JEE Mains (12th)

Sample Paper - IV

DURATION : 180 Minutes | **M. MARKS : 300**

ANSWER KEY

PHYSICS

1.	(2)
2.	(2)
3.	(1)
4.	(4)
5.	(3)
6.	(3)
7.	(4)
8.	(4)
9.	(3)
10.	(1)
11.	(2)
12.	(2)
13.	(4)
14.	(1)
15.	(3)
16.	(3)
17.	(3)
18.	(2)
19.	(1)
20.	(2)
21.	(10)
22.	(6)
23.	(30)
24.	(2)
25.	(4)
26.	(2)
27.	(3)
28.	(3)
29.	(4)
30.	(2)

CHEMISTRY

31.	(4)
32.	(2)
33.	(2)
34.	(1)
35.	(4)
36.	(1)
37.	(1)
38.	(4)
39.	(3)
40.	(3)
41.	(3)
42.	(1)
43.	(3)
44.	(1)
45.	(4)
46.	(2)
47.	(3)
48.	(3)
49.	(3)
50.	(4)
51.	(15)
52.	(100)
53.	(4)
54.	(6)
55.	(5825)
56.	(6)
57.	(273)
58.	(8)
59.	(6)
60.	(2)

MATHEMATICS

61.	(3)
62.	(1)
63.	(4)
64.	(4)
65.	(4)
66.	(1)
67.	(3)
68.	(2)
69.	(2)
70.	(4)
71.	(2)
72.	(1)
73.	(3)
74.	(2)
75.	(1)
76.	(1)
77.	(1)
78.	(3)
79.	(1)
80.	(3)
81.	(1)
82.	(16)
83.	(310)
84.	(2)
85.	(225)
86.	(3)
87.	(31650)
88.	(2)
89.	(11)
90.	(1)

PHYSICS

1. **(2)**

For $BC = 0$,

$$a = \frac{2g}{2+5+1} = \frac{g}{4} = \frac{10}{4}$$

$$= \frac{20}{8}\,\text{ms}^{-2}$$

For $BC = 2m$,

$$a = \frac{(2+1)g}{2+5+1} = \frac{3g}{8}$$

$$= \frac{30}{8}\,\text{ms}^{-2}$$

2. **(2)**

$$a_1 = \frac{2mg - mg}{m} = g;\ a_2 = \frac{2mg - mg}{3m} = \frac{g}{3}$$

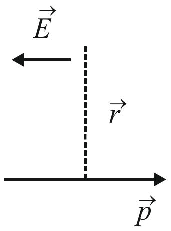
$$a_3 = \frac{mg + mg - mg}{2m} = \frac{g}{2}; a_1 > a_3 > a_2$$

3. **(1)**

Just after release $T = 0$ due to non–impulsive nature of spring. So acceleration of both blocks will be $g\downarrow$

4. **(4)**

$$\hat{E} = -\hat{p}$$



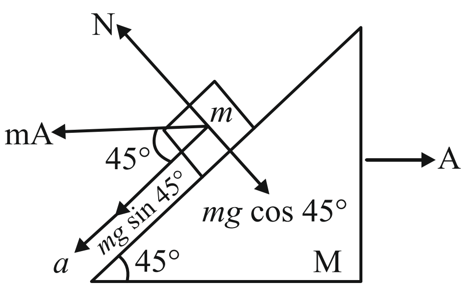
$$\Rightarrow \hat{E} = -\left[\frac{-\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{14}}\right]$$

$$\vec{E}| = \frac{k|\vec{p}|}{r^3}$$

$\hat{E}$ is parallel to $(\hat{i} + 3\hat{j} - 2\hat{k})$

5. **(3)**

a is acceleration of block w.r.t. wedge



For blocks:

N + mA sin 45° = mg cos 45° …(i)

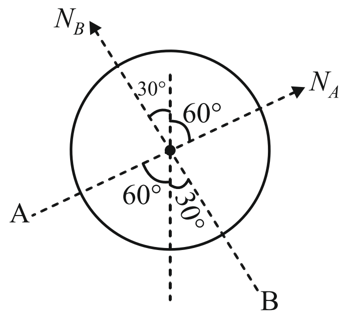
For wedge: N sin 45° = MA…(ii)

From (i) and (ii)

$A = 1\ \text{m/s}^2$

6. **(3)**

For equilibrium N_A cos 60° + N_B cos 30° = Mg and N_A sin 60° = N_B sin 30°



On solving $N_B = \sqrt{3}N_A;\ N_A = \frac{Mg}{2}$

7. **(4)**

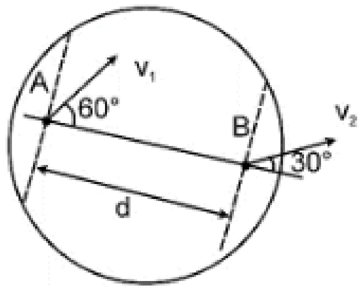
In equilibrium, torques of forces mg and Mg about an axis passing through O balance each other.

$$mg\frac{L}{2}\sin 30° = Mg\frac{L}{2}\sin 60°$$

$$\Rightarrow \frac{M}{m} = \sqrt{3}$$

8. **(4)**

For rigid body separation between two point remains same.



v_1 cos60° = v_2 cos30°

$$\frac{v_1}{2} = \frac{\sqrt{3}v_2}{2}$$

$\Rightarrow v_1 = \sqrt{3}v_2$

$$\omega_{disc} = \left|\frac{v_2\sin 30° - 4v_1\sin 60°}{d}\right|$$

$$= \left|\frac{\frac{v_2}{2} - \frac{\sqrt{3}v_1}{2}}{d}\right| = \left|\frac{v_2 - \sqrt{3}\times\sqrt{3}v_2}{2d}\right|$$

$$= \frac{2v_2}{2d} = \frac{v_2}{d}$$

$$\omega_{disc} = \frac{v_2}{d}$$

9. **(3)**

$$\vec{E} = K\frac{\vec{p}}{r^3}\sqrt{3\cos^2\theta + 1}$$

$$\Rightarrow \theta = \pi/2 \quad (0, d, 0)$$

$$\therefore \quad \vec{E} = \frac{-k\vec{p}}{d^3}$$

10. **(1)**

$$VBI = iR_{eq}$$

$$\because \quad R_{eq} = \frac{4}{3}\Omega + 1.7 = 3\Omega$$

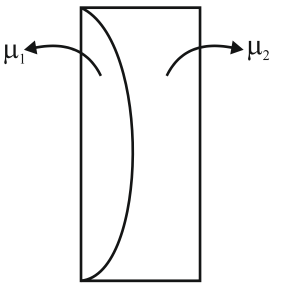
$$i = \frac{(BLV)}{R_{eq}} = \frac{(1)\left(5\times10^{-2}\right)\times10^{-2}}{3}$$

$$= \frac{5}{3}\times10^{-4}\ \text{A} \simeq 1.7\times10^{-4}\ \text{A}$$

$$= 170\mu\text{A}$$

11. **(2)**

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$



$$= (\mu_1 - 1)\left(\frac{1}{R}\right) + (\mu_2 - 1)\left(-\frac{1}{R}\right)$$

$$\frac{R}{f} = (\mu_1 - 1) + (1 - \mu_2) = (\mu_1 - \mu_2)$$

12. **(2)**

$$\rho gh = \rho aL$$

$$\therefore h = \frac{aL}{g}$$

13. **(4)**

See the typed example 12.

$$\tan\theta = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} = \tan 30° = \frac{1}{\sqrt{3}}$$

Solving this equation we get,

$$\frac{\rho_1}{\rho_2} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

14. **(1)**

For 1st minima

$$\sqrt{d^2 + (2d)^2} - 2d = \frac{\lambda}{2}$$

$$\Rightarrow \sqrt{5}d - 2d = \frac{\lambda}{2}$$

$$\Rightarrow d = \frac{\lambda}{2(\sqrt{5}-2)}$$

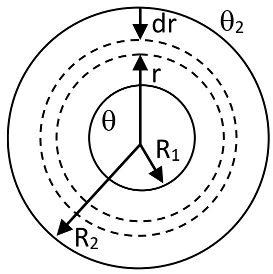
15. **(3)**

$$h_{Top} = \frac{2H + H}{2} = 1.5H$$

Since, this point lies in the tank. So hole should be made at this point.

16. **(3)**

$$\theta_1 - \theta_2 = \Delta\theta \qquad \frac{\theta_1 - \theta}{\int_{R_1}^{R}\frac{dr}{K4\pi r^2}} = \frac{\theta_1 - \theta_2}{\int_{R_1}^{R_2}\frac{dr}{K4\pi r^2}}$$



$$\frac{\Delta\theta/2}{\frac{1}{4\pi K_1}\left[\frac{1}{R_1} - \frac{1}{R}\right]} = \frac{\Delta\theta}{\frac{1}{4\pi K_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

$$\Rightarrow \quad R = \frac{2R_1R_2}{R_1 + R_2}$$

17. **(3)**

$$R = \int_{r_1}^{r_2}\frac{dr}{k.4\pi r^2} = \frac{1}{4\pi k}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$\Rightarrow \frac{(r_2 - r_1)}{4\pi k r_1 r_2}$$

Rate of heat flow

$$= \frac{T_2 - T_1}{R} = \frac{(T_1 - T_2)}{(r_2 - r_1)}\cdot 4\pi k r_1 r_2$$

$$\Rightarrow \propto \frac{r_1 r_2}{r_2 - r_1}$$

18. **(2)**

$x_1 = A\sin(\omega t + \phi_1)$

$x_2 = A\sin(\omega t + \phi_2)$

$$\Rightarrow |x_1 - x_2| = 2a\sin\left(2\omega t + \frac{\phi_1 + \phi_2}{2}\right)\cos\left(\frac{\phi_1 - \phi_2}{2}\right)$$

To maximize $|x_1 - x_2|$:

$$\sin\left(2\omega t + \frac{\phi_1 + \phi_2}{2}\right) = 1$$

$$\Rightarrow a\sqrt{2} = 2a\times1\times\cos\left(\frac{\phi_1 - \phi_2}{2}\right)$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \cos\left(\frac{\phi_1 - \phi_2}{2}\right)$$

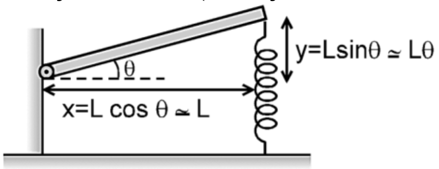
$$\Rightarrow \frac{\pi}{4} = \frac{\phi_1 - \phi_2}{2}$$

$$\Rightarrow \phi_1 - \phi_2 = \frac{\pi}{2}$$

19. **(1)**

Restoring torque:

$\tau = ky\,L = KL^2\theta$ (since $y \cong L\theta$ from figure)



$$\Rightarrow\ kL^2\theta = \frac{mL^2}{3}.\alpha$$

$$\Rightarrow\ \alpha = \frac{3k}{m}.\theta = \omega^2\theta \qquad \Rightarrow\ \omega = \sqrt{\frac{3k}{m}}$$

20. **(2)**

$$I_{\max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

$$I_{\min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

$$\therefore \frac{I_{\max} + I_{\min}}{I_{\max} - I_{\min}} = \frac{2\left(I_1 + I_2\right)}{4 \times \sqrt{I_1 I_2}}$$

$$= \frac{1}{2} \times \frac{\left(\frac{I_1}{I_2} + 1\right)}{\sqrt{\frac{I_1}{I_2}}}$$

$$= \frac{1}{2} \times \frac{\left(\frac{1}{4} + 1\right)}{\left(\frac{1}{2}\right)}$$

$$= \frac{5}{4} = \frac{2 \times 2 + 1}{1 + 3}$$

$$\therefore \frac{\alpha}{\beta} = \frac{2}{1} = 2$$

21. **(10)**

Applying pseudo force ma and resolving it. Applying $F_{\text{net}} = ma_x$ for x–direction.

$ma\cos\theta - (f_1 + f_2) = ma_x$

$ma\cos\theta - \mu N_1 - \mu N_2 = ma_x$

$ma\cos\theta - \mu\, ma\sin\theta - \mu\, mg = ma_x$

$\Rightarrow a_x = a\cos\theta - \mu a\sin\theta - \mu g$

$$= \left(25 \times \frac{4}{5}\right) - \left(\frac{2}{5} \times 25 \times \frac{3}{4}\right) - \left(\frac{2}{5} \times 10\right) = 10\ \text{m/s}^2$$

22. **(6)**

$$\frac{\rho_1}{\rho_2} = \frac{\mu_1 B_1 \sin 90}{\mu_2 B_2 \sin 90}$$

$$= \frac{1.2 \times 10^{-30} \times 5 \times 10^4}{2.4 \times 10^{-30} \times 15 \times 10^4} = \frac{1}{6}$$

23. **(30°)**

Time of flight $t = \dfrac{2u\sin(2\theta - \theta)}{g\cos\theta} = \dfrac{2u}{g}\tan\theta$

$$AB = \frac{2u^2\tan\theta}{g\sqrt{3}} = \frac{u}{\sqrt{3}}t$$

$$u_H = u\cos 2\theta$$

$$AC = u_H.t = u\cos 2\theta t$$

$$AB = \frac{AC}{\cos\theta} = \frac{u\cos 2\theta t}{\cos\theta}$$

Compare equations (i) and (ii)

$$\frac{\cos 2\theta}{\cos\theta} = \frac{1}{\sqrt{3}} \text{ or } \theta = 30°$$

24. **(2)**

$S_A = h$

and $S_B = 2h$

$V_A = 4$ m/s $\qquad V_B = 8$ m/s

From energy conservation decreases in P.E. of A = Increase in P.E. B + Increase in K.E. of both A and B

$$300h = 50 \times 2h + \frac{1}{2} \times 30 \times 4^2 + \frac{1}{2} \times 5 \times 8^2$$

$300h = 100h + 240 + 160$

$200h = 400$

$h = 2m$

25. **(4)**

When ball at maximum height block and ball has equal velocity

So, Using momentum conservation

$P_i = mv$

$P_f = 2mv_0 \qquad$ (v_0 final velocity)

$P_i = P_t \qquad mv = 2\ mv_0$

$$V_0 = \left(\frac{V}{2}\right)$$

Using energy conservation

$$\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}I\omega^2 + \frac{1}{2}2mv_0^2 + mgh$$

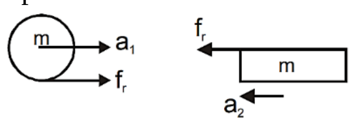
$(I = mR^2) \qquad v = \omega R$

$$\frac{1}{2}mv^2 = \frac{1}{2}2mv_0^2 + mgh$$

$$v^2 - 2\frac{v^2}{4} = 2gh \qquad \left(h = \frac{v^2}{4g}\right)$$

26. **(2)**

FBD for sphere & block



$$a_1 = \frac{f_r}{m} = \frac{\mu mg}{m} \qquad a_2 = \frac{f_r}{m} = \frac{\mu mg}{m}$$

$$\vec{a}_1 = \mu g\hat{i} \qquad \vec{a}_2 = -\mu g\hat{i}$$

$a_{rel} = \vec{a}_1 - \vec{a}_2 = 2\mu g\hat{i}$

27. (3)

$U(r) = U_0 r^4$

$F(r) = -\frac{dU(r)}{dr} = -4U_0 r^3$

$\frac{mv_n^2}{r_n} = 4U_0 r_n^3$ …(i)

$mvr_n = n\frac{h}{2\pi}$ …(ii)

$r_n \propto n^{\frac{1}{3}}$

28. (3)

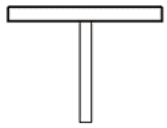
Position of 1st maxima is $\frac{3}{2}\frac{\lambda D}{a}$

⇒ According to given values, required separation

$= \frac{3}{2} \times (655\text{ nm} - 650\text{ nm}) \times \frac{2\text{m}}{0.5\text{ mm}}$

⇒ Required separation = 3×10^{-5} m.

29. (4)



(Thermal resistance) $R = \frac{L}{KA} = 5.0\frac{K}{W}$

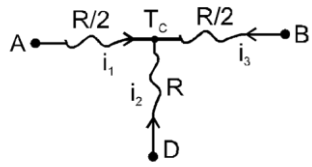
$\frac{T_A - T_C}{R/2} + \frac{T_D - T_C}{R} + \frac{T_B - T_C}{R/2} = 0$

$2T_A - 2T_C + T_D - T_C + 2T_B - 2T_C = 0$

$200 + 25 + 0 = 5T_C$

$T_C = \frac{225}{5} = 45$

$i_{CD} = \frac{T_C - T_D}{R} = \frac{45 - 25}{5} = 4\text{W}$



30. (2)

$R = \frac{\sqrt{2mK}}{qB}$

So $\frac{r_d}{r_p} = \frac{\sqrt{m_d}/q_d}{\sqrt{m_p}/q_p}$

$= \sqrt{2}$

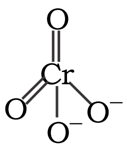
So $x = 2$

CHEMISTRY

31. (4)

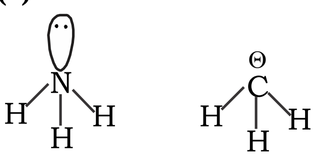
$\Delta P = \frac{h}{4\pi} \times \frac{1}{\Delta x}$

32. (2)



It show resonance therefore all bond are equivalent.

33. (2)



34. (1)

Theoretical

35. (4)

Since 'R' is universal gas constant, so it do not depend upon P, T & V.

36. (1)

If number of solute molecule increase then concentration increases and hence osmotic pressure increases.

37. (1)

$\underset{1-\alpha}{CO_2(g)} \rightleftharpoons \underset{\alpha}{CO(g)} + \underset{\alpha/2}{\frac{1}{2}O_2(g)}$

$K = \frac{\left(\frac{\alpha}{2}\right)^{1/2} \cdot \alpha}{1-\alpha}$ Here $(1 - \alpha = 1)$

$K = \frac{\alpha^{3/2}}{\sqrt{2}}$

38. (4)

Conceptual

39. (3)

$\underset{1\text{ mole}}{Na^+} + \underset{1\text{ mole}=1F}{e^-} \longrightarrow Na(s)$

40 **(3)**

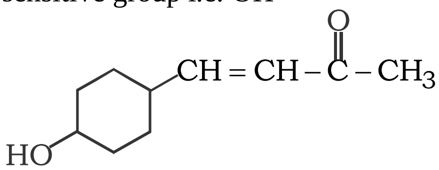
$CaCO_3 \rightarrow \underset{Basic}{CaO} + \underset{Acidic}{CO_2}$

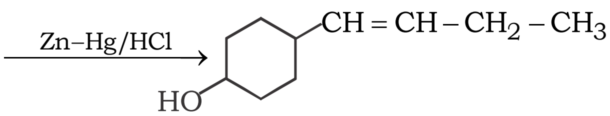
41. **(3)**

Highly pure hydrogen is obtained by the electrolysis of water.

42. **(1)**

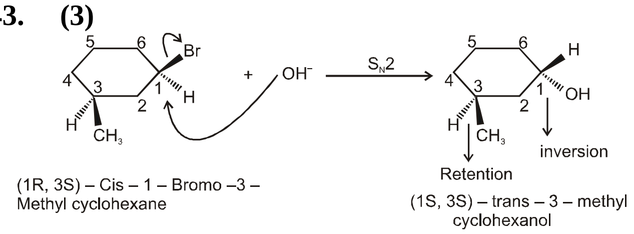
ZnHg/HCl can't be used due to the presence of acid sensitive group i.e. OH





and Na/Liq. NH_3 and $NABH_4$ convert –CO– into –CH(OH)–

43. **(3)**



44. **(1)**

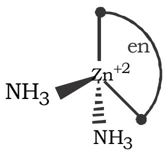
Orthosubstituted benzoic acid stronger acid then benzoic acid due to ortho effect.

45. **(4)**

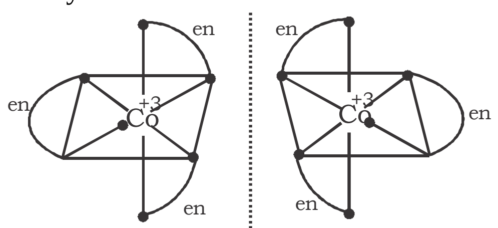
More activating and ortho para directing group will decide the orientation of incoming electrophile (NO_2^+).

46. **(2)**

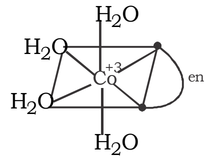
Only option (2) is having non–super imposable mirror image & hence one optical isomer.

(1) 

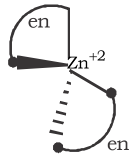
No. optical isomer. It is Tetrahedral with a plane of symmetry

(2) 

Optical isomer

(3) 

Horizontal plane is plane of symmetry

(4) 

No. optical isomer, it is tetrahedral with a plane of symmetry

47. **(3)**

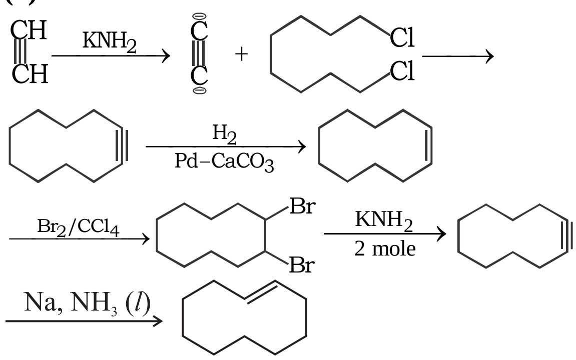
Neutral complex. No ions,

value < 10 $ohm^{-1}cm^2mol^{-1}$

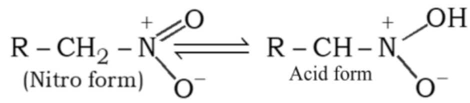
48. **(3)**

In XeF_n possible value of n is 2, 4, 6, 8, then compound should be XeF_2 (linear), XeF_4 (square planar), XeF_6 (capped octahedral) so in this case trigonal planar molecules does not possible.

49. **(3)**



50. **(4)**



51. **(15)**

Number of electrons in OF is 17.

Therefore bond order is 1.5

$1.5 \times 10 = 15$

52. **(100)**

$r_x = K\sqrt{\frac{1}{m}}, \quad r_{He} = K\sqrt{\frac{1}{4}}$

$K\sqrt{\frac{1}{m}} = \frac{1}{5}K\sqrt{\frac{1}{4}}$

$m = 100$

53. **(4)**

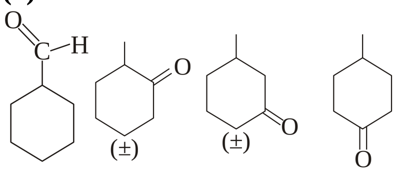
$$H_3C-CH=\underset{CH_3}{\overset{CH_3}{C}}-\underset{CH_3}{C}=CH-\underset{CH_3}{CH}-C_2H_5 \xrightarrow[\text{ozonolysis}]{O_3/Zn/H_2O}$$

3,4,6-trimethylocta-2,4-diene

$$H_3C-\overset{O}{\overset{\|}{C}}-H + CH_3-\overset{O}{\overset{\|}{C}}-\overset{O}{\overset{\|}{C}}-CH_3 + H-\overset{O}{\overset{\|}{C}}-\underset{(\pm)CH_3}{CH}-C_2H_5$$

So total of four products are obtained.

54. **(6)**



Total six reactants on Wolff Kishner reduction will give methylcyclohexane.

55. **(5825)**

$$H_2SO_4 + BaCl_2 \xrightarrow{L.R} BaSO_4\downarrow + 2HCl$$

	H_2SO_4	$BaCl_2$	$BaSO_4$	$2HCl$
initial mole	1	0.5	0	0
final mole	0.5	0	0.5	1

58.25 × 100 = 5825

56. **(6)**

$$Cr \longrightarrow 2Cr^{+3} + 6e^-$$

$$6e + 3Fe^{+2} \longrightarrow 3Fe$$

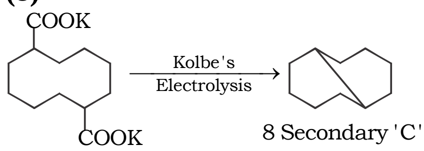
$$2Cr + 3Fe^{+2} \longrightarrow 2Cr^{+3} + 3Fe$$

57. **(0)**

$$\Delta S_f^o = \frac{\Delta H_f^o}{T_f}$$

$$T_f = \frac{1435}{5.26} = 273K \quad \text{or } 0^oC$$

58. **(8)**



59. **(6)**

$$CH_3-\overset{O}{\overset{\|}{C}}-H \xrightarrow[\text{[O]}]{K_2Cr_2O_7} CH_3-\overset{O}{\overset{\|}{C}}-OH \xrightarrow{Ca(OH)_2}$$

$$Ca(CH_3-\overset{O}{\overset{\|}{C}}-O^-)_2 \xrightarrow{\Delta} CH_3-\overset{O}{\overset{\|}{C}}-CH_3$$

$$\xrightarrow{\text{dil. }OH^-} CH_3-\overset{O}{\overset{\|}{C}}-CH=C(CH_3)_2$$

60. **(2)**

$3d^8$

Ni^{+2} (weak field ligand) $\qquad$ Ni^{+2} (strong field)

MATHEMATICS

61. **(3)**

$$I = \int_0^2 f(x)dx$$

$$= \left[xf(x)\right]_0^2 - \int_0^2 xf'(x)dx$$

$$= 2e^2 - \int_0^2 xf'(x)dx \qquad \ldots(1)$$

Put $I_1 = \int_0^2 xf'(x)dx \qquad \ldots(i)$

Using properties $\int_a^b f(x)dx$

$$= \int_a^b f(a+b-x)dx$$

$$I_1 = \int_0^2 (2-x)f'(2-x)dx$$

$$= \int_0^2 (2-x)f'(x)dx \qquad \ldots(ii)$$

Adding (i) and (ii) we get

$$2I_1 = 2\int_0^2 f'(x)dx \Rightarrow I_1 = \left[f(x)\right]_0^2$$

$f(2) - f(0) = e^2 - 1$

From (A) I = $2e^2 - e^2 + 1 = e^2 + 1$

62. **(1)**

$$I = \int_1^3 \left[x^2 - 2x + 1 - 3\right]dx$$

$$= \int_1^3 \left[(x-1)^2 - 3\right]dx$$

$$= \int_1^3 \left[(x-1)^2\right]dx - 3\int_1^3 dx \qquad \ldots(1)$$

$$I_1 = \int_1^3 \left[(x-1)^2\right]dx$$

Put $(x-1)^2 = t$

$2(x-1)\,dx = dt$

$$dx = \frac{dt}{2\sqrt{t}}$$

$$I_1 = \frac{1}{2}\int_0^4 [t]\frac{dt}{\sqrt{t}}$$

$$I_1 = \frac{1}{2}\left[0\int_0^1 \frac{dt}{\sqrt{t}} + \int_1^2 \frac{dt}{\sqrt{t}} + 2\int_2^3 \frac{dt}{\sqrt{t}} + 3\int_3^4 \frac{dt}{\sqrt{t}}\right]$$

$$= \frac{1}{2}\left\{\left|\frac{t^{\frac{-1}{2}+1}}{-\frac{1}{2}+1}\right|_1^2 + 2\left|\frac{t^{\frac{-1}{2}+1}}{-\frac{1}{2}+1}\right|_2^3 + 3\left|\frac{t^{\frac{-1}{2}+1}}{-\frac{1}{2}+1}\right|_3^4\right\}$$

$$= 5 - \sqrt{2} - \sqrt{3}$$

Hence from (1)

$$= 5 - \sqrt{2} - \sqrt{3} - 6 = -1 - \sqrt{2} - \sqrt{3}$$

63. (4)

$$x - 2y = 1,\ x - y + kz = -2,\ ky + 4z = 6$$

$x - 2y + 0.z - 1 = 0$

$x - y + kz + 2 = 0$

$0x - ky + 4z - 6 = 0$

$$\Delta = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix}$$

$= 1(-4 - k^2) - 2(-4 + 0)$

$= -4 - k^2 + 8 = -k^2 + 4$

$\therefore k^2 = 4 \Rightarrow k = \pm 2$

For unique solution $\Delta \neq 0 \Rightarrow k \neq \pm 2$

$$\Delta_1 = \begin{vmatrix} 1 & -2 & 0 \\ -2 & -1 & k \\ 6 & k & 4 \end{vmatrix} = -(k+10)(k+2)$$

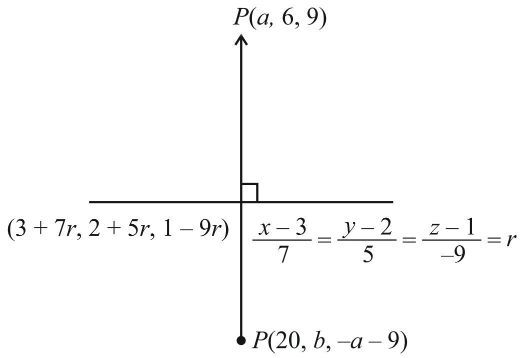
For no solution

$\Delta = 0,\ \Delta_1 \neq 0$

$k = 2$

64. (4)

$$\frac{a+20}{2} = 3 + 7r$$



$a + 20 = 6 + 14r$ …(i)

$$\frac{b+6}{2} = 2 + 5r$$

$b + 6 = 4 + 10r$

$b = -2 + 10r$ …(ii)

$$\frac{9-a-9}{2} = 1 - 9r$$

$-a = 2 - 18r$

$a = 18r - 2$ …(iii)

Solving (i) and (iii) we get

$20 + 18r - 2 = 6 + 14r$

$4r = 8 - 20 = -12$

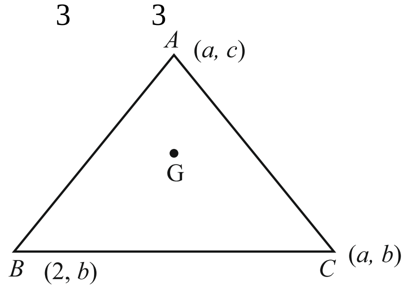
$r = -3$

$\therefore a = 14 + 14(-3) = -56$ and $b = -2 - 30 = -32$

$|a + b| = |-56 - 32| = 88$

65. (4)

$$\frac{a+2+a}{3} = \frac{10}{3}$$



$2a + 2 = 10$

$2a = 8 \Rightarrow a = 4$ …(i)

and $\dfrac{c+b+b}{3} = \dfrac{7}{3}$

$2b + c = 7$ …(ii)

Since a, b, c are in A.P.

$2b = a + c$

From (i) $\therefore 2b = 4 + c$ …(iii)

Solving (ii) and (iii)

$4 + c + c = 7$

$2c = 3$

$$c = \frac{3}{2}$$

$$\therefore 2b = 4 + \frac{3}{2} = \frac{11}{2}$$

$$\therefore b = \frac{11}{4}$$

As per question

$$\alpha + \beta = \frac{-b}{a} \text{ and } \alpha\beta = \frac{1}{a}$$

$$= \frac{\frac{-11}{4}}{4} = -\frac{11}{16},\ \alpha\beta = \frac{1}{4}$$

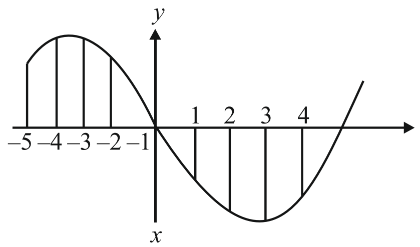
$$\alpha^2 + \beta^2 - \alpha\beta = (\alpha + \beta)^2 - 3\alpha\beta$$

$$= \frac{121}{256} - \frac{3}{4}$$

$$= \frac{121 - 192}{256} = \frac{-71}{256}$$

66. (1)

Let $f: R \to R$



$$f'(x)=\begin{cases}-55, & x<-5\\ 6x^2-6x-120, & -5<x<4\\ 6x^2-6x-36, & x>4\end{cases}$$

$$=\begin{cases}-55, & x<-5\\ 6(x-5)(x+4), & -5<x<4\\ 6(x-3)(x+2), & x>4\end{cases}$$

$f'(x)$ increasing in $x\in(-5,\ -4)\cup(4,\ \infty)$

67. (3)

Total possibilities = $2^5 \times 2^5$

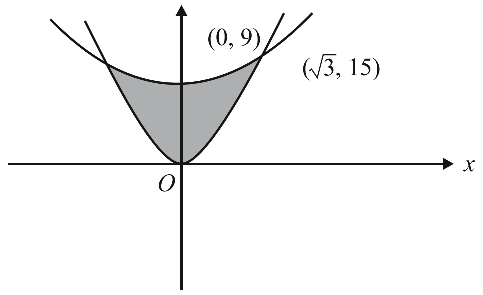
Favorable case = ${}^5C_2 \times 3^3 = 10 \times 3^3$

$\therefore$ required probability $=\dfrac{10\times 3^3}{2^5\times 2^5}$

$$=\frac{5\times 27}{2^9}=\frac{135}{2^9}$$

68. (2)

Area of shaded region



$$=2\int_0^{\sqrt{3}}\left(2x^2+9-5x^2\right)dx$$

$$=2\int_0^{\sqrt{3}}\left(9-3x^2\right)dx$$

$$=\int_0^{\sqrt{3}}\left(3-x^2\right)dx$$

$$=6\left[3x-\frac{x^3}{3}\right]_0^{\sqrt{3}}$$

$$=6\left[\frac{9\sqrt{3}-3\sqrt{3}}{3}\right]=12\sqrt{3}$$

69. (2)

$A^T = A$ and $B^T = -B$

$C = A^2B^2 - B^2A^2$

$C^T=\left(A^2B^2\right)^T-\left(B^2A^2\right)^T=B^2A^2-A^2B^2$

$C^T = -C$. Hence C, is skew symmetric matrix

$\therefore$ $det(C) = 0$

Hence, system have infinite solutions

70. (4)

$x+\sqrt{3}y=2\sqrt{3}$, and $m=-\dfrac{1}{\sqrt{3}}$, $C=2$

$(1)\ \dfrac{x^2}{9}-\dfrac{y^2}{1}=1\Rightarrow c=\sqrt{a^2m^2-b^2}$

$=\sqrt{\dfrac{9}{2}\times\dfrac{1}{3}-\dfrac{1}{2}}=1$, not possible

$(2)\ y^2=\dfrac{1}{6\sqrt{3}}x,\ c=\dfrac{a}{m}=\dfrac{1}{24\sqrt{3}}\times\left(\dfrac{-1}{\sqrt{3}}\right)$

$=-\dfrac{1}{72}$, not possible

$(3)\ c=a\sqrt{1+m^2}=\sqrt{7}\times 2$

$=2\sqrt{7}$, not possible

$(4)\ \dfrac{x^2}{9}+\dfrac{y^2}{1}=1,\ C=\sqrt{9m^2+1}$

$=\sqrt{9\times\dfrac{1}{3}+1}=2$

71. (2)

$${}^{n+1}C_2+2\left({}^2C_2+{}^3C_2+{}^4C_2...+{}^nC_2\right)$$

$${}^{n+1}C_2+2\sum_{r=1}^{n-1}{}^{r+1}C_2$$

$$={}^{n+1}C_2+2\sum_{r=1}^{n-1}\frac{(r+1)!}{(r-1)!2!}={}^{n+1}C_2+\sum_{r=1}^{n-1}(r+1)r$$

$$=\frac{(n+1)!}{2!(n+1)!}+\frac{(n-1)n\left(2(n-1)+1\right)}{6}+\frac{(n-1)n}{2}$$

$$=\frac{n(n+1)}{2}+\frac{n(n-1)(2n-1)}{6}+\frac{n(n-1)}{2}$$

$$=\frac{n}{2}\left[n+1+n-1+\frac{(n-1)(2n-1)}{3}\right]$$

$$=\frac{n}{2}\left[2n+\frac{2n^2-3n+1}{3}\right]$$

$$=\frac{n\left(6n+2n^2-3n+1\right)}{6}$$

$$=\frac{\left(2n^2+3n+1\right)}{6}$$

$$=\frac{n(2n+1)(n+1)}{6}$$

72. (1)

$$\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)-1+\lambda\left(\vec{r}.\left(\hat{i}-2\hat{j}\right)+2\right)=0$$

$$\vec{r}.\left((1+\lambda)\hat{i}+(1-2\lambda)+\hat{j}\right)-1+2\lambda=0$$

$\therefore$ point $(1, 0, 2) = \hat{i} + 2\hat{k}$

$\therefore 1 + \lambda + 2 - 1 + 2\lambda = 0 \Rightarrow \lambda = -\dfrac{2}{3}$

$\therefore \vec{r}\left(\dfrac{\hat{i}}{3} + \dfrac{7\hat{j}}{3} + \hat{k}\right) - \dfrac{7}{3} = 0$

$\vec{r}.\left(\hat{i} + 7\hat{j} + 3\hat{k}\right) = 7$

73. **(3)**

$\sim\left(\sim p \wedge (p \vee q)\right) = pV\left(\sim p \wedge \sim q\right)$

$= (pv \sim p) \wedge (pv \sim q)$

$= pv \sim q$

74. **(2)**

$\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$

$f(x) f''(x) = f'(x) f'(x)$

$\dfrac{f''(x)}{f'(x)} = \dfrac{f'(x)}{f(x)}$, integrating on both sides

$\ln|f'(x)| = \ln|f(x)| + \ln|c|$

$f'(x) = cf(x),\ f'(0) = cf(0) \Rightarrow c = 2$

$\dfrac{f'(x)}{f(x)} = 2$, again integrating on both side

$\ln f(x) = 2^x + k$

$f(x) = e^{2x+k}$

$f(0) = e^k = e^k = 1 \Rightarrow k = 0$

$\therefore f(x) = e^{2x}$ $\quad [\because e = 2.718]$

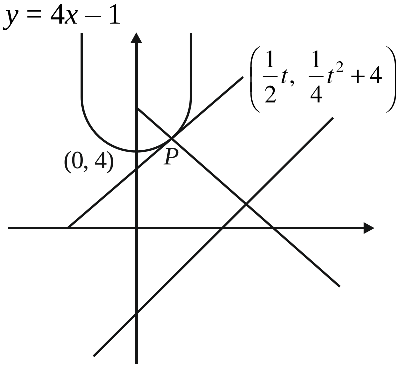
$\therefore e^2 \in (6, 9)$

75. **(1)**

$y = x^2 + 4$

$x^2 = y - 4$

$y = 4x - 1$



$PQ = \dfrac{\left|4 \times \frac{1}{2}t - \frac{1}{4}t^2 - 4 - 1\right|}{\sqrt{17}}$

$p = \dfrac{\left|t^2 - 8t - 20\right|}{4\sqrt{17}}$

$\dfrac{dp}{dt} = \dfrac{1}{4\sqrt{17}}(2t - 8)$ for maximum/minimum,

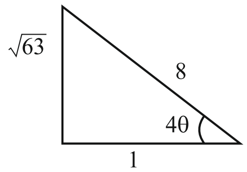
$\dfrac{dp}{dt} = 0 \Rightarrow t = 4$

also $\dfrac{d^2p}{dt^2} > 0$ at $t = 4$

hence closest point becomes at $t = 4$ is (2, 8)

76. **(1)**

$\tan\left(\dfrac{1}{4}\sin^{-1}\dfrac{\sqrt{63}}{8}\right)$



Put $\dfrac{1}{4}\sin^{-1}\dfrac{\sqrt{63}}{8} = \theta$

$\sin 4\theta = \dfrac{\sqrt{63}}{8} \quad \therefore \cos 4\theta = \dfrac{1}{8}$

$2\cos^2 2\theta = \dfrac{1}{8} + 1 \Rightarrow \cos 2\theta = \dfrac{3}{4}$

$\Rightarrow \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta} = \dfrac{3}{4} \Rightarrow$ (adding 1 both side)

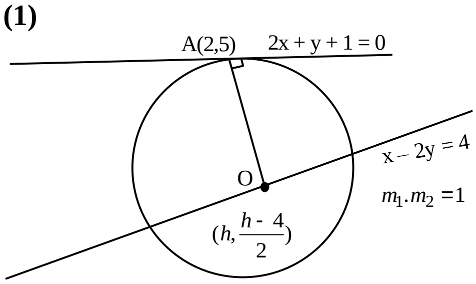
$\dfrac{2}{\sec^2\theta} = \dfrac{7}{4}$

$\cos^2\theta = \dfrac{7}{8} \Rightarrow \sec^2\theta = \dfrac{8}{7}$

$\tan^2\theta = \dfrac{8}{7} - 1 = \dfrac{1}{7}$

$\tan\theta = \dfrac{1}{\sqrt{7}}$

77. **(1)**



$\left(\dfrac{h - \frac{(h-4)}{2}}{2 - h}\right)(2) = -1$

$h = 8$

centre (8, 2)

Radius = $\sqrt{(8-2)^2 + (2-5)^2} = 3\sqrt{5}$

78. **(3)**

$y = ax^2 + bx + c$

$a + b + c = 2$

$\therefore \dfrac{dy}{dx} = 2ax + b \Rightarrow \left.\dfrac{dy}{dx}\right|_{(0,\,0)} = b = 1$

It passes through (0, 0)

$0 = 0 + 0 + c \Rightarrow c = 0$
$a = 1$
$\therefore\ a = 1, b = 1, c = 0$

79. **(1)**

Let $I=\int_0^{10}\frac{[\sin 2\pi x]}{e^{x-[x]}}dx=\int_0^{10}\frac{[\sin 2\pi x]}{e^{\{x\}}}dx$

Function $f(x)=\frac{[\sin 2\pi_x]}{e^{\{x\}}}$ is periodic with period '1' Therefore

$I=10\int_0^1\frac{[\sin 2\pi x]}{e^{\{x\}}}dx$

$=10\int_0^1\frac{[\sin 2\pi x]}{e^x}dx$

$=10\left(\int_0^{1/2}\frac{[\sin 2\pi x]}{e^x}dx+\int_{1/2}^1\frac{[\sin 2\pi x]}{e^x}dx\right)$

$=10\left(0+\int_{1/2}^1\frac{(-1)}{e^x}dx\right)$

$=-10\int_{1/2}^1 e^{-x}dx$

$=10\left(e^{-1}-e^{-1/2}\right)$

Now, $10\cdot e^{-1}-10\cdot e^{-1/2}=\alpha e^{-1}+\beta e^{-1/2}+\gamma$

$\Rightarrow \alpha=10,\beta=-10,\gamma=0$

$\Rightarrow \alpha+\beta+\gamma=0$

80. **(3)**

$x\frac{dy}{dx}+y=bx^4$

$\frac{dy}{dx}+\frac{y}{x}=bx^3$

$\text{I.F.}=e^{\int\frac{1}{x}dx}=e^{\ln x}=x$

$xy=b\int x^4dx=\frac{bx^5}{5}+c$

at $(1,\ 2)$, $1.2=\frac{b}{5}+c\Rightarrow c=2-\frac{b}{5}$

$\therefore \int_1^2 f(x)dx=\int_1^2\left(\frac{bx^4}{5}+\frac{10-b}{5x}\right)dx$

$=\left[\frac{bx^5}{25}+\left(\frac{10-b}{5}\right)\ln x\right]_1^2$

As per question

$\frac{32b}{25}+(10-b)\ln 2-\frac{b}{32}=\frac{62}{5}$

$\frac{31}{25}b+(10-b)\ln 2=\frac{62}{5}$

$(10-b)\ln 2=\frac{62}{5}-\frac{31}{25}b=\frac{31}{5}\left(2-\frac{b}{5}\right)$

$\therefore\ b = 10$

81. **(1)**

$L_1=\frac{x-\lambda}{1}=\frac{y-\frac{1}{2}}{\frac{1}{2}}=\frac{z}{-\frac{1}{2}}$

$L_2=\frac{x}{1}=\frac{y+2\lambda}{1}=\frac{z-\lambda}{1}$

$SD=\left|\frac{(\vec a_1-\vec a_2).(\vec b_1\times\vec b_2)}{|\vec b_1\times\vec b_2|}\right|$

$\vec b_1\times\vec b_2=\begin{vmatrix}\hat i & \hat j & \hat k\\ 2 & 1 & -1\\ 1 & 1 & 1\end{vmatrix}=2\hat i-3\hat j+\hat k$

$\frac{\vec b_1\times\vec b_2}{|\vec b_1\times\vec b_2|}=\frac{2\hat i-3\hat j+\hat k}{\sqrt{14}}$

$\vec b_2\times\vec b_1=(0-\lambda)\hat i+\left(-2\lambda-\frac{1}{2}\right)\hat j+(\lambda-0)\hat k$

$\therefore SD=\frac{\left|-2\lambda+3\left(2\lambda+\frac{1}{2}\right)+\lambda\right|}{\sqrt{14}}=\frac{\left|5\lambda+\frac{3}{2}\right|}{\sqrt{14}}$

$\therefore \frac{\left|5\lambda+\frac{3}{2}\right|}{\sqrt{14}}=\frac{\sqrt7}{2\sqrt2}\Rightarrow\left|5\lambda+\frac{3}{2}\right|=\frac{7}{2}$

$\therefore 5\lambda+\frac{3}{2}=\frac{7}{2}\Rightarrow\therefore 5\lambda=2\Rightarrow\lambda=\frac{2}{5}$

or $5\lambda+\frac{3}{2}=-\frac{7}{2}\Rightarrow 5\lambda=-5\Rightarrow\lambda=-1$

$\therefore |\lambda|=1$

82. **(16)**

$f(x)+f(x+1)=2$

$\Rightarrow f(x)$ is periodic with period = 2

$I_1=\int_0^8 f(x)dx=4\int_0^2 f(x)dx$

$=4\int_0^1(f(x)+f(1+x))dx=8$

Similarly, $I_2=2\times2=4$

$I_1+2I_2=16$

83. **(310)**

$\frac{2^{21}\left(\frac{-1+i\sqrt3}{2}\right)^{21}}{(\sqrt2)^{24}\left(\frac{1-i}{\sqrt2}\right)^{24}}+\frac{2^{21}\left(\frac{1+\sqrt3 i}{2}\right)^{21}}{(\sqrt2)^{21}\left(\frac{1+i}{\sqrt2}\right)^{24}}=k$

$2^9\frac{e^{14\pi i}}{e^{-i6\pi}}+2^9\frac{e^{i7\pi}}{e^{i6\pi}}=k$

$2^9\left[\frac{1}{1}+\frac{-1}{1}\right]=k$

$0 = k$

$\therefore n=[|k|]=0$

$$\sum_{j=0}^{5}(j+5)^2-\sum_{j=0}^{5}(j+5)$$

$$=\sum_{j=0}^{5}(j+5)(j+5-1)=\sum_{j=0}^{5}(j+5)(j+4)$$

$$=\sum_{j=0}^{5}\left(j^2+9j+20\right)$$

$$=\frac{5\times6\times11}{6}+\frac{9\times5\times6}{2}+20\times6$$

55 + 135 + 120 = 310

84. **(2)**

$$(x+1)^2+|x-5|=\frac{27}{4}$$

Case I :- $x < 5$

$$x^2+2x+1-x+5=\frac{27}{4}$$

$$x^2+x+6=\frac{27}{4}\Rightarrow 4x^2+4x-3=0$$

$$(2x+3)(2x-1)=0\Rightarrow x=-\frac{3}{2},\ \frac{1}{2}$$

Case II $x \geq 5$

$$x^2+2x+1+x-5=\frac{27}{4}$$

$$x^2+3x-4=\frac{27}{4}$$

$4x^2 + 12x - 43 = 0$

$$x=\frac{-12\pm\sqrt{144+43\times16}}{2\times4}$$

$$=\frac{-12\pm\sqrt{832}}{8}\ (\text{rejected})$$

Because $x > 5$

85. **(225)**

Let $P(h, k)$ $\sqrt{(h-5)^2+k^2}=\sqrt{(h+5)^2+k^2}$

$$h^2-10h+25+k^2=9h^2+90h+225+9k^2$$

$$8h^2+8k^2+100h+200=0$$

$$x^2+y^2+\frac{25}{2}x+25=0$$

$$r^2=\frac{625}{16}-25=\frac{625-400}{16}=\frac{225}{16}$$

$$16r^2=16\times\frac{225}{16}=225$$

86. **(3)**

Let a, ar, ar^2, ar^3 are in G.P.

$$a+ar+ar^2+ar^3=\frac{65}{12}$$

$$\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}+\frac{1}{ar^3}=\frac{65}{18}$$

$$\frac{\dfrac{a\left(1-r^4\right)}{1-r}}{\dfrac{\frac{1}{a}\left(1-\frac{1}{r^4}\right)}{1-\frac{1}{r}}}=\frac{65}{12}\times\frac{18}{65}$$

$$(ar)^2 r=\frac{3}{2}\Rightarrow a=\frac{2}{3},\ r=\frac{3}{2}$$

$$\therefore \text{third term}=ar^2=\frac{2}{9}\times\frac{9}{4}=\frac{3}{2}$$

$$\therefore \alpha=\frac{3}{2}\Rightarrow 2\alpha=3$$

87. **(31650)**

A	B	C
–	–	1
–	–	2
–	–	3

Number of groups = ${}^{10}C_1(2^9 - 2) = 5100$

Number of groups = ${}^{10}C_2(2^8 - 2) = 11430$

Number of groups = ${}^{10}C_3(2^7 - 2) = 15120$

Total number of groups = 31650

88. **(2)**

$$af(x)+\alpha f=bx+\frac{\beta}{x}\qquad \ldots\text{(i)}$$

Replace x by $\frac{1}{x}$

$$af\left(\frac{1}{x}\right)+\alpha f(x)=\frac{b}{x}+\beta x\qquad \ldots\text{(ii)}$$

$$a\left(f(x)+f\left(\frac{1}{x}\right)\right)+\alpha\left(f(x)+f\left(\frac{1}{x}\right)\right)$$

$$=b\left(x+\frac{1}{x}\right)+\beta\left(x+\frac{1}{x}\right)$$

$$(a+\alpha)\left(f(x)+f\left(\frac{1}{x}\right)\right)$$

$$=(b+\beta)\left(x+\frac{1}{x}\right)$$

$$\therefore\ \frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$$

$$=\frac{b+\beta}{a+\alpha}=\frac{2}{1}=2$$

89. **(11)**

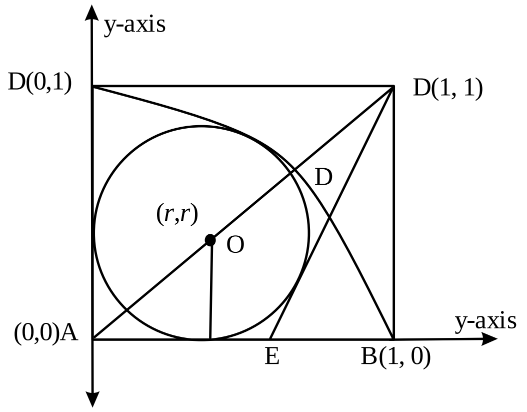
$$\sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$$

$$= \frac{9+k^2}{10} - \left(\frac{9+k}{10}\right) < 10$$

$$\Rightarrow k < \frac{10\sqrt{10}}{3} + 1 \Rightarrow k \le 11$$

$\therefore$ maximum value of k is 11

90. **(1)**



Here $AO + OD = 1$ or $(\sqrt{2}+1)r = 1$

$$\Rightarrow r = \sqrt{2-1}$$

Equation of circle $(x-r)^2 + (y-r)^2 = r^2$

Equation of CE

$$y - 1 = m(x-1)$$

$$mx - y + 1 - M = 0$$

It is tangent to circle

$$\therefore \left|\frac{mr - r + 1 - m}{\sqrt{m^2+1}}\right| = r$$

$$\left|\frac{(m-1)r + 1 - m}{\sqrt{m^2+1}}\right| = r$$

$$\frac{(m-1)^2(r-1)^2}{m^2+1} = r^2$$

Put $r = \sqrt{2} - 1$

On solving $m = 2-\sqrt{3}, 2+\sqrt{3}$

Taking greater slope of CE as $2+\sqrt{3}$

$$y - 1 = (2+\sqrt{3})(x-1)$$

Put $y = 0$

$$-1 = (2+\sqrt{3})(x-1)$$

$$\frac{-1}{2+\sqrt{3}} \times \left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right) = x - 1$$

$$x - 1 = \sqrt{3} - 1$$

$$EB = 1 - x = 1 - (\sqrt{3} - 1)$$

$$EB = 2 - \sqrt{3}$$