NCERT Solutions for Class 10 Maths Chapter 8 Exercise 8.4: NCERT Solutions for Class 10 Maths Chapter 8 Exercise 8.4 provide a detailed understanding of the application of trigonometric identities to solve various problems.

These solutions are created to help students develop a strong foundation in trigonometry, enabling them to approach problems systematically. By practicing these solutions students can enhance their problem-solving skills and prepare effectively for their board exams.

NCERT Solutions for Class 10 Maths Chapter 8 Exercise 8.4 Overview

Exercise 8.4 of Chapter 8, Introduction to Trigonometry focuses on applying trigonometric concepts to solve problems involving heights and distances. This exercise emphasizes the use of trigonometric ratios such as sine, cosine, and tangent to calculate unknown heights, distances, or angles.

The problems require understanding scenarios described in the questions, drawing appropriate diagrams, and solving them using trigonometric identities and formulas. Key elements of this exercise include:

1. **Angles of Elevation and Depression**: Understanding and calculating angles formed when looking up or down at an object.
2. **Problem-solving**: Using trigonometric ratios to find the height of an object, distance between two points, or other related quantities.
3. **Visual Interpretation**: Translating verbal descriptions into diagrams to aid calculations.

NCERT Solutions for Class 10 Maths Chapter 8 Exercise 8.4 PDF

The **NCERT Solutions for Class 10 Maths Chapter 8 Exercise 8.4** are now available in PDF format, providing step-by-step solutions to all the problems in this exercise. This PDF is designed to help students understand and apply trigonometric concepts effectively, including solving problems involving heights, distances, angles of elevation, and angles of depression. The detailed explanations and solved examples ensure clarity and assist in exam preparation.

You can access the PDF from the link provided below to enhance your understanding and practice at your own pace.

NCERT Solutions for Class 10 Maths Chapter 8 Exercise 8.4 PDF

NCERT Solutions for Class 10 Maths Chapter 8 Introduction to Trigonometry Exercise 8.4

Below is the NCERT Solutions for Class 10 Maths Chapter 8 Introduction to Trigonometry Exercise 8.4

Solve the followings Questions.

1. Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

We know that

$\operatorname{cosec}^2 A = 1 + \cot^2 A$

$$\frac{1}{\cos ec^2 A} = \frac{1}{1+\cot^2 A}$$

$$\sin^2 A = \frac{1}{1+\cot^2 A}$$

$$\sin A = \pm\frac{1}{\sqrt{1+\cot^2 A}}$$

$\sqrt{1+\cot^2 A}$ will always be positive as we are adding two positive quantities.

Therefore

$$\sin A = \frac{1}{\sqrt{1+\cot^2 A}}$$

we know that

$$\tan A = \frac{\sin A}{\cos A}$$

However

$$\cot A = \frac{\cos A}{\sin A}$$

Therfore tan A = 1/cot A

Also $\sec^2 A = 1+\tan^2 A$

$$= 1 + \frac{1}{\cot^2 A}$$

$$= \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

2. Write all the other trigonometric ratios of ∠A in terms of sec A.

Answer:

$\sin^2 A + \cos^2 A = 1$

$\text{cosec}^2 A = 1 + \cot^2 A$

$\sec^2 A = 1 + \tan^2 A$

We know that,

cos A = 1/sec A .....Equation (1)

Also,

$\sin^2 A + \cos^2 A = 1$ (trigonometric identity)

$\sin^2 A = 1 - \cos^2 A$ (By transposing)

Using value of cos A from Equation (1) and simplifying further

$\sin A = \sqrt{}1 - (1 / \sec A)^2$

$= \sqrt{}(\sec^2 A - 1) / \sec^2 A$

$= \sqrt{(\sec^2 A - 1)} / \sec A$ ....Equation (2)

$\tan^2 A + 1 = \sec^2 A$ (Trigonometric identity)

$\tan^2 A = \sec^2 A - 1$ (By transposing)

$\tan A = \sqrt{(\sec^2 A - 1)}$ ... Equation (3)

cot A = cosA/sinA

$= (1/\sec A) / [\sqrt{(\sec^2 A - 1)}/\sec A]$ .....(By substituting the values from Equations (1) and (2))

$= 1 / \sqrt{(\sec^2 A - 1)}$

cosec A = 1/sin A

$= \sec A / \sqrt{(\sec^2 A - 1)}$ (By substituting from Equation (2) and simplifying)

3. Evaluate :

(i) $(\sin^2 63° + \sin^2 27°)/(\cos^2 17° + \cos^2 73°)$
(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

Answer:

(i) $(\sin^2 63° + \sin^2 27°)/(\cos^2 17° + \cos^2 73°)$

$$\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^\circ 73^\circ}$$

$$= \frac{[\sin(90^\circ - 27^\circ)]^2 + \sin^2 27^\circ}{[\cos(90^\circ - 73^\circ)]^2 + \cos^2 73^\circ}$$

$$= \frac{[\cos 27^\circ]^2 + \sin^2 27^\circ}{[\sin 73^\circ]^2 + \cos^2 73^\circ}$$

$$= \frac{\cos^2 27^\circ + \sin^2 27^\circ}{\sin^2 73^\circ + \cos^2 73^\circ}$$

= 1/1 (As $\sin^2 A + \cos^2 A = 1$)

= 1

(ii) sin 25° cos 65° + cos 25° sin 65°

sin25° cos65° + cos25° sin65°

=(sin 25°) {cos(90°-25°)}+cos 25°{sin(90°-25)}

=(sin 25°)(sin 25°) + (cos 25°)(cos 25°)

= $\sin^2 25° + \cos^2 25°$

= 1 (As $\sin^2 A + \cos^2 A = 1$)

Choose the correct option. Justify your choice.

4. (i) 9 $\sec^2 A$ - 9 $\tan^2 A$ =

(A) 1 (B) 9 (C) 8 (D) 0

(ii) (1 + tan θ + sec θ) (1 + cot θ - cosec θ)

(A) 0 (B) 1 (C) 2 (D) - 1

(iii) (secA + tanA) (1 - sinA) =

(A) secA (B) sinA (C) cosecA (D) cosA

(iv) 1+$\tan^2 A$/1+$\cot^2 A$ =

(A) $\sec^2 A$ (B) -1 (C) $\cot^2 A$ (D) $\tan^2 A$

Answer:

(i) **(i) 9 $\sec^2 A$ - 9 $\tan^2 A$ =**

(A) 1 (B) 9 (C) 8 (D) 0

$\sin^2 A + \cos^2 A = 1$

$\text{cosec}^2 A = 1 + \cot^2 A$

$\sec^2 A = 1 + \tan^2 A$

(i) $9 \sec^2 A - 9 \tan^2 A$

$= 9 (\sec^2 A - \tan^2 A)$

$= 9 \times 1$ [By using the identity, $1 + \sec^2 A = \tan^2 A$]

$= 9$

Thus, option (B) is the correct answer.

(ii) (1 + tan θ + sec θ) (1 + cot θ - cosec θ)

(A) 0 (B) 1 (C) 2 (D) - 1

L.H.S $= (1 + \cot \theta - \text{cosec}\, \theta)(1 + \tan \theta + \sec \theta)$

$$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$$

$$= \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)$$

$$= \frac{1}{\sin\theta\cos\theta}\left(\begin{matrix}\sin\theta\cos\theta + \sin^2\theta + \sin\theta + \cos^2\theta \\ + \sin\theta\cos\theta + \cos\theta - \cos\theta - \sin\theta - 1\end{matrix}\right)$$

$$= \frac{1}{\sin\theta\cos\theta}\left(2\sin\theta\cos\theta + \left(\sin^2\theta + \cos^2\theta\right) - 1\right)$$

$$= \frac{1}{\sin\theta\cos\theta}(2\sin\theta\cos\theta + 1 - 1)$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$= 2$

$=$ R.H.S

(iii) (secA + tanA) (1 - sinA) =

(A) secA (B) sinA (C) cosecA (D) cosA

(secA + tanA) (1 − sinA)

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A}$$

= cosA

Hence, alternative cosA is correct

(iv) 1+tan²A/1+cot²A =

(A) $\sec^2 A$ (B) -1 (C) $\cot^2 A$ (D) $\tan^2 A$

$$\text{LHS} = \left(\frac{1 + \tan^2 A}{1 + \cot^2 A}\right) = \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

$$\text{RHS} = \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \left(\frac{1 - \tan A}{1 - \frac{1}{\tan A}}\right)^2$$

$$= \left(\frac{1 - \tan A}{\frac{\tan A - 1}{\tan A}}\right)^2 = \left(\frac{1 - \tan A}{\tan A - 1} \times \tan A\right)^2 = (-\tan A)^2 = \tan^2 A$$

LHS = RHS.

5. Prove the following identities, where the angles involved are acute angles for which the

expressions are defined.

(i) $(\text{cosec } \theta - \cot \theta)^2 = (1-\cos \theta)/(1+\cos \theta)$

(ii) $\cos A/(1+\sin A) + (1+\sin A)/\cos A = 2 \sec A$

(iii) $\tan \theta/(1-\cot \theta) + \cot \theta/(1-\tan \theta) = 1 + \sec \theta \text{ cosec } \theta$

[Hint : Write the expression in terms of sin θ and cos θ]

(iv) $(1 + \sec A)/\sec A = \sin^2 A/(1-\cos A)$

[Hint : Simplify LHS and RHS separately]

(v) $(\cos A – \sin A+1)/(\cos A+\sin A–1) = \text{cosec } A + \cot A$, using the identity $\text{cosec}^2 A = 1+\cot^2 A$.

(vi) $\sqrt{1 + \sin A/1 - \sin A} = \sec A + \tan A$

(vii) $(\sin \theta - 2\sin^3\theta)/(2\cos^3\theta-\cos \theta) = \tan \theta$

(viii) $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2 = 7+\tan^2 A+\cot^2 A$

(ix) $(\text{cosec } A – \sin A)(\sec A – \cos A) = 1/(\tan A+\cot A)$

[Hint : Simplify LHS and RHS separately]

(x) $(1+\tan^2 A/1+\cot^2 A) = (1-\tan A/1-\cot A)^2 = \tan^2 A$

Answer:

(i) $(\text{cosec } \theta - \cot \theta)^2 = (1-\cos \theta)/(1+\cos \theta)$

L.H.S. = $(1 – \cos \theta)/(1 + \cos \theta)$

$= (1 – \cos \theta)/(1 + \cos \theta) \times (1 – \cos \theta)/(1 – \cos \theta)$

$= (1 – \cos \theta)^2/(1 – \cos^2 \theta)$

$= (1 – \cos \theta)^2/\sin^2 \theta$

$= [1 – \cos \theta/\sin \theta]^2$

$= [1 /\sin \theta – \cos \theta/\sin \theta]^2$

$= [\text{cosec } \theta - \cot \theta]^2$

$= [- (\cot \theta - \text{cosec } \theta)]^2$

$= (\cot \theta – \text{cosec } \theta)^2$

= R.H.S.

(ii) $\cos A/(1+\sin A) + (1+\sin A)/\cos A = 2 \sec A$

LHS = Cos A/(1+ sin A) + (1+ sin A)/cos A

$$= \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$$

$$= \frac{\cos^2 A + (1+\sin A)^2}{\cos A(1+\sin A)}$$

$$= \frac{\cos^2 A + 1 + 2\sin A + \sin^2 A}{\cos A(1+\sin A)}$$

$$= \frac{1+1+2\sin A}{\cos A(1+\sin A)}$$

$$= \frac{2(1+\sin A)}{\cos A(1+\sin A)}$$

$$= \frac{2}{\cos A} = 2\sec A$$

(iii) tan θ/(1-cot θ) + cot θ/(1-tan θ) = 1 + sec θ cosec θ

[Hint : Write the expression in terms of sin θ and cos θ]

$$\text{LHS} = \frac{\tan\theta}{(1-\cot\theta)} + \frac{\cot\theta}{(1-\tan\theta)}$$

$$= \frac{\tan\theta}{\left(1-\frac{\cos\theta}{\sin\theta}\right)} + \frac{\cot\theta}{\left(1-\frac{\sin\theta}{\cos\theta}\right)}$$

$$= \frac{\sin\theta\tan\theta}{(\sin\theta-\cos\theta)} + \frac{\cos\theta\cot\theta}{(\cos\theta-\sin\theta)}$$

$$= \frac{\sin\ \theta\ \times \frac{\sin\theta}{\cos\theta}\cos\theta \times \frac{\cos\theta}{\sin\theta}}{(\sin\theta-\cos\theta)}$$

$$= \frac{\frac{\sin^2\theta}{\cos\theta}\frac{\cos^2\theta}{\sin\theta}}{(\sin\theta-\cos\theta)}$$

$$\sin^3\theta - \cos^3\theta$$

$$=\frac{\sin^3\theta-\cos^3\theta}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$$

$$=\frac{(\sin\theta-\cos\theta)\left(\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta\right)}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$$

$$=\frac{1+\sin\theta\cos\theta}{\cos\theta\sin\theta}$$

$$=\frac{1}{\cos\theta\sin\theta}+\frac{\sin\theta\cos\theta}{\cos\theta\ \sin\theta}$$

$$=\frac{1}{\cos\theta\sin\theta}+\frac{\sin\theta\cos\theta}{\cos\theta\sin\theta}$$

$$=\sec\theta\cos ec\ \theta+1$$

$$=1+\sec\theta\ \cos ec\ \theta$$

=RHS

(iv) (1 + sec A)/sec A = $\sin^2$A/(1-cos A)

[Hint : Simplify LHS and RHS separately]

We know that, $\sin^2\theta+\cos^2\theta=1$

$$\frac{1+\sec\theta}{\sec\theta}=\frac{1+\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}}$$

$$=\frac{\frac{\cos\theta+1}{\cos\theta}}{\frac{1}{\cos\theta}}$$

$$=\frac{1+\cos\theta}{1}$$

Multiplying the numerator and denominator by $(1-\cos\theta)$ we have

$$\frac{1+\sec\theta}{\sec\theta} = \frac{(1+\cos\theta)(1-\cos\theta)}{1-\cos\theta}$$

$$= \frac{1-\cos^2\theta}{1-\cos\theta}$$

$$= \frac{\sin^2\theta}{1-\cos\theta}$$

(v) (cos A–sin A+1)/(cos A+sin A–1) = cosec A + cot A,using the identity $cosec^2A = 1+cot^2A$

L.H.S = cos A - sin A + 1/(cos A + sin A - 1)

Diving both numerator and denominator by sin A

= [cos A/sin A - sin A/sin A + 1/sin A]/[cos A/sin A + sin A/sin A - 1/sin A]

We know that the trigonometric functions,

cot (x) = cos(x)/sin (x) = 1/tan (x)

cosec(x) = 1/sin (x)

We get,

⇒ (cot A - 1 + cosec A) / (cot A + 1 - cosec A)

⇒ cot A - (1 - cosec A) / cot A+ (1 - cosec A)

We know that, 1 + cot^2 A = $Cosec^2$ A

Hence multiplying [cot A - (1 - cosec A)] in numerator and denominator

$= [(\cot A) - (1 - \operatorname{cosec} A) \times (\cot A) - (1 - \operatorname{cosec} A)]/[(\cot A) + (1 - \operatorname{cosec} A) - \times (\cot A) - (1 - \operatorname{cosec} A)]$

$= [\cot A - (1 - \operatorname{cosec} A)]^2/[(\cot A)^2 - (1 - \operatorname{cosec} A)^2]$

$= [\cot^2 A + (1 - \operatorname{cosec} A)^2 - 2\cot A(1 - \operatorname{cosec} A)]/[\cot^2 A - (1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A)]$

$= (\cot^2 A + 1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A - 2\cot A + 2\cot A \operatorname{cosec} A)/(\cot^2 A - (1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A))$

$= (2\operatorname{cosec}^2 A + 2\cot A \operatorname{cosec} A - 2\cot A - 2\operatorname{cosec} A)/(\cot^2 A - 1 - \operatorname{cosec}^2 A + 2\operatorname{cosec} A)$

$= 2\operatorname{cosec} A(\operatorname{cosec} A + \cot A) - 2(\cot A + \operatorname{cosec} A)/(\cot^2 A - \operatorname{cosec}^2 A - 1 + 2\operatorname{cosec} A)$

$= (\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)/(-1 - 1 + 2\operatorname{cosec} A)$

$= (\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)/(2\operatorname{cosec} A - 2)$

$= \operatorname{cosec} A + \cot A$

= R.H.S

(vi) √1 + sin A/1 - sin A = sec A+ tan A

LHS = 1 + sin A/(1 - sin A) .....(1)

Multiplying and dividing by (1 + sin A)

⇒ (1 + sin A)(1 + sin A/1 - sin A)(1 + sin A)

$= (1 + \sin A)^2/(1 - \sin^2 A)$ $[a^2 - b^2 = (a - b)(a + b)]$

$= (1 + \sin A)/1 - \sin^2 A$

$= 1 + \sin A/\cos^2 A$

= 1 + sin A/cos A

= 1/cos A + sin A/cos A

= sec A + tan A

= R.H.S

(vii) $(\sin\theta - 2\sin^3\theta)/(2\cos^3\theta - \cos\theta) = \tan\theta$

L.H.S = $(\sin\theta - 2\sin^3\theta)/(2\cos\theta - \cos\theta)$

Taking Sin θ and Cos θ common in both numerator and denominator respectively.

$\sin\theta\,(1 - 2\sin^2\theta)/\cos\theta\,(2\cos^2\theta - 1)$

By Identity $\sin^2 A + \cos^2 A = 1$ hence, $\cos^2 A = 1 - \sin^2 A$ and substituting this in the above equation,

$\Rightarrow \sin\theta\,(1 - 2\sin^2\theta)/\cos\theta\,\{2(1 - \sin^2\theta) - 1\}$

$= \sin\theta\,(1 - 2\sin^2\theta)/\cos\theta(2 - 2\sin^2\theta - 1)$

$= \sin\theta\,(1 - 2\sin^2\theta)/\cos\theta(1 - 2\sin^2\theta)$

$= \sin\theta/\cos\theta$

$= \tan\theta$

= R.H.S

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7+\tan^2 A+\cot^2 A$

L.H.S =-$(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2$

By using $(a + b)^2 = a^2 + 2ab + b^2$

$\Rightarrow \sin^2 A + \text{cosec}^2 A + 2\sin A \text{ cosec } A + \cos^2 A + \sec^2 A + 2\cos A \sec A$

By rearranging and using sec A = 1/cos A and cosec A = 1/sin A

$\Rightarrow (\sin^2 A + \cos^2 A) + (\text{cosec}^2 A + \sec^2 A) + 2 \sin A (1/\sin A) +-2\cos A (1/\cos A)$

Hence $(\sin^2 A + \cos^2 A) = 1$, $\text{cosec}^2 A = (1 + \cot^2 A)$ and $(\sec^2 A - \tan^2 A) = 1$

$\Rightarrow 1 + 1 + \cot^2 A + 1 + \tan^2 A + 2 + 2$

$= 7 + \tan^2 A + \cot^2 A$

= R.H.S

(ix) (cosec A – sin A)(sec A – cos A) = 1/(tan A+cotA)

[Hint : Simplify LHS and RHS separately]

L.H.S = (cosec A - sin A)(sec A - cos A) .....Equation (1)

We know that the trigonometric functions,

sec (x) = 1/cos (x)

cosec (x) = 1/sin (x)

By substituting the above relations in Equation (1)

⇒ (1/sin A - sin A)(1/cos A - cos A)

= $(1 - \sin^2 A)/\sin A \times (1 - \cos^2 A)/\cos A$

= $\cos^2 A \sin^2 A/\sin A \cos A$

= sin A cos A/1

= $\sin A \cos A/(\sin^2 A + \cos^2 A)$ [$(\sin^2 A + \cos^2 A) = 1$]

= $1/\sin^2 A + \cos^2 A$ [Dividing numerator and denominator by (sin A cos A)]

= $1/ [(\sin^2 A/\sin A \cos A) + (\cos^2 A/\sin A \cos A)]$

= 1/[(sin A/cos A) + (cos A/sin A)]

= 1/(tan A + cot A)

= RHS

(x) $(1+\tan^2 A/1+\cot^2 A) = (1-\tan A/1-\cot A)^2 = \tan^2 A$

Taking LHS, $(1 + \tan^2 A)/(1 + \cot^2 A)$

$= \sec^2 A/\text{cosec}^2 A$

$= (1/\cos^2 A)/(\sin^2 A)$

$= (1/\cos^2 A) \times (\sin^2 A/1)$

$= \tan^2 A$

= RHS

Taking,-$(1 - \tan A/1 - \cot A)^2$

$= [(1 - \tan A)/(1 - 1/\tan A)]^2$

$= [(1 - \tan A)/(\tan A - 1)/\tan A]^2$

$= ((1 - \tan A) \times \tan A(\tan A - 1))^2$

$=(-\tan A)^2$

$= \tan^2 A$

= R.H.S

Hence, L.H.S = R.H.S.

Benefits of Solving NCERT Solutions for Class 10 Maths Chapter 8 Exercise 8.4

- **Conceptual Clarity**: By solving the problems in Exercise 8.4, students gain a deeper understanding of trigonometric ratios, particularly in relation to heights and distances. This solidifies their grasp of essential concepts such as angles of elevation and depression, helping them apply these concepts accurately in different scenarios.
- **Improved Problem-Solving Skills**: The exercise provides a range of problems that require logical thinking and application of trigonometric formulas. By practicing these, students develop problem-solving skills and learn to approach mathematical challenges methodically.
- **Boosts Confidence**: As students work through the problems and understand the step-by-step solutions, they build confidence in their ability to tackle similar questions in exams, reducing exam anxiety.

- **Helps in Exam Preparation**: The NCERT solutions for Exercise 8.4 are structured to match the exam pattern. Solving these problems regularly helps students familiarize themselves with the type of questions that might appear in board exams, making them well-prepared for their exams.
- **Better Time Management**: By practicing with these solutions, students learn to solve problems within a given time frame, which is a crucial skill during exams. This helps them manage time effectively and solve questions efficiently.
- **Comprehensive Understanding**: The step-by-step solutions in the NCERT PDF make it easier to grasp the process of solving trigonometric problems, ensuring a thorough understanding of the topic and its applications.
- **Revision Aid**: These solutions are a great resource for revision, allowing students to quickly refresh their memory of important formulas, concepts, and methods before exams.