

**RD Sharma Solutions Class 10 Maths Chapter 1 Exercise 1.1:** RD Sharma Solutions for Class 10 Maths Chapter 1 Exercise 1.1 provides a solid foundation in understanding real numbers and their properties. This exercise covers essential concepts like the Euclidean division lemma, prime factorization and the Fundamental Theorem of Arithmetic.

By practicing these solutions students can enhance their problem-solving skills focusing on understanding the structure and classification of numbers. The step-by-step explanations in RD Sharma solutions help simplify complex problems, making it easier for students to grasp each concept thoroughly. Working through Exercise 1.1 builds a strong base for the more advanced topics in mathematics, ensuring students are well-prepared for exams and real-world applications.

## **RD Sharma Solutions Class 10 Maths Chapter 1 Exercise 1.1 Overview**

RD Sharma Solutions for Class 10 Maths Chapter 1 Exercise 1.1 are prepared by subject experts of Physics Wallah provides students with a clear and structured approach to understanding the fundamentals of real numbers.

The Physics Wallah experts have created these solutions to simplify complex concepts with step-by-step explanations, helping students build confidence and improve their problem-solving skills.

## **RD Sharma Solutions Class 10 Maths Chapter 1 Exercise 1.1 PDF**

RD Sharma Solutions for Class 10 Maths Chapter 1 Exercise 1.1 PDF provides a detailed guide to mastering the basics of real numbers including essential concepts like the Euclidean division lemma and prime factorization.

With clear explanations and structured problem-solving techniques the PDF helps students understand and apply the Fundamental Theorem of Arithmetic ensuring thorough preparation for exams. You can access the PDF link below to solve these expertly created solutions in detail.

**RD Sharma Solutions Class 10 Maths Chapter 1 Exercise 1.1 PDF**

## **RD Sharma Solutions Class 10 Maths Chapter 1 Real Numbers Exercise 1.1**

Below is the RD Sharma Solutions Class 10 Maths Chapter 1 Real Numbers Exercise 1.1 -

**1. If  $a$  and  $b$  are two odd positive integers, such that  $a > b$ , then prove that one of the two numbers  $(a+b)/2$  and  $(a-b)/2$  is odd, and the other is even.**

**Solution:**

We know that any odd positive integer is of form  $4q+1$ , or  $4q+3$  for some whole number  $q$ .

Now that it's given  $a > b$

So, we can choose  $a = 4q+3$  and  $b = 4q+1$ .

$$\therefore (a+b)/2 = [(4q+3) + (4q+1)]/2$$

$$\Rightarrow (a+b)/2 = (8q+4)/2$$

$$\Rightarrow (a+b)/2 = 4q+2 = 2(2q+1) \text{ which is clearly an even number.}$$

Now, doing  $(a-b)/2$

$$\Rightarrow (a-b)/2 = [(4q+3)-(4q+1)]/2$$

$$\Rightarrow (a-b)/2 = (4q+3-4q-1)/2$$

$$\Rightarrow (a-b)/2 = (2)/2$$

$$\Rightarrow (a-b)/2 = 1, \text{ which is an odd number.}$$

Hence, one of the two numbers  $(a+b)/2$  and  $(a-b)/2$  is odd, and the other is even.

**2. Prove that the product of two consecutive positive integers is divisible by 2.**

**Solution:**

Let's consider two consecutive positive integers as  $(n-1)$  and  $n$ .

$$\therefore \text{Their product} = (n-1) n$$

$$= n^2 - n$$

And then, we know that any positive integer is of form  $2q$  or  $2q+1$ . (From Euclid's division lemma for  $b = 2$ )

So, when  $n = 2q$

We have,

$$\Rightarrow n^2 - n = (2q)^2 - 2q$$

$$\Rightarrow n^2 - n = 4q^2 - 2q$$

$$\Rightarrow n^2 - n = 2(2q^2 - q)$$

Thus,  $n^2 - n$  is divisible by 2.

Now, when  $n = 2q+1$

We have,

$$\Rightarrow n^2 - n = (2q+1)^2 - (2q+1)$$

$$\Rightarrow n^2 - n = (4q^2 + 4q + 1 - 2q - 1)$$

$$\Rightarrow n^2 - n = (4q^2 + 2q)$$

$$\Rightarrow n^2 - n = 2(2q^2 + q)$$

Thus,  $n^2 - n$  is divisible by 2 again.

Hence, the product of two consecutive positive integers is divisible by 2.

### **3. Prove that the product of three consecutive positive integers is divisible by 6.**

**Solution:**

Let  $n$  be any positive integer.

Thus, the three consecutive positive integers are  $n$ ,  $n+1$  and  $n+2$ .

We know that any positive integer can be of form  $6q$ , or  $6q+1$ ,  $6q+2$ ,  $6q+3$ ,  $6q+4$ , or  $6q+5$ .  
(From Euclid's division lemma for  $b=6$ ).

So,

For  $n = 6q$ ,

$$\Rightarrow n(n+1)(n+2) = 6q(6q+1)(6q+2)$$

$$\Rightarrow n(n+1)(n+2) = 6[q(6q+1)(6q+2)]$$

$$\Rightarrow n(n+1)(n+2) = 6m, \text{ which is divisible by 6. } [m = q(6q+1)(6q+2)]$$

For  $n = 6q+1$ ,

$$\Rightarrow n(n+1)(n+2) = (6q+1)(6q+2)(6q+3)$$

$$\Rightarrow n(n+1)(n+2) = 6[(6q+1)(3q+1)(2q+1)]$$

$$\Rightarrow n(n+1)(n+2) = 6m, \text{ which is divisible by 6. } [m = (6q+1)(3q+1)(2q+1)]$$

For  $n = 6q+2$ ,

$$\Rightarrow n(n+1)(n+2) = (6q+2)(6q+3)(6q+4)$$

$$\Rightarrow n(n+1)(n+2) = 6[(3q+1)(2q+1)(6q+4)]$$

$$\Rightarrow n(n+1)(n+2) = 6m, \text{ which is divisible by 6. } [m = (3q+1)(2q+1)(6q+4)]$$

For  $n = 6q+3$ ,

$$\Rightarrow n(n+1)(n+2) = (6q+3)(6q+4)(6q+5)$$

$$\Rightarrow n(n+1)(n+2) = 6[(2q+1)(3q+2)(6q+5)]$$

$$\Rightarrow n(n+1)(n+2) = 6m, \text{ which is divisible by 6. } [m = (2q+1)(3q+2)(6q+5)]$$

For  $n = 6q+4$ ,

$$\Rightarrow n(n+1)(n+2) = (6q+4)(6q+5)(6q+6)$$

$$\Rightarrow n(n+1)(n+2) = 6[(3q+2)(3q+1)(2q+2)]$$

$$\Rightarrow n(n+1)(n+2) = 6m, \text{ which is divisible by 6. } [m = (3q+2)(3q+1)(2q+2)]$$

For  $n = 6q+5$ ,

$$\Rightarrow n(n+1)(n+2) = (6q+5)(6q+6)(6q+7)$$

$$\Rightarrow n(n+1)(n+2) = 6[(6q+5)(q+1)(6q+7)]$$

$$\Rightarrow n(n+1)(n+2) = 6m, \text{ which is divisible by 6. } [m = (6q+5)(q+1)(6q+7)]$$

Hence, the product of three consecutive positive integers is divisible by 6.

#### 4. For any positive integer $n$ , prove that $n^3 - n$ is divisible by 6.

**Solution:**

Let  $n$  be any positive integer. And any positive integer can be of form  $6q$ , or  $6q+1$ ,  $6q+2$ ,  $6q+3$ ,  $6q+4$ , or  $6q+5$ . (From Euclid's division lemma for  $b=6$ )

$$\text{We have } n^3 - n = n(n^2 - 1) = (n-1)n(n+1),$$

For  $n = 6q$ ,

$$\Rightarrow (n-1)n(n+1) = (6q-1)(6q)(6q+1)$$

$$\Rightarrow (n-1)n(n+1) = 6[(6q-1)q(6q+1)]$$

$$\Rightarrow (n-1)n(n+1) = 6m, \text{ which is divisible by 6. } [m = (6q-1)q(6q+1)]$$

For  $n = 6q+1$ ,

$$\Rightarrow (n-1)n(n+1) = (6q)(6q+1)(6q+2)$$

$$\Rightarrow (n-1)n(n+1) = 6[q(6q+1)(6q+2)]$$

$$\Rightarrow (n-1)n(n+1) = 6m, \text{ which is divisible by 6. } [m = q(6q+1)(6q+2)]$$

For  $n = 6q+2$ ,

$$\Rightarrow (n-1)n(n+1) = (6q+1)(6q+2)(6q+3)$$

$$\Rightarrow (n-1)n(n+1) = 6[(6q+1)(3q+1)(2q+1)]$$

$$\Rightarrow (n-1)n(n+1) = 6m, \text{ which is divisible by 6. } [m = (6q+1)(3q+1)(2q+1)]$$

For  $n = 6q+3$ ,

$$\Rightarrow (n-1)n(n+1) = (6q+2)(6q+3)(6q+4)$$

$$\Rightarrow (n-1)n(n+1) = 6[(3q+1)(2q+1)(6q+4)]$$

$$\Rightarrow (n-1)n(n+1) = 6m, \text{ which is divisible by 6. } [m = (3q+1)(2q+1)(6q+4)]$$

For  $n = 6q+4$ ,

$$\Rightarrow (n-1)n(n+1) = (6q+3)(6q+4)(6q+5)$$

$$\Rightarrow (n-1)n(n+1) = 6[(2q+1)(3q+2)(6q+5)]$$

$$\Rightarrow (n-1)n(n+1) = 6m, \text{ which is divisible by 6. } [m = (2q+1)(3q+2)(6q+5)]$$

For  $n = 6q+5$ ,

$$\Rightarrow (n-1)n(n+1) = (6q+4)(6q+5)(6q+6)$$

$$\Rightarrow (n-1)n(n+1) = 6[(6q+4)(6q+5)(q+1)]$$

$$\Rightarrow (n-1)n(n+1) = 6m, \text{ which is divisible by 6. } [m = (6q+4)(6q+5)(q+1)]$$

Hence, for any positive integer  $n$ ,  $n^3 - n$  is divisible by 6.

**5. Prove that if a positive integer is of form  $6q + 5$ , then it is of form  $3q + 2$  for some integer  $q$ , but not conversely.**

**Solution:**

Let  $n = 6q+5$  be a positive integer for some integer  $q$ .

We know that any positive integer can be of form  $3k$ , or  $3k+1$ , or  $3k+2$ .

$\therefore q$  can be  $3k$  or,  $3k+1$  or,  $3k+2$ .

If  $q = 3k$ , then

$$\Rightarrow n = 6q+5$$

$$\Rightarrow n = 6(3k)+5$$

$$\Rightarrow n = 18k+5 = (18k+3)+2$$

$$\Rightarrow n = 3(6k+1)+2$$

$$\Rightarrow n = 3m+2, \text{ where } m \text{ is some integer.}$$

If  $q = 3k+1$ , then

$$\Rightarrow n = 6q+5$$

$$\Rightarrow n = 6(3k+1)+5$$

$$\Rightarrow n = 18k+6+5 = (18k+9)+2$$

$$\Rightarrow n = 3(6k+3)+2$$

$$\Rightarrow n = 3m+2, \text{ where } m \text{ is some integer.}$$

If  $q = 3k+2$ , then

$$\Rightarrow n = 6q+5$$

$$\Rightarrow n = 6(3k+2)+5$$

$$\Rightarrow n = 18k+12+5 = (18k+15)+2$$

$$\Rightarrow n = 3(6k+5)+2$$

$$\Rightarrow n = 3m+2, \text{ where } m \text{ is some integer.}$$

Hence, if a positive integer is of form  $6q + 5$ , then it is of form  $3q + 2$  for some integer  $q$ .

Conversely,

Let  $n = 3q+2$

And we know that a positive integer can be of form  $6k$ , or  $6k+1$ ,  $6k+2$ ,  $6k+3$ ,  $6k+4$ , or  $6k+5$ .

So, if  $q = 6k+1$ , then

$$\Rightarrow n = 3q+2$$

$$\Rightarrow n = 3(6k+1)+2$$

$$\Rightarrow n = 18k + 5$$

$$\Rightarrow n = 6m+5, \text{ where } m \text{ is some integer.}$$

So, if  $q = 6k+2$ , then

$$\Rightarrow n = 3q+2$$

$$\Rightarrow n = 3(6k+2)+2$$

$$\Rightarrow n = 18k + 6 + 2 = 18k+8$$

$$\Rightarrow n = 6(3k + 1) + 2$$

$$\Rightarrow n = 6m+2, \text{ where } m \text{ is some integer}$$

Now, this is not of form  $6q + 5$ .

Therefore, if  $n$  is of form  $3q + 2$ , then it is necessary won't be of form  $6q + 5$ .

**6. Prove that the square of any positive integer of form  $5q + 1$  is of the same form.**

**Solution:**

Here, the integer ' $n$ ' is of form  $5q+1$ .

$$\Rightarrow n = 5q+1$$

On squaring it,

$$\Rightarrow n^2 = (5q+1)^2$$

$$\Rightarrow n^2 = (25q^2+10q+1)$$

$$\Rightarrow n^2 = 5(5q^2+2q)+1$$

$$\Rightarrow n^2 = 5m+1, \text{ where } m \text{ is some integer. [For } m = 5q^2+2q\text{].}$$

Therefore, the square of any positive integer of form  $5q + 1$  is of the same form.

**7. Prove that the square of any positive integer is of the form  $3m$  or  $3m + 1$  but not of the form  $3m + 2$ .**

**Solution:**

Let any positive integer 'n' be of form  $3q$  or  $3q+1$  or  $3q+2$ . (From Euclid's division lemma for  $b = 3$ )

If  $n = 3q$ ,

Then, on squaring

$$\Rightarrow n^2 = (3q)^2 = 9q^2$$

$$\Rightarrow n^2 = 3(3q^2)$$

$$\Rightarrow n^2 = 3m, \text{ where } m \text{ is some integer } [m = 3q^2]$$

If  $n = 3q+1$ ,

Then, on squaring

$$\Rightarrow n^2 = (3q+1)^2 = 9q^2 + 6q + 1$$

$$\Rightarrow n^2 = 3(3q^2 + 2q) + 1$$

$$\Rightarrow n^2 = 3m + 1, \text{ where } m \text{ is some integer } [m = 3q^2 + 2q].$$

If  $n = 3q+2$ ,

Then, on squaring

$$\Rightarrow n^2 = (3q+2)^2 = 9q^2 + 12q + 4$$

$$\Rightarrow n^2 = 3(3q^2 + 4q + 1) + 1$$

$$\Rightarrow n^2 = 3m + 1, \text{ where } m \text{ is some integer } [m = 3q^2 + 4q + 1].$$

Thus, it is observed that the square of any positive integer is of the form  $3m$  or  $3m + 1$  but not of the form  $3m + 2$ .

**8. Prove that the square of any positive integer is of form  $4q$  or  $4q + 1$  for some integer  $q$ .**

**Solution:**

Let 'a' be any positive integer.

Then,

According to Euclid's division lemma,

$$a = bq + r$$

According to the question, when  $b = 4$ .

$$a = 4k + r, 0 < r < 4$$

When  $r = 0$ , we get,  $a = 4k$

$$a^2 = 16k^2 = 4(4k^2) = 4q, \text{ where } q = 4k^2$$

When  $r = 1$ , we get,  $a = 4k + 1$

$$a^2 = (4k + 1)^2 = 16k^2 + 1 + 8k = 4(4k^2 + 2k) + 1 = 4q + 1, \text{ where } q = k(4k + 2)$$

When  $r = 2$ , we get,  $a = 4k + 2$

$$a^2 = (4k + 2)^2 = 16k^2 + 4 + 16k = 4(4k^2 + 4k + 1) = 4q, \text{ where } q = 4k^2 + 4k + 1$$

When  $r = 3$ , we get,  $a = 4k + 3$

$$a^2 = (4k + 3)^2 = 16k^2 + 9 + 24k = 4(4k^2 + 6k + 2) + 1$$

$$= 4q + 1, \text{ where } q = 4k^2 + 6k + 2$$

Therefore, the square of any positive integer is either of the form  $4q$  or  $4q + 1$  for some integer  $q$ .

**9. Prove that the square of any positive integer is of form  $5q$  or  $5q + 1$ ,  $5q + 4$  for some integer  $q$ .**

**Solution:**

Let 'a' be any positive integer.

Then,

According to Euclid's division lemma,

$$a = bq + r$$

According to the question, when  $b = 5$ .

$$a = 5k + r, 0 < r < 5$$

When  $r = 0$ , we get  $a = 5k$

$$a^2 = 25k^2 = 5(5k^2) = 5q, \text{ where } q = 5k^2$$

When  $r = 1$ , we get  $a = 5k + 1$

$$a^2 = (5k + 1)^2 = 25k^2 + 1 + 10k = 5k(5k + 2) + 1 = 5q + 1, \text{ where } q = k(5k + 2)$$

When  $r = 2$ , we get  $a = 5k + 2$

$$a^2 = (5k + 2)^2 = 25k^2 + 4 + 20k = 5(5k^2 + 4k) + 4 = 4q + 4, \text{ where } q = 5k^2 + 4k$$

When  $r = 3$ , we get  $a = 5k + 3$

$$a^2 = (5k + 3)^2 = 25k^2 + 9 + 30k = 5(5k^2 + 6k + 1) + 4$$

$$= 5q + 4, \text{ where } q = 5k^2 + 6k + 1$$

When  $r = 4$ , we get  $a = 5k + 4$

$$a^2 = (5k + 4)^2 = 25k^2 + 16 + 40k = 5(5k^2 + 8k + 3) + 1$$

$$= 5q + 1, \text{ where } q = 5k^2 + 8k + 3$$

Therefore, the square of any positive integer is of form  $5q$  or  $5q + 1$  or  $5q + 4$  for some integer  $q$ .

**10. Show that the square of an odd integer is of form  $8q + 1$  for some integer  $q$ .**

**Solution:**

From Euclid's division lemma,

$$a = bq + r; \text{ where } 0 < r < b$$

Putting  $b = 4$  for the question,

$$\Rightarrow a = 4q + r, 0 < r < 4$$

For  $r = 0$ , we get  $a = 4q$ , which is an even number.

For  $r = 1$ , we get  $a = 4q + 1$ , which is an odd number.

On squaring,

$$\Rightarrow a^2 = (4q + 1)^2 = 16q^2 + 1 + 8q = 8(2q^2 + q) + 1 = 8m + 1, \text{ where } m = 2q^2 + q$$

For  $r = 2$ , we get  $a = 4q + 2 = 2(2q + 1)$ , which is an even number.

For  $r = 3$ , we get  $a = 4q + 3$ , which is an odd number.

On squaring,

$$\Rightarrow a^2 = (4q + 3)^2 = 16q^2 + 9 + 24q = 8(2q^2 + 3q + 1) + 1$$

$$= 8m + 1, \text{ where } m = 2q^2 + 3q + 1$$

Thus, the square of an odd integer is of form  $8q + 1$  for some integer  $q$ .

**11. Show that any positive odd integer is of the form  $6q + 1$  or  $6q + 3$  or  $6q + 5$ , where  $q$  is some integer.**

**Solution:**

Let 'a' be any positive integer.

Then from Euclid's division lemma,

$$a = bq + r; \text{ where } 0 < r < b$$

Putting  $b = 6$ , we get

$$\Rightarrow a = 6q + r, 0 < r < 6$$

For  $r = 0$ , we get  $a = 6q = 2(3q) = 2m$ , which is an even number. [ $m = 3q$ ]

For  $r = 1$ , we get  $a = 6q + 1 = 2(3q) + 1 = 2m + 1$ , which is an **odd** number. [ $m = 3q$ ]

For  $r = 2$ , we get  $a = 6q + 2 = 2(3q + 1) = 2m$ , which is an even number. [ $m = 3q + 1$ ]

For  $r = 3$ , we get  $a = 6q + 3 = 2(3q + 1) + 1 = 2m + 1$ , which is an **odd** number. [ $m = 3q + 1$ ]

For  $r = 4$ , we get  $a = 6q + 4 = 2(3q + 2) = 2m$ , which is an even number. [ $m = 3q + 2$ ]

For  $r = 5$ , we get  $a = 6q + 5 = 2(3q + 2) + 1 = 2m + 1$ , which is an **odd** number. [ $m = 3q + 2$ ]

Thus, from the above, it can be seen that any positive odd integer can be of the form  $6q + 1$  or  $6q + 3$  or  $6q + 5$ , where  $q$  is some integer.

**12. Show that the square of any positive integer cannot be of the form  $6m + 2$  or  $6m + 5$  for any integer  $m$ .**

**Solution:**

Let the positive integer = a

According to Euclid's division algorithm,

$a = 6q + r$ , where  $0 \leq r < 6$

$$a^2 = (6q + r)^2 = 36q^2 + r^2 + 12qr \quad [ \because (a+b)^2 = a^2 + 2ab + b^2 ]$$

$$a^2 = 6(6q^2 + 2qr) + r^2 \quad \dots (i), \text{ where } 0 \leq r < 6$$

When  $r = 0$ , substituting  $r = 0$  in Eq.(i), we get

$$a^2 = 6(6q^2) = 6m, \text{ where, } m = 6q^2 \text{ is an integer.}$$

When  $r = 1$ , substituting  $r = 1$  in Eq.(i), we get

$$a^2 = 6(6q^2 + 2q) + 1 = 6m + 1, \text{ where, } m = (6q^2 + 2q) \text{ is an integer.}$$

When  $r = 2$ , substituting  $r = 2$  in Eq.(i), we get

$$a^2 = 6(6q^2 + 4q) + 4 = 6m + 4, \text{ where, } m = (6q^2 + 4q) \text{ is an integer.}$$

When  $r = 3$ , substituting  $r = 3$  in Eq.(i), we get

$$a^2 = 6(6q^2 + 6q) + 9 = 6(6q^2 + 6q) + 6 + 3$$

$$a^2 = 6(6q^2 + 6q + 1) + 3 = 6m + 3, \text{ where, } m = (6q^2 + 6q + 1) \text{ is integer.}$$

When  $r = 4$ , substituting  $r = 4$  in Eq.(i), we get

$$a^2 = 6(6q^2 + 8q) + 16$$

$$= 6(6q^2 + 8q) + 12 + 4$$

$$\Rightarrow a^2 = 6(6q^2 + 8q + 2) + 4 = 6m + 4, \text{ where, } m = (6q^2 + 8q + 2) \text{ is integer.}$$

When  $r = 5$ , substituting  $r = 5$  in Eq.(i), we get

$$a^2 = 6(6q^2 + 10q) + 25 = 6(6q^2 + 10q) + 24 + 1$$

$$a^2 = 6(6q^2 + 10q + 4) + 1 = 6m + 1, \text{ where, } m = (6q^2 + 10q + 4) \text{ is integer.}$$

Hence, the square of any positive integer cannot be of the form  $6m + 2$  or  $6m + 5$  for any integer  $m$ .

Hence Proved.

**13. Show that the cube of a positive integer of form  $6q + r$ ,  $q$  is an integer and  $r = 0, 1, 2, 3, 4, 5$  is also of the form  $6m + r$ .**

**Solution:**

Given,  $6q + r$  is a positive integer, where  $q$  is an integer and  $r = 0, 1, 2, 3, 4, 5$

Then, the positive integers are of form  $6q, 6q+1, 6q+2, 6q+3, 6q+4$  and  $6q+5$ .

Taking cube on L.H.S and R.H.S,

For  $6q$ ,

$$(6q)^3 = 216 q^3 = 6(36q)^3 + 0$$

$$= 6m + 0, \text{ (where } m \text{ is an integer } = (36q)^3)$$

For  $6q+1$ ,

$$(6q+1)^3 = 216q^3 + 108q^2 + 18q + 1$$

$$= 6(36q^3 + 18q^2 + 3q) + 1$$

$$= 6m + 1, \text{ (where } m \text{ is an integer } = 36q^3 + 18q^2 + 3q)$$

For  $6q+2$ ,

$$(6q+2)^3 = 216q^3 + 216q^2 + 72q + 8$$

$$= 6(36q^3 + 36q^2 + 12q + 1) + 2$$

$$= 6m + 2, \text{ (where } m \text{ is an integer } = 36q^3 + 36q^2 + 12q + 1)$$

For  $6q+3$ ,

$$(6q+3)^3 = 216q^3 + 324q^2 + 162q + 27$$

$$= 6(36q^3 + 54q^2 + 27q + 4) + 3$$

$$= 6m + 3, \text{ (where } m \text{ is an integer } = 36q^3 + 54q^2 + 27q + 4)$$

For  $6q+4$ ,

$$(6q+4)^3 = 216q^3 + 432q^2 + 288q + 64$$

$$= 6(36q^3 + 72q^2 + 48q + 10) + 4$$

$$= 6m + 4, \text{ (where } m \text{ is an integer } = 36q^3 + 72q^2 + 48q + 10)$$

For  $6q+5$ ,

$$(6q+5)^3 = 216q^3 + 540q^2 + 450q + 125$$

$$= 6(36q^3 + 90q^2 + 75q + 20) + 5$$

$$= 6m + 5, \text{ (where } m \text{ is an integer } = 36q^3 + 90q^2 + 75q + 20)$$

Hence, the cube of a positive integer of form  $6q + r$ ,  $q$  is an integer and  $r = 0, 1, 2, 3, 4, 5$  is also of the form  $6m + r$ .

**14. Show that one and only one out of  $n, n + 4, n + 8, n + 12$  and  $n + 16$  is divisible by 5, where  $n$  is any positive integer.**

**Solution:**

According to Euclid's division Lemma,

Let the positive integer =  $n$

And,  $b=5$

$n = 5q+r$ , where  $q$  is the quotient and  $r$  is the remainder.

$0 < r < 5$  implies that the remainder may be 0, 1, 2, 3, 4 and 5.

Therefore,  $n$  may be in the form of  $5q, 5q+1, 5q+2, 5q+3$ , and  $5q+4$ .

So, this gives us the following cases:

CASE 1:

When,  $n = 5q$

$$n+4 = 5q+4$$

$$n+8 = 5q+8$$

$$n+12 = 5q+12$$

$$n+16 = 5q+16$$

Here,  $n$  is only divisible by 5.

CASE 2:

When,  $n = 5q+1$

$$n+4 = 5q+5 = 5(q+1)$$

$$n+8 = 5q+9$$

$$n+12 = 5q+13$$

$$n+16 = 5q+17$$

Here,  $n + 4$  is only divisible by 5.

CASE 3:

When,  $n = 5q+2$

$$n+4 = 5q+6$$

$$n+8 = 5q+10 = 5(q+2)$$

$$n+12 = 5q+14$$

$$n+16 = 5q+18$$

Here,  $n + 8$  is only divisible by 5.

CASE 4:

When,  $n = 5q+3$

$$n+4 = 5q+7$$

$$n+8 = 5q+11$$

$$n+12 = 5q+15 = 5(q+3)$$

$$n+16 = 5q+19$$

Here,  $n + 12$  is only divisible by 5

CASE 5:

When,  $n = 5q+4$

$$n+4 = 5q+8$$

$$n+8 = 5q+12$$

$$n+12 = 5q+16$$

$$n+16 = 5q+20 = 5(q+4)$$

Here,  $n + 16$  is only divisible by 5.

So, we can conclude that one and only one out of  $n$ ,  $n + 4$ ,  $n + 8$ ,  $n + 12$  and  $n + 16$  is divisible by 5.

Hence Proved.

**15. Show that the square of an odd integer can be of form  $6q + 1$  or  $6q + 3$  for some integer  $q$ .**

**Solution:**

Let 'a' be an odd integer and  $b = 6$ .

According to Euclid's algorithm,

$$a = 6m + r \text{ for some integer } m \geq 0$$

And  $r = 0, 1, 2, 3, 4, 5$  because  $0 \leq r < 6$ .

So, we get,

$$a = 6m \text{ or, } 6m + 1 \text{ or, } 6m + 2 \text{ or, } 6m + 3 \text{ or, } 6m + 4 \text{ or } 6m + 5$$

Thus, we are choosing for  $a = 6m + 1$  or,  $6m + 3$  or  $6m + 5$  for it to be an odd integer.

For  $a = 6m + 1$ ,

$$(6m + 1)^2 = 36m^2 + 12m + 1$$

$$= 6(6m^2 + 2m) + 1$$

$$= 6q + 1, \text{ where } q \text{ is some integer and } q = 6m^2 + 2m.$$

For  $a = 6m + 3$

$$(6m + 3)^2 = 36m^2 + 36m + 9$$

$$= 6(6m^2 + 6m + 1) + 3$$

$$= 6q + 3, \text{ where } q \text{ is some integer and } q = 6m^2 + 6m + 1$$

For  $a = 6m + 5$ ,

$$(6m + 5)^2 = 36m^2 + 60m + 25$$

$$= 6(6m^2 + 10m + 4) + 1$$

$$= 6q + 1, \text{ where } q \text{ is some integer and } q = 6m^2 + 10m + 4.$$

Therefore, the square of an odd integer is of form  $6q + 1$  or  $6q + 3$  for some integer  $q$ .

Hence Proved.

**16. A positive integer is of the form  $3q + 1$ ,  $q$  is a natural number. Can you write its square in any form other than  $3m + 1$ ,  $3m$  or  $3m + 2$  for some integer  $m$ ? Justify your answer.**

**Solution:**

No, we cannot write its square in any form other than  $3m + 1$ ,  $3m$  or  $3m + 2$  for some integer  $m$ .

Justification:

By Euclid's Division Lemma,

$$a = bq + r, 0 \leq r < b$$

Here,  $a$  is any positive integer and  $b = 3$ ,

$$\Rightarrow a = 3q + r$$

So,  $a$  can be of form  $3q$ ,  $3q + 1$  or  $3q + 2$ .

Now, for  $a = 3q$

$$(3q)^2 = 3(3q^2) = 3m \text{ [where } m = 3q^2\text{]}$$

for  $a = 3q + 1$

$$(3q + 1)^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1 = 3m + 1 \text{ [where } m = 3q^2 + 2q\text{]}$$

for  $a = 3q + 2$

$$(3q + 2)^2 = 9q^2 + 12q + 4 = 9q^2 + 12q + 3 + 1 = 3(3q^2 + 4q + 1) + 1$$

$$= 3m + 1 \text{ [where } m = 3q^2 + 4q + 1\text{]}$$

Thus, the square of a positive integer of form  $3q + 1$  is always of the form  $3m + 1$  or  $3m$  for some integer  $m$ .

**17. Show that the square of any positive integer cannot be of the form  $3m + 2$ , where  $m$  is a natural number.**

**Solution:**

Let the positive integer be ' $a$ '

According to Euclid's division lemma,

$$a = bm + r$$

According to the question, we take  $b = 3$

$$a = 3m + r$$

So,  $r = 0, 1, 2$ .

When  $r = 0$ ,  $a = 3m$ .

When  $r = 1$ ,  $a = 3m + 1$ .

When  $r = 2$ ,  $a = 3m + 2$ .

Now,

When  $a = 3m$

$$a^2 = (3m)^2 = 9m^2$$

$$a^2 = 3(3m^2) = 3q, \text{ where } q = 3m^2$$

When  $a = 3m + 1$

$$a^2 = (3m + 1)^2 = 9m^2 + 6m + 1$$

$$a^2 = 3(3m^2 + 2m) + 1 = 3q + 1, \text{ where } q = 3m^2 + 2m$$

When  $a = 3m + 2$

$$a^2 = (3m + 2)^2$$

$$a^2 = 9m^2 + 12m + 4$$

$$a^2 = 3(3m^2 + 4m + 1) + 1$$

$$a^2 = 3q + 1 \text{ where } q = 3m^2 + 4m + 1$$

Therefore, the square of any positive integer cannot be of form  $3q + 2$ , where  $q$  is a natural number.

Hence Proved.

## **Benefits of Solving RD Sharma Solutions Class 10 Maths Chapter 1 Exercise 1.1**

Solving the RD Sharma Solutions for Class 10 Maths Chapter 1 Exercise 1.1 provides several benefits for students, enhancing their understanding and confidence in foundational mathematics. Here are some key advantages:

**Strengthens Core Concepts:** This exercise introduces fundamental ideas like real numbers, prime factorization, and the Euclidean division lemma. Working through these problems helps build a strong base for more advanced chapters.

**Improves Problem-Solving Skills:** By practicing these solutions, students develop a systematic approach to problem-solving. RD Sharma methodical steps encourage logical thinking, which is important for solving complex math problems.

**Boosts Exam Readiness:** The clear and concise solutions prepare students for board exams by covering important questions that often appear in exams. The exercises also help in time management and accuracy, both crucial for performing well in exams.

**Enhances Analytical Abilities:** Solving exercises like these promotes analytical thinking, allowing students to understand the 'why' behind each solution. This deeper understanding is useful in tackling unfamiliar problems.

**Builds Confidence:** With expert-prepared solutions, students can practice independently and verify their answers, which builds confidence and helps reduce math-related anxiety.