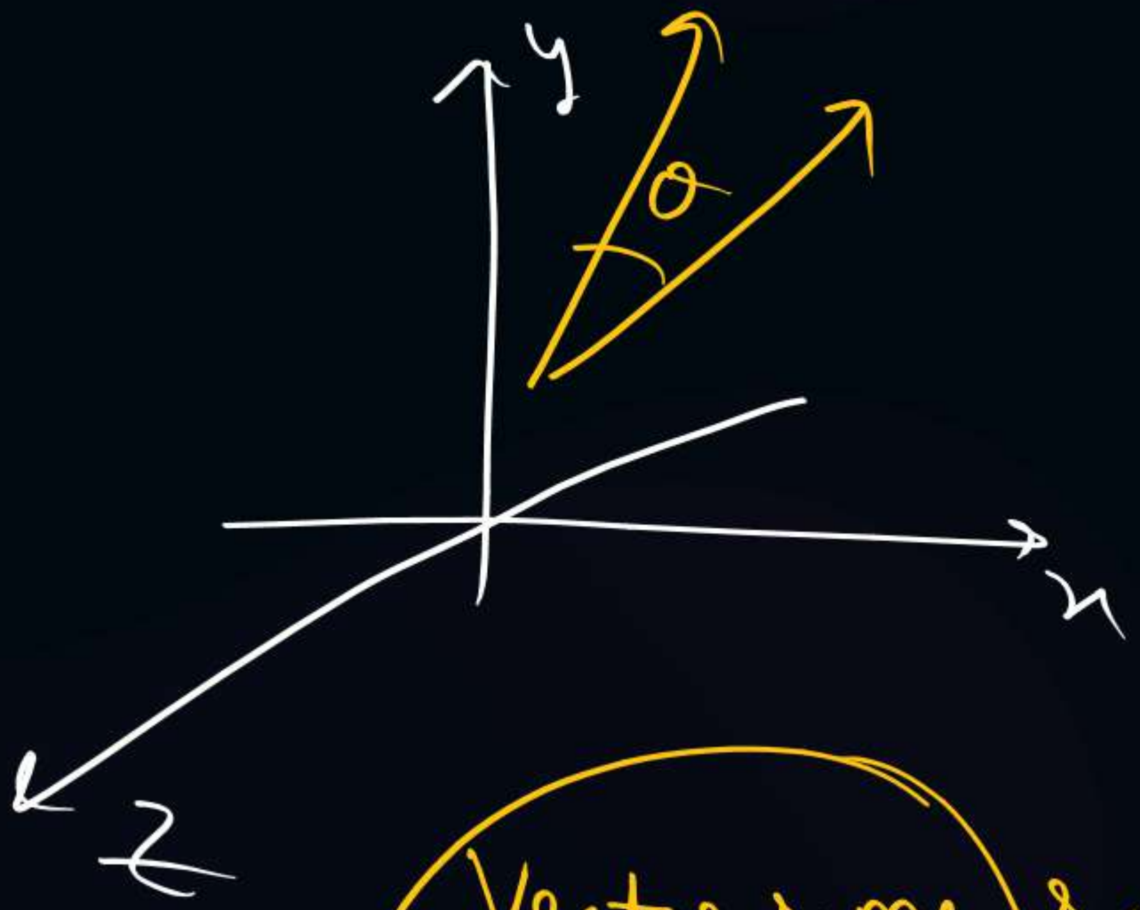
(1) Impulse, pressure and area

(2) Impulse and area

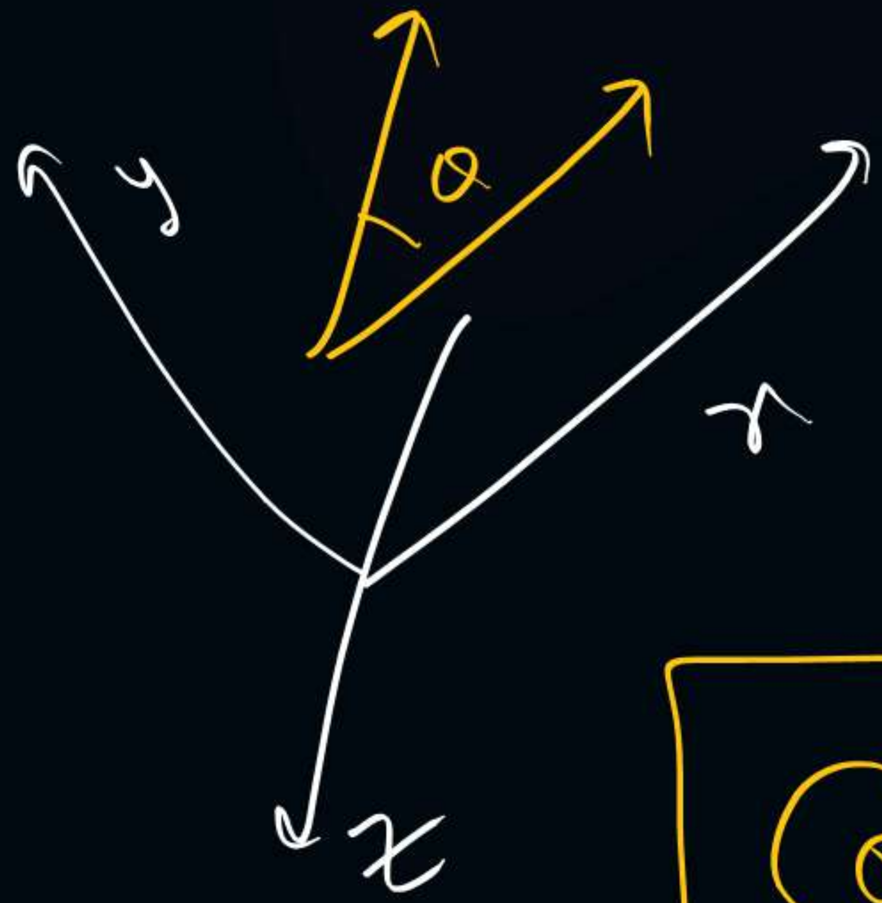
(3) Area and gravitational potential

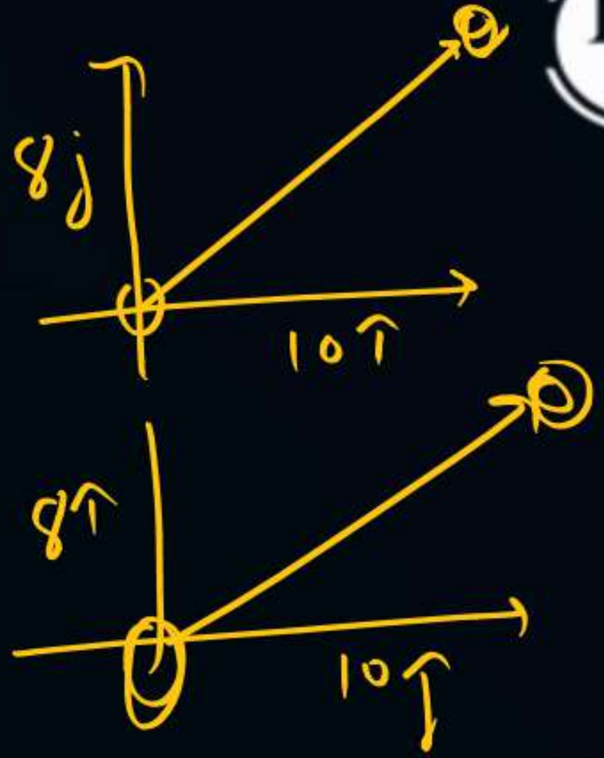
(4) Impulse and pressure

Orientation of axis



Vector same — same
Magnitude,
Addition
Angle





Components change

QUESTION - 03

Difficulty Level : Hard

Which one of the following statements is true?

(NCERT Exemplar)

1. A scalar quantity is the one that is conserved in a process.
2. A scalar quantity is the one that can never take negative values.
3. A scalar quantity is the one that does not vary from one point to another in space.
4. A scalar quantity has the same value for observers with different orientations of the axes.

2.3 – Vector Representation

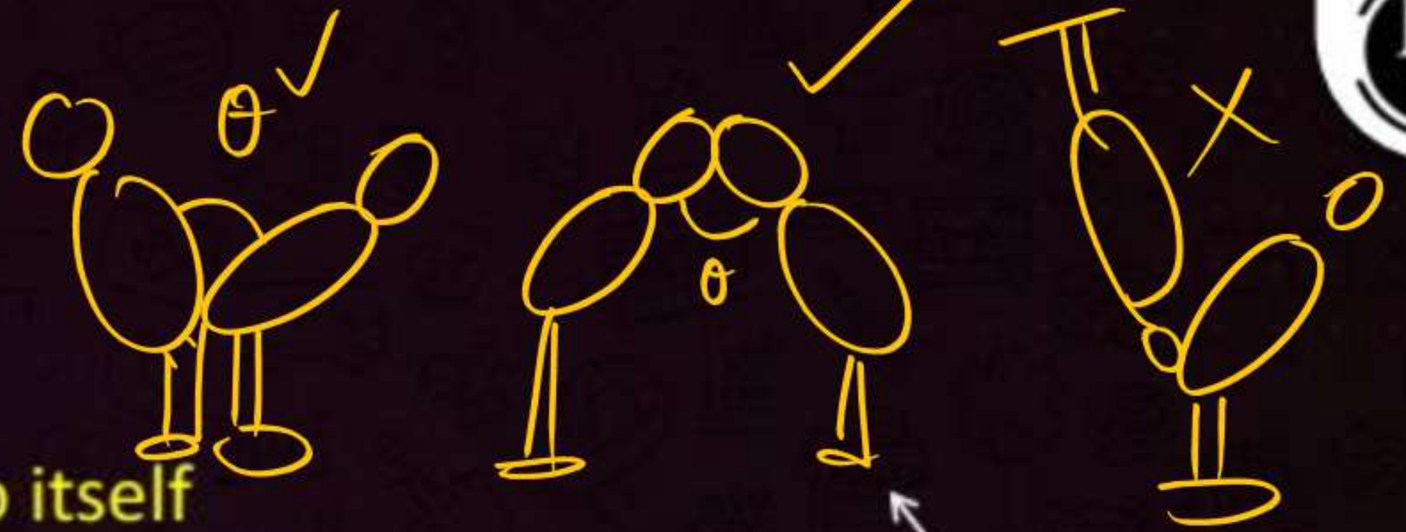
While Typing $\vec{A}$ or **A** → (Bold) → (NCERT)

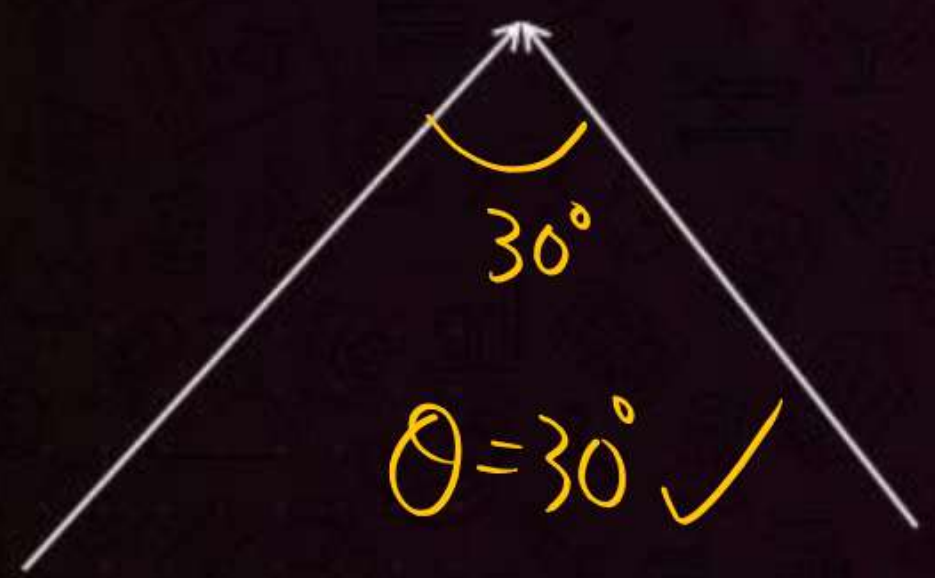
While Drawing → head (direction)
↳ Tail (Magnitude)

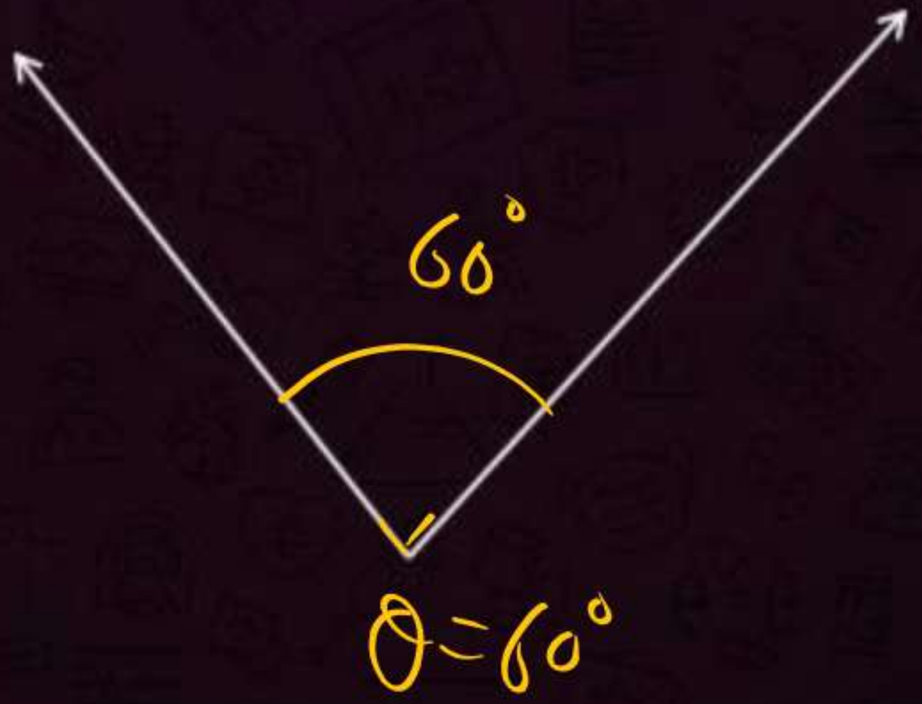
Head of Vector represents direction while tail represents magnitude

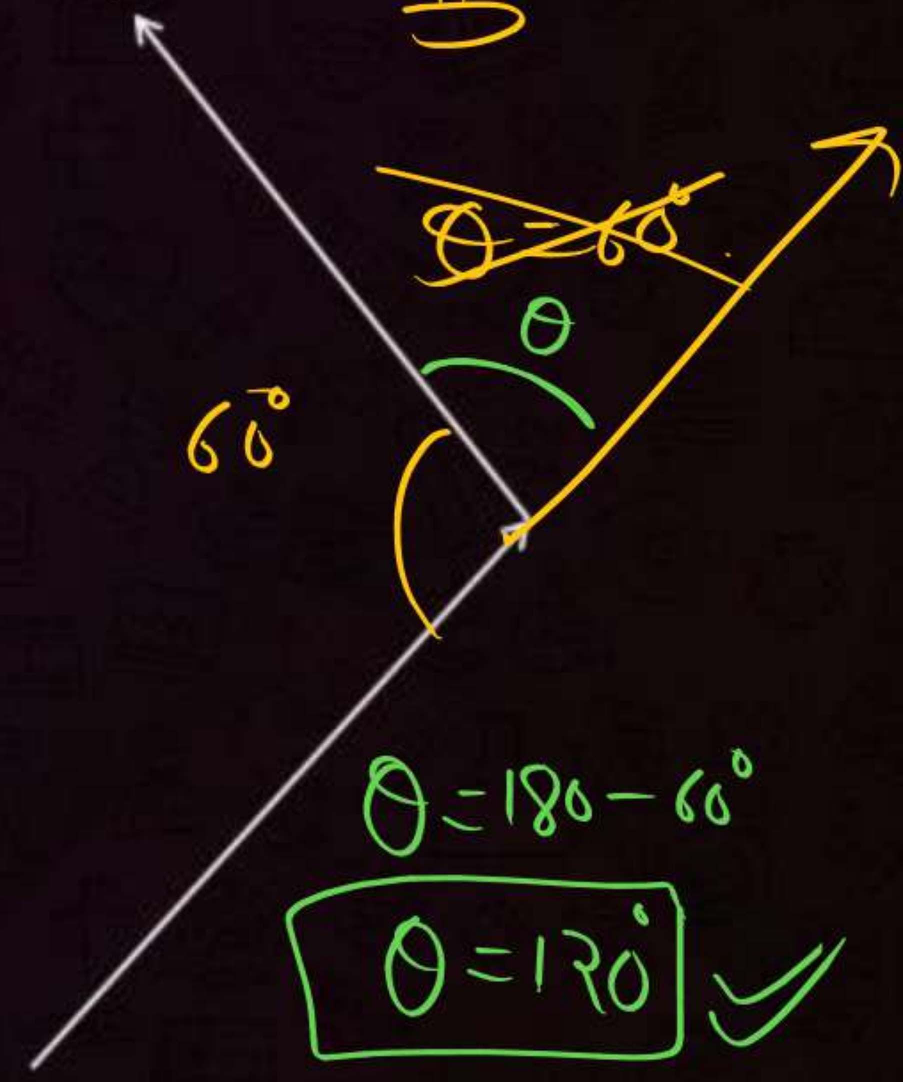
2.4 – Angle between 2 Vectors

- Join head to head ✓ θ ✓
- Join head to tail ✗ (180 − θ)
- Join tail to tail ✓ θ ✓
- A vector is not changed if it is slid parallel to itself



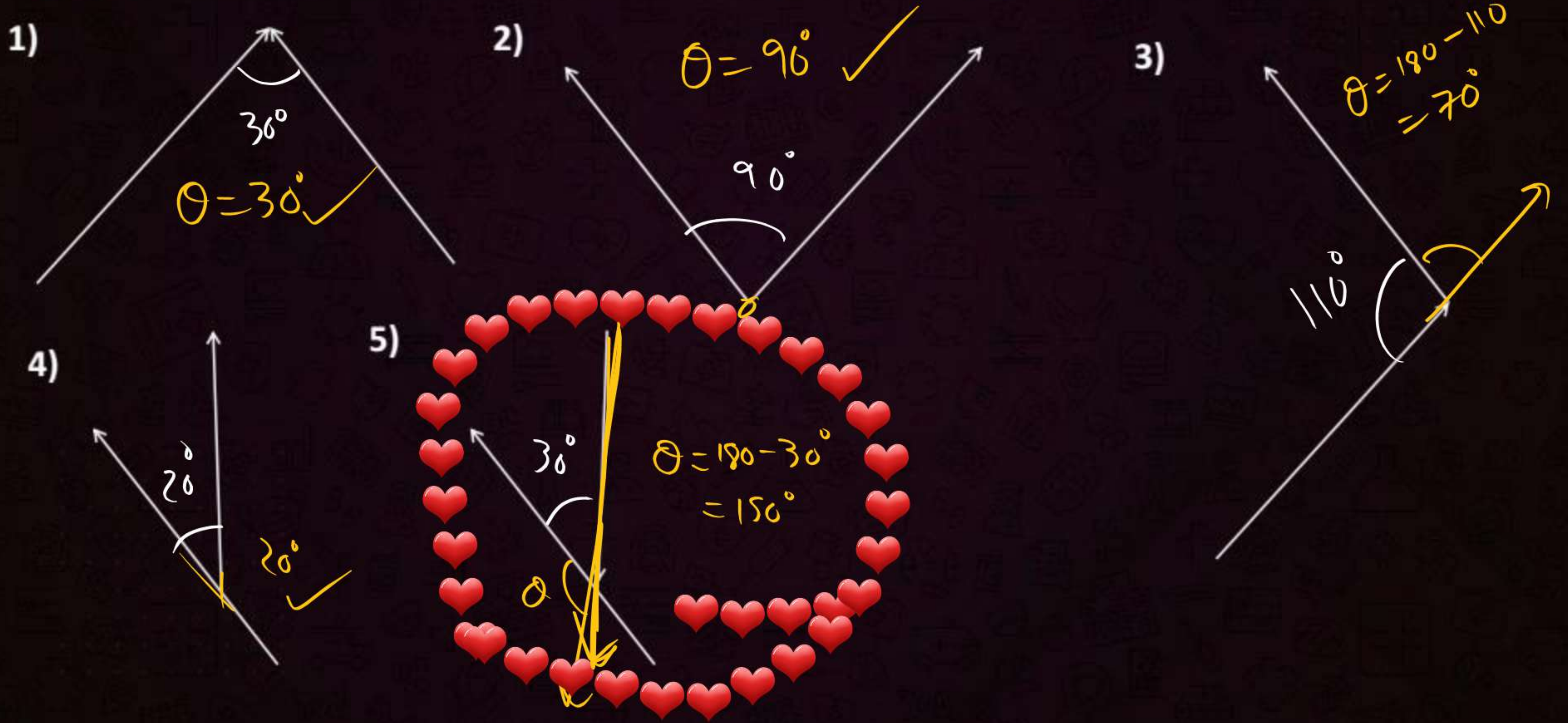




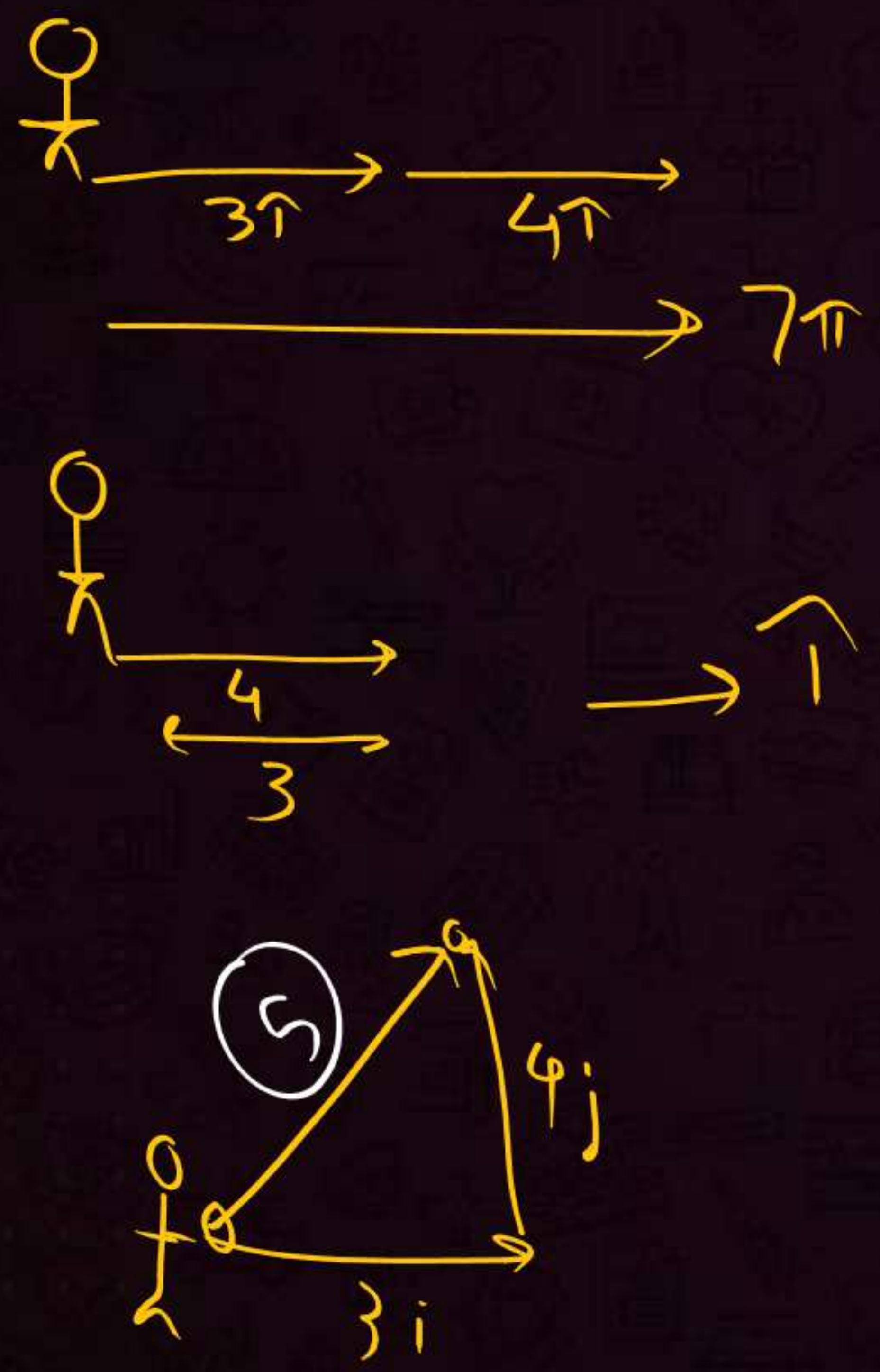


Puppy 1 – Angle between 2 Vectors

Difficulty Level : Easy



2.5 – Range of Vector Addition

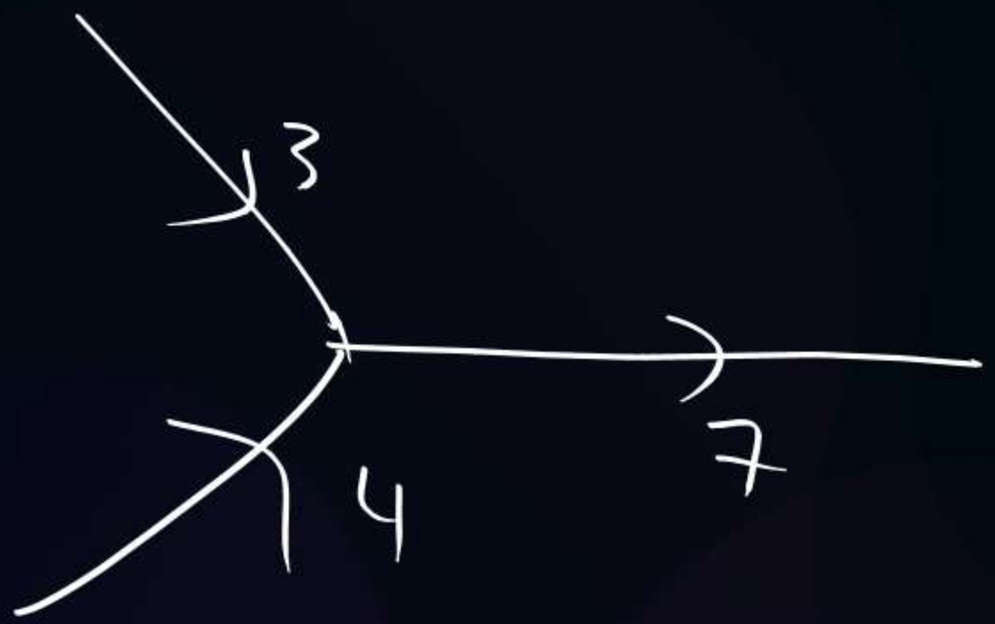


$$\vec{3} + \vec{4} = 7, 5, 1$$

Depend on direction

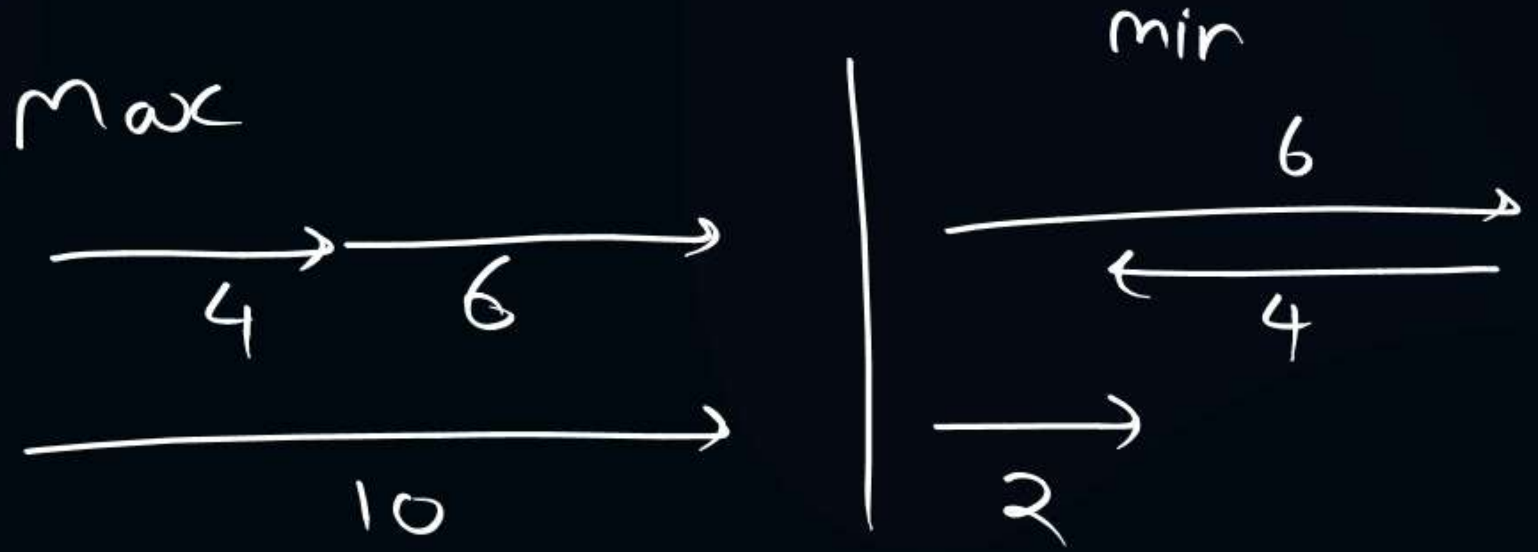
3 Mango + 4 Manjo = 7 Manjo

Current ⟶ Scalar



Range

$\vec{4} + \vec{6} =$



$\vec{a} + \vec{b} \rightarrow$ Max $(a+b)$ ✓ Angle is 0°	Min $(a-b)$ Angle 180°

Puppy 2 – Range of Vector Addition

Difficulty Level : Easy

Find maximum and minimum values obtained by adding following vector quantities whose magnitudes are given

1) 3 , 3.
(0 to 6)

2) 4 , 5.
(1 to 9)

3) 5 , 10.
(5 to 15)

4) 8 , 10
(2 to 18)

5) a , b.
a − b to a + b

6) a , 3a.
(2a to 4a)

7) √2 , 1.
$\sqrt{2}-1$ to $\sqrt{2}+1$

8) 11 , √4
$11-\sqrt{4}$, $11+\sqrt{4}$
11 − 2 , 11 + 2
9, 13

QUESTION - 04

Difficulty Level : Easy

Out of the following the resultant of which cannot be 4 N?

(NCERT Based)

1. 2 N and 2N
2. 2 N and 4 N
3. 2 N and 6N
4. 2 N and 8 N

Range of Vector Addition - Theory

Magnitude of **A + B** can be less than, more than or even equal to magnitude of A and B

Maximum value of **|A + B|** is equal to |A| + |B| , when $\theta = 0^0$

Maximum value of magnitude of **A + B** is equal to Magnitude of **A** Magnitude of **B**

Minimum value of **|A + B|** is equal to $-(|A| + |B|)$, when $\theta = 180^0$ or π

|A + B| = |A| , Only when B = 0

$$\vec{x} = -10000$$

$$|\vec{x}| = 10000$$

QUESTION - 05

Difficulty Level : ~~Easy~~ Yodha

It is found that $|\vec{A}+\vec{B}| = |\vec{A}|$. This necessarily implies

end

(NCERT Exemplar)

1. $\vec{B} = 0$
2. $\vec{A}, \vec{B}$ are antiparallel
3. $\vec{A}, \vec{B}$ are perpendicular
4. $\vec{A} . \vec{B} \leq 0$

QUESTION - 06

Difficulty Level : Medium

Let vector $\vec{C} = \vec{A} + \vec{B}$.

(HCV OBJECTIVE II)

1. $|\vec{C}|$ is always greater than $|\vec{A}|$
2. It is possible to have $|\vec{C}| < |\vec{A}|$ and $|\vec{C}| < |\vec{B}|$
3. C is always equal to A + B
4. C is never equal to A + B

$\vec{3} + \vec{4} = (1, 2, 3, 4, 5, 6, 7)$

must, only, always, never → hamesha
ek bhi exception nahi

may be, it is possible → 1 bhi case me horha hai
toh chalega (Shayad)

QUESTION - 07

Difficulty Level : Hard

Which of the sets given below may represent the magnitudes of three vectors adding to zero?

(HCV OBJECTIVE I)

1. 2, 4, 8
2. 4, 8, 16
3. 1, 2, 1
4. 0.5, 1, 2

$\vec{a}+\vec{b}+\vec{c}=0$ $(a, b < c)$ Only when $a+b \geq c$

2.6 – Special Types of Vectors

Equal Vectors

- Same Magnitude and Direction

$$2\hat{i} + 4\hat{j} = a\hat{i} + b\hat{j}$$

$a = 2$ $b = 4$

$\frac{a}{b} = ??$ $\frac{2}{4} = \frac{1}{2}$

Negative Vectors / Opposite

- Same Magnitude and opposite Direction

$\vec{A}$ & $-\vec{A}$ are -ve vectors

Q $\vec{A} = \hat{i} + 5\hat{j}$

$\vec{B} = a\hat{i} + b\hat{j}$

$\vec{A}$ is opposite of $\vec{B}$

$a = -1$

$b = -5$

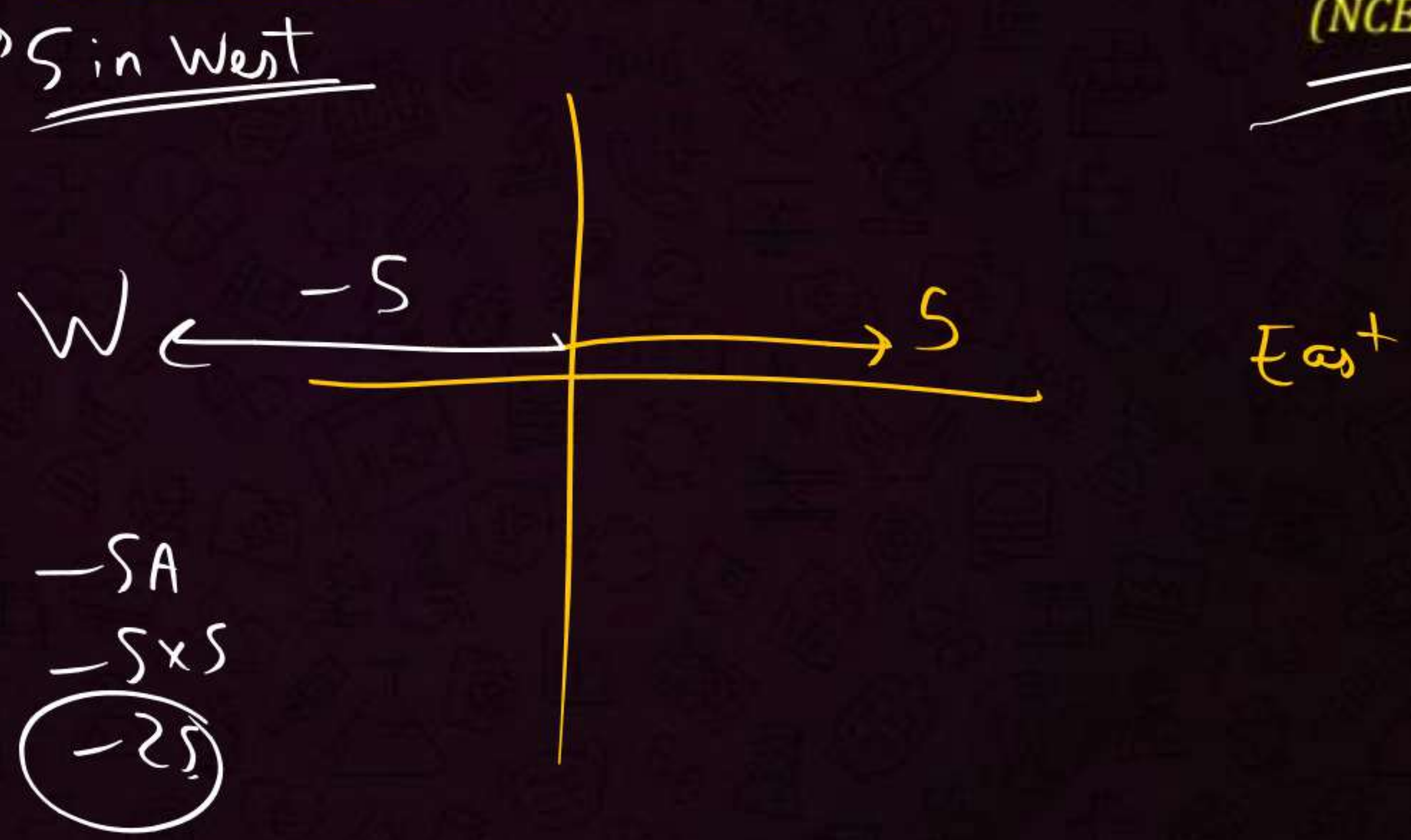
$\frac{a}{b} = ?? = \frac{-1}{-5} = \frac{1}{5}$

Difficulty Level : Easy

QUESTION - 08

If $\vec{A}$ is a vector of magnitude 5 units due east. What is the magnitude and direction of a vector $-5\,\vec{A}$? *(NCERT Based)*

1. 5 units due east
2. 25 units due west
3. 5 units due west
4. 25 units due east



Parallel Vectors

- Same Direction but Magnitude may/may not be same

(All equal vectors are ∥)

(All ∥ are equal) → False

Anti Parallel Vectors

- Opposite Direction but Magnitude may/may not be same

(All opposite vectors are anti∥)

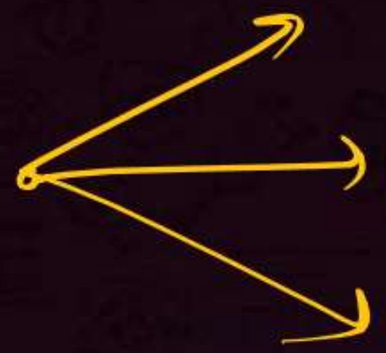
(All anti∥ are opposite) →

Orthogonal/Perpendicular Vectors

- Perpendicular Direction

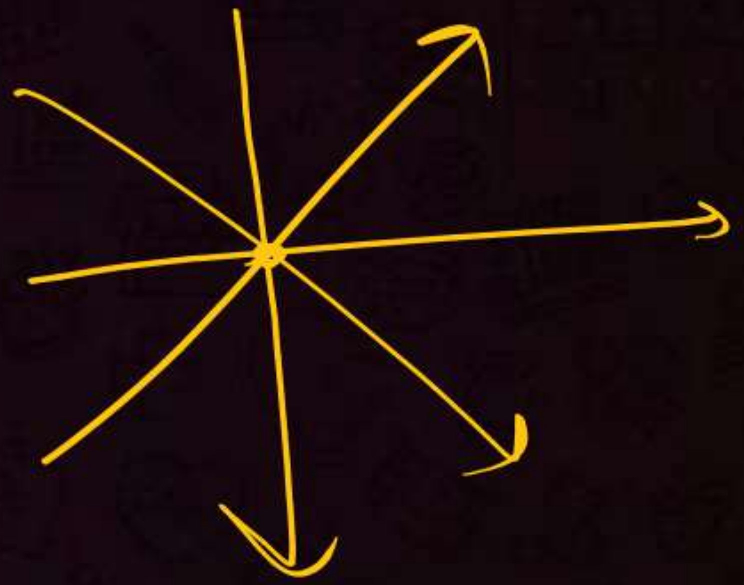
Co initial Vectors

- Starts from same position



Con current Vectors

- May not start from same position but at all cross at same point



Colinear Vectors

- Lie in same line

Coplanar Vectors

- Lie in same plane
- 2 vectors are always co planar
- 3 Vectors are coplanar only when there volume enclosed is zero

Unit Vector

- Magnitude of one

$\hat{i}, \hat{j}, \hat{k}$

These are used to give directions.

Zero Vector / Null Vector

- **Zero Magnitude & in any direction** (arbitary)

$\vec{0}$ → Any direction

QUESTION - 09

Difficulty Level : Medium

Which of the following is not a property of a null vector?

(NCERT Based)

1. $\vec{A} + \vec{0} = \vec{A}$
2. $\lambda \vec{0} = \vec{0}$, where λ is a scalar
3. $0 \vec{A} = \vec{A}$
4. $\vec{A} - \vec{A} = \vec{0}$

QUESTION - 10

Difficulty Level : Easy

If $|\vec{A} + \vec{B}| = |\vec{A}| + |\vec{B}|$ then angle between A and B will be:

PYQ - (2001)

(1) 90°

(2) 120°

(3) 0°

(4) 60°

2.6 – Some Rules for Vectors (Theory)

- **A vector is not changed if it is slid parallel to itself**

A vector is changed if it is rotated

Only 2 vectors can be added or subtracted to give a vector

Vector can be multiplied or divided by scalar

Vector can be multiplied by another vector but cannot be divided

- **2 vectors are always co planar**
- **3 Vectors are coplanar only when there volume enclosed is zero**

QUESTION - 11

Difficulty Level : Easy

nw

A vector is not changed if

(NCERT Based)

1. it is displaced parallel to itself.
2. it is rotated through an arbitrary angle.
3. it is cross-multiplied by a unit vector.
4. it is multiplied by an arbitrary scalar

QUESTION - 12

Difficulty Level : Easy

HW

A vector is not changed if

(HCV OBJECTIVE I)

1. it is rotated through an arbitrary angle
2. it is multiplied by an arbitrary scalar
3. it is cross multiplied by a unit vector
4. it is slid parallel to itself ✓

PUPPY POINTS-1

Scalar
- → Magnitude
- → Independent of axis ✗

Vector
- → Magnitude ✓
- → Direction ✓
- → Law of +n ✓

Current
Moment of Inertia
Stress
strain } Scalar

Area } → Scalar/Vector

Angle btw

θ θ π-θ π-θ

$\vec{a}+\vec{b} = (a-b)$ to $(a+b)$
180° 0°

Tail → Head (Direction)
(Magnitude)

θ is independent of axis

---

* Vector can be moved ∥ ✓
Vector cannot be rotated ✗

~~$\vec{A}+\vec{B}=\vec{A} \Rightarrow \vec{B}=0$~~

All equal are ∥
All negative are anti∥
Not vice versa

Equal → same M, D
Negative → same M, opp D
∥ → same D
anti ∥ → opp D
orthogonal → ⊥
co initial
concurrent
colinear → →
coplanar (same plane)
-2 vectors are always coplanar
-Sum of 2 unequal coplanar cannot be 0

Vector, +, θ, Scalar,

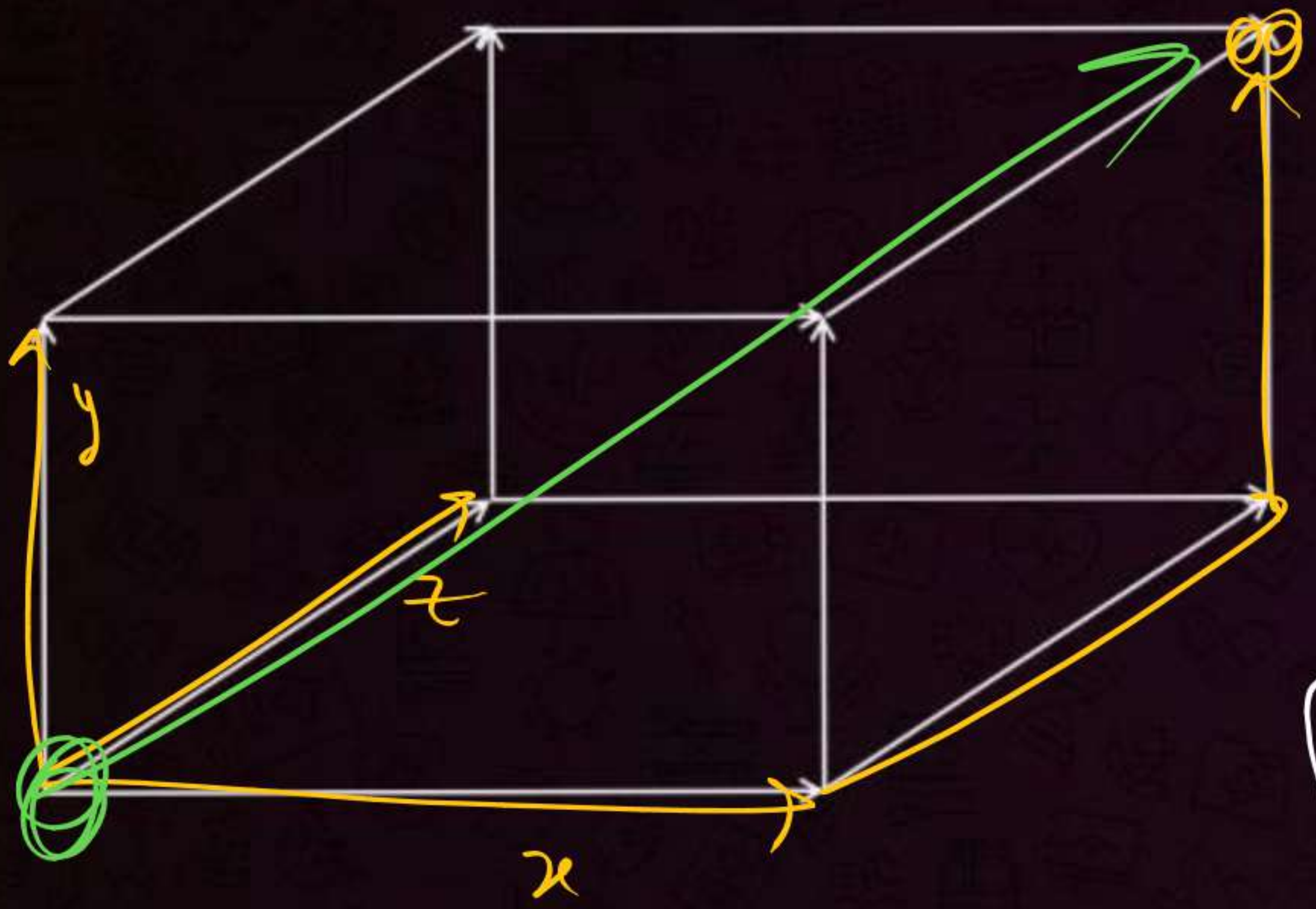
Change - components

Part 3 – Magnitude and Unit Vector

$$3^2 + 4^2 = 5^2$$
$$6^2 + 8^2 = 10^2$$
$$5^2 + 12^2 = 13^2$$
$$7^2 + 24^2 = 25^2$$

$$\sqrt{10^2 + 10^2} = 10\sqrt{2}$$
$$\sqrt{x^2 + x^2} = x\sqrt{2}$$
$$\sqrt{9^2 + 9^2} = 9\sqrt{2}$$
$$\sqrt{5^2 + 5^2} = 5\sqrt{2}$$
$$\sqrt{x^2 + x^2 + x^2} = x\sqrt{3}$$
$$\sqrt{5^2 + 5^2 + 5^2} = 5\sqrt{3}$$
$$\sqrt{10^2 + 10^2 + 10^2} =$$

3.1 - Magnitude of Vectors



$$\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$|\vec{A}| = \sqrt{x^2 + y^2 + z^2}$$

$| \; |$ → mode - magnitude

Magnitude of Vectors – Symbol and Formula

Magnitude of $\vec{A} = |\vec{A}|$ or A

Puppy 3 – Magnitude of Vectors

Difficulty Level : Medium

Find Magnitude of the following vectors

1) $|15i| = 15$

2) $|110j| = 110$

3) $|120k| = 120$

4) $|i + j| = \sqrt{1^2+1^2} = \sqrt{2}$

5) $|10j + 10k| = 10\sqrt{2}$

6) $|2k + 2i| = 2\sqrt{2}$

7) $|3i + 4j| = \sqrt{3^2+4^2} = \sqrt{5^2} = 5$ ✓

8) $|5j + 12k| = 13$

9) $|7k + 24i| = 25$

10) $|5i + 5j + 5k| = \sqrt{5^2+5^2+5^2} = 5\sqrt{3}$

11) $|i - j + k| = \sqrt{3}$ ✓

12) $|-12k - 12i + 12j| = 12\sqrt{3}$

13) $|5i + 3j + 4k| = \sqrt{3^2+4^2+5^2}$

$\sqrt{5^2+5^2} = 5\sqrt{2}$ ✓

14) $|-3j + 4k - 12i| = \sqrt{3^2+4^2+12^2}$

$\sqrt{5^2+12^2} = \sqrt{13^2} = 13$ ✓

$$|3\hat{i}+4\hat{j}+5\hat{k}| = \sqrt{3^2+4^2+5^2}$$

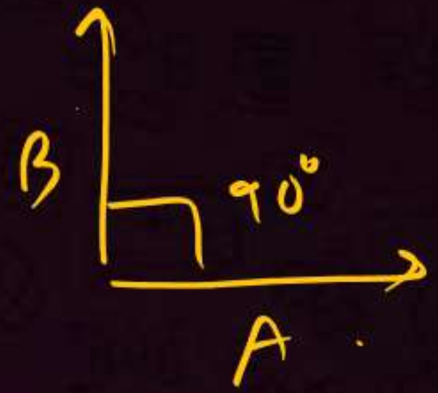
$$= \sqrt{5^2+5^2} = 5\sqrt{2}$$

$$|3\hat{i}+4\hat{j}+12\hat{k}| = \sqrt{3^2+4^2+12^2}$$

$$\sqrt{5^2+12^2} = 13$$

Magnitude of Vectors – Theory

Value of $|A + B| = (A^2 + B^2)^{1/2}$, when $\theta = 90^0$ or $\pi/2$



$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2}$

~~$|A + B| = |A|$, Only when $B = 0$~~

Magnitude is always taken as positive for both positive and negative vectors

Magnitude will not change even when you change the reference axis or origin. It is always constant

QUESTION - 13

Difficulty Level : Hard

A situation may be described by using different sets of coordinate axes having different orientations. Which of the following do not depend on the orientation of the axes?

(HCV OBJECTIVE II)

1. the value of a scalar
2. component of a vector
3. a vector
4. the magnitude of a vector

QUESTION - 14

Difficulty Level : Medium

It is found that $|\vec{A} + \vec{B}| = |\vec{A}|$. This necessarily implies

(NCERT Exemplar)

(1) $\vec{B} = 0$

(2) $\vec{A}, \vec{B}$ are antiparallel

(3) $\vec{A}, \vec{B}$ are perpendicular

(4) $\vec{A} . \vec{B} \leq 0$

QUESTION - 15

Difficulty Level : Medium

Three forces of magnitudes 6 N, 6 N and $\sqrt{72}$ N act at corner of cube along three sides as shown in figure. Resultant of these forces is

(NCERT Based)

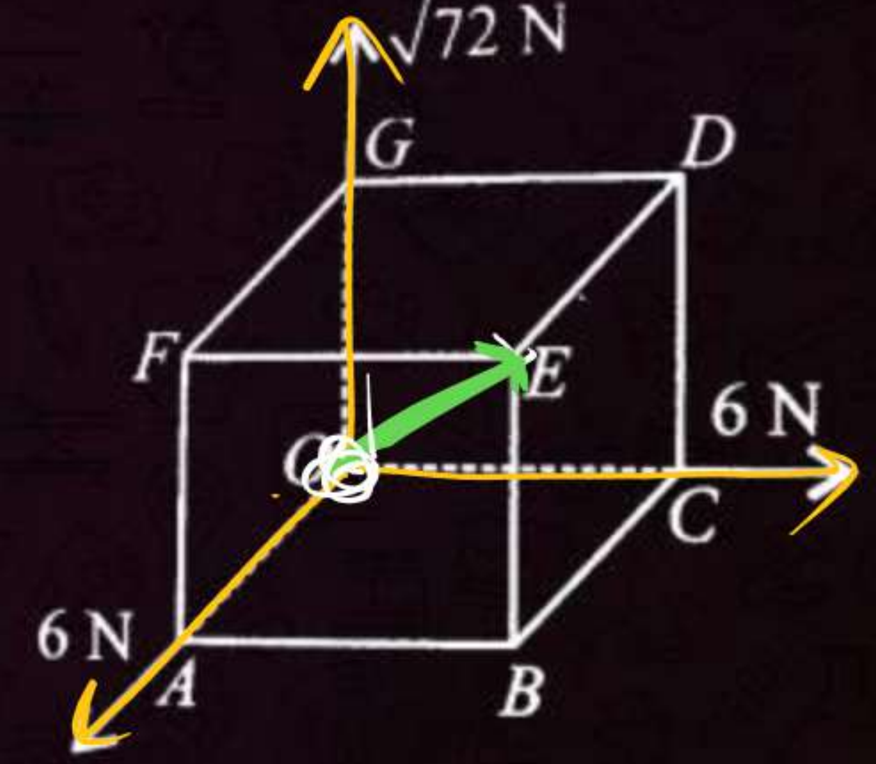
(1) 12 N angle OB

(2) 18 N along OA

(3) 18 N along OC

(4) 12 N along OE

$$\sqrt{6^2+6^2+72}$$
$$\sqrt{36+36+72}$$
$$\sqrt{144}$$
$$=12\text{ N}$$



3.2 – Unit Vector

x	$\hat{i}$
y	$\hat{j}$
z	$\hat{k}$
$-x$	$-\hat{i}$
$-y$	$-\hat{j}$
$-z$	$-\hat{k}$

→ Magnitude = 1

→ Use to give direction

→ represented by ($\hat{a}$) → cap

Vector = (Magnitude)(Direction)

$$\text{Direction} = \frac{\text{Vector}}{\text{Magnitude}}$$

$$\frac{(5\hat{i})}{5} = \hat{i}$$

$$\hat{A} = \frac{\vec{A}}{|A|}$$

Puppy 4 – Unit Vectors

Difficulty Level : Medium

$\hat{a} = \frac{\vec{a}}{|a|}$

Find Unit Vector along the following vectors

1) 15i $\hat{i}$

2) 110j $\hat{j}$

3) 120k $\hat{k}$

4) i + j $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

5) 10j + 10k $\frac{10j+10k}{10\sqrt{2}}$

6) 2k + 2i $\frac{2k+2i}{2\sqrt{2}}$

7) 3i + 4j

8) 5j + 12k

9) 7k + 24i

10) 5i + 5j + 5k

11) i - j + k

12) -12k - 12i + 12j

13) 5i + 3j + 4k

14) $\frac{-3j + 4k - 12i}{13}$

QUESTION - 16

Difficulty Level : Easy

If $\hat{n}$ is a unit vector in the direction of the vector $\vec{A}$, then

(NCERT Based)

(1) $\hat{n} = \frac{\vec{A}}{|\vec{A}|}$

(2) $\hat{n} = \frac{|\vec{A}|}{\vec{A}}$

(3) $\hat{n} = |\vec{A}|\,\vec{A}$

(4) $\hat{n} = \vec{A}$

QUESTION - 17

Difficulty Level : HARD

A unit vector is represented as $(0.8\hat{\imath} + b\hat{\jmath} + 0.4\hat{k})$. Hence the value of '$b$' must be

(NCERT Based)

(1) 0.4

(2) $\sqrt{0.6}$

(3) 0.2

(4) $\sqrt{0.2}$ ✓

$$\sqrt{0.8^2 + b^2 + 0.4^2} = 1$$

$$0.64 + b^2 + 0.16 = 1 \longrightarrow (\text{kyu??})$$

$$b^2 + 0.80 = 1$$

$$b^2 = 1 - 0.8 = 0.2$$

$$b = \sqrt{0.2}$$

Difficulty Level : Hard

QUESTION - 18

add kro

A unit vector in the direction of resultant vector of $\vec{A} = -2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ and $\vec{B} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k}$ is (NCERT Based)

(1) $\frac{-2\hat{\imath}+3\hat{\jmath}+\hat{k}}{\sqrt{35}}$

(2) $\frac{\hat{\imath}+2\hat{\jmath}-4\hat{k}}{\sqrt{35}}$

(3) $\frac{-\hat{\imath}+5\hat{\jmath}-3\hat{k}}{\sqrt{35}}$

(4) $\frac{-3\hat{\imath}+\hat{\jmath}+5\hat{k}}{\sqrt{35}}$

$\hat{R} = \frac{\vec{R}}{|R|}$

(I) $A = -2\hat{\imath} + 3\hat{\jmath} + \hat{k}$

$B = \hat{\imath} + 2\hat{\jmath} - 4\hat{k}$

$\bar{A} + \bar{B} = -\hat{\imath} + 5\hat{\jmath} - 3\hat{k}$

$|A+B| = \sqrt{1^2+5^2+3^2}$

$= \sqrt{1+25+9}$

$= \sqrt{35}$

PUPPY POINTS - 1

Scalar
→ Magnitude
→ Independent of axis

Vector
→ Magnitude
→ Direction
→ Law of +ⁿ

Current
Moment of Inertia
Stress
strain] Scalar

Area] → Scalar / vector

Angle btw

$\theta \quad \theta \quad \pi-\theta \quad \pi-\theta$

$\vec{a}+\vec{b} = (-(a+b)$ to $(a+b))$
180° 0°

Tail → Head (Direction)
(Magnitude)

θ is independent of axis

---

* Vector can be moved ∥
Vector cannot be rotated
$\vec{A}+\vec{B}=\vec{A} \Rightarrow B=0$
All equal are ∥
All negative are anti∥]
Not vice versa

Equal → same M, D
Negative → same M, opp D
∥ → same D
anti ∥ → opp D
orthogonal → ⊥
Co initial
Concurrent
Colinear → →
Coplanar (same plane)
- 2 vectors are always coplanar
- Sum of 2 unequal cannot be 0

$\vec{A}$ - vector ✓
$|\vec{A}|$ → magnitude ✓
$\hat{A}$ → unit vector ✓
$\hat{A} = \frac{\vec{A}}{|A|}$ ✓

Triplets (Pytha.)
3, 4, 5
6, 8, 10
5, 12, 13
7, 24, 25
1, 1, $\sqrt{2}$
$\sqrt{3}$, 1, 2
1, 1, 1 → $\sqrt{3}$

$\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$ ✓
$|\vec{A}| = \sqrt{x^2+y^2+z^2}$ ✓
$\hat{A} = \frac{\vec{A}}{|A|} = \frac{x\hat{i}+y\hat{j}+z\hat{k}}{\sqrt{x^2+y^2+z^2}}$ ✓
$|5\hat{i}+3\hat{j}+5\hat{k}| = 5\sqrt{2}$ ✓
$|5\hat{i}+3\hat{j}+12\hat{k}| = 13$ ✓

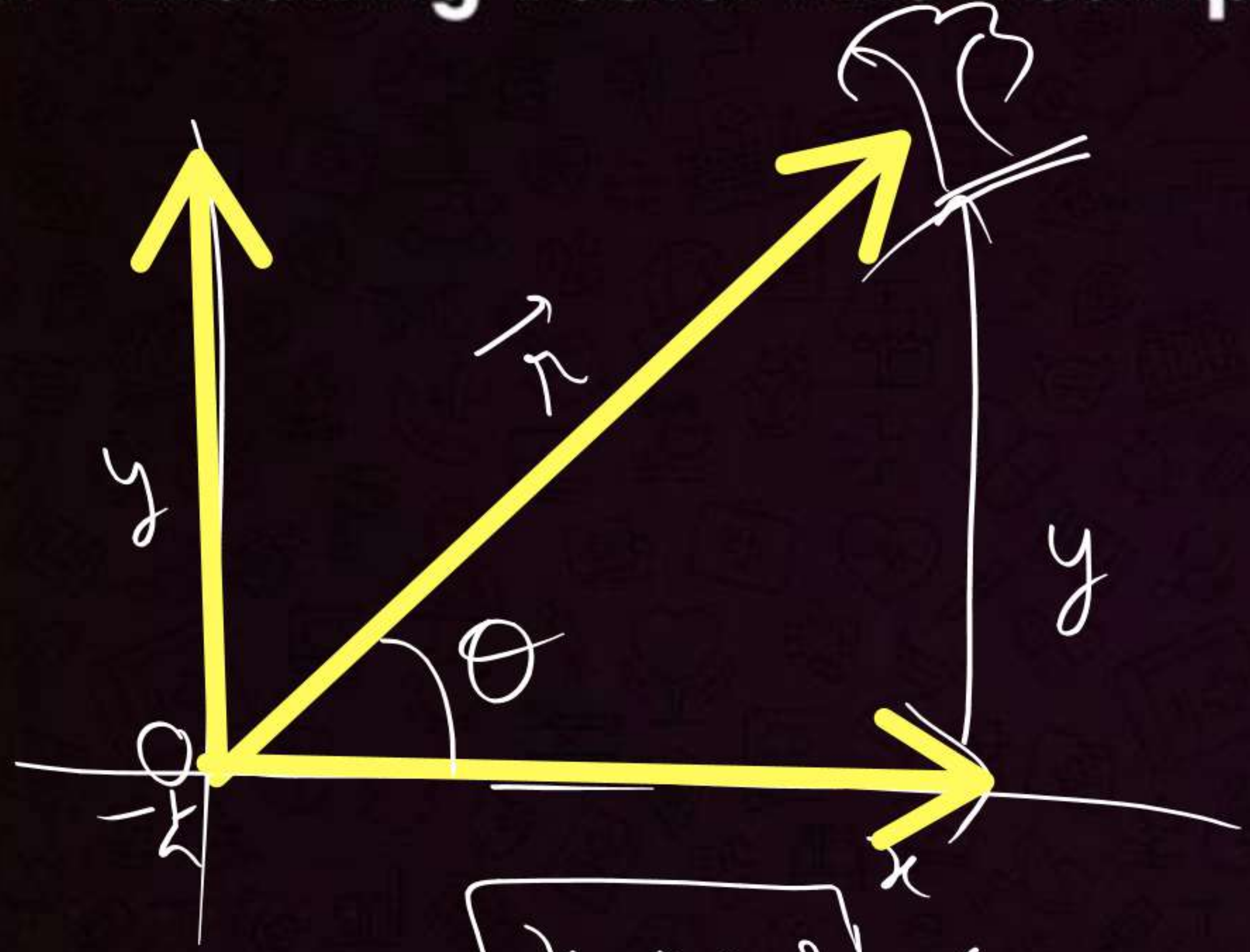
Part 4 – Resolution of Vectors

$\sin 0^\circ = 0$
$\sin 30 = \text{half}$
$\sin 45 = \text{half}\sqrt{2}$
$\sin 60 = \text{half}\sqrt{3}$
$\sin 90 = 1$

=

$\cos 0^\circ = 1$
$\cos 30^\circ = \text{half}\sqrt{3}$
$\cos 45 = \text{half}\sqrt{2}$
$\cos 60 = \text{half}$
$\cos 90 = 0$

4.1 – Breaking Vector into 2 components



$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$\left(\frac{1}{\cos C}\right)$$

$$\vec{r} = x\hat{i} + y\hat{j}$$

Components

Puppy 5 – Resolution of Vectors

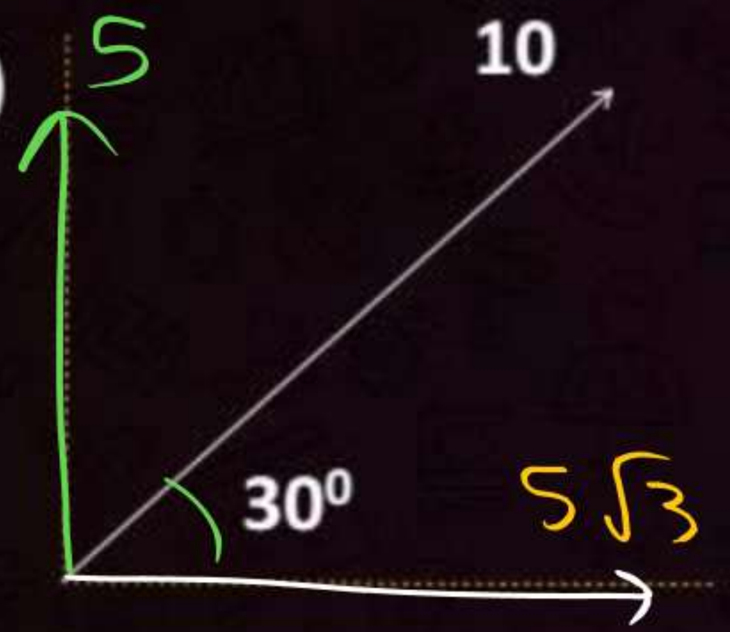
Difficulty Level : Easy

$\sin 37^\circ = \frac{1}{5} \times 3$ | $\cos 37^\circ = \frac{1}{5} \times 4$

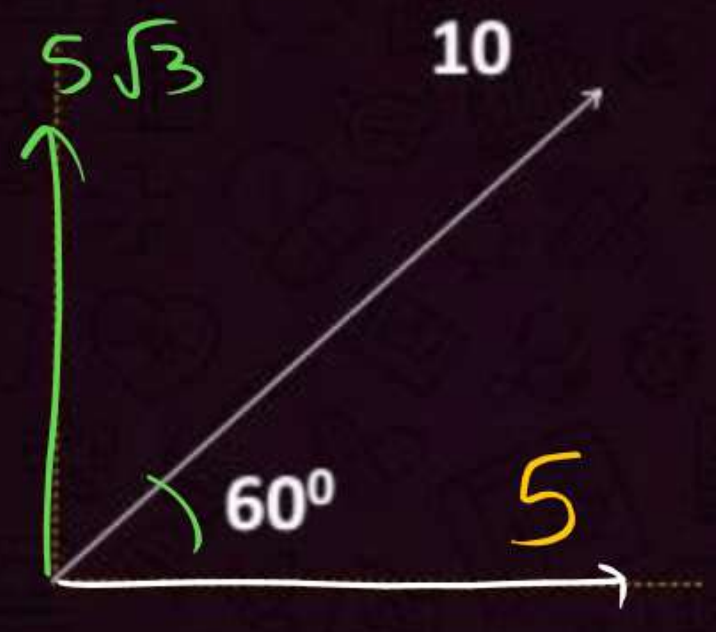
$\sin 53^\circ = \frac{1}{5} \times 4$ | $\cos 53^\circ = \frac{1}{5} \times 3$

Break into x and y components

1)



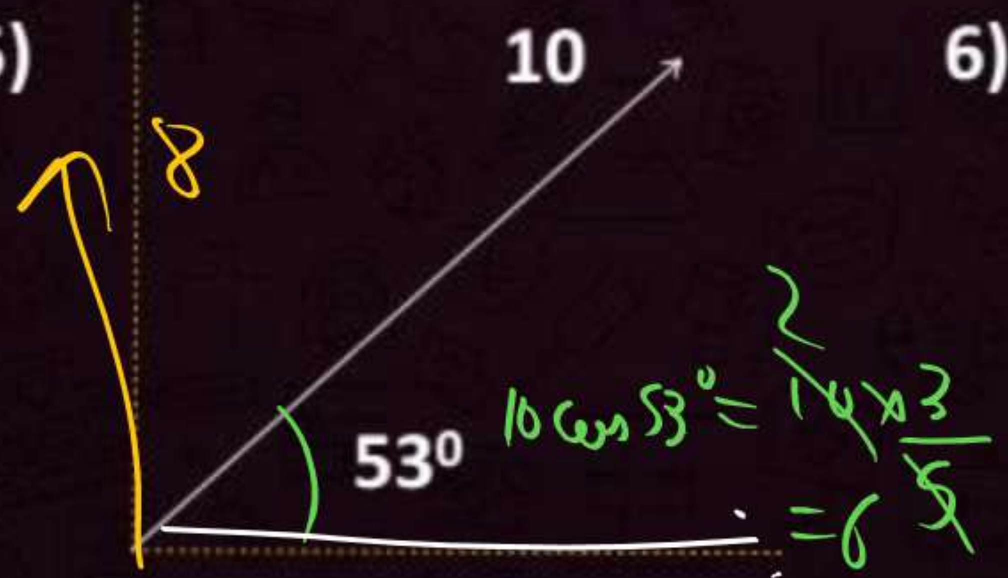
2)



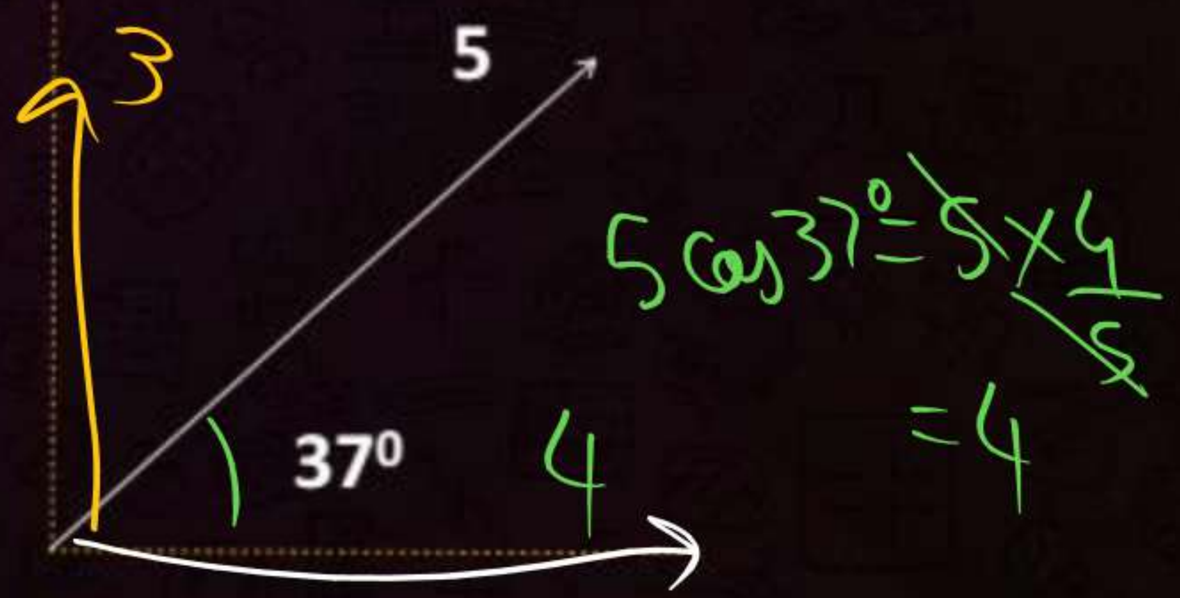
3)

4)

5)



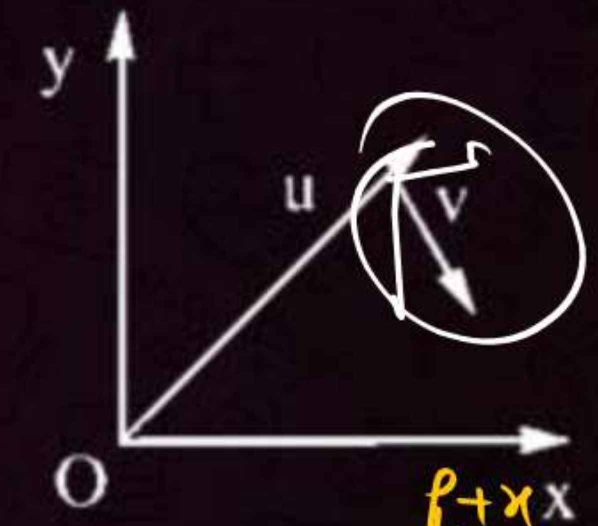
6)



QUESTION - 19

Difficulty Level : Medium

Figure shows the orientation of two vectors $\vec{u}$ and $\vec{v}$ in the xy plane. If $\vec{u} = a\hat{i} + b\hat{j}$ and $\vec{v} = p\hat{i} + q\hat{j}$. Which of the following is correct? *(NCERT Exemplar)*



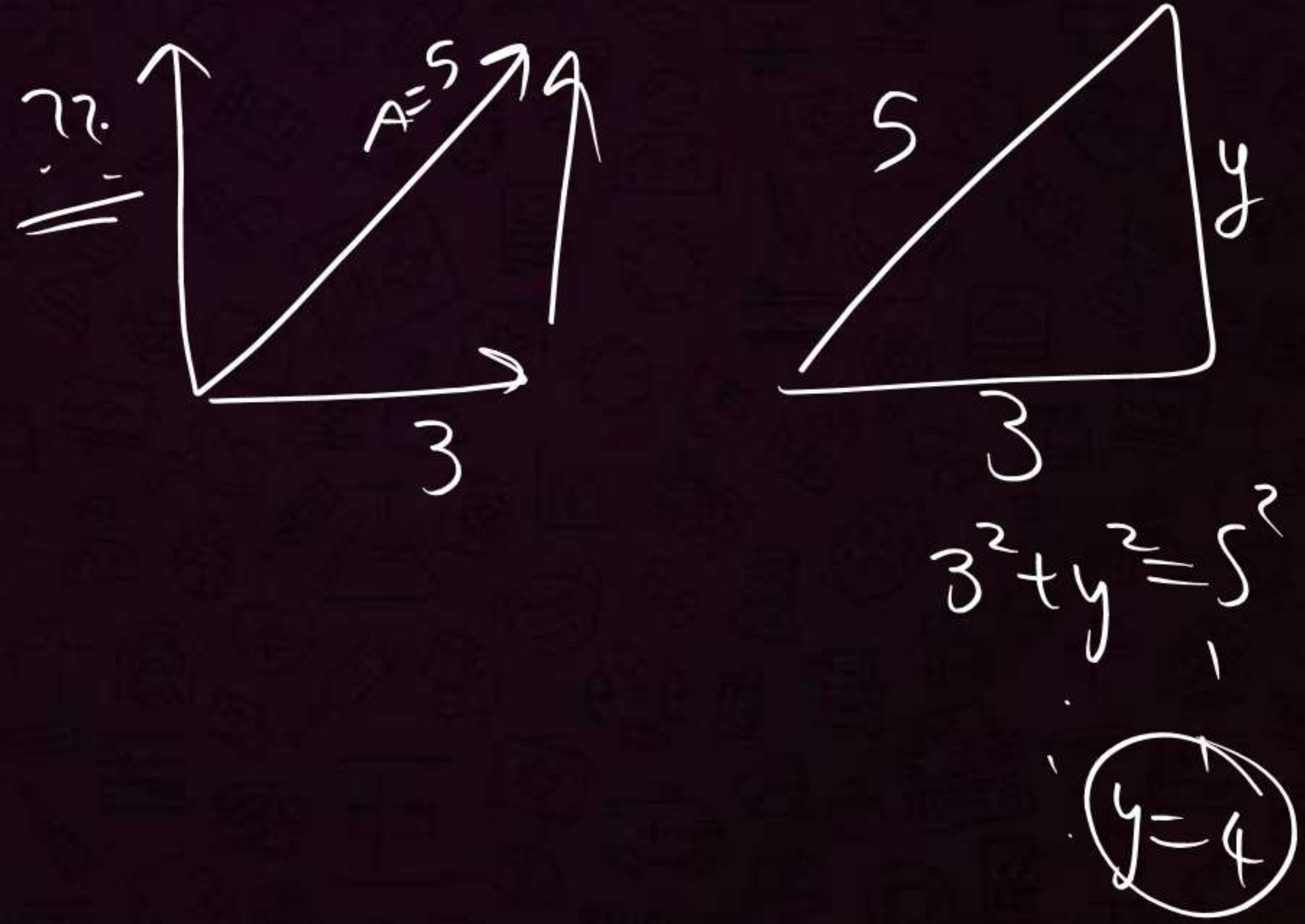
1. a and p are positive while b and q are negative.
2. a, p and b are positive while q is negative.
3. a, q and b are positive while p is negative.
4. a, b, p and q are all positive.

Difficulty Level : Medium

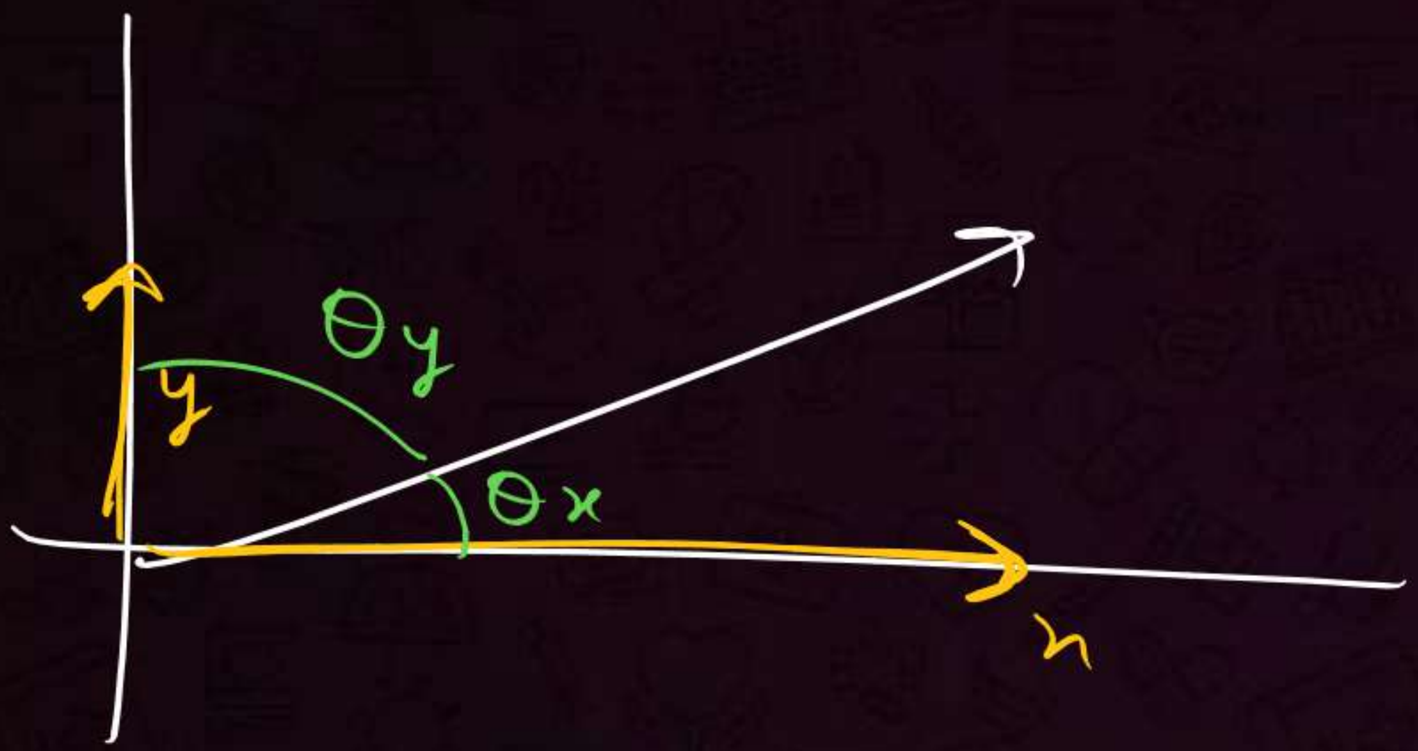
QUESTION - 20

~~The~~ The magnitude of the x-component of vector $\vec{A}$ is 3 and the magnitude of vector $\vec{A}$ is 5. What is the magnitude of the y-component of vector $\vec{A}$? *(NCERT Based)*

1. 3
2. 4
3. 5
4. 8



4.2 – Comparing angles with components ☆ Theory



$x > y$

$\theta_x < \theta_y$

QUESTION - 21

Difficulty Level : Easy

The component of a vector $\vec{r}$ along x-axis will have maximum value if

(NCERT Exemplar)

1. $\vec{r}$ is along positive y-axis
2. $\vec{r}$ is along positive x-axis
3. $\vec{r}$ makes an angle of 45° with the x-axis
4. $\vec{r}$ is along negative y-axis

QUESTION - 22

Difficulty Level : Hard

The component of a vector is

(HCV OBJECTIVE I)

1. always less than its magnitude
2. always greater than its magnitude
3. always equal to its magnitude
4. none of these

less than or equal to

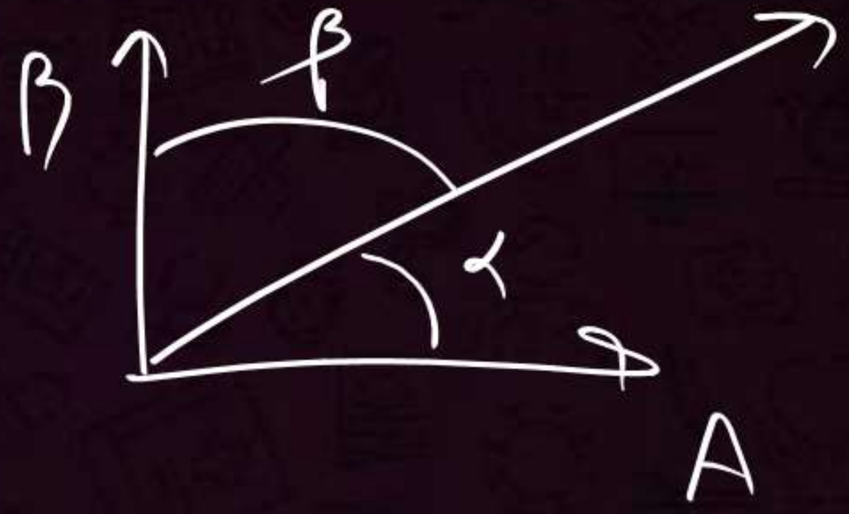
QUESTION - 23

Difficulty Level : Hard

The resultant of $\vec{A}$ and $\vec{B}$ makes an angle α with $\vec{A}$ and β with $\vec{B}$,

(HCV OBJECTIVE I)

1. $\alpha < \beta$
2. $\alpha < \beta$ if $A < B$
3. $\alpha < \beta$ if $A > B$
4. $\alpha < \beta$ if $A = B$



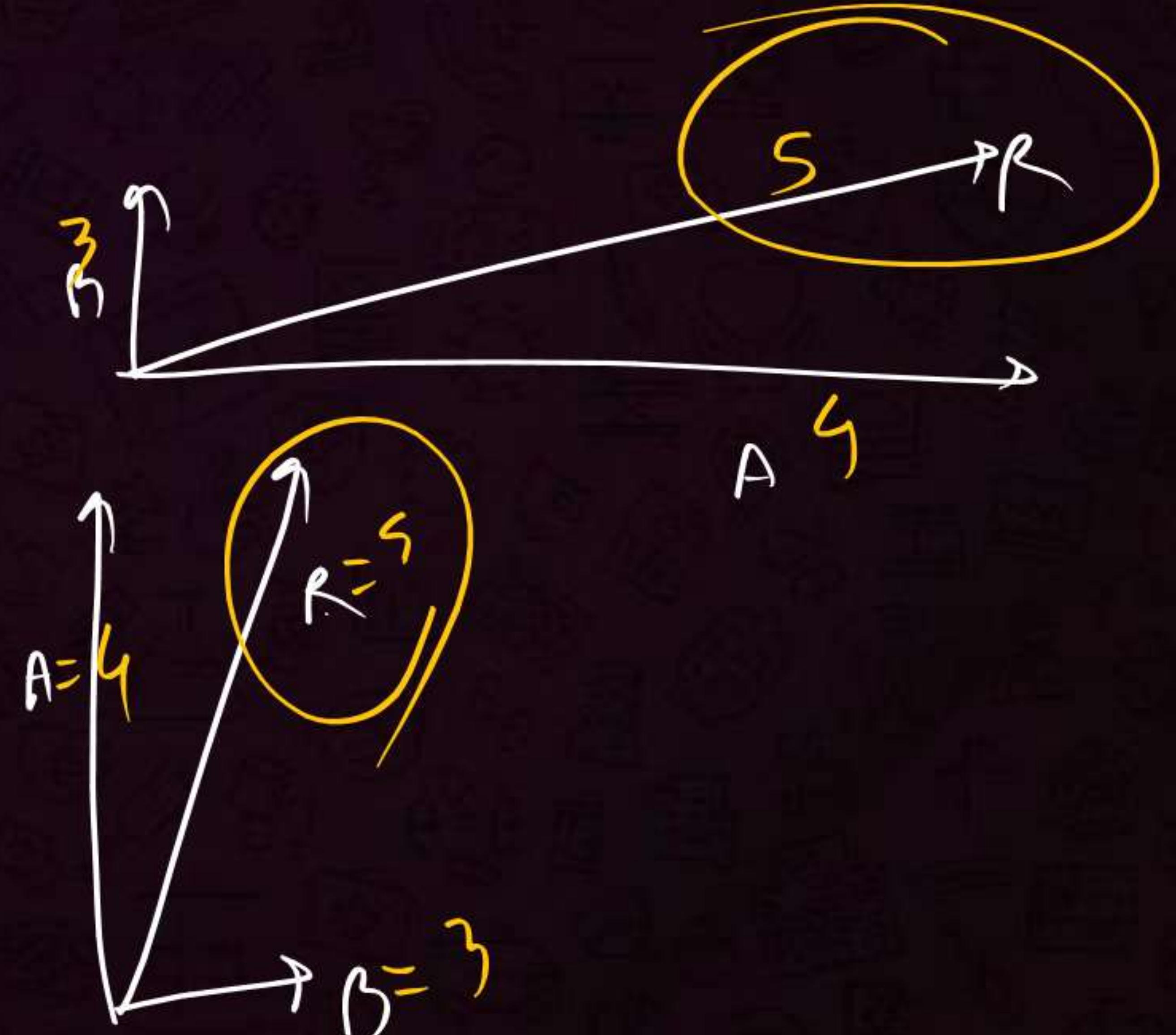
QUESTION - 24

Difficulty Level : Medium

Two vectors $\vec{A}$ and $\vec{B}$ inclined at an angle θ have a resultant $\vec{R}$ which makes an angle α with $\vec{A}$. If the directions of $\vec{A}$ and $\vec{B}$ are interchanged, the resultant will have the same

(NCERT Based)

1. direction
2. magnitude
3. direction as well as magnitude
4. none of these



PUPPY POINTS-1

Scalar
→ Magnitude
→ Independent of axis

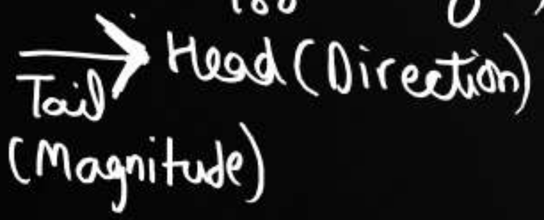
Vector
→ Magnitude
→ Direction
→ Law of +n

Current
Moment of Inertia
Stress
strain
] Scalar

Area] → Scalar/vector

Angle btw

$\vec{a}+\vec{b} = (-(a+b)$ to $(a+b))$
180° 0°



θ is independent of axis

---

* Vector can be moved ∥
Vector cannot be rotated
$\vec{A}+\vec{B}=\vec{A} \Rightarrow B=0$
All equal are ∥
All negative are anti∥]
Not vice versa

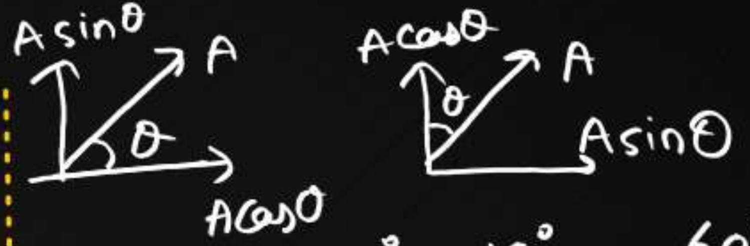
Equal → same M, D
Negative → same M, opp D
∥ → same D
anti ∥ → opp D
orthogonal → ⊥
Co initial
Concurrent
Colinear → →
Coplanar (same plane)
- 2 vectors are always
- Sum of 2 unequal coplanar cannot be 0

$\vec{A}$ – vector
$|\vec{A}|$ → magnitude
$\hat{A}$ → unit vector
$\hat{A} = \frac{\vec{A}}{|A|}$

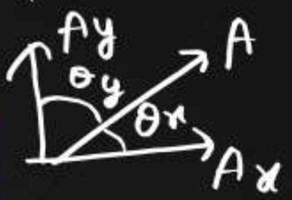
Triplets (Pytha.)
3, 4, 5
6, 8, 10
5, 12, 13
7, 24, 25
1, 1, $\sqrt{2}$
$\sqrt{3}$, 1, 2
1, 1, 1 → $\sqrt{3}$

$\vec{A} = x\hat{i}+y\hat{j}+z\hat{k}$
$|\vec{A}| = \sqrt{x^2+y^2+z^2}$
$\hat{A} = \frac{\vec{A}}{|A|} = \frac{x\hat{i}+y\hat{j}+z\hat{k}}{\sqrt{x^2+y^2+z^2}}$

$|5\hat{i}+3\hat{j}+5\hat{k}| = 5\sqrt{2}$
$|5\hat{i}+3\hat{j}+12\hat{k}| = 13$



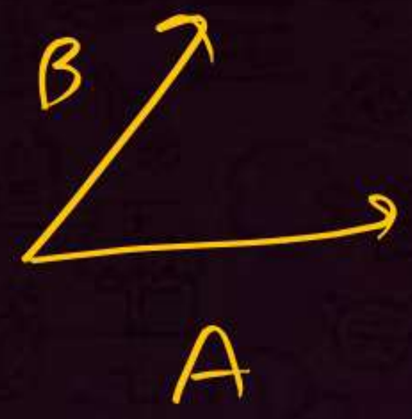
	0	30°	45°	60°	90°	37°	53°
sin	0	half	half√2	half√3	1	3/5	4/5
cos	1	half√3	half√2	half	0	4/5	3/5
tan	0	1/√3	1	√3	∞	3/4	4/3



$\theta_y > \theta_x$
$A_y < A_x$
(chota angle, bada component)

Part 5 – Operations + - on Vectors

$\vec{A} + \vec{B}$



$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

☆ Subraction of vector is also vector addition

5.1 – Analytical Addition & Subtraction

$\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{B} = 8\hat{i} + 2\hat{j} + 5\hat{k}$

$\vec{A} + \vec{B} = 10\hat{i} + 5\hat{j} + 9\hat{k}$

$\vec{B} - \vec{A}$

$8\hat{i} + 2\hat{j} + 5\hat{k}$

$-\ 2\hat{i} - 3\hat{j} - 4\hat{k}$

$6\hat{i} - \hat{j} + \hat{k}$

QUESTION - 25

Difficulty Level : Medium

Two vectors are given by $\vec{A} = (3\hat{\imath} + \hat{\jmath} + 3\hat{k})$ and $\vec{B} = (3\hat{\imath} + 5\hat{\jmath} - 2\hat{k})$. Find the third vector $\vec{C}$ if $\vec{A} + 3\vec{B} - \vec{C} = 0$.

(NCERT Based)

(1) $(12\hat{\imath} + 14\hat{\jmath} + 12\hat{k})$

(2) $(13\hat{\imath} + 17\hat{\jmath} + 12\hat{k})$

(3) $(12\hat{\imath} + 16\hat{\jmath} - 3\hat{k})$ ✓

(4) $(15\hat{\imath} + 13\hat{\jmath} + 4\hat{k})$

$\vec{A} + 3\vec{B} - \vec{C} = 0$

$\vec{C} = \vec{A} + 3\vec{B}$

$\vec{A} = 3\hat{\imath} + \hat{\jmath} + 3\hat{k}$

$3\vec{B} = 9\hat{\imath} + 15\hat{\jmath} - 6\hat{k}$

$= 12\hat{\imath} + 16\hat{\jmath} - 3\hat{k}$

QUESTION - 26

Difficulty Level : Easy

The components of the sum of two vectors $2\hat{i} + 3\hat{j}$ and $2\hat{i} + 3\hat{k}$ along x and y directions respectively are *(NCERT Based)*

1. 2 and 5
2. 4 and 6
3. 2 and 6
4. 4 and 3

QUESTION - 27

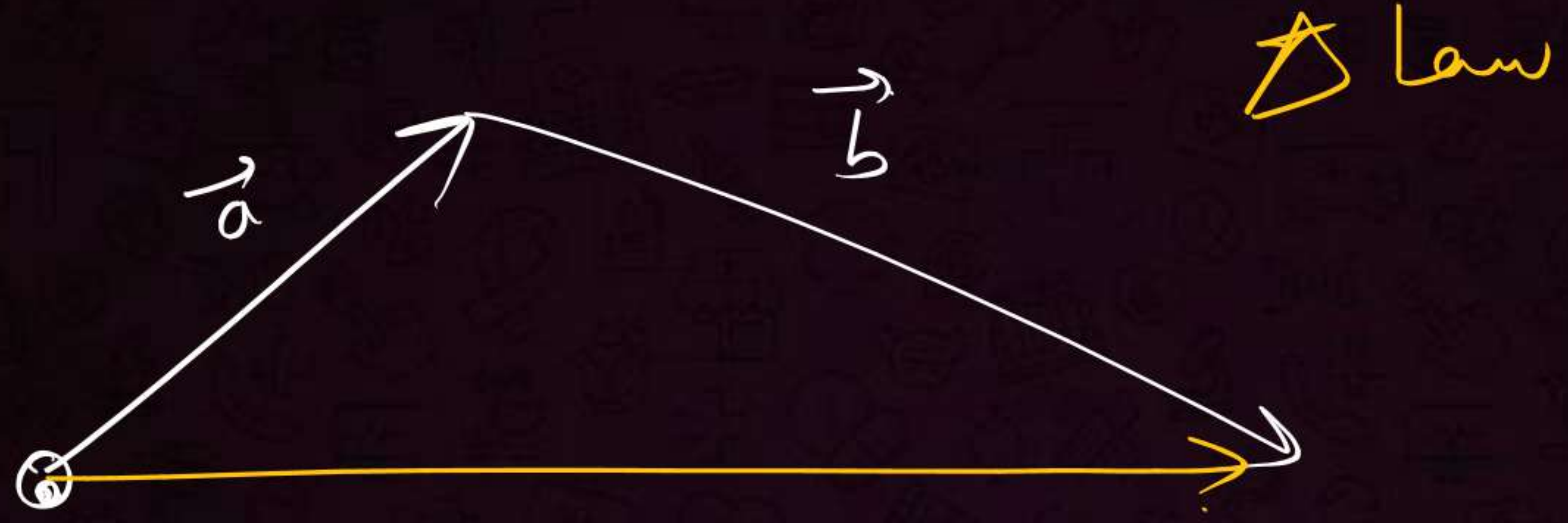
HW

Difficulty Level : Easy

$A = i + j + k$, $B = 3i + 2j + k$, $C = 2i + j - k$, Find value of $2B + C - A$

(NCERT Based)

5.2 – Diagrammatic Addition



QUESTION - 28

Difficulty Level : Hard

Which of the following quantities is dependent of the choice of orientation of the coordinate axes?

(NCERT Based)

1. $\vec{A} + \vec{B}$
2. $A_x + B_y$
3. $|\vec{A} + \vec{B}|$
4. Angle between $\vec{A}$ and $\vec{B}$

Orientation of axis

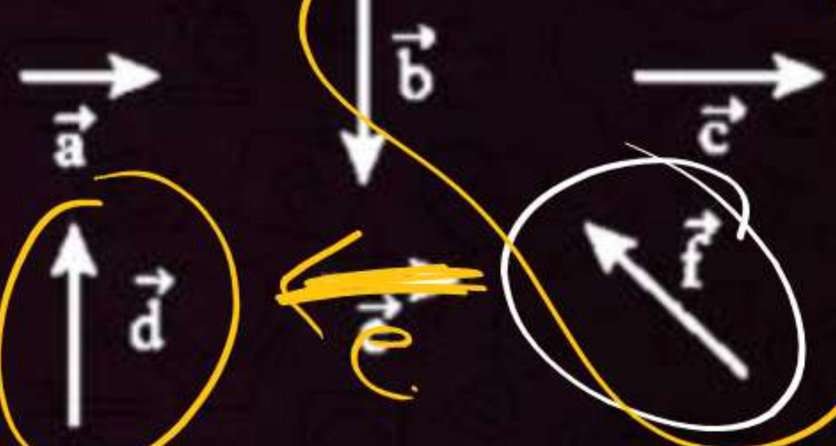
Only components depend krte

Difficulty Level : Easy

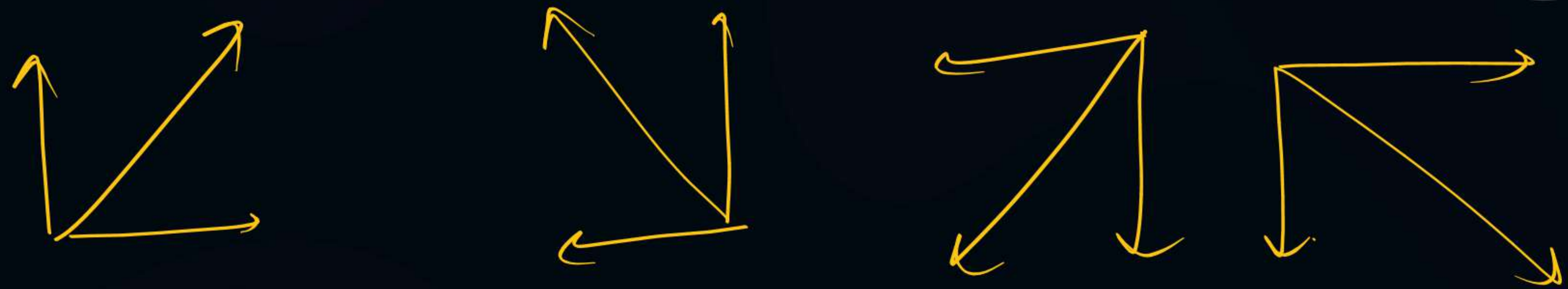
QUESTION - 29

Six vectors, $\vec{a}$ to $\vec{f}$ have the magnitudes and directions indicated in the figure. Which of the following statements are true?

PYQ - (2010)



1. $\vec{b} + \vec{e} = \vec{f}$
2. $\vec{b} + \vec{c} = \vec{f}$
3. $\vec{d} + \vec{c} = \vec{f}$
4. $\vec{d} + \vec{e} = \vec{f}$



$$\vec{f} = (b) + (c)$$

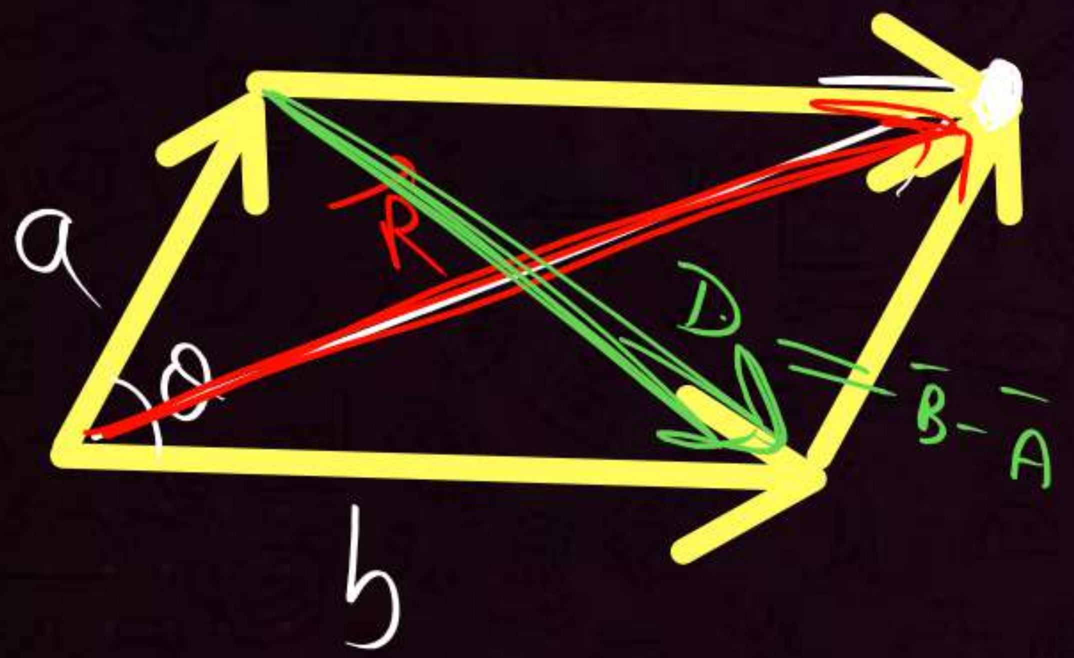
5.3 – Vector Subtraction

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

5.4 – Parallelogram Law

When two vectors are acting simultaneously at a point, the resultant vector is represented by the diagonal of a parallelogram drawn from that point

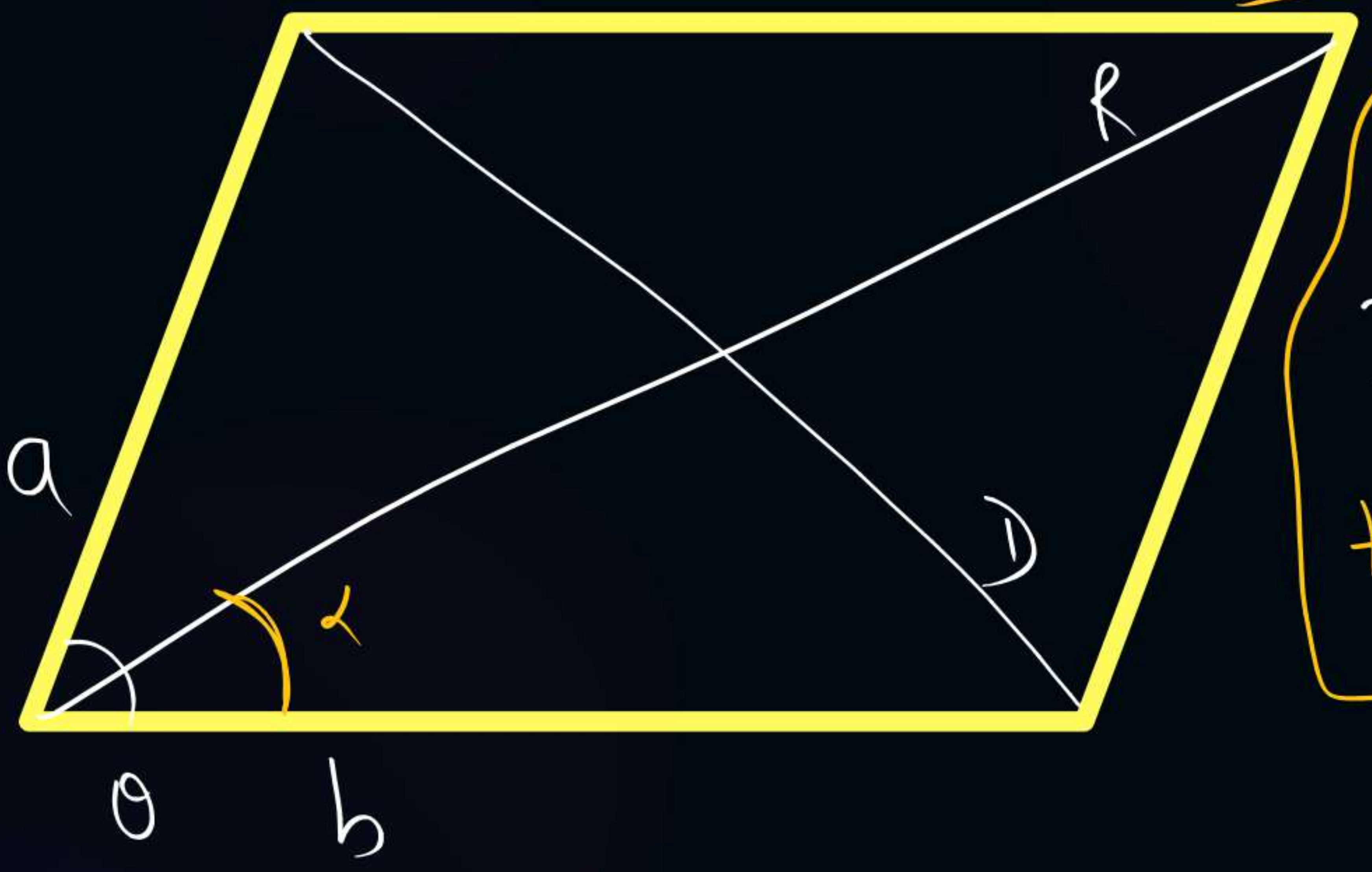
When two vectors are acting simultaneously at a point, the difference vector is represented by the other diagonal of a parallelogram.



$$\vec{R} = \vec{a} + \vec{b} = \sqrt{a^2 + b^2 + 2ab\cos\theta}$$

$$\vec{B} = \vec{b} - \vec{a} = \sqrt{a^2 + b^2 - 2ab\cos\theta}$$

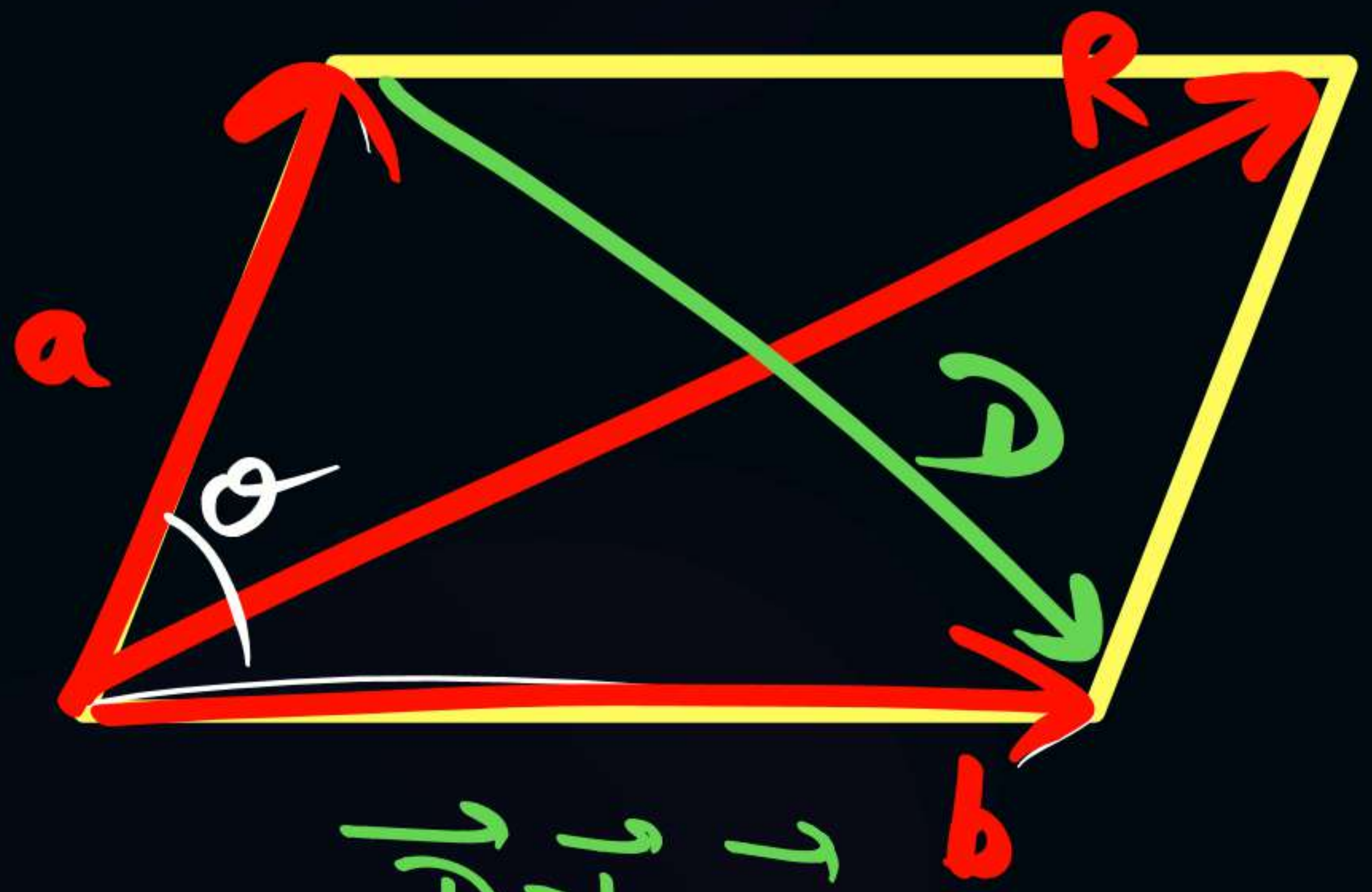
Geometry



$$R = \sqrt{a^2 + b^2 + 2ab\cos\theta}$$

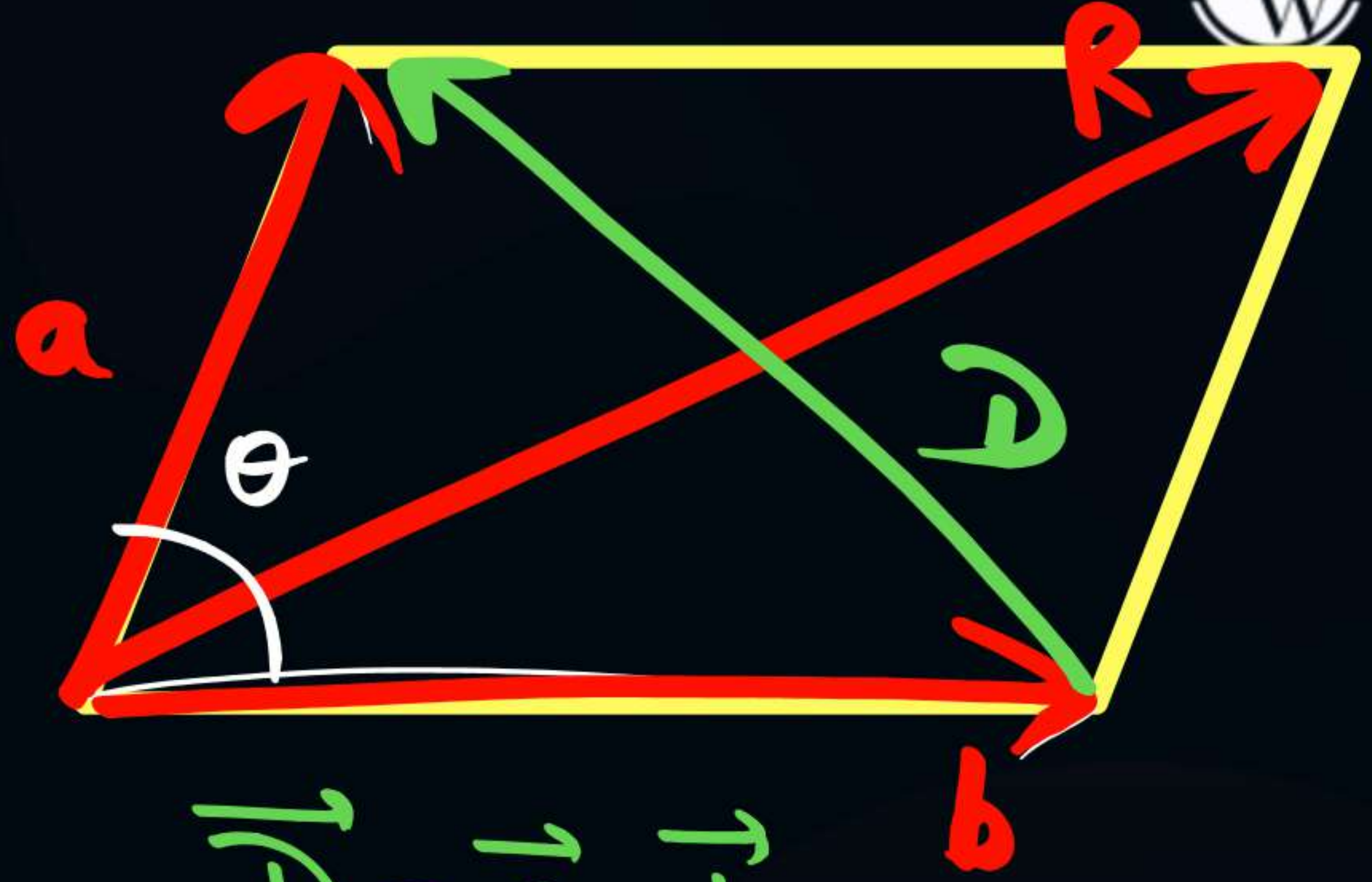
$$D = \sqrt{a^2 + b^2 - 2ab\cos\theta}$$

$$\tan\alpha = \frac{a\sin\theta}{b + a\cos\theta}$$



$$\vec{D} = \vec{b} - \vec{a}$$

$$\vec{R} = \vec{a} + \vec{b}$$

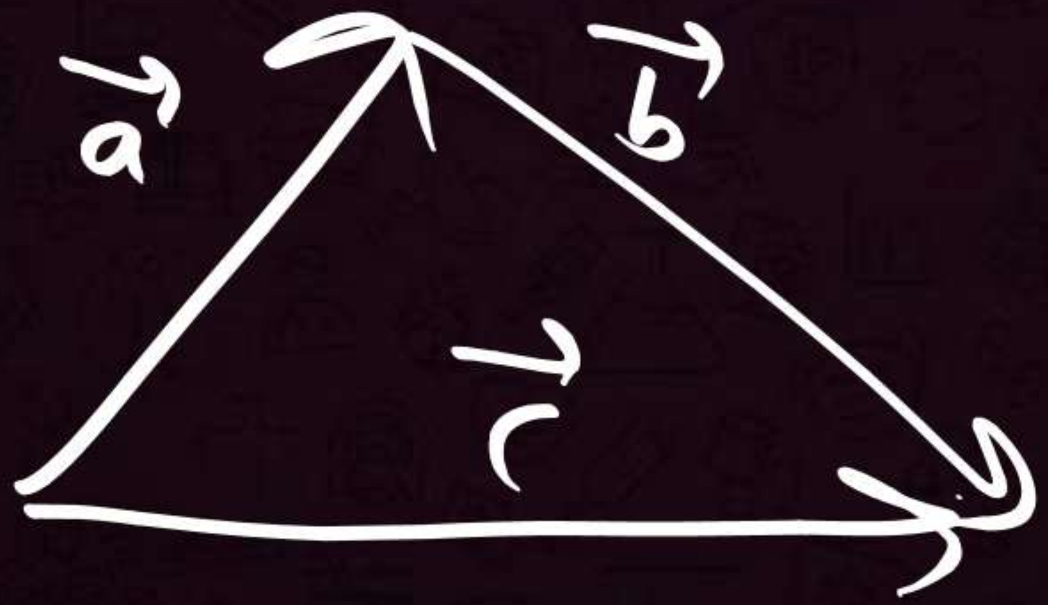


$$\vec{D} = \vec{a} - \vec{b}$$

$$\vec{R} = \vec{a} + \vec{b}$$

5.5 – Triangle Law

1. Represent the two vectors as two sides of a triangle, with the order of magnitude and direction.
2. Place the tail of the second vector at the head of the first vector.
3. Draw the third side of the triangle from the tail of the first vector to the head of the second vector.
4. The third side of the triangle represents the resultant vector in both magnitude and direction.

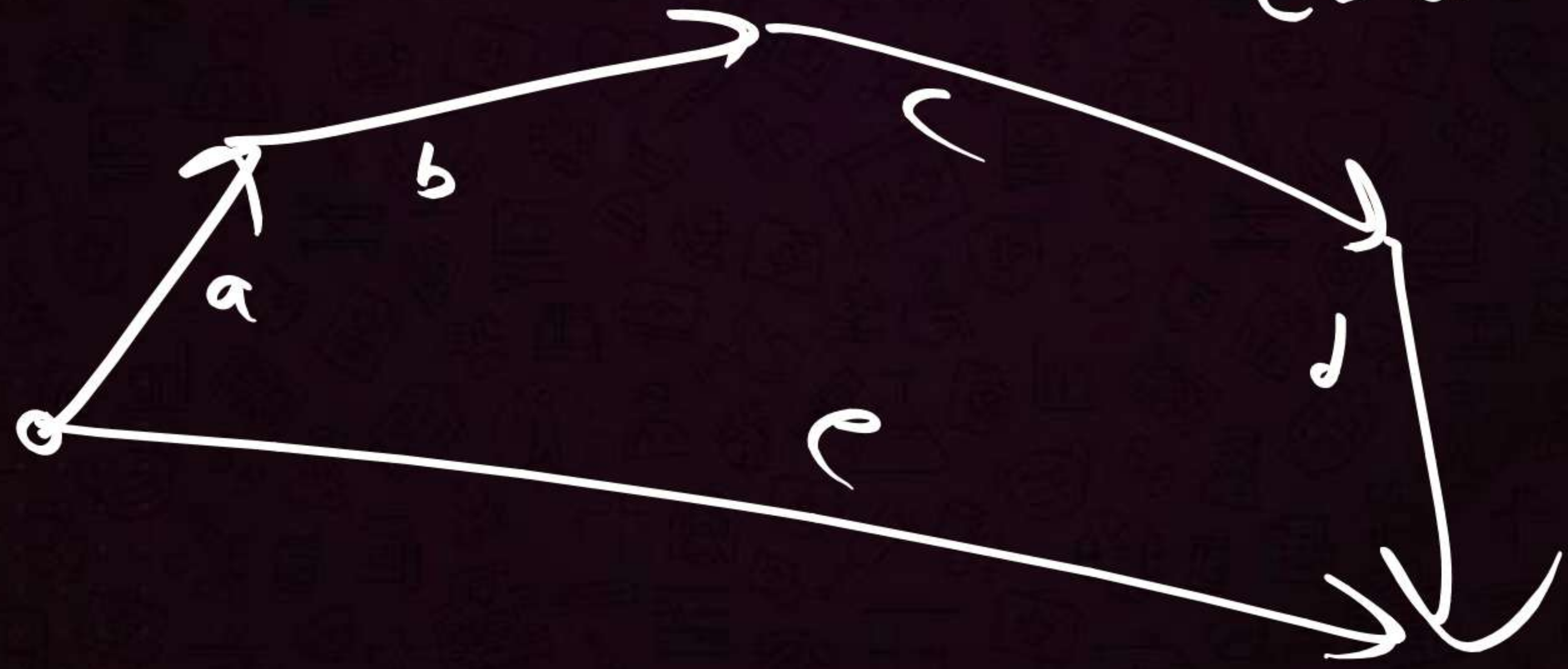


$$\vec{c} = \vec{a} + \vec{b}$$

5.6 – Polygon Law

Polygon law of vector addition states that if a number of vectors can be represented in magnitude and direction by the sides of a polygon taken in the same order, then their resultant is represented in magnitude and direction by the closing side of the polygon taken in the opposite order.

$\vec{e} = \vec{a} + \vec{b} + \vec{c} + \vec{d}$



QUESTION - 30

Difficulty Level : YODHA

Given $\vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{0}$, which of the following statements is not correct?

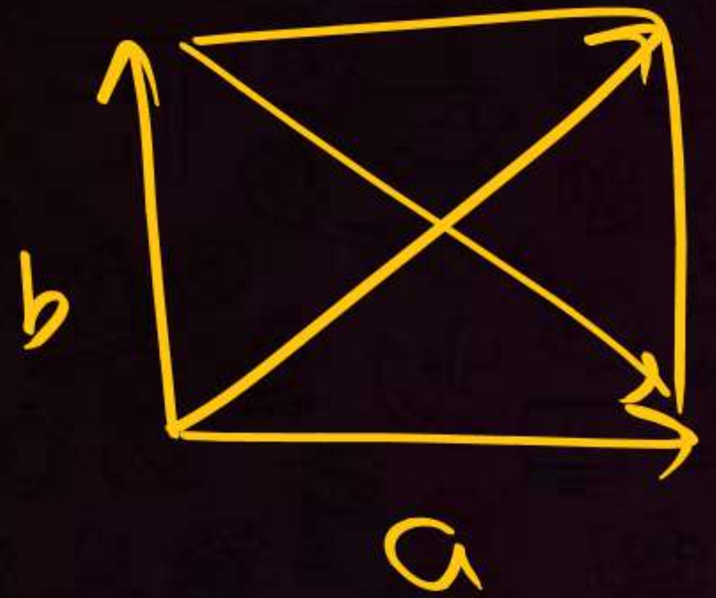
(1) $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$ must each be a null vector.

(2) The magnitude of $(\vec{A} + \vec{C})$ equals the magnitude of $(\vec{B} + \vec{D})$.

(3) The magnitude of $\vec{A}$ can never be greater than the sum of the magnitudes of $\vec{B}$, $\vec{C}$ and $\vec{D}$.

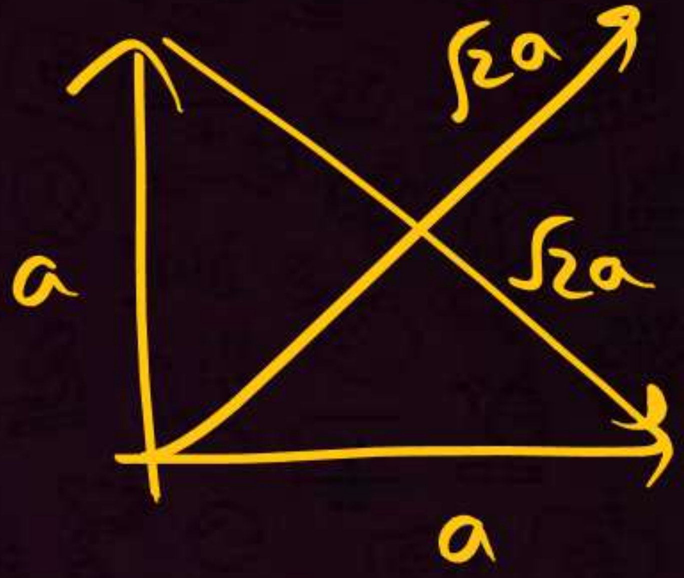
(4) $\vec{B} + \vec{C}$ must lie in the plane of $\vec{A}$ and $\vec{D}$ if $\vec{A}$ and $\vec{D}$ are not collinear and in the line of $\vec{A}$ and $\vec{D}$ if they are collinear.

5.7 – Addition when angle is 90^0



$$R = \sqrt{a^2 + b^2}$$

$$D = \sqrt{a^2 + b^2}$$



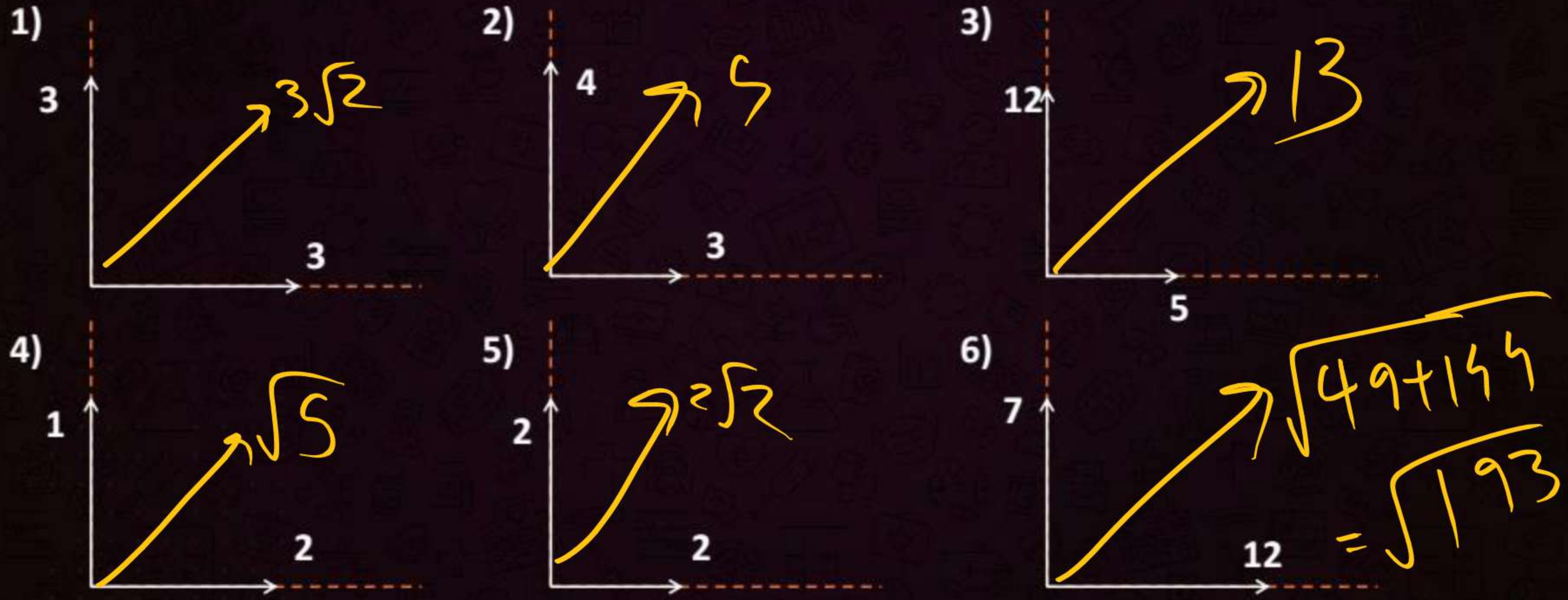
$$R = \sqrt{2}a$$

$$D = \sqrt{2}a$$

Puppy 6 – Addition of Vectors

Difficulty Level : Easy

Find Resultant and Difference of 2 vectors



QUESTION - 31

Difficulty Level : Easy

The magnitude of vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 3, 4 and 5 units respectively. If $\vec{A} + \vec{B} = \vec{C}$, the angle between $\vec{A}$ and $\vec{B}$ is:

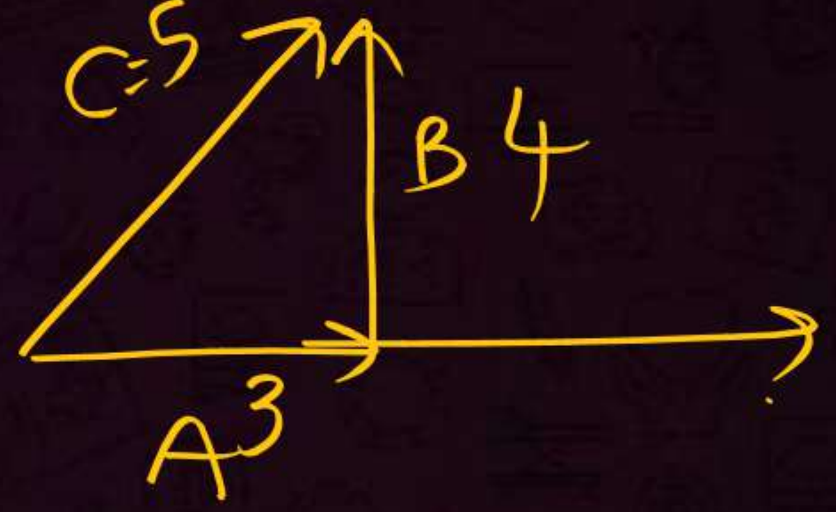
PYQ - (1988)

(1) $\pi/2$

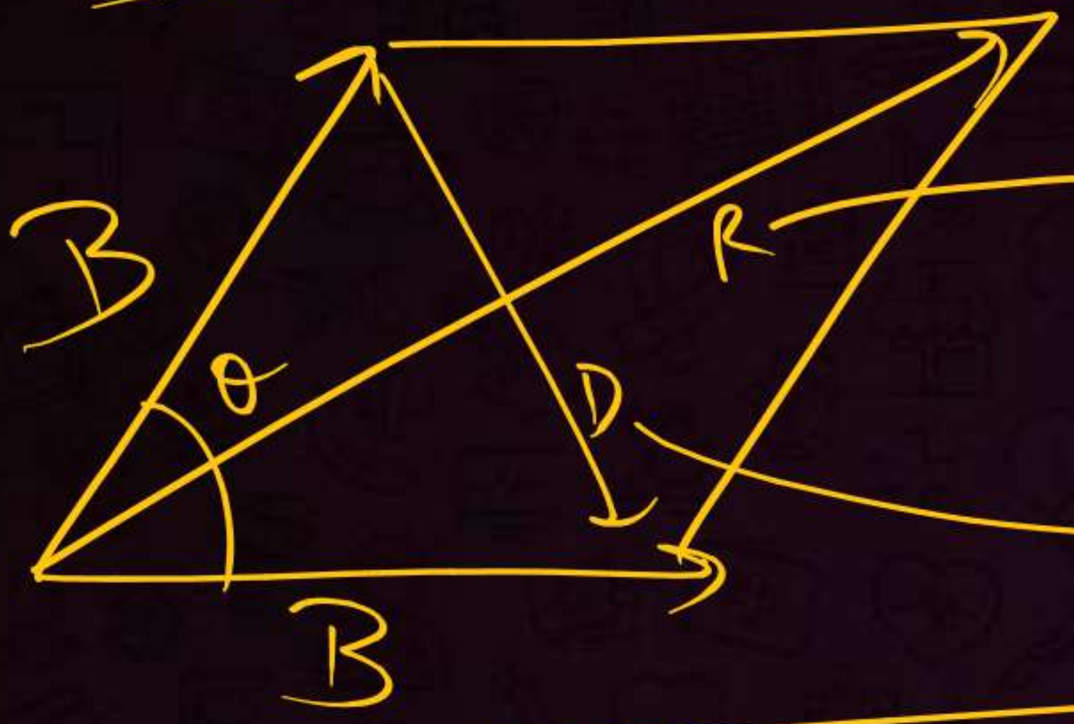
(2) $\cos^{-1}(0.6)$

(3) $\tan^{-1}(7/5)$

(4) $\pi/4$



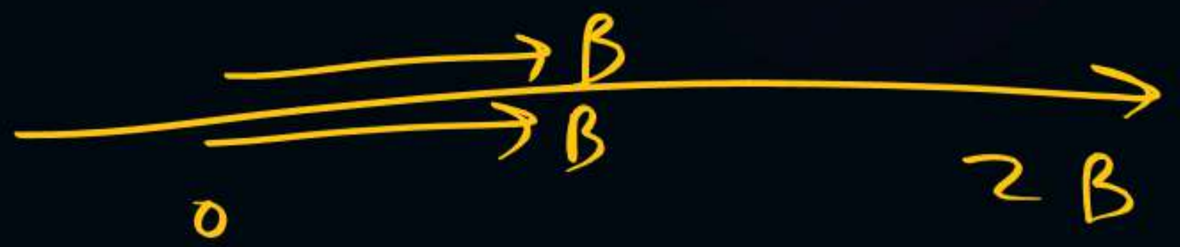
5.8 – Rhombus/Babli Addition

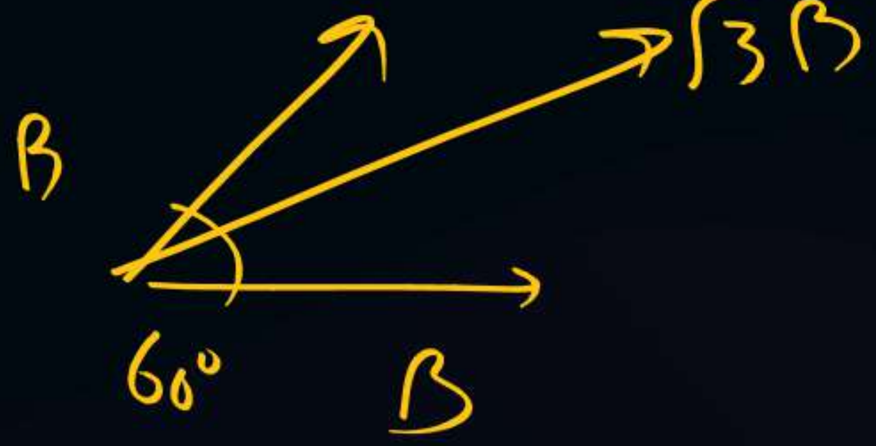


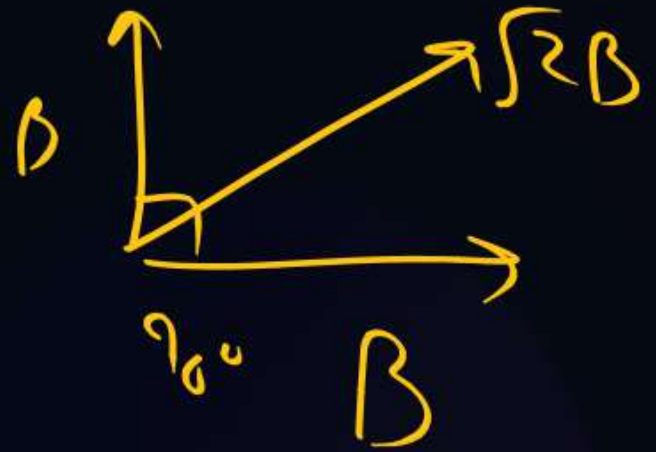
$$R = 2B\cos\left(\frac{\theta}{2}\right)$$

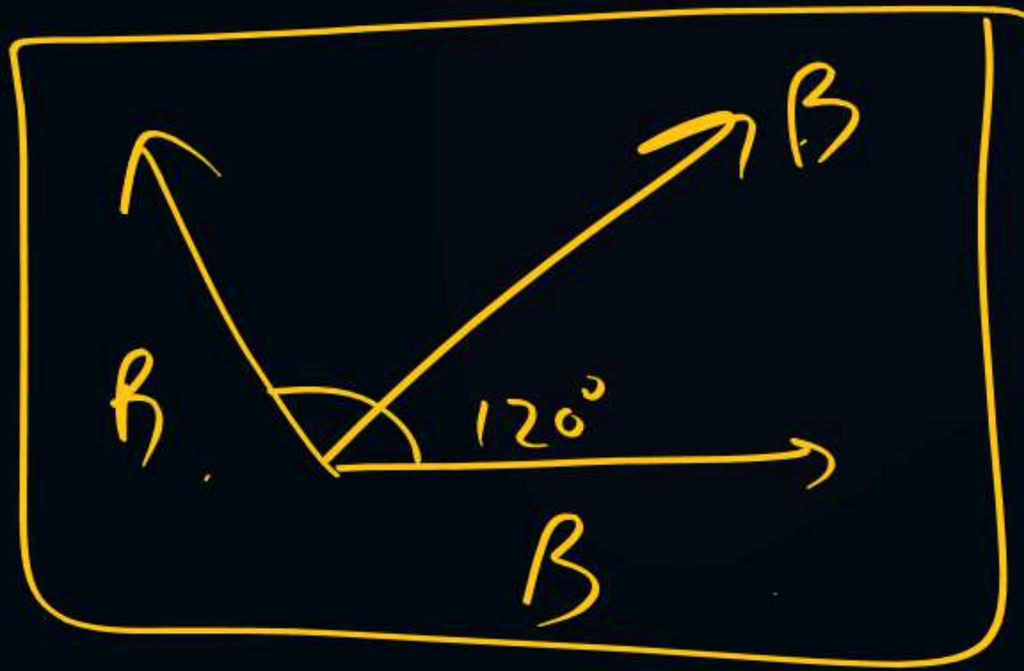
$$D = 2B\sin\left(\frac{\theta}{2}\right)$$

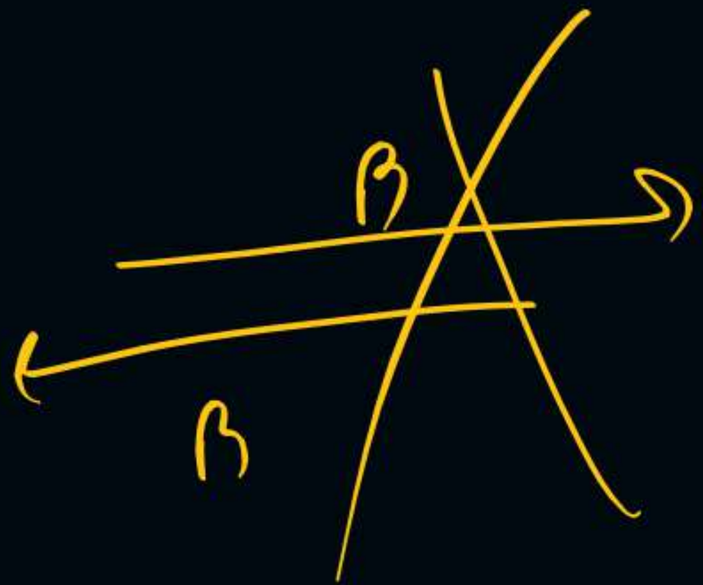
$\theta = 0^\circ$ / 60° / 90° / 120° / 180°





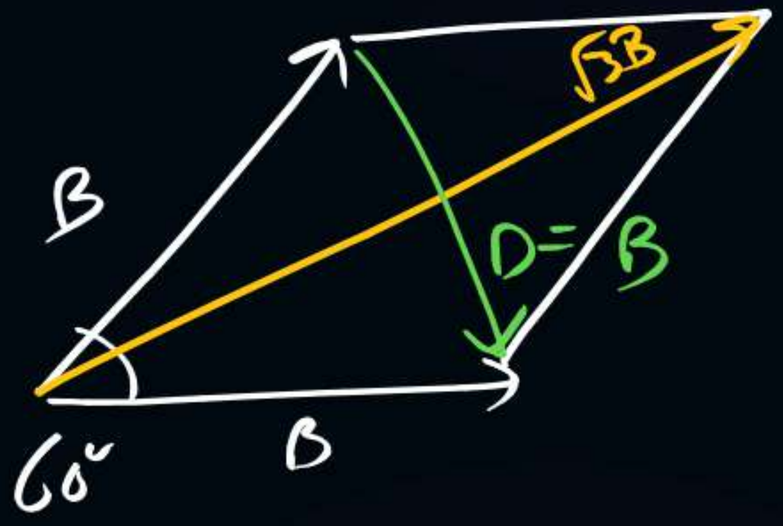


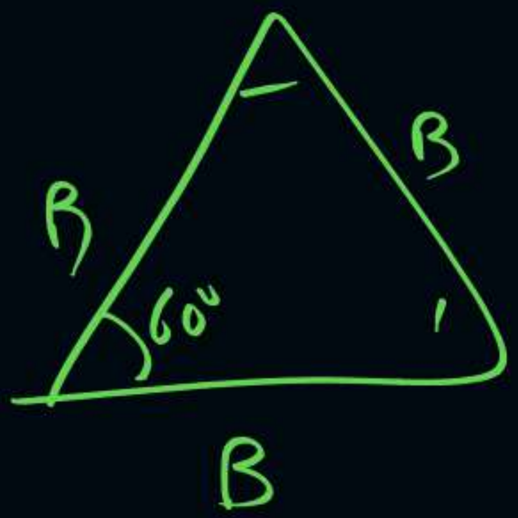


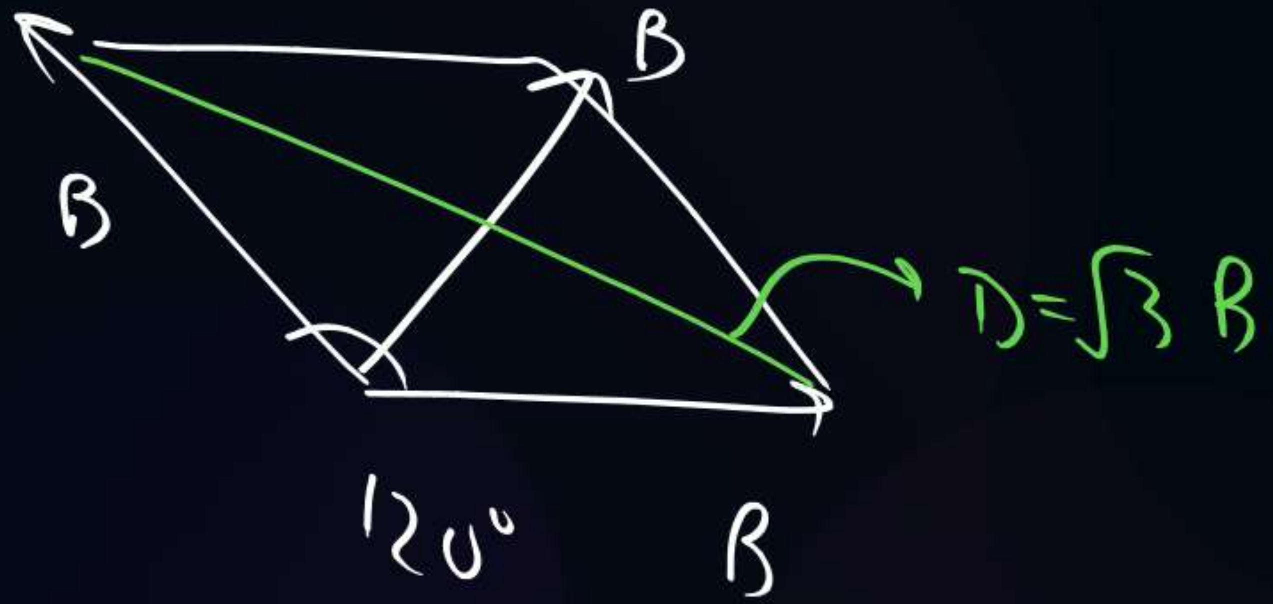


$180^\circ = \text{null } \vec{O}$

B
B
2B
B
B
180°
Null
$\vec{0}$
B
$\sqrt{3}$B
60°
B
B
$\sqrt{2}$B
B
B
B
B
120°
B







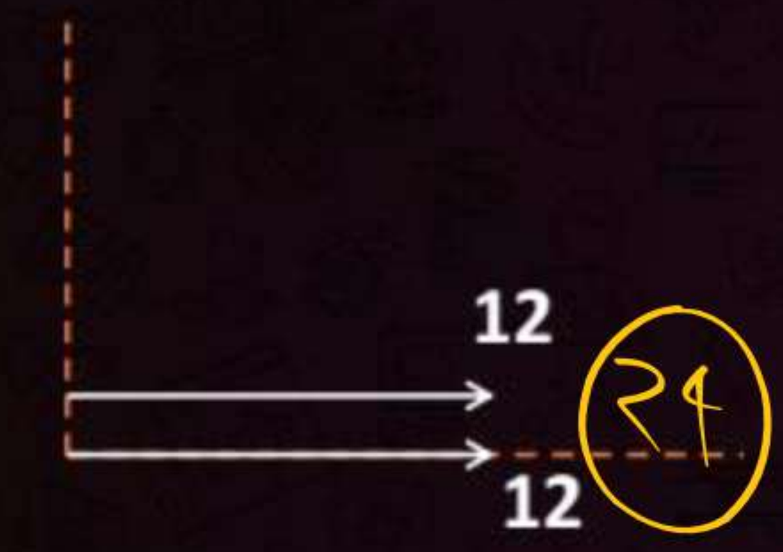
$R = 2B\cos\left(\frac{\theta}{2}\right)$ $D = 2B\sin\left(\frac{\theta}{2}\right)$

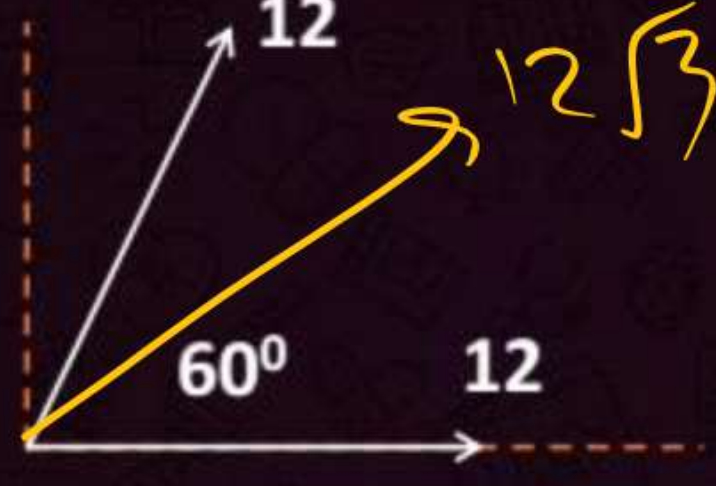
θ	R	D
0°	2B	×
60°	√3B	B
90°	√2B	√2 B
120°	B	√3 B
180°	0	2B

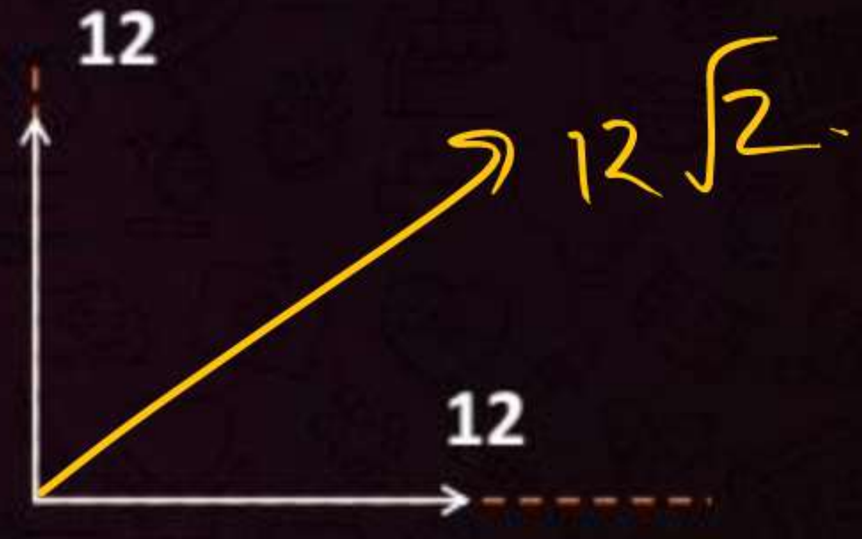
Puppy 7 – Babli Addition

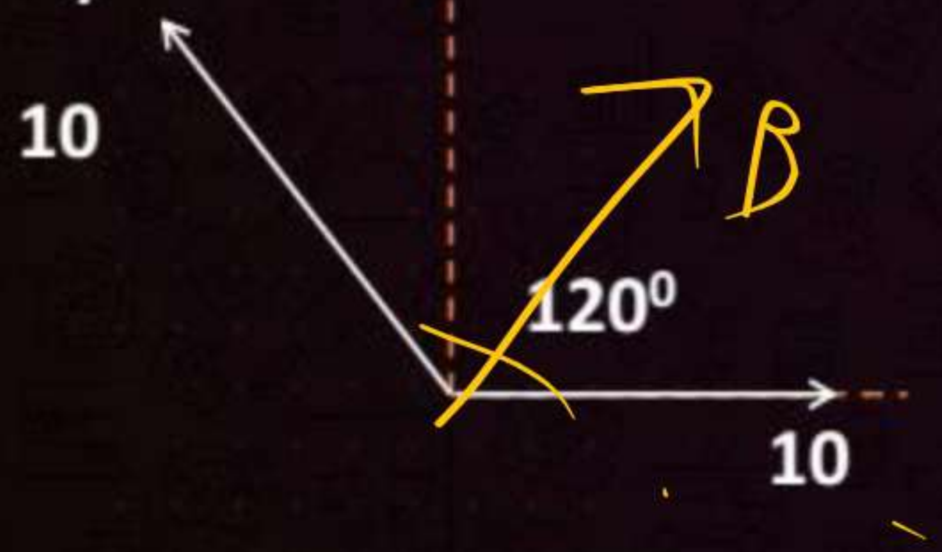
Difficulty Level : Easy

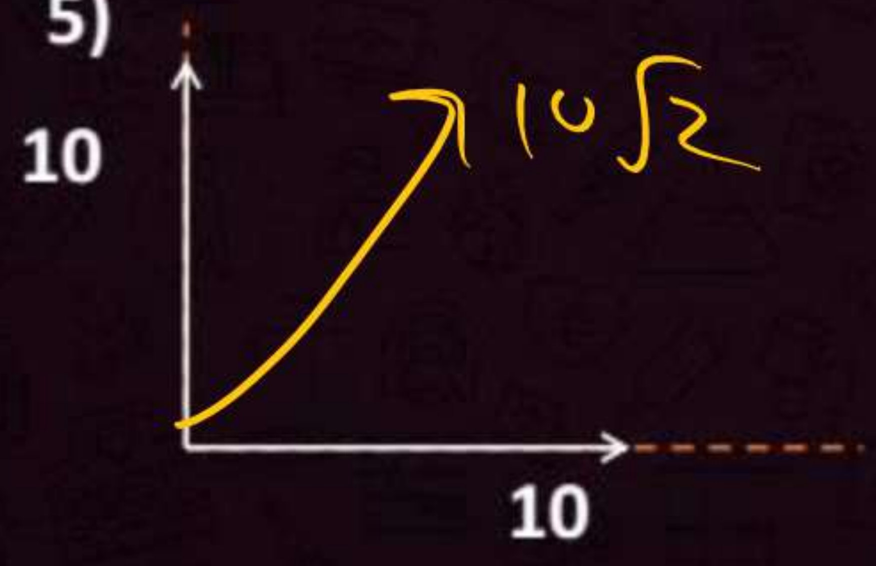
Find Resultant and Difference of 2 vectors

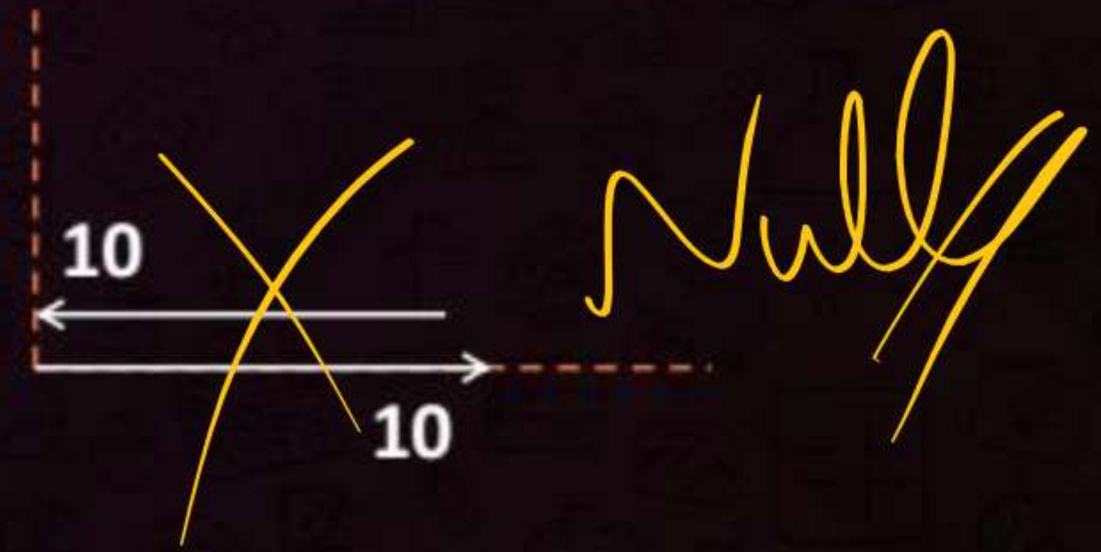
1)


2)


3)


4)


5)


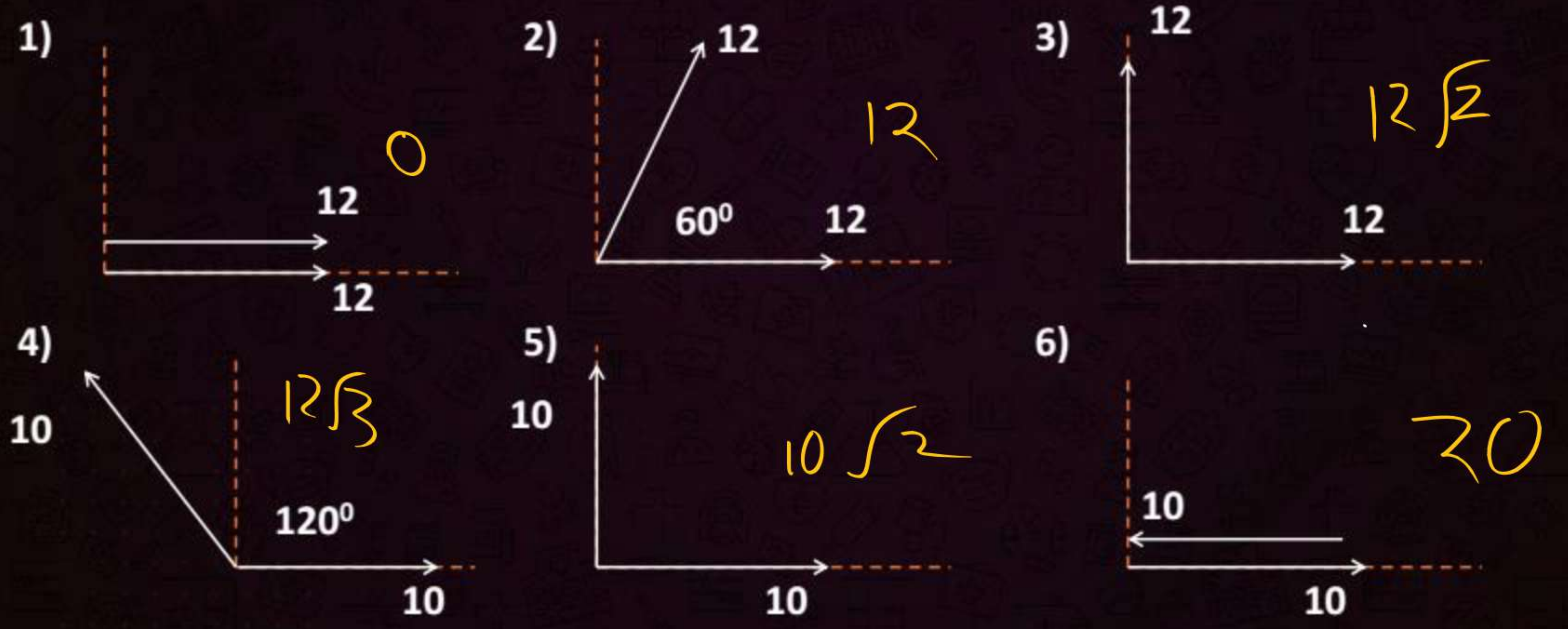
6)


Puppy 7 – Babli Addition

Difficulty Level : Easy

HW

Find ~~Resultant and~~ Difference of 2 vectors



Puppy 8 – Babli Addition

Difficulty Level : Hard

1) $|\vec{A} + \vec{B}| = A + B$. What is the angle between A and B

2) $\vec{A} + \vec{B} = \vec{C}$ and $A + B = C$. What is the angle between A and B

3) $|\vec{A} + \vec{B}| = A - B$. What is the angle between A and B

4) $\vec{A} + \vec{B} = \vec{C}$ and $A - B = C$. What is the angle between A and B

5) $|\vec{A} + \vec{B}| = \sqrt{(A^2 + B^2)}$. What is the angle between A and B

6) $\vec{A} + \vec{B} = \vec{C}$ and $\sqrt{(A^2 + B^2)} = C$. What is the angle between A and B

7) $\vec{A} + \vec{B} = \vec{C}$ and $A^2 + B^2 = C^2$. What is the angle between A and B

$$|\vec{A}+\vec{B}| = A+B$$

$$\vec{10}+\vec{5} = 5 \text{ to } 15$$

$15 \rightarrow 0°$ Max

$5 \rightarrow 180°$ min

$$|\vec{A}+\vec{B}| = A+B \quad 0°$$

$$|\vec{A}+\vec{B}| = A-B \quad 180°$$

$$|\vec{A}+\vec{B}| = \sqrt{A^2+B^2} \quad 90°$$

Puppy 9 – Babli Addition

Difficulty Level : Hard

Q. Magnitude of 2 vectors A and B are equal and resultant C is 2 times that of A. Find the Angle between A and B as well as A and C.

Q10. Magnitude of 2 vectors A and B are equal and resultant C is √3 times that of A. Find the Angle between A and B as well as A and C.

Q11. Magnitude of 2 vectors A and B are equal and resultant C is √2 times that of A. Find the Angle between A and B as well as A and C.

Q12. Magnitude of 2 vectors A and B are equal and resultant is also equal. Find the Angle between A and B as well as A and resultant.

Q13. Magnitude of 2 vectors A and B are equal and resultant is null vector . Find the Angle between A and B as well as A and C.

QUESTION - 32

Difficulty Level : Easy

Q. Magnitude of Resultant of 2 vectors of equal to magnitude of the individual vectors. What is the angle between them

PYQ - (2003)

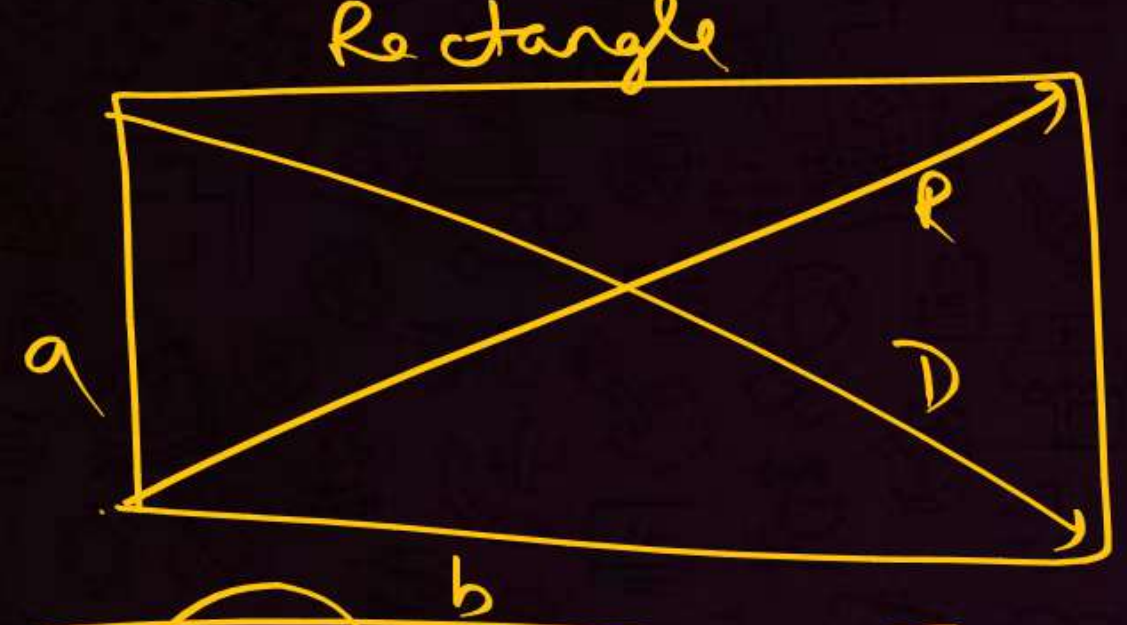
(A) 0^0

(B) 180^0

(C) 120^0

(D) 60^0

5.9 Diagonals based Problems

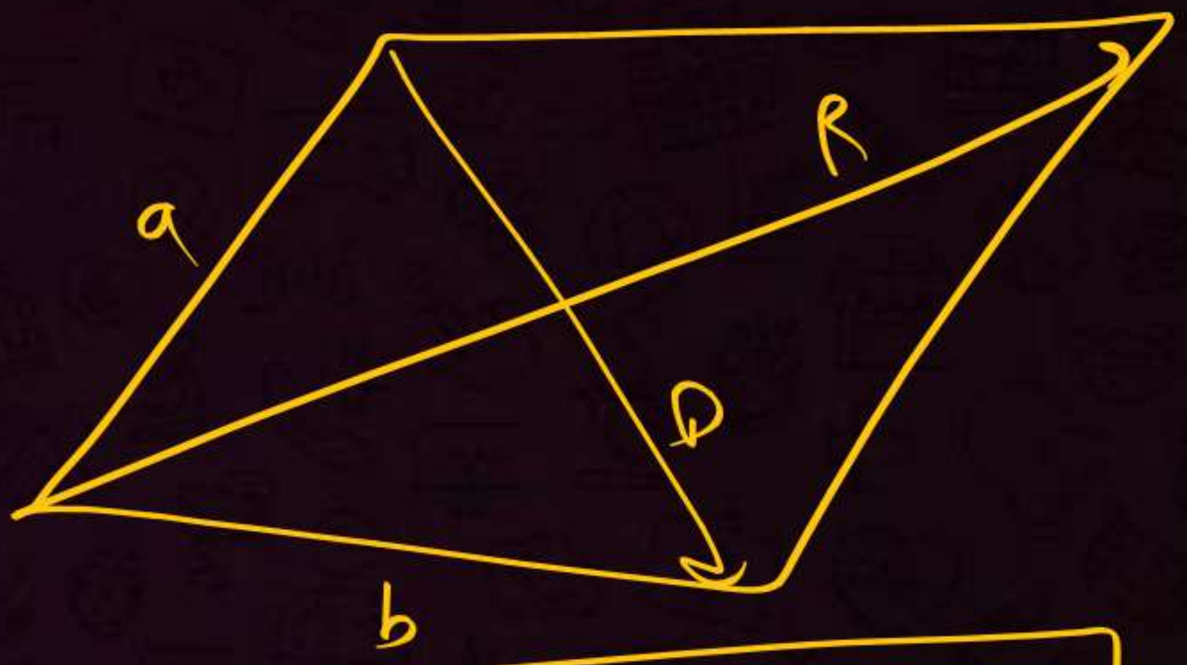


$a \perp b$ $\quad$ $R = D$

$$|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$$

Then ∠ btw A & B is

$= 90°$



$a = b$ $\quad$ $R \perp D$

If $\vec{A}+\vec{B}$ is $\perp$ $\vec{A}-\vec{B}$

then (a) $|A| = 2|B|$

(b) $|A| = 3|B|$

(c) $|A| = |B|$ ✓

Puppy 10 – Rhombus & Rectangle

Difficulty Level : Medium

1. $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$. What is the angle between $\vec{A}$ and $\vec{B}$

R = D   A ⊥ B   90°

2. $|\vec{A}| = |\vec{B}|$. What is the angle between $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$.

A = B   R ⊥ D   90°

QUESTION - 33

Difficulty Level : Medium

If the magnitude of sum of two vectors is equal to the magnitude of difference of the two vectors, the angle between these vectors is:

PYQ - (2016)

The vectors $\vec{A}$ and $\vec{B}$ are such that $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$. The angle between the two vectors is:

PYQ - (2006)

(1) 1°

(2) 90°

(3) 45°

(4) 180°

QUESTION - 34

Difficulty Level : Medium

The vector sum of two forces is perpendicular to their vector differences. In that case, the forces:

PYQ - (2003)

1. Are equal to each other
2. Are equal to each other in magnitude
3. Are not equal to each other in magnitude
4. Cannot be predicted

R ⊥ D

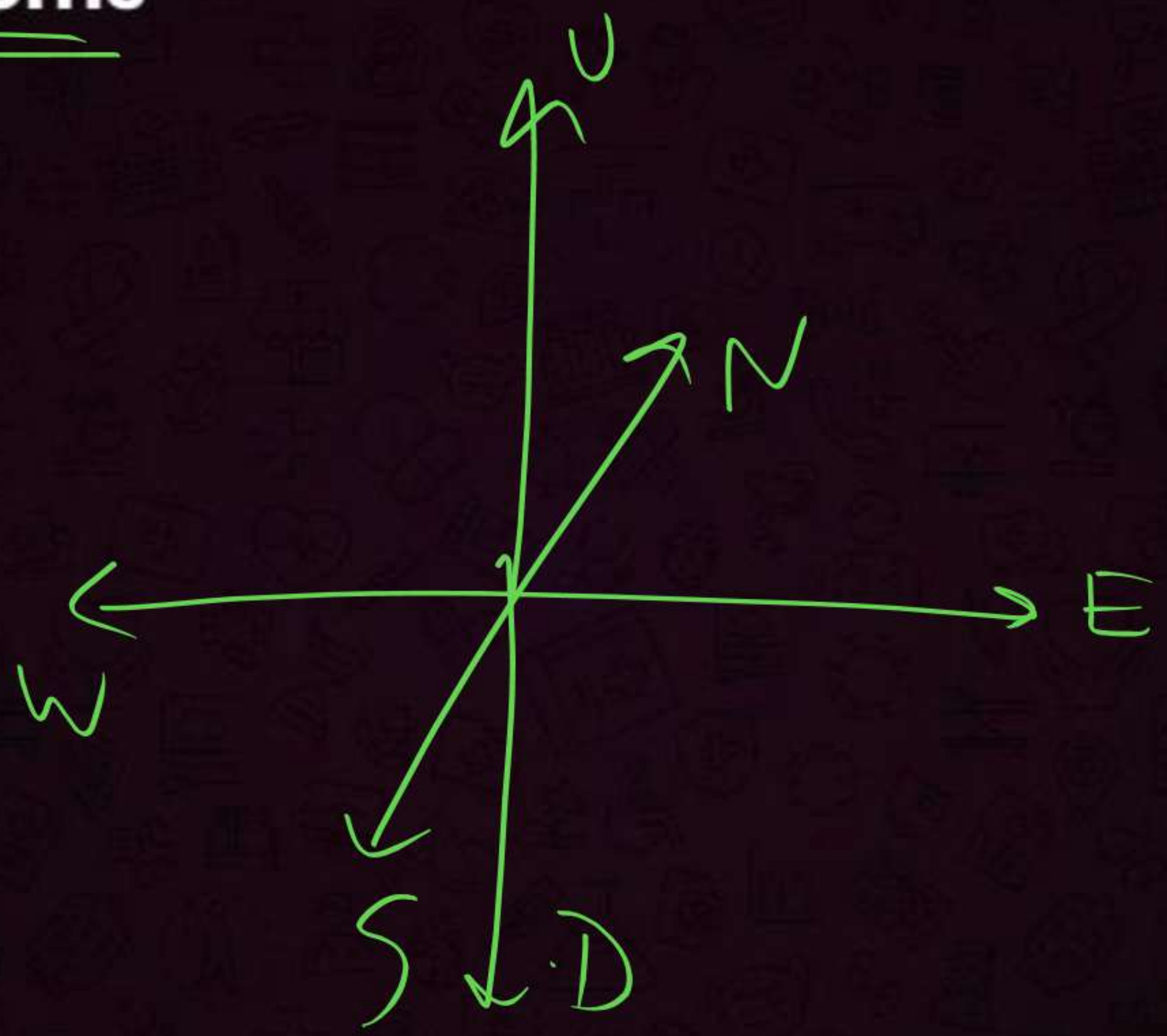
a = b

Puppy 11 – All Addition

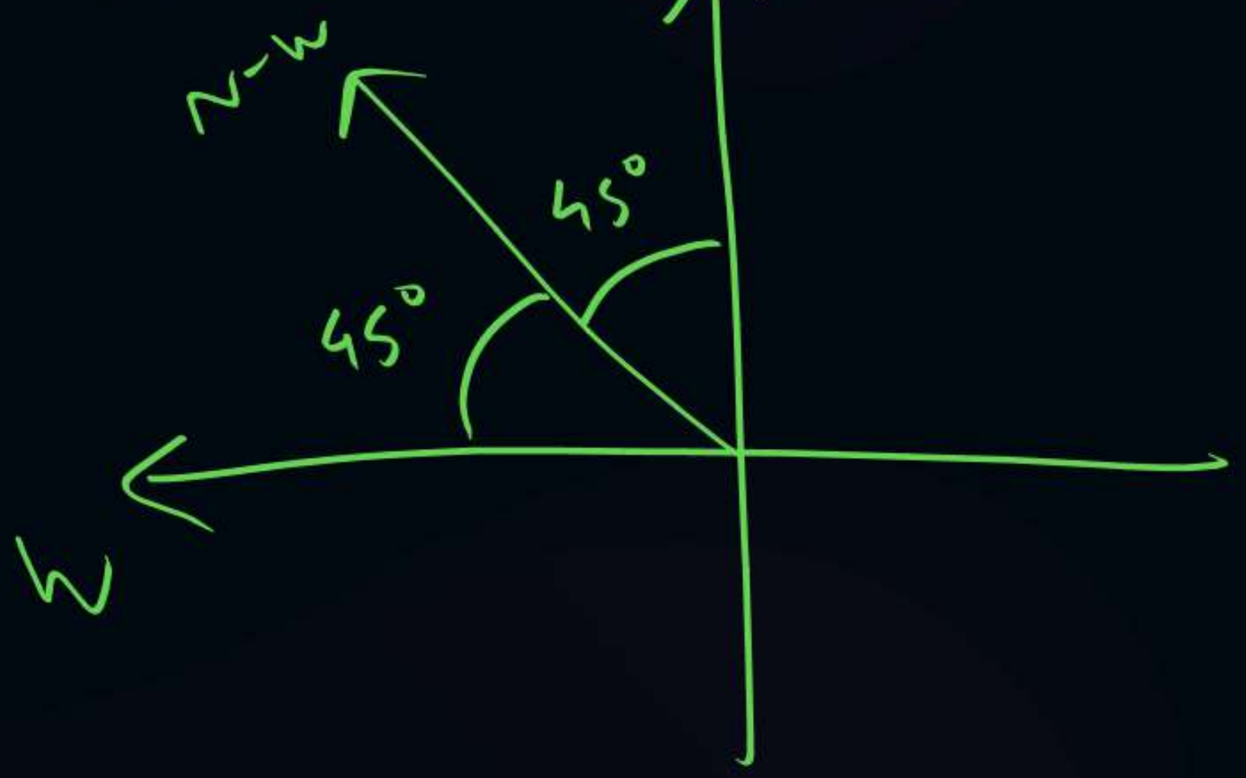
Difficulty Level : Easy

HW

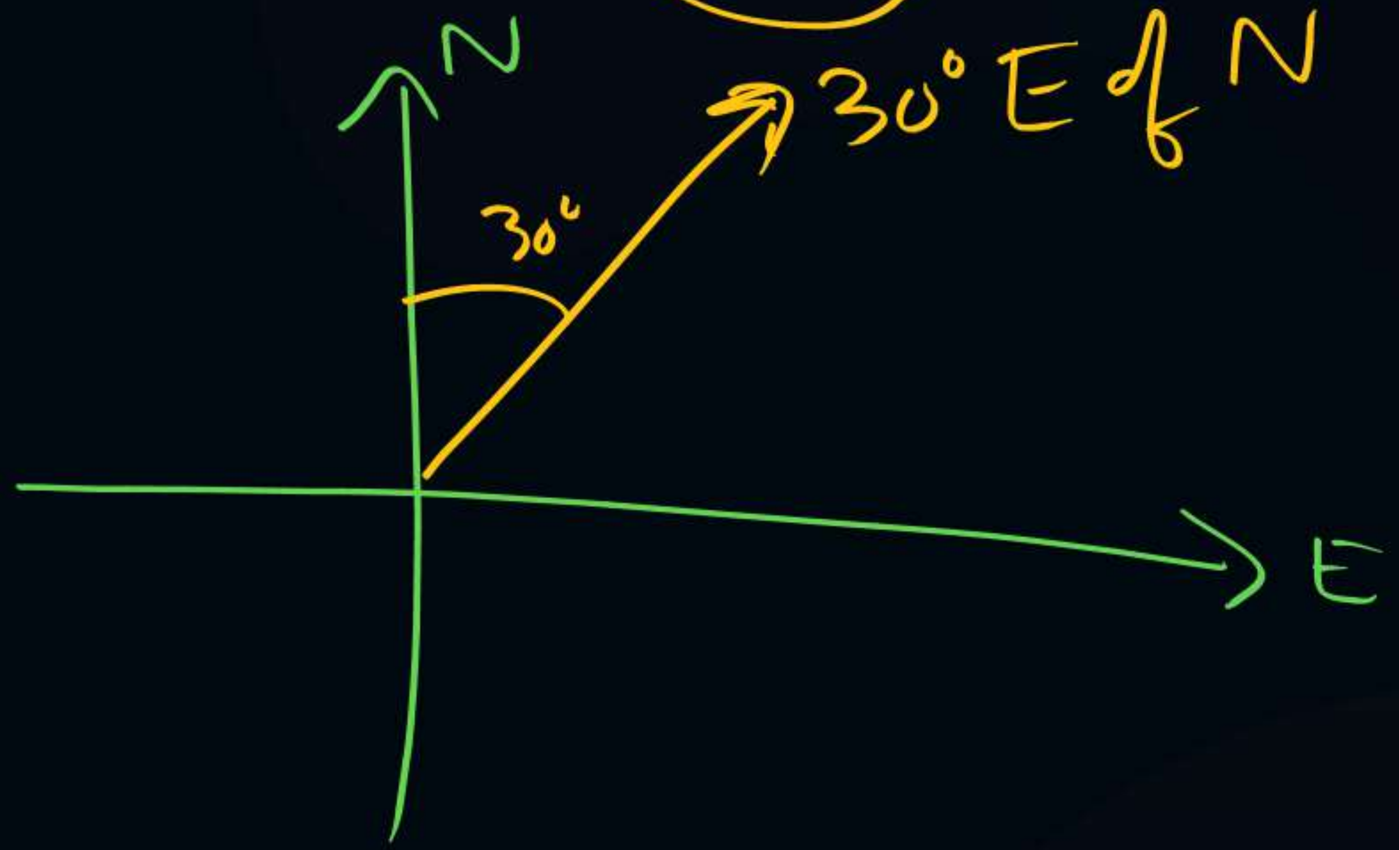
5.10 NEWS Problems



North-West



30° E of N

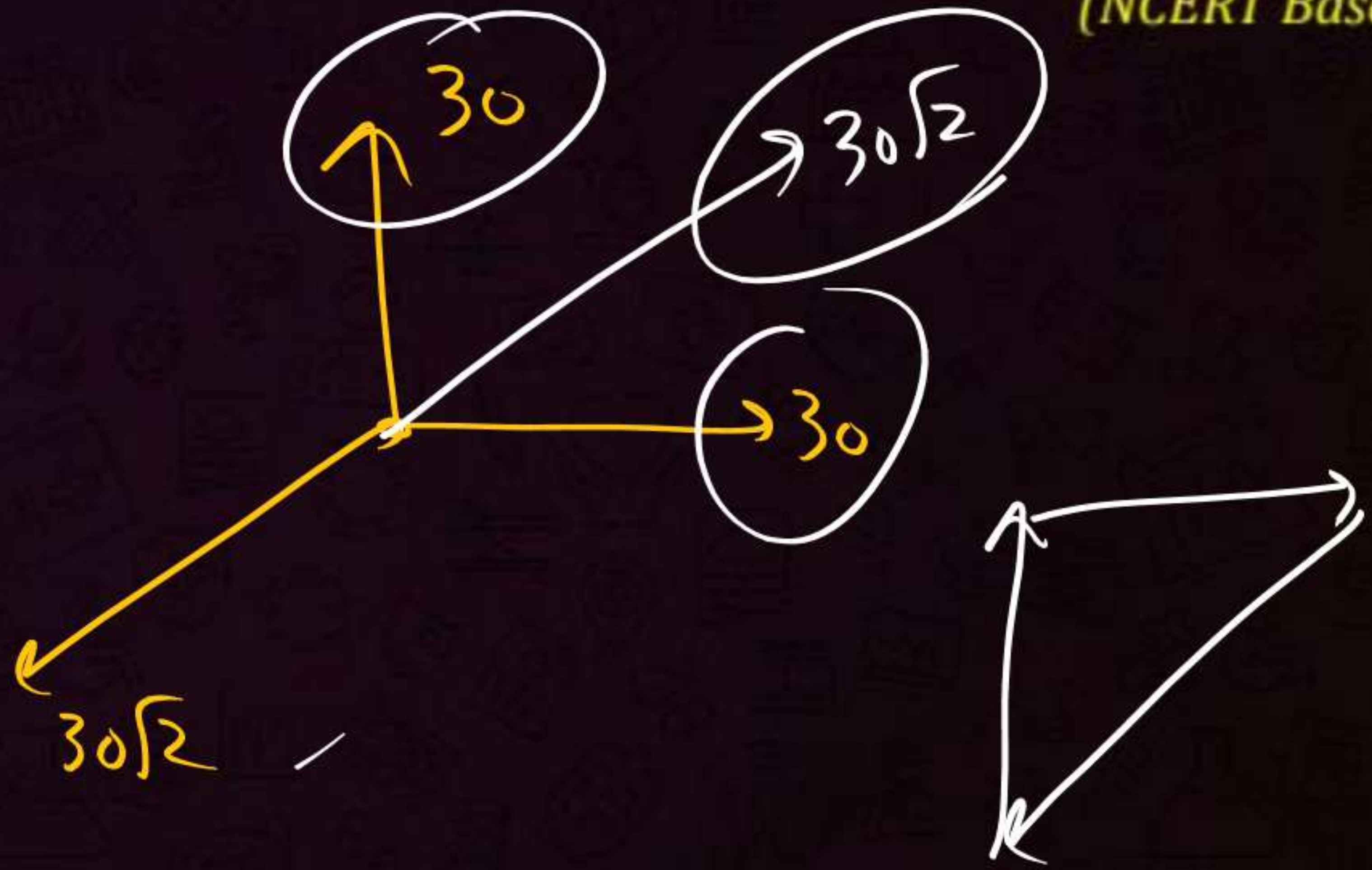


QUESTION - 35

Difficulty Level : Medium

A person moves 30 m north, then 30 m east, then $30\sqrt{2}$ m south-west. His displacement from the original position is

(NCERT Based)



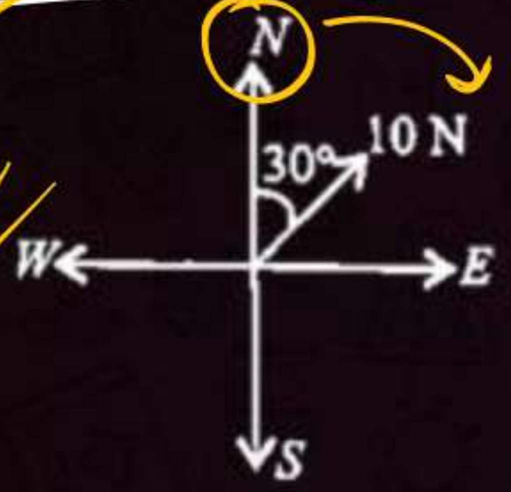
1. zero
2. 28 m towards south
3. 10 m towards west
4. 15 m towards east

QUESTION - 36

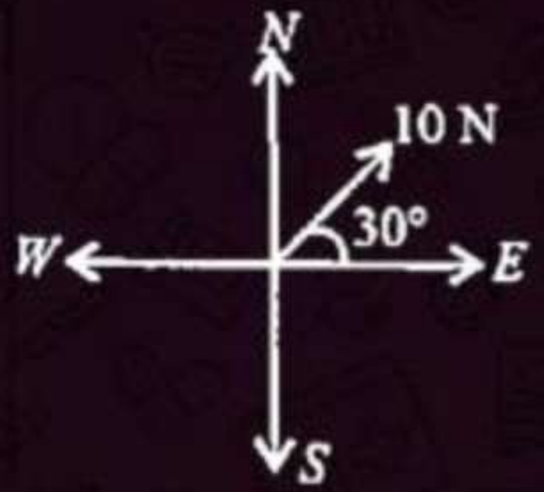
Difficulty Level : Hard

Which of the following figures represents the force of 10 N in a direction of 30°east of north?

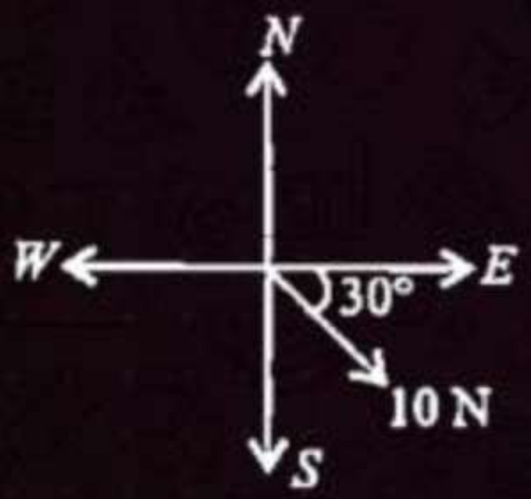
(1)



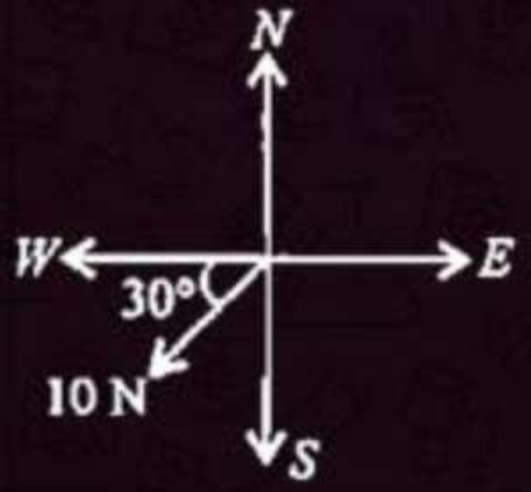
(2)



(3)



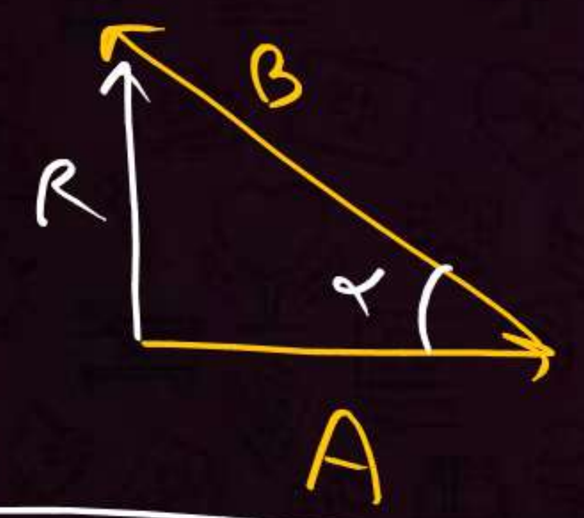
(4)



5.12 Special Case $\vec{A} + \vec{B} = \vec{R}$ R is perpendicular to A

Q Sum of 2 vectors is ⊥ to one of them

fix

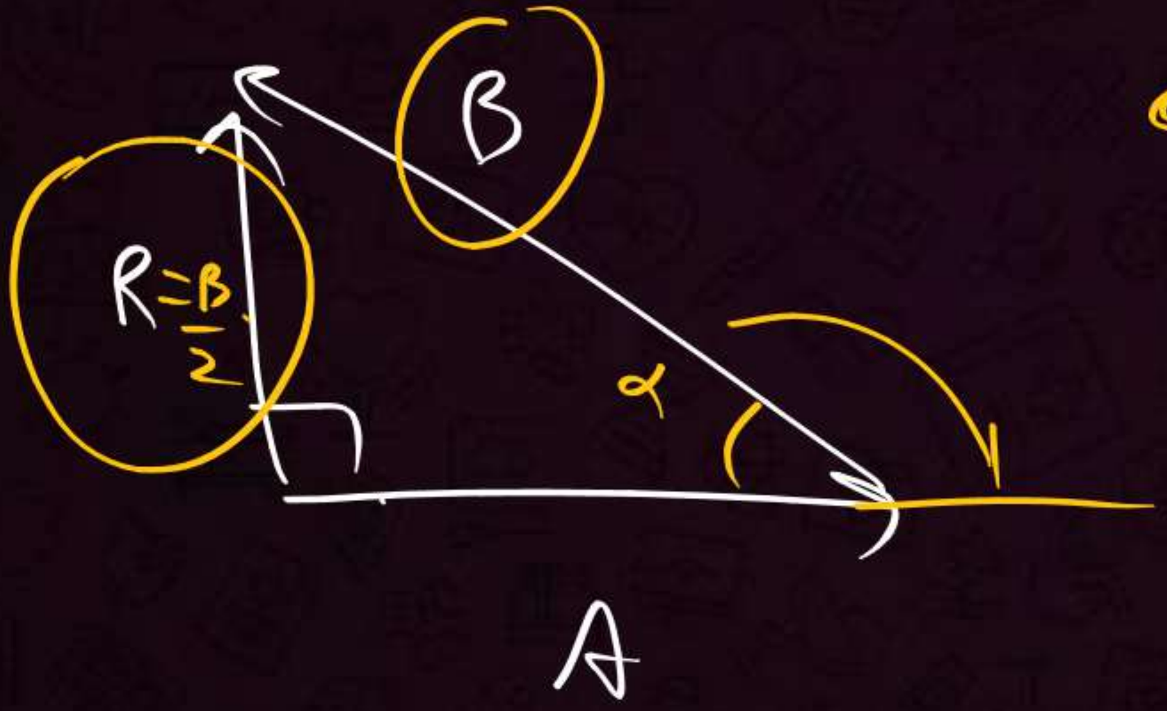


∠ bet A & B is $180 - \alpha$

QUESTION - 37

Difficulty Level : YODHA

The resultant of two vectors A and B is perpendicular to the vector A and its magnitude is equal to half the magnitude of vector B . The angle between A and B is



$$\sin\alpha = \frac{R}{B} = \frac{B}{2B} \qquad \alpha = 30^\circ$$

$$\theta = 180 - \alpha = 150^\circ$$

PUPPY POINTS – 2

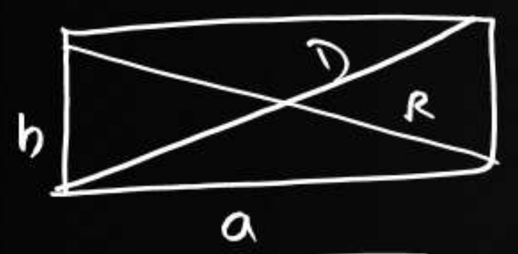
$\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$

$\vec{B} = x\hat{i} + y\hat{j} + c\hat{k}$

$\vec{A} + \vec{B} = \vec{B} + \vec{A}$

$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$

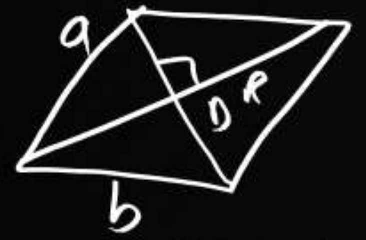
$\vec{A} - \vec{B} = -(\vec{B} - \vec{A})$



$R = D = \sqrt{a^2+b^2}$

Equal diagonals in rectangle

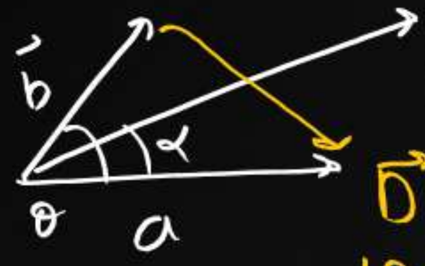
$a \perp b$



a = b Rhombus

$R \perp D$

Only vectors can be added or subtracted to give vectors.



$\vec{R} = \vec{a} + \vec{b}$

$|R| = \sqrt{a^2+b^2+2ab\cos\theta}$

$\vec{D} = \vec{a} - \vec{b}$

$|D| = \sqrt{a^2+b^2-2ab\cos\theta}$

$\tan\alpha = \dfrac{b\sin\theta}{a+b\cos\theta}$

Babli Addition

θ	0°	60°	90°	120°	180°	θ
R	2B	√3B	√2B	B	0	$2B\cos\left(\frac{\theta}{2}\right)$
D	0	B	√2B	√3B	2B	$2B\sin\left(\frac{\theta}{2}\right)$

$\vec{A} + \vec{B} = A + B \quad \theta = 0^\circ$

$\vec{A} + \vec{B} = A - B \quad \theta = 180^\circ$

$\vec{A} + \vec{B} = \sqrt{A^2+B^2} \quad \theta = 90^\circ$

$\vec{A} + \vec{B} = \sqrt{A^2+B^2+AB} \quad \theta = 60^\circ$

$\vec{A} + \vec{B} = \sqrt{A^2+B^2-AB} \quad \theta = 120^\circ$

$\vec{A} + \vec{B} = \sqrt{A^2+B^2+\sqrt{3}AB} \quad \theta = 30^\circ$

$\vec{A} + \vec{B} = \sqrt{A^2+B^2-\sqrt{3}AB} \quad \theta = 150^\circ$

$\vec{A} + \vec{B} = \sqrt{A^2+B^2+\sqrt{2}AB} \quad \theta = 45^\circ$

$\theta = 135^\circ$

HW

If A = B, θ =

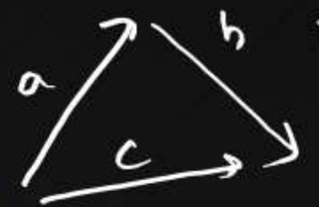
$|\vec{A}+\vec{B}| = 2A \quad 0^\circ$

$|\vec{A}+\vec{B}| = \sqrt{3}A \quad 60^\circ$

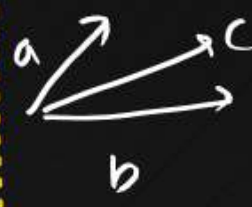
$|\vec{A}+\vec{B}| = \sqrt{2}A \quad 90^\circ$

$|\vec{A}+\vec{B}| = A \quad 120^\circ$

$|\vec{A}+\vec{B}| = 0 \quad 180^\circ$



$\vec{a} + \vec{b} = \vec{c}$ Triangle Law ✓

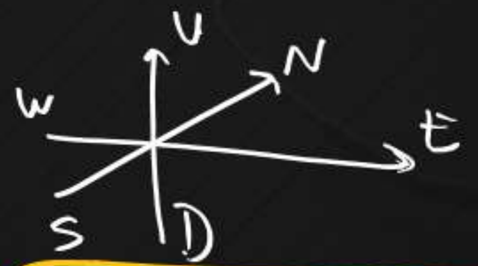


$\bar{a} + \bar{b} = \bar{c}$ ‖gm Law ✓

$\vec{d} = \vec{a} + \vec{b} + \vec{c}$ Polygon Law ✓

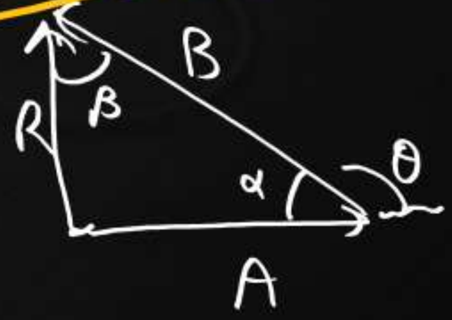
$|\vec{A}| = |\vec{B}|$, then rhombus, $\vec{A} + \vec{B}$ is ⊥ to $\vec{A} - \vec{B}$

$|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$, then rectangle, $|\vec{A}| = |\vec{B}|$



30° E of N ✓

$\vec{A} + \vec{B} = \vec{R}$ $\quad \vec{R} \perp \vec{A}$



$B^2 = A^2 + R^2$

Angle $\theta = 180 - \alpha$

$\tan\alpha = \dfrac{R}{A}$

Part 6 – Operations x ÷ on Vectors

6.1 – Multiplication of Scalar by Vector

Division

allowed ✓

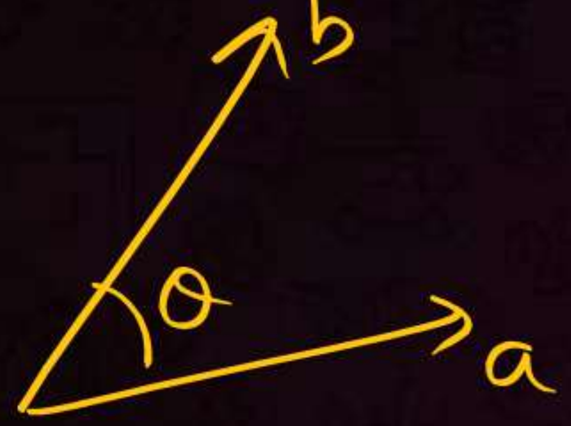
$$\vec{A} = \uparrow$$

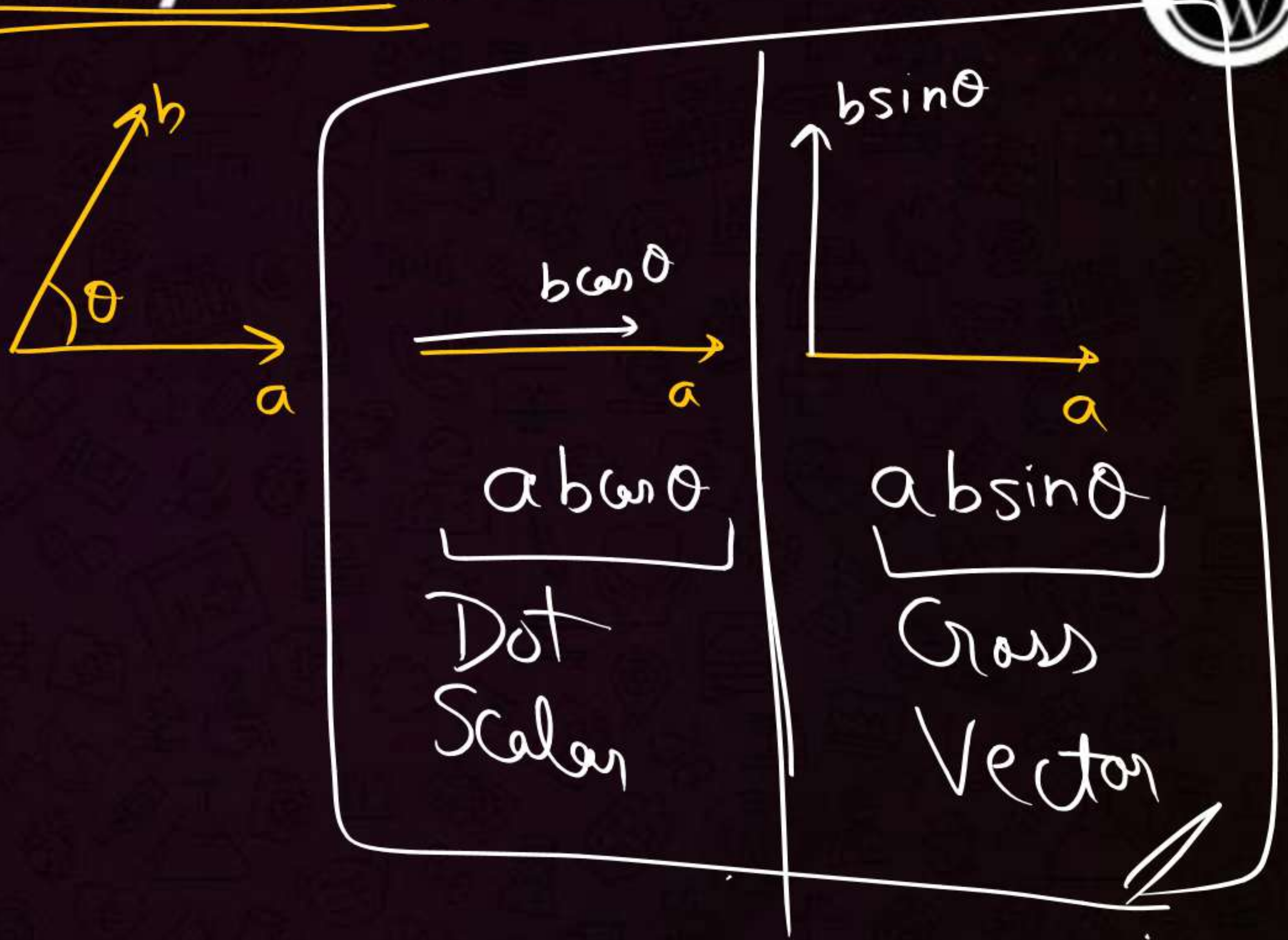
$$5\vec{A} = 5\uparrow$$

$$\frac{\vec{A}}{5} = \frac{\uparrow}{5}$$

→ Vector ✓

6.2 – Multiplication of Vector by Vector





6.3 – Scalar Product / Dot Product

$\vec{P} = a\hat{i} + b\hat{j} + c\hat{k}$

$\vec{Q} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{P}\cdot\vec{Q} = ax + by + cz$$

$\hat{i}\cdot\hat{i} = 1 \qquad \hat{i}\cdot\hat{j} = 0$

$\hat{j}\cdot\hat{j} = 1 \qquad \hat{j}\cdot\hat{k} = 0$

$\hat{k}\cdot\hat{k} = 1 \qquad \hat{k}\cdot\hat{i} = 0$

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$\vec{A}\cdot\vec{B} = 0$ —
1. $A = 0$
2. $B = 0$
3. $\cos\theta = 0$, $\theta = 90^\circ$

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|A||B|}$$

6.4 – Finding Angle between 2 vectors

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|A|\,|B|}$$

QUESTION - 38

Difficulty Level : Medium

The angle between the two vectors $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ will be

PYQ - (1994)

$\vec{A} \cdot \vec{B} = 9 + 16 - 25 = 0$

(1) 90°

(2) 180°

(3) zero

(4) 45°

QUESTION - 39

Difficulty Level : Medium

The angle between $\vec{A} = \hat{\imath} + \hat{\jmath}$ and $\vec{B} = \hat{\imath} + \hat{\jmath}$ is

(NCERT Exemplar)

(1) 45°

(2) 90°

(3) –45°

(4) 180°

$A \cdot B = (\hat{i} + \hat{j})(\hat{i} - \hat{j})$

$1 - 1 = 0$

QUESTION - 40

Difficulty Level : Medium

If $\vec{A} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{B} = -\hat{i} + \hat{j} - 2\hat{k}$, then angle between $\vec{A}$ and $\vec{B}$ is

(NCERT Based)

1. $\frac{\pi}{2}$
2. 0
3. π
4. $\frac{\pi}{3}$

$A \cdot B = -1 + 2 + 2$

$\vec{A} \cdot \vec{B} = 3$

$|A| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$

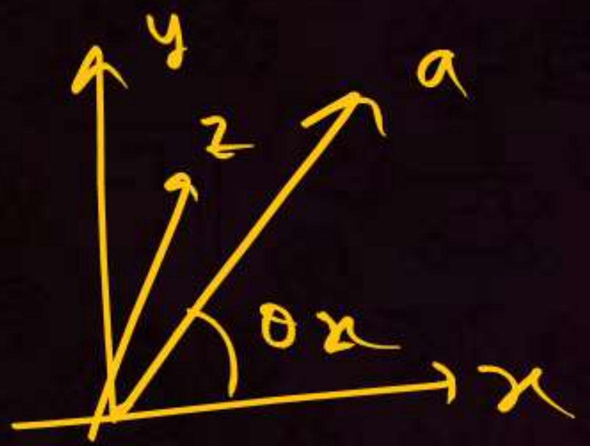
$|B| = \sqrt{6}$

$\cos\theta = \frac{A \cdot B}{|A||B|} = \frac{3}{\sqrt{6}\sqrt{6}} = \frac{3}{6} = \frac{1}{2}$

$\cos\theta = \frac{3}{6} = \frac{1}{2}$

$\theta = 60^\circ$

6.5 – Direction cosines



$$\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\cos\theta_x = \frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$\cos\theta_y = \frac{y}{\sqrt{x^2+y^2+z^2}}$$

$$\cos\theta_z = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

QUESTION - 41

Difficulty Level : Easy

If a vector $\vec{A}$ makes an angles α, β and γ respectively with the x, y and z axes respectively. Then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is equal to *(NCERT Based)*

(1) 0

(2) 1

(3) 2

(4) 3

QUESTION - 42

Difficulty Level : Medium

$x=1 \quad y=1 \quad z=1$

The direction cosines $\hat{\imath} + \hat{\jmath} + \hat{k}$ of are

(NCERT Based)

(1) 1, 1, 1

(2) 2, 2, 2

(3) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

(4) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ ✓

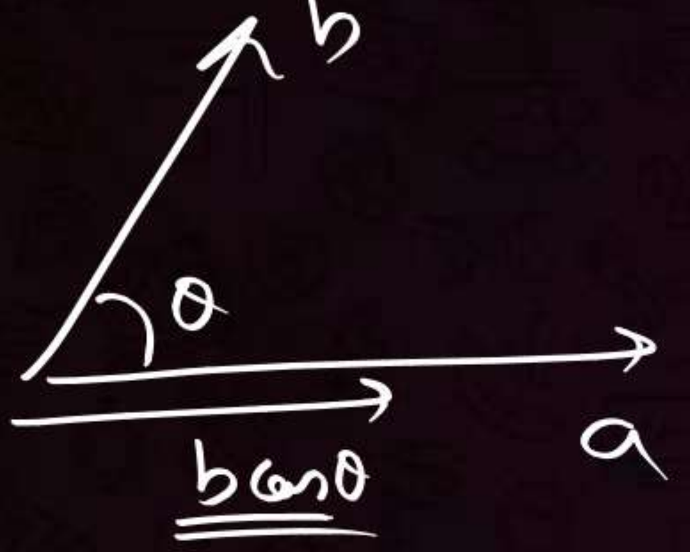
$$\cos\theta_x = \frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$= \frac{1}{\sqrt{1^2+1^2+1^2}}$$

$$= \frac{1}{\sqrt{3}}$$

6.6 – Scalar Projections & Vector Projection

(Useless)
(Less than 2%)



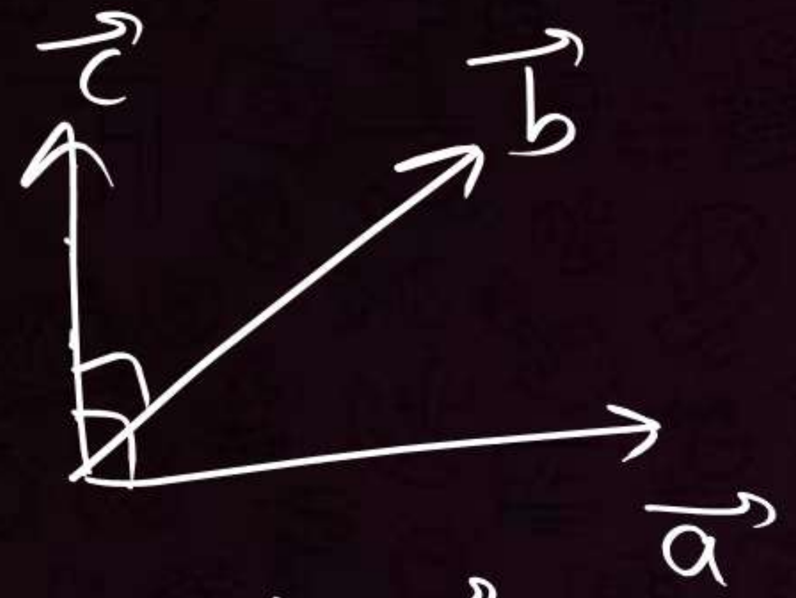
$b\cos\theta \longrightarrow$ Scalar Projection (SP)

$b\cos\theta\,\hat{a} \longrightarrow$ Vector Projection (VP)

$$SP = B\cos\theta = B\,\frac{\vec{A}\cdot\vec{B}}{AB}$$

$$VP = B\cos\theta\,\hat{A} = \left(\frac{\vec{A}\cdot\vec{B}}{A}\,\hat{A}\right)$$

6.7 – Vector Product / Cross Product



$\vec{c} = \vec{a} \times \vec{b}$

$\vec{c} \perp \vec{a}$ & $\vec{c} \perp \vec{b}$

$\vec{c} \perp (\vec{a}+\vec{b})$

$(\vec{a}-\vec{b})$

$(20\vec{a} - 10\vec{b})$

$$\vec{a} \times \vec{b} = ab\sin\theta\, \hat{n}$$

$\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$

$\vec{a} \times \vec{b}$

2 | 3 4 2 3 | 4

1 | 1 2 1 1 | 2

-4 -4 -3

6 4 2

$2\hat{i} + 0\hat{j} - \hat{k}$

$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$$

$+3 \quad -2 \quad -4$

$2 \quad 3 \quad 1 \quad 2$

$-1 \quad 2 \quad 2 \quad -1$

$4 \quad 6 \quad -1$

$$7\hat{i} + 4\hat{j} - 5\hat{k}$$

$\vec{A} = i + j + k$

$\vec{B} = 2\hat{i} + 2\hat{j} + 2k$

$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n} = 0$

If $\theta = 0^\circ$ or 180°

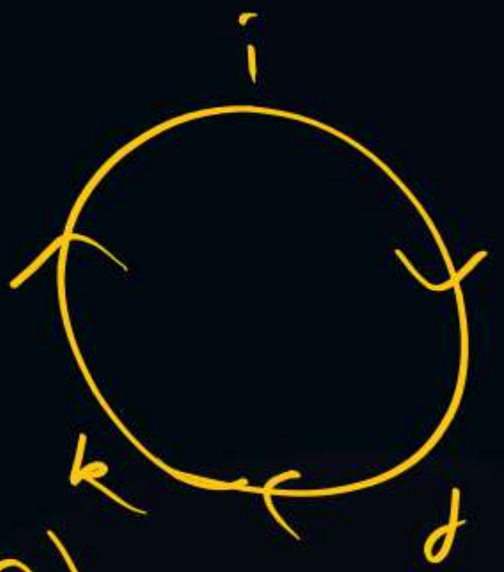
Parallel & Anti-parallel

$\hat{i} \times \hat{i} = 0$

$\hat{j} \times \hat{j} = 0$

$\hat{k} \times \hat{k} = 0$

i j k



$\hat{i} \times \hat{j} = \hat{k}$ $\quad$ $\hat{j} \times \hat{i} = -\hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$ $\quad$ $\hat{k} \times \hat{j} = -\hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$ $\quad$ $\hat{i} \times \hat{k} = -\hat{j}$

6.8 – NEWS Direction

HW → 3 times → less than 3 sec

N x N = 0	S x N = 0	E x N = U	W x N = D	U x N = W	D x N = E
N x S = 0	S x S = 0	E x S = D	W x S = U	U x S = E	D x S = W
N x E = D	S x E = U	E x E = 0	W x E = 0	U x E = N	D x E = S
N x W = U	S x W = D	E x W = 0	W x W = 0	U x W = S	D x W = N
N x U = E	S x U = W	E x U = S	W x U = N	U x U = 0	D x U = 0
N X D = W	S X D = E	E X D = N	W X D = S	U X D = 0	D X D = 0

QUESTION - 43

Difficulty Level : Hard

A vector $\vec{A}$ points vertically upward and $\vec{B}$ points towards north. The vector product $\vec{A} \times \vec{B}$ is

(HCV OBJECTIVE I)

$\vec{U} \times \vec{N}$

(1) along west

(2) along east

(3) zero

(4) vertically downward

QUESTION - 44

Difficulty Level : Hard

The magnitude of the vector product of two vectors $|\vec{A}|$ and $|\vec{B}|$ may be

(HCV OBJECTIVE II)

1. greater than AB
2. equal to AB
3. less than AB
4. equal to zero
5. ~~All of the above~~

QUESTION - 45

Difficulty Level : Easy

hw

The resultant of $\vec{A} \times 0$ will be equal to

PYQ - (1992)

(1) Zero

(2) A

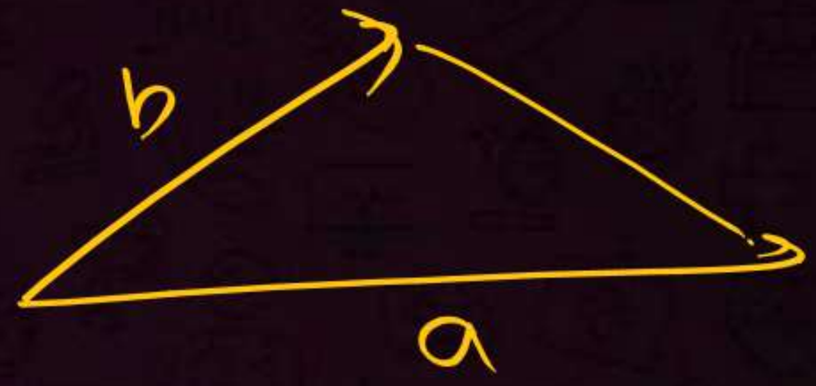
(3) Zero vector

(4) Unit vector

6.9 – Area of Triangle and Parallelogram

(useless) (×)

$$\text{Area}\parallel^{gm} = |\vec{a} \times \vec{b}|$$



$$\text{Area}\,\Delta = \frac{1}{2}\,|\vec{a} \times \vec{b}|$$

6.10 – Identities of Vectors

Commutative $\vec{A}+\vec{B}=\vec{B}+\vec{A}$ ✓

Associative $(\vec{A}+\vec{B})+\vec{C}=\vec{A}+(\vec{B}+\vec{C})$ ✓

$\vec{A}+\vec{B}=\vec{B}+\vec{A}$ ✓

$\vec{A}-\vec{B}=\vec{B}-\vec{A}$ ✗

$A-B=-(B-A)$

$\vec{A}\times\vec{B}=\vec{B}\times\vec{A}$ ✗

$(A\times B)=-(B\times A)$

$\vec{A}.\vec{B}=\vec{B}.\vec{A}$ ✓

$(\vec{A}+\vec{B})+\vec{C}=\vec{A}+(\vec{B}+\vec{C})$ ✓

$(A\times B)\times C=A\times(B\times C)$ ✗

$(A.B).C=A.(B.C)$

$(\vec{A}\times\vec{B}).\vec{C}=\vec{A}.(\vec{B}\times\vec{C})$ ✓

6.11 – Finding a vector perpendicular/ parallel

Parallel

$$\begin{pmatrix} \hat{i} + \hat{j} + \hat{k} \\ 2\hat{i} + 2\hat{j} + 2\hat{k} \end{pmatrix}$$

Anti Parallel

$$\begin{pmatrix} 3\hat{i} - 2\hat{j} + 5\hat{k} \\ -9\hat{i} + 6\hat{j} - 15\hat{k} \end{pmatrix} \quad -\frac{1}{3}$$

$$\vec{A} \cdot \vec{B} = 0$$

then A & B are perpendicular

$$2\hat{i} + 3\hat{j} \longrightarrow \hat{k}$$

$$\longrightarrow 3\hat{i} - 2\hat{j}$$

$$\star \begin{pmatrix} a\hat{i} + b\hat{j} \\ b\hat{i} - a\hat{j} \end{pmatrix}$$

Q A vector ⊥ to $8\hat{i} + 3\hat{k}$ is

(A) $\hat{j}$ ✓

(B) $3\hat{i} - 8\hat{k}$ ✓

(C)

(D)

Find a vector ⊥ to $\vec{A}$ & $\vec{B}$

$$\vec{C} = \vec{A} \times \vec{B}$$

(A & B ⊥ then $\vec{A} \cdot \vec{B} = 0$)

QUESTION - 46

Difficulty Level : Medium

Which of the following pairs of vectors are parallel?

(NCERT Based)

(1) $\vec{A} = \hat{i} - 2\hat{j}; \vec{B} = \hat{i} - 5\hat{j}$

(2) $\vec{A} = \hat{i} - 10\hat{j}; \vec{B} = 2\hat{i} - 5\hat{j}$

(3) $\vec{A} = \hat{i} - 5\hat{j}; \vec{B} = \hat{i} - 10\hat{j}$

(4) $\vec{A} = \hat{i} - 5\hat{j}; \vec{B} = 2\hat{i} - 10\hat{j}$

QUESTION - 47

Difficulty Level : Medium

If a vector $2\hat{\imath} + 3\hat{\jmath} + 8\hat{k}$ is perpendicular to the vector $4\hat{\jmath} - 4\hat{\imath} + \alpha\hat{k}$ then the value of α is

PYQ - (2005)

1. 1/2
2. –1/2
3. 1
4. –1

$2\hat{i} + 3\hat{j} + 8\hat{k}$

$-4\hat{i} + 4\hat{j} + \alpha\hat{k}$

$$-8 + 12 + 8\alpha = 0$$

$$4 + 8\alpha = 0$$

$$8\alpha = -4$$

$$\alpha = -\frac{4}{8} = -\frac{1}{2}$$

QUESTION - 48

Difficulty Level : Medium

The value of λ for which the two vectors $\vec{a} = 5\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ are perpendicular to each other is *(NCERT Based)*

1. 2
2. –2
3. 3
4. –3

$5 - 2\lambda + 1 = 0$

$3 \; 6 = 2\lambda$

QUESTION - 49

Difficulty Level : Medium

Given: $\vec{A} = 2\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{B} = -6\hat{i} + 9\hat{j} + 3\hat{k}$ which of the following statements is correct?

(NCERT Based)

(1) $\vec{A}$ and $\vec{B}$ are equal vectors.

(2) $\vec{A}$ and $\vec{B}$ are parallel vectors.

(3) $\vec{A}$ and $\vec{B}$ are perpendicular vectors.

(4) None of these

QUESTION - 50

Difficulty Level : Medium

The angle between force $\vec{F} = (3\hat{i} + 4\hat{j} - 5\hat{k})$ unit and displacement $\vec{d} = (5\hat{i} + 4\hat{j} + 3\hat{k})$ unit is

(NCERT Based)

hw

1. $\cos^{-1}(0.16)$
2. $\cos^{-1}(0.32)$
3. $\cos^{-1}(0.24)$
4. $\cos^{-1}(0.64)$

6.12 – Mix Questions

Puppy 9 – dot & cross

Difficulty Level : Hard

$\tan\theta = \frac{|\bar{A}\times\bar{B}|}{\bar{A}\cdot\bar{B}}$

If $|\vec{A}\times\vec{B}| = \sqrt{3}\,\vec{A}.\vec{B}$ then the value of angle between A and B is 60°

If $\sqrt{3}\,|\vec{A}\times\vec{B}| = \vec{A}.\vec{B}$ then the value of angle between A and B is 30°

If $|\vec{A}\times\vec{B}| = \vec{A}.\vec{B}$ then the value of angle between A and B is 45°

If $4|\vec{A}\times\vec{B}| = 3\vec{A}.\vec{B}$ then the value of angle between A and B is 37°

If $3|\vec{A}\times\vec{B}| = 4\,\vec{A}.\vec{B}$ then the value of angle between A and B is 53°

$\tan 37° = 3/4$

$\tan 53 = 4/3$

QUESTION - 51

Difficulty Level : Hard

If $|\vec{A} \times \vec{B}| = \sqrt{3}\, \vec{A}.\vec{B}$ then the value of $|\vec{A} + \vec{B}|$ is $= \sqrt{A^2 + B^2 + 2AB\cos 60^\circ}$

$\sqrt{A^2 + B^2 + 2AB\frac{1}{2}}$

PYQ - (2004)

(1) $\left(A^2 + B^2 + \frac{AB}{\sqrt{3}}\right)^{1/2}$

(2) $A + B$

(3) $(A^2 + B^2 + \sqrt{3}AB)^{1/2}$

(4) $(A^2 + B^2 + AB)^{1/2}$

$\tan\theta = \frac{|A \times B|}{A \cdot B} = \sqrt{3}$

$\theta = 60^\circ$

$|\vec{A}\times\vec{B}| = \vec{A}\cdot\vec{B}$ → $\theta = 45^\circ$

then value of $|\vec{A}+\vec{B}| =$

(A) $\sqrt{A^2+B^2}$

(B) $\sqrt{A^2+B^2+2AB}$

(C) $\sqrt{A^2+B^2+\sqrt{2}AB}$ ✓

(D) $\sqrt{A^2+B^2+\sqrt{3}AB}$

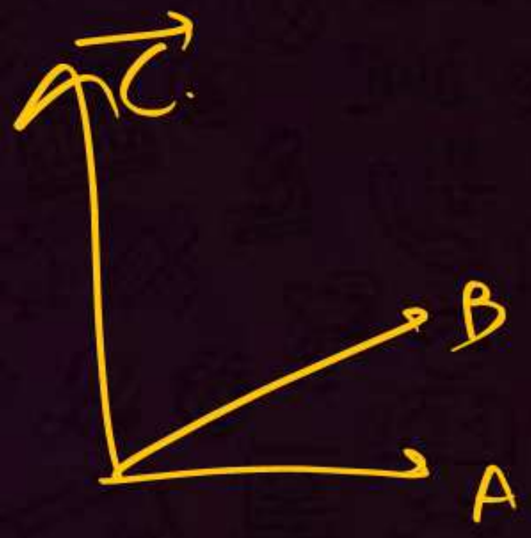
$A^2+B^2+\cancel{2}^{\sqrt{2}}\dfrac{AB}{\cancel{\sqrt{2}}}$

QUESTION - 52

Difficulty Level : Medium

If the angle between $\vec{A}$ and $\vec{B}$ is θ, the value of the product $(\vec{B} \times \vec{A}) . \vec{A}$ is equal to

PYQ - (2005)



1. $BA^2 \sin\theta$
2. $BA^2 \cos\theta$
3. $BA^2 \sin\theta \cos\theta$
4. zero

QUESTION - 53

Difficulty Level : Medium

If $\left|\vec{a}.\vec{b}\right| = \left|\vec{a} \times \vec{b}\right|$, then angle between $\vec{a}$ and $\vec{b}$ will be:

(NCERT Based)

(1) 30°

(2) 45°

(3) 60°

(4) 90°

QUESTION - 54

Difficulty Level : ~~Hard~~ Yodha

(NCERT Based)

If $\vec{A} + \vec{B} + \vec{C} = 0$, then $\vec{A} \times \vec{B}$ is

(1) $\vec{B} \times \vec{C}$ ✓

(2) $\vec{C} \times \vec{B}$

(3) $\vec{A} \times \vec{C}$

(4) none of these

A, B, C are coplanar

$$\vec{A} + \vec{B} + \vec{C} = 0$$

$$\vec{A} \times \vec{B} + \vec{B} \times \vec{B} + \vec{C} \times \vec{B} = 0 \times \vec{B}$$

$$\vec{B} \times \vec{B} = 0, \quad 0 \times \vec{B} = 0$$

$$\vec{A} \times \vec{B} = -\vec{C} \times \vec{B}$$

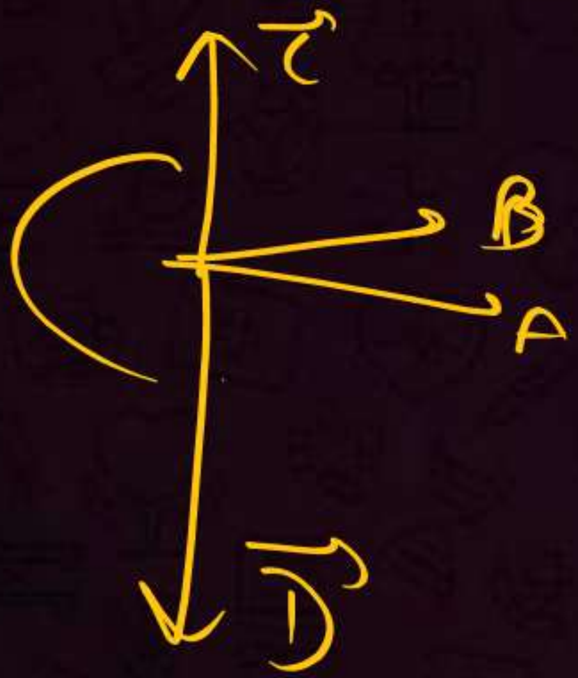
$$\vec{A} \times \vec{B} = \vec{B} \times \vec{C}$$

QUESTION - 55

Difficulty Level : Hard

Given $\vec{C} = \vec{A} \times \vec{B}$ and $\vec{D} = \vec{B} \times \vec{A}$. What is the angle between $\vec{C}$ and $\vec{D}$?

(NCERT Based)



1. 30°
2. 60°
3. 90°
4. 180°

QUESTION - 56

Difficulty Level : YODHA

P W

Challenge

Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ add up to zero. Find which is false.

(NCERT Exemplar)

1. $(\vec{A} \times \vec{B}) \times \vec{C}$ is not zero unless $\vec{B}$, $\vec{C}$ are parallel.
2. $(\vec{A} \times \vec{B}) . \vec{C}$ is not zero unless $\vec{B}$, $\vec{C}$ are parallel.
3. If $\vec{A}$, $\vec{B}$, $\vec{C}$ define a plane, $(\vec{A} \times \vec{B}) \times \vec{C}$ is in that plane.
4. $(\vec{A} \times \vec{B}) . \vec{C} = |\vec{A}|\,|\vec{B}|\,|\vec{C}| \rightarrow C^2 + A^2 + B^2$.

QUESTION - 57

Difficulty Level : YODHA

Chanllenge 2

Resultant of two vectors $\vec{A}$ and $\vec{B}$ is of magnitude P. If $\vec{B}$ is reversed, then resultant is of magnitude Q. What is the value of $P^2 + Q^2$?

(NCERT Based)

(1) $2(A^2 + B^2)$

(2) $2(A^2 - B^2)$

(3) $A^2 - B^2$

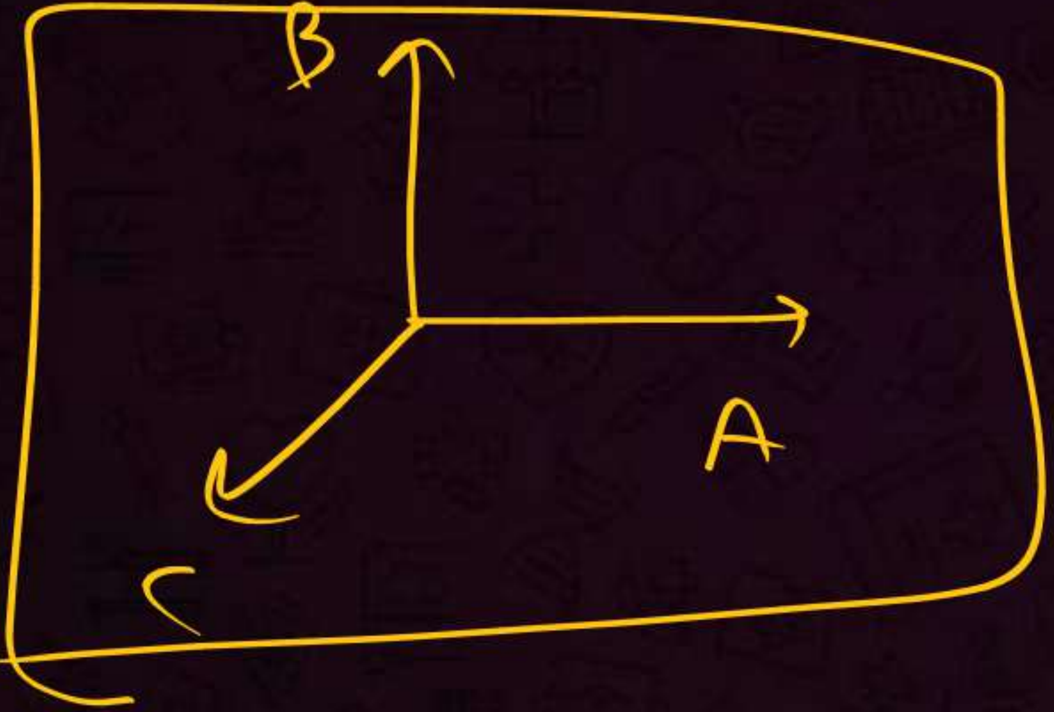
(4) $A^2 + B^2$

QUESTION - 58

Difficulty Level : YODHA

Vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are such that $\vec{A}\cdot\vec{B} = 0$ and $\vec{B}\cdot\vec{C} = 0$. Then the vector parallel to $\vec{A}$ is

(NCERT Based)



C ⊥ A

1. $\vec{A} \times \vec{B}$
2. $\vec{B} + \vec{C}$
3. $\vec{B} \times \vec{C}$
4. $\vec{B}$ and $\vec{C}$

PUPPY POINTS - 3

$(\vec{V}) \times$ Scalar = Vector

$\vec{A}$ multiplied by $\vec{B}$

Dot — Scalar

Cross — Vector

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|A||B|}$ $\left\{ \begin{matrix} 0 \text{ is} \\ \theta = 90^\circ \end{matrix} \right\}$

$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

$\bar{A} = a\hat{i} + b\hat{j} + c\hat{k}$

$\bar{B} = x\hat{i} + y\hat{j} + z\hat{k}$

$\bar{A}\cdot\bar{B} = ax + by + cz$

$\vec{B}$ on $\vec{A}$ Scalar $= B\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|A|}$

Vector $= B\cos\theta\hat{A} = \left(\dfrac{\vec{A}\cdot\vec{B}}{|A|}\right)\hat{A} = \dfrac{(\bar{A}\cdot\bar{B})\vec{A}}{|A|^2}$

$\cos\theta_x = x/|a| = x/\sqrt{x^2+y^2+z^2}$

$\cos\theta_y = y/|a| = y/\sqrt{x^2+y^2+z^2}$

$\cos\theta_z = z/|a| = z/\sqrt{x^2+y^2+z^2}$

$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$\sin^2\theta_x + \sin^2\theta_y + \sin^2\theta_z = 2$

$\vec{A}\times\vec{B} = |A||B|\sin\theta\,\hat{n}$

$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

$\vec{A}\times\vec{B}$ is $\perp$ to plane of $\vec{A}$ & $\vec{B}$

$\bar{A}\times\bar{B} = 0$ if $\theta = 0^\circ / 180^\circ$

$i\times j = k$ $\quad$ $j\times i = -k$

$j\times k = i$ $\quad$ $k\times j = -i$

$k\times i = j$ $\quad$ $i\times k = -j$

$i\times i = j\times j = k\times k = 0$

$\tan\theta = \dfrac{|\bar{A}\times\bar{B}|}{A\cdot B}$

Area $\Delta = \dfrac{1}{2}|\bar{A}\times\bar{B}|$

Area $||gm = |\bar{A}\times\bar{B}|$

$\bar{A}+\bar{B} = \bar{B}+\bar{A}$

$\bar{A}+(\bar{B}+\bar{C}) = \bar{A}+(\bar{B}+\bar{C})$

$\bar{A}-\bar{B} = -(\bar{B}-\bar{A})$

$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

$\vec{A}\times\vec{B} = -(\vec{B}\times\vec{A})$

$(\vec{A}\cdot\vec{B})C \neq \bar{A}(\bar{B}\cdot\bar{C})$

$(\bar{A}\times\bar{B})\times\bar{C} \neq \bar{A}\times(\bar{B}\times\bar{C})$

$(\bar{A}\times\bar{B})\cdot\bar{C} = \bar{A}\cdot(\bar{B}\times\bar{C}) \rightarrow$ VTP ✓

$\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$

$\vec{B} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{A}\parallel\vec{B}$ if $\dfrac{a}{x} = \dfrac{b}{y} = \dfrac{c}{z}$ → +ve $\parallel$ / −ve anti $\parallel$

$\vec{A}\perp\vec{B}$ if $A\cdot B = 0 \Rightarrow ax + by + cz = 0$

A vector $\vec{C}\perp$ to both A & B is $\vec{C} = \vec{A}\times\vec{B}$

$a\hat{i} + b\hat{j}$ is $\perp$ to $b\hat{i} - a\hat{j}$ & $\hat{k}$

Part 7 – Theory & Assertion and Reason

(Assertion and Reason) - 60

Assertion: Two vectors are said to be equal if, and only if, they have the same magnitude and the same direction.

Reason: Addition and subtraction of scalars make sense only for quantities with same units.

1. If both A and R are true and reason is the correct explanation of assertion.
2. If both A and R are true but reason is not the correct explanation of assertion.
3. If assertion is true but reason is false.
4. If both assertion and reason are false.

(Assertion and Reason) 61

Assertion: Vector addition is commutative.

Reason: Two vectors may be added graphically using head-to-tail method or parallelogram method.

1. If both A and R are true and reason is the correct explanation of assertion.
2. If both A and R are true but reason is not the correct explanation of assertion.
3. If assertion is true but reason is false.
4. If both assertion and reason are false.

(Assertion and Reason) 62

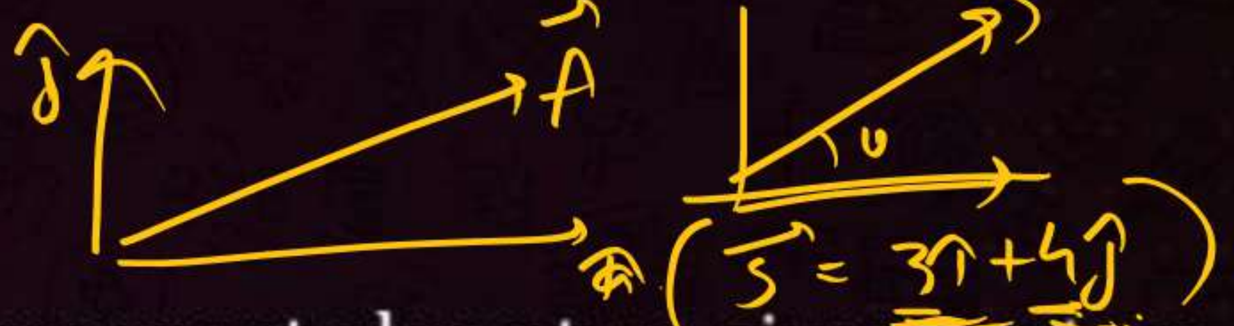
$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

Assertion : The difference of two vectors $\vec{A}$ and $\vec{B}$ can be treated as the sum of two vectors.

Reason: Subtraction of vectors can be defined in terms of addition of vectors.

1. If both A and R are true and reason is the correct explanation of assertion.
2. If both A and R are true but reason is not the correct explanation of assertion.
3. If assertion is true but reason is false.
4. If both assertion and reason are false.

(Assertion and Reason) 63



Assertion: A vector $\vec{A}$ can be resolved into component along two given vectors $\vec{a}$ and $\vec{b}$ lying in the same plane.

Reason: $\vec{A} = \lambda\vec{a} + \mu\vec{b}$ where λ and μ are real numbers.

1. If both A and R are true and reason is the correct explanation of assertion.
2. If both A and R are true but reason is not the correct explanation of assertion.
3. If assertion is true but reason is false.
4. If both assertion and reason are false.

(Assertion and Reason) 64

Assertion: Magnitude of the resultant of two vectors may be less than the magnitude of either vector.

Reason: The resultant of two vectors is obtained by means of law of parallelogram of vectors.

1. If both A and R are true and reason is the correct explanation of assertion.
2. If both A and R are true but reason is not the correct explanation of assertion.
3. If assertion is true but reason is false.
4. If both assertion and reason are false.

(Assertion and Reason) 65

Assertion: If $\hat{\imath}$ and $\hat{\jmath}$ are unit vectors along x-axis and y-axis respectively, the magnitude of vector $\hat{\imath} + \hat{\jmath}$ will be $\sqrt{2}$.

Reason: Unit vectors are used to indicate direction only.

1. If both A and R are true and reason is the correct explanation of assertion.
2. If both A and R are true but reason is not the correct explanation of assertion.
3. If assertion is true but reason is false.
4. If both assertion and reason are false.

(Assertion and Reason) 66

Assertion: $\vec{A} \times \vec{B}$ is perpendicular to both $\vec{A} + \vec{B}$ as well as $\vec{A} - \vec{B}$.

Reason: $\vec{A} + \vec{B}$ as well $\vec{A} - \vec{B}$ lie in the plane containing $\vec{A}$ and $\vec{B}$, but $\vec{A} \times \vec{B}$ lies perpendicular to the plane containing $\vec{A}$ and $\vec{B}$ both.

1. If both A and R are true and reason is the correct explanation of assertion.
2. If both A and R are true but reason is not the correct explanation of assertion.
3. If assertion is true but reason is false.
4. If both assertion and reason are false.

(Assertion and Reason) 67

Assertion: Angle between $\hat{\imath} + \hat{\jmath}$ and $\hat{\imath}$ is $45°$.

Reason: $\hat{\imath} + \hat{\jmath}$ is equally inclined to both $\hat{\imath}$ and $\hat{\jmath}$ and the angle between $\hat{\imath}$ and $\hat{\jmath}$ is $90°$.

1. If both A and R are true and reason is the correct explanation of assertion.
2. If both A and R are true but reason is not the correct explanation of assertion.
3. If assertion is true but reason is false.
4. If both assertion and reason are false.

(Assertion and Reason) 68

Assertion: If θ be the angle between $\vec{A}$ and $\vec{B}$, then

$$\tan\theta = \frac{\vec{A}\times\vec{B}}{\vec{A}.\vec{B}}$$

Reason: $\vec{A} \times \vec{B}$ is perpendicular to $\vec{A}.\vec{B}$.

1. If both A and R are true and reason is the correct explanation of assertion.
2. If both A and R are true but reason is not the correct explanation of assertion.
3. If assertion is true but reason is false.
4. If both assertion and reason are false.

(Assertion and Reason) 69

Assertion: If $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, then the angle between $\vec{A}$ and $\vec{B}$ is $90°$.

Reason: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$.

1. If both A and R are true and reason is the correct explanation of assertion.
2. If both A and R are true but reason is not the correct explanation of assertion.
3. If assertion is true but reason is false.
4. If both assertion and reason are false.

(Assertion and Reason) 70

Assertion: Minimum number of non-equal vectors in a plane required to give zero resultant is three.

Reason: If $\vec{A} + \vec{B} = \vec{C} = \vec{0}$, then they must lie in one plane.

1. If both A and R are true and reason is the correct explanation of assertion.
2. If both A and R are true but reason is not the correct explanation of assertion.
3. If assertion is true but reason is false.
4. If both assertion and reason are false.

(Assertion and Reason) 71

Assertion: Vector addition of two vectors $\vec{A}$ and $\vec{B}$ is com- mutative.

Reason: : $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

1. If both A and R are true and reason is the correct explanation of assertion.
2. If both A and R are true but reason is not the correct explanation of assertion.
3. If assertion is true but reason is false.
4. If both assertion and reason are false.

(Assertion and Reason) 72

Assertion: $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

Reason: Dot product of two vectors is commutative.

(1) If both A and R are true and reason is the correct explanation of assertion.

(2) If both A and R are true but reason is not the correct explanation of assertion.

(3) If assertion is true but reason is false.

(4) If both assertion and reason are false.

(Assertion and Reason) 73

Assertion: $\vec{\tau} = \vec{r} \times \vec{F}$ and $\vec{\tau} \neq \vec{F} \times \vec{r}$

Reason: Cross product of vectors is commutative.

1. If both A and R are true and reason is the correct explanation of assertion.
2. If both A and R are true but reason is not the correct explanation of assertion.
3. If assertion is true but reason is false.
4. If both assertion and reason are false.

(Assertion and Reason) 74

Assertion: A physical quantity cannot be called a vector if its magnitude is zero.

Reason: A vector has both non-zero magnitude and direction.

(1) If both A and R are true and reason is the correct explanation of assertion.

(2) If both A and R are true but reason is not the correct explanation of assertion.

(3) If assertion is true but reason is false.

(4) If both assertion and reason are false.

(Assertion and Reason) 75

Assertion: The sum of two vectors can be zero.

Reason: The vectors cancel each other, when they are equal and opposite.

(1) If both A and R are true and reason is the correct explanation of assertion.

(2) If both A and R are true but reason is not the correct explanation of assertion.

(3) If assertion is true but reason is false.

(4) If both assertion and reason are false.

(Assertion and Reason) 76

Assertion: Two vectors are said to be like vectors if they have the same direction but different magnitude.

Reason: Vector quantities do not have a specific direction.

1. If both A and R are true and reason is the correct explanation of assertion.
2. If both A and R are true but reason is not the correct explanation of assertion.
3. If assertion is true but reason is false.
4. If both assertion and reason are false.

(Assertion and Reason) 77

Assertion: The scalar product of two vectors can be zero.

Reason: If two vectors are perpendicular to each other, their scalar product will be zero.

1. If both A and R are true and reason is the correct explanation of assertion.
2. If both A and R are true but reason is not the correct explanation of assertion.
3. If assertion is true but reason is false.
4. If both assertion and reason are false.

(Assertion and Reason) 78

Assertion: Multiplying any vector by a scalar is a meaningful operation.

Reason: Product of vector and scalar is always a vector.

(1) If both A and R are true and reason is the correct explanation of assertion.

(2) If both A and R are true but reason is not the correct explanation of assertion.

(3) If assertion is true but reason is false.

(4) If both assertion and reason are false.

(Assertion and Reason) 79

Assertion: A null vector is a vector whose magnitude is zero and direction is arbitrary.

Reason: A null vector does not exist.

1. If both A and R are true and reason is the correct explanation of assertion.
2. If both A and R are true but reason is not the correct explanation of assertion.
3. If assertion is true but reason is false.
4. If both assertion and reason are false.

(Assertion and Reason) 80

Assertion: If dot product and cross product of $\vec{A}$ and $\vec{B}$ are zero, it implies that one of the vector $\vec{A}$ or $\vec{B}$ must be a null vector.

Reason: Null vector is a vector with zero magnitude.

1. If both A and R are true and reason is the correct explanation of assertion.
2. If both A and R are true but reason is not the correct explanation of assertion.
3. If assertion is true but reason is false.
4. If both assertion and reason are false.

Part 7 – PUPPY POINTS

PUPPY POINTS-1

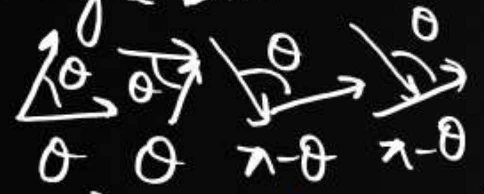
Scalar
- → Magnitude
- → ~~Independent of axis~~

Vector
- → Magnitude
- → Direction
- → Law of $+^n$

Current, Moment of Inertia, Stress, strain] Scalar

Area] → Scalar/vector

Angle btw



$\vec{a}+\vec{b} = (a-b)$ to $(a+b)$
(180° … 0°)

Tail → Head (Direction)
(Magnitude)

- * Vector can be moved ∥
- Vector cannot be rotated
- ~~$\vec{A}+\vec{B}-\vec{A} \to \vec{B}=0$~~

All equal are ∥
All negative are anti∥] Not vice versa

- Equal → same M, D
- Negative → same M, opp D
- ∥ → same D
- anti∥ → opp D
- orthogonal → ⊥
- coinitial
- concurrent
- colinear →→
- coplanar (same plane)

- 2 vectors are always coplanar
- Sum of 2 unequal coplanar cannot be 0
- 3 vector coplanar (Vol = 0)

$\vec{A}$ - vector

$|\vec{A}|$ → magnitude

$\hat{A}$ → unit vector

$\hat{A} = \frac{\vec{A}}{|A|}$

Triplets (Pytha.)
- 3, 4, 5
- 6, 8, 10
- 5, 12, 13
- 7, 24, 25
- 1, 1, $\sqrt{2}$
- $\sqrt{3}$, 1, 2
- 1, 1, 1 → $\sqrt{3}$

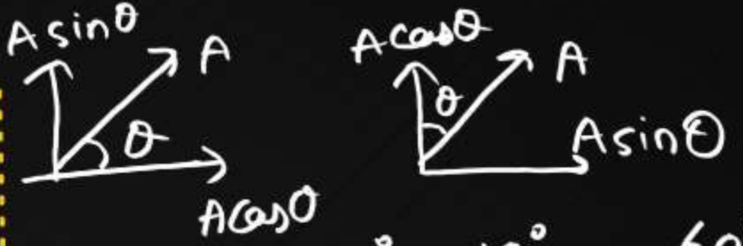
$\vec{A} = x\hat{i}+y\hat{j}+z\hat{k}$

$|\vec{A}| = \sqrt{x^2+y^2+z^2}$

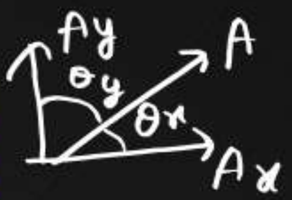
$\hat{A} = \frac{\vec{A}}{|A|} = \frac{x\hat{i}+y\hat{j}+z\hat{k}}{\sqrt{x^2+y^2+z^2}}$

$|4\hat{i}+3\hat{j}+5\hat{k}| = 5\sqrt{2}$

$|4\hat{i}+3\hat{j}+12\hat{k}| = 13$



	0	30°	45°	60°	90°	37°	53°
sin	0	half	half √2	half √3	1	3/5	4/5
cos	1	half √3	half √2	half	0	4/5	3/5
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	×	3/4	4/3



$\theta_y > \theta_x$
$A_y < A_x$ (chota angle, bada component)

$|\vec{A}+\vec{B}| = |\vec{A}|$
1. $B = 0$
2. A & B are antiparallel & $B = 2A$
3. $\cos\theta = \frac{-B}{2A}$

PUPPY POINTS – 2

$\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$

$\vec{B} = x\hat{i} + y\hat{j} + c\hat{k}$

$\vec{A} + \vec{B} = \vec{B} + \vec{A}$ ✓

$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$ ✓

$\vec{A} - \vec{B} = -(\vec{B} - \vec{A})$

$R = D = \sqrt{a^2 + b^2}$

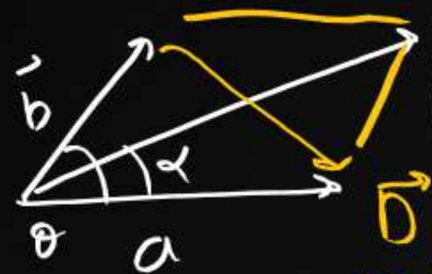
Equal diagonals in rectangle

$a \perp b$

$a = b$ Rhombus

$R \perp D$

Only vectors can be added or subtracted to give vectors.



$\vec{R} = \vec{a} + \vec{b}$

$|R| = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

$\vec{D} = \vec{a} - \vec{b}$

$|D| = \sqrt{a^2 + b^2 - 2ab\cos\theta}$

$$\tan\alpha = \frac{b\sin\theta}{a + b\cos\theta}$$

Babli Addition

θ	0°	60°	90°	120°	180°	θ
R	2B	$\sqrt{3}B$	$\sqrt{2}B$	B	0	$2B\cos\left(\frac{\theta}{2}\right)$
D	0	B	$\sqrt{2}B$	$\sqrt{3}B$	2B	$2B\sin\left(\frac{\theta}{2}\right)$

$\vec{A} + \vec{B} = A + B \quad \theta = 0°$

$\vec{A} + \vec{B} = A - B \quad \theta = 180°$

$\vec{A} + \vec{B} = \sqrt{A^2 + B^2} \quad \theta = 90°$

$\vec{A} + \vec{B} = \sqrt{A^2 + B^2 + AB} \quad \theta = 60°$

$\vec{A} + \vec{B} = \sqrt{A^2 + B^2 - AB} \quad \theta = 120°$

$\vec{A} + \vec{B} = \sqrt{A^2 + B^2 + \sqrt{3}AB} \quad \theta = 30°$

$\vec{A} + \vec{B} = \sqrt{A^2 + B^2 - \sqrt{3}AB} \quad \theta = 150°$

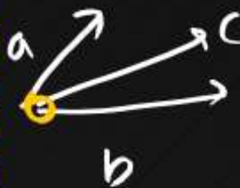
$\vec{A} + \vec{B} = \sqrt{A^2 + B^2 + \sqrt{2}AB} \quad \theta = 45°$

$\theta = 135°$

If A = B, θ =

$\lvert\vec{A} + \vec{B}\rvert = 2A$	0°
$\lvert\vec{A} + \vec{B}\rvert = \sqrt{3}A$	60°
$\lvert\vec{A} + \vec{B}\rvert = \sqrt{2}A$	90°
$\lvert\vec{A} + \vec{B}\rvert = A$	120°
$\lvert\vec{A} + \vec{B}\rvert = 0$	180°

$\vec{a} + \vec{b} = \vec{c}$ Triangle Law ✓



$\bar{a} + \bar{b} = \bar{c}$ llgm Law ✓

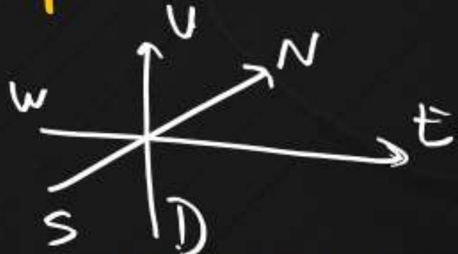
$\vec{d} = \vec{a} + \vec{b} + \vec{c}$ Polygon Law ✓

$R \perp D$

$|\vec{A}| = |\vec{B}|$, then rhombus, $\vec{A} + \vec{B}$ is $\perp$ to $\vec{A} - \vec{B}$

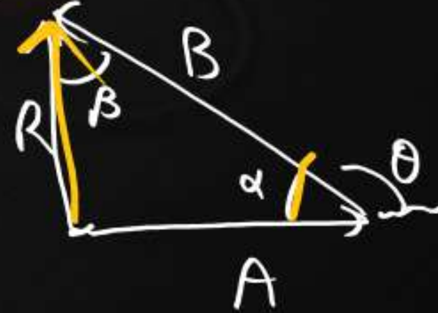
$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, then rectangle, $|\bar{A}| \perp |\bar{B}|$

$R = D$



30° E of N

$\vec{A} + \vec{B} = \vec{R} \qquad \vec{R} \perp \vec{A}$



$B^2 = A^2 + R^2$ ✓

Angle $\theta = 180 - \alpha$

$\tan\alpha = \frac{R}{A}$

PUPPY POINTS - 3

$(\vec{V}) \times$ Scalar = Vector

$\vec{A}$ multiplied by $\vec{B}$

Dot — Scalar

Cross — Vector

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$ $\left\{\begin{matrix} 0 \text{ is} \\ \theta = 90^\circ \end{matrix}\right\}$

$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

$\bar{A} = a\hat{i} + b\hat{j} + c\hat{k}$

$\bar{B} = x\hat{i} + y\hat{j} + z\hat{k}$

$\bar{A}\cdot\bar{B} = ax + by + cz$

$\vec{B}$ on $\vec{A}$ Scalar $= B\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|A|}$

Vector $= B\cos\theta\hat{A} = \frac{(\vec{A}\cdot\vec{B})}{|A|}\hat{A} = \frac{(\bar{A}\cdot\bar{B})\vec{A}}{|A|^2}$

$\cos\theta_x = x/|a| = x/\sqrt{x^2+y^2+z^2}$

$\cos\theta_y = y/|a| = y/\sqrt{x^2+y^2+z^2}$

$\cos\theta_z = z/|a| = z/\sqrt{x^2+y^2+z^2}$

$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$\sin^2\theta_x + \sin^2\theta_y + \sin^2\theta_z = 2$

$\vec{A}\times\vec{B} = |A||B|\sin\theta\,\hat{n}$

$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

$\vec{A}\times\vec{B}$ is $\perp$ to plane of $\vec{A}$ & $\vec{B}$

$\bar{A}\times\bar{B} = 0$ if $\theta = 0^\circ/180^\circ$

$i\times j = k$ $\quad$ $j\times i = -k$

$j\times k = i$ $\quad$ $k\times j = -i$

$k\times i = j$ $\quad$ $i\times k = -j$

$i\times i = j\times j = k\times k = 0$

$\tan\theta = \frac{|\bar{A}\times\bar{B}|}{A\cdot B}$

Area $\Delta = \frac{1}{2}|\bar{A}\times\bar{B}|$

Area $\|gm = |\bar{A}\times\bar{B}|$

$(\vec{A}\times\vec{B})\cdot(\vec{A}\pm\vec{B}) = 0$

$\bar{A}+\bar{B} = \bar{B}+\bar{A}$ ✓(C)

$\bar{A}+(\bar{B}+\bar{C}) = \bar{A}+(\bar{B}+\bar{C})$ ✓(A)

$\bar{A}-\bar{B} = -(\bar{B}-\bar{A})$

$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$ ✓(C)

$\vec{A}\times\vec{B} = -(\vec{B}\times\vec{A})$

$(\vec{A}\cdot\vec{B})C \neq \bar{A}(\bar{B}\cdot\bar{C})$ ✗

$(\bar{A}\times\bar{B})\times\bar{C} \neq \bar{A}\times(\bar{B}\times\bar{C})$ ✗

$(\bar{A}\times\bar{B})\cdot\bar{C} = \bar{A}\cdot(\bar{B}\times\bar{C}) \to$ VTP ✓

$\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$

$\vec{B} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{A} \parallel \vec{B}$ if $\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$ → +ve ∥ ; → −ve anti ∥

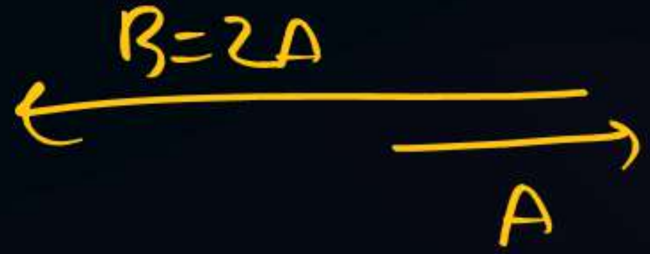
$\vec{A} \perp \vec{B}$ if $A\cdot B = 0 \Rightarrow ax + by + cz = 0$

A vector $\vec{C} \perp$ to both A & B is $\vec{C} = \vec{A}\times\vec{B}$

$(a\hat{i} + b\hat{j})$ is $\perp$ to $b\hat{i} - a\hat{j}$ & $\hat{k}$

$$|\vec{A}+\vec{B}| = |A|$$

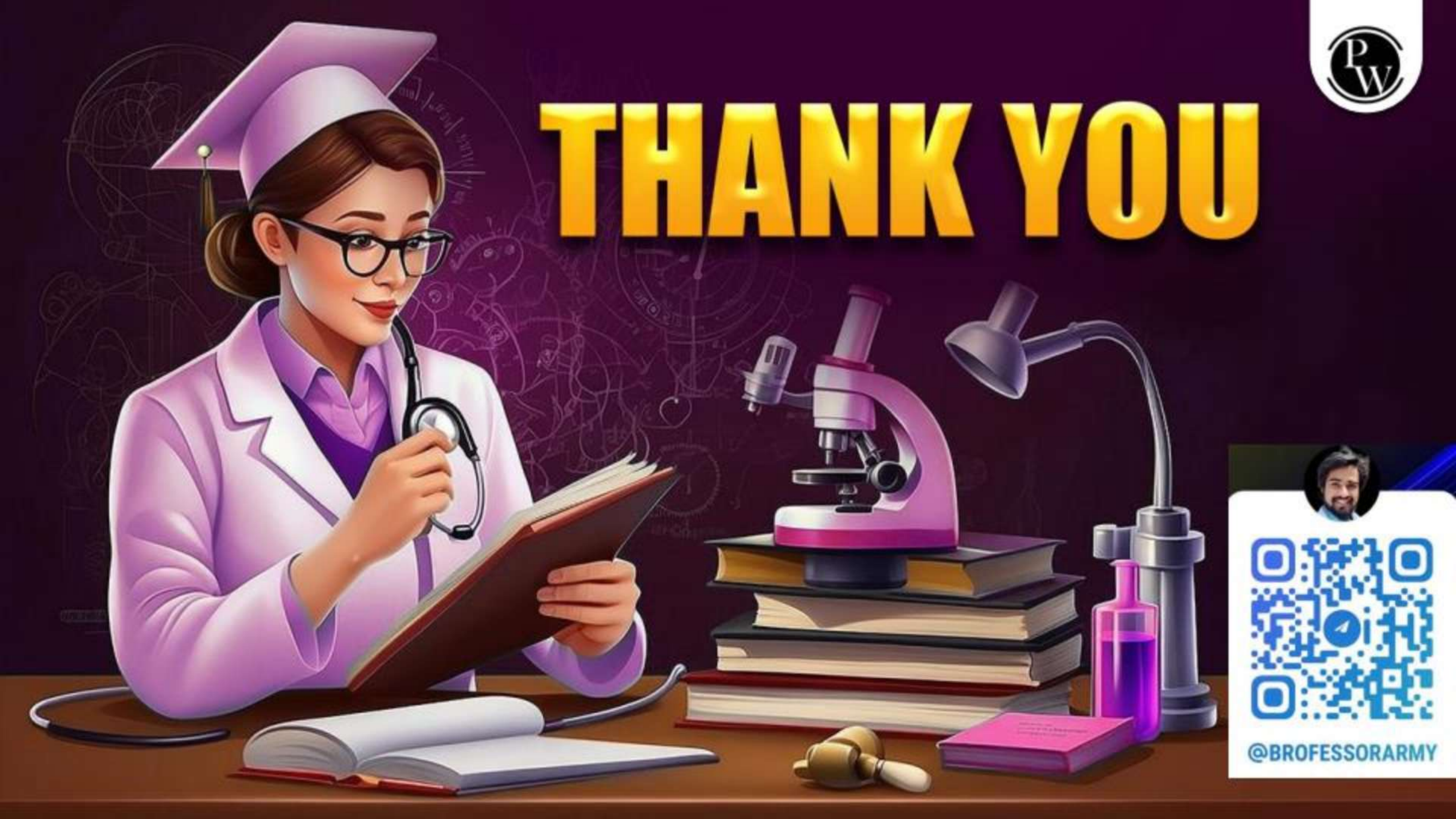
① $B = 0$ ✓



② Antiparallel ✓

③ $\sqrt{A^2+B^2+2A\cos\theta} = A$

$\longrightarrow \cos\theta = \dfrac{-B}{2A}$ ✓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