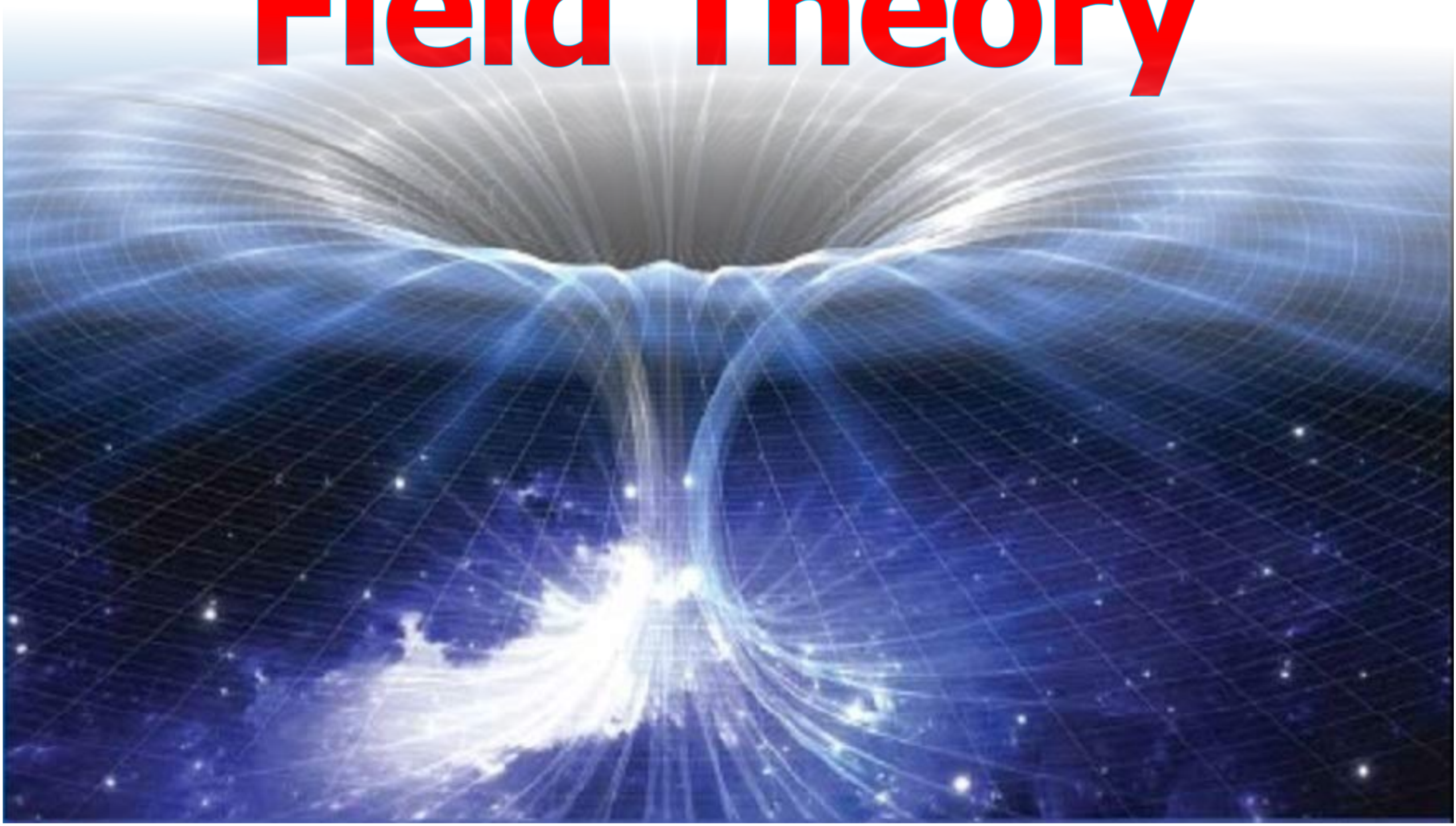
Electromagnetic Field Theory



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ELECTROMAGNETIC FIELD THEORY

INDEX

1 CO-ORDINATE SYSTEM

1.1. Introduction

1. **Vector :** It has magnitude and direction. In addition, it follows Vector Law of Addition.
   e.g. :- Electric field, Magnetic Field, Force etc.

2. **Scalar :** It has magnitude and no direction. It does not follow Vector Law of Addition.
   eg :– Current, Distance, Potential etc.

3. **Tensor :** It has magnitude and direction. It does not follow Vector Law of Addition. It shows different values in different directions at the same point.
   e.g.:– Conductivity, Resistivity, Refractive Index.

4. **Unit Vector:** It is the vector which has unit magnitude and directed along increasing direction of parameters.

1.2. Equation of line in 3-dimentional

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}=constant$$

Or

$$\frac{x-x_2}{x_2-x_1}=\frac{y-y_2}{y_2-y_1}=\frac{z-z_2}{z_2-z_1}=constant$$

Equation of Plane In 3-Dimentional.

$$ax+by+cz=d$$

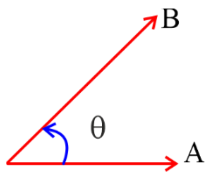
Eg. $3x+4y=5$

Equation of line in two-dimensional. However, equation of plane in three-dimensional.

1. X = Constant.
   (a) A plane is parallel to Y and Z-axis.
   (b) Y and Z-axis is tangential component.
   (c) X axis is normal component.
2. Y = Constant.
   (a) A plane is parallel to X and Z-axis.
   (b) X and Z axis is tangential component.
   (c) Y axis is normal component

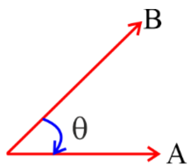
3. Z = Constant
   (a) A plane is parallel to X and Y-axis.
   (b) X and Y-axis is tangential components.
   (c) Z axis is normal components.

1.2.1. Cross Product



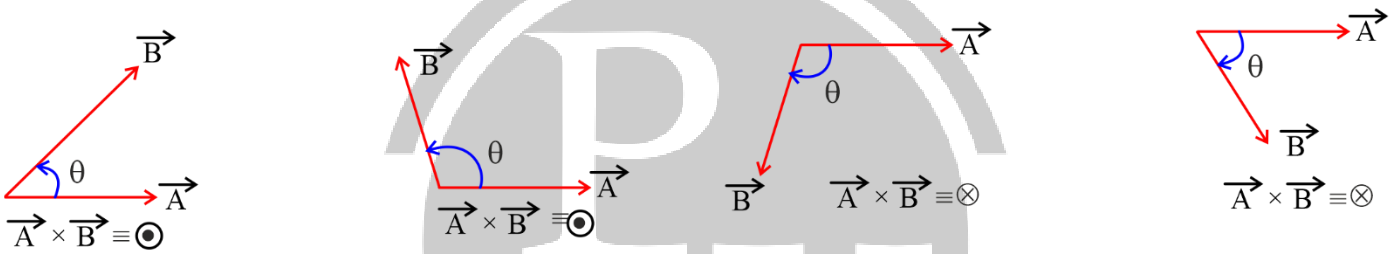
$\vec{A}\times\vec{B} \equiv AB\sin\theta \odot$

$\odot \equiv$ Outward direction (Anticlockwise)



$\vec{B}\times\vec{A} \equiv AB\sin\theta \otimes$

$\otimes \equiv$ Inward direction

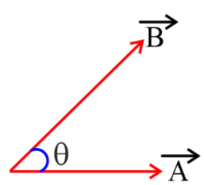


$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

1.2.2. Dot Product

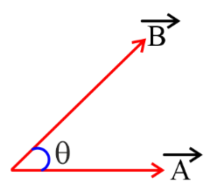
$$\vec{A}\bullet\vec{B} = a_1b_1 + a_2b_2 + a_3b_3$$

1. Projection of B along Vector $\vec{A}$



A||ar = B cos θ

2. Projection of vector $\vec{B}$ along Vector $\vec{A}$.



$\vec{A}$||ar = B cos θ $\hat{A}$

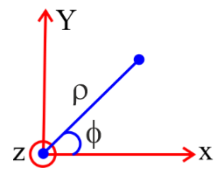
3. Projection of vector $\vec{B}$ perpendicular to Vector $\vec{A}$.

$\vec{A}_{\perp r} = \vec{B} - (B\cos\theta)\hat{A}$.

Point Conversion

1. Cartesian to cylindrical

$$\rho = \sqrt{X^2 + Y^2}, \quad \phi = \tan^{-1}\left(\frac{Y}{X}\right), \quad Z = Z$$



2. Cylindrical to Cartesian

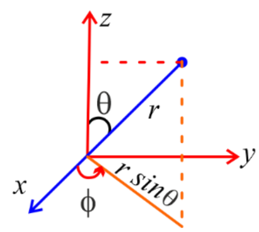
$$X = \rho\cos\phi, \; Y = \rho\sin\phi, \; Z = Z$$

3. Cartesian to spherical

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$



4. Spherical to Cartesian

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

5. Cylindrical to Spherical

$$r = \sqrt{\rho^2 + z^2}, \; \theta = \cos^{-1}\left(\frac{z}{\sqrt{\rho^2 + z^2}}\right), \phi = \phi$$

6. Spherical to Cylindrical

$$\rho = r\sin\theta, \; \phi = \phi, \; z = r\cos\theta.$$

Unit vector Conversion

1. Cartesian to Cylindrical

$$\begin{bmatrix} \hat{a}_\rho \\ \hat{a}_\phi \\ \hat{a}_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix}$$

2. Cylindrical to Cartesian

$$\begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{a}_\rho \\ \hat{a}_\phi \\ \hat{a}_z \end{bmatrix}$$

3. Cartesian to Spherical

$$\begin{bmatrix} \hat{a}_r \\ \hat{a}_\theta \\ \hat{a}_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix}$$

4. Spherical to Cartesian

$$\begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} \hat{a}_r \\ \hat{a}_\theta \\ \hat{a}_\phi \end{bmatrix}$$

5. Spherical to Cylindrical.

$$\begin{bmatrix} \hat{a}_\rho \\ \hat{a}_\phi \\ \hat{a}_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} \hat{a}_r \\ \hat{a}_\theta \\ \hat{a}_\phi \end{bmatrix}$$

6. Cylindrical to Spherical

$$\begin{bmatrix} \hat{a}_r \\ \hat{a}_\theta \\ \hat{a}_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{a}_\rho \\ \hat{a}_\phi \\ \hat{a}_z \end{bmatrix}$$

1.3. Differential Length, Area and Volume

Parameter			Coefficient		
u	v	w	h_1	h_2	h_3
x	y	z	1	1	1
ρ	ϕ	z	1	ρ	1
r	θ	ϕ	1	r	$r\sin\theta$

1. Differential Length

It is a vector quantity and directed along tangential direction.

In general form.

$$\vec{dl} = h_1 du\hat{a}_u + h_2 dv\hat{a}_v + h_3 dw\hat{a}_w$$

(a) Cartesian co-ordinate system.

$$\vec{dl} = dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z$$

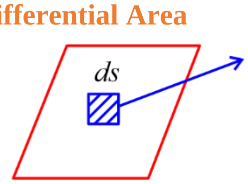
(b) Cylindrical Co-ordinate system.

$$\vec{dl} = d\rho\hat{a}_\rho + \rho d\phi\hat{a}_\phi + dz\hat{a}_z$$

(c) Spherical co-ordinate system.

$$\vec{dl} = dr\hat{a}_r + rd\theta\hat{a}_\theta + r\sin\theta d\phi\hat{a}_\phi$$

2. Differential Area



$$d\vec{s} = ds\hat{a}_n$$

$$\hat{a}_n = unit\ normal\ to\ the\ surface$$

It is a vector quantity and directed normal to the surface.
In general Form

$$d\vec{S} = h_2 h_3 dvdw\hat{a}_u + h_1 h_3 dudw\hat{a}_v + h_1 h_2 dudv\hat{a}_v$$

(a) Cartesian co-ordinate system.

$$d\vec{S} = dydx\hat{a}_x + dxdz\hat{a}_y + dxdy\hat{a}_z$$

(b) Cylindrical co-ordinate system.

$$d\vec{S} = \rho d\phi dz\hat{a}_\rho + d\rho dz\hat{a}_\phi + \rho d\rho d\phi\hat{a}_z$$

(c) Spherical co-ordinate system3.

$$d\vec{S} = r^2 \sin\theta d\theta d\phi\hat{a}_r + r\sin\theta drd\phi\hat{a}_\theta + rdrd\theta\hat{a}_\phi$$

3. Differential Volume

In general form

$$dv = h_1 h_2 h_3 dudvdw$$

(a) Cartesian co-ordinate system :–

$$dv = dxdydz$$

(b) Cylindrical co-ordinate system :–

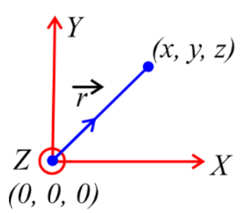
$$dv = \rho d\rho d\phi dz$$

(c) Spherical co-ordinate system.

$$dv = r^2 \sin\theta drd\theta d\phi$$

1.3.1. Position Vector

(a) Cartesian co-ordinate system



$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

(b) Cylindrical Co-ordinate system.

$\vec{r} = \rho\hat{a}_\rho + z\hat{a}_z$ → 'Z' is an axis

$\vec{r} = \rho\hat{a}_\rho + y\hat{a}_y$ → 'Y' is an axis

$\vec{r} = \rho\hat{a}_\rho + x\hat{a}_x$ → 'X' is an axis

(c) Spherical Co-ordinate system :

$$\vec{r} = r\hat{a}_r$$

(d) Position vector in 2D

$$\vec{\rho} = \rho\hat{a}_\rho = (x_2 - x_1)\hat{a}_x + (y_2 - y_1)\hat{a}_y \rightarrow 'Z' axis$$

$$\vec{\rho} = \rho\hat{a}_\rho = (y_2 - y_1)\hat{a}_y + (z_2 - z_1)\hat{a}_z \rightarrow 'X' axis$$

$$\vec{\rho} = \rho\hat{a}_\rho = (x_2 - x_1)\hat{a}_x + (z_2 - z_1)\hat{a}_z \rightarrow 'Y' axis$$

❑❑❑

2 VECTOR CALCULUS

2.1. Introduction

I. Differential Form

1. Gradient of scalar function
2. Divergence.
3. Curl.
4. Laplacian operator.

II. Integral Form

1. Open line integration
   (a) Path independent
   (b) Path dependent
2. Closed Line integration.
3. Closed Surface integration.
4. Volume integration.

Gradient of scalar function 'f': –

1. **Definition**: It is a vector quantity which gives maximum rate of change of scalar function 'f' and is directed normal to surface 'f' or scalar function 'f'.

$$\vec{\nabla} f = \left.\frac{df}{dl}\right|_{\max} \hat{a}_n$$

2. **Formulae**:

$$\vec{\nabla} f = \frac{1}{h_1}\frac{\partial f}{\partial u}\hat{a}_u + \frac{1}{h_2}\frac{\partial f}{\partial v}\hat{a}_v + \frac{1}{h_3}\frac{\partial f}{\partial w}\hat{a}_w$$

(a) Cartesian Co-ordinate system.

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{a}_x + \frac{\partial f}{\partial y}\hat{a}_y + \frac{\partial f}{\partial z}\hat{a}_z$$

(b) Cylindrical Co-ordinate system.

$$\vec{\nabla} f = \frac{\partial f}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{a}_\phi + \frac{\partial f}{\partial z}\hat{a}_z$$

(c) Spherical Co-ordinate system.

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{a}_\phi$$

3. Physical Significance:

(a) Maximum rate of change of scalar function ‘f’ will be given by gradient of scalar function ‘f’.

$$\left.\frac{df}{dl}\right|_{\max} = \left|\vec{\nabla} f\right|$$

(b) To find directional derivate.

$$D.D = \vec{\nabla} f . \hat{A}$$

(c) It gives unit normal on surface.

$$\hat{a}_n = \frac{\vec{\nabla} f}{\left|\vec{\nabla} f\right|}$$

(d) To find angle between the surfaces.

(e) $\vec{A} = \vec{\nabla} f$

$$'f' \equiv Scalar\ function\ of\ vector \vec{A}$$

4. Properties of Gradient

(a) $\vec{\nabla}(f+g) = \vec{\nabla} f + \vec{\nabla} g$

(b) $\vec{\nabla}(fg) = f\vec{\nabla} g + g\vec{\nabla} f$

(c) $\vec{\nabla}\left(\frac{f}{g}\right) = \frac{g\vec{\nabla} f - f\vec{\nabla} g}{g^2}$

5. Application:

(a) $\vec{\nabla} r = \hat{a}_r$

(b) $\vec{\nabla}\left(\frac{1}{r}\right) = \frac{-\vec{r}}{r^3}$

(c) $\vec{\nabla}\left(r^n\right) = \left(nr^{n-2}\right)\vec{r}$

(d) $\vec{\nabla}(lnr) = \frac{\vec{r}}{r^2}$

(e) $\vec{\nabla}\left(r^2 + lnr\right) = \left(2r + \frac{1}{r}\right)\hat{a}_r$

(f) $\vec{\nabla}\left(r^2 lnr\right) = r\left(2 + \ln r\right)\hat{a}_r$

(h) $\vec{\nabla}\left(\frac{lnr}{r^2}\right) = \left(\frac{r - 2rlnr}{r^4}\right)\hat{a}_r$

Divergence:

1. Definition: It gives total outward flux per unit volume.

$$\vec{\nabla}.\vec{A} = \frac{\oiint \vec{A}.d\vec{S}}{\lim\limits_{\Delta v \to 0} \Delta v}$$

(a) $\vec{\nabla}.\vec{A} \equiv Divergence\ at\ a\ point.$

(b) $\oiint \vec{A} \bullet d\vec{S} \equiv Divergence\ in\ a\ range.$

(c) $\oiint \vec{A} \bullet d\vec{S} = \int_{v} (\vec{\nabla} \bullet \vec{A}) dv \Rightarrow$ *Divergence theorem.*

2. Formulae:

$$\vec{\nabla} \bullet \vec{A} = \frac{1}{h_1 h_2 h_3}\left(\frac{\partial(h_2 h_3 A_u)}{\partial u} + \frac{\partial(h_1 h_3 A_v)}{\partial v} + \frac{\partial(h_1 h_2 A_w)}{\partial w}\right)$$

(a) Cartesian Co-ordinate System

$$\vec{\nabla} \bullet \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

(b) Cylindrical Co-ordinate System.

$$\vec{\nabla} \bullet \vec{A} = \frac{1}{\rho}\left(\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{\partial A_\phi}{\partial \phi} + \frac{\partial(\rho A_z)}{\partial z}\right)$$

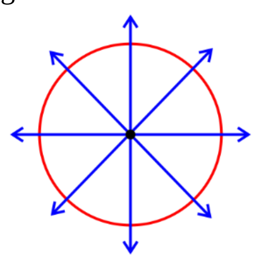
(c) Spherical Co-ordinate system.

$$\vec{\nabla} \bullet \vec{A} = \frac{1}{r^2 \sin\theta}\left(\frac{\partial(r^2 \sin\theta A_r)}{\partial r} + \frac{\partial(r \sin\theta A_\theta)}{\partial \theta} + \frac{\partial(r A_\phi)}{\partial \phi}\right)$$

3. Physical Significance:

It gives outward flux.

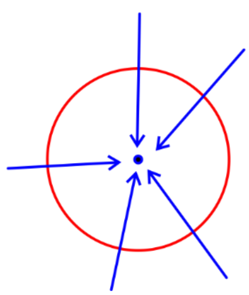
(a)



Outward Flux ≠ 0
Inward Flux = 0
Net Flux ≠ 0

$\vec{\nabla} \bullet \vec{A} > 0$

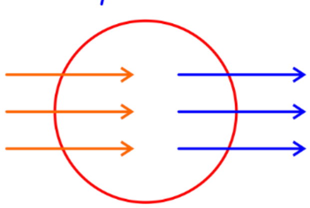
(b)



Outward Flux = 0
Inward Flux ≠ 0
Net Flux ≠ 0

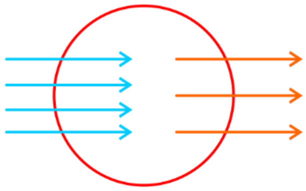
$\vec{\nabla} \bullet \vec{A} < 0$

(c)



Outward Flux
= Inward Flux
Net Flux = 0

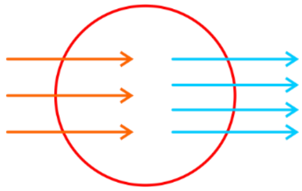
$\vec{\nabla} \bullet \vec{A} = 0$

(d) 

Outward Flux < Inward Flux

Net Flux ≠ 0

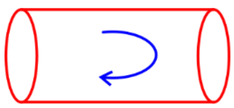
$\vec{\nabla} \cdot \vec{A} < 0$

(e) 

Outward Flux > Inward Flux

Net Flux ≠ 0

$\vec{\nabla} \cdot \vec{A} > 0$

(f) 

Outward Flux = 0
Inward Flux = 0
Net Flux = 0

$\vec{\nabla} \cdot \vec{A} = 0$

4. Properties

(a) $\vec{\nabla} \cdot \left(\vec{A} + \vec{B}\right) = \vec{\nabla} \cdot \vec{A} + \vec{\nabla} \cdot \vec{B}$

(b) $\vec{\nabla} \cdot \left(f\,\vec{A}\right) = f\left(\vec{\nabla} \cdot \vec{A}\right) + \vec{A} \cdot \left(\vec{\nabla} f\right)$

(c) $\vec{\nabla} \cdot \left(\dfrac{\vec{A}}{f}\right) = \dfrac{f\left(\vec{\nabla} \cdot \vec{A}\right) - \vec{A} \cdot \left(\vec{\nabla} f\right)}{f^2}$

5. Application

(a) $\vec{\nabla} \cdot \vec{r} = 3$

(b) $\vec{\nabla} \cdot \left(r^n \hat{a}_r\right) = (n+2) r^{n-1}$

Curl:

1. Definition:

$$\vec{\nabla} \times \vec{A} = \frac{\oint \vec{A} \cdot \vec{dl}}{\lim_{\Delta S \to 0} \Delta S}\,\hat{n}$$

Curl gives total Motive Force due to $\vec{A}$ per unit area. And it is directed normal to the rotatory plane.

(a) $\vec{\nabla} \times \vec{A}$ = Curl at a point

(b) $\oint \vec{A} \cdot \vec{dl}$ = Curl in a range

(c) $\oint \vec{A} \cdot \vec{dl} = \iint \left(\vec{\nabla} \times \vec{A}\right) \cdot d\vec{S} \Rightarrow$ Stoke's theorem

2. Formulae:

$$\vec{\nabla} \times \vec{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \hat{a}_u & h_2 \hat{a}_v & h_3 \hat{a}_w \\ \dfrac{\partial}{\partial u} & \dfrac{\partial}{\partial v} & \dfrac{\partial}{\partial w} \\ h_1 A_u & h_2 A_v & h_3 A_w \end{vmatrix}$$

(a) Cartesian Co-ordinate system

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

(b) Cylindrical Co-ordinate System.

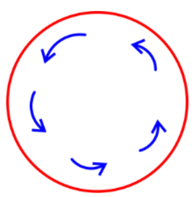
$$\vec{\nabla} \times \vec{A} = \frac{1}{\rho} \begin{vmatrix} \hat{a}_\rho & \rho \hat{a}_\phi & \hat{a}_z \\ \dfrac{\partial}{\partial \rho} & \dfrac{\partial}{\partial \phi} & \dfrac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

(c) Spherical Co-ordinate system

$$\vec{\nabla} \times \vec{A} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta \hat{a}_\phi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

3. Physical Significance: – It gives rotation.

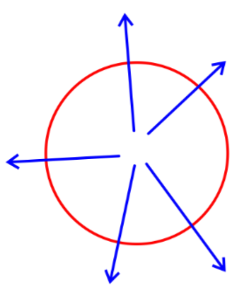
(a)



$\vec{\nabla} \times \vec{A} \neq \vec{0}$

$\vec{\nabla} \bullet \vec{A} = 0$

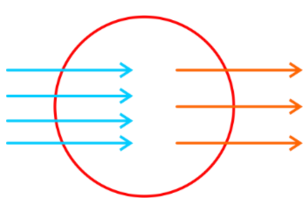
(b)



$\vec{\nabla} \times \vec{A} = \vec{0}$

$\vec{\nabla} \bullet \vec{A} \neq 0$

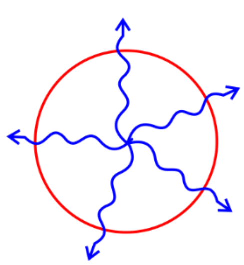
(c)



$\vec{\nabla} \times \vec{A} = \vec{0}$

$\vec{\nabla} \bullet \vec{A} \neq 0$

(d)



$\vec{\nabla} \times \vec{A} \neq \vec{0}$

$\vec{\nabla} \bullet \vec{A} \neq 0$

4. Properties:

(a) $\vec{\nabla}\times\left(\vec{A}+\vec{B}\right)=\vec{\nabla}\times\vec{A}+\vec{\nabla}\times\vec{B}$

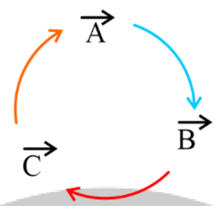
(b) $\vec{\nabla}\times\left(f\,\vec{A}\right)=f\left(\vec{\nabla}\times\vec{A}\right)-\vec{A}\times\left(\vec{\nabla}f\right)$

(c) $\vec{\nabla}\times\left(\dfrac{\vec{A}}{f}\right)=\dfrac{f\left(\vec{\nabla}\times\vec{A}\right)+\vec{A}\times\left(\vec{\nabla}f\right)}{f^2}$

Mixed Product:

1. Scalar Product:

(a) $\vec{A}\bullet\left(\vec{B}\times\vec{C}\right)=\vec{B}\bullet\left(\vec{C}\times\vec{A}\right)=\vec{C}\bullet\left(\vec{A}\times\vec{B}\right)$



$$\vec{A}\bullet\left(\vec{B}\times\vec{C}\right)=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

(b) $\vec{A}\bullet\left(\vec{B}\times\vec{C}\right)=0\Rightarrow\left[\vec{A}\vec{B}\vec{C}\right]=0$

Then $\vec{A},\vec{B}\,\&\,\vec{C}$ are independent vectors and they do not lie in a single plane.

(c) Volume of parallelepiped

$\left|\vec{A}\bullet\left(\vec{B}\times\vec{C}\right)\right|$

2. Vector Product:

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right)=\left(\vec{A}\bullet\vec{C}\right)\vec{B}-\left(\vec{A}\bullet\vec{B}\right)\vec{C}$$

3. Mixed product with Del operator.

(a) $\vec{\nabla}\left(\vec{\nabla}\,f\right)=Doesnot\ exist$

(b) $\vec{\nabla}\bullet\left(\vec{\nabla}\,f\right)=\nabla^2 f$

(c) $\vec{\nabla}\times\left(\vec{\nabla}\,f\right)=\vec{0}$

(d) $\vec{\nabla}\left(\vec{\nabla}\bullet\vec{A}\right)=exist$

(e) $\vec{\nabla}\bullet\left(\vec{\nabla}\bullet\vec{A}\right)=does\ not\ exist$

(f) $\vec{\nabla}\times\left(\vec{\nabla}\bullet\vec{A}\right)=does\ not\ exist$

(g) $\vec{\nabla}\left(\vec{\nabla}\times\vec{A}\right)=does\ not\ exist$

(h) $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{A}\right)=\vec{\nabla}\left(\vec{\nabla}\bullet\vec{A}\right)-\nabla^2\vec{A}$

(i) $\vec{\nabla}\bullet\left(\vec{\nabla}\times\vec{A}\right)=0$

Some Important Identities

1. $\vec{\nabla}\left(f+g\right)=\vec{\nabla}f+\vec{\nabla}g$
2. $\vec{\nabla}\left(fg\right)=f\vec{\nabla}g+g\vec{\nabla}f$

3. $\vec{\nabla}\left(\frac{f}{g}\right)=\frac{g\vec{\nabla}f-f\vec{\nabla}g}{g^2}$

4. $\vec{\nabla}\bullet\left(\vec{A}+\vec{B}\right)=\vec{\nabla}\bullet\vec{A}+\vec{\nabla}\bullet\vec{B}$

5. $\vec{\nabla}\bullet\left(f\vec{A}\right)=f\vec{\nabla}\bullet\vec{A}+\vec{A}\bullet\vec{\nabla}f$

6. $\vec{\nabla}\left(\frac{\vec{A}}{f}\right)=\frac{f\left(\vec{\nabla}\bullet\vec{A}\right)-\vec{A}\bullet\vec{\nabla}f}{f^2}$

7. $\vec{\nabla}\bullet\left(\vec{A}\times\vec{B}\right)=\left(\vec{\nabla}\times\vec{A}\right)\bullet\vec{B}-\left(\vec{\nabla}\times\vec{B}\right)\bullet\vec{A}$

8. $\vec{\nabla}\times\left(\vec{A}+\vec{B}\right)=\vec{\nabla}\times\vec{A}+\vec{\nabla}\times\vec{B}$

9. $\vec{\nabla}\times\left(f\,\vec{A}\right)=f\left(\vec{\nabla}\times\vec{A}\right)-\vec{A}\times\left(\vec{\nabla}f\right)$

10. $\vec{\nabla}\times\left(\frac{\vec{A}}{f}\right)=\frac{f\left(\vec{\nabla}\times\vec{A}\right)+\vec{A}\times\left(\vec{\nabla}f\right)}{f^2}$

11. $\vec{\nabla}\bullet\left(\vec{\nabla}f\right)=\nabla^2 f$

12. $\vec{\nabla}\times\left(\vec{\nabla}f\right)=\vec{0}$

13. $\vec{\nabla}\bullet\left(\vec{\nabla}\times\vec{A}\right)=0$

14. $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{A}\right)=\vec{\nabla}\left(\vec{\nabla}\bullet\vec{A}\right)-\nabla^2\vec{A}$

Laplacian Operator:

1. Laplacian operator with scalar function 'f'

$$\nabla^2 f=\vec{\nabla}\bullet\left(\vec{\nabla}f\right)$$

$$=\frac{1}{h_1h_2h_3}\left(\frac{\partial\left(\frac{h_2h_3}{h_1}\frac{\partial f}{\partial u}\right)}{\partial u}+\frac{\partial\left(\frac{h_1h_3}{h_2}\frac{\partial f}{\partial v}\right)}{\partial v}+\frac{\partial\left(\frac{h_1h_2}{h_3}\frac{\partial f}{\partial w}\right)}{\partial w}\right)$$

(a) Cartesian co-ordinate system

$$\nabla^2 f=\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 f}{\partial z^2}$$

(b) Cylindrical co-ordinate system.

$$\nabla^2 f=\frac{\partial^2 f}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial f}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2 f}{\partial\phi^2}+\frac{\partial^2 f}{\partial z^2}$$

(c) Spherical co-ordinate system :–

$$\nabla^2 f=\frac{\partial^2 f}{\partial r^2}+\frac{2}{r}\frac{\partial f}{\partial r}+\frac{1}{r^2}\frac{\partial^2 f}{\partial\theta^2}+\frac{\cot\theta}{r^2}\frac{\partial f}{\partial r}+\frac{1}{\left(r\sin\theta\right)^2}\frac{\partial^2 f}{\partial\phi^2}$$

Note: When scalar function '*f*' is function of 'r' only. Then, $\nabla^2 f(r) = \frac{1}{r}\frac{df(r)}{dr} + \frac{d^2 f(r)}{dr^2}$

(d) (i) $\nabla^2 r^n = n(n+1)r^{n-2}$ (ii) $\nabla^2 r = \frac{2}{r}$

(iii) $\nabla^2\left(\frac{1}{r}\right) = 0$ (iv) $\nabla^2 lnr = \frac{1}{r^2}$

2. Laplacian operator with vector $\vec{A}$

$$\nabla^2\vec{A} = \frac{1}{h_1^2}\frac{\partial^2\vec{A}}{\partial u^2} + \frac{1}{h_2^2}\frac{\partial^2\vec{A}}{\partial V^2} + \frac{1}{h_3^2}\frac{\partial^2\vec{A}}{\partial w^2}$$

(a) Cartesian co-ordinate system.

$$\nabla^2\vec{A} = \frac{\partial^2\vec{A}}{\partial x^2} + \frac{\partial^2\vec{A}}{\partial y^2} + \frac{\partial^2\vec{A}}{\partial z^2}$$

(b) Cylindrical co-ordinate system

$$\nabla^2\vec{A} = \frac{\partial^2\vec{A}}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2\vec{A}}{\partial \phi^2} + \frac{\partial^2\vec{A}}{\partial z^2}$$

(c) Spherical co-ordinate system.

$$\nabla^2\vec{A} = \frac{\partial^2\vec{A}}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2\vec{A}}{\partial \theta^2} + \frac{1}{(r\sin\theta)^2}\frac{\partial^2\vec{A}}{\partial \phi^2}$$

Some Important points:

(a) $\vec{\nabla} r = \hat{a}_r$

(b) $\vec{\nabla}\bullet\vec{r} = 3$

(c) $\vec{\nabla}\times\vec{r} = \vec{0}$

(d) $\nabla^2 r = \frac{2}{r}$

(e) $\nabla^2\vec{r} = \vec{0}$

(f) $\vec{\nabla} f(r) = \frac{df(r)}{dr}\hat{a}_r$

(g) $\nabla^2 f(r) = \frac{2}{r}\frac{df(r)}{dr} + \frac{d^2 f(r)}{dr^2}$

(h) $\vec{\nabla}\bullet\vec{A} = \vec{0}$ then $\vec{A}$ is solenoidal and diversion less.

(i) $\vec{\nabla}\times\vec{A} = \vec{0}$ then $\vec{A}$ is irrational, conservative and path independent vector.

(j) $\nabla^2\phi = 0$ (Laplacian Equation).

2.3. Path dependent open Line Integral.

1. $\int \vec{P} \cdot \vec{dl}$

Condition for path dependent open Line Integral is given as.

$$\vec{\nabla} \times \vec{P} \neq \vec{0}$$

Then the above integrals will be solved using parameterization process.

Example:

The Line Integral of the Vector Field

$\vec{F} = 5xz\hat{i} + (3x^2 + 2y)\hat{j} + x^2 z\hat{k}$ along a path from (0, 0, 0) to (1, 1, 1) parameterized by (t, t^2, t) is …...

Solution:

$x = t \Rightarrow dx = dt$

$y = t^2 \Rightarrow dy = 2tdt$

$z = t \Rightarrow dz = dt$

$$\int \vec{F} \cdot \vec{dl} = \int 5xzdx + \int (3x^2 + 2y)dy + \int x^2 zdz$$

$$= \int_0^1 5t^2 dt + \int_0^1 (3t^2 + 2t^2)(2tdt) + \int_0^1 t^3 dt$$

$$= \frac{5}{3} + \frac{10}{4} + \frac{1}{4} = \frac{20+33}{12} = \frac{53}{12} = 4.41$$

2. $\int \vec{P} \cdot \vec{dl}$

Condition for path independent open Line Integral is given by $\vec{\nabla} \times \vec{P} = \vec{0}$

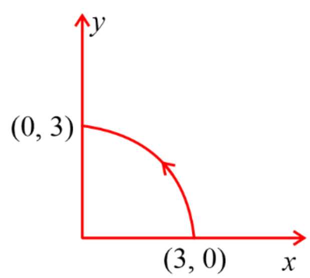
So, open line integral is performed through straight line.

Question:

As shown in the figure, C is the arc from the point (3, 0) to the point (0, 3) on the circle $x^2 + y^2 = 9$. The value of the integral.

$$\int_C (y^2 + 2yx)dx + (2xy + x^2)dy$$

is …….



Solution:

Method I: - Basic Method

$$x^2 + y^2 = 9$$

$$x = \pm\sqrt{9 - y^2}\ ,\ y = \pm\sqrt{9 - x^2}$$

Since, the curve lies in 1st quadrant of xy plane. Hence,

$$x = \sqrt{9-y^2}\ ,\ y = \sqrt{9-x^2}$$

$$I = \int_3^0 (9-x^2) + 2\left[\left(\sqrt{9-x^2}\right)x\right]dx + \int_0^3 \left[2\left(\sqrt{9-y^2}\right)y + (9-y^2)\right]dy = 0$$

Method II: - Parameterization Process.

$$x^2 + y^2 = 9 \ \Rightarrow x = 3\cos\theta,\ y = 3\sin\theta$$

$$x = 3 \text{ to } 0 \ \Rightarrow \theta = 0 \text{ to } \frac{\pi}{2}$$

$$y = 0 \text{ to } 3 \ \Rightarrow \theta = 0 \text{ to } \frac{\pi}{2}$$

$$I = \int_3^0 (y^2 + 2yx)dx + \int_0^3 (2xy + x^2)dy$$

$$= 3\int_0^{\frac{\pi}{2}} (9\sin^2\theta + 18\sin\theta\cos\theta)(-\sin\theta d\theta) + 3\int_0^{\frac{\pi}{2}} (9\cos^2\theta + 18\sin\theta\cos\theta)(\cos\theta d\theta) = 0$$

Method III: - To check path independent or dependent.

$$\int (y^2 + 2yx)dx + (2xy + x^2)dy$$

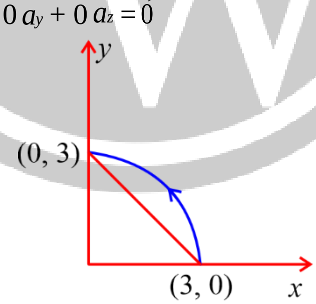
$$= \int \left((y^2 + 2yx)a_x + (x^2 + 2xy)a_y\right) \bullet \left(dx a_x + dy a_y\right)$$

$$= \int \vec{P} \bullet \vec{dl}$$

$$\vec{V} \times \vec{P} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 + 2xy & 2xy + x^2 & 0 \end{vmatrix}$$

$$= (0 - 0)\,a_x - (0 - 0)\,a_y + (2x + 2y - 2y - 2x)\,a_z$$

$$= 0\,a_x + 0\,a_y + 0\,a_z = \vec{0}$$



So, the open line integral will be through straight line.

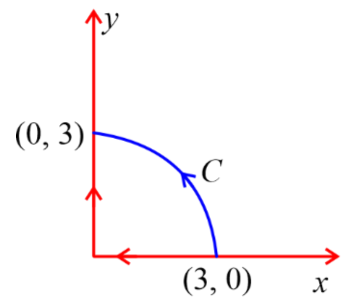
$$\frac{x-3}{3-0} = \frac{y-0}{0-3} \Rightarrow x = -y + 3 \Rightarrow y = 3 - x$$

$$I = \int (y^2 + 2xy)dx + (2xy + x^2)dy$$

$$\int_3^0 \left[(3-x)^2 + 2x(3-x)\right]dx + \int_0^3 \left[2(3-y)y + (3-y)^2\right]dy = 0$$

Method IV:

Open line integral is not performed through curve '*C*' but through along *x*-axis and then along *y*-axis.



Along x-axis, $y = 0$, $x = 3$ to 0, $dx \neq 0$, $dy = 0$

$$I_1 = \int\left[(0)^2 + 2x\times 0\right]dx + \int\left[2x(0) + x^2\right](0) = 0$$

Along y-axis, $x = 0$, $y = 0$ to 3, $dx = 0$, $dy \neq 0$

$$I_2 = \int_0^3\left[2y(0) + y^2\right](0) + \int_3^0\left[(0)^2 + 2(0)y\right]dy = 0$$

$\therefore$ $$I = I_1 + I_2 = 0 + 0 = 0$$

3. $\oint \vec{A}\bullet\vec{dl}$ = closed line integral

To solve above integral, we use closed line integral or open surface integral.

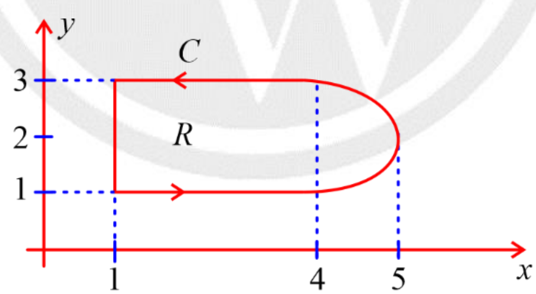
$$\oint \vec{A}\bullet\vec{dl} = \iint\left(\vec{\nabla}\times\vec{A}\right)\bullet\vec{ds}$$

Question:

Consider the line integral

$$\int_C (xdy - ydx)$$

The integral being taken in a counter clock-wise direction over the closed curve 'C' that forms the boundary of the region 'R' shown in the figure below. The region 'R' is the area enclosed by the union of a 2×3 rectangle and a semicircle of radius 1. The line integral evaluates to –



Solution:

Method I: Using closed line integral

Path 1. $x = 1$ to 4, $y = 1$, $dx \neq 0$, $dy = 0$

$$I_1 = \int_1^4 -y\,dx\Big|_{y=1} = -3$$

Path 2. Along semicircle centre = (4, 2)

$$(x-4)^2 + (y-2)^2 = (1)^2$$

$$x = \cos\phi + 4,\ y = \sin\phi + 2$$

$$y = 1 \text{ to } 3 \Rightarrow 2 + \sin\phi = 1 \text{ to } 3$$

$$\sin\phi = -1 \text{ to } 1 \Rightarrow \phi = -\frac{\pi}{2} \text{ to } \frac{\pi}{2}$$

$$x = 4 + \cos\phi \Rightarrow dx = -\sin\phi\, d\phi$$

$y = 2 + \sin\phi \Rightarrow dy = \cos\phi d\phi$

$$I_2 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (4+\cos\phi)(\cos\phi d\phi) - \int (2+\sin\phi)(-\sin\phi d\phi)$$

$$I_2 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4\cos\phi d\phi + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\sin\phi d\phi = 8 + \pi$$

Path 3.

$x = 4$ to 1, $y = 3$, $dx \neq 0$, $dy \neq 0$

$$I_3 = \int_4^1 -y\,dx\Big|_{y=3} = 3 \times 3 = 9$$

Path 4.

$x = 1$, $y = 3$ to 1, $dx = 0$, $dy \neq 0$

$$I_4 = \int_3^1 x\,dy\Big|_{x=1} = -2$$

$\therefore$ $I = I_1 + I_2 + I_3 + I_4$

$= -3 + (\pi + 8) + 9 - 2 = \pi + 12$

Method 2: - Using Stoke's theorem or Green's theorem

$$\oint \vec{A} \cdot \vec{dl} = \iint \left(\nabla \times \vec{A}\right) \cdot \vec{ds}$$

$$\oint (xdy - ydx) = \oint (-ya_x + xa_y) \cdot (dxa_x + dya_y)$$

$$\vec{A} = -ya_x + xa_y$$

$$\nabla \times \vec{A} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & x & 0 \end{vmatrix}$$

$= [1 - (-1)]\, a_z = 2\, a_z$

Since, the given curve line in XY plane. Hence differential area will be written as

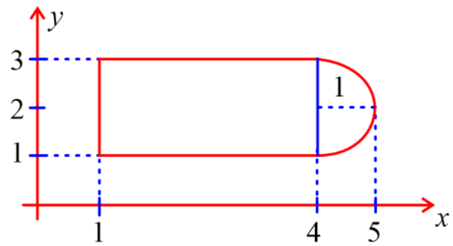
$$d\vec{s} = dx\,dy\,a_z\Big|_{z=0}$$

$$\oint (-y\,a_x + x\,a_y) \cdot (dx\,a_x + dy\,a_y)$$

$$= \iint 2a_z \cdot dx\,dy\,a_z = 2\iint dx\,dy$$

$= 2$ (Area of Curve)

$= 2$ (Area of rectangle + Area of semicircle)



$$=2\left(2\times3+\frac{\pi(1)^2}{2}\right)=\pi+12$$

Note: As we have seen that open surface integral is simpler method than closed line integral. So, we generally use Stoke's or Green theorem to solve closed line integral.

4. Closed Surface Integral or volume integrals.

$$\oiint \vec{A}\cdot d\vec{s}=\int_{\upsilon}\left(\vec{\nabla}\cdot\vec{A}\right)d\upsilon$$

$\Rightarrow$ Divergence theorem

Volume integral is easier than closed surface integral.

Question:

Consider a closed surface S surrounding a volume V. If $\vec{r}$ is the position vector of a point inside S, with n the unit normal on 'S', the value of the integral

$$\oiint 5\vec{r}\cdot nds \text{ is ....}$$

(A) 3 V

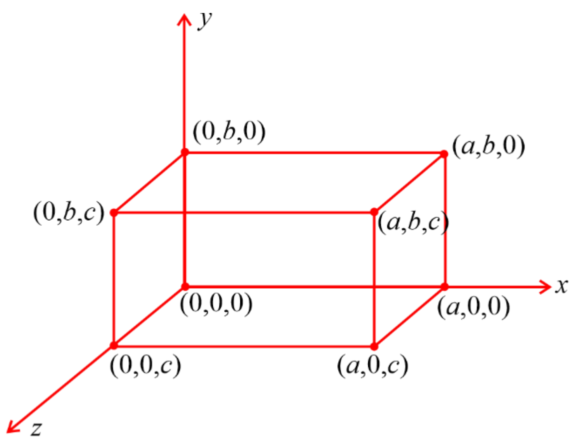
(B) 5 V

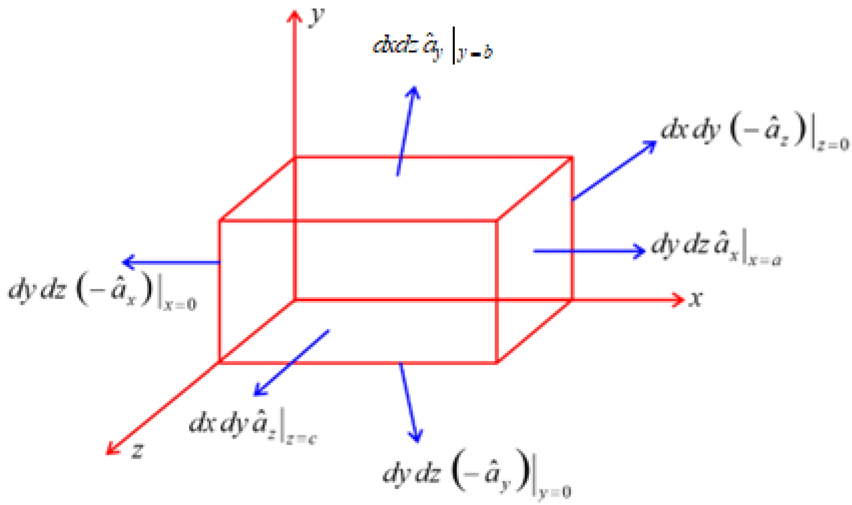
(C) 10 V

(D) 15 V

Solution:

Method I: - Assuming closed as a cuboid (cartesian co-ordinate system).





$$\oiint 5\vec{r}.d\vec{s}$$

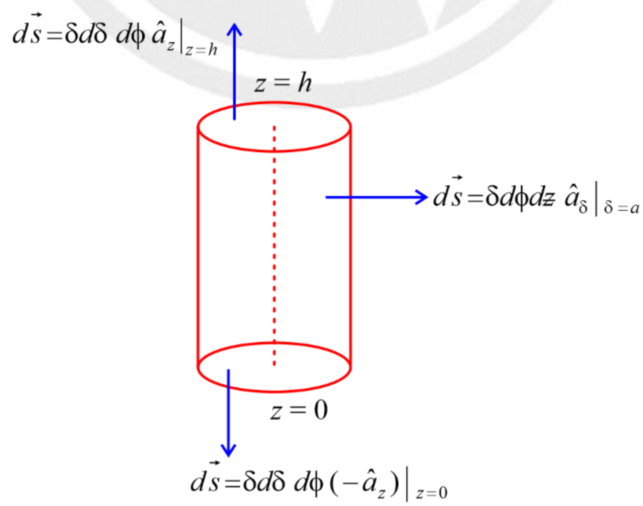
$$= \iint 5(x\hat{i}+yj+zk).dy\,dz\,a_x\Big|_{x=a} + 5\iint (x\hat{i}+yj+zk).dy\,dz(-a_x)\Big|_{x=0} + 5\iint (x\hat{i}+yj+zk).dx\,dz\,a_y\Big|_{y=b}$$

$$+\ 5\iint (x\hat{i}+yj+zk).dx\,dz(-a_y)\Big|_{y=0} + 5\iint (x\hat{i}+yj+zk).dx\,dy\,a_z\Big|_{z=c} + 5\iint (x\hat{i}+yj+zk).dx\,dy(-a_z)\Big|_{z=0}$$

$$=5\int_0^b\int_0^c x\,dy\,dz\Big|_{x=a} - 5\int_0^b\int_0^c x\,dy\,dz\Big|_{x=0} + 5\int_0^a\int_0^c y\,dx\,dz\Big|_{y=b} - 5\int_0^a\int_0^c y\,dx\,dz\Big|_{y=0} + 5\int_0^a\int_0^b z\,dx\,dy\Big|_{z=c} - 5\int_0^a\int_0^b z\,dx\,dy\Big|_{z=0}$$

$= 5abc - 0 + 5abc - 0 + 5abc - 0 = 15abc = 15\text{V}$

Method II: - Assuming closed surface as a cylinder (cylindrical co-ordinate system).



$$\oiint 5\vec{r}\bullet d\vec{s}$$

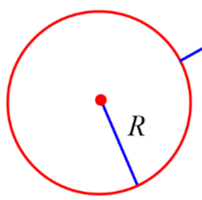
$$= 5\iint(\rho a_\rho + z a_z)\bullet \rho\,d\phi\,dz\,a_\rho\Big|_{\rho=a} + 5\iint(\rho a_\rho + z a_z)\bullet(-\rho\,d\rho\,d\phi\,a_z)\Big|_{z=0} + 5\iint(\rho a_\rho + z a_z)\bullet(\rho\,d\rho\,d\phi\,a_z)\Big|_{z=h}$$

$$= 5\int_0^{2\pi}\int_0^h \rho^2 d\phi\,dz\Big|_{\rho=a} + 5\int_0^a\int_0^{2\pi} \rho z\,d\rho\,d\phi\Big|_{z=h} - 5\int_0^a\int_0^{2\pi} \rho z\,d\rho\,d\phi\Big|_{z=0}$$

$= 5\,a^2 2\pi h + 5h\,\dfrac{a^2}{2} 2\pi - 0$

$= 10\pi a^2 h + 5\pi a^2 h = 15\pi a^2 h = 15V$

Method III: - Assuming closed surface as a sphere (Spherical co-ordinate system).



$$\vec{ds} = r^2 \sin\theta d\theta d\phi\ \hat{a}_r\Big|_{r=R}$$

$$\oiint 5\vec{r}\bullet d\vec{s} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} 5(r a_r)\bullet(r^2 \sin\theta\, d\theta\, d\phi\, a_r)\Big|_{r=R}$$

$$= 5R^3(2)\ (2\pi) = 20\pi R^3$$

$$= 15\left(\frac{4\pi R^3}{3}\right)$$

$$= 15V$$

Method IV: - Consider a cuboid as volume.

$$I = \oiint 5\vec{r}\bullet d\vec{s} = \int_V \left(\vec{\nabla}\bullet 5\vec{r}\right) dv$$

$$\vec{r} = x\hat{i} + y\,j + zk$$

$$\vec{\nabla}\bullet\vec{r} = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial x} + \frac{\partial z}{\partial z}$$

$$= 1 + 1 + 1 = 3$$

$$I = 5\int_V \left(\vec{\nabla}\bullet\vec{r}\right) dv$$

$$= 5 \times 3 \int_v dv$$

$$= 15\int_0^a\int_0^b\int_0^c dx\,dy\,dz$$

$$= 15abc$$

Method V: - consider a cylindrical as volume.

$$\vec{r} = \rho a_\rho + z a_z$$

$$\vec{\nabla}.\vec{r} = \frac{1}{\rho}\left(\frac{\partial(\rho\bullet\rho)}{\partial\rho} + \frac{\partial(0)}{\partial\phi} + \frac{\partial(\rho\bullet z)}{\partial z}\right)$$

$$= \frac{1}{\rho}(2\rho + 0 + \rho) = 3$$

$$I = \int_v \left(\vec{\nabla} \bullet 5\vec{r}\right) dv$$

$$= 5\int_v \left(\vec{\nabla} \bullet \vec{r}\right) dv$$

$$= 5 \times 3 \int_0^a \int_0^{2\pi} \int_0^h \rho d\rho d\phi dz$$

$$= 15\ (\pi a^2 h)$$

$$= 15V$$

Method VI: - Consider a spherical as volume.

$$\vec{r} = r a_r,\ \vec{\nabla} \bullet \vec{r} = \frac{1}{r^2 \sin\theta}\left[\frac{\partial (r^2 \sin\theta \bullet r)}{\partial r} + 0 + 0\right] = 3$$

$$I = \int_v \vec{\nabla} \bullet (5\vec{r})\, dv$$

$$= 15\int_0^R \int_0^{\pi} \int_0^{2\pi} r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= 15\left(\frac{4\pi R^3}{3}\right)$$

$$= 15V$$

Note:
(a) In all three-co-ordinate system, closed surface integration in spherical co-ordinate is easier.
(b) Volume integral is easier than closed surface integration.

❑❑❑

3 MAXWELL EQUATION

3.1. Type of Medium

(A) Free space/air.

- Volume charge density (ρ_v) = 0
- Conductivity (σ) = 0 (perfect insulator)
- Dipole moment (P) $\neq$ 0
- Displacement current density (J_d) $\neq$ 0
- $\in = \in_o = \frac{1}{36\pi} \times 10^{-9} F/m = 8.85 \times 10^{-12} F/m$
- $\mu = \mu_o = 4\pi \times 10^{-7} H/m$
- $\epsilon_r = 1$, $\mu_r = 1$

(B) Lossless Dielectric/perfect dielectric: -

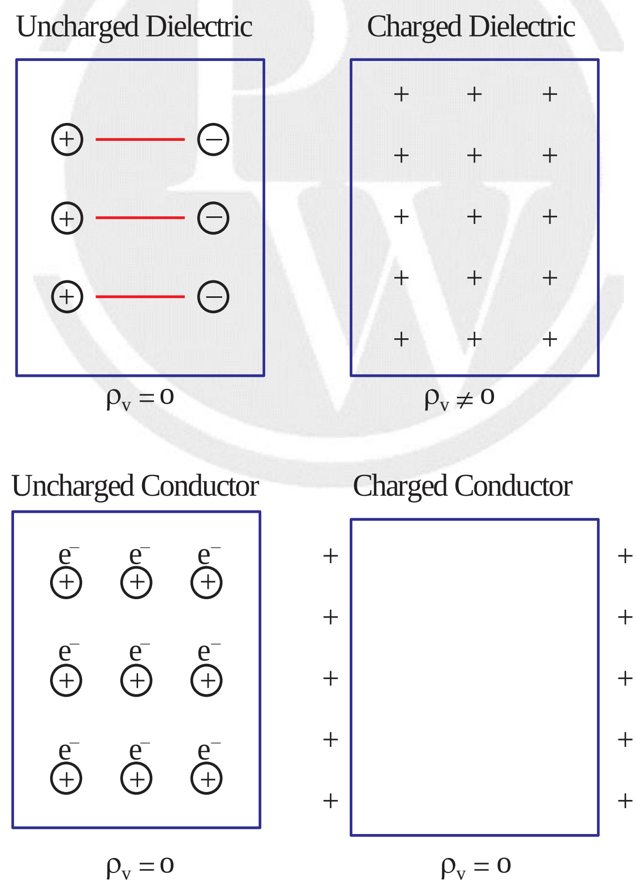
- Dielectric is a type of insulator in which it has dipole.
- Perfect dielectric means perfect insulator.
- $\rho_v = 0$,
- Conductivity of dielctric (σ_d) = 0
- Dipole moment (P) $\neq$ 0
- Displacement current (J_d) $\neq$ 0
- $\in = \in_0 \in_r$
- $\mu = \mu_o \mu_r$
- In general $\epsilon_r > 1$ and $\mu_r = 1$ (Non-magnetic).

(C) Lossy Dielectric/Imperfect dielectric: -

- $\rho_v = 0$
- $\sigma_d \neq 0$ (It gives conductor loss)

- $\in = \in_0 \in_r$
- $\mu = \mu_o \mu_r$
- In general $\epsilon_r > 1$ and $\mu_r = 1$ (Non-magnetic).
- Dipole moment (P) $\neq 0$
- $J_d \neq 0$ (due to dipoles)
- Conduction current density (J_c) $\neq 0$

  → due to conductivity of lossy dielectric
- $\frac{\sigma_d}{\omega \in} < \frac{1}{100}$ ⇒ Low loss dielectric
- $\frac{1}{100} < \frac{\sigma_d}{\omega \in} < 100$ ⇒ Medium-loss dielectric
- $\frac{\sigma_d}{\omega \in} > 100$ ⇒ *High*-loss dielectric



but $\rho_s \neq 0$ (surface charge density).

(D) Conductor:

- $\rho_v = 0$, $\sigma_c \neq 0$ (Very high)

- σ_c = conductivity of conductor.
- Conductor is very high loss dielectric.
- Perfect conductor: - $\sigma_c = \infty$ (Very-very high)
  eg: - Gold, Silver etc.
- Good conductor: - σ_c = very high
  eg: - Brass, conductor etc.
- Poor conductor: - σ_c = high
  eg: - Aluminium

3.2. Types of Conductors

(A) Perfect Electric Conductor (PEC)

E = 0, H = maximum

(B) Perfect Magnetic Conductor (PMC)

E = maximum, H = 0

(C) Super Conductor

E = 0, H = 0

3.3 Properties of Medium

ϵ, μ & σ are known as consecutive property.

(A) Linear: If consecutive property of any medium does not depends upon strength of field, then that medium is linear.

$\sigma, \mu, \epsilon \neq f(E, H) \rightarrow$ Linear Medium $\sigma, \mu, \epsilon = f(E, H) \rightarrow$ Non-Linear Medium

$\epsilon_r = 5 \rightarrow$ Linear $\quad \epsilon_r = \frac{E}{10} + \frac{E^2}{1023} + \frac{H^3}{20319} + \ldots.. \Rightarrow$ Non-Linear

(B) Homogeneous Medium: If consecutive property of any medium does not depends on the point in space, them that medium is homogeneous.

$\epsilon_r = 5 \rightarrow$ Homogeneous

$\epsilon_r = 10x\,(x + 9) \rightarrow$ Inhomogeneous

$\epsilon_r = f\,(10f + 1) \rightarrow$ Inhomogeneous

$\epsilon_r, \mu_r, \sigma = f\,(x, y, z, \delta, \phi, z\, r, \theta, \phi) \rightarrow$ Inhomogeneous

$\epsilon_r, \mu_r, \sigma \neq f\,(x, y, z, \delta, \phi, z\, r, \theta, \phi) \rightarrow$ Inhomogeneous

(C) Isotropic Medium: If consecutive property of any medium does not depends upon direction, them that medium is isotropic.

Case I: Isotropic Medium

$$\vec{D} = \epsilon \vec{E}, \quad \epsilon_r = 3$$

$$\vec{D} = 3\epsilon_o \vec{E}, \Rightarrow \left(D_x\hat{i} + D_y\hat{j} + D_z\hat{k}\right) = 3\epsilon_o\left(E_x\hat{i} + E_y\hat{j} + E_z\hat{k}\right)$$

$$D_x = 3\epsilon_o E_x,\ D_y = 3\epsilon_0 E_y,\ D_z = 3\epsilon_o E_z$$

Since Er is same in all direction. Hence medium is isotropic.

$$\begin{bmatrix} Dx \\ Dy \\ Dz \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} E_o \begin{bmatrix} Ex \\ Ey \\ Ez \end{bmatrix}$$

↳ Scalar Matrix ⟶ Isotropic Medium

Case II: Uni-isotropic Medium

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \epsilon_o \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

↳ Diagonal Matrix

$D_x = 3\epsilon_o E_x$, $D_y = 4\epsilon_o E_y$, $D_z = 5\epsilon_o E_z$

Case III: Anisotropic Medium

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \epsilon_o \begin{bmatrix} 2 & 3 & 4 \\ 5 & 7 & 9 \\ 6 & 3 & 1 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

$D_x = 2\epsilon_o E_x + 3\epsilon_o E_y + 4\epsilon_o E_z$

$D_y = 5\epsilon_o E_x + 7\epsilon_o E_y + 9\epsilon_o E_z$

$D_z = 6\epsilon_o E_x + 3\epsilon_o E_y + \epsilon_o E_z$

Note: All homogeneous are isotropic and all isotropic are homogeneous.

$$H \rightleftharpoons I$$

(D) Non-Dispersive: - If consecutive property of any medium does not depend upon frequency then that medium is non-dispersive.

$\sigma = 2$ ⟶ Non-dispersive

$\sigma = 5\omega\epsilon$ ⟶ dispersive

3.4. Electric Gauss Law

Total electric flux through closed surface is equal to algebraic sum of charges enclosed.

$$\oiint \vec{D} \cdot \vec{ds} = Q_{enc}$$

(a) $\vec{\nabla} \cdot \vec{D} = \rho_V \rightarrow$ Point Form of Gauss law

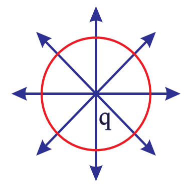
(b) $\oiint \vec{D} \cdot \vec{ds} = \iiint (\vec{\nabla} \cdot \vec{D}) dv = \iiint \rho_V dv$

Gauss theorem or divergence theorem

(c) $\oiint \vec{D} \cdot \vec{ds} \equiv$ Total Flux through Closed Surface.

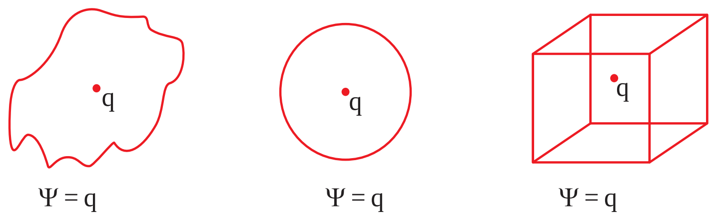
(d) $\oiint \vec{E} \cdot \vec{ds} \equiv$ Total number of electric field lines.

(e) Gaussian surface must be closed.

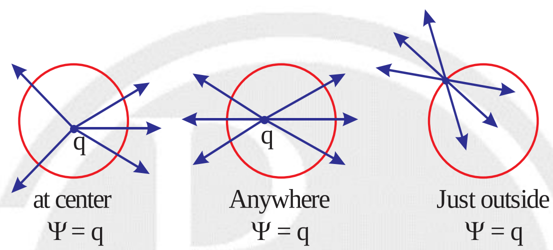


$\Psi = q$

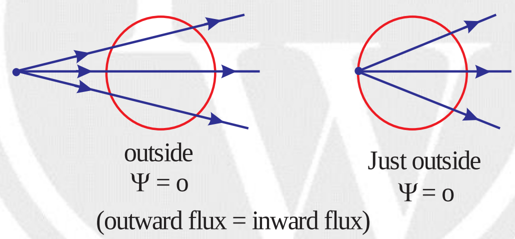
(f) Gaussian surface may be irregular/regular.



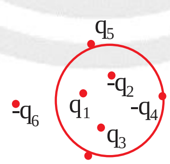
(g) The charge will be placed anywhere inside the Gaussian surface.



(h) When charge is placed outside the Gaussian surface.



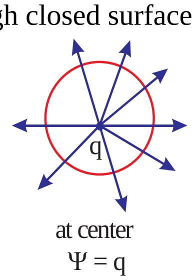
(i) $Q_{enc} \equiv$ algebraic sum of the charges enclosed.



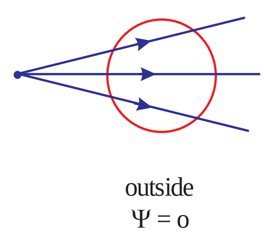
$Q_{enc} = q_1 - q_2 + q_3 - q_4$

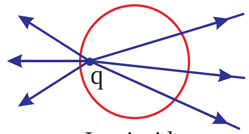
(j) Total flux through closed surface

(i)

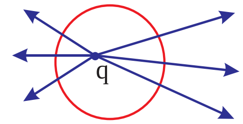


(ii)

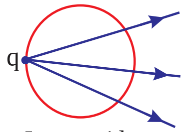


(iii) 

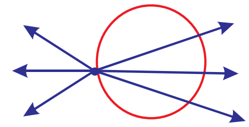
Just inside
$\Psi = q$

(iv) 

Anywhere inside
$\Psi = q$

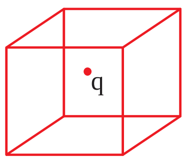
(v) 

Just outside
$\Psi = 0$

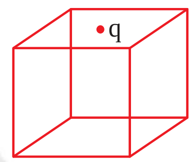
(vi) 

at the Surface
$\Psi = \frac{q}{2}$

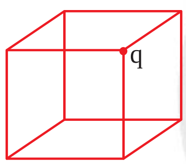
(k) Total flux through cube, when charge is placed at

(i) 

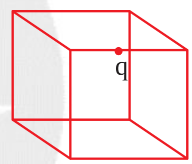
$\Psi = q$
(Flux through one cube).

(ii) 

$\Psi = \frac{q}{2}$
(Flux through one cube).

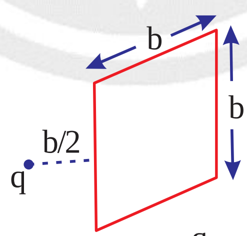
(iii) 

$\Psi = \frac{q}{8}$
(Flux through one cube).

(iv) 

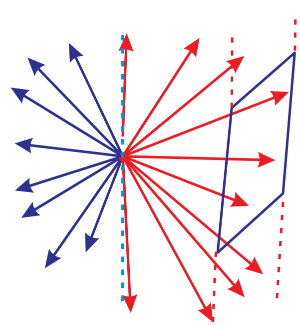
$\Psi = \frac{q}{4}$
(Flux through one cube).

(l) Flux through the plane



$\Psi = \frac{q}{6}$

(m) Flux through infinite length plane



$\Psi = \frac{q}{2}$

3.5. Magnetic Gauss Law

Total magnetic flux through any closed surface is zero.

$$\oiint \vec{B}\bullet d\vec{s} = 0$$

(a) Point form $\Rightarrow \vec{\nabla}\bullet\vec{B} = 0$

(b) Integral form $\Rightarrow \vec{B}\bullet d\vec{s} = 0$

(c) $\oiint \vec{B}.d\vec{s} = 0$, $\vec{\nabla}.\vec{B} = 0$
- → Divergericeless
- → Solenoidal
- → Magnetic monopole does not exist
- → Originating and terminating point are not defined

3.6. Electric Field Conservative/KVL.

$$\oint \vec{E}.d\vec{l} = 0,\ \vec{\nabla}\times\vec{E} = \vec{0}$$

Work done in a closed path is zero.

$\oint \vec{E}.d\vec{l} = 0$, $\vec{\nabla}\times\vec{E} = \vec{0}$
- → Irrotational
- → Path independent
- → Conservative
- → KVL
- → Does not exist in a closed loop

3.7. Ampeare Circuital Law

Total magneto motive force in a loop is equal to algebraic sum of current enclose.

$$\oint \vec{H}.d\vec{l} = I_{enc}, \vec{\nabla}\times\vec{H} = \vec{J}.$$

(a) If loop is closed in anticlockwise or counter clockwise direction, then

(i) Outward current is taken as positive.

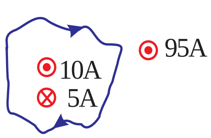
(ii) Inward current is taken as negative.

(b) If loop is closed in clockwise direction then

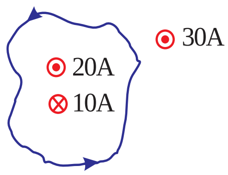
(i) Inward current is taken as negative.

(ii) Outward current is taken as positive.

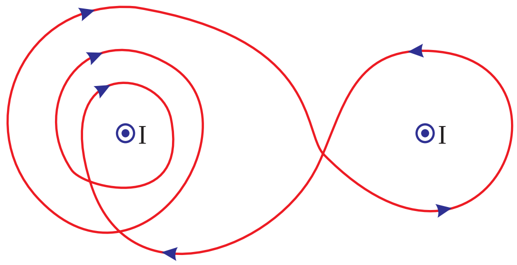
(c)



$$\oint \vec{H}.\vec{dl} = Ienc$$
$$= -10 + 5 = 5A$$

(d) 

$$\oint \vec{H}.\vec{dl} = I_{enc}$$
$$= -20 - 10 = 10A$$

(e) 

$$\oint \vec{H}.\vec{dl} = I_{enc} = -3I - I = -2I$$

3.8. Maxwell Equation in Statics

Integral Form	Point Form
(a) $\oint \vec{D}.d\vec{l} = Q_{enc}$	$\vec{\nabla} \times \vec{D} = \rho_V$

Electric Gauss Law

(b) $\oint \vec{E}.\vec{dl} = 0$	$\vec{\nabla} \times \vec{E} = \vec{0}$

Electric field conservative

(c) $\oiint \vec{B}.d\vec{s} = 0$	$\vec{\nabla} \times \vec{B} = 0$

Magnetic Gauss Law

(d) $\oint \vec{H}.\vec{dl} = I_{enc}$	$\vec{\nabla} \times \vec{H} = \vec{J}$

Ampeare Circuital Law

3.9. Maxwell Equation in Ideal and Practical Medium

		Ideal	Practical
(a)	$\oiint \vec{D}.d\vec{s} = Q_{enc}$	✓	✓
(b)	$\vec{\nabla} \cdot \vec{D} = \rho_V$	✓	×
(c)	$\oiint \vec{E} \cdot d\vec{s} = \frac{Qenc}{\epsilon}$	✓	×

(d)	$\vec{\nabla} \times \vec{E} = \frac{\rho_v}{\epsilon}$	✓	×
(e)	$\oint \vec{E} \cdot \vec{dl} = 0$	✓	✓
(f)	$\vec{\nabla} \times \vec{E} = \vec{0}$	✓	✓
(g)	$\oint \vec{D} \cdot \vec{dl} = 0$	✓	×
(h)	$\vec{\nabla} \times \vec{D} = \vec{0}$	✓	×
(i)	$\oiint \vec{B} \cdot d\vec{s} = 0$	✓	✓
(j)	$\vec{\nabla} \cdot \vec{B} = 0$	✓	✓
(k)	$\oiint \vec{H} \cdot d\vec{s} = 0$	✓	×
(l)	$\vec{\nabla} \cdot \vec{H} = 0$	✓	×
(m)	$\oint \vec{H} \cdot \vec{dl} = I_{enc}$	✓	✓
(n)	$\vec{\nabla} \times \vec{H} = \vec{J}$	✓	✓
(o)	$\oint \vec{B} \cdot \vec{dl} = \mu I_{enc}$	✓	×
(p)	$\vec{\nabla} \times \vec{B} = \mu \vec{J}$	✓	×

- **Ideal Medium:** Linear, Homogeneous, and Isotropic
- **Practical Medium:** Non-linear, Inhomogeneous and an isotropic.

3.10. Maxwell Equation in Different types of Medium

(a) Charge Free Medium

$$Q = 0,\ \rho_L = 0,\ \rho_s = 0,\ \rho_v = 0$$

- $\oiint \vec{D} \cdot d\vec{s} = 0$ $\qquad \vec{\nabla} \cdot \vec{D} = 0$
- $\oint \vec{E} \cdot \vec{dl} = 0$ $\qquad \vec{\nabla} \times \vec{E} = \vec{0}$
- $\oiint \vec{B} \cdot d\vec{s} = 0$ $\qquad \vec{\nabla} \cdot \vec{B} = 0$
- $\oint \vec{H} \cdot \vec{dl} = I_{enc}$ $\qquad \vec{\nabla} \times \vec{H} = \vec{J}$

(b) Current Free Medium

$I = 0, K = 0, J = 0$

- $\oiint \vec{D} \cdot d\vec{s} = Qenc$ $\qquad \vec{\nabla} \cdot \vec{D} = \rho_V$
- $\oint \vec{E} . \vec{dl} = 0$ $\qquad \vec{\nabla} \times \vec{E} = \vec{0}$
- $\oiint \vec{B} \cdot d\vec{s} = 0$ $\qquad \vec{\nabla} \cdot \vec{B} = 0$
- $\oint \vec{H} . \vec{dl} = 0$ $\qquad \vec{\nabla} \times \vec{H} = \vec{0}$

(c) Source Free Medium

$Q = 0, \rho_L = 0, \rho_S = 0, \rho_V = 0$

$I = 0, K = 0, J = 0$

- $\oiint \vec{D} \cdot d\vec{s} = 0$ $\qquad \vec{\nabla} \cdot \vec{D} = 0$
- $\oint \vec{E} \cdot \vec{dl} = 0$ $\qquad \vec{\nabla} \times \vec{E} = \vec{0}$
- $\oiint \vec{B} \cdot d\vec{s} = 0$ $\qquad \vec{\nabla} \cdot \vec{B} = 0$
- $\oint \vec{H} \cdot \vec{dl} = 0$ $\qquad \vec{\nabla} \times \vec{H} = \vec{0}$

3.11. To Find Unknowns of $\vec{E}, \vec{H}$ & V.

(a) $\vec{\nabla} \cdot \vec{B} = 0 \rightarrow$ Unknown of $\vec{B}$

(b) $\vec{\nabla} \times \vec{E} = \vec{0} \rightarrow$ Unknown of $\vec{E}$

(c) $\nabla^2 V = 0 \rightarrow$ Unknown of V

(d) $\rho_V = \vec{\nabla} \cdot \vec{D}$

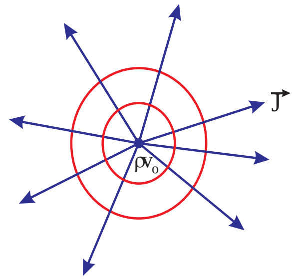
(e) $\vec{J} = \vec{\nabla} \times \vec{H}$

3.12. Continuity Equation/KCL

- It gives flow of charge in a medium.

- Continuity Equation in point form.

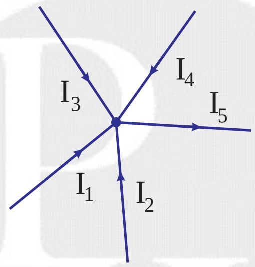
$$\vec{\nabla} \bullet \vec{J} = \frac{-\partial \rho_V}{\partial t}$$



- Continuity Equation in integral form

$$\oiint \vec{J} \bullet \vec{ds} = -\iiint_{\vartheta} \left(\frac{\partial \rho_v}{\partial t} \right) dv$$

- KCL

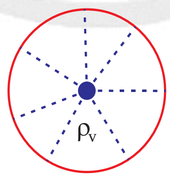


At node

$$\sum I_i = 0$$

$$\Rightarrow \vec{\nabla} . \vec{J} = 0$$
$$\Rightarrow \oiint \vec{J} . \vec{ds} = 0 \quad \mapsto \text{KCL}$$

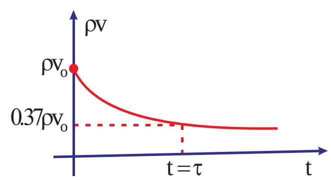
- Equation of flow of charge in a medium



(a) $\rho_V = \rho_{Vo} e^{-\frac{\sigma}{\varepsilon} t}$ (b) $\rho_S = \rho_{So} e^{-\frac{\sigma}{\varepsilon} t}$

(c) $\rho_L = \rho_{Lo} e^{-\frac{\sigma t}{\varepsilon}}$ (d) $Q = Q_o e^{-\frac{\sigma t}{\varepsilon}}$

Where $\frac{\varepsilon}{\sigma} = \tau$ (Relaxation time constant)

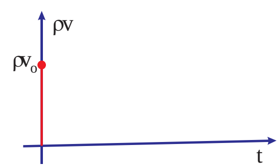


- Flow of charge in conductor.

  σ is very high

  So, τ (Relaxation time constant) = 0

  Hence, $\rho_V = \rho_{vo} e^{-\frac{\sigma}{\varepsilon}t} \simeq 0$



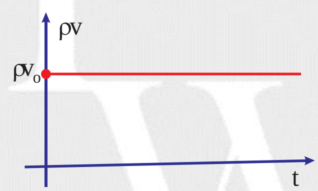
Very fast flow of charge in a conductor. So, charge does not reside inside conductor.

- Flow of charge inside insulator.

  $\sigma_d = 0$ (very low)

So, τ (Relaxation time constant) = ∞

$$\rho_V = \rho_{vo} e^{-\frac{\sigma}{\varepsilon}t} = \rho_{vo}$$



There is no flow of charge inside perfect insulator.

3.13. Laplace's Equation/Poisson Equation

(a) Poisson Equation in point form

(i) For Ideal Medium

$$\nabla^2 V = \frac{-\delta_V}{\varepsilon}$$

(ii) For Practical Medium

$$\vec{\nabla}.\left(\varepsilon\left(\vec{\nabla}V\right)\right) = -\delta_V$$

$$\varepsilon\left(\nabla^2 V\right) + \left(\vec{\nabla}_V\right).\left(\vec{\nabla}_\varepsilon\right) = -\delta_V$$

(b) Laplace Equation

$\delta_V \rightarrow$ Free space/air/vacuum/uncharged/ dielectric/conductor/singular charge distribution/cavity/source free medium charge free medium.

$\nabla^2 V = 0 \Rightarrow$ Laplacian Equation

3.14. Difference between Laplacian & Poisson Equation

Laplace's Equation		Poisson Equation	
(a)	$\nabla^2 V = 0$	(i)	$\nabla^2 V = \frac{-\rho_V}{\varepsilon}$
(b)	$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$	(ii)	$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{-\rho_V}{\varepsilon}$
(c)	Complementary function	(iii)	Complementary function + Particular integral
(d)	Unique solution and follow uniqueness theorem	(iv)	Does not follow uniqueness theorem
(e)	Linear Equation	(v)	Non- Linear Equation
(f)	Homogeneous Equation	(vi)	Non- Homogeneous Equation

3.15. Magnetic Force

$$\vec{F}_m = I\left(\vec{l} \times \vec{B}\right)$$

$\vec{l} \equiv$ length of wire and directed along current direction.

3.16. Magnetic Energy Density

$$\mu_m = \frac{1}{2}\vec{B}.\vec{H} = \frac{1}{2}\mu H^2 = \frac{B^2}{2\mu}$$

$$\mu_m = \frac{1}{2}\vec{J}.\vec{A} \text{ where } \vec{A} \equiv \text{Magnetic vector potential}$$

3.17. Faraday's Law

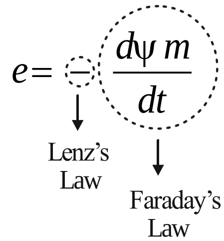
(a) Faraday's 1st Law (Qualitative Analysis):

When magnetic field lines cuts conductor, then an electromotive force will be developed, which is known as induced electromotive force.

(b) Faraday's 2nd Law (Quantitative Analysis): -

The induced electromotive force is directly proportional to time rate of charge of flux.

$$e \alpha \frac{d\psi m}{dt} \Rightarrow \text{Faraday's Law}$$



(C) Faradays and Lenz's Law in integral form

$$\oint \vec{E} \bullet d\vec{l} = \iint -\frac{\partial \vec{B}}{\partial t} \bullet d\vec{s}$$

(d) Faraday's and Lenz's Law in point form

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

(e)

- ψ_m = Magnetic Flux

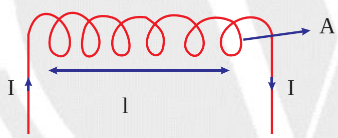
  N = Number of turns

  l = length of solenoid

  ϕ_m = Magnetic flux due to one turn

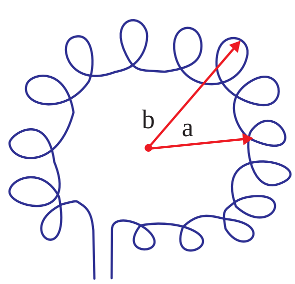
  A = cross-sectional area of solenoid

(f)



- l = (N-1)d
- $H = \frac{NI}{l}$
- $B = \mu H = \frac{\mu NI}{l}$
- $\phi_m = BA = \frac{\mu NIA}{l}$
- $\psi_m = LI$ (Weber)
- $e = \frac{-d\psi_m}{dt} = \frac{-\mu N^2 A}{l}\left(\frac{di}{dt}\right) = -L\frac{dI}{dt}$
- $V = -e = +L\frac{dI}{dt}$

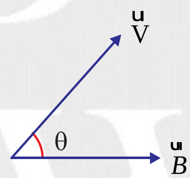
(g) Toroid



- $r = \frac{a+b}{2}$
- $l = 2\pi r$
- $H = \frac{NI}{2\pi r}$
- $B = \mu H = \frac{\mu NI}{2\pi r}$
- $\phi_m = \left(\frac{\mu NIA}{2\pi r}\right)$
- $\psi_m = N\phi_m = \left(\frac{\mu N^2 A}{2\pi r}\right) I = LI$
- $V = -e = +\frac{d\psi_m}{dt} = L\frac{dI}{dt}$
- $L = \frac{\mu N^2 A}{2\pi r}$

Lorentz's Force

- $\vec{F}_{net} = q\left(\vec{V} \times \vec{B}\right) + q\vec{E}_{in}$
- $\vec{E}_{in} = \vec{B} \times \vec{V}$
- $e = BVL \sin\theta$



$e = \left(\vec{V} \times \vec{B}\right).\vec{l}$

3.18. Modified Ampere's Circuital Law:

(a) Conduction current: - It is due to flow of electron. It is due to conductivity.

- Since, conductivity of conductor is very high. Hence, conduction current is very high.
- Since, conductivity of dielectric is very low. Hence, conduction current is very low. (Leakage Current)

(b) Displacement current: - It is due to rotation of dipoles.

- Since, number of dipoles inside conductor is very low. Hence, displacement current is very low.
- Since, number of dipoles inside dielectric is very high. Hence, displacement current is very high.

(c)

Conductor	Dielectric
$I_c \gg I_d$	$I_c \ll I_d$

(d) Ampeare's circuit law is modified by maxwell

$$\oint \vec{H} \bullet \vec{dl} = I_{enc} \Rightarrow \text{In statics}$$

$$\oint \vec{H} \bullet \vec{dl} = I_c + I_d \Rightarrow \text{In time varrying}$$

(e) Modified Ampeare's circuital low in integral low

$$\oint \vec{H} \bullet \vec{dl} = I_c + I_d$$

(f) Modified Ampeare's circuital law in point form

$$\vec{\nabla} \times \vec{H} = \vec{J_c} + \vec{J_d}$$

(g) $\vec{J_c}$ **= Conduction current density =** $\sigma \vec{E}$

$$\vec{J_d} = \frac{\partial \vec{D}}{\partial t} = \varepsilon \frac{\partial \vec{E}}{\partial t} = \text{Displacement current density}$$

(h) $$I_c = \iint \vec{J}_c . d\vec{s}$$

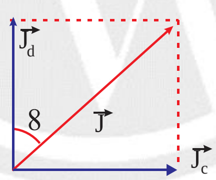
$$I_d = \frac{\partial \psi_e}{dt} \equiv \text{Time rate of change of electric flux per unit time.}$$

(i)

$$J = \sqrt{J_c^2 + J_d^2}$$

$$\tan \delta = \frac{J_c}{J_d}$$

$$I = I_c + I_d$$



(j) $$\vec{\nabla} \times \vec{H} = \vec{J}_c + \vec{J}_d = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} = (\sigma + j\omega\varepsilon)\vec{E}$$

(k) $$\oint \vec{H} \bullet \vec{dl} = \iint \left(\sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} \right) \bullet d\vec{s} = \iint (\sigma + j\omega\varepsilon) \vec{E} \bullet d\vec{s}$$

3.19. Maxwell's Equation in time Varying Fields

(a) $\oiint \vec{D} \bullet d\vec{s} = Q_{enc}$ (i) $\vec{\nabla} \bullet \vec{D} = \rho_V$

Electric Gauss Law

(b) $\oiint \vec{B} \bullet d\vec{s} = 0$ (ii) $\vec{\nabla} \bullet \vec{B} = 0$

Magnetic Gauss Law

(c) $\oint \vec{E} \bullet \vec{dl} = -\iint \frac{\partial \vec{B}}{\partial t} \bullet d\vec{s}$ (iii) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -jw\mu\vec{H}$

Faraday's and Lenz's Law

(d) $\oint \vec{H} \bullet \vec{dl} = I_c + I_d$ (iv) $\vec{\nabla} \times \vec{H} = \sigma\vec{E} + \varepsilon\frac{\partial \vec{E}}{\partial t} = (\sigma + jw\varepsilon)\vec{E}$

Modified Ampeare Circuital

3.20. Magnetic Vector Potential and Magnetic scaler Potential

(a) Magnetic scalar potential: -

$$\vec{J} = \vec{0} \Rightarrow \text{Volume current density} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{0} \Rightarrow \vec{H} = -\vec{\nabla} V_m$$

$$\boxed{V_m = -\int \vec{H} \cdot \vec{dl}} \rightarrow \text{Magnetic scalar potential}$$

(b) Magnetic Vector Potential

- $\vec{B} = \vec{\nabla} \times \vec{A}$ ($\vec{A} \equiv$ Magnetic Vector Potential)
- $\vec{A} = \frac{\mu}{4\pi}\int \frac{I\vec{dl}}{R}$
- $\vec{A} = \frac{\mu}{4\pi}\iint \frac{\vec{k}ds}{R}$
- $\vec{A} = \frac{\mu}{4\pi}\iiint \frac{\vec{J}dv}{R}$
- $\oint \vec{A} \bullet \vec{dl} = \iint (\vec{\nabla} \times \vec{A}) \bullet d\vec{s} = \iint \vec{B} . d\vec{s} = \psi_m = \text{Magentic flux}$
- $\vec{\nabla}(\vec{\nabla} \bullet \vec{A}) - \nabla^2 \vec{A} = \mu\vec{J}$

(a) In statics $\boxed{\vec{\nabla} \bullet \vec{A} = 0}$ $\therefore \nabla^2 \vec{A} = -\mu\vec{J}$

(b) In time varying $\left(\vec{\nabla}(\vec{\nabla} \bullet \vec{A})\right) = \mu\varepsilon\frac{\partial^2 \vec{A}}{\partial t^2}$

$$\nabla^2 \vec{A} - \mu\varepsilon\frac{\partial^2 \vec{A}}{\partial t^2} = -\mu\vec{J}$$

❒❒❒

4 Electromagnetic Wave

4.1. Time Harmonic Equation

(a) Lossless Medium

$$\nabla^2 \vec{H} = \mu \in \frac{\partial^2 \vec{H}}{\partial t^2} \Rightarrow \nabla^2 \vec{H} = \frac{1}{v_P^2} \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \mu \in \frac{\partial^2 \vec{E}}{\partial t^2} \Rightarrow \nabla^2 \vec{E} = \frac{1}{v_P^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

Where, v_P = Phase velocity in a medium.

(b) Lossy Medium

$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu \in \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{H} = \mu\sigma \frac{\partial \vec{H}}{\partial t} + \mu \in \frac{\partial^2 \vec{H}}{\partial t^2}$$

Loss : $\mu\sigma \frac{\partial \vec{E}}{\partial t}, \quad \mu\sigma \frac{\partial \vec{H}}{\partial t}$

Harmonic : $\mu \in \frac{\partial^2 \vec{E}}{\partial t^2}, \quad \mu \in \frac{\partial^2 \vec{H}}{\partial t^2}$

4.2. Wave Equation/Helmholtz Equation/Telegraph Equation

(a) Lossy Medium

$$\nabla^2 \vec{E} = \gamma \vec{E}, \; \nabla^2 \vec{H} = \gamma^2 \vec{H}$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$\gamma = \alpha + j\beta$$

$$\alpha \equiv \text{ attenuation constant } \left(\frac{\text{Neper}}{\text{m}}\right)$$

$$\beta \equiv \text{ Phase constant } \left(\frac{\text{radian}}{\text{m}}\right)$$

$$\gamma \equiv \text{ Propagation constant } \left(\frac{1}{\text{m}}\right)$$

(b) Lossless medium

$$\nabla^2\vec{E}-\left(\omega^2\mu\in\right)\vec{E}=\vec{0}$$

$$\nabla^2\vec{H}-\left(\omega^2\mu\in\right)\vec{H}=\vec{0}$$

$$\gamma=j\omega\sqrt{\mu\in},\alpha=0,\beta=\omega\sqrt{\mu\in}$$

4.3. Equation of Electric Field and Magnetic Field

- $\vec{E}=E_0e^{-\alpha z}e^{-j\beta z}e^{j\omega t}\ a_x$ (Complex Form)
- $\vec{H}=H_0e^{-\alpha z}e^{-j\beta z}e^{j\omega t}a_y$ (Complex Form)
- $\vec{E}=E_0e^{-\alpha z}\cos(\omega t-\beta z)\hat{a}_x$
- $\vec{H}=H_0e^{-\alpha z}\cos(\omega t-\beta z)\hat{a}_y$
- $E_0=\text{Amplitude of Electric Field}\left(\frac{V}{m}\right)$
- $\alpha=\text{Attenuation constant}\left(\frac{\text{Neper}}{m}\right)$
- $e^{-\alpha z}=\text{Attenuation Factor}$
- $H_0=\text{Amplitude of Magnetic Field}\left(\frac{A}{m}\right)$
- $\beta=\frac{2\pi}{\lambda}\Rightarrow$ Phase Constant/Wave Number

4.4. Intrinsic Impedance/Wave Impedance.

- It relates conversion from Electric Field to Magnetic Field or vice versa.
- It is impeadance offered by medium during conversion Electric Field to Magnetic Field or vice versa.
- $\eta\equiv$ Intrinsic Impedance (Ω)

Lossy Medium

$$\eta=\frac{|E|}{|H|}=\frac{E_0^+}{H_0^+}=\frac{-E_0^-}{H_0^-}=\frac{j\omega\mu}{\gamma}=\frac{\gamma}{\sigma+j\omega\in}=\sqrt{\frac{j\omega\mu}{\sigma+j\omega\in}}$$

Lossless Medium

$$\eta=\frac{\omega\mu}{\beta}=\frac{\beta}{\omega\in}=\sqrt{\frac{\mu}{\in}}$$

Derivation of Intrinsic Impeadance.

$$\left.\begin{aligned}\vec{E}&=E_0^+e^{-\gamma z}e^{j\omega t}a_x\\ \vec{H}&=H_0^+e^{-\gamma z}e^{j\omega t}a_y\end{aligned}\right\}\text{Wave travelling in +z direction}$$

(a) $\vec{\nabla}\times\vec{H}=(\sigma+j\omega\in)\vec{E}$ (Modified Ampeare Circuital Law)

$$\frac{E_0^+}{H_0^+}=\frac{\gamma}{\sigma+j\omega\in}$$

(b) $\vec{\nabla}\times\vec{H}=-j\omega\mu\vec{H}$ (Faraday's + Lenz's Law)

$$\frac{E_0^+}{H_0^+}=\frac{j\omega\mu}{\gamma}$$

$$\left.\begin{aligned}\vec{E}&=E_0^-e^{\gamma z}e^{j\omega t}\hat{a}_x\\ \vec{H}&=H_0^-e^{\gamma z}e^{j\omega t}\hat{a}_y\end{aligned}\right\}\text{ Wave travelling in -z direction.}$$

(a) $\vec{\nabla}\times\vec{E}=-j\omega\mu\vec{H}$ (Faraday's + Lenz's Law)

$$\frac{-E_0^-}{H_0^-}=\frac{j\omega\mu}{\gamma}$$

(b) $\vec{\nabla}\times\vec{H}=(\sigma+j\omega\in)\vec{E}$

$$\frac{-E_0^-}{H_0^-}=\frac{\gamma}{\sigma+j\omega\in}\text{(Modified Ampeare circuital Law)}$$

4.5. Phase Velocity and Group Velocity

(a) Phase Velocity: The velocity which is along the propagation direction and defined at single frequency.

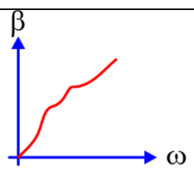
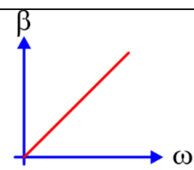
$$\vec{v}_p=\frac{\omega}{\beta}a_p$$

(b) Group Velocity: The velocity which is along the propagation direction and defined for multiple frequency.

$$\vec{v}_g=\frac{d\omega}{d\beta}a_p$$

* $a_p\equiv$ unit vector directed along propagation direction.

(c) Phase Velocity and Group Velocity in Dispersive and Non-Dispersive Medium.

Dispersive	Non-Dispersive
(a) $v_p=f(\omega)$ Phase velocity depends upon frequency of operation.	(a) $v_p\neq f(\omega)$ Phase velocity does not depend upon frequency of operation.
(b) $v_p\neq v_g$	(b) $v_p=v_g$
(c) $\beta\propto\omega$ (NOT)	(c) $\beta\propto\omega$
(d) 	(d) 

(d) $v_g = \dfrac{v_p}{1 - \dfrac{\omega}{v_p}\dfrac{dv_p}{d\omega}}$

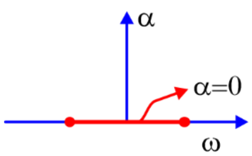
(i) $v_p \neq f(\omega) \Rightarrow \dfrac{dv_p}{d\omega} = 0 \Rightarrow v_g = v_p$

(ii) $v_p = f(\omega)$

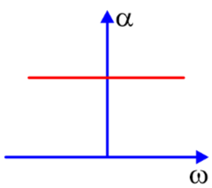
Case I: $\dfrac{dv_p}{d\omega} > 0 \Rightarrow v_g > v_p$

Case II: $\dfrac{dv_p}{d\omega} < 0 \Rightarrow v_g < v_p$

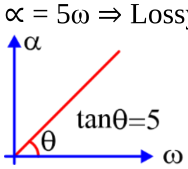
(e)
- $\propto = 0 \Rightarrow$ Lossless and Distortionless.



- $\propto$ = Constant $\Rightarrow$ Lossy and Distortionless.



- $\propto = 5\omega \Rightarrow$ Lossy and Distortive.



- $\propto = 0 \Rightarrow$ Lossless, $\propto = f(\omega) \Rightarrow$ Distortive
  $\propto \neq 0 \Rightarrow$ Lossy, $\propto \neq f(\omega) \Rightarrow$ Distortionless

* **All lossline medium is distortionless medium but all distortionless medium is not lossless medium.**

4.6. Wave Parameters of Different Medium

	Free Space or Air	Lossless or Perfect Dielectric	Lossy or Imperfect dielectric	High Loss or Conductor
α	0	0	$\dfrac{\sigma_d}{2}\sqrt{\dfrac{\mu}{\epsilon}}$	$\sqrt{\pi\mu f\sigma_c}$
	Lossless and Distortionless	Lossless and Distortionless	Lossy and Distortionless	Lossy and Distortive
β	$\beta_0 = \omega\sqrt{\mu_0\epsilon_0}$	$\omega\sqrt{\mu\epsilon} = \beta_0\sqrt{\mu_r\epsilon_r}$	$\omega\sqrt{\mu\epsilon} = \beta_0\sqrt{\mu_r\epsilon_r}$	$\sqrt{\pi f\mu\sigma_C}$
	Non-Dispersive	Non-Dispersive	Non-Dispersive	Dispersive
λ	$\lambda_0 = \dfrac{2\pi}{\beta_0}$	$\dfrac{2\pi}{\beta} = \dfrac{\lambda_0}{\sqrt{\mu_r\epsilon_r}}$	$\dfrac{2\pi}{\beta} = \dfrac{\lambda_0}{\sqrt{\mu_r\epsilon_r}}$	$\dfrac{2\pi}{\beta}$

v_p	$\dfrac{1}{\sqrt{\mu_0 \in_0}} = C_0$	$\dfrac{1}{\sqrt{\mu \in}} = \dfrac{C_0}{\sqrt{\mu_r \in_r}}$	$\dfrac{1}{\sqrt{\mu \in}} = \dfrac{C_0}{\sqrt{\mu_r \in_r}}$	$\sqrt{\dfrac{2\omega}{\mu\sigma_C}}$
v_g	v_p	v_p	v_p	$2v_p$
$v_p.v_g$	C_0^2	$\dfrac{C_0^2}{\mu_r \in_r}$	$\dfrac{C_0^2}{\mu_r \in_r}$	$\dfrac{4\omega}{\mu\sigma_C}$
η	$\sqrt{\dfrac{\mu_0}{\in_0}} = 120\pi = 377$ (Resistive)	$\sqrt{\dfrac{\mu}{\in}} = 120\pi\sqrt{\dfrac{\mu_r}{\in_r}} =$ $377\sqrt{\dfrac{\mu_r}{\in_r}}$ (Resistive)	$\sqrt{\dfrac{\mu}{\in}} + \dfrac{j\sigma_d\sqrt{\mu}}{2\omega \in^{1.5}} = \text{R+jX}$ (Complex and Inductive)	$\sqrt{\dfrac{\omega\mu}{2\sigma_C}} + j\sqrt{\dfrac{\omega\mu}{2\sigma_C}} = R_S + jX_S$ (Complex and Inductive)

4.7. Wave in Space

(a) $\vec{\beta} = \beta_x \hat{a}_x + \beta_y \hat{a}_y + \beta_z \hat{a}_z$

$\beta = \sqrt{\beta_x^2 + \beta_y^2 + \beta_z^2}$

(b) $\dfrac{1}{\lambda^2} = \dfrac{1}{\lambda_x^2} + \dfrac{1}{\lambda_y^2} + \dfrac{1}{\lambda_z^2}$

(c) $\dfrac{1}{V_p^2} = \dfrac{1}{V_{p_x}^2} + \dfrac{1}{V_{p_y}^2} + \dfrac{1}{V_{p_z}^2}$

4.8. Tangent Loss

(a) Lossless medium (σ = 0)

$$\vec{\nabla} \times \vec{H} = \vec{J}_d = \in \frac{\partial \vec{E}}{\partial t} = j\omega \in \vec{E}$$

$$= j\omega \in_0 \in_r \vec{E}$$

Where $\in_r$ is real quantity.

(b) Lossy medium

$$\vec{\nabla} \times \vec{H} = \underset{\substack{\Downarrow \\ Loss}}{\vec{J}_c} + \underset{\substack{\Downarrow \\ Storage}}{\vec{J}_d}$$

$$\vec{\nabla} \times \vec{H} = (\sigma + j\omega \in)\vec{E} = j\omega \in_0 (\in_r - j\frac{\sigma}{\omega \in_0})\vec{E}$$

$$\vec{\nabla} \times \vec{H} = j\omega_0 \in_0 \in_r^* \vec{E}$$

$\in_r^*$ = Complex form of relative permittivity

$$\in_r^* = \in_r' - j\in_r'' = \in_r - j\frac{\sigma}{\omega \in_0}$$

Where $\in_r^{'} = \in_r$ gives storage

$$\in_r^{''} = \frac{\sigma}{\omega \in_0} \text{ gives loss}$$

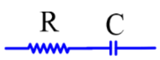
(c) Tangent Loss or Loss Tangent

$$\tan\delta = \frac{J_c}{J_d} = \frac{\text{Loss}}{\text{Storage}}$$

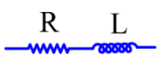
$$\tan\delta = \frac{J_c}{J_d} = \frac{I_c}{I_d} = \frac{\sigma}{\omega \in_0 \in_r} = \frac{\in_r^{''}}{\in_r^{'}}$$

(d) Tangent Loss related to network Circuit.

(i) $\tan\delta = \frac{R}{X_C} = \frac{V_R}{V_C} = \frac{P}{Q_C}$

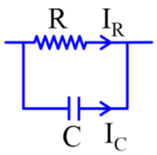


(ii) $\tan\delta = \frac{R}{X_L} = \frac{V_R}{V_L} = \frac{P}{Q_L}$



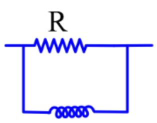
(iii) $Y = G + jB_C$

$$\tan\delta = \frac{G}{B_C} = \frac{X_C}{R} = \frac{P}{Q_C} = \frac{I_R}{I_C}$$



(iv) $Y = G - jB_L$

$$\tan\delta = \frac{G}{B_L} = \frac{X_L}{R} = \frac{P}{Q_L} = \frac{I_R}{I_L}$$



(e) $y = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\in)}$

$2\alpha\beta = \omega\mu\sigma$

$\beta^2 - \alpha^2 = \omega^2\mu\in$

$$\tan\delta = \frac{\sigma}{\omega\in} = \frac{2\alpha\beta}{\beta^2 - \alpha^2}$$

(f) $\eta = \sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\in}} \Rightarrow$ Intrinsic Impedance

$$\theta_\eta = \frac{\frac{\pi}{2} - \tan^{-1}\left(\frac{\omega\in}{\sigma}\right)}{2}$$

$$\Rightarrow \frac{\omega\in}{\sigma} = \cot(2\theta_\eta)$$

$$\Rightarrow \frac{\sigma}{\omega\in} = \tan(2\theta_\eta)$$

$$\Rightarrow \tan\delta = \tan(2\theta_\eta)$$

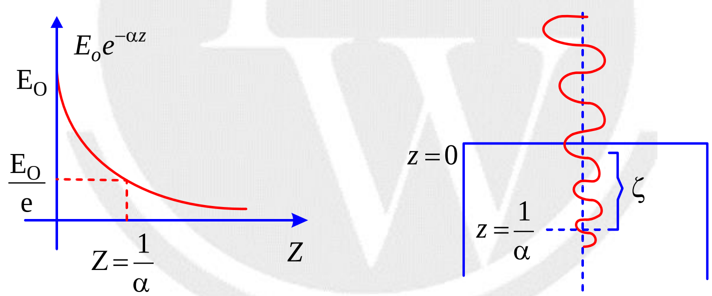
$$\therefore \delta = 2\theta_\eta = 2(\angle E - \angle H)$$

4.9. Skin depth and Depth of Penetration.

Skin depth ⇒ μm to cm

Depth of Penetration ⇒ cm to m.

$$\vec{E} = \underbrace{E_0 e^{-\alpha z}}_{\text{decreases with distance}} \underbrace{\cos(\omega t - \beta z)}_{\text{Constant.}} a_x$$

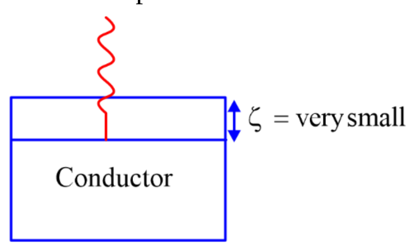


(a) Skin Depth: The distance at which the amplitude of electric field becomes $\frac{1}{e}$ times of its initial value.

$$E_0 \xrightarrow{z=d} E_0 e^{-\alpha d} = \frac{E_0}{e} \Rightarrow \alpha d = 1 \Rightarrow d = \frac{1}{\alpha}$$

$\therefore$ $\xi \equiv$ Skin depth $\Rightarrow$ $\xi = \dfrac{1}{\alpha}$

For high loss medium or conductor, attenuation constant is very high (10^6 to 10^8).
Hence, ξ is very small (μm). So, it is called as skin depth.



Electric field remains at the surface of conductor. It will not penetrate inside conductor.

$$\alpha = \beta = \frac{2\pi}{\lambda} = \sqrt{\pi f \mu \sigma_C}$$

$$\eta = R_S + jX_S; \; R_S = X_S = \sqrt{\frac{\omega\mu}{2\sigma_C}}$$

$$\xi = \frac{1}{\alpha} = \frac{1}{\beta} = \frac{1}{\sqrt{\pi f \mu \sigma_C}} = \frac{\lambda}{2\pi} = \frac{V_p}{\omega} = \frac{1}{\sigma_C R_S}$$

(b) Depth of Penetration:

For Low Lossy medium, attenuation constant is very low. Hence, electric field penetrates very high to that medium.

$$\alpha = \frac{\sigma_d}{2}\sqrt{\frac{\mu}{\epsilon}}$$

$$\xi = \frac{2}{\sigma_d}\sqrt{\frac{\epsilon}{\mu}}$$

(c) Lossless and Perfect Dielectric:

$\sigma_d = 0 \Rightarrow \alpha = 0 \Rightarrow$ Depth of penetration = ∞.

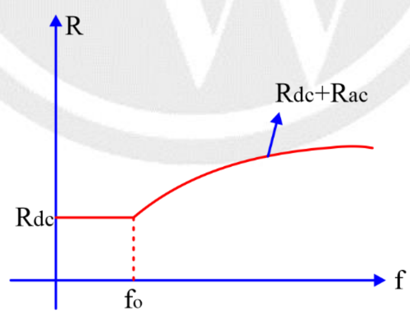
(d) Perfect Conductor

$\sigma_c = \infty \Rightarrow \alpha = \infty \Rightarrow$ Skin depth = 0

(e) Surface Resistance

It is found at high frequency.

$$R_S = \sqrt{\frac{\mu\omega}{2\sigma_C}}$$



$$R_{dc} = \frac{\rho l}{A}, \qquad R_{ac} = \sqrt{\frac{\omega\mu}{2\sigma_C}}$$

Relation between Neper and dB Scale.

Attenuation Factor = AF = $e^{-\alpha d}$

$$AF|_{dB} = -20\log_{10} e^{-\alpha d} = 20\alpha d \log_{10} e = 8.68\alpha d \, dB$$

$$\frac{AF|_{dB}}{d} = +8.68\alpha\left(\frac{dB}{m}\right) = \alpha\left(\frac{\text{Neper}}{m}\right)$$

$1\,\text{Neper} = 8.68\,dB$

4.10. Angle of Wave Impedance or Intrinsic Impedance.

$$\theta_\eta = \frac{90° - \tan^{-1}\left(\frac{\omega \in}{\sigma}\right)}{2}$$

$$\eta = \frac{E}{H} \Rightarrow \theta_\eta = \angle E - \angle H$$

(a) Lossless Dielectric / Free Space / Air

$$\sigma = 0 \Rightarrow \theta_\eta = 0$$

$\therefore \angle E - \angle H = 0 \Rightarrow \angle E = \angle H$

Hence, Phase of Electric Field and Magnetic Field have same.

(b) Conductor / High Loss Medium

$$\theta_\eta = 45°$$

$\Rightarrow \angle E - \angle H = 45°$

Hence, Electric Field leads Magnetic Field by $\frac{\pi}{4}$

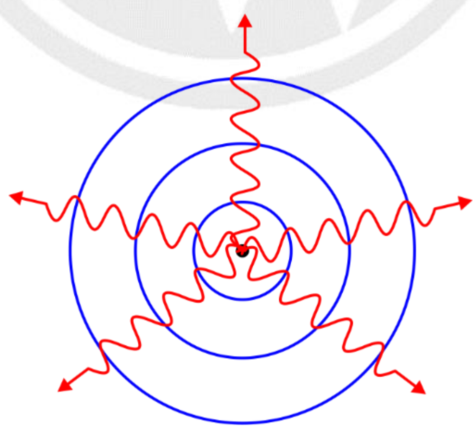
Magnetic Field lags Electric Field by $\frac{\pi}{4}$

(c) Low Loss / Medium Loss Dielectric

$0 < \theta_\eta < 45°$

4.11. Pointing Vector

(a) Propagation Loss $= -\oiint \left(\vec{E} \times \vec{H}\right).d\vec{S}$



(b) Conductor Loss or Ohmic Loss $\equiv -\iiint \left(\sigma E^2\right) dv$

(c) $\frac{\partial}{\partial t}\left(\int_v (\mu_e + \mu_m) dv\right) = -\oiint_S \left(\vec{E} \times \vec{H}\right) \bullet d\vec{S} - \int_v \sigma E^2 dv$

The above equation represents flow of Electromagnetic energy in any medium.

(d) Poynting Vector $\equiv \vec{E} \times \vec{H}\left(\frac{\text{Watt}}{m^2}\right)$

Instantaneous Poynting Vector $\equiv \vec{P}(t) = \vec{E}(t) \times \vec{H}(t)$

Instantaneous Power Density $\equiv \vec{E}(t) \times \vec{H}(t)$

Complex Poynting Vector $\equiv \vec{E}(t) \times \vec{H}(t)$

(e) $\text{Re}(\vec{P}) = \text{Re}(\vec{E}) \times \text{Re}(\vec{H})$

$\text{Re}(\vec{P}) \equiv$ Real part of Poynting Vector

(f) Average Poynting Vector. $\left(\vec{P}_{avg}\right)$

(i) Lossy Medium

$$|\eta| = \frac{E_{rms}}{H_{rms}}, \eta = \text{ Complex + inductive}$$

$$E_{rms} = E_0 e^{-\alpha z}, H_{rms} = H_0 e^{-\alpha z}$$

$$\vec{P}_{avg} = \frac{E_0 H_0}{2} e^{-2\alpha z} \cos\theta_\eta a_p = \frac{E_0^2}{2|\eta|} e^{-2\alpha z} \cos\theta_\eta a_p$$

$$= \frac{|\eta| H_0^2}{2} e^{-2\alpha z} \cos\theta_\eta a_p = E_{rms} H_{rms} \cos\theta_\eta a_p$$

$$= \frac{E_{rms}^2}{|\eta|} \cos\theta_\eta a_p = |\eta| H_{rms}^2 \cos\theta_\eta a_p$$

(ii) Lossless Medium $\left(\sigma = 0, \alpha = 0, \theta_\eta = 0\right)$

$$\vec{P}_{avg} = \frac{E_0 H_0}{2} a_p = \frac{E_0^2}{2\eta} a_p = \frac{\eta H_0^2}{2} a_p = E_{rms} H_{rms} a_p = \frac{E_{rms}^2}{\eta} a_p = \eta H_{rms}^2 a_p$$

(iii) $\vec{P}_{avg} = \frac{1}{2}\text{Re}\left(\vec{E} \times \vec{H}^*\right)$

(iv) Power (W) = $\iint \vec{P}_{avg} \bullet d\vec{S}\,(\text{Watt})$

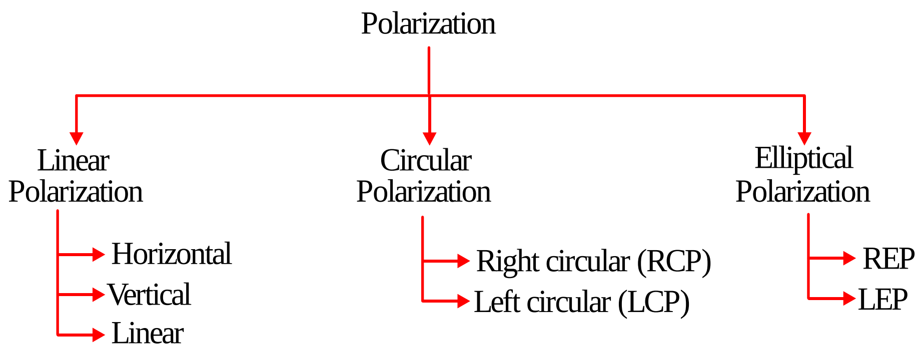
Total Power $(W_0) = \oiint \vec{P}_{avg} \bullet d\vec{S}\,(\text{Watt})$

(v) $E \to \alpha, \omega, \beta, \lambda, V_p, \ f = \frac{\omega}{2\pi}$

$H \to \alpha, \omega, \beta, \lambda, V_p, \ f = \frac{\omega}{2\pi}$

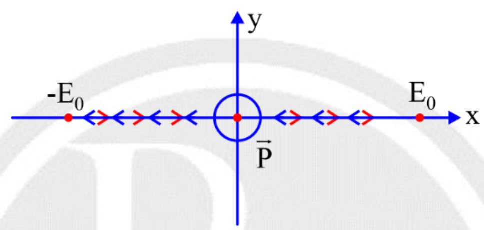
$P \to 2\alpha, 2\omega, 2\beta, 0.5\lambda, V_p, \ f = \frac{\omega}{\pi}$

4.12. Polarization of Electromagnetic Wave



(a) Horizontal Polarization

$$\vec{E} = E_0 \cos(\omega t - \beta z) a_x$$

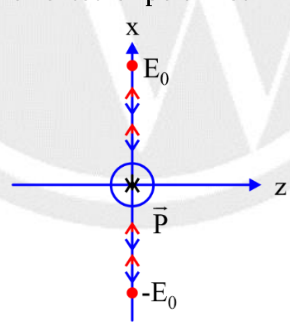


Electric field is propagating in (z) direction and oriented or polarized in (x) direction.

(b) Vertical Polarization.

$$\vec{E} = E_0 \cos(\omega t + \beta y)(-a_x)$$

Electric field is propagating in (-y) direction and oriented or polarized in (-x) direction.



(c) Linear Polarization.

$$\vec{E} = E_1 \cos(\omega t - \beta z) a_x + E_2 \cos(\omega t - \beta z) a_y$$

Wave is linearly Polarized in X and Y, propagating in +z direction.

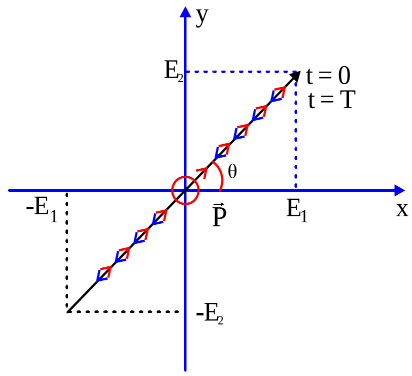
Tilt angle (θ) :- The angle subtend by Electromagnetic Wave at z = 0, t = 0 with

X – Axis ≡ (XY Plane)

Y – Axis ≡ (YZ Plane)

Z – Axis ≡ (ZX Plane)

$$\tan\theta = \frac{E_2}{E_1}$$



(d) Circular Polarization $(E_1 = E_2, \Delta\phi = \pm 90°)$

Step 1. Thumb represent direction of Propagation.

Step 2. Remaining Finger gives rotation.

Step 3. Right hand gives Right Circular Polarization where as Left hand gives Left Circular Polarization.

(i) Right Circular Polarization

$$\vec{E} = E_0 \cos(\omega t - \beta z) a_x + E_0 \sin(\omega t - \beta z) a_y$$

$$Z = 0 \Rightarrow \vec{E} = E_0 \cos(\omega t) a_x + E_0 \sin(\omega t) a_y$$

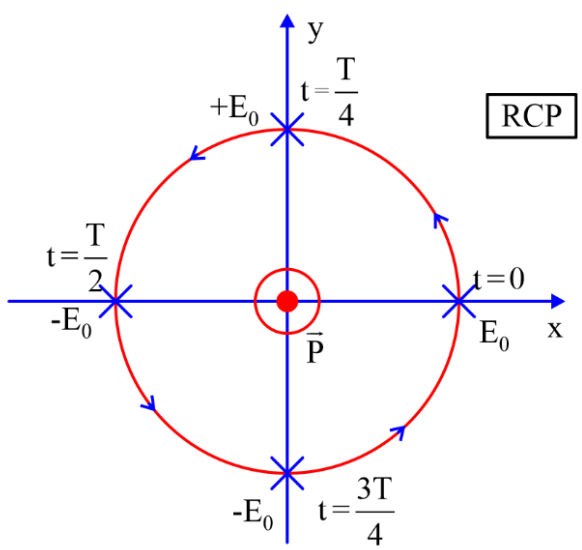
$$t = 0 \Rightarrow \vec{E} = E_0 a_x$$

$$t = \frac{T}{4} \Rightarrow \vec{E} = E_0 a_y$$

$$t = \frac{T}{2} \Rightarrow \vec{E} = -E_0 a_x$$

$$t = \frac{3T}{4} \Rightarrow \vec{E} = -E_0 a_y$$

$$t = T \Rightarrow \vec{E} = E_0 a_x$$



(ii) Left Circular Polarization.

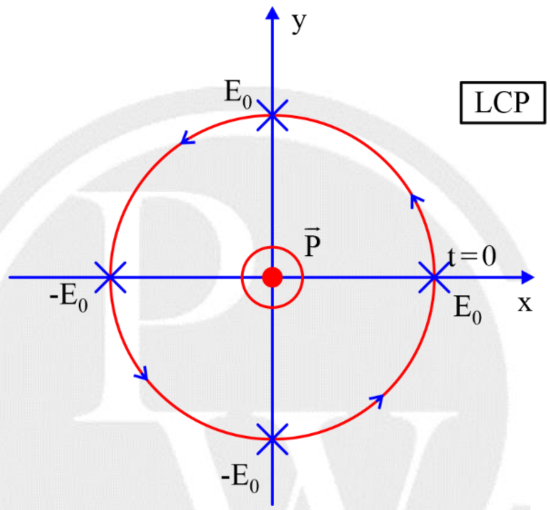
$$\vec{E} = E_0 \cos(\omega t - \beta z) a_x - E_0 \sin(\omega t - \beta z) a_y$$

$$t = 0 \Rightarrow \vec{E} = E_0 a_x$$

$$t = \frac{T}{4} \Rightarrow \vec{E} = -E_0 a_y$$

$$t = \frac{T}{2} \Rightarrow \vec{E} = -E_0 a_x$$

$$t = \frac{3T}{4} \Rightarrow \vec{E} = E_0 a_y$$

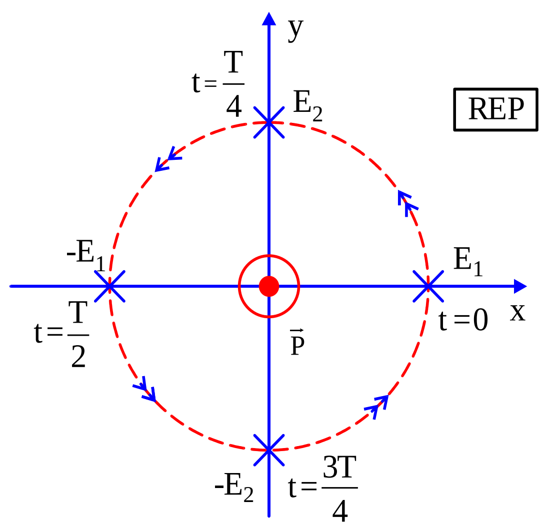


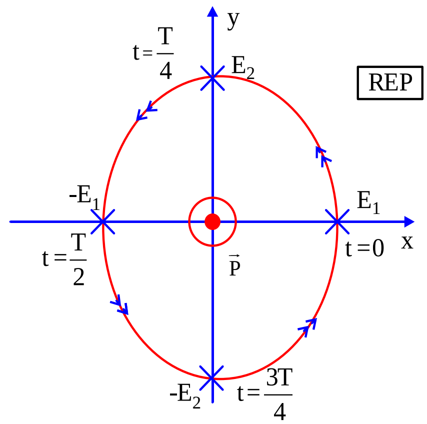
(e) Elliptical Polarization

$E_1 = E_2 \;\; \Delta\phi \neq \pm 90°, \pm 180°$ $\quad E_1 \neq E_2, \;\; \Delta\phi \neq \pm 180°$ $\quad E_1 \neq E_2 \;\; \Delta\phi = \pm 90°$

(i) Right Elliptical Polarization $(E_1 \neq E_2)$

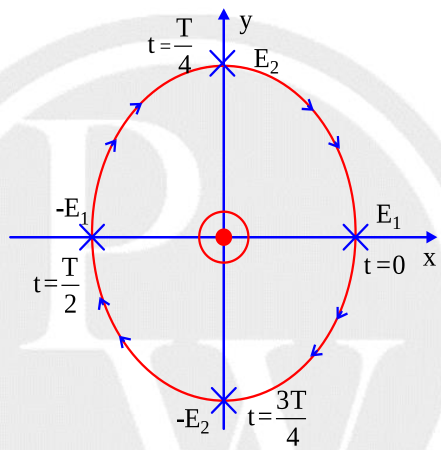
$$\vec{E} = E_1 \cos(\omega t - \beta z) a_x + E_2 \sin(\omega t - \beta z) a_y$$



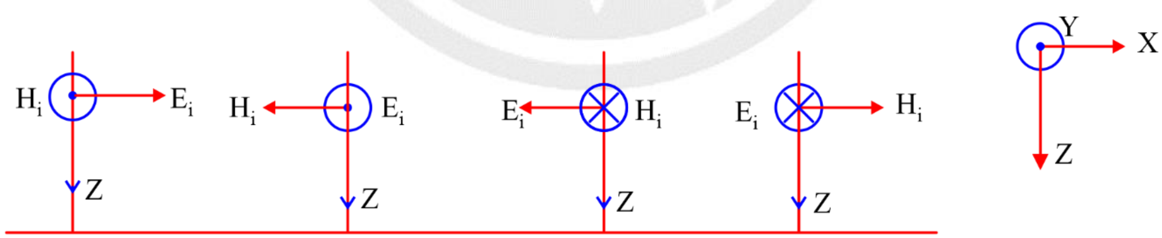


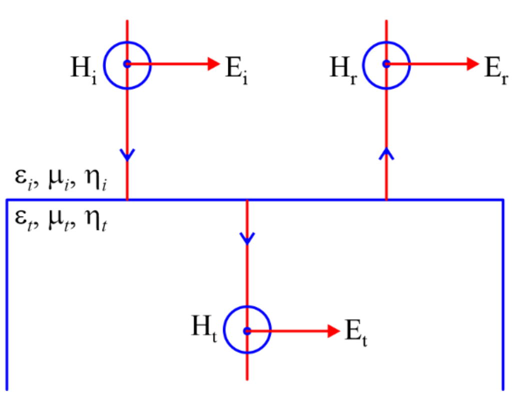
(ii) Left Elliptical Polarization $(E_1 \neq E_2)$

$$\vec{E} = E_1 \cos(\omega t - \beta z) a_x - E_2 \sin(\omega t - \beta z) a_y$$



4.13. Normal Incidence





(a) Formulae

(i) $\Gamma_E = -\Gamma_H$

(ii) $\tau_E = 1 + \Gamma_E$

(ii) $\tau_H = 1 + \Gamma_H = 1 - \Gamma_E$

(iv) $\eta_t = \eta_0\sqrt{\frac{\mu_t}{\epsilon_t}}, \eta_i = \eta_0\sqrt{\frac{\mu_i}{\epsilon_i}}$

(v) $\Gamma_E = \frac{\eta_t - \eta_i}{\eta_t + \eta_i}$

(vi) $\Gamma_E = \frac{\sqrt{\mu_t \epsilon_i} - \sqrt{\mu_i \epsilon_t}}{\sqrt{\mu_t \epsilon_i} + \sqrt{\mu_i \epsilon_t}}$

Non-Magnetic Material $\mu_i = \mu_t = 1$

(vii) $\Gamma_E = \frac{\sqrt{\epsilon_i} - \sqrt{\epsilon_t}}{\sqrt{\epsilon_i} + \sqrt{\epsilon_t}}$

(viii) $\beta \propto \sqrt{\epsilon_r} \Rightarrow \Gamma_E = \frac{\beta_i - \beta_t}{\beta_i + \beta_t}$

(ix) $\lambda \propto \frac{1}{\sqrt{\epsilon_r}} \Rightarrow \Gamma_E = \frac{\lambda_t - \lambda_i}{\lambda_t + \lambda_i}$

(x) $V_P \propto \frac{1}{\sqrt{\epsilon_r}} \Rightarrow \Gamma_E = \frac{V_{P_t} - V_{P_i}}{V_{P_t} + V_{P_i}}$

(xi) $\eta \propto \frac{1}{\sqrt{\epsilon_r}} \Rightarrow \Gamma_E = \frac{\eta_t - \eta_i}{\eta_t + \eta_i}$

(xii) $n \propto \sqrt{\epsilon_r} \Rightarrow \Gamma_E = \frac{n_i - n_t}{n_i + n_t}$ (n =Refractive Index)

Magnetic – Magnetic Material $\epsilon_t = \epsilon_i = 1$

(xiii) $\Gamma_E = \frac{\sqrt{\mu_t} - \sqrt{\mu_i}}{\sqrt{\mu_t} + \sqrt{\mu_i}}$

(xiv) $\beta \propto \sqrt{\mu_r} \Rightarrow \Gamma_E = \frac{\beta_t - \beta_i}{\beta_t + \beta_i}$

(xv) $\lambda \propto \frac{1}{\sqrt{\mu_r}} \Rightarrow \Gamma_E = \frac{\lambda_i - \lambda_t}{\lambda_i + \lambda_t}$

(xvi) $V_P \propto \frac{1}{\sqrt{\mu_r}} \Rightarrow \Gamma_E = \frac{V_{P_i} - V_{P_t}}{V_{P_i} + V_{P_t}}$

(xvii) $\eta \propto \sqrt{\mu_r} \Rightarrow \Gamma_E = \frac{\eta_t - \eta_i}{\eta_t + \eta_i}$

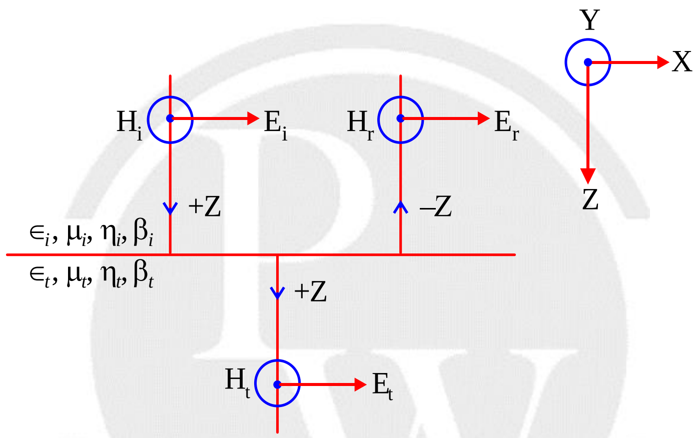
(xviii) $n \propto \sqrt{\mu_r} \Rightarrow \Gamma_E = \dfrac{n_t - n_i}{n_t + n_i}$

(xix) $\vec{P}_r = -|\Gamma_E|^2 \vec{P}_i$

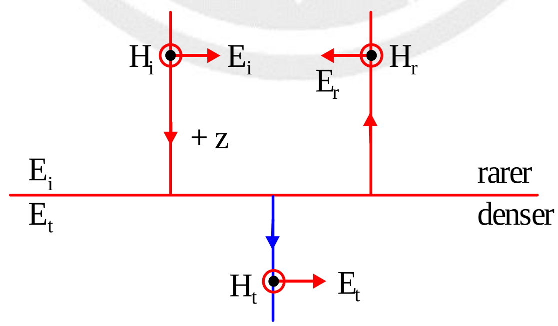
(xx) $\vec{P}_t = -\left(1-|\Gamma_E|^2\right)\vec{P}_i$

(b) Representation of electric field and Magnetic field.

$$\vec{E}_i = E_o \cos(wt - \beta_i z)\hat{a}_x \qquad \vec{E}_r = \Gamma_E E_o \cos(wt + \beta_i z)\hat{a}_x$$
$$\vec{E}_t = \tau_E E_o \cos(wt - \beta_t z)\hat{a}_x \qquad \vec{H}_i = H_o \cos(wt - \beta_i z)\hat{a}_y$$
$$\vec{H}_r = \Gamma_H H_o \cos(wt + \beta_i z)\hat{a}_y \qquad \vec{H}_t = \tau_H H_o \cos(wt - \beta_t z)\hat{a}_y$$

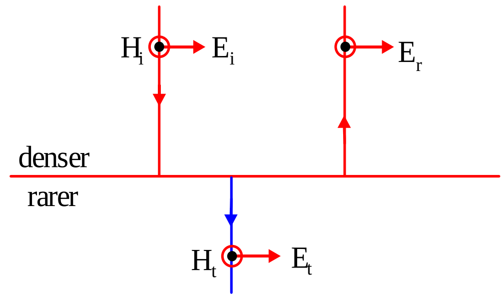


(c) When wave travels from rarer to denser medium.



- $n_i < n_t \Rightarrow \in_i < \in_t \qquad (\because n \propto \sqrt{\in_r})$
- $\Gamma_E = \dfrac{\sqrt{\in_i} - \sqrt{\in_t}}{\sqrt{\in_i} + \sqrt{\in_t}} < 0 \Rightarrow \Gamma_H = -\Gamma_E > 0$
- $\dfrac{E_r}{E_i} = \Gamma_E < 0 \Rightarrow \dfrac{E_r}{E_i} < 0 \Rightarrow$ E_r & E_i are of opposite sign.
- $\dfrac{H_r}{H_i} = \Gamma_H > 0 \Rightarrow \dfrac{H_r}{H_i} > 0 \Rightarrow$ H_r & H_i are of same sign.

(d) When wave travels from denser to rarer medium.



- $n_i > n_t \Rightarrow \in_i > \in_t \qquad \left(\because n \alpha \sqrt{\in_r}\right)$
- $\Gamma_E = \dfrac{\sqrt{\in_i} - \sqrt{\in_t}}{\sqrt{\in_i} + \sqrt{\in_t}} > 0 \Rightarrow \Gamma_H = -\Gamma_E < 0$
- $\dfrac{E_r}{E_i} = \Gamma_E > 0 \Rightarrow \dfrac{E_r}{E_i} > 0 \Rightarrow$ E_r & E_i are of same sign.

  $\dfrac{H_r}{H_i} = \Gamma_H < 0 \Rightarrow \dfrac{H_r}{H_i} < 0 \Rightarrow$ H_r & H_i are of opposite sigh.

(e) Range of Reflection and Transmission coefficient.

- $\Gamma \in [-1,1]$
- $\tau \in [0,2]$

(f) When wave travels from air to conductor.

- $\eta_i = \eta_o \sqrt{\dfrac{\mu_r}{\in_r}}$

  $\eta_t = \eta_o \sqrt{\dfrac{j\omega\mu}{\sigma_c + j\omega \in}}$

Since σ_c is very high. Hence η_t value will be in the range of milli ohm.

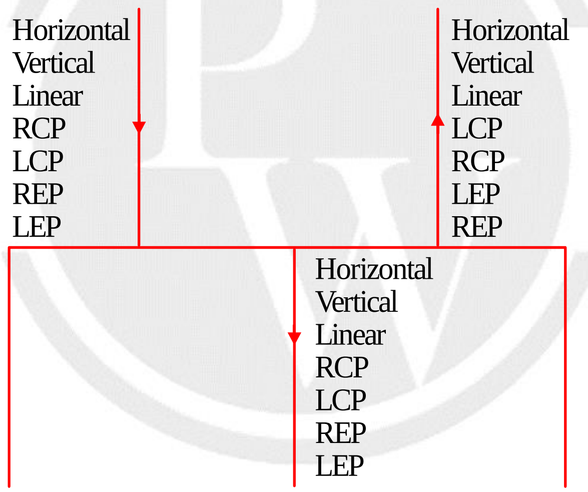
- $\eta_i >>>> \eta_t$
- $\Gamma_E = \dfrac{\eta_t - \eta_i}{\eta_t + \eta_i} = -1 \Rightarrow \Gamma_H = -\Gamma_E = 1$
- $\tau_E = 1 + \Gamma_E = 0$ & $\tau_H = 1 + \Gamma_H = 2$
- $\dfrac{E_r}{E_i} = -1 \Rightarrow E_r = -E_i$
- $\dfrac{E_t}{E_i} = 0 \Rightarrow E_t = 0 \rightarrow$ Minima

- $\frac{H_r}{H_i} = 1 \Rightarrow H_r = H_i$
- $\frac{H_t}{H_i} = 2 \Rightarrow H_t = 2H_i \rightarrow$ Maxima
- Reflected and Incident Electric Field are out of phase.
- Electric Field at the surface of conductor is '0' (minima).
- Magnetic Field at the surface is maximum.
- Hence, Electric Field and Magnetic Field on the surface of conductor is having 90° phase difference,

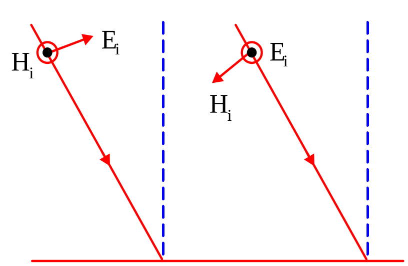
i.e. Electric Field at the surface of conductor lags Magnetic Field at the surface of conductor by 90°.

- Inside Conductor, Electric Field leads Magnetic Field by $\frac{\pi}{4}$.
- $\vec{P}_r = -\vec{P}_i$
- $\vec{P}_t = \vec{0}$

(g) Effect of polarization during normal incidence

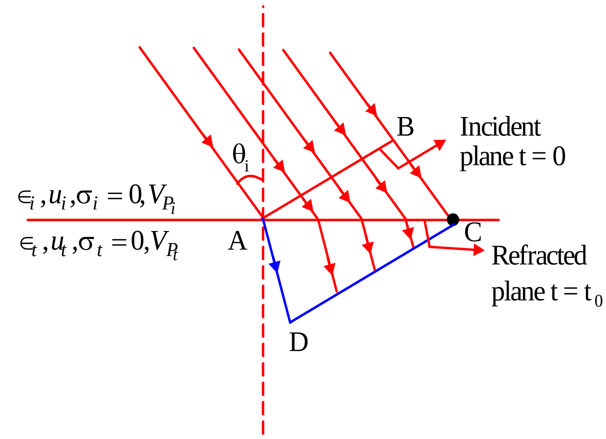


4.14. Oblique Incidence



(I) Parallel Polarization	(A) Perpendicular polarization
(II) Vertical	(B) Horizontal
(III) P - polarization	(C) S - polarization

(a) Snell' law:



(i) $\sqrt{\mu_r \in_t}\sin\theta_t = \sqrt{\mu_i \in_i}\sin\theta_i \rightarrow$ Lossless Medium

Non-magnetic $\mu_i = \mu_t = 1$

(ii) $\sqrt{\in_t}\sin\theta_t = \sqrt{\in_i}\sin\theta_i$ (iii) $\beta \propto \sqrt{\in_r} \Rightarrow \beta_t \sin\theta_t = \beta_i \sin\theta_i$

(iv) $\lambda \propto \frac{1}{\sqrt{\in_r}} \Rightarrow \frac{\sin\theta_t}{\lambda_t} = \frac{\sin\theta_i}{\lambda_i}$ (v) $V_P \propto \frac{1}{\sqrt{\in_r}} \Rightarrow \frac{\sin\theta_t}{V_{pt}} = \frac{\sin\theta_i}{V_{pi}}$

(vi) $\eta \propto \frac{1}{\sqrt{\in_r}} \Rightarrow \frac{\sin\theta_t}{\eta_t} = \frac{\sin\theta_i}{\eta_i}$ (vii) $n \propto \sqrt{\in_r} \Rightarrow n_t \sin\theta_t = n_i \sin\theta_i$

Magnetic – Magnetic $\in_i = \in_t = 1$

$$\sqrt{\mu_i}\sin\theta_i = \sqrt{\mu_t}\sin\theta_t$$

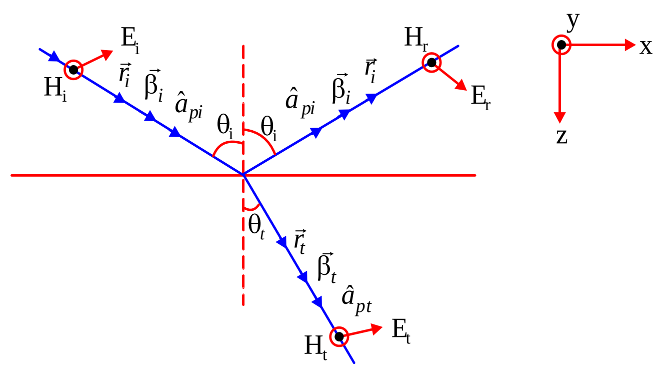
(viii) $\beta \propto \sqrt{\mu_r} \Rightarrow \beta_i \sin\theta_i = \beta_t \sin\theta_t$ (ix) $\lambda \propto \frac{1}{\sqrt{\mu_r}} \Rightarrow \frac{\sin\theta_i}{\lambda_i} = \frac{\sin\theta_t}{\lambda_t}$

(x) $V_P \propto \frac{1}{\sqrt{\mu_r}} \Rightarrow \frac{\sin\theta_i}{V_{P_i}} = \frac{\sin\theta_t}{V_{P_t}}$

(xi) $\eta \propto \sqrt{\mu_r} \Rightarrow \eta_i \sin\theta_i = \eta_t \sin\theta_t$

(xii) $n \propto \sqrt{\mu_r} \Rightarrow n_i \sin\theta_i = n_t \sin\theta_t$

4.15. Parallel/Vertical/P-Polarization



- $\vec{r}_i = x\hat{i} + z\hat{k}$
- $\vec{r}_r = x\hat{i} - z\hat{k}$
- $\vec{r}_t = x\hat{i} + z\hat{k}$
- $\hat{a}_{P_i} = \sin\theta_i \hat{i} + \cos\theta_i \hat{k}$
- $\hat{a}_{P_r} = \sin\theta_i \ \hat{i} - \cos\theta_i \ \hat{k}$
- $\hat{a}_{P_t} = \sin\theta_t \ \hat{i} + \cos\theta_t \ \hat{k}$
- $\vec{\beta}_i = \beta_i \sin\theta_i \ \hat{i} + \beta_i \cos\theta_i \hat{k}$
- $\vec{\beta}_r = \beta_i \sin\theta_i \ \hat{i} + \beta_i \cos\theta_i \hat{k}$
- $\vec{\beta}_t = \beta_t \sin\theta_t \hat{i} + \beta_t \cos\theta_t \hat{k}$
- $\beta_i = \dfrac{\omega}{c_o}\sqrt{\mu_i \in_i}$
- $\beta_t = \dfrac{\omega}{c_o}\sqrt{\mu_t \in_t}$
- $\eta_i = \eta_o \sqrt{\dfrac{\mu_i}{\in_i}}$
- $\eta_t = \eta_o \sqrt{\dfrac{\mu_t}{\in_t}}$

Summary:

(a) $\Gamma_H = -\Gamma_E$

(b) $\tau_H = 1 + \Gamma_H = 1 - \Gamma_E$

(c) $\tau_E = (1 + \Gamma_E)\left(\dfrac{\cos\theta_i}{\cos\theta_t}\right)$

(d) $\Gamma_E = \dfrac{\eta_t \cos\theta_t - \eta_i \cos\theta_i}{\eta_t \cos\theta_t + \eta_i \cos\theta_i}$

(e) $\Gamma_E = \dfrac{\sqrt{\mu_t \in_i}\cos\theta_t - \sqrt{\mu_i \in_t}\cos\theta_i}{\sqrt{\mu_t \in_i}\cos\theta_t + \sqrt{\mu_i \in_t}\cos\theta_i}$

For non-magnetic ($\mu_i = \mu_t = 1$)

(f) $\Gamma_E = \dfrac{\sqrt{\in_i}\sec\theta_i - \sqrt{\in_t}\sec\theta_t}{\sqrt{\in_i}\sec\theta_i + \sqrt{\in_t}\sec\theta_t}$

(g) $\beta \propto \sqrt{\in_r} \Rightarrow \Gamma_E = \dfrac{\beta_i \sec\theta_i - \beta_t \sec\theta_t}{\beta_i \sec\theta_i + \beta_t \sec\theta_t}$

(h) $\lambda \propto \dfrac{1}{\sqrt{\in_r}} \Rightarrow \Gamma_E = \dfrac{\lambda_t \cos\theta_t - \lambda_i \cos\theta_i}{\lambda_t \cos\theta_t + \lambda_i \cos\theta_i}$

(i) $V_P \propto \dfrac{1}{\sqrt{\in_r}} \Rightarrow \dfrac{V_{P_t} \cos\theta_t - V_{P_i} \cos\theta_i}{V_{P_t} \cos\theta_t + V_{P_i} \cos\theta_i}$

(j) $\eta \propto \dfrac{1}{\sqrt{\in_r}} \Rightarrow \Gamma_E = \dfrac{\eta_t \cos\theta_t - \eta_i \cos\theta_i}{\eta_t \cos\theta_t + \eta_i \cos\theta_i}$

(k) $n \propto \sqrt{\in_r} \Rightarrow \Gamma_E = \dfrac{n_t \cos\theta_t - n_i \cos\theta_i}{n_t \cos\theta_t + n_i \cos\theta_i}$

For Magnetic medium $(\in_i = \in_t = 1)$

(l) $\Gamma_E = \dfrac{\sqrt{\mu_t} \cos\theta_t - \sqrt{\mu_i} \cos\theta_i}{\sqrt{\mu_t} \cos\theta_t + \sqrt{\mu_i} \cos\theta_i}$

(m) $\beta \propto \sqrt{\mu_r} \Rightarrow \Gamma_E = \dfrac{\beta_t \cos_t - \beta_i \cos\theta_i}{\beta_t \cos_t + \beta_i \cos\theta_i}$

(n) $\lambda \propto \dfrac{1}{\sqrt{\mu_r}} \Rightarrow \Gamma_E = \dfrac{\lambda_i \sec\theta_i - \lambda_t \sec\theta_t}{\lambda_i \sec\theta_i + \lambda_t \sec\theta_t}$

(o) $V_P \propto \dfrac{1}{\sqrt{\mu_r}} \Rightarrow \Gamma_E = \dfrac{V_{P_i} \sec\theta_i - V_{P_t} \sec\theta_t}{V_{P_i} \sec\theta_i + V_{P_t} \sec\theta_t}$

(p) $\eta \propto \sqrt{\mu_r} \Rightarrow \Gamma_E = \dfrac{\eta_t \cos\theta_t - \eta_i \cos\theta_i}{\eta_t \cos\theta_t + \eta_i \cos\theta_i}$

(q) $n \propto \sqrt{\mu_r} \Rightarrow \Gamma_E = \dfrac{n_t \cos\theta_t - n_i \cos\theta_i}{n_t \cos\theta_t + n_i \cos\theta_i}$

(r) Representation of Electric Field and Magnetic Field

⊙ $\vec{E}_i = E_o\left(\cos\theta_i\, \hat{i} - \sin\theta_i \hat{k}\right) e^{jwt}\, e^{-j\vec{\beta}_i \bullet \vec{r}_i}$

$\vec{r}_i = x\hat{i} + z\hat{k}$

$\vec{\beta}_i \bullet \vec{r}_i = \left(\beta_i \sin\theta_i\right)x + \left(\beta_i \cos\theta_i\right)z$

⊙ $\vec{E}_r = \Gamma_E E_0\left(\cos\theta_i \hat{i} + \sin\theta_i \hat{k}\right) e^{jwt}\, e^{-j\vec{\beta}r \bullet \vec{r}_r}$

⊙ $\vec{E}_r = \tau_E E_0\left(\cos\theta_t \hat{i} - \sin\theta_t \hat{k}\right) e^{jwt}\, e^{-j\vec{\beta}_t \bullet \vec{r}_t}$

⊙ $\vec{H}_i = \dfrac{E_o}{\eta_i} e^{jwt}\, e^{-j\vec{\beta}_i \bullet \vec{r}_i}$

⊙ $\vec{H}_r = \Gamma_H \left(\dfrac{E_o}{\eta_i}\right) e^{jwt}\, e^{-j\vec{\beta}_r \bullet \vec{r}_r}$

⊙ $\vec{H}_t = \tau_H \left(\frac{E_o}{\eta_i}\right) e^{jwt} e^{-j\vec{\beta}_t \cdot \vec{r}_t}$

(s) Power Density

⊙ $\vec{P}_i = \frac{E_0^2}{2\eta_i} \hat{a}_{P_i}$, ⊙ $\vec{P}_r = \frac{|\Gamma_E|^2 E_o^2}{2\eta_i} \hat{a}_{P_r}$

⊙ $\vec{P}_t = \left(1 - |\Gamma_E|^2\right) \frac{\cos\theta_i}{\cos\theta_t} \left(\frac{E_o^2}{2\eta_i}\right) \hat{a}_{P_t}$

⊙ $\frac{P_r}{P_i} = |\Gamma_E|^2$ ⊙ $\frac{P_t}{P_i} = \left(1 - |\Gamma_E|^2\right)\left(\frac{\cos\theta_i}{\cos\theta_t}\right)$

(t) When electromagnetic wave travels from air to conductor.

⊙ $\eta_i = \eta_o \sqrt{\frac{\mu_r}{\epsilon_r}}$ ⊙ $\eta_t = \sqrt{\frac{jw\mu}{\sigma_c + jw\varepsilon}}$

⊙ Since, σ_c is very high. Hence, η_t value will be found in the range of milli ohm.

⊙ $\eta_i >>> \eta_t$

⊙ $\Gamma_E = \frac{\eta_t \cos\theta_t - \eta_i \cos\theta_i}{\eta_t \cos\theta_t + \eta_i \cos\theta_i} = -1$

$\Rightarrow \Gamma_H = -\Gamma_E = 1 \quad \Rightarrow \tau_E = 1 + \Gamma_E = 0 \;\; \& \tau_H = 1 + \Gamma_H = 2$

⊙ $\tau_E = 1 + \Gamma_E = 0 \;\; \& \tau_H = 1 + \Gamma_H = 2$

⊙ $\frac{E_r}{E_t} = -1 \Rightarrow E_r = -E_i$

⊙ $\frac{E_r}{E_t} = 0 \Rightarrow E_t = 0 \rightarrow Minima$

⊙ $\frac{H_r}{H_i} = 1 \Rightarrow H_r = H_i$

⊙ $\frac{H_t}{H_i} = 2 \Rightarrow H_t = 2H_i \rightarrow Maxima$

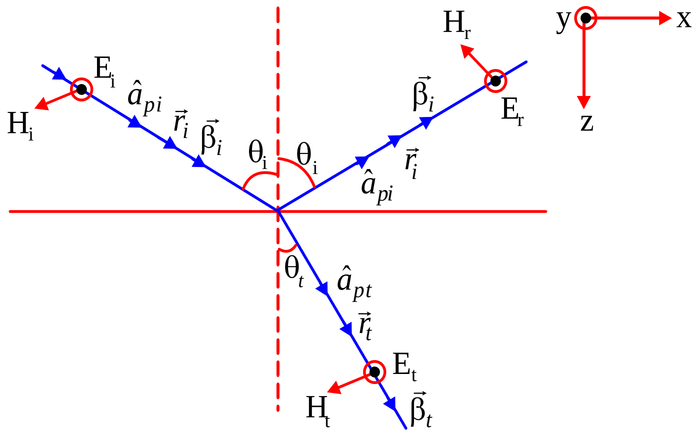
⊙ Reflected and Incident Electric Field are out of phase.

⊙ Hence, Electric Field and Magnetic Field or the surface of conductor is having 90° phase difference.
i.e. Electric Field at the surface of conductor lags Magnetic Field at the surface of conductor by 90°.

⊙ Inside conductor, Electric Field leads magnetic field by $\frac{\pi}{4}$

⊙ $\vec{P}_r = -\vec{P}_i$ ⊙ $\vec{P}_t = \vec{0}$

4.16. Perpendicular polarization/Horizontal polarization/S-Polarization



- $\vec{r}_i = x\hat{i} + z\hat{k}$
- $\vec{r}_r = x\hat{i} - z\hat{k}$
- $\vec{r}_t = x\hat{i} + z\hat{k}$
- $\hat{a}_{P_i} = \sin\theta_i \hat{I} + \cos\theta_i \hat{k}$
- $\hat{a}_{P_r} = \sin\theta_i \ \hat{i} - \cos\theta_i \ \hat{k}$
- $\hat{a}_{P_t} = \sin\theta_t \ \hat{i} + \cos\theta_t \ \hat{k}$
- $\vec{\beta}_i = \beta_i \sin\theta_i \ \hat{i} + \beta_i \cos\theta_i \hat{k}$
- $\vec{\beta}_r = \beta_i \sin\theta_i \ \hat{i} + \beta_i \cos\theta_i \hat{k}$
- $\vec{\beta}_t = \beta_t \sin\theta_t \hat{i} + \beta_t \cos\theta_t \hat{k}$
- $\beta_i = \dfrac{\omega}{c_o}\sqrt{\mu_i \in_i}$
- $\beta_t = \dfrac{\omega}{c_o}\sqrt{\mu_t \in_t}$
- $\eta_i = \eta_o\sqrt{\dfrac{\mu_i}{\in_i}}$
- $\eta_t = \eta_o\sqrt{\dfrac{\mu_t}{\in_t}}$

Summary:

(a) $\Gamma_H = -\Gamma_E$

(b) $\tau_H = (1+\Gamma_H)\left(\dfrac{\cos\theta_i}{\cos\theta_t}\right)$

(c) $\tau_E = 1 + \Gamma_E$

(d) $\Gamma_E = \dfrac{\eta_t \sec\theta_t - \eta_i \sec\theta_i}{\eta_t \sec\theta_t + \eta_i \sec\theta_i}$

(e) $\Gamma_E = \dfrac{\sqrt{\mu_t \in_i}\sec\theta_t - \sqrt{\mu_i \in_t}\sec\theta_i}{\sqrt{\mu_t \in_i}\sec\theta_t + \sqrt{\mu_i \in_t}\sec\theta_i}$

For Non-Magnetic ($\mu_i = \mu_t = 1$)

(f) $$\Gamma_E = \frac{\sqrt{\epsilon_i}\cos\theta_i - \sqrt{\epsilon_t}\cos\theta_t}{\sqrt{\epsilon_i}\cos\theta_i + \sqrt{\epsilon_t}\cos\theta_t}$$

(g) $$\beta \propto \sqrt{\epsilon_r} \Rightarrow \Gamma_E = \frac{\beta_i\cos\theta_i - \beta_t\cos\theta_t}{\beta_i\cos\theta_i + \beta_t\cos\theta_t}$$

(h) $$\lambda \propto \frac{1}{\sqrt{\epsilon_r}} \Rightarrow \Gamma_E = \frac{\lambda_t\sec\theta_t - \lambda_i\sec\theta_i}{\lambda_t\sec\theta_t + \lambda_i\sec\theta_i}$$

(i) $$V_P \propto \frac{1}{\sqrt{\epsilon_r}} \Rightarrow \Gamma_E = \frac{V_{P_t}\sec\theta_t - V_{P_i}\sec\theta_i}{V_{P_t}\sec\theta_t + V_{P_i}\sec\theta_i}$$

(j) $$\eta \propto \frac{1}{\sqrt{\epsilon_r}} \Rightarrow \Gamma_E = \frac{\eta_t\sec\theta_t - \eta_i\sec\theta_i}{\eta_t\sec\theta_t + \eta_i\sec\theta_i}$$

(k) $$n \propto \sqrt{\epsilon_r} \Rightarrow \Gamma_E = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t}$$

For magnetic medium $(\epsilon_i = \epsilon_t = 1)$

(l) $$\Gamma_E = \frac{\sqrt{\mu_t}\sec\theta_t - \sqrt{\mu_i}\sec\theta_i}{\sqrt{\mu_t}\sec\theta_t + \sqrt{\mu_i}\sec\theta_i}$$

(m) $$\beta \propto \sqrt{\mu_r} \Rightarrow \Gamma_E = \frac{\beta_t\sec_t - \beta_i\sec\theta_i}{\beta_t\sec_t + \beta_i\cos\theta_i}$$

(n) $$\lambda \propto \frac{1}{\sqrt{\mu_r}} \Rightarrow \Gamma_E = \frac{\lambda_i\cos\theta_i - \lambda_t\cos\theta_t}{\lambda_i\cos\theta_i + \lambda_t\cos\theta_t}$$

(o) $$V_P \propto \frac{1}{\sqrt{\mu_r}} \Rightarrow \Gamma_E = \frac{V_{P_i}\cos\theta_i - V_{P_i}\cos\theta_t}{V_{P_i}\cos\theta_i + V_{P_i}\cos\theta_t}$$

(p) $$\eta \propto \sqrt{\mu_r} \Rightarrow \Gamma_E = \frac{\eta_t\sec\theta_t - \eta_i\sec\theta_i}{\eta_t\sec\theta_t + \eta_i\sec\theta_i}$$

(q) $$n \propto \sqrt{\mu_r} \Rightarrow \Gamma_E = \frac{n_t\sec\theta_t - n_i\sec\theta_i}{n_t\sec\theta_t + n_i\sec\theta_i}$$

(r) Representation of Electric Field and Magnetic Field

- $\odot\ \vec{E}_i = E_o e^{jwt} e^{-j\vec{\beta}_i \cdot \vec{r}_i} \hat{a}_y$
- $\odot\ \vec{E}_r = \Gamma_E E_o e^{jwt} e^{-j\vec{\beta}_r \cdot \vec{r}_r} \hat{a}_y$
- $\odot\ \vec{E}_t = \tau_E E_o e^{jwt} e^{-j\vec{\beta}_t \cdot \vec{r}_t} \hat{a}_y$
- $\odot\ \vec{H}_i = \frac{E_o}{\eta_i}\left(-\cos\theta_i\,\hat{i} + \sin\theta_i\hat{k}\right)e^{jwt} e^{-j\vec{\beta}_i \cdot \vec{r}_i}$
- $\odot\ \vec{H}_r = \Gamma_E \frac{E_o}{\eta_i}\left(-\cos\theta_i\,\hat{i} - \sin\theta_i\,\hat{k}\right)e^{jwt} e^{-j\vec{\beta}_r \cdot \vec{r}_r}$
- $\odot\ \vec{H}_t = \tau_H \frac{E_o}{\eta_i}\left(-\cos_t\,\hat{i} + \sin\theta_t\hat{k}\right)e^{jwt} e^{-j\vec{\beta}_t \cdot \vec{r}_t}$

(s) Power Density

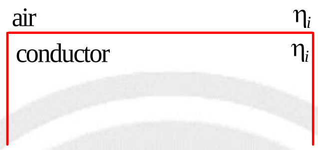
⊙ $\vec{P}_i = \dfrac{E_0^2}{2\eta_i}\hat{a}_{P_i}$

⊙ $\vec{P}_r = \dfrac{|\Gamma_E|^2 E_o^2}{2\eta_i}\hat{a}_{P_t}$

⊙ $\vec{P}_t = \left(1-|\Gamma_E|^2\right)\dfrac{E_o^2}{2\eta_t}\hat{a}_{P_t}$

⊙ $\dfrac{\mathrm{P_r}}{P_i} = |\Gamma_E|^2$ ⊙ $\dfrac{\mathrm{P}_t}{P_i} = \left(1-|\Gamma_E|^2\right)\left(\dfrac{\cos\theta_i}{\cos\theta_t}\right)$

(t) When electromagnetic wave travels from air to conductor.



⊙ $\eta_i = \eta_o\sqrt{\dfrac{u_r}{\epsilon_r}}$

⊙ $\eta_t = \eta_o\sqrt{\dfrac{jwu}{\sigma_c + jw\epsilon}}$

Since, σ_c is very high. Hence, η_t value will be found in the range of milli ohm.

$\eta_i >>> \eta_t$

$$\Gamma_E = \frac{\eta_t \sec\theta_t - \eta_i \sec\theta_i}{\eta_t \sec\theta_t + \eta_i \sec\theta_i} = -1$$

$\Rightarrow \Gamma_H = -\Gamma_E = 1$

⊙ $\tau_E = 1 + \Gamma_E = 0$

⊙ $\tau_E = (1+\Gamma_E)\left(\dfrac{\cos\theta_i}{\cos\theta_t}\right) = \dfrac{2\cos\theta_i}{\cos\theta_t}$

⊙ $\dfrac{E_r}{E_i} = -1 \Rightarrow E_r = -E_i$

⊙ $\dfrac{E_t}{E_i} = 0 \Rightarrow E_t = 0 \rightarrow Minima$

⊙ $\dfrac{H_r}{H_i} = 1 \Rightarrow H_r = H_i$

⊙ $\dfrac{H_t}{H_i} = \dfrac{2\cos\theta_i}{\cos\theta_t} \Rightarrow H_t = \left(\dfrac{2\cos\theta_i}{\cos\theta_t}\right)H_i \rightarrow Maxima$

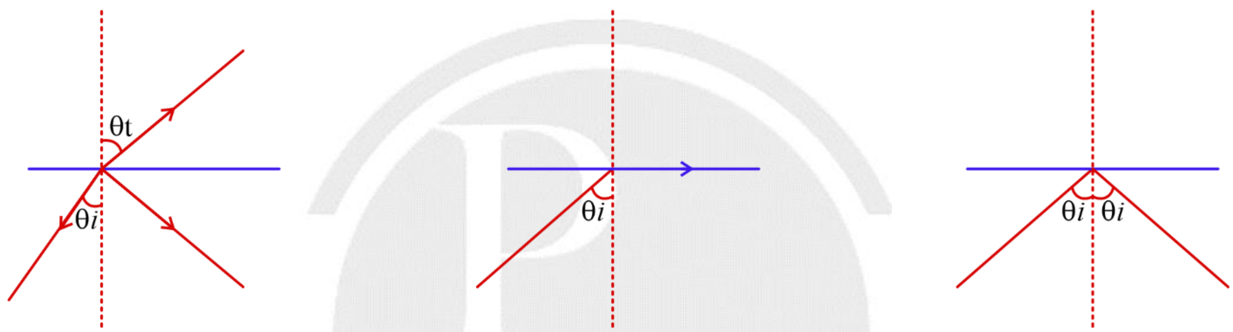
⊙ Reflected and Incident Electric Field are out of phase.

⊙ Electric Field at the surface of conductor is '0' (Minima).

- Magnetic Field at the surface is maximum.
- Hence, Electric Field and Magnetic Field on the surface of conductor is having 90° phase difference.
- Electric Field at the surface of conductor lags Magnetic Field at the surface of conductor by 90°.
- Inside conductor, Electric Field loads Magnetic Field by $\frac{\pi}{4}$.
- $\vec{P}_r = -\vec{P}_i$ ⊙ $\vec{P}_t = \vec{0}$

4.17. Critical angle and total integral reflection (TIR)

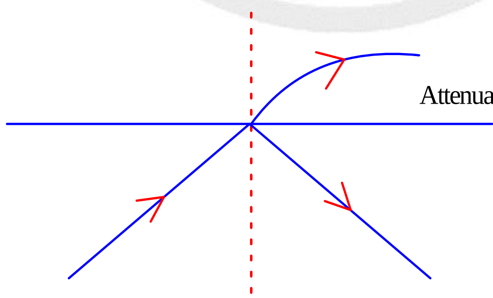
When wave travels from denser to rarer medium, then at some angle, the electromagnetic wave starts gazing or travelling along the surface. So that angle is known as critical angle.



$\theta_i < \theta_c$ | $\theta_i = \theta_c$ | $\theta_i > \theta_c$

Transmission + Reflection | Wave travels along the surface | No Transmission + Only Reflection (TIR)

- $\theta_i > \theta_c \Rightarrow \theta_t =$ Imaginary angle or does not exist.



- In general medium $\sin\theta_c = \sqrt{\frac{\mu_t \in_t}{\mu_i \in_i}}$
- In non-magnetic medium $\sin\theta_c = \sqrt{\frac{\in_t}{\in_i}}$
- In magnetic medium $\sin\theta = \sqrt{\frac{\mu_t}{\mu_i}}$

4.18. Brewster's Angle

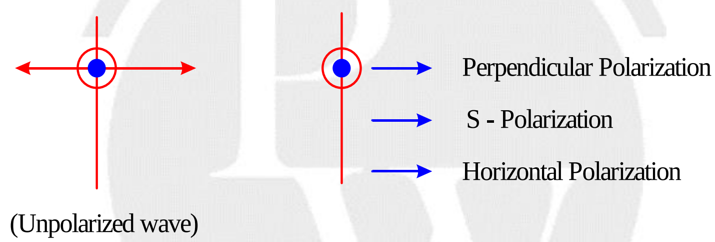
$$\theta_i = \theta_B$$

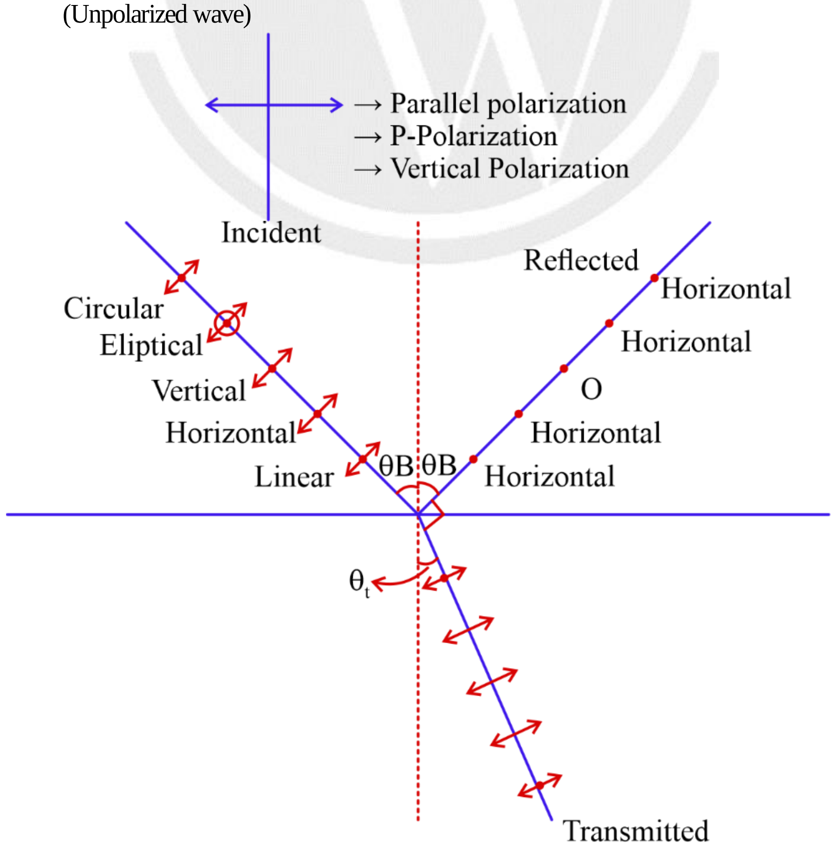
- No Reflection only Transmission
- The angle of incidence at which Electromagnetic wave is only transmitted but not reflected.

Case I : For Non-magnetic Medium

- $\sqrt{\epsilon_i}\sin\theta_B = \sqrt{\epsilon_t}\sin\theta_t$
- at $\theta_i = \theta_B, \Gamma_E = 0, \tau_E = 1$
- For S-polarization $\theta_i = \theta_B$ (does not exist)
- For P-Polarization, $\theta_i = \theta_B$ exists.

$$\tan\theta_B = \sqrt{\frac{\epsilon_t}{\epsilon_i}}$$





- $\tan\theta_B = \sqrt{\frac{\epsilon_t}{\epsilon_i}}$, For P-Polarization only, not for S-Polarization.
- $\theta_B + \theta_t = 90°$ i.e., Reflected and Transmitted wave are perpendicular to each other.

Case II : For General Medium

- $\sqrt{\mu_i \epsilon_i}\sin\theta_B = \sqrt{\mu_t \epsilon_t}\sin\theta_t$
- For S-polarization, $\sin\theta_B = \sqrt{\frac{(\mu_t \epsilon_i - \mu_i \epsilon_t)\epsilon_t}{\mu_i(\epsilon_i^2 - \epsilon_t^2)}}$
- For P-Polarization, $\sin\theta_B = \sqrt{\frac{(\mu_i \epsilon_t - \mu_t \epsilon_i)\mu_t}{(\mu_i^2 - \mu_t^2)\epsilon_i}}$
- For general medium, Brewster's angle will exist for both S-and P-Polarization
- Their Brewster's angles are different for same medium.
- For non-magnetic medium Brewster's angle will exist for S-polarization. Whereas it will not exist for P-polarization.

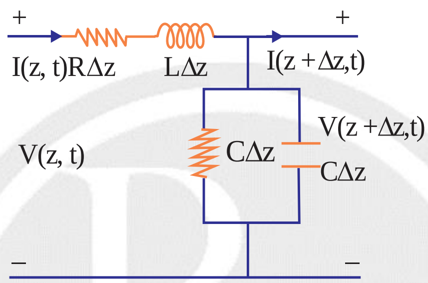
4.19. Difference between Brewster's angle and Critical angle.

	Critical Angle		Brewster's Angle
(a)	$\theta_i > \theta_c \rightarrow$ Total internal reflection	(a)	$\theta_i = \theta_B \rightarrow$ Brewster's Angle
(b)	$\theta_i > \theta_c \rightarrow$ Many angle	(b)	$\theta_i = \theta_B \rightarrow$ Only one angle
(c) (d)	Only Reflection No Transmission. Wave should travel from denser to rarer medium	(c) (d)	Only Transmission No Reflection No constrain for EM wave transmission
(e)	$\sin\theta_c = \sqrt{\frac{\epsilon_t}{\epsilon_i}}$	(e)	$\tan\theta_B = \sqrt{\frac{\epsilon_t}{\epsilon_i}}$
(f)	Critical angle exist for both P-polarization and S-polarization.	(f)	For non-magnetic medium, Brewster's angle exist only for S-polarization only
(g)	$\lvert\Gamma\rvert = 1, \tau = 0$	(g)	$\lvert\Gamma\rvert = 0, \tau = 1$

❑❑❑

5 TRANSMISSION LINE

5.1. Introduction



- $\frac{-dV(z)}{dz} = (R + j\omega L) I(z)$
- $\frac{-dI(z)}{dz} = (G + j\omega C) V(z)$
- $\frac{d^2 I(z)}{dz^2} = y^2 I(z)$
- $\frac{d^2 V(z)}{dz^2} = y^2 V(z)$

$\Big\}$ VI – Wave equation.

- $\frac{d^2 I(z,t)}{dz^2} = \left(\frac{1}{V_p}\right)^2 \frac{d^2 I(z,t)}{dt^2}$
- $\frac{d^2 V(zt)}{dz^2} = \left(\frac{1}{V_p}\right)^2 \frac{d^2 V(z,t)}{dt^2}$

$\Big\}$ Time harmonic Equation.

Here, V_p = phase velocity.

1. Transmission Line Parameter:

(a) R = Resistance per unit length

(i) It is distributed along the transmission line.

(ii) It is due to conductor loss.

(iii) It is in series.

(iv) It gives attenuation in the voltage.

(b) G = Conductance per unit length.

(i) It is distributed in between the transmission line.

(ii) It is due to dielectric loss.

(iii) It is in parallel.

(iv) It gives attenuation in the current.

(c) L = Inductance per unit length.

(i) It is distributed along the transmission line.

(ii) It is due to storage of magnetic field.

(iii) It is in series.

(iv) It gives phase shift in voltage.

(d) C = Capacitance per unit length.

(i) It is distributed in between the transmission line.

(ii) It is due to storage of electric field.

(iii) It is in parallel.

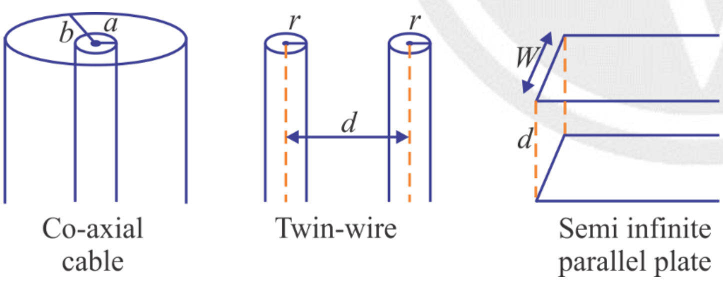
(iv) It gives phase shift in current.

(e)
$$R \xleftarrow{\alpha} V \angle\theta \xrightarrow{\beta} L$$
$$G \xleftarrow{\alpha} I \angle\theta \xrightarrow{\beta} C$$

(f) $LC = \mu E$

(g) $\frac{G}{C} = \frac{\sigma d}{E}$

(h)



(i)

	Semi-Infinite Parallel Plate	Co-axial cable	Twin wire
R	$\frac{2Rs}{W}$	$\frac{Ra}{2\pi a} + \frac{Rs}{2\pi b}$	$\frac{2Rs}{2\pi r}$
C	$\frac{WE}{d}$	$\frac{2\pi E}{\ln\left(\frac{b}{a}\right)}$	$\frac{\pi E}{\ln\left(\frac{d}{r}\right)}$
L	$\frac{\mu d}{W}$	$\frac{2\mu}{2\pi}\ln\left(\frac{b}{a}\right)$	$\frac{2\mu}{2\pi}\ln\left(\frac{d}{r}\right)$

G	$\frac{W\sigma_d}{d}$	$\frac{2\pi\sigma_d}{\ln\left(\frac{b}{a}\right)}$	$\frac{2\pi\sigma_d}{\ln\left(\frac{d}{r}\right)}$
Z_o	$\left(\sqrt{\frac{\mu}{E}}\right)\left(\frac{d}{W}\right)$	$\frac{60\sqrt{\mu_r}}{\sqrt{\epsilon_r}}\ln\left(\frac{b}{a}\right)$	$\frac{120\sqrt{\mu_x}}{\sqrt{\epsilon_r}}\ln\left(\frac{d}{r}\right)$

2. Intrinsic Impedance: -

(a) $Z_o = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-} = \frac{R + jwL}{\gamma} = \frac{\gamma}{G + j\omega C} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$

(b) It is defined at every point of transmission line.

(c) It is independent from length of the transmission line.

(d) Z_0 = depends upon σ_d, σ_c, μ, $\angle$ (medium) and geometry of transmission line.

3. Lossless Transmission Line:

R = 0 $\rightarrow$ Perfect conductor $\rightarrow \sigma_c \rightarrow \infty$

G = 0 $\rightarrow$ Perfect dielectric $\rightarrow \sigma_d = 0 \rightarrow \tan\delta = 0$

(a) $\gamma = \sqrt{(R + j\omega L)(G + j\omega L)} = \sqrt{(R + j\omega L)(G + j\omega L)} = j\omega\sqrt{LC}$

$\alpha = 0 \Rightarrow$ Lossless and Distortionless

$\beta = \omega\sqrt{LC} \Rightarrow$ Non-Dispersive

$\beta = \omega\sqrt{LC} = \omega\sqrt{\mu\varepsilon} = \beta_0\sqrt{\mu_r\varepsilon_r}$

(b) $\lambda = \frac{\lambda_0}{\sqrt{\mu_r\varepsilon_r}}$

(c) $V_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c_o}{\sqrt{\mu_r\varepsilon_r}}$

(d) $V_g = \frac{1}{\sqrt{LC}}$

(e) $V_p \cdot V_g = \frac{1}{LC} = \frac{c_0^2}{\mu_r\varepsilon_r}$

(f) $Z_o = \sqrt{\frac{L}{C}}$

(g) α, β, λ, V_p, $V_g \rightarrow$ Medium dependent

$Z_0 \rightarrow$ medium and geometry depend

4. Distortionless Transmission Line

$LG = RC$

(a) $\alpha = \sqrt{RG} = G\sqrt{\frac{L}{C}}$ → Lossy and Distortionless

$\beta = \omega\sqrt{LC} = \omega\sqrt{\mu\varepsilon} = \beta_0\sqrt{\mu_r\varepsilon_r}$ → Non-dispersive

(b) $V_P = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c_o}{\sqrt{\mu_r\varepsilon_r}}$

(c) $V_g = \frac{1}{\sqrt{LC}}$

(d) $V_p.\ V_g = \frac{1}{\sqrt{LC}} = \frac{c_o^2}{\mu_r\varepsilon_r}$

(e) $Z_o = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$

5. Some important formulae

(a) $\Gamma v = \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$

(b) $\Gamma_I = \frac{Z_0 - Z_L}{Z_0 + Z_L}$

(c) $\Gamma_{(x)} = \frac{Z_L - Z_0}{Z_L + Z_0} e^{-j2\beta x}$

(d) $Z(x) = Z_0\left(\frac{1+\Gamma_{(x)}}{1-\Gamma_{(x)}}\right)$

(e) $\Gamma_{(x)} = \frac{Z(x) - Z_0}{Z(x) + Z_0}$

(f) $Z(x)\ Z_0 = \left(\frac{Z_L + jZ_0 \tan(\beta x)}{Z_0 + jZ_L \tan(\beta x)}\right)$ (For lossless Tx-line)

(g) $2(x) = Z_0\left(\frac{Z_L + Z_O \tan n(\gamma x)}{Z_o + Z_L \tan n(\gamma x)}\right)$ (For lossy Tx-line)

6. Impedance:

(a) $Z_{OC} = -jZ_o \cot \beta l$

(b) $Z_{SC} = jZ_o \tan \beta l$

(c) $Z_{OC} \cdot Z_{SC} = (Z_o)^2$

(d) For $Z_L = Z_0$, $Z_{in} = Z_0$

Length	Z_{SC}	Z_{OC}
$O < l < \frac{\lambda}{4}$	Inductive	Capacitive
$\frac{\lambda}{4} < l < \frac{\lambda}{2}$	Capacitive	Inductive
$\frac{\lambda}{2} < l < \frac{3\lambda}{4}$	Inductive	Capacitive
$\frac{3\lambda}{4} < l < \lambda$	Capacitive	Inductive

(e)

Series Resonance		**Parallel Resonance**	
Z_{SC}	Z_{OC}	Z_{SC}	Z_{OC}
$l = \frac{n\lambda}{2}$	$l = \frac{(2n+1)\lambda}{4}$	$l = \frac{(2n+1)\lambda}{4}$	$l = \frac{n\lambda}{2}$
$\lambda = \frac{2l}{n}$	$\lambda = \frac{4l}{(2n+1)}$	$\lambda = \frac{4l}{(2n+1)}$	$\lambda = \frac{2l}{n}$
$f_o = \frac{nV_p}{2l}$	$f_o = \frac{(2n+1)V_p}{4l}$	$f_o = \frac{(2n+1)V_p}{4l}$	$f_o = \frac{nV_p}{2l}$

(f) $l = \frac{(2n+1)\lambda}{4} \Rightarrow Z_{in} = \frac{Z_o^2}{Z_L} \Rightarrow \bar{Z}_{in} = \frac{1}{\bar{Z}_L}$ impedance in version

(g) Normalised Impedance

Input impedance $\Rightarrow \bar{Z}_{in} = \frac{Z_{in}}{Z_o}$

Normalised load impedance $\Rightarrow \bar{Z}_L = \frac{Z_L}{Z_o}$

(h) $l = \frac{n\lambda}{2} \Rightarrow Z_{in} = Z_L$

(i) $l = \frac{\lambda}{8}, \frac{5\lambda}{8}, \frac{9\lambda}{8}, \ldots$

$$Z_{in} = Z_o\left(\frac{Z_L + jZ_o}{Z_o + jZ_L}\right)$$

(j) $l = \frac{3\lambda}{8}, \frac{7\lambda}{8}, \frac{11\lambda}{8}, \ldots$

$$Z_{in} = Z_o\left(\frac{Z_L - jZ_o}{Z_o - jZ_L}\right)$$

5.2. VSWR

SWR : Standing wave ratio

$$V_{SWR} = \frac{V_{max}}{V_{min}}$$

$$I_{SWR} = \frac{I_{max}}{I_{min}}$$

$$E_{SWR} = \frac{E_{max}}{E_{min}}$$

$$H_{SWR} = \frac{H_{max}}{H_{min}}$$

$$\rightarrow \rho = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$Z_L < Z_0$ $\quad$ $Z_L > Z_0$

$Z_L = 0$ $\quad$ $Z_L = Z_0$ $\quad$ $Z_L = \infty$

Case I: $Z_L = 0 \Rightarrow$ Short Circuit

1. $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{0 - Z_0}{0 + Z_0} = -1 = 1e^{j\pi} \Rightarrow |\Gamma_L| = 1, \theta_\Gamma = \pi$

2. $V_{max} = V_f(1 + \Gamma_L) = 2V_f$

   $V_{min} = V_f(1 - \Gamma_L) = 0$

3. $V_{SWR} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \infty$

4. $Z_{max} = \frac{(2n\pi + \pi)\lambda}{4\pi} = \frac{(2n+1)\lambda}{4}$

   $Z_{max} = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \frac{7\lambda}{4}, \frac{9\lambda}{4}, \ldots$

   $\uparrow$

   1st Maxima

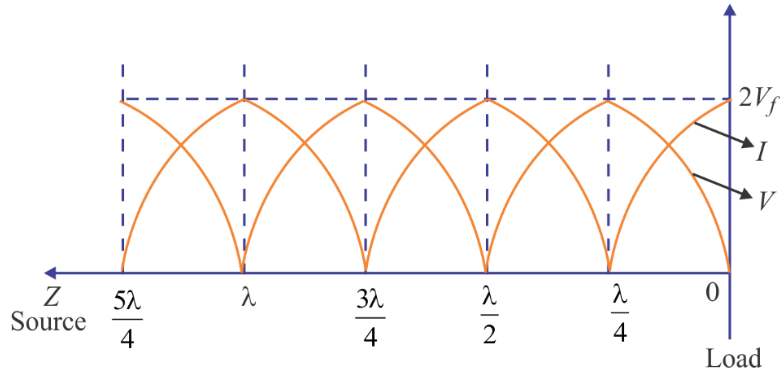
5. $Z_{min} = \frac{((2n+1)\pi + \pi)\lambda}{4\pi} = \frac{(n+1)\lambda}{2}$

   $0, \quad \lambda/2, \quad \frac{3\lambda}{2}, \ldots$

   $\downarrow \quad \downarrow$

   Z_{min} = 1st Minima

6.



Case II: $Z_L < Z_0$

1. $\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0} = -K = Ke^{j\pi} \Rightarrow \begin{array}{l} |\Gamma_L| = K \\ \underline{\theta^{\Gamma} = \pi} \end{array} \quad 0 < K < 1$

2. $V_{\max} = V_f(1 + K) < 2V_f$

   $V_{\min} = V_f(1 - K) > 0$

3. $V_{SWR} = \dfrac{1+K}{1-K}$

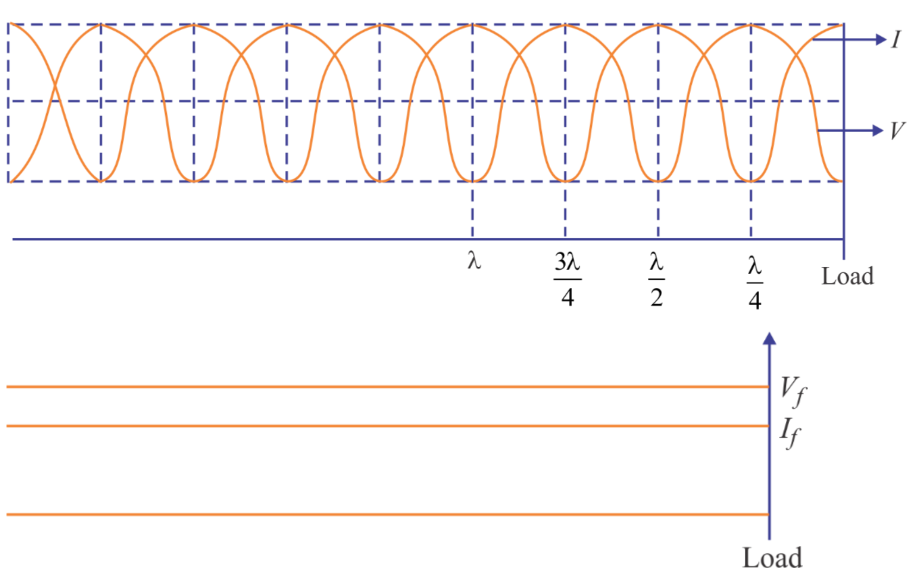
4. $Z_{\max} = \dfrac{(2n\pi + \pi)\lambda}{4\pi} = \dfrac{(2n+1)\lambda}{4}$

   $Z_{\max} = \dfrac{\lambda}{4}, \dfrac{3\lambda}{4}, \dfrac{5\lambda}{4}, \ldots$

5. $Z_{\min} = \dfrac{((2n+1)\pi + \pi)\lambda}{4\pi} = \dfrac{n\lambda}{2}$

   $Z_{\min} = 0, \dfrac{\lambda}{2}, \lambda, \dfrac{3\lambda}{2}, 2\lambda, \ldots$

6.



Case III: $Z_L > Z_0$

1. $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = K \Rightarrow \begin{matrix} |\Gamma_L| = K \\ \theta_\Gamma = 0 \end{matrix}, \quad 0 < K < 1$

2. $V_{max} = V_f(1 + K) < 2V_f = V_f(1 - K) > 0$

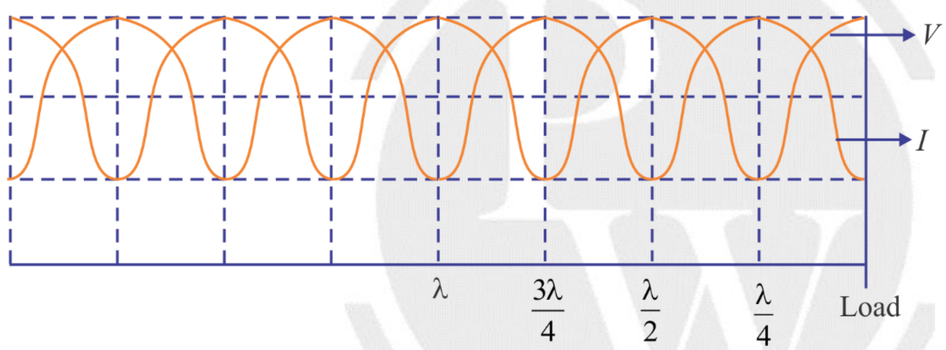
3. $V_{SWR} = \frac{1+K}{1-K}$

4. $Z_{max} = \frac{(2n\pi + 0)\lambda}{4\pi} = \frac{n\lambda}{2}$

   $Z_{max} = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda, \ldots$

5. $Z_{min} = \frac{(2n\pi + \pi)\lambda}{4} = \frac{(2n+1)\lambda}{4}$

   $Z_{min} = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$

6. 

Case IV: $\Gamma_L = \infty \Rightarrow$ Open circuit

1. $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{1 - Z_0/Z_L}{1 + Z_0/Z_L} = 1 \Rightarrow |\Gamma_L| = 1, \ \theta_\Gamma = 0$

2. $V_{max} = V_f(1 + |\Gamma_L|) = 2V_f$

   $V_{min} = V_f(1 - |\Gamma_L|) = 0$

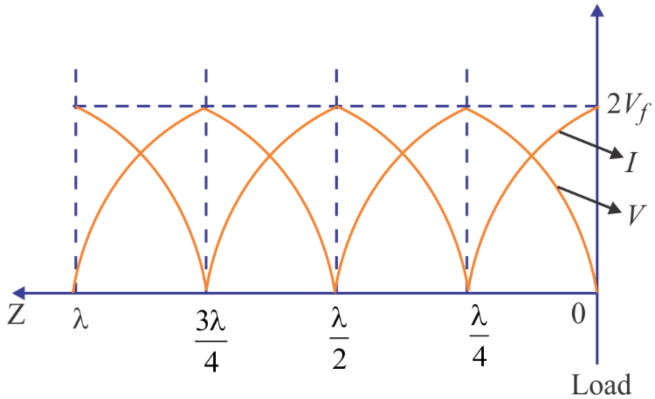
3. $V_{SWR} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \infty$

4. $Z_{max} = \frac{(2n\pi + 0)\lambda}{4\pi} = \frac{n\lambda}{2}$

   $Z_{max} = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda, \ldots$

5. $Z_{min} = \frac{((2n+1)\pi + 0)\lambda}{4\pi} = \frac{(2n+1)\lambda}{4}$

   $Z_{min} = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$

6. 

1. Distance between two successive maxima = λ/2

   Distance between two successive minima = λ/2

   Distance between two successive maxima & minima = λ/4

2.

V	I	Z	Z
Maximum	Minimum	Maximum	$Z_{max} = \frac{(2n\pi + \theta_\Gamma)\lambda}{4\pi}$
Minimum	Maximum	Minimum	$Z_{min} = \frac{((2n+1)\pi + \theta_\Gamma)\lambda}{4\pi}$

3. (a) Complete matched or matched

$$\Gamma_L = 0,\ VSWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = 1$$

   (b) Complete mismatched

$$|\Gamma_L| = 1,\ VSWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \infty$$

   For example: S/C, O/C, pure inductive, pure resistive.

   (c) Mismatched:

$$-1 < \Gamma_L < 1 \quad \Rightarrow 1 < \text{VSWR} < \infty$$

4. VSWR ∈ [1, ∞)

$$\Gamma \in [-1, 1]$$

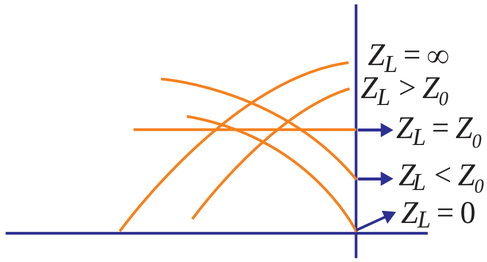
5. $\Gamma(z) = \Gamma_L e^{-j2fz} = |\Gamma_L| e^{+j\theta_\Gamma} e^{-j\theta Z} = |\Gamma_L| e^{j(\theta_\Gamma - 2fz)}$

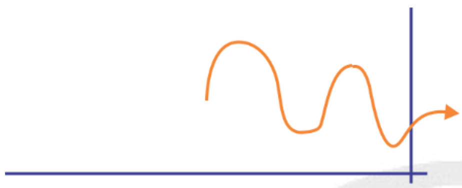
   Magnitude of reflection coefficient does not depend on distance. But phase of reflection coefficient depends upon distance.

$$VSWR = \frac{1+|\Gamma(Z)|}{1-|\Gamma(Z)|} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

VSWR does not depends upon distance.

6.





7.

8. (a) In case of maxima at load, $Z_L > Z_0$.

$$\Gamma_L = \frac{Z_L - Z_O}{Z_L + Z_O} \Rightarrow |\Gamma_L| = \frac{(Z_L - Z_O)}{(Z_L + Z_O)}$$

$$VSWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{Z_L}{Z_O}$$

(b) In case of minima at load ($Z_L < Z_O$)

$$\Gamma_L = \frac{Z_L - Z_O}{Z_L + Z_O} < 0$$

$$|\Gamma_L| = -\Gamma_L = \frac{Z_O - Z_L}{Z_O + Z_L}$$

$$VSWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{Z_O}{Z_L}$$

$$VSWR = \rho = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \Rightarrow |\Gamma_L| = \frac{\rho-1}{\rho+1}$$

$VSWR = \infty$
- → Maxima at load $\Rightarrow Z_L = \infty,\ \Gamma_L = 1$
- → Minima at load $\Rightarrow Z_L = 0,\ \Gamma_L = -1$

$VSWR = \text{finite} = \rho$
- → Maxima at load $\Rightarrow Z_L > Z_O,\ VSWR = \frac{Z_L}{Z_O},\ |\Gamma_L| = \Gamma_L$
- → Minima at load $\Rightarrow Z_L < Z_O,\ VSWR = \frac{Z_O}{Z_L},\ |\Gamma_L| - \Gamma_L$

12. In ideal case, matched $\Rightarrow |\Gamma_L| = 0$, VSWR = 1

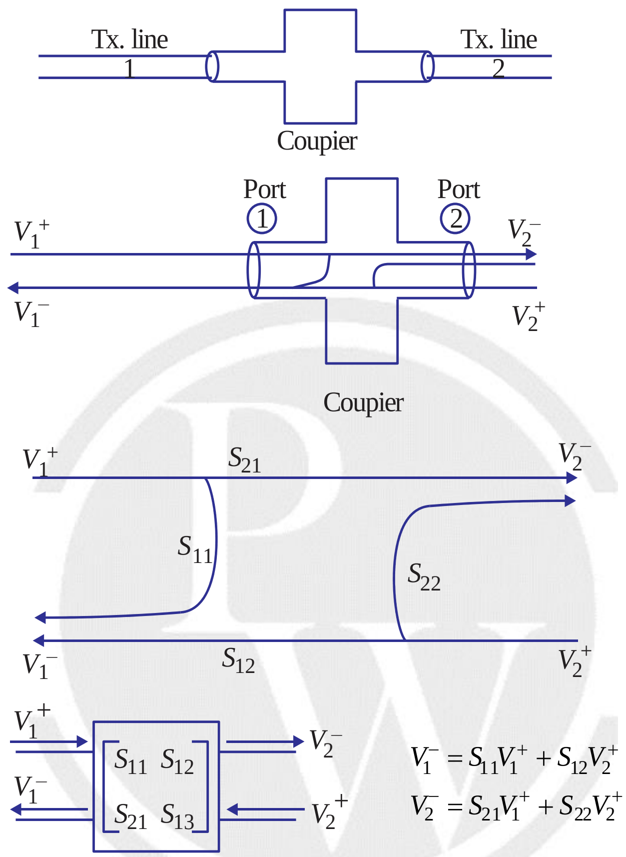
In practical case, matched $\Rightarrow 1 \le$ VSWR ≤ 2.

Scattering Matrix:

In Y, Z, h, g parameter, open circuit and short circuit conditions are required. But in transmission line, short and open circuit cannot be achieved throughout the transmission line.

So, S-parameter concept is introduced in microwave devices.

⇒ S-parameter for coupler.



(a) When $V_1^+ = 0 \Rightarrow$ Port (1) matched

$$S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{V_1^+=0}$$

S_{12} = Transmission coefficient at port (2) due to port (2) when port (1) matched.

= Reverse voltage gain

$$S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{V_1^+=0}$$

S_{22} = Reflection coefficient at port (2) due to port (1) when port (1) matched

= Output reflection coefficient

(b) When $V_2^+ = 0 \Rightarrow$ Port (2) matched

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0}$$

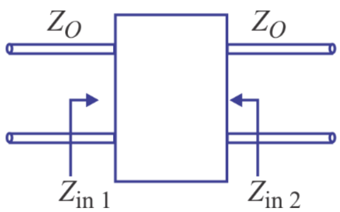
S_{11} = Reflection coefficient at port (1) due to port (1) when port (2) is matched

= Input reflection coefficient

$$S_{21} = \left.\frac{V_2^-}{V_2^+}\right|_{V_2^+=0}$$

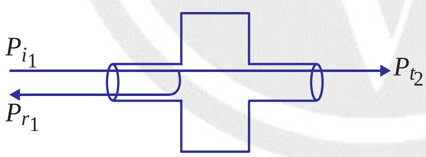
S_{21} = Transmission coefficient at port (2) due to port (1) when port (2) is matched

= Forward voltage gain

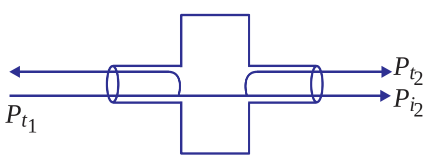


1. $S_{11} = \Gamma_1 = \dfrac{Z_{in_1} - Z_O}{Z_{in_1} + Z_O}$ 2. $S_{21} = \tau_1 = 1 + \Gamma_1$ 3. $S_{22} = \Gamma_1 = \dfrac{Z_{in_2} - Z_O}{Z_{in_2} + Z_O}$ 4. S12 = $\tau_2 = 1 + \Gamma_2$	(a) For Symmetry $Z_{in_1} = Z_{in_2}$ $\Rightarrow \boxed{S_{11} = S_{22}}$ (b) For reciprocal Forward gain = Reverse gain $\boxed{S_{12} = S_{21}}$

For Lossless:



$$S_{11} = \sqrt{\frac{P_{r_1}}{P_{i_1}}},\ S_{21} = \sqrt{\frac{P_{t_1}}{P_{i_1}}}$$



$$S_{22} = \sqrt{\frac{P_{t_2}}{P_{i_2}}},\ S_{12} = \sqrt{\frac{P_{t_1}}{P_{i_2}}}$$

For lossless device

$$P_{i_1} = P_{r_1} + P_{t_2} \Rightarrow \quad 1 = \frac{P_{r_1}}{P_{i_1}} + \frac{P_{t_2}}{P_{i_2}} \Rightarrow 1 = |S_{11}|^2 + |S_{21}|^2$$

$$P_{i_2} = P_{r_2} + P_{t_2} \Rightarrow \ 1 = |S_{22}|^2 + |S_{12}|^2$$

For lossy device:

$$P_{i_1} > P_{r_1} + P_{t_2} \Rightarrow |S_{11}|^2 + |S_{21}|^2 < 1$$

$$P_{i_2} > P_{r_2} + P_{t_2} \Rightarrow |S_{22}|^2 + |S_{12}|^2 < 1$$

For Amplifier

$$P_{i_1} < P_{r_1} + P_{t_2} \Rightarrow |S_{11}|^2 + |S_{21}|^2 > 1$$

$$P_{i_2} < P_{r_2} + P_{t_1} \Rightarrow |S_{11}|^2 + |S_{12}|^2 > 1$$

For Lossless

S*S = I

$$\begin{bmatrix} S_{11}^* & S_{21}^* \\ S_{12}^* & S_{22}^* \end{bmatrix}\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} |S_{11}|^2 + |S_{21}|^2 = 1 \\ |S_{12}|^2 + |S_{22}| = 1 \end{bmatrix} \rightarrow \text{Unitar property} \begin{vmatrix} S_{11}^* S_{12} + S_{21}^* S_{22} = 0 \\ S_{12}^* S_{11} + S_{22}^* S_{21} = 0 \end{vmatrix} \rightarrow \text{zero property}$$

1. Return loss at port (1) = – 20 $\log_{10}|S_{11}|$
2. Return loss at port (2) = – 20 $\log_{10}|S_{22}|$
3. Gain at port (2) = 20 $\log_{10}|S_{21}|$
4. Gain at port (1) = 20 $\log_{10}|S_{12}|$
5. Intertion loss at port (1) = $-20\log_{10}\left(\frac{(S_{11})^2}{1-|S_{11}|^2}\right)$

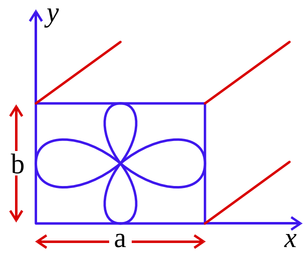
   Intertion loss at port (2) = $-20\log_{10}\left(\frac{(S_{22})^2}{1-|S_{22}|^2}\right)$

❑❑❑

6 WAVEGUIDE

6.1. Introduction

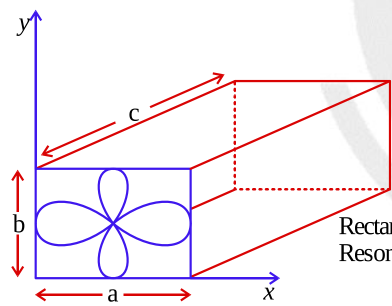
- Waveguide directs the wave in one direction



Rectangular waveguide.

		E	H
X direction	Standing wave	sin (min)	cos (max)
Y direction	Standing wave	sin (min)	cos (max)
Z direction	Propagating wave	$e^{-j\beta_z z}$	$e^{-j\beta_z z}$

-



Rectangular Resonator.

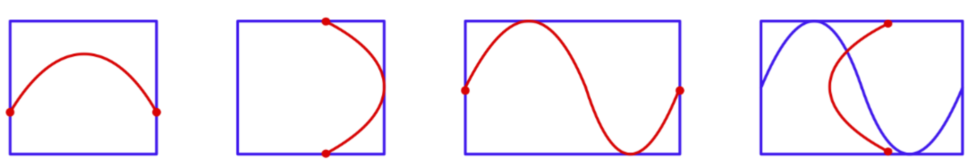
		E	H
X direction	Standing wave	Min.	Max.
Y direction	Standing wave	Min.	Max.
Z direction	Standing wave	Max.	Min.

(1) Analysis of propagation constants.

(a) $f < f_c \Rightarrow \beta_z = -j\sqrt{\beta_c^2 - \beta^2}$

(b) $f = f_c \Rightarrow \beta_z = 0$

No Propagation only Oscillation



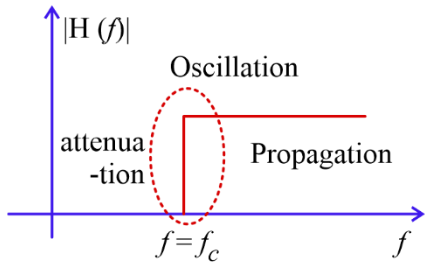
m = 1, n = 0   m = 0, n = 1   m = 2, n = 0   m = 2, n = 1

(c) $f > f_c \Rightarrow \beta_z = \sqrt{\beta^2 - \beta_c^2} \neq 0$ Only Propagation occurs.

2. (a) $f < f_c$ & $\lambda > \lambda_c$ Attenuation, Evancent mode.

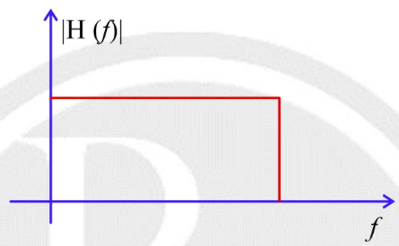
(b) $f = f_c$, & $\lambda = \lambda_c$ Only Attenuation

(c) $f > f_c$ & $\lambda < \lambda_c$ Propagation



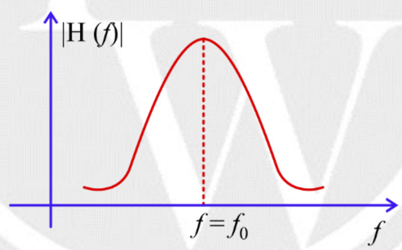
Rectangular Waveguide

(3) Transmission Line:



It will act as a low pass filter. It will behave like an all pass filter within Frequency range.

(4) Resonator:



It will act as a band pass filter.

(5) Cut-off frequency and Cut-off wavelength.

$$f_c = \left(\frac{C_0}{2\sqrt{\mu_r \in_r}}\right)\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$\lambda_c = \frac{2\sqrt{\mu_r \in_r}}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{p}\right)^2}}$$

(6) λ_x = Wavelength in X-direction.

λ_y = Wavelength in Y-direction.

λ_z = Wavelength in Z-direction

λ_g = Guided wavelength

λ = Operating wavelength.

$$\boxed{\lambda_g = \lambda_z}$$

- $\lambda_x = \frac{2a}{m}, \qquad \lambda_y = \frac{2b}{n}$
- $\lambda_z = \lambda_g = \frac{\lambda}{\cos\theta}$
- $\frac{1}{\lambda^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}$
- θ = tilt angle
- $\sin\theta = \frac{f_c}{f} = \frac{\lambda}{\lambda_c} = \frac{\omega_c}{\omega} = \frac{\beta_c}{\beta}$
- $\beta_x = \frac{m\pi}{a}, \beta_y = \frac{n\pi}{b}, \beta_z = \beta\cos\theta$
- $\beta_c = \sqrt{\beta_x^2 + \beta_y^2} = \beta\sin\theta$.

(7) Phase velocity and group velocity.

$\bar{V}_p$ = phase velocity inside waveguide

$\bar{V}_g$ = group velocity inside waveguide

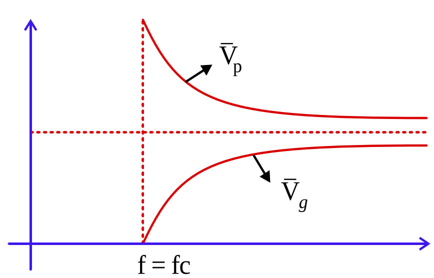
V_p = phase velocity in free space

V_g = group velocity in free space

- $\bar{V}_p = \frac{V_p}{\cos\theta}, \bar{V}_g = V_p\cos\theta$ for $f > f_c$
- $V_p = \frac{C_0}{\sqrt{\mu_r \in_r}}$
- $\bar{V}_p = 0 \qquad f < f_c$

  $\bar{V}_p = \infty \qquad f = f_c$

  $\bar{V}_p = \frac{V_P}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \qquad f > f_c$

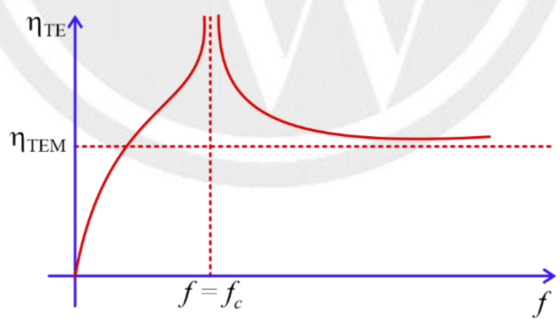


- $\bar{V}_P . \bar{V}_g = \frac{C_0^2}{\mu_r \in_r}$

- Inside waveguide, $\bar{V}_P > V_P$
- Inside waveguide, $\bar{V}_g < V_P$
- At very high frequency, $\bar{V}_P = \bar{V}_g = V_P$

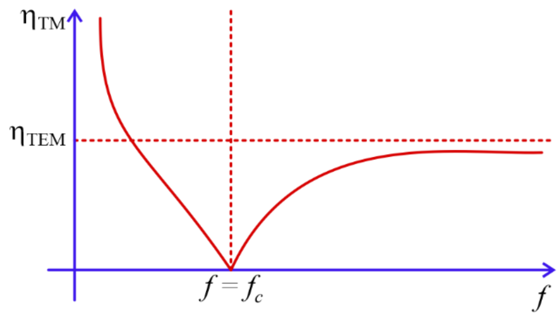
(8) Intrinsic Impedance:

- η_{TEM} = Wave Impedance in free space or TEM wave.
- η_{TE} = Wave Impedance in TE mode
- η_{TM} = Wave Impedance in TM mode
- $\eta_{TEM} = \eta_0 \sqrt{\dfrac{\mu_r}{\epsilon_r}}$
- η_{TE} →
  - $-jx$ (Capacitive)
  - ∞ Open circuit (Parallel resonance)
  - $\dfrac{\eta_{TEM}}{\sqrt{1-\left(\dfrac{f_c}{f}\right)^2}}$ $f > f_c$



- $\eta_{TE} > \eta_{TEM}$
- At very high frequency, $\eta_{TE} = \eta_{TEM}$

η_{TM} →
- jx $f < f_c$ Inductive
- 0 $f = f_c$ Short circuit (series resonance)
- $\eta_{TEM}\sqrt{1-\left(\dfrac{f_c}{f}\right)^2}$ $f > f_c$



- $\eta_{TM} < \eta_{TEM}$
- At very high frequency, $\eta_{TM} = \eta_{TEM}$
- $\eta_{TE} \cdot \eta_{TM} = \eta_{TEM}^2 = \eta_0^2 \left(\frac{\mu_r}{\in_r}\right)$
- $\eta_{TE} = \frac{\eta_{TEM}}{\cos\theta}$ $\qquad f > f_c$
- $\eta_{TM} = (\eta_{TEM})\cos\theta$ $\qquad f > f_c$

(9) Some Important Terms

(i) Dominant mode: The mode which has lowest cut-off frequency and highest cut-off wavelength, is known as dominant mode.

- For a > b → TE_{10} dominant mode → $f_c = \frac{C_0}{2a}$
- For a < b → TE_{01} dominant mode → $f_c = \frac{C_0}{2b}$
- For c > a > b → TE_{101} dominant mode → $f_r = \frac{C_0}{2\sqrt{\mu_r \in_r}}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{c}\right)^2}$
- For c > b > a → TE_{011} dominant mode → $f_r = \frac{C_0}{2\sqrt{\mu_r \in_r}}\sqrt{\left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}$

(ii) Single mode frequency operation

(a) For $a > b \rightarrow \frac{C_0}{2a} < f < \frac{C_0}{2b} \rightarrow 2b < \lambda < 2a$

(b) For $a < b \rightarrow \frac{C_0}{2b} < f < \frac{C_0}{2a} \rightarrow 2a < \lambda < 2b$

(iii) Non-Existence Mode:

The mode which does not exist.

(a) Rectangular waveguide:

TEoo, TMoo, TMon, TMmo

(b) Rectangular resonator

TEooo, TEmoo, TEono, TEmno, TEoop, TMooo, TMoop, TMmop, TMmoo, TMono, TMonp

(iv) Existence Mode:

The mode which exist

(a) Rectangular waveguide

TEon, TEmo, TEmn, TMmn

(b) Rectangular resonator:

TEonp, TEmop, TEmnp, TMmno, TMmnp

(v) Evanescent Mode:

The mode which exist but operating frequency is less than cutt-off frequency.

(vi) Degenerate Mode:

- Two modes have same cutt-off frequency
- These modes are existance mode
- These modes exist in a square waveguide (a = b) and cubic resonator (a = b = c).

e.g. TEmn ≡ TEnm, TEon ≡ TEmo

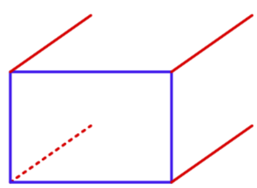
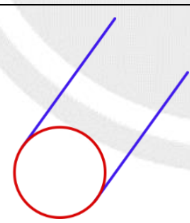
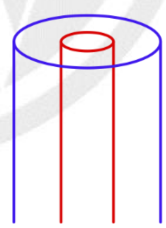
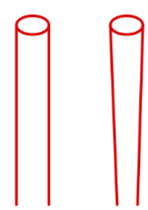
TMmn ≡ TMnm,

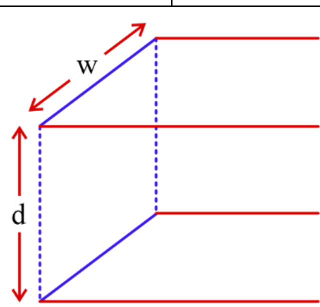
TEmop ≡ TEonp ≡ TEpom ≡ TEopm.

(vii) TEM mode:

- The mode which has no cutt-off frequency is known as TEM mode.

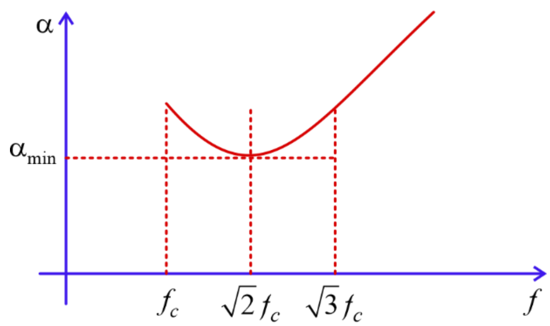
- TEoo, TMoo, TEooo, TMooo are TEM mode

				
•	Rectangular	Circular	Co-axial	Twin-wire
•	Single conductor	Single conductor	Double conductor	Double conductor
•	$f_c \neq o$	$f_c \neq o$	$f_c = o$	$f_c = o$
•	TEM does not exist	TEM does not exist	TEM exist	TEM exist

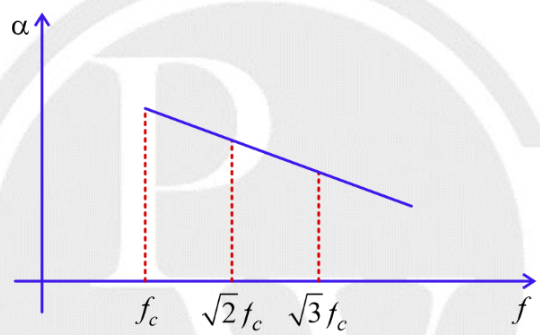


- Semi-infinite parallel plate waveguide
- $f_c = 0$
- TEM exist

(viii) Attenuation Constant in Rectangular Waveguide and Circular Waveguide.



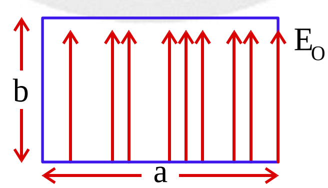
This graph is valid for all modes of rectangular and circular waveguide except TE_{01} mode in circular waveguide.



TE_{01} mode of circular waveguide has lowest cutt-off frequency

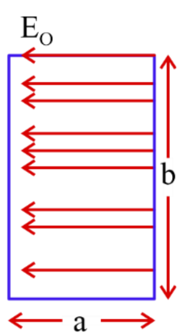
(10) Open circuit voltage:

- For a > b → TE_{10} mode is dominant



$V_{OC} = E_0 b$

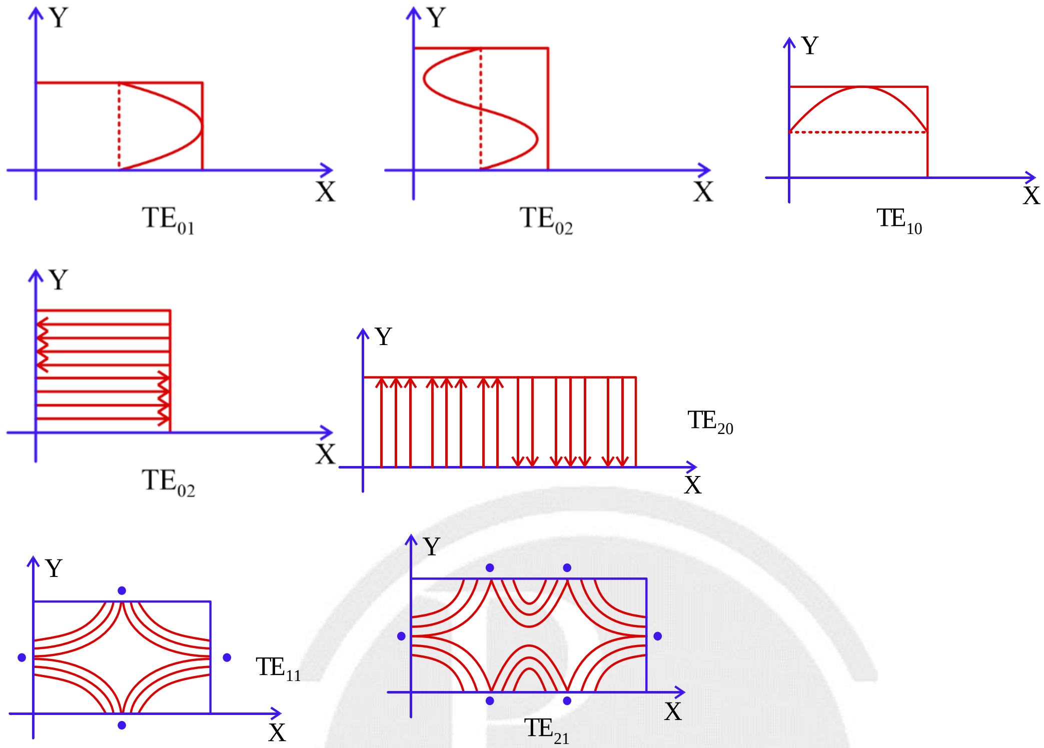
- For a < b → TE_{01} mode is dominant



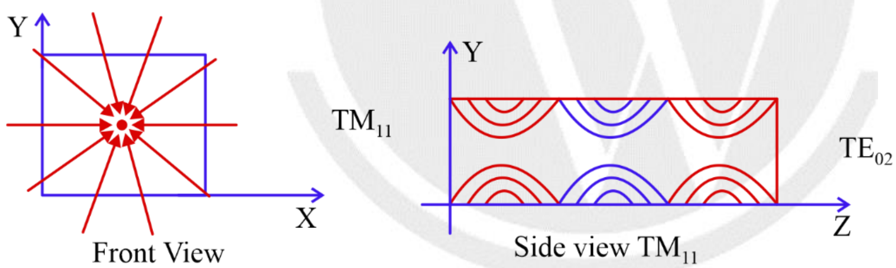
$V_{OC} = E_0 a$

(11) Electric Field Pattern:

(a)



(b) TM mode :



7 ANTENNA

6.1. Antenna and its Radiation Pattern.

- **Antenna:** It is a device which converts electrical signal into EM wave & EM wave into electrical signal.
- Some Important terms related to antenna.
  - **W_o:** Total Radiated Power (Watt)
  - **P_{rad} or P_{av}:** Time average power density or time average poynting vector or Radiated power density (Watt/m^2)
  - **$U(\theta, \phi)$:** Radiated power Intensity or Radiation Pattern (Watt/steradian)
  - **Ω_{eff}:** effective solid angle (Steradian)
  - **θ_E:** Half power beam width or elevation angle. (radian)
  - **ϕ_H or θ_H:** Half power beam width or azimuthal angle. (radian)
  - **D or G_D:** Directive Gain
  - **D_o** = Maximum directive gain or directivity.
  - **G_P:** Power gain
  - **G_{Po}:** Maximum Power gain.
  - **e_c:** efficiency due to conductor loss.
  - **e_d:** efficiency due to dielectric loss.
  - **e_{cd}:** efficiency due to conductor and dielectric loss.
  - **e_r:** efficiency due to reflection
  - **R_r:** Radiation Resistance (Ω)
  - **R_l:** Loss Resistance (Ω)
  - **P_{in}:** Direct of transmitting Antenna.
  - $\hat{\rho}_t$ **:** Direct on transmitting Antenna.
  - $\hat{\rho}_r$ **:** Direction of receiving Antenna.
  - **PLF:** Polarization Loss Factor
  - **A_{eff}:** Effective Aperture Area (m^2)
  - **L_{phy}:** Physical length (m)

- **l_{eff} or l_{av}:** Effective length or average length (m)
- **V_{OC}:** Open circuit Voltage (Volt)

1. Solid Angle

(a) $d\Omega = \sin\theta d\theta d\phi$

(b) $\Omega_{sphere} = 4\pi$

(c) $\Omega_{eff} = \theta_E \times \theta_H \rightarrow$ Non-uniform cone

$= \theta^2{}_E \rightarrow$ Uniform cone

(d) 1 Steradian = 1 radian × 1 radian

$$= \frac{(180°)^2}{\pi^2} = 3282.8$$

2. Radiated Power Density:

$$\vec{P}_{av} = \vec{P}_{rad} = \frac{E_{rms}^2}{\eta}\hat{a}_p = E_{rms} \cdot H_{rms}\hat{a}_p$$

$$= \frac{E_o H_o}{2}\hat{a}_p$$

The strength of EM wave transmitted by the antenna which depends upon distance (r) and direction (θ, ϕ) is radiated power density.

3. Radiated Power Intensity:

$$U(\theta,\phi) = \frac{dW}{d\Omega}\left(\frac{\text{Watt}}{\text{Steradian}}\right)$$

- $P_{rad} = \frac{dW_0}{dS}$
- $W_o = \int\int P_{rad}\, r^2 \sin\theta d\theta d\phi$
- $U(\theta,\phi) = \frac{dW_o}{d\Omega}$
- $W_o = \int\int U(\theta,\phi)\sin\theta d\theta d\phi$

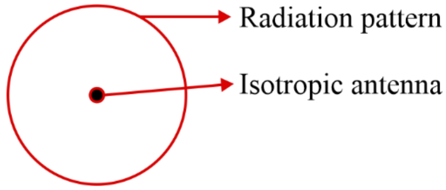
4. Isotropic Antenna:

(a) It is point size antenna.

(b) It is ideal Antenna.

(c) It is reference Antenna.

(d) It is reference Antenna

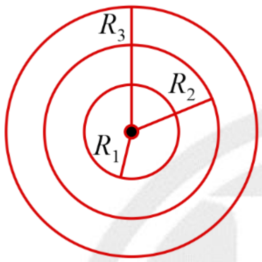


(e) W_o = Total power radiated by isotropic

(f) $\Omega_{sphere} = 4\pi$ Steradian

(g) $U_o(\theta, \phi) = \dfrac{W_o}{4\pi}$

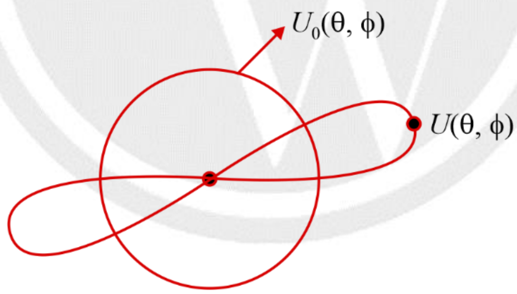
(h)



$$\left.\begin{aligned} P_1 &= \frac{W_o}{4\pi R_1^2} \\ P_2 &= \frac{W_o}{4\pi R_2^2} \\ P_3 &= \frac{W_o}{4\pi R_3^2} \end{aligned}\right\} \text{Radiated Power Density}$$

(i) $D_o = 1,\ G_{P_0} = 1,\ e_{cd} = 1,\ e_r = 1,\ e_t = 1$

5. Directive Gain And Directivity



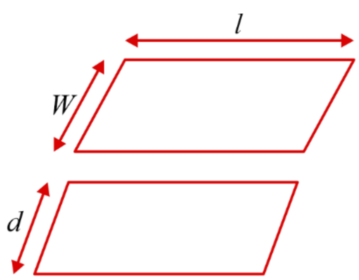
- $D = \dfrac{\text{Radiation Pattern of general antenna}}{\text{Radiation Pattern of isotropic antenna}}$
- $D = \dfrac{U(\theta, \phi)}{U_o(\theta, \phi)} = \dfrac{4\pi U(\theta, \phi)}{W_o} = \dfrac{4\pi U(\theta, \phi)}{\oint\oint U(\theta, \phi) \sin\theta \, d\theta \, d\phi}$
- For General Antenna → $U(\theta, \phi) = U_o(\theta, \phi)$
- $D_o = D_{max} = \dfrac{4\pi U(\theta, \phi)|_{max}}{U_o(\theta, \phi)}$
- $0 \le D \le D_o,\ D_o \ge 1$
  - $D_0 > 1 \rightarrow$ General Antenna
  - $D_0 = 1 \rightarrow$ Isotropic Antenna

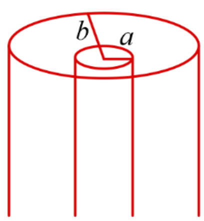
- $D_o = \frac{4\pi}{\Omega_{eff}} = \frac{4\pi}{\theta_E \cdot \theta_H} \rightarrow$ Non-uniform conical beam

  $= \frac{4\pi}{\theta_E^2} \rightarrow$ Uniform conical beam
- $D = f_1(\theta, \varphi) \rightarrow$ directional antenna.
- $D = f_1(\theta)$ or $f_1(\varphi) \rightarrow$ Omnidirectional Antenna.
- $D = K =$ constant $\rightarrow$ All directional antenna.

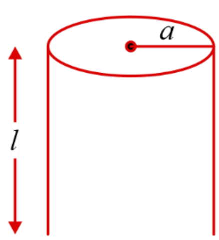
6. R_L = Loss Resistance

$$R_S = \sqrt{\frac{wu}{2\sigma_c}},\ R = \frac{2R_S}{W},\ R_L = \left(\frac{2R_S}{W}\right)l$$



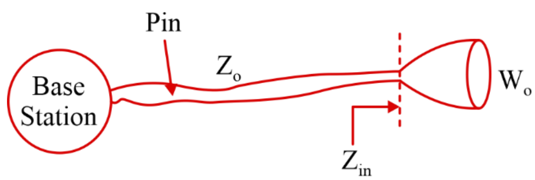


$$R_L = \left(\frac{R_S}{2\pi a} + \frac{R_S}{2\pi b}\right)l$$



$$R_L = \left(\frac{R_S}{2\pi a}\right)l$$

7. Efficiency:

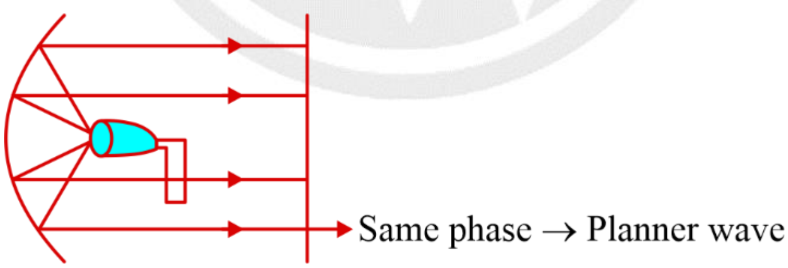


- $e_c = 1$ (Perfect conductor)
- $e_c = \frac{R_r}{R_r + R_L} \rightarrow$ (Good or Poor Conductor)

- $e_d = 1 \rightarrow$ Perfect dielectric
- $0 < e_c < 1 \rightarrow$ Conductor loss
- $0 < e_d < 1 \rightarrow$ Dielectric loss
- $e_r = (^2)$
- $e_r = \left(1 - |\Gamma|^2\right)$
- $e_r = 1 \rightarrow$ Perfectly matched
- $0 < e_r < 1 \rightarrow$ Mismatched
- $e_r = 0 \rightarrow$ Perfectly mismatched
- $e_{cd} = e_c \cdot e_d = \dfrac{R_r}{R_r + R_L}$
- $e_t = e_{cd} \cdot e_r = \dfrac{R_r}{R_r + R_L}\left(1 - |\Gamma|^2\right)$
- $W_0 = e_t P_{in}$
- $G_{Po} = e_t D_o$
- $D_o|_{dB} = \log_{10} D_o$
- $G_{P_o}\Big|_{dB} = 10\log_{10} G_{P_o}$
- $PLF|_{dB} = 10\log_{10} PLF$

8. Parabolic Antenna

- $A_{eff} = \dfrac{\lambda^2}{4\pi} D_o$
- $A_{eff} = \dfrac{\lambda^2}{4\pi} D_o (PLF)$



- Area efficiency = $\dfrac{A_{eff}}{A_{phy}}$
- $G_{P_o} = e_t D_o = e_A e_t \pi^2 \left(\dfrac{d}{\lambda}\right)^2$
- $G_{P_o} = 6.5\left(\dfrac{d}{\lambda}\right)^2 \rightarrow$ Gain of parabolic antenna
- First Null Beam width = FNBW = $\left(\dfrac{70\lambda}{d}\right)^0$

- Half Power Beam width = HPBW = $\left(\dfrac{58\lambda}{d}\right)^0$

Cassegrain Antenna

(a) A Horn antenna is used as feed antenna.

(b) A secondary reflector which is hyperbolic in Shape.

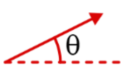
(c) A primary reflector which is paraboloid

9. Polarization Loss Factor (PLF):

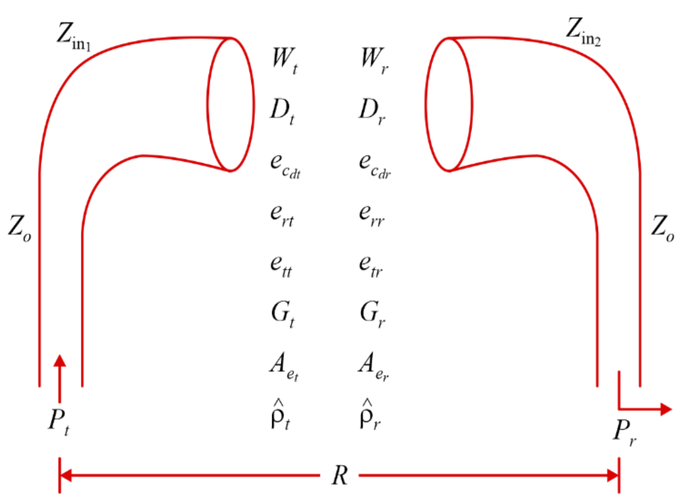
$\hat{\rho}_t$ = Direction of transmitting Antenna

$\hat{\rho}_r$ = Direction of receiving Antenna

$$\text{PLF} = |\hat{\rho}_t \cdot \hat{\rho}_r|^2$$

O-Polarization Received = 100%, Loss = 0%		X-Polarization Received = 0%, Loss = 100%	
T_x	R_x	T_x	R_x
H	H	H	V
V	V	V	H
			
RCP	LCP	RCP	RCP
LCP	RCP	LCP	LCP
LEP	REP	REP	REP
REP	LEP	REP	REP

10. Friis Formulae:



- $f\lambda = V_P$
- $e_{tt} = \dfrac{W_t}{P_t}$
- $e_{tr} = \dfrac{P_r}{W_r}$
- $G_t = e_{tt} D_t$
- $G_r = e_{tr} D_r$
- $e_{rt} = 1 - |\Gamma_t|^2$
- $e_{rr} = 1 - |\Gamma_r|^2$
- $\Gamma_t = \dfrac{Z_{in_1} - Z_o}{Z_{in_2} + Z_o}$
- $\Gamma_r = \dfrac{Z_{in_2} - Z_o}{Z_{in_2} + Z_o}$
- $e_{c_{dt}} = \dfrac{R_{r_t}}{R_{r_t} + R_{L_t}}$
- $e_{c_{dr}} = \dfrac{R_{rr}}{R_{rr} + R_{Lr}}$
- $e_{tt} = e_{cdt}\, e_{rt}$
- $e_{tr} = e_{cdr}\, e_{rr}$
- $\text{PLF} = |\hat{\rho}_t \cdot \hat{\rho}_r|^2$
- $A_{et} = \dfrac{\lambda^2}{4\pi} D_t$
- $A_{er} = \dfrac{\lambda^2}{4\pi} D_r$
- Power density at the receiving antenna due to transmitting Antenna when transmitting antenna is isotropic = $\dfrac{W_t}{4\pi R^2}$
- Power density at the receiving antenna due to transmitting antenna when transmitting antenna is not isotropic = $\dfrac{W_t D_t}{4\pi R^2}$
- Power density of the receiving antenna in terms of Electric field = $\dfrac{E_{rms}^2}{\eta}$

- where $D_o = 1$ (Isotropic)
- $D_o = 1.5$ (dipole antenna)
- $D_o = 1.63$ ($l = \lambda/2 \rightarrow$ half wave dipole).
- Power Received

$$W_r = \frac{W_t D_t A_{er}}{4\pi R^2} = \frac{W_t A_{et} A_{er}}{R^2 \lambda^2}$$

➢ $P_r = \frac{P_t D_t D_r}{(4\pi R)^2}\left(\frac{R_{rt}}{R_{rt} + R_{Lt}}\right)(1 - |\Gamma_t|^2)\left(\frac{R_{rr}}{R_{rr} + R_{Lr}}\right)(1 - |\Gamma_r|^2)\text{PLF}$

➢ $P_r = \frac{P_t G_t F_r}{\left(\frac{4\pi R}{\lambda}\right)^2}$

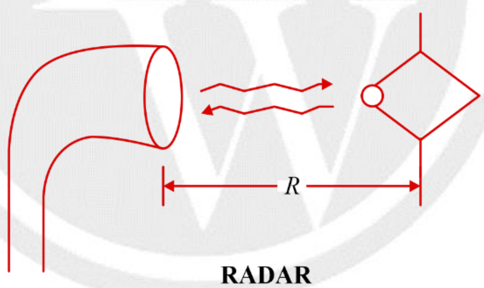
Link Formulae

$$P_r|_{dB} = P_t|_{dB} + G_t|_{dB} + G_r|_{dB} - 20\log_{10}\left(\frac{4\pi R}{\lambda}\right)$$

➢ Path Loss = $20\log_{10}\left(\frac{4\pi R}{\lambda}\right)$

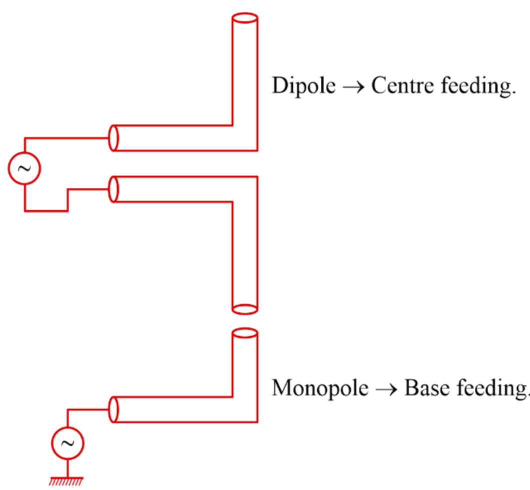
➢ $P_r = \frac{W_t D_t^2 \sigma^2 + \lambda^2}{64\pi^3 R^4}$

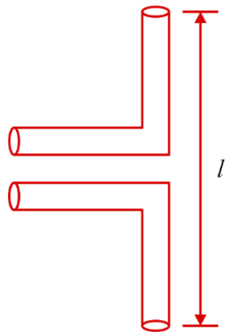
σ^2 = Cross-Sectional area of object.



RADAR

11. Dipole Antenna





$l = \frac{\lambda}{2} \rightarrow$ Half wave dipole antenna

$l = \frac{\lambda}{4} \rightarrow$ Quater wave dipole antenna.

12. Types Of Dipole Antenna

(a) Infinitesimal dipole antenna/Hertzian dipole.

$$dl < \frac{\lambda}{50}$$

(b) Small dipole antenna/short dipole antenna

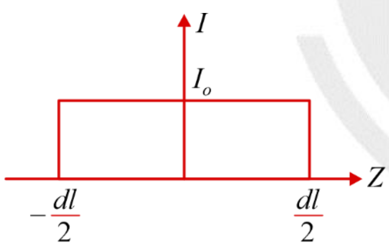
$$\frac{\lambda}{50} < l < \frac{\lambda}{10}$$

(c) Large dipole antenna

$$l > \frac{\lambda}{10}$$

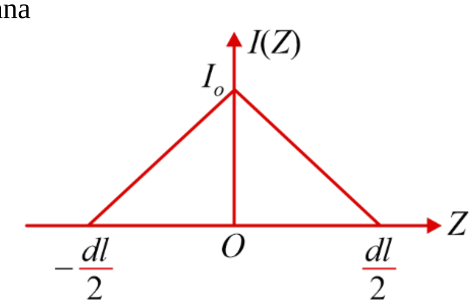
13. Current Distribution In Dipole Antenna

(a)



$\rightarrow$ Hertizian dipole antenna.

(b) Small/Short dipole Antenna

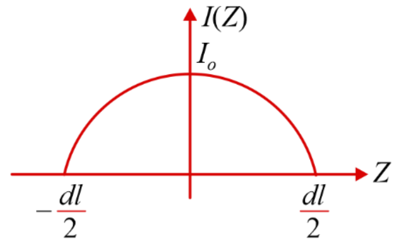


$$I_{av} = \frac{I_o}{2}$$

$$I(Z) = I_o\left(1 + \frac{2Z}{l}\right) \quad -\frac{l}{2} \le Z \le 0$$

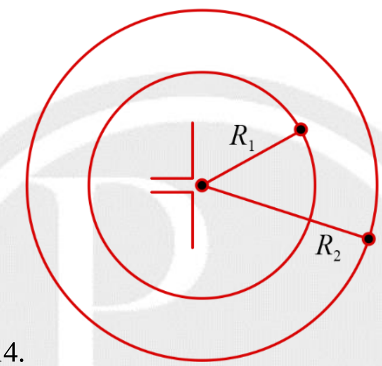
$$= I_o\left(1 - \frac{2Z}{l}\right) \quad 0 \le Z \le \frac{l}{2}$$

(c) Large dipole Antenna:



$$I(Z) = I_o \sin\left(K\left(1 + \frac{2Z}{l}\right)\right), -\frac{l}{2} \le Z \le 0$$

$$= I_o \sin\left(K\left(1 - \frac{2Z}{l}\right)\right) \quad 0 \le Z \le \frac{l}{2}$$



14.

(a) $r > R_1$ → Near field

(b) $R_1 < r < R_2$ → Fresnel's region.

(c) $r > R_2$ → Far field.

(d) $R_1 = 0.63\sqrt{\frac{l_3}{\lambda}},\ R_2 = \frac{2l^2}{\lambda}$

(e) The distance at which radiation field and inductive field.

$$\beta r = 1 \rightarrow \boxed{r = \frac{\lambda}{2\pi}}$$

15. Far Field of Hertizian Dipole Antenna

$$\vec{E} = \frac{I_o dl}{4\pi r}(\sin\theta)(\eta)(j\beta)e^{-j\beta r}\hat{a}_\theta$$

$$\vec{H} = \frac{I_o dl}{4\pi r}(\sin\theta)(j\beta)e^{-j\beta r}\hat{a}_\phi$$

$$\eta = \frac{E_\theta}{H_\phi}$$

(a) $U(\theta, \phi) = K\sin^2\theta$ → Omnidirectional antenna.

(b) $R_r = 80\pi^2\left(\frac{dl}{\lambda}\right)^2$ → Radiation Resistance.

(c) $D = \frac{3}{2}\sin^2\theta \rightarrow$ Directive Gain.

(d) $D_o = \frac{3}{2} \rightarrow$ Directivity

16. Radiation Resistance

(a) Hertzian dipole Antenna

$$R_r = 80\pi^2\left(\frac{dl}{\lambda}\right)^2$$

(b) Small dipole antenna.

$$R_r = 20\pi^2\left(\frac{l}{\lambda}\right)^2$$

(c) Half wave dipole antenna.

$R_r = 73\ \Omega$
$Z_{in} = (73 + j42.5)\ \Omega$
$D_o = 1.63$

(d) Quarter water dipole Antenna.

$Z_{in} = (36.5 + j21.25)\ \Omega$
$D_o = 3.26$

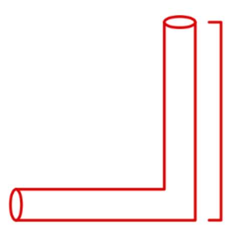
(e) $l = \frac{\lambda}{2} \rightarrow R_r = 73\,\Omega$

$$l = \frac{\lambda}{4} \rightarrow R_r = 36.5\,\Omega$$

$$l = \frac{\lambda}{8} \rightarrow R_r = 18.25\,\Omega$$

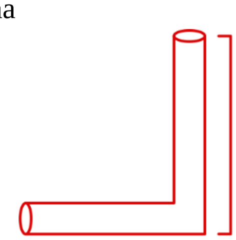
$$l = \frac{\lambda}{16} \rightarrow R_r = 20\pi^2\left(\frac{\frac{\lambda}{16}}{\lambda}\right)^2 = \frac{5\pi^2}{64}$$

(f) Hertzian monopole antenna.



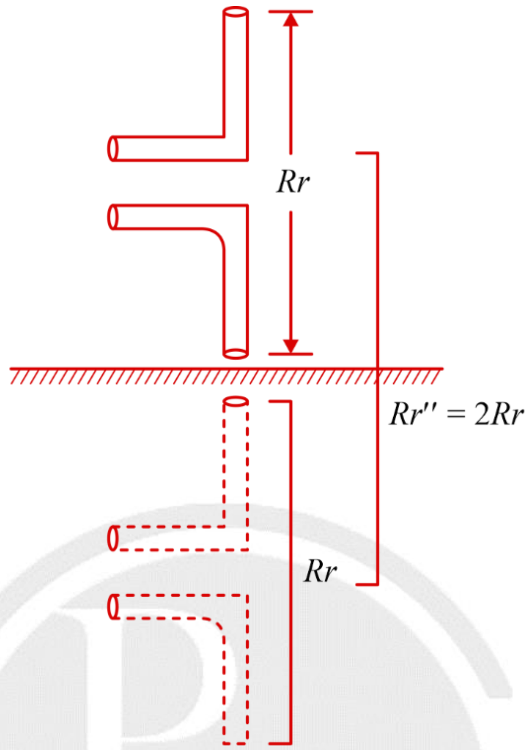
$$R_r' = \frac{R_r}{2} = 40\pi^2\left(\frac{dl}{\lambda}\right)^2$$

(g) Small/Short Monopole Antenna



$$R_r' = 10\pi^2\left(\frac{l}{\lambda}\right)^2$$

(g) Grounded Antenna



(i) Hertzian Grounded dipole antenna

$$R_r'' = 2R_r' = 160\pi^2\left(\frac{dl}{\lambda}\right)^2$$

(j) Hertzian Grounded monopole antenna

$$R_r'' = 2R_r' = 80\pi^2\left(\frac{dl}{\lambda}\right)^2$$

(k) Small/short Grounded dipole Antenna

$$R_r'' = 2R_r = 40\pi^2\left(\frac{l}{\lambda}\right)^2$$

(l) Small/Short Grounded monopole antenna.

$$R_r'' = 2R_r' = 20\pi^2\left(\frac{l}{\lambda}\right)^2$$

(m) Folded Antenna (n-fold)

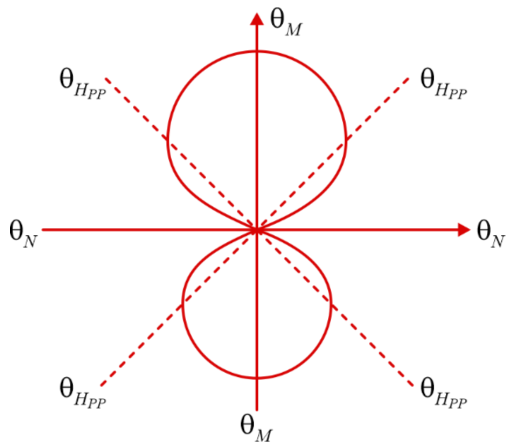
$$R_r' = n^2 R_r$$

R_r' = radiation resistance due to n fold

R_r = radiation resistance due to one fold

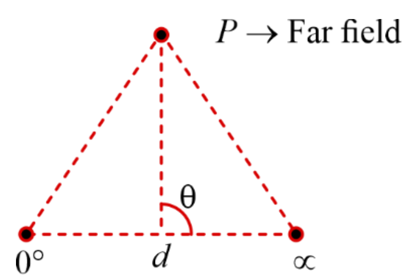
17. Radiation Pattern:

(a) Hertzian Dipole/small dipole



- $\theta_M = \frac{\pi}{2}, \frac{3\pi}{2}$
- $\theta_N = 0, \pi$
- $\theta_{HPP} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
- $\text{HPBW} \equiv \frac{\pi}{2}$, ie. $\theta_H = \frac{\pi}{2}, \theta_E = \frac{\pi}{2}$
- $\Omega_{eff} = \frac{\pi^2}{4}$
- $A_{eff} = \frac{\lambda^2}{\Omega_{eff}} = \frac{4\lambda^2}{\pi^2}$
- $F_{NBW} = \pi$ i.e. $F_{NBW} = 2(\text{HPBW})$

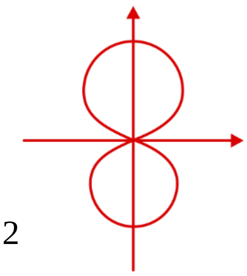
(b) Two element array of antenna



$$|E_P| = 2E_o \left|\cos\left(\frac{\psi}{2}\right)\right|$$

$$\psi = Bd\cos\theta + \propto$$

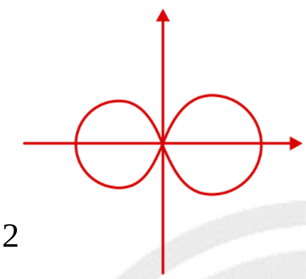
- $d = \frac{\lambda}{2}, \alpha = 0$



Board side array

Number of Lobes = $\frac{4d}{\lambda} = 2$

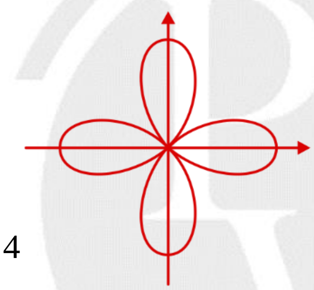
- $d = \frac{\lambda}{2}, \alpha = \pi$



End fire array

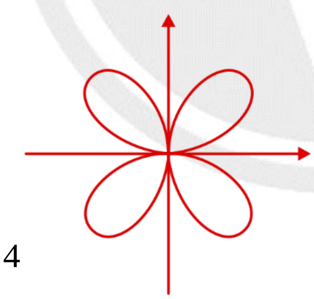
Number of lobes = $\frac{4d}{\lambda} = 2$

- $d = \lambda, \propto = 0$



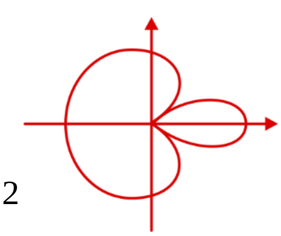
Number of lobes = $\frac{4d}{\lambda} = 4$

- $d = \lambda, \alpha = \pi$



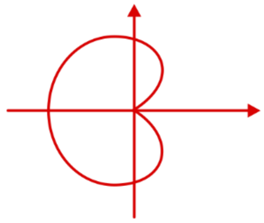
Number of lobes = $\frac{4d}{\lambda} = 4$

- $d = \frac{\lambda}{2}, \alpha = \frac{\pi}{2}$



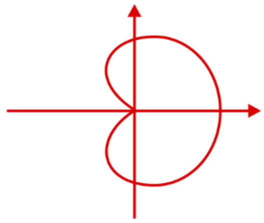
Number of lobes = $\frac{4d}{\lambda} = 2$

- $d = \frac{\lambda}{4}, \alpha = \frac{\pi}{2}$



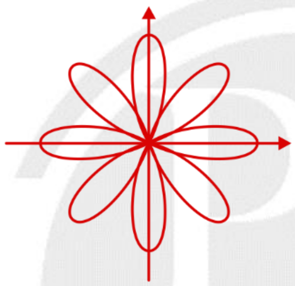
Number of lobes = $\frac{4d}{\lambda} = 1$

- $d = \frac{\lambda}{4}, \alpha = -\frac{\pi}{2}$



Number of lobes = $\frac{4d}{\lambda} = 1$

- $d = 2\lambda, \alpha = 0$



Number of lobes = $\frac{4d}{\lambda} = 8$

Note:-Number of lobes = $\frac{4d}{\lambda}$

(c) N-element Linear array

$$|\vec{E}_P| = E_o \left| \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} \right|$$

(d) Vertical Grounded dipole antenna.

Number of Lobes = $\frac{2h}{\lambda} + 1$

h = distance of dipole antenna from Ground.

(e) Horizontal Grounded dipole antenna

Number of Lobes = $\frac{2h}{\lambda}$

h = distance of dipole antenna from ground.

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